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Optimal Monetary Policy with Imperfect Common Knowledge

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Abstract

We study optimal nominal demand policy in a flexible price economy with monopolistic competition where firms have imperfect common knowledge about the shocks hitting the economy. Information imperfections emerge endogenously because firms are assumed to have finite (Shannon) capacity to process information. We then ask how policy that minimizes a quadratic objective in output and prices depends on firms' processing capacity. When price setting decisions of firms are strategic complements, we find that policy should nominally accommodate white noise mark-up shocks for a large range of capacity values. This finding is robust to the policy maker observing shocks imperfectly or being uncertain about firms' processing capacity. When mark-up shocks are persistent, accommodation may even have to increase in the medium term but has to decrease in the long-run, thereby generating a hump-shaped price response and a slow reduction in output. Instead, when prices are strategic substitutes, policy tends to react with nominal demand contractions to mark-up shocks. In addition, there might exist discontinuities between common knowledge equilibria and equilibria with small amounts of imperfect common knowledge.

Keywords: optimal policy, information frictions, imperfect common knowledge, higher order beliefs, Shannon capacity

JEL-Class.No.: E31, E52, D82

"The peculiar character of the problem of rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess."

Friedrich A. Hayek (1945)

1 Introduction

Decentralized economic activity, as it takes place in modern market economies, tends to generate decentralized knowledge, i.e. knowledge which is not necessarily shared among all agents that might find it potentially relevant.

Most economic models, however, derive policy recommendations under the assumption that private agents share a common information set. In the realm of monetary policy, for example, information asymmetries between private agents have not yet received much attention, and the literature has mainly focused on asymmetries between the private sector and the policy maker (e.g. Svensson and Woodford (2002a, 2002b)).

This paper considers optimal monetary policy when private agents do not share a common information set and thereby seeks to close in part this gap in the monetary policy literature.

Presented is a simple model with imperfectly competitive firms, flexible prices, and a policy maker using nominal demand to minimize the quadratic deviations of output and prices from target. The novel feature of the model is that firms (and potentially also the policy maker) possess private information about the shocks hitting the economy and that for such a setting optimal nominal demand policy is determined.

As pointed out earlier by Keynes (1936) and Phelps (1983), disparate information sets coupled with the assumption that agents hold rational expectations generate substantial difficulties: optimal decision making then requires that agents formulate so-called higher order beliefs, i.e. beliefs about the beliefs of others and beliefs about what the others believe about others, and so on ad infinitum.[1] This is the case because agents' optimal decisions typically depend on the choices of other agents and, thus, on other agents' beliefs.[2]

Despite these difficulties, a number of recent papers successfully pioneered methods to determine rational expectations equilibria in imperfect common

[1] Morris and Shin (2000) have shown that agents do not necessarily have to formulate such higher order beliefs. In binary action games, optimal decisions can be generated by holding simple uniform beliefs about other agents' *actions.*

[2] In the present model such dependencies arise from price competition between firms.

knowledge environments, most notably Townsend (1983b, 1983a), Sargent (1991), Binder and Pesaran (1998), Kasa (2000), and Woodford (2002), and the recent literature on global games, see Morris and Shin (2000).

While the present paper in many respects is simpler than these earlier contributions it adds to them by solving an optimal policy problem for a private sector rational expectations equilibrium with imperfect common knowledge and by generating the imperfect common knowledge structure endogenously. The latter is achieved by assuming that agents can process information only at a finite rate, as will be explained further below.

In related papers Morris and Shin (2003) and Amato and Shin (2003) also derive normative implications for imperfect common knowledge settings by analyzing the welfare effects of disclosing public information. Ball et al. (2002) analyze optimal monetary policy with disparate information by assuming that some agents set prices based on lagged information. The assumed information lags, however, do not generate imperfect common knowledge.

As mentioned before, imperfect common knowledge arises in the model because firms are assumed to process information only at a finite rate. This causes them to make idiosyncratic processing errors because processing all information perfectly would require the ability to process information at an infinite rate. Following Shannon (1948) and Sims (2001) this situation is modeled by assuming that firms receive information through an information channel with finite *channel capacity*. A channel capacity parameter then conveniently parametrizes firms' ability to process information: when capacity is equal to zero, firms cannot process information at all; as capacity increases the amount of information processed by firms increases, and the ability to process information becomes perfect (in the limit) as capacity becomes infinite.[3]

There are several advantages in modelling information frictions with the help of information channels. Firstly, as mentioned before, the information structure turns out endogenous. Firms can *choose* which variables to observe with what precision through their channels. The information structure will thus react to the policy pursued by the monetary policy maker. In the language of Kalman filtering one may describe this as a situation where agents *choose* their observation equation and where the observation noise is determined by the channel capacity to limit the information content of the signal. Secondly, information channels preserve the linear quadratic nature of the policy problem considered in the paper and thereby allow for simple closed form solutions.[4]

[3] Information should here be understood in the sense of information theory, i.e. as the difference in entropy between prior and posterior beliefs.

[4] This would not be the case if the variance of the observation error was taken as the primitive parameter characterizing information frictions, as is common in the literature on global games.

The main finding of the paper is that the presence of differential information may have stark implications for optimal monetary policy decisions. This is the case because differential information strongly affects the equilibrium outcome of the price setting game between firms and the reaction to policy decisions.

Suppose, for example, that firms' prices are strategic complements, i.e. that the optimal price of some firm increases, ceteris paribus, with the average price in the economy. With a common information set such complementarities require (and allow for) a high degree of coordination between price setters in reaction to shocks. Coordination, however, becomes difficult in the presence of differential information, as firms are uncertain about the decisions of other firms who base their price decisions on (slightly) different information sets. As shown in the paper, prices will then considerably underreact to shocks even if information frictions generate only relatively small amounts of differential information.

The underreaction to shocks in the presence of strategic complementarities implies that nominal demand policy has the ability to stabilize output since prices also underreact to policy decisions. Consequently, optimal policy nominally accommodates mark-up shocks for a large range of intermediate values of the channel capacity. This result is shown to be robust to a number of extensions, such as imperfect observations of shocks by the policy maker or uncertainty of the policy maker about the value of the private sector's ability to process information.

When mark-up shocks are persistent, the optimal policy reaction in response to these shocks varies over time. As information about the shocks dissipates in the economy, coordination between price setters improves and nominal demand policy eventually becomes ineffective in stabilizing output. Optimal policy can then be characterized by increasing amounts of accommodation in the medium term and nominal demand contraction in the long-run. A persistent mark-up shock thereby generates a hump-shaped price response and a slowly decreasing output level, although prices are perfectly flexible.

The paper also briefly analyzes the case where firms' prices are strategic substitutes. Firms then tend to react with price cuts in response to aggregate mark-up shocks which induces optimal policy to react with nominal demand contractions in response to these shocks. Moreover, we find that when the degree of strategic substitutability is sufficiently strong, there might be a discontinuity between perfect information rational expectations equilibria and rational expectations equilibria with (arbitrarily) small amounts of imperfect common knowledge.

