Maarten Bosker

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Evidence from a nonstationary dynamic panel data analysis
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E.M. Bosker*†

ABSTRACT

The effectiveness of the important role for money in the monetary policy of the European Central Bank (ECB) is usually assessed by looking at time series estimates of the eurozone money demand equation. This implicitly calls for a choice of aggregation method to construct data series long enough to obtain meaningful econometric results. The choice of aggregation method can affect the results of such studies. This study tries to avoid the issue of aggregation by adopting a (nonstationary) dynamic panel method that uses the data for each of the eurozone countries by itself. Specific tests are developed to test for differences in the long run money demand equation across eurozone countries. The results show that differences in the money demand equation across the eurozone countries are likely to exist. A drawback of the method used is that it is not able to quantify these differences, which makes it difficult to give specific implications for the ECB’s monetary policy. The found differences could however influence the time series estimates and through these have implications for the ECB’s monetary policy.

Key words: money demand, eurozone, dynamic panel estimation
JEL codes: C12, C32, C33, E41, E52

*I am especially grateful to Jean-Pierre Urbain, Franz Palm and Peter Vlaar for their comments. Moreover I would like to thank Peter van Els and two anonymous referees for their suggestions and Peter Keus for statistical assistance. This research is part of the author’s master thesis written while at Maastricht University.
†m.bosker@econ.uu.nl, Utrecht School of Economics, Utrecht University, Vredenburg 138, 3511 BG, Utrecht, The Netherlands, tel: +31 30 2539800, fax: +31 30 2537373
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INTRODUCTION

The primary objective of the European System of Central Banks (ESCB) shall be to maintain price stability in the medium term (Article 105(1) of the Treaty establishing the European Community). In accordance with this primary objective, the main elements of the stability-oriented monetary policy strategy of the Governing Council of the European Central Bank (ECB), as announced in October and December 1998, are aimed at maintaining price stability. Price stability is defined as a year-on-year increase in the Harmonised Index of Consumer Prices for the eurozone below 2%. As of May 8th 2003, the Governing Council has extended this definition to below, but close to, 2%, to underline the ECB’s commitment to guard against deflation. One of the ‘two pillars’ announced to achieve the strategy’s objective is a prominent role for money. Money is believed to constitute a natural, firm and reliable ‘nominal anchor’ for monetary policy aiming at the maintenance of price stability (ECB Monthly Bulletin, January 1999).

The development of the price level is believed to be a monetary phenomenon in the long run and useful information about future price movements may thus be revealed by the development of the amount of money held. It can offer important signals when setting the monetary policy. This and the fact that the monetary policy strategy of the ECB is aimed at the medium term, requires the definition of a monetary aggregate which is a stable and reliable indicator of the price level over the medium term. The Governing Council of the ECB has decided to give M3, the broad money aggregate, a prominent role in the monetary policy strategy. It has chosen M3, because it is believed to have better properties than more narrow money aggregates in terms of stability and information content with respect to price developments in the long term (ECB Monthly Bulletin, February 1999). The Governing Council even announced a quantitative reference value for the growth of M3 of 4.5% per year (ECB Monthly Bulletin, January 1999). This reference value is not a monetary target but deviations from the reference value are closely analysed in the context of other economic data in order to obtain useful information regarding possible risks to price stability. Recently (ECB Press Release, May 8th 2003), the Governing Council announced that this reference value will not be reviewed every year, keeping it fixed at 4.5% per year. To be able to say that setting a reference value for the growth of M3 is consistent with the main objective of price stability in the medium term, there must exist a stable relationship between M3 and the price level at this time horizon.

The aim of this paper is to conduct empirical research in order to test for the existence of such a stable relationship at the eurozone level. If evidence of such a stable relationship cannot be found, this would shed serious doubts on the role for money in the monetary policy of the ECB. It would imply that the ECB bases its monetary policy on the incorrect assumption of the existence of a stable relationship
between M3 and the price level, causing damage to its credibility and subsequently the effectiveness of its monetary policy.

In order to do this the relatively new econometric framework of nonstationary panel data estimation is used to answer the question whether or not a stable money demand equation exists. This technique has the advantage that it makes the issue of which type of aggregation method to use irrelevant. Instead of using aggregate European data, each country’s individual series are used in the analysis. Another nice feature is that the framework allows for testing the equality of the long run relations across the countries in the panel. If a long run money demand equation is found for each country in the panel, this econometric framework can be used to test for and, if found, identify differences in that relation between the countries (or groups of countries), that have handed over their monetary policy to the ECB in joining the European Monetary Union. As the ECB is only in charge of monetary policy as of 1999, it would not be very surprising to find such differences. If differences are found, identifying them is important as that allows the policy makers at the ECB to take account of these differences when deciding on a meaningful monetary policy.

The paper is organised as follows. Section 2 introduces the economic model and gives a short review of past research stating also the econometric methods that have been used in the past in the analysis of money demand. In Section 3 the nonstationary panel framework is introduced, giving an overview of recent developments in that field of econometrics and explaining the choice of test that is used in this paper. Furthermore the estimation results using this econometric framework are presented and new tests are developed where existing tests are not appropriate for answering some of the empirical questions. Section 5 concludes.

2 MODEL AND ECONOMETRIC METHODS

The existence of a long run money demand equation is typically assessed in the context of standard economic theories of money demand. As indicated in Ericsson (1998), in standard theories of money demand, money may be demanded for at least two reasons. First, out of transaction motives, it can be used as an inventory to smooth differences between income and expenditure streams and second as an asset among other assets in a portfolio. Both reasons for money to be demanded, lead to the following long run specification,

\[ \frac{M_t}{P} = g(I,R) \]  \hspace{1cm} (1)
where $M^d$ is nominal money demanded, $P$ is the price level, $I$ a scale variable and $R$ a vector of returns on various assets. The theory suggests that $g(\cdot, \cdot)$ is increasing in $I$, decreasing in those asset returns included in $R$ that are excluded from $M$, and increasing in those asset returns in $R$ that are included in $M$.

In the literature one commonly finds (1) in the following log-linear form:

$$m^d - p = \alpha + \beta y + \gamma_1 R^{out} + \gamma_2 R^{own} + \delta \Delta p,$$

where lowercase letters indicate variables in logs, $\alpha$, $\beta$, $\gamma_1$, $\gamma_2$, and $\delta$ are coefficients and $\Delta$ is the difference operator. $R^{out}$ are the rates of return on financial assets alternative to money and $R^{own}$ the return on the components included in the definition of money itself. As argued in Fair (1987) both the outside and own rate should not be transformed in logs. $y$ is the real gross domestic product (GDP), a choice of scale variable that is very common in the existing literature, although one can also find studies in which it is chosen to be financial wealth, e.g. Fase and Winder (1998). Finally $\Delta p$ is the inflation rate (growth rate of the GDP deflator, i.e. nominal GDP divided by real GDP), included as a proxy for the return on goods alternative to money.