The paper is organized as follows. Section 2 introduces the monopolistically competitive economy and section 3 briefly describes optimal policy for two common knowledge benchmarks where shocks are either perfectly observable or completely unobservable. Information channels are introduced in section 4 which summarizes relevant results from information theory. Section 5 then determines the rational expectations equilibrium with imperfect common knowledge and section 6 characterizes optimal monetary policy. A conclusion summarizes the main findings.

2 A simple Lucas-type model

Consider a flexible price economy with a continuum of monopolistically competitive firms $i \in [0,1]$ and a central bank that (imperfectly) controls nominal demand. Except for the information assumptions the model described below is standard.

As is well known, e.g. Woodford (2001a), the log-linearized first order conditions for firms in monopolistic competition deliver a standard pricing equation of the form

$$p_t(i) = E\left[p_t + \xi y_t | I_t^i\right] + \varepsilon_t^i, \tag{1}$$

where $p_t(i)$ denotes the log of firm i's profit maximizing price, p_t the log average price ($p_t = \int p_t(i)di$), y_t the log output gap ($y_t = \log \frac{Y_t}{\overline{Y}}$, where $\overline{Y}$ is the output level emerging in the absence of any aggregate shocks), and I_t^i the information available to firm i.

The stochastic component ε_t^i in equation (1) is a firm-specific mark-up shock. This shock is assumed to have an idiosyncratic component ϕ_t^i and a component ε_t that is common to all firms:

$$\varepsilon_t^i = \varepsilon_t + \phi_t^i.$$

The idiosyncratic component is thought to represent an efficient variation in the relative prices of the goods of different firms. The cross-sectional distribution of the ϕ_t^i is assumed to be time-invariant with zero mean. The common mark-up shock ε_t is given by

$$\varepsilon_t \sim iiN(0, \sigma_\varepsilon^2)$$

and is a source of aggregate price level risk in the economy, which generates an incentive for the policy maker to intervene.

To simplify the exposition, it will be assumed that the mark-up shock ε_t^i of a single firm does not convey any information about the average shock ε_t. This is the case when the variance of the common mark-up shock is small relative to the variance in the innovation of the process ϕ_t^i, denoted by σ_ϕ^2, i.e. whenever

$$\sigma_\varepsilon^2 / \sigma_\phi^2 \approx 0.$$

This assumption is made for analytical convenience and insures that all information about ε_t enters firms' information sets via the information channels.[5]

The parameter $\xi > 0$ in equation (1) is crucial since it determines whether firms' prices are strategic complements or substitutes. This can be seen by defining the (log) nominal spending gap q_t as

$$q_t = y_t + p_t, \tag{2}$$

and using it to substitute y_t in equation (1):

$$p_t(i) = E\left[(1-\xi)p_t + \xi q_t | I_t^i\right] + \varepsilon_t^i. \tag{3}$$

For $\xi \leq 1$ prices are strategic complements since each firm's optimal price is (weakly) increasing with the average price level for a given level of nominal demand q_t. For $\xi > 1$ prices become strategic substitutes since each firm's optimal price is then decreasing with the average price level.[6]

We now describe the demand side of the economy. For simplicity we assume that the central bank (imperfectly) controls nominal demand q_t, i.e.

$$q_t = q_t^* + \delta_t,$$

where q_t^* is the target level chosen by the central bank and $\delta_t \sim iiN(0, \sigma_\delta^2)$ is a control error that realizes after the policy maker has determined q_t^* but before firms decide about prices.

Control of nominal spending can be achieved in various ways. In an economy featuring a quantity equation it may be established through control of nominal money balances. Alternatively, nominal demand could be controlled by setting an appropriate level for the nominal interest rate, which is the policy instrument used by most central banks today. Which of these instruments is effectively used does not matter for the results in this paper.

2.1 Central bank objective function

As shown in Woodford (2001a) and Ball et. al. (2002), the welfare function of the representative household in models with monopolistic competition can be approximated by a quadratic loss function that contains an output gap variable and a term measuring inefficient cross-sectional dispersion in firms' relative prices.[7] Intuitively, the first term captures inefficient deviations of aggregate

[5] None of the qualitative results depend on this assumption as long as ε_t^i does not fully reveal ε_t.

[6] This is the case because a nominal price increase reduces real demand and thereby real production costs by so much that it is optimal to reduce prices.

[7] This assumes that the natural output rate is (up to a first order approximation error) equal to the efficient output level, see Woodford (2001a).

output from the first best output level, while the second term captures inefficiencies in the production of aggregate output that are generated by relative prices that do not properly reflect the relative marginal costs of production.

Since the present model does not contain any nominal price rigidities, inefficient cross-sectional price dispersion emerges purely due to the idiosyncratic elements in firms' information sets.[8] Such price dispersion would indeed be welfare reducing if consumers adapted the consumption levels of different goods to the relative price movements generated by these idiosyncratic information differences. This, however, would amount to assuming that consumers ultimately know firms' private information sets, which seems highly unlikely.

Consumers who do not know the idiosyncratic elements in firms' information sets would have to base their consumption decisions on the systematic part of firms' price decisions, i.e. firms' pricing rules, which is what will be assumed subsequently. This implies that information-based price dispersion is not welfare reducing and would leave the output gap as the unique element of a welfare-based loss function.

The loss function that will be used in this paper, however, will also include a price level objective, i.e.

$$\min_{q_t^*} E\left[\sum_{t=0}^{\infty} \beta^t \left(y_t^2 + p_t^2\right) \mid I_t^{CB}\right]. \tag{4}$$

The presence of the price level term in (4) is motivated by a concern that the flexible price assumption might be unrealistically strong. In particular, suppose that there exists a (non-modeled) sector in the economy where prices are sticky. Then, if all sectors are hit by the same aggregate shocks, price level movements in the flexible price sector measure price dispersion between the flexible and sticky price sector. Since such dispersion is inefficient, including the price level into the objective functions seems justified on welfare grounds.

We now make some further remarks about the central bank's maximization problem. Firstly, note that the central bank optimizes conditional on its information set I_t^{CB} which contains (potentially noisy) information about the aggregate mark-up shock ε_t but no information about the control error δ_t. Secondly, the output gap target is equal to zero in all periods, which implies that the central bank has no incentive to raise output above its natural rate. Finally, the central bank will commit to q_t^* before firms set their prices. Consequently, real effects of nominal demand policy will be due to the *systematic* variation in monetary policy only.

[8] Recall that the dispersion created by the noise term ϕ_t^i is assumed to be efficient, e.g. representing firm level productivity shocks.

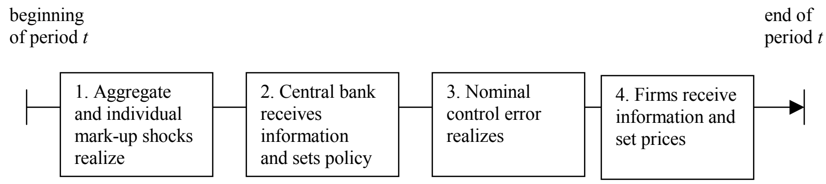

Figure 1: Sequence of events

2.2 Time-line

Before going into the details of the analysis we briefly summarize the sequence of events within each period. As illustrated in figure 1, the period starts with the realization of the aggregate and idiosyncratic mark-up shocks. The central bank then receives a (potentially noisy) signal of theses shocks and determines its policy q_t^*. Thereafter, the nominal control error δ_t realizes. Finally, firms receive (potentially noisy) information about the shocks and the central bank's policy choice and then simultaneously decide about prices.

3 Optimal policy in two benchmark settings

This section considers two common knowledge settings with rather extreme informational assumptions. In the first setting it is assumed that firms perfectly observe aggregate mark-up shocks ε_t and nominal spending shocks δ_t; in the second setting we assume that firms do not observe shocks at all. The optimal policies for these settings will serve as useful benchmarks when analyzing policy in environments where firms have imperfect common knowledge about these shocks.

3.1 Benchmark I: Perfectly observable shocks

Suppose firms perfectly observe the shocks ε_t and δ_t, the policy maker perfectly observes ε_t, and this is common knowledge. Based on equation (3) firm i's optimal price $p_t(i)$ can be expressed as

$$\begin{aligned} p_t(i) &= E\left[(1-\xi)p_t + \xi q_t | I_t^i\right] + \varepsilon_t^i \\ &= E\left[(1-\xi)p_t | I_t\right] + \xi q_t + \varepsilon_t^i. \end{aligned} \tag{5}$$

where the second line uses the fact that all firms share the same information set and assumes that q_t is a function of the shocks ε_t and δ_t and, thus, perfectly observed by agents.

Integrating equation (5) over $i \in [0,1]$ and taking conditional expectations with respect to I_t delivers

$$E[p_t|I_t] = q_t + \frac{1}{\xi}\varepsilon_t, \tag{6}$$

which shows that agents can pin down average expectations as a function of the random variables ε_t and q_t. By substituting equation (6) into (5), integrating over i, and using one more time the definition of q_t, one can determine how prices and output depend on shocks and policy decisions:

$$p_t = q_t + \frac{1}{\xi}\varepsilon_t \tag{7}$$

$$y_t = -\frac{1}{\xi}\varepsilon_t. \tag{8}$$

As one would expect, nominal demand q_t affects the price level but has no effect on output. This is an example of the classical monetary neutrality result that holds in models without nominal rigidities and information asymmetries (Lucas (1972)).

Since nominal demand policy has no effect on output, optimal policy stabilizes prices:

$$q_t^* = -\frac{1}{\xi}\varepsilon_t. \tag{9}$$

Under optimal policy prices then fluctuate in response to the nominal control error δ_t and output depends on the aggregate mark-up shock ε_t only.

3.2 Benchmark II: Unobservable shocks

We now consider the case where firms have no information about the realization of mark-up shocks and control errors. The policy maker continues to observe mark-up shocks perfectly.

Firms' expectations about shocks are then given by the mean values of shocks, which are equal to zero. Equation (1) and the definition of q_t then imply

$$p_t = \varepsilon_t \tag{10}$$

$$y_t = q_t - \varepsilon_t. \tag{11}$$

Since nominal demand policy and spending shocks are unobserved, they come as a 'surprise' and affect real variables only. Optimal policy then seeks to stabilize output, which is achieved by nominally accommodating mark-up shocks:

$$q_t^* = \varepsilon_t.$$

Under optimal policy, prices now respond to mark-up shocks and output is driven by control errors, which is the opposite of the full information setup considered in the previous section.

4 Imperfect common knowledge and information channels

From here on we consider the more realistic case of economic agents that do not share a common information set.

We assume that each firm (and potentially also the policy maker) receives information through a so-called information channel that is contaminated with idiosyncratic noise.[9] The presence of noise will generate private information about the shocks hitting the economy.

Information channels allow for the transmission of information from the source to the receiver in a similar way as telephone or modem lines do. However, due to the presence of noise, the information arriving at the receiver (the channel output) does not perfectly reveal the information at the source (the channel input). Noise may arise, for example, from limited attention on the part of the receiver or from interpretation errors due to background noise in the channel.

An information channel can be characterized by its capacity $K \geq 0$. The capacity places an upper bound on the amount of information that can be transmitted via the channel, as will be made precise below. Channel capacity is a simple technology parameter, like the TFP-parameter in a production function, that depends on channel features such as the number of signals the channel can transmit per period of time, the number of letters in the channel's alphabet, the probability with which the respective letters are transmitted correctly, etc.

An attractive feature of information channels is that they limit only the overall amount of information flowing to agents while agents decide which random variables to observe with what precision. Agents, thus, *choose* the information structure subject to the constraint imposed by the capacity limit. This causes the information structure to be endogenous since agents' information choices depend on the stabilization policy pursued by the central bank and the parameters characterizing the economy.

Readers familiar with Kalman filtering may think of this situation as one where agents *choose* their observation equation and where the information noise is determined by the capacity of the available information channel.

In the next section we give a brief introduction to real-valued Gaussian information channels. Such channels will be used in the latter part of the paper. Readers interested in a more detailed treatment may consult the textbook of Cover and Thomas (1991) or the, very accessible, original contribution of Shannon (1948).

[9] Information channels have been introduced by Shannon (1948). See Cover and Thomas (1991) for a textbook treatment and Sims (2001) for an application in macroeconomics.

4.1 The real-valued Gaussian channel

Suppose a firm must choose a price $p \in R$ to maximize a quadratic profit function of the form

$$\max_p -E\left[(p - \zeta' Z)^2 |I\right], \tag{12}$$

where $Z \sim N(0, V)$ is a vector of shocks driving the economy and where the profit maximizing price $\zeta' Z$ depends on these shocks. The vector ζ' may thereby be a function of the parameters of the underlying economic model, the policy pursued by the central bank, and other factors that the agent takes as given.

The profit function (12) can be interpreted as a quadratic approximation to the firm's profit function. Such a quadratic approximation is convenient because it leads to linear decision rules that mimic the log-linearized first order conditions in equation (1).

Suppose the information set I is exogenous. The solution to the above problem is then trivially given by

$$p^* = E[\zeta' Z|I],$$

and the expected loss equals

$$-Var(\zeta' Z|I). \tag{13}$$

Now instead, suppose that the firm can choose its information structure I but must receive information about $\zeta' Z$ through an information channel with capacity $K \in [0, \infty)$.

The channel coding theorems (e.g. theorem 8.7.1 in Cover and Thomas (1991)) state that channel capacity K places a limit on the amount of entropy reduction that can be achieved by the channel.[10] Formally,

$$H(\zeta' Z) - H(\zeta' Z|s) < K, \tag{14}$$

where $H(\zeta' Z)$ denotes the entropy of the random variable $\zeta' Z$ prior to observing the channel output signal s and $H(\zeta' Z|s)$ the entropy after observing the signal.