Past research on money demand (see Golinelli and Pastorello (2001) for a good overview up to 1999) in the eurozone has focused on the loglinear long run money demand specification as in (2). Analysing money demand in the setup of equation (2) poses a number of questions. What variables are chosen to appear in the equation? How to choose the countries included in the sample, the sample period and periodicity of the data? What econometric framework to use in estimating the parameters of (2)?

Since the ECB made clear that it would take M3 as the relevant indicator for money, most studies choose M3 to be the dependent variable of equation (2). Some researchers advocated the inclusion of cross border holdings in the definition of money, but, as shown in Fagan and Henry (1998), this inclusion does not improve the estimation results.

As mentioned before, GDP is most often chosen to represent the scale variable in (2). The use of GDP is supported by the demand for money based on transaction motives, consequently it can be argued to be suitable for more narrow definitions of money. The portfolio theory of asset demand suggests the use of financial wealth as a choice of scale variable more suitable for analysing broader money definitions. However, the lack of reliable financial wealth data is often used to justify the use of GDP for the scale variable.
As for the choice of own and outside rates, most papers use the 3-month interest rate as a measure of the own rate, as this is included in the definition of M3, and the 10-year government bond yield (or a proxy) as a measure of the outside rate. One also finds studies in which only one of the two rates or only the term spread, \((R^{out} - R^{own})\), is included in the money demand equation. Finally inflation is measured by the growth rate of the GDP-deflator, i.e. nominal GDP divided by real GDP.

With very few exceptions most studies take the end of the seventies as the beginning of their sample period. The European Monetary System (EMS) began in that period and the Exchange Rate Mechanism (ERM) was established (March 1979). The periodicity of the time series data is usually quarterly. Empirical research has mainly focused on the eurozone, where some studies exclude Luxembourg from the analysis, the reason being the unavailability of data (Beyer et al., 2001). Its exclusion can also be justified by its small quantitative importance (Clausen, 1998) and the fact that, due to the monetary union between Belgium and Luxembourg, Luxembourg’s M3 is probably already included in Belgian M3 (Beyer et al. 2001).

The usual econometric framework in which (2) is estimated is that of a time series setup. As most research finds that all the variables in (2) are non-stationary, the money demand equation is usually estimated in a cointegration framework. The estimation of the cointegrating relationship differs per author. Most recently the Johansen (1995) framework (Vlaar and Schuberth, 1999; Coenen and Vega, 1999; Brand and Cassola, 2000; Fagan and Henry, 1998) is used, while in the past the Engle and Granger (1987) two-step-approach was popular.

Most recent empirical studies focus on eurozone money demand only. First eurozone-wide aggregates are constructed using a certain method of aggregation (as prior to the introduction of the euro only national accounts are available), and then the cointegration techniques are ‘unleashed’ on these constructed area-wide aggregates (Coenen and Vega, 1999; Brand and Cassola, 2000; Vlaar, 2004). In this analysis the first step, obtaining aggregate time series for the eurozone long enough to be able to make reliable estimates that do not suffer from small sample distortion, is crucial. This complicates the approach, as the choice of aggregation method is not unimportant. Different aggregation techniques can lead to different results (see Bosker, 2003).

The empirical approach taken in this paper is to avoid these aggregational issues and use a dynamic panel approach in estimating the money demand equation. The need to choose for a certain aggregation method does no longer exist, as all series for each individual country, expressed in local currency, are used in the estimation. Another very nice property of this econometric framework is that increasing the amount of information available by analysing time series across similar cross-sections,
i.e. adapting a panel approach, leads to tests with better power properties (Levin and Lin, 1992). Hereby overcoming the problem that in the usual time series framework, the power of the usual cointegration tests is shown to depend on the time span of the data series used.

Besides these advantages from an econometric point of view, a nice feature of the panel setup is that it explicitly takes account of the national differences in financial structures and markets and country specific structural breaks by using the national series. Both effects are less evident when using eurozone aggregates, as their impact is ‘canceled out’ in the aggregation process. As noted by Dedola, Gaiotti and Silipo (2001), these country specific effects may result in differences in national money demand functions. If the monetary policy of the ECB has different consequences for different countries of the eurozone, it can be very helpful to identify and take account of these differences. The panel setup allows for testing homogeneity restrictions on the resulting money demand equations if similar relations are found for the countries in the panel. It allows for explicit tests that check whether or not the same long run relations hold for all eurozone countries or subgroups of eurozone countries. If substantial cross-country differences are found, looking also at national data can be of importance for policymakers.

3 NONSTATIONARY PANEL DATA, METHODS AND RESULTS

Econometric inference using nonstationary panels is a field that has emerged only recently. The interest in this field has increased over the last few years, starting with papers by Quah (1994) and Levin and Lin (1994), who test for unit roots within a panel setup. The main reason for this is the fact that tests for stationarity or cointegration within a time series setup tend to suffer from low power if the time span of the data used is not large enough, a phenomenon that is not very rare for macroeconomic data. Using a panel setup adds additional observations by drawing data from among similar cross-sections to increase the power of the tests. Furthermore a panel setup allows for the exploitation of similarities across the individuals in the panel. By pooling the information of several individuals, the panel setup may provide better evidence on the (non)existence of economic relationships.

The panel cointegration literature mainly focuses on residual based tests (Kao, 1999; Banerjee, 1999; Pedroni, 2002) that are based on pooling the individual residual based test statistics. These residual based tests however only focus on the question whether or not cointegration is present in the panel. The same drawbacks as in the case of residual based tests for cointegration in a time series setup apply for these tests. The estimated cointegrating relation may differ with the ordering of the variables and
the choice of normalisation of the cointegrating relation. The tests also do not allow testing for the number of cointegrating relations. There are however some papers that do derive tests for the number of cointegrating relations within a panel setup (Larsson, Lyhagen and Löthgren, 2001; Groen and Kleibergen, 2002). These tests do not suffer from the drawbacks described above, but are only useful (i.e. the test has the correct size for, or requires, fixed N) for panels with a small cross-section dimension compared to the time dimension1.

Another important issue that has received a lot of attention in the nonstationary panel literature is the assumption of cross-sectional independence between the units in the panel. Most existing residual based tests use this assumption to be able to get a nice asymptotic distribution for the test statistic as the cross-section dimension goes to infinity. The cross-sectional independence allows for the use of standard asymptotic tools, like the (standard) Central Limit Theorem. As shown in Monte Carlo experiments, see O’Connell (1998) inappropriately assuming cross-sectional independence can severely distort the size of panel tests in a nonstationary world. Recently some tests have been proposed that do allow for the units in the panel to have some form of dependency (Bai and Ng, 2003; Chang, 2002; Groen and Kleibergen, 2002; Larsson and Lyhagen, 2000).

Studies that use a nonstationary panel framework to test for the existence of a long run money demand equation are relatively few. Mark and Sul (2002) apply a residual based cointegration test, Pedroni’s (1999) panel t-test, on a panel of nineteen countries. Golinelli and Pastorello (2001) and Dedola, Gaiotti and Silipo (2001) also apply a residual based cointegration test on countries in the eurozone (excluding Greece or Luxembourg and Greece from the analysis). Focarelli (2000) applies a bootstrap biased-correction procedure, correcting the Mean Group estimator of Pesaran and Smith (1995). All papers apply a panel cointegration test that does not allow for cross-dependence of the units in the panel and that does not explicitly test for the cointegrating rank. As mentioned before inappropriately assuming cross-sectional independence can lead to a severe distortion of the size of the test when this assumption is not valid.

In this paper the test of Groen and Kleibergen (2002) is used. This test does allow for cross-sectional transitory dependence among the units in the panel. A nice feature, as the data for the countries in our panel are evidently not independent of each other. Looking only at the correlation between the individual time series gives evidence of this (see Table 1 for the correlation between the individual

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1 As (at the moment) the eurozone includes twelve countries, this does not seem to constitute a big problem. One might even be more doubtful regarding panel cointegration tests that rely on large N asymptotics.
GDP growth rate series, similar correlations are found for the other variables). The next section will come back to this issue.

Table 1 Correlation matrix of the individual GDP growth rate series

<table>
<thead>
<tr>
<th></th>
<th>AU</th>
<th>BE</th>
<th>ES</th>
<th>FI</th>
<th>FR</th>
<th>GE</th>
<th>GR</th>
<th>IER</th>
<th>IT</th>
<th>LU</th>
<th>NE</th>
<th>PT</th>
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<tr>
<td>AU</td>
<td>1</td>
<td>0.34</td>
<td>0.28</td>
<td>0.05</td>
<td>0.40</td>
<td>0.41</td>
<td>0.13</td>
<td>0.14</td>
<td>0.24</td>
<td>0.21</td>
<td>0.28</td>
<td>0.23</td>
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<tr>
<td>BE</td>
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<td>0.44</td>
<td>0.18</td>
<td>0.46</td>
<td>0.25</td>
<td>0.27</td>
<td>0.30</td>
<td>0.31</td>
<td>0.28</td>
<td>0.27</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>ES</td>
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<td>0.08</td>
<td>0.33</td>
<td>0.12</td>
<td>0.35</td>
<td>0.21</td>
<td>0.07</td>
<td>0.33</td>
<td>0.29</td>
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</tr>
<tr>
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<td>0.35</td>
<td>0.09</td>
<td>0.20</td>
<td>0.24</td>
<td>0.30</td>
<td>0.06</td>
<td>0.07</td>
<td>0.32</td>
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<tr>
<td>FR</td>
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<td>0.41</td>
<td>0.16</td>
<td>0.37</td>
<td>0.45</td>
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<td>0.24</td>
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<td>GE</td>
<td>1</td>
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<td>0.22</td>
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<tr>
<td>GR</td>
<td>1</td>
<td>0.11</td>
<td>0.13</td>
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<td>0.08</td>
<td>0.24</td>
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<td>IER</td>
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<td>0.11</td>
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<tr>
<td>IT</td>
<td>1</td>
<td>0.16</td>
<td>0.18</td>
<td>0.04</td>
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<td>LU</td>
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<td>0.33</td>
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<td>NE</td>
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Another nice feature of the test is that it does not suffer from the drawbacks that apply to residual based tests and it allows testing for the cointegrating rank. Finally the fact that the test should be used only for panels with a relatively small cross-sectional dimension compared to the time-dimension, does not seem to be a big problem as the cross-section dimension in the panel used is fixed at ten (excluding Luxembourg and Greece due to data problems) and the time span of the data is from 1979:2 – 2002:4.


The test developed by Groen en Kleibergen (2002) (hereafter GK-2002) is based on the vector error correction model (VECM) framework of Johansen (1995) for each of the individuals in the panel. A VECM for each individual is stacked into one system, a Panel VECM,

$$\Delta Y = \begin{pmatrix} \Phi_1 & 0 & \cdots & 0 \\ 0 & \Phi_2 & \cdots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \Phi_N \end{pmatrix} Y_{t-1} + \varepsilon_t = \Phi A Y_{t-1} + \varepsilon_t$$

(3)

where $\Phi_N$ is a $Nm \times Nm$ matrix containing the individual $\Phi_i$, $i = 1,\ldots,N$, $m \times m$ matrices (m is the number of variables), relating for each individual $\Delta y_{it}$ to $y_{i,t-1}$. $Y_{t-1} = (y_{1,t-1} \ldots y_{N,t-1})'$, $\Delta Y_t = Y_t - Y_{t-1}$ and
\( \epsilon_i = (\epsilon_{i1}', \ldots, \epsilon_{Nm}') \) are \( Nm \times 1 \) vectors. The vector \( \epsilon_i \) contains the \( Nm \times 1 \) disturbance vectors for each individual VECM and it is distributed \( \epsilon_i \sim N(0, \Omega) \) with the \( Nm \times Nm \) nondiagonal covariance matrix,

\[
\Omega = \begin{pmatrix}
\Omega_{11} & \cdots & \Omega_{1N} \\
\vdots & \ddots & \vdots \\
\Omega_{N1} & \cdots & \Omega_{NN}
\end{pmatrix}
\]  

(4)

where \( \Omega_{ij} = \text{Cov}(\epsilon_{ij}, \epsilon_{ij}) \) is not restricted to be zero for any \( i,j = 1, \ldots, N \). Note that notation is changed with respect to the previous section. Where \( y \) previously denoted GDP, here \( y_{it} \) denotes a vector of all variables contained in the VECM, i.e. here \( y_{it} = (m-p, y, i, l, p') \).

As can be seen, the model as specified in (3) does allow for cross-sectional dependency through the covariance matrix of the disturbance terms (4), i.e. transitory cross-sectional dependence. However as the off-diagonal elements of the \( \Phi_A \) matrix are set equal to zero, the model is a restricted version of the model in which the off-diagonal elements are left to be estimated. These restrictions on the \( \Phi_A \) matrix impose that there is no linear dependence between the variables of individual \( i \) and lags of the variables of individual \( j \), for \( j \neq i \), i.e. no persistent cross-sectional dependence. By imposing the restrictions, cross-dependence in the panel is only allowed through the non-diagonal covariance structure of the error term (4). Larsson and Lyhagen (1999) do allow for non-zero off-diagonal elements, which also implicitly allows for the possibility of cointegration between series of different individuals in the panel. The reason why these restrictions are imposed here is that if these restrictions are not imposed, all the off-diagonal elements of the \( \Phi_A \) matrix would also have to be estimated. This is very likely not to be efficient due to the large number of parameters that have to be estimated in the estimation procedure.

The GK-2002 test for cointegration is developed to test for a common cointegrating rank for each individual \( i \) within the panel, i.e. \( \text{rank}(\Phi_i) = r \) for each \( i = 1, \ldots, N \). Cointegration within the panel imposes a rank reduction on the different \( \Phi_i \)'s in (3) identical to all individuals in the panel,

\[
\Delta Y_i = \begin{pmatrix}
B_1 A_1' & 0 & \cdots & 0 \\
0 & B_2 A_2' & \cdots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & B_N A_N'
\end{pmatrix} Y_{i-1} + \epsilon_i = \Phi_y Y_{i-1} + \epsilon_i
\]  

(5)
where the $m \times r$ matrices $A_i$ contain the cointegrating vectors and the $m \times r$ matrices $B_i$ the loading factors, the adjustment factors to the long run equilibrium, for each of the $i = 1, \ldots, N$ individuals.