Intuitively, entropy is a measure of the uncertainty about a random variable. Stated in these terms, equation (14) provides a bound for the maximum uncertainty reduction that can be achieved by observing the channel output s. Since the entropy $H(\zeta' Z)$ is determined by the distribution of $\zeta' Z \sim N(0, \zeta' V \zeta)$ and, thus, taken as given, equation (14) simply implies that $H(\zeta' Z|s)$ must lie above a certain threshold.

[10] The entropy $H(X)$ of a continous random variable X is defined as $H(X) = -\int \ln(x)p(x)dx$ where $p(x)$ is the probability density function of X and where the convention is to take $ln(x)p(x) = 0$ when $p(x) = 0$.

What is the optimal information structure that fulfills this entropy constraint? Equation (13) shows that the expected loss associated with any information structure is equal to $Var(\xi' Z|s)$. Thus, choosing the optimal information structure is identical to minimizing this conditional variance subject to the constraint that $H(\zeta' Z|s)$ is above the threshold.

Shannon (1948) shows that Gaussian variables minimize the variance for a given entropy.[11] Thus, if possible, the observation noise should be Gaussian such that the posterior distribution $\xi' Z|s$ is Gaussian and has the minimum variance property.

We will assume that the coding allows for such kind of Gaussian noise, i.e. there exists a way to map the realizations of $\zeta' Z$ into a sequence of input signals from the channel's alphabet such that the observation noise generated by the incorrect transmission of signals is Gaussian and independent across the realization of the input signal.

When this is the case, optimal use of the channel implies that the channel output signal s has a simple representation of the form

$$s = \zeta' Z + \eta, \tag{15}$$

where $\eta \sim N(0, \sigma_\eta^2)$ is the Gaussian observation noise and σ_η^2 is the infimum variance satisfying the channel capacity constraint

$$\ln Var(\zeta' Z) - \ln Var(\zeta' Z|s) < 2K. \tag{16}$$

Constraint (16) follows from equation (14) and the fact that the entropy of a Gaussian random variable is equal to one half its log variance plus some constant.

From the updating formula for normal random variables[12] and the capacity constraint (16) it follows that the observation noise has (infimum) variance

$$\sigma_\eta^2 = \frac{1}{e^{2K} - 1} Var(\zeta' Z).$$

Firms' expectations after observing the signal are then given by

$$E[\zeta' Z|s] = k \cdot s, \tag{17}$$

where the Kalman gain k is

$$k = \frac{Var(\zeta' Z)}{Var(\zeta' Z) + \sigma_\eta^2} = (1 - e^{-2K}). \tag{18}$$

[11] Shannon solves the dual problem of maximizing entropy for a given variance.

[12] $Var(\zeta' Z|s) = Var(\zeta' Z) - Var(\zeta' Z)^2 / (Var(\zeta' Z) + \sigma_\eta^2)$

Note that the Kalman gain in equation (18) is independent of the variance of $\zeta' Z$ and, thus, independent of policy (ζ). This feature will be crucial later on since it helps preserve the linear quadratic nature of the policy maker's optimization problem. It would be absent, however, if the variance of the observation noise was taken as the primitive parametrizing information frictions, as is commonly done in the literature.

The gain $k \in [0, 1]$ in equation (18) will be used in the remaining part of the paper as an index of how well agents observe their environment. When $k = 0$ firms receive no information since $\sigma_\eta^2 = \infty$. Conversely, if $k = 1$ firms observe perfectly since $\sigma_\eta^2 = 0$. At intermediate values of k the variance of the observation noise is positive but finite and decreasing in k.

5 Rational expectations equilibrium with imperfect common knowledge

In this section we endow each firm with an information channel of given capacity and solve for a rational expectations equilibrium (REE) in which firms choose profit maximizing prices *and* optimal information structures. As it turns out, there is a unique rational expectations equilibrium.

Solving for the rational expectations equilibrium is not a trivial task. Since the observation noise generated by the information channels is idiosyncratic, firms do not know what other firms have observed and must formulate beliefs about other agents' beliefs, which leads to a system of higher order beliefs. Firms can rationally formulate higher order beliefs because the stochastic properties of the observation noise are assumed to be common knowledge.

The remainder of this section is structured as follows. Section 5.1 determines how profit-maximizing prices depend on firms' higher-order beliefs. Section 5.2 then derives the rational expectations equilibrium with imperfect common knowledge where firms gather information optimally.

5.1 Price setting with imperfect common knowledge

We first have to introduce some notation to be able to refer to firms' expectations of various order.

Let $x_{t|t}^{(n)}(i)$ denote firm i's n-th order expectation of x_t, where the 'zero-th order expectations' are given by the variable itself, i.e.

$$x_{t|t}^{(0)}(i) = x_t.$$

Expectations of order $n + 1$ are then obtained by averaging the n-th order expectations over i and applying the expectations operator, i.e.

$$x_{t|t}^{(n+1)}(i) = E[\int x_{t|t}^{(n)}(i)di|I_t^i].$$

Therefore, $x^{(1)}_{t|t}(i)$ denotes the familiar (first order) expectation $E[x_t|I^i_t]$; the second order expectations $x^{(2)}_{t|t}(i)$ denote i's expectations of the average (first order) expectations; the third order expectations $x^{(3)}_{t|t}(i)$ denote i's expectations of the average second order expectations, etc.

With this notation the price setting equation (3) can be expressed as

$$p^{(0)}_{t|t}(i) = (1-\xi)p^{(1)}_{t|t}(i) + \xi q^{(1)}_{t|t}(i) + \varepsilon^i_t. \tag{19}$$

Iterating on equation (19) by taking repeatedly the average over i and the conditional expectations $E[\cdot|I^i_t]$, one obtains

$$p_t(i) = E\left[\sum_{n=0}^{\infty}(1-\xi)^n\left(\xi q^{(n)}_{t|t} + (1-\xi)\varepsilon^{(n)}_{t|t}\right)|I^i_t\right] + \varepsilon^i_t, \tag{20}$$

where $x^{(n)}_{t|t} = \int x^{(n)}_{t|t}(i)\mathrm{di}$ denotes the average expectations of order n.

Equation (20) expresses firm i's profit maximizing price as a function of first and higher order expectations of

$$\xi q_t + (1-\xi)\varepsilon_t.$$

5.2 REE and optimal information structure

This section determines agents' optimal information structure and characterizes the rational expectations equilibrium.

Equation (20) and the discussion in section 4 imply that agents wish to observe the term

$$\sum_{n=0}^{\infty}(1-\xi)^n\left(\xi q^{(n)}_{t|t} + (1-\xi)\varepsilon^{(n)}_{t|t}\right) \tag{21}$$

as precisely as possible through their information channels.

Equation (21) shows that firms' seek to observe a combination of the fundamental shocks and agents' higher-order expectations about these shocks. The latter implies that to construct a rational expectations equilibrium one has to determine a fixed point in the space of beliefs where for given expectations the signals obtained about these expectations exactly generate them.

A much simpler way to proceed, however, is to let agents observe only the fundamentals

$$\xi q_t + (1-\xi)\varepsilon_t. \tag{22}$$

They can then construct the higher order beliefs in (21) using their (noisy) observation of these fundamentals. As shown in the appendix, this leads to the same equilibrium outcome, but equilibrium is much simpler to derive and easier to interpret.

We proceed by assuming that firms' observation equation is given by

$$s_t^i = (\xi q_t + (1-\xi)\varepsilon_t) + \eta_t^i, \tag{23}$$

where η_t^i is an idiosyncratic observation error that we take to be normally distributed for the reasons discussed in section 4.

Note that nominal demand q_t in equation (23) will depend on the control error δ_t and, provided policy reacts to mark-up shocks, on ε_t and possible central bank observation errors about ε_t. The weights given to the shocks ε_t and δ_t in equation (23), thus, depend on the policy pursued by the central bank, which shows that the information structure is truly endogenous to the model.

From equation (17) it follows that

$$E[\xi q_t + (1-\xi)\varepsilon_t | s_t^i] = k s_t^i, \tag{24}$$

where $k = 1 - e^{2K}$, see equation (18).

Integrating equation (24) over i, using equation (23) to substitute s_t^i, and taking the expectations $E[\cdot | I_t^i]$ delivers

$$E[\xi q_{t|t}^{(1)} + (1-\xi)\varepsilon_{t|t}^{(1)} | I_t^i] = k^2 s_t^i,$$

while applying the same operations n times delivers

$$E[\xi q_{t|t}^{(n)} + (1-\xi)\varepsilon_{t|t}^{(n)} | I_t^i] = k^{n+1} s_t^i. \tag{25}$$

The previous equation reveals that agents' higher order expectations (rationally) react less strongly to the signal s_t^i than expectations of lower order. This is the case because firms are increasingly uncertain about the expectations of higher order. This feature will become important later on.

Provided $|(1-\xi)k| < 1$, e.g. if prices are strategic complements, one can use expression (25) to substitute the expectations in equation (20) to derive the rational expectations equilibrium price level:

$$p_t = \frac{k}{1-(1-\xi)k}(\xi q_t + (1-\xi)\varepsilon_t) + \varepsilon_t. \tag{26}$$

The equilibrium output level follows from equation (2).

The expression for the equilibrium price level (26) has a number of interesting implications. Firstly, is unique which shows that there is a unique rational expectations equilibrium. Secondly, since $\frac{\partial p_t}{\partial q_t} < 1$ as long as $k < 1$, information imperfections imply that nominal demand policy has real effects. Thirdly, since $\frac{\partial p_t}{\partial q_t}$ is increasing in ξ, strategic complementarities strengthen the real effects of nominal demand policy in the presence of imperfect common knowledge. Finally, note that for $k = 1$ and $k = 0$ equation (26) reduces to the common knowledge benchmarks (7) and (10), respectively.

6 Optimal stabilization policy

This section derives optimal nominal demand policy when firms have imperfect common knowledge about shocks.

We first consider the case where prices are strategic complements ($\xi \leq 1$), which is a sufficient condition for equation (26) to hold. Section 6.1 determines how optimal policy depends on firms' ability to process information k and on the complementarity parameter ξ. Section 6.2 then extends the setting to situations where the central bank is uncertain about the private sector's ability to process information and section 6.3 considers the case where mark-up shocks display persistence. Finally, section 6.4 discusses optimal policy when firms' prices are strategic substitutes ($\xi > 1$).

6.1 The baseline case

Since there are no intertemporal links, the policy maker's stabilization problem consists of a sequence of static maximization problems of the form:[13]

$$\max_{q_t^*} E[-p_t^2 - y_t^2 | I_t^{CB}] \tag{27a}$$

$$s.t.$$

$$p_t = \frac{k}{1-(1-\xi)k}\left(\xi(q_t^* + \delta_t) + (1-\xi)\varepsilon_t\right) + \varepsilon_t \tag{27b}$$

$$y_t = q_t^* + \delta_t - p_t. \tag{27c}$$

The policy maker's problem is linear-quadratic despite the presence of information noise in the private sector. This is the case because the Kalman gains k in equation (27b) are independent of policy.[14] This feature implies that certainty equivalence holds and that the results derived below hold independently from central bank information imperfections about the shocks ε_t.

[13] Equations (27b) and (27c) follow from equation (26) and the definitions of q_t and q_t^*.

[14] If the variance of observation noise was specified exogenously this property would be lost and closed form solutions would be unavailable, even for the relative simple policy problem at hand.