The test for a common cointegrating rank $r$ developed by GK-2002 tests the following:

$$H_0 : \Phi_B \ vs. \ H_1 : \Phi_A$$

(6)

where $\Phi_B$ is as in (5) and $\Phi_A$ is as in (3). GK-2002 develops maximum likelihood estimators of the cointegrating vectors, the loading factors and the covariance matrix of the disturbance term using Generalised Method of Moments estimators. Using these estimators they construct the following likelihood ratio statistics to test for a common cointegrating rank among the individuals in the panel.

$$LR(\Phi_B | \Phi_A) = 2 \left[ I \left( \hat{\Phi}_A, \hat{\Omega}(\hat{\Phi}_A) \right) - I \left( \hat{\Phi}_B, \hat{\Omega}(\hat{\Phi}_B) \right) \right]$$

(7)

where $I \left( \hat{\Phi}_\Theta, \hat{\Omega}(\hat{\Phi}_\Theta) \right)$, $\Theta \in \{A,B\}$, is the value of the maximised log-likelihood function. Now keeping $N$ fixed and letting $T$ go to infinity the limiting distribution of the test statistic in (7) equals,

$$LR \left( \Phi_B | \Phi_A \right) \Rightarrow \sum_{i=1}^{N} \text{tr} \left( \int dB_{m-r,i} B'_{m-r,i} \left[ \int B_{m-r,i} B'_{m-r,i} \right]^{-1} \int B_{m-r,i} dB'_{m-r,i} \right)$$

(8)

where $B_{m,r,i}$ is a $(m-r)$-dimensional Brownian motion for individual $i$ with an identity covariance matrix. As can be seen in (8) the limiting distribution as $T \to \infty$ of the likelihood ratio statistic (7) is the sum of $N$ individual limiting distributions for the Johansen trace statistic.

GK-2002 extend the result for the more general case of model (3) that allows for deterministic components and higher order dynamics,

$$\Delta Y_t = \alpha + \delta t + \Phi_\delta Y_{t-1} + \Gamma W_t + \varepsilon_t$$

(9)

where $\alpha = (\alpha_1, \ldots, \alpha_N)'$ is a vector containing constants, $\delta t = (\delta_1, \ldots, \delta_N)' t$ represents the individual linear trends, $W_t = (\Delta y_{1,t-1}', \ldots, \Delta y_{k_1,t-1}', \ldots, \Delta y_{N,t-1}', \ldots, \Delta y_{N,k_2}' )$ contains the $k_i$ lagged first differences of individual $i$, and
contains the parameters for each of the individual lagged differences.

The higher order dynamics and deterministic components can be concentrated out in the usual way through OLS regressions of $\Delta y_{i,t-1}$ and $y_{i,t-1}$ on the included higher order dynamics and deterministic components for each of the individuals $i$ (see Johansen, 1995). In doing so, one implicitly assumes heterogeneity across individuals with respect to the deterministic components and short run dynamics. Now the test statistic can be computed using the maximum likelihood estimators where $\Delta y_{i,t-1}$ and $y_{i,t-1}$ in (3) are replaced by the residuals of the corresponding OLS regressions for each individual. The limiting distributions are only affected by the inclusion of deterministic components. Including higher order dynamics does not affect them as those only affect the short-run properties of the model. The change in limiting distribution depends on the type of deterministic components included. As in the time series case, five different cases (see Johansen, 1995) can be distinguished. One can allow for:

1. zero mean in the cointegrating relations and non-zero mean in the data
2. non-zero mean in both the cointegrating relations and the data
3. non-zero mean in the cointegrating relations and a linear trend in the data
4. linear trend in both the cointegrating relations and the data
5. linear trend in the cointegrating relations and a quadratic trend in the data

As looking at the data in the available sample (see Appendix A) provides some evidence of linear trending behavior and a non-zero mean in the cointegrating relations will not be excluded a priori, the main focus in this paper will be case 3. A choice that is not uncommon in the empirical literature on money demand (Brand and Cassola, 2000; Vlaar, 2004). Subsequently the limiting distribution of the likelihood ratio statistic is changed in the following way,

$$LR(\Phi_b | \Phi_d) \Rightarrow \sum_{i=1}^{N} tr\left( dB_{m-r,i} S_i^{-1} S_i^{-1} dB_{m-r,i}^t \right)$$ (11)
where in \( S_j \equiv B_{m-r,j} - \int B_{m-r,d} \). As can be seen the likelihood ratio statistic (11) remains the sum of the \( N \) individual limiting distributions of the Johansen trace statistic adjusted for the presence of deterministic components as in case 3.

In their paper GK-2002 focus on case 2 and case 4 and calculate critical values for these two cases. As the point of interest here is case 3, critical values had to be obtained through simulation. To do this discrete-time approximations to (11) were used. First the \( N \) individual limiting distributions are approximated by their discrete-time counterparts, i.e.

\[
tr\left(\sum_{t=1}^{T} \Delta x_{t,j} F_{it}^\prime \left(\sum_{t=1}^{T} F_{it} F_{it}^\prime\right)^{-1} \sum_{t=1}^{T} F_{it} \left(\Delta x_{t,j}\right)^\prime\right)
\]

where the Brownian motions are replaced by Gaussian random walks and \( T \) is the number of discrete-time approximations. More explicitly, let \( x_t \) denote an \((m-r)\)-vector whose elements follow a random walk with \( N(0,I) \) innovations and \( x_0 = 0 \). Then \( \Delta x_{t,j} = x_{t,j} - x_{t-1,j} \) and \( F_{it} = x_{t,j} - \frac{1}{T} \sum_{t=1}^{T} x_{t,j} \). Finally the discrete-time approximation of (11) is calculated as the sum of all \( N \) individual approximations. To obtain the critical values, the above is repeated 50,000 times for \( T = 1000 \). Thus 50,000 discrete-time approximations of (11) are obtained and the critical values are calculated as being the \( q \)th (\( q = 99\%, 95\% \) or 90\%) quantile of these 50,000 replications. Table 2 shows the critical values thus obtained for a cointegrating rank up to four.

Table 2 Critical values for the GK-2002 test with \( N = 10 \)

<table>
<thead>
<tr>
<th>r</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>415.92</td>
<td>398.79</td>
<td>389.32</td>
</tr>
<tr>
<td>2</td>
<td>255.46</td>
<td>241.34</td>
<td>234.13</td>
</tr>
<tr>
<td>3</td>
<td>135.17</td>
<td>124.00</td>
<td>118.17</td>
</tr>
<tr>
<td>4</td>
<td>52.58</td>
<td>45.02</td>
<td>41.43</td>
</tr>
</tbody>
</table>

Equipped with the above critical values, the tests for common cointegrating rank can be done using a panel of eurozone countries. Appendix A provides a quick glance at the data used in the empirical analysis. The panel used to conduct these test is one including observations on all eurozone countries except Luxembourg and Greece. These two countries are excluded due to the beforementioned reason of data problems. To determine the cointegrating rank the likelihood ratio statistic as in (7) is
calculated for the cases where \( r = 1, 2, 3, 4 \). As can be seen in Table 3 below, the tests indicate a common cointegrating rank of three.

<table>
<thead>
<tr>
<th>( r )</th>
<th>Test-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>723.14**</td>
</tr>
<tr>
<td>2</td>
<td>341.75**</td>
</tr>
<tr>
<td>3</td>
<td>100.69</td>
</tr>
<tr>
<td>4</td>
<td>7.87</td>
</tr>
</tbody>
</table>

Note: (**), (*) denotes significance at a 1%, 5% level.

This finding is similar to the cointegrating rank found in studies using the European aggregates in a time series setting (Bosker, 2003; Vlaar, 2004). However by using the European aggregates, it is implicitly assumed that the relationships found, hold for all member countries of the eurozone. In the panel case, the relationships found can differ per individual member of the panel, as each individual \( i \) has its own \( A_i \) matrix containing the individual specific cointegrating vectors.

3.1.1 Testing for homogeneity of the cointegration space

However one can test for the equality of the cointegrating vectors for all individuals. In order to do so, one has to estimate the following regression (after again first concentrating out the deterministic components and higher order dynamics):

\[
\Delta Y_i = \begin{pmatrix}
B_1A' & 0 & \cdots & 0 \\
0 & B_2A' & \cdots & \vdots \\
\vdots & \vdots & \ddots & 0 \\
0 & \cdots & 0 & B_NA'
\end{pmatrix}Y_{t-1} + \varepsilon_i = \Phi_cY_{t-1} + \varepsilon_i
\]

(13)

where the matrix containing the cointegrating vectors is the same for all individuals, i.e. \( A_i = A \) for all \( i \in \{1, \ldots, N\} \). Groen and Kleibergen (2002) develop a test that is based on the likelihood ratio test to test for the validity of a homogeneous cointegrating space for all individuals in the panel. As in the time series case, the cointegrating rank is fixed before testing and the resulting test statistic is distributed as a \( \chi^2((N-1)r(m-r)) \) variable, i.e.

\textsuperscript{2} To capture the higher order dynamics, \( k_i \) is set to one for all \( i \). This choice is of lag length is made based on evidence from country-specific optimal lag length selection procedures based on the HQ criterion (results available upon request).
As the test for the common cointegrating rank indicated a rank of three, the test for a homogeneous
cointegrating space across individuals is done keeping the cointegrating rank fixed at three. The
resulting test statistic is 318.13, which is quite a bit larger than the corresponding $F_{2(54)}$
critical value, i.e. 86.0. The hypothesis of a homogeneous cointegrating space across individuals is clearly
rejected, indicating differences in the long run relationships across the different members of the
eurozone.

As the ten countries included in our analysis of the eurozone do not share the same economic history,
the rejection of a homogeneous cointegrating space may not be that surprising. Some countries joined
the European Union later than others. This might have caused some countries to already integrate
more economically (and non-economically) than others. Also countries have shown historical
differences in economic policy and economic development. The economic policy of some countries
(e.g. Germany, the Netherlands) was much more stable than others (Spain, Portugal). These historical
economic differences between countries in the eurozone might explain the rejection of a homogeneous
cointegrating space. However it also gives rise to another question, i.e. ‘Can we identify groups of
countries within the eurozone that share the same long run developments?’. In order to answer this
question, a test is needed that tests for the homogeneity of the cointegrating space for predefined
groups of countries.

To do this the test developed by Groen and Kleibergen (2002) to test for a homogeneous cointegrating
space across all individuals is adapted in the way the likelihood under the null is calculated. Under the
null of $s$, 1 < $s$ < N, groups with the same cointegrating space each group containing $n_1$, $n_2$, ..., $n_{s-1}$, $n_s$
individuals respectively and $\sum_{i=1}^{s'} n_i = N$ such that all individuals are assigned to one group, the
following model is estimated (again after first concentrating out the deterministic components and
higher order dynamics):

\[
LR(\Phi \left| \Phi_{g} \right.) = 2 \left[ l(\hat{\Phi}_{g}, \hat{\Omega}(\hat{\Phi}_{g})) - l(\hat{\Phi}_{c}, \hat{\Omega}(\hat{\Phi}_{c})) \right] - \chi^2((N-1)r(m-r))
\] (14)

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resulting test statistic is 318.13, which is quite a bit larger than the corresponding $\chi^2(54)$ 1% critical
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space across all individuals is adapted in the way the likelihood under the null is calculated. Under the
null of $s$, 1 < $s$ < N, groups with the same cointegrating space each group containing $n_1$, $n_2$, ..., $n_{s-1}$, $n_s$
individuals respectively and $\sum_{i=1}^{s'} n_i = N$ such that all individuals are assigned to one group, the
following model is estimated (again after first concentrating out the deterministic components and
higher order dynamics):

\[
LR(\Phi \left| \Phi_{g} \right.) = 2 \left[ l(\hat{\Phi}_{g}, \hat{\Omega}(\hat{\Phi}_{g})) - l(\hat{\Phi}_{c}, \hat{\Omega}(\hat{\Phi}_{c})) \right] - \chi^2((N-1)r(m-r))
\] (14)

3 Based on individual country analyses, Fase and Winder (1993) already found considerable differences in the nature of the
demand for money.
To test for the validity of a homogeneous cointegrating space within each of the s groups, a likelihood ratio test is used. As in the case of testing for a homogeneous cointegrating space across all individuals in the panel, the common cointegrating rank is fixed, and the resulting distribution of the test statistic is again $\chi^2$ but with degrees of freedom equal to $\frac{N-s}{r(m-r)}$, i.e.

$$LR(\Phi_G | \Phi_B) = 2 \left[ l(\hat{\Phi}_G, \hat{\Omega}_G) - l(\hat{\Phi}_B, \hat{\Omega}_B) \right] \sim \chi^2((N-s)r(m-r)) \quad (16)$$

To calculate the maximised value of the likelihood function under specification (15), the procedure used by Groen and Kleibergen (2002) is adapted (see Appendix B.1. for the details). The above outlined test procedure is done for several group specifications. Table 4 shows the results of the tests.