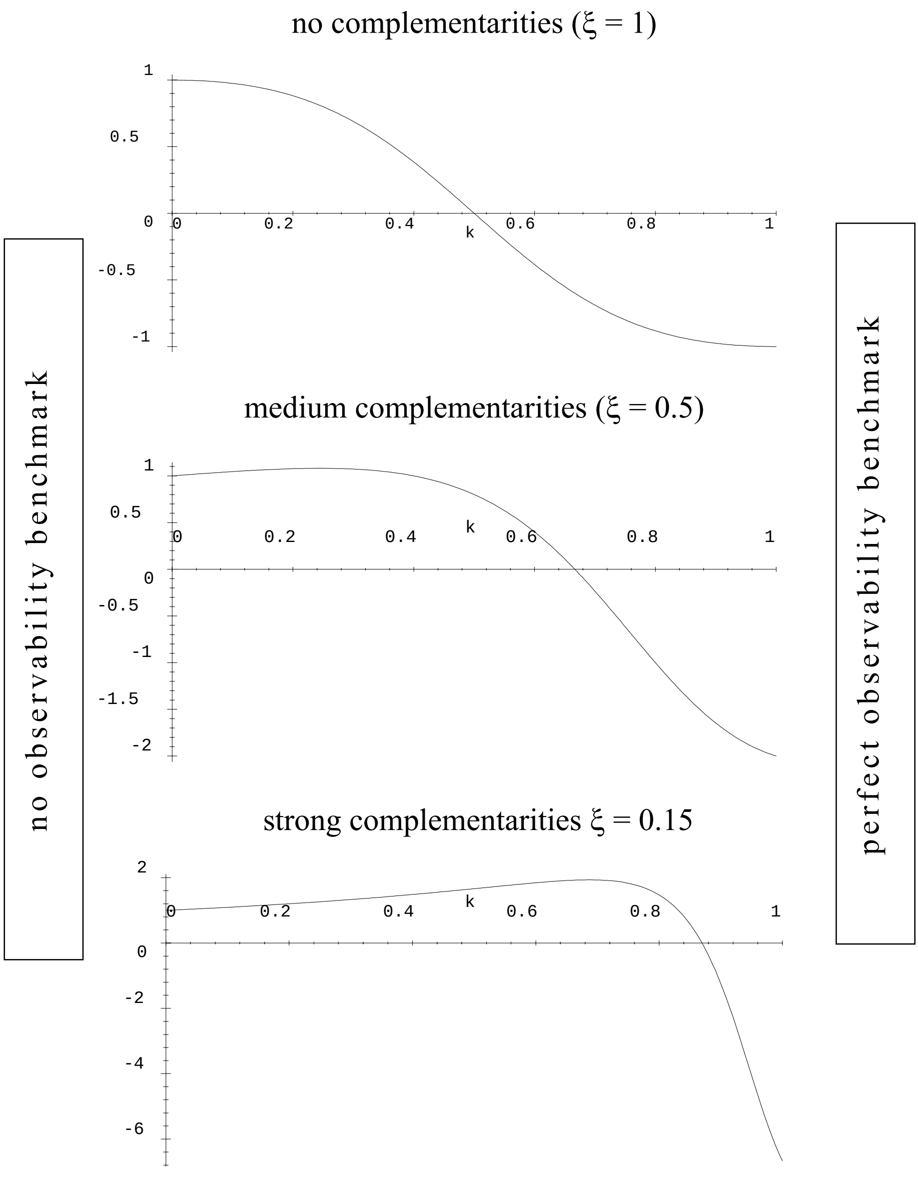

Figure 2: Optimal policy reaction coefficient

The solution to (A) is readily calculated to be

$$q_t^* = a(k,\xi) \cdot E[\varepsilon_t | I_t^{CB}], \tag{28}$$

where the reaction coefficient

$$a(k,\xi) = -\frac{\xi k - (1-k)}{(\xi k)^2 + (1-k)^2} \tag{29}$$

depends in a non-trivial way on firms' ability to process information (k) and on the complementarity parameter (ξ).

The expression for the reaction coefficient (29) shows that optimal policy is continuous when making the transition from the common knowledge benchmarks to an imperfect common knowledge setting: the coefficient converges to its benchmark values for $k \to 0$ and $k \to 1$, respectively. This feature is a consequence of the continuity of the equilibrium price level (26) with respect to k but may be lost when prices are strategic substitutes, as shown in section 6.4.

From expression (29) follows that optimal policy tends to nominally accommodate mark-up shocks ($a > 0$) in the presence of strategic complementarities. In particular, policy is accommodative whenever:

$$k < \frac{1}{1+\xi}.$$

Strong strategic complementarities (small values of ξ) cause policy to nominally accommodate mark-up shocks for a large range of k values, since prices then react relatively little to nominal demand variations, see equation (26). The policy maker then seeks to stabilize the output level.

Figure 2 illustrates the situation by depicting the reaction coefficient for intermediate values of k and various degrees of strategic complementarity. Woodford (2001b) suggests that $\xi = 0.15$ is a plausible parameter value for the U.S. economy. In this case policy is accommodative as long as $k < 0.869$. Figure 2 also shows that the optimal reaction coefficient may even have to increase with k, provided $\xi < 1$. Thus, it may be optimal to accommodate mark-up shocks more strongly in economies where firms receive more information about shocks.

Intuition for the latter finding can be obtained by considering the optimal reaction coefficients for the case that the policy maker pursues only one objective at the time, namely either the stabilization of the output gap or the stabilization of the price level.

Consider the case of pure output stabilization. The optimal reaction coefficient is then given by

$$a_y = \frac{1}{1-k},$$

which implies that nominal accommodation should increase with k. Clearly, higher values of k imply that firms receive more information about nominal demand variations. Therefore, a larger share of these variations gets translated into price movements. Policy can counteract this effect and close the output gap by increasingly accommodating shocks.[15]

Next, consider the case of pure price level stabilization. The optimal reaction coefficient is then given by

$$a_p = -\frac{1}{k\xi} < 0.$$

The optimal coefficient is now negative and depends on k and ξ. In particular, smaller values of ξ and k require a more negative reaction coefficient, which can be explained as follows.

At low values of k firms' do not observe nominal demand variations very well and, thus, react weakly to policy. As a result, a more negative reaction coefficient is required to undo any given mark-up shock to prices.

At small values of ξ higher-order beliefs are relatively important for firms' price setting decision, see equation (20). Since higher order beliefs react less strongly to the signals received by agents, see equation (25), a strong policy reaction is required to achieve any given change in the price level.

The optimal reaction coefficient for the policy maker pursuing output *and* price level stability will be a convex combination between the positive coefficient required for output stabilization and the negative coefficient required for price level stabilization.

As k increases the weight on the price level coefficient must increase because the trade-off between the two policy objectives becomes more favorable to the price level objective: at larger values of k output stabilization leads to increased price level variability, which works against output stabilization; at the same time price level stabilization leads to lower output variability, which works in favor of price level stabilization.

If, as k increases, this trade-off becomes more favorable to price level stabilization fast enough, then the optimal policy reaction coefficient will decrease with k.[16] The speed at which the trade-off is shifted, however, depends crucially on the parameter ξ. For low values of ξ price level stabilization remains unattractive even at intermediate values of k since price level stabilization requires strong output movements due to the sluggishness of higher order beliefs.[17] As a result, the increase of a_y in k carries over to the case where the policy maker pursues output and price level stability. This explains why nominal accommodation may have to increase with k in the presence of strategic complementarities.

[15] Policy successfully stabilizes output at the target as long as $k < 1$. Deviations from target occur only in response to control errors.

[16] This holds independently from the fact that a_p is increasing in k and is a consequence of the different signs of a_p and a_y.

[17] In the extreme case where $\xi \to 0$ the optimal reaction coefficient for an output *and* price

6.2 Uncertainty about the degree of information frictions

This section analyzes optimal policy when the policy maker is uncertain about the capacity constraint faced by the private sector. Such uncertainty is likely to be important in real-world policy situations.

Let $\mu(\cdot)$ denote the policy maker's beliefs about the value of $k \in [0,1]$. Then, in analogy to equation (28), the optimal reaction coefficient is given by

$$a(\mu(\cdot), \xi) = -\frac{\xi E[k] - (1 - E[k])}{\xi^2 E[k^2] + 1 - 2E[k] + E[k^2]}, \tag{30}$$

where the expectations are computed using beliefs $\mu(\cdot)$.

Equations (29) and (30) show that for a given mean belief $E[k]$ the reaction coefficient has the same sign independently of the degree of uncertainty about k. Thus, mean-preserving spreads in beliefs do not alter the sign of the optimal reaction coefficient.

Equations (29) and (30) also show that certainty equivalence fails to hold in this case. Since

$$E[k^2] \geq E^2[k], \tag{31}$$

a mean-preserving increase in the spread of beliefs causes the optimal reaction coefficient to decrease in absolute value. Uncertainty about k, thus, leads to a less strong optimal policy reaction.