### Table 4 Tests for group-wise homogeneous cointegration spaces

<table>
<thead>
<tr>
<th>Group specification(s)</th>
<th>Test statistic $\chi^2$(dgf)</th>
<th>1% critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(AU,GE), (BE,NL), (ES,PT), (FI,IER), (FR,IT)</td>
<td>152.01</td>
<td>$\chi^2(30)$ 50.89</td>
</tr>
<tr>
<td>(AU,GE), (BE,NL), (ES,PT,FR,IT), FI, IER</td>
<td>138.48</td>
<td>$\chi^2(30)$ 50.89</td>
</tr>
<tr>
<td>(BE,GE,NL,FR,IT), (ES,PT), (AU,FI), IER</td>
<td>213.19</td>
<td>$\chi^2(36)$ 58.62</td>
</tr>
<tr>
<td>(AU,GE,GE,NL), FR, IT, ES, FI, IER, PT</td>
<td>139.70</td>
<td>$\chi^2(18)$ 34.81</td>
</tr>
<tr>
<td>(AU,GE), (BE,NL), (ES,PT), FI, IER, FR, IT</td>
<td>116.66</td>
<td>$\chi^2(18)$ 34.81</td>
</tr>
<tr>
<td>(AU,GE), (ES,PT), FI, IER, FR, IT, BE, NL</td>
<td>78.58</td>
<td>$\chi^2(12)$ 26.22</td>
</tr>
</tbody>
</table>
As can be seen in Table 4, none of the tests shows evidence of a homogeneous cointegrating space across certain groups of eurozone countries. However as the main focus of this paper is the verification of the existence of a stable long run money demand equation across eurozone countries, the above tests can be viewed as too restrictive. Under the null of homogeneity of the cointegrating space for groups of or all eurozone countries, all three cointegrating vectors are equal. Now only one of these cointegrating vectors, say the first, represents the money demand equation. Thus when the only thing to be verified is the homogeneity of the money demand equation, testing for the homogeneity of all three cointegrating vectors can be argued too restrictive.

Before testing the homogeneity of the money demand equation, the cointegrating space has to be identified such that one of the cointegrating vectors can be said to represent a long-term money demand equation. The issue of identification however is more complicated than in the case of a time series setup, as several ‘problems’ arise in identifying the cointegrating space(s). In a ‘worst case scenario’ the cointegrating space for each individual country in the panel is identified in a different way, leaving a very large number of possible combinations of identifying restrictions. Can a common identification be accepted? If not, how to find the right identification for each of the cointegrating spaces? As one can imagine, the issue of identification is not that straightforward as it might seem at first hand.

3.1.2 Identifying the individual cointegrating spaces within a Panel VECM

To identify the cointegrating space, \((m-r)\)-restrictions have to be placed on each cointegrating vector. Econometrically any type of \((m-r)\)-restrictions can be chosen, however as in the time series case, economic theory may help to suggest some likely specifications. The Fisher equation, relating the interest rate to inflation and the expectations theory of the term structure, relating long and short term interest to each other, are examples of good candidates. In imposing the Fisher equation and/or the term structure more restrictions are imposed than actually needed to identify the cointegrating space. These restrictions are called overidentifying restrictions and their validity can be tested.

To (over)identify the cointegrating spaces, restrictions are imposed on each single vector of the \(A_i\)-matrices as in (5) resulting in the identified \(\tilde{A}_i\)-matrices:

\[
\tilde{A}_i = \left( V_{i1} \tilde{a}_{i1} \; V_{i2} \tilde{a}_{i2} \; \ldots \; V_{iq} \tilde{a}_{iq} \right) \quad i = 1, \ldots, N
\]  

where \(V_{iq}\), \(i = 1, \ldots, N\), \(q = 1, \ldots, r\), are the matrices that ‘contain’ the restriction(s) on the \(q^{th}\) cointegrating vector of the \(A\)-matrix of the \(i^{th}\) individual and \(\tilde{a}_{iq}\), \(i = 1, \ldots, N\), \(q = 1, \ldots, r\) are vectors
containing the coefficients of the restricted $q^{th}$ cointegrating vector of the $A$-matrix of individual $i$. For example, in our case with $k = 5$ and $r = 3$, the following specification of $V_q$ and $\tilde{a}_q$ would impose the restriction that the last two elements of the $q^{th}$ cointegrating vector of the $A$-matrix of the $i^{th}$ individual are equal to zero:

\[
V_q = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\quad \text{and} \quad
\tilde{a}_q = \begin{pmatrix}
a_{q,1} \\
a_{q,2} \\
a_{q,3}
\end{pmatrix}
\]

where $a_{q,1}$, $a_{q,2}$ and $a_{q,3}$ denote the first, second and third respective element of the cointegrating vector. Now under imposing all restrictions on all individual cointegrating spaces in the panel VECM, each individual $A_i$-matrix, the model would look like (5) with the $A_i$-matrices replaced by the definition in (17) in order to test for the validity of the overidentifying restrictions:

\[
LR(\Phi_{BI} \mid \Phi_B) = 2[\ell(\hat{\Phi}_B, \hat{\Omega}(\hat{\Phi}_B)) - \ell(\tilde{\Phi}_{BI}, \hat{\Omega}(\tilde{\Phi}_{BI}))] \sim \chi^2(\sum_{i=1}^{N} z_i)
\]

where $\Phi_{BI}$ is as $\Phi_B$ (5) but with the $\tilde{A}_i$-matrices instead of the $A_i$-matrices and $z_i$ is the number of overidentifying restrictions imposed on the $A$-matrix of individual $i$. The procedure to obtain the maximised likelihood is similar to the procedure used in section 3.1.1. Its details can be found in Appendix B.2.

To delimit the number of possible combinations of identifying restrictions, it is chosen to confine attention to combinations of (over-)identifying that adhere to the following conditions:

**Condition 1**

1.1 The first cointegrating vector represents a long run money demand equation.
1.2 The second cointegrating vector represents a Fisher equation.
1.3 The third cointegrating vector represents the expectations theory of the term structure.

A nice property of these conditions, besides delimiting the number of possible identifications, is that all three of them are based on economic theory, representing a money demand equation, Fisher equation and stationary term structure respectively. The money demand equation is the main focus of
this paper. The Fisher equation suggests a stable relation between nominal and real interest rate, i.e. the interest rate and inflation rate cointegrate. And the expectations theory of the term structure states that the relation between short and long run interest rates should be a stable, i.e. they cointegrate, ensuring no arbitrage profits in the short or long run. Note that the second and third condition each impose one overidentifying restriction. Ideally the same specification of the cointegrating space for all countries can be accepted. Table 5 below shows the results of tests for such a common specification.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Test statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m-p, y, i), (i, i), (i, Δp)</td>
<td>83.25</td>
</tr>
<tr>
<td>(m-p, y, i), (i, i), (i, Δp)</td>
<td>91.53</td>
</tr>
<tr>
<td>(m-p, y, Δp), (i, i), (i, Δp)</td>
<td>68.96</td>
</tr>
</tbody>
</table>

Note: all test statistics are $\chi^2(20)$-distributed; corresponding 1% critical value is 37.6.

As Table 5 clearly indicates, no common specification can be accepted that adheres to Condition 1. This complicates matters, as the number of possible specifications that allow every country or groups of countries to have their own specification, is quite large even under Condition 1. Dropping Condition 1 would complicate finding the right identification(s) even more as the number of possible combinations of (over)identifying restrictions would get very large.

To get an indication of possible differences in specification, a look at each country’s individual cointegrating space is taken. Assuming a cointegrating rank of three, a specification of the cointegrating space of an individual country is looked for. For most countries individually a specification cannot be rejected at a 1% level that adheres to Condition 1. However in the case of Spain and Finland this does not hold. Concerning the money demand equation, for all countries except Ireland the money demand equation relates real M3, real GDP and the long (or short) term interest rate to each other. For Ireland instead of one of the interest rates, inflation enters the money demand equation. Taking these results, found for the individual countries, as an indication, several different combinations of overidentifying restrictions are tested, still assuming Condition 1. Several different combinations of restrictions are tested. Table 6 shows the results.

---

4 Although formal tests for cointegrating rank indicate cross-country differences in cointegrating rank.
5 The results stated in this paragraph are available upon request. They merely serve as an indication that could be helpful in identifying the cointegrating spaces in the Panel VECM.
Table 6 Tests for possible combinations of overidentifying restrictions.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Identified Countries</th>
<th>Test statistic, $\chi^2$(df)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m-p,y,i), (i,y,il), (i,Ap)</td>
<td>AU,BE,GE,FR,IT,NL</td>
<td>29.23 [0.004], $\chi^2$(12)</td>
</tr>
<tr>
<td>(m-p,y,i), (i,y,il), (i,Ap)</td>
<td>AU,BE,GE,FR,IT,NL,IER</td>
<td>35.07 [0.002], $\chi^2$(14)</td>
</tr>
<tr>
<td>(m-p,y,i), (i,y,il), (i,Ap)</td>
<td>AU,BE,GE,FR,IT,NL,PT</td>
<td>41.59 [0.002], $\chi^2$(14)</td>
</tr>
<tr>
<td>(m-p,y,i), (i,y,il), (i,Ap)</td>
<td>AU,BE,GE,FR,IT,NL,IER(m-p,y,Ap)</td>
<td>35.01 [0.001], $\chi^2$(14)</td>
</tr>
<tr>
<td>(m-p,y,i), (i,y,il), (i,Ap)</td>
<td>AU,BE,GE,FR,IT,NL,ES,</td>
<td>48.03 [0.000], $\chi^2$(14)</td>
</tr>
<tr>
<td>(m-p,y,i), (i,y,il), (i,Ap)</td>
<td>AU,BE,GE,FR,IT,NL,PT,IER(m-p,y,Ap)</td>
<td>48.27 [0.000], $\chi^2$(16)</td>
</tr>
<tr>
<td>(m-p,y,i), (i,y,il), (i,Ap)</td>
<td>AU,BE,GE,FR,IT,NL,PT,IER</td>
<td>48.25 [0.000], $\chi^2$(16)</td>
</tr>
</tbody>
</table>

Note: p-values in brackets. Replacing the long term by the short term interest in the first and third cointegrating vector leads to similar results.

As can be seen in Table 6, even at a 1% level each of the specifications is clearly rejected. Finding a suitable specification of the cointegrating spaces (even when it is left unidentified for some countries), that allows for the identification of a long run money demand equation is not that easy within the Panel VECM framework.

However imposing equality of the long run money demand equation might still be accepted if tested for. Imposing the corresponding restrictions on the identified cointegrating spaces increases the number of degrees of freedom. This increase in the number of degrees of freedom might result in the acceptance of the null hypothesis if the imposing of the restrictions does not decrease the maximised value of the likelihood function too much. After having identified the first cointegrating vector as representing a possible long run money demand equation, the equivalence of this first cointegrating vector across individuals can be tested using a similar procedure as described in 3.1.1 to calculate the maximised value of the likelihood function under the null hypothesis.

3.1.3 Testing for homogeneity of the first cointegrating vector only

Assuming a cointegrating rank of r, the model, under the null of homogeneity of the first cointegrating equation across the individuals in each of the $s$, $1 \leq s < N$, subgroups ($s = 1$ denotes the case of

---

6 For simplicity attention is focused merely on testing for equivalence of the first cointegrating vector. The procedure as described here can easily be modified to allow for tests on other cointegrating vectors or groups of cointegrating vectors.
homogeneity of the first cointegrating equation across all individuals in the panel) assigned to one group, is similar to (15).

\[
\Delta Y_t = \begin{pmatrix}
B_1 & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & B_{n_1} \hat{A}'_1 \\
& & & \ddots & 0 \\
& & & 0 & B_{n_{s+1}} \hat{A}'_s \\
& & & & \ddots & 0 \\
& & & & 0 & B_{n} \hat{A}'_n \\
\end{pmatrix}
\]

Again the deterministic components and higher order dynamics are concentrated out before estimating (20). As in (17) a certain identification is needed to be able to say that a test for the equivalence of the first cointegrating vector is indeed a test for the equivalence of a meaningful long term relation. Therefore in (20) the \( \hat{A}_i \)-matrices are defined as,

\[
\hat{A}_i = \begin{pmatrix}
V_{i1} \hat{a}_{i1} & V_{i2} \hat{a}_{i2} & \ldots & V_{i} \hat{a}_{i} \\
\end{pmatrix}
\]

for all individuals \( j \) in group \( i, i \in \{1, \ldots, s\} \) (21)

The definition of the \( \hat{A}_i \)-matrices as in (21) allows for an individual specific 2\(^{nd}\), 3\(^{rd}\), \ldots, \( r^{th} \) cointegrating vector but restricts the first cointegrating vector to be equal for all individuals in a group. Note that in order to have the same first cointegrating vector for a groups of individuals, the imposed (over)identification on this vector in a specific group has to be the same for every individual in that group.

Testing for the validity of the homogeneity of the first cointegrating vector across groups can be done using a similar procedure as described for the case of testing for the homogeneity of the whole cointegrating space across groups. That is, after having calculated the maximum likelihood under the null hypothesis, i.e. specification (20) (see Appendix B.3. for details), the following test statistic can be constructed
\[
LR(\Phi_{\Theta_1} | \Phi_{\theta}) = 2\left[ l(\hat{\Phi}_{\theta}, \hat{\Omega}(\Phi_{\theta})) - l(\hat{\Phi}_{\Theta_1}, \hat{\Omega}(\hat{\Phi}_{\Theta_1})) \right] \sim \chi^2 \left( \sum_{j=1}^{s} (n_j - 1)f_j + \sum_{i=1}^{N} z_i \right) \tag{22}
\]

where \( \sum_{i=1}^{N} z_i \) are the number of restrictions imposed in order to overidentify the cointegrating spaces (equal to zero in case of no overidentifying restrictions) and for \( j = 1, \ldots, s \), \( n_j \) denotes the number of individuals in group \( j \) and \( f_j \) the number of elements of the first cointegrating vector restricted to be equal across the individuals in group \( j \).

Table 7a shows the test results obtained when testing for the equivalence of the first cointegrating vector, when the cointegrating spaces are identified as in the two cases with the highest p-value in Table 6.