Using equations (30) and (31) one can compute a lower bound (in absolute terms) for the optimal reaction coefficient that is consistent with any given mean belief $E[k]$. This bound is given by

$$b = \frac{1 - E[k] - E[k]\xi}{1 - E[k] + E[k]\xi^2}. \tag{32}$$

The optimal reaction coefficient is equal to b if the dispersion of beliefs is maximal but is larger (in absolute terms) otherwise.[18]

Interestingly, if strategic complementarities are pervasive, i.e. if ξ is small, then $b \approx 1$ for almost all values of $E[k]$.[19] High amounts of uncertainty about k

target is given by

$$\lim_{\xi \to 0} a(k, \xi) = \frac{1}{1-k}$$

which is the coefficient for a pure output target. Thus, for $\xi \to 0$ the weight on the output coefficient never decreases with k.

[18] For given mean beliefs $E[k]$ maximal dispersion is achieved by assigning probability $E[k]$ to $k = 1$ and probability $1 - E[k]$ to $k = 0$.

[19] This fails to hold when $E[k]$ is sufficiently close to one.

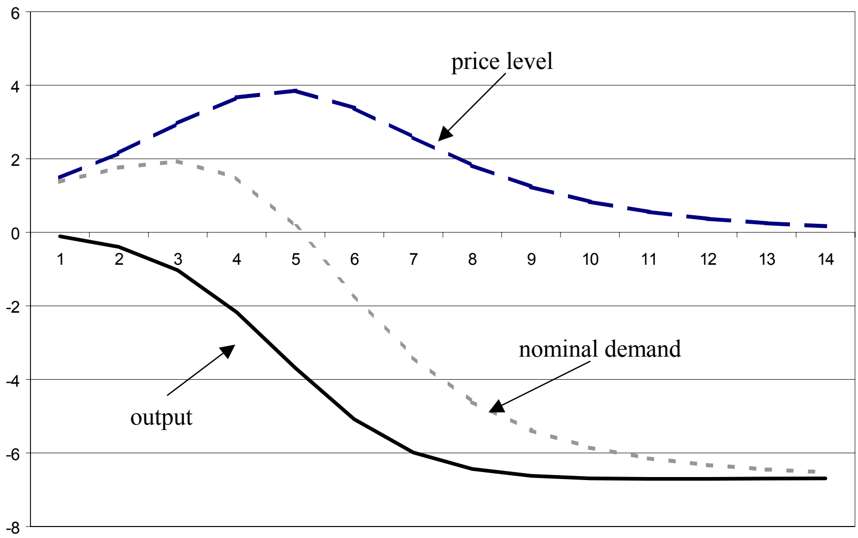

Figure 3: Impulse responses under optimal policy ($\xi = 0.15$, $K = 0.2$)

coupled with strong strategic complementarities, thus, require the policy maker to accommodate mark-up shocks one-for-one with a nominal demand increase. Optimal policy is then identical to the case where shocks are completely unobservable, i.e. where $k = 0$ with probability one.

6.3 Persistent mark-up shocks

The policy derived in the previous sections can be interpreted as optimal short-run policy since mark-up shocks have been assumed to be white noise. This section asks how policy has to react in the medium to long-run when mark-up shocks are persistent.

To analyze the issue we suppose that mark-up shocks arrive according to a Poisson process with probability $\alpha \in (0, 1)$ where the value of new shocks is independent of previous shocks, i.e.

$$\varepsilon_t = \begin{cases} \varepsilon_{t-1} \text{ with probability } \alpha \\ iiN(0, \sigma_\varepsilon^2) \text{ with probability } 1 - \alpha \end{cases}$$

Furthermore, suppose that the arrival of new shocks is common knowledge. This implies that firms are uncertain only about the realization of the shock and about other firms' expectations of the shock, but not about whether a new shock has arrived and whether other firms have noticed the arrival.

Finally, we assume the control error δ_t to be equal to zero and the central bank to observe mark-up shocks perfectly. The presence of control errors or central bank observation errors unnecessarily complicates the analysis.[20]

Suppose a new shock hits the economy in period $t = 1$. Since current policy choices do not constrain future choices and since future choices do not affect the current trade-off, optimal policy in period $t = 1$ is the same as with white noise shocks. Equation (28) then implies that the optimal reaction coefficient is given by

$$a_1 = a(k, \xi) = a(1 - e^{-2K}, \xi).$$

In periods $t \geq 2$ firms possess prior information about the shock from the signals they observed in earlier periods. Since receiving t times a signal via an information channel with capacity K is identical to receiving a single signal from a channel with capacity $t \cdot K$, the optimal reaction coefficient for period t is given by:[21]

$$a_t = a(1 - e^{-2tK}, \xi).$$

Figure 3 displays optimal nominal demand policy and the resulting behavior of output and prices for a persistent mark-up shock of one unit.[22] Optimal policy generates a hump-shaped price response and a gradually declining output level.

Policy initially accommodates the mark-up shock to stabilize output. In the subsequent periods accommodation becomes even stronger for the reasons discussed in section 6.1.[23] Over time, however, firms become less uncertain about the value of the shock and about how other firms perceive it. Nominal demand variations are then increasingly ineffective in stabilizing output and the policy maker starts to stabilize the price level by contracting nominal demand.

6.4 Strategic substitutes

This section briefly discusses the case where prices are strategic substitutes.

We first consider the case $\xi \leq 2$, where equation (26) continues to hold and where the optimal reaction coefficient continues to be given by (29). Figure

[20] The presence of iid control errors, for example, would cause agents to shift attention (i.e. the weights in the observation equation) from the mark-up shock in early periods to the control errors in later periods.

[21] This follows from the fact that the capacity constraint is linear in the entropies and the capacity parameter.

[22] The figure assumes that $\xi = 0.15$ and $K = 0.2$. Output and prices for period t can be calculated from equations (27b) and (27c) by setting $q_t^* = a_t$, $k = 1 - e^{-2tK}$, and $\delta_t = 0$.

[23] If the value of K is large enough, nominal accommodation may not increase initially, see figure 2.

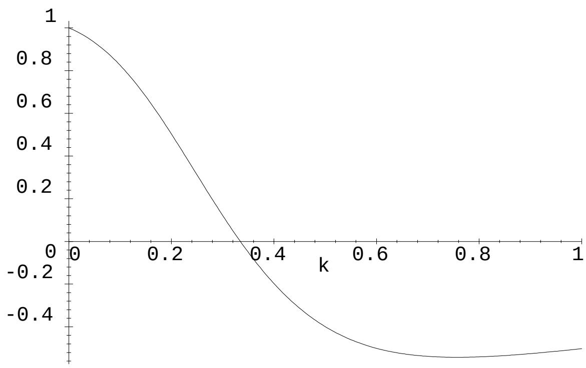

Figure 4: Optimal reaction coefficient with strategic substitutability ($\xi = 2$)

4 depicts the optimal reaction coefficient for various values of k when $\xi = 2$. Unlike with strategic substitutabilities, the reaction coefficient now tends to be negative, showing that optimal policy tends to react with nominal demand contractions in response to mark-up shocks. This feature emerges because strategic substitutabilities imply that firms react with price cuts in response to positive mark-up shocks, see equation (20).

When $\xi > 2$ rational expectations equilibria with common knowledge may not be robust to the introduction of (arbitrarily) small amounts of imperfect common knowledge. In particular, for the linearized model rational expectations equilibria cease to exist for all $k < 1$ sufficiently close to one. This is the case because the sum of higher order expectations in the price setting equation (20) does then not converge, see equation (25), which induces firms to choose either very large or very small prices.[24] It implies that there exists a discontinuity in the transition from the perfect information benchmark to a setting with arbitrarily small amounts of imperfect common knowledge.[25] Analyzing this discontinuity further, however, would require analyzing a non-linear model, which is beyond the scope of this paper.

Interestingly, the no-observation benchmark does not display a similar discontinuity. If information frictions are sufficiently severe, i.e. for $k < \frac{1}{\xi - 1}$, linear rational expectations equilibria start again to exist since higher order expectations then react sufficiently sluggishly, causing the sum of expectations to converge.

[24] Technically, the prices implied by equation (20) are equal to $\pm\infty$. This extreme feature, however, arises because we consider linearized first order conditions. It does not need to hold in the underlying non-linear model.

[25] See Kajii and Morris (1997) for a game-theoretic example where the unique equilibrium is not robust to imperfect common knowledge.

7 Conclusions

This paper shows that optimal policy that seeks to stabilize the quadratic deviations of output and prices from their target values may, at first sight, prescribe rather surprising measures when the underlying economy is characterized by strategic complementarities and imperfect common knowledge about shocks. In particular, the optimal policy recommendations differ strongly from the optimal ones under perfect information. This is the case because the high amounts of coordination that can be achieved with perfect information largely overstates agents' ability to coordinate in the presence of differential information. As a result, optimal policy with imperfect common knowledge resembles much more the policy that is optimal when shocks are unobservable. These conclusions are reversed, however, when firms' prices are strategic substitutes. If rational expectations equilibria exist, optimal policy then tends to look more like in perfect information settings.

A Appendix

The text assumes that agents receive a signal about (22). Below we show that the rational expectations equilibrium (REE) derived under this assumption is unaffected when allowing agents to receive a signal about (21) instead.

Let f_t denote the infinite sum in equation (21). Equation (25) implies that in the rational expectations equilibrium where agents observe (22):

$$f_t = \sum_{n=0}^{\infty} ((1-\xi)k)^n (\xi q_t + (1-\xi)\varepsilon_t) \tag{33}$$

and

$$\begin{aligned} E[f_t|s_t^i] &= k \sum_{n=0}^{\infty} ((1-\xi)k)^n \left(\xi q_t + (1-\xi)\varepsilon_t + \eta_t^i\right) \\ &= k \sum_{n=0}^{\infty} ((1-\xi)k)^n (\xi q_t + (1-\xi)\varepsilon_t) + \frac{k}{1-(1-\xi)k}\eta_t^i. \end{aligned} \tag{34}$$

Now instead suppose that agents observe

$$\widetilde{s}_t^i = f_t + \widetilde{\eta}_t^i.$$

Expectations are then given by

$$\begin{aligned} E[f_t|\widetilde{s}_t^i] &= k\widetilde{s}_t^i \\ &= kf_t + k\widetilde{\eta}_t^i \\ &= k \sum_{n=0}^{\infty} ((1-\xi)k)^n (\xi q_t + (1-\xi)\varepsilon_t) + k\widetilde{\eta}_t^i. \end{aligned} \tag{35}$$

where the last line uses the fact that in the REE (33) holds. The expectations in (35) are identical to ones in (34) if

$$\widetilde{\eta}_t^i = \frac{1}{1-(1-\xi)k}\eta_t^i,$$

which follows from the fact that the agent faces the same channel capacity constraint, independently of which object is observed. Equation (20) together with $E[f_t|\widetilde{s}_t^i] = E[f_t|s_t^i]$ then implies that agents set the same profit maximizing price, independently of whether they observe (21) or (22).

References

Amato, Jeffrey and Hyun Song Shin, "Public and Private Information in Monetary Policy Models," *London School of Economics mimeo*, 2003.

Ball, Laurence, Gregory Mankiw, and Ricardo Reis, "Monetary Policy for Inattentive Economies," 2002. Harvard University mimeo.

Binder, Michael and Hashem Pesaran, "Decision Making in the Presence of Heterogeneous Information and Social Interactions," *Journal of Economic Dynamics and Control*, 1998, *39*, 1027–1052.

Cover, Thomas M. and Joy A. Thomas, *Elements of Information Theory*, New York: John Wiley and Sons, 1991.

Hayek, Friedrich A., "The Use of Knowledge in Society," *American Economic Review*, 1945, *35 (4)*, 519–530.

Kajii, Atsushi and Stephen Morris, "The Robustness of Equilibria to Incomplete Information," *Econometrica*, 1997, *65*, 1283–1309.

Kasa, Kenneth, "Forecasting the Forecasts of Others in the Frequency Domain," *Review of Economic Dynamics*, 2000, *3*, 726–756.

Keynes, John M., *The General Theory of Employment, Interest, and Money*, New York: Hartcourt Brace and World, 1936.

Lucas, Robert E., "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 1972, *4*, 103–124.

Morris, Stephen and Hyun Song Shin, "Global Games: Theory and Applications," *Conference Volume of the Eighth World Congress of the Econometric Society, Seattle, (forthcoming)*, 2000.

____ **and** ____ , "Social Value of Public Information," *American Economic Review (forthcoming)*, 2003.

Phelps, Edmund S., “The Trouble with "Rational Expectations" and the Problem of Inflation Stabilization,” in R. Frydman and E. Phelps, eds., *Individual Forecasting and Aggregate Outcomes*, Cambridge: Cambridge University Press, 1983, pp. 31–41.

Sargent, Thomas J., “Equilibrium with Signal Extraction from Endogenous Variables,” *Journal of Economic Dynamics and Control*, 1991, *15*, 245–273.

Shannon, C. E., “A Mathematical Theory of Communication,” *The Bell System Technical Journal*, 1948, *27*, 623–656.

Sims, Christopher, “Implications of Rational Inattention,” 2001. Princeton University Mimeo.

Svensson, Lars and Michael Woodford, “Indicator Variables for Optimal Policy under Asymmetric Information,” 2002. Princeton University.

—— **and** —— , “Indicator Variables for Optimal Policy,” 2002. Princeton University.

Townsend, Robert M., “Equilibrium Theory with Learning and Disparate Information,” in R. Frydman and E. Phelps, eds., *Individual Forecasting and Aggregate Outcomes*, Cambridge: Cambridge University Press, 1983, pp. 169–202.

—— , “Forecasting the Forecasts of Others,” *Journal of Political Economy*, 1983, *91*, 546–588.

Woodford, Michael, “Inflation Stabilization and Welfare,” *NBER Working Paper No. 8071*, 2001.

—— , “Optimizing Models with Nominal Rigidities,” *Princeton University Mimeo*, 2001.

—— , “Imperfect Common Knowledge and the Effects of Monetary Policy,” in P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, eds., *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honour of Edmund S. Phelps*, Princeton: Princeton University Press, 2002.

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