Table 7a Testing for the equivalence of the long run money demand relation

<table>
<thead>
<tr>
<th>Specification</th>
<th>Identified Countries (same money demand)</th>
<th>Test statistic, ( \chi^2(dgf) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m-p,y,i), (i,i), (i,\Delta p)</td>
<td>(AU,BE,GE,FR,IT,NL)</td>
<td>130.34 [0.000], ( \chi^2(22) )</td>
</tr>
<tr>
<td>(m-p,y,i), (i,i), (i,\Delta p)</td>
<td>(AU,BE,GE,FR,IT,NL),IER(m-p,y,\Delta p)</td>
<td>131.31 [0.000], ( \chi^2(24) )</td>
</tr>
</tbody>
</table>

Note: p-values in brackets

Table 7b Testing for the equivalence of the term structure

<table>
<thead>
<tr>
<th>Specification</th>
<th>Identified Countries (same term structure)</th>
<th>Test statistic, ( \chi^2(dgf) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m-p,y,i), (i,i), (i,\Delta p)</td>
<td>(AU,BE,GE,FR,IT,NL)</td>
<td>34.88 [0.006], ( \chi^2(17) )</td>
</tr>
<tr>
<td>(m-p,y,i), (i,i), (i,\Delta p)</td>
<td>(AU,BE,GE,FR,IT,NL),IER(m-p,y,\Delta p)</td>
<td>38.46 [0.005], ( \chi^2(19) )</td>
</tr>
</tbody>
</table>

Note: p-values in brackets

The equivalence of the first cointegrating vector, representing a long run money demand equation, is clearly rejected in both cases. The two cases in Table 7b, where the equality of the cointegrating vector representing the term structure is tested across six countries, are shown to illustrate that the increase in degrees of freedom (by imposing more restrictions) can lead to an increase in the significance level at which the null hypothesis can be accepted (p-values). However, note that also in this case the null hypothesis cannot be accepted at a 1% level.
4 CONCLUSIONS

One of the ‘pillars’ of the ECB’s monetary policy aimed at the maintenance of price stability in the medium term is a prominent role for the amount of money held, as inflation is believed to be a monetary phenomenon in the long run. This prominent role for money is based on the assumption that a stable relation exists between money and inflation. This paper tries to verify the existence of such a stable relation and performs some tests on it using a Panel VECM framework.

Using this particular econometric framework leads to results that are a bit twofold. Using the GK-2002 test for common cointegrating rank clearly indicates a common cointegrating rank of three. The variables included in the analysis of the money demand equation (real M3, real GDP, the long and short term interest rates and inflation), share three long run relations. This finding reinforces the choice for a dynamic panel method that allows for a cointegrating rank larger than one. Applying a dynamic panel method that restricts the number of possible long run relations, e.g. a residual based testing procedure, seems inappropriate. Tests for the equivalence of the cointegrating spaces of the countries included in the Panel VECM clearly show that (group-wise) homogeneity of these cointegrating spaces is rejected. This indicates differences in the long run relationships between the countries (or groups of countries) in the eurozone.

However, this finding per se does not give an indication as to whether a long run money demand equation exists for each country and more specifically if these money demand relations are similar for some (groups of) countries in the eurozone. To be able to draw meaningful conclusions about long run money demand, the cointegrating spaces of each of the countries have to be identified in such a way that one of the three cointegrating vectors specifies a long run money demand equation. This identification poses several problems. The number of possible combinations of (over)identifying restrictions is quite large and testing only one combination already requires a substantial amount of computational time. To restrict the number of possibilities, attention is confined to specifications that are reasonable from an economic perspective (see Condition 1). This, however, does not result in the acceptance of a common specification across the countries of the eurozone. Also when leaving some countries’ cointegrating space unidentified and allowing for differing identifications per country, an identification of a long term money demand equation for each of the ‘identified’ countries can not be found. Not surprisingly a homogeneous money demand equation, which could have been accepted

Evidence from tests using ‘smaller’ panels consisting of subgroups of eurozone countries also shows difficulties identifying each of the individual cointegrating spaces. Only when focusing on ‘the D-mark’ area, consisting of Germany, Austria and the Netherlands, a (common) identification is found. Subsequent tests on the cointegrating vector representing the long run
due to the increase in degrees of freedom obtained by imposing this extra restriction, is clearly rejected.

The effect of national differences in financial structures and markets and country specific structural breaks seem to have an influence on the specific long run relations, shared by the five variables included in the analysis, for each eurozone member country respectively. However, as no specification for the cointegrating spaces is found that can be accepted when tested for, the Panel VECM framework does not (yet) provide results that are able to quantify these differences. This inability to provide quantitative results may be because the Panel VECM setup is too severely affected by country-specific misspecification (some countries’ individual time series analyses do not support the existence of three cointegrating relations, e.g. Spain and Finland) or/and country-specific data irregularities (e.g. Finland’s GDP, Italy’s M3)\(^8\), that seem to be more relevant for countries with a more tumultuous monetary history.

It is however tempting to say that cross-country differences do exist in the eurozone with respect to long run money demand, confirming earlier panel results by Golinelli and Pastorello (2000) and time series results by Fase and Winder (1993). Not being able to quantify these differences is a major drawback of the Panel VECM framework\(^9\), as it does not allow for specific comments and/or suggestions for policy makers to take account of these differences. Direct implications for the ECB’s monetary policy are difficult to indicate. Still the existence of differences in the long run relations among the eurozone countries in itself is of interest. Other (newer) dynamic nonstationary panel methods that can provide better quantitative results therefore constitute an interesting direction for future research. It would also be interesting to see whether these differences are disappearing over the years after the introduction of the euro. Due to the short period that the ECB has been in charge of monetary policy, the differences found in this study can be mere artefacts of the short period that the eurozone countries have shared a single monetary policy. To see whether or not these differences smooth over time is an interesting topic for future research to look at.

---

\(^8\) Allowing for country specific short term dynamics may be able to filter out the country specific effects more effectively, leaving only the ’core’ relations. This, and also allowing for country specific deterministic components (this might change the distribution of the test statistics\(^9\)), may lead to better quantitative results.

\(^9\) It may also be due to poor size/power properties of the test given the (relatively) large N-dimension and the number of variables included. However the problem of finding quantitative results remains valid also when using ‘smaller’ panels, only a panel consisting of ’D-mark’-countries (see footnote 9) provides quantitative results. This may be viewed as an indication that cross-country differences are a more important reason for the lack of quantitative results.
Also, and maybe even more important, the difference in the long term relations between the countries that comprise the eurozone, as shown by the differences in their cointegrating spaces, may have an effect on the results that are obtained by studies that do a time series analysis using aggregate eurozone data. Using aggregates implicitly ‘aggregates away’ country specific effects as their impact is smoothed by the method of aggregation itself. Some care has therefore to be taken when basing policy decisions on the results obtained from the aggregate analysis. Dedola et al. (2000) and Golinelli and Pastorello (2001) both claim, based on not very rigorous results from residual based testing procedures, that the results obtained from the area-wide aggregates are still useful. In case of Johansen type estimation procedures the impact of using aggregates, constructed from individual series that share different long run relations on the disaggregated level, is not yet clear. Identifying the impact of these differences is therefore another important issue to be looked at in future studies on eurozone money demand.
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APPENDIX A  THE DATA

Table A1 Country Codes

<table>
<thead>
<tr>
<th>Code</th>
<th>Country</th>
<th>Code</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>AU</td>
<td>Austria</td>
<td>GR</td>
<td>Greece</td>
</tr>
<tr>
<td>BE</td>
<td>Belgium</td>
<td>IER</td>
<td>Ireland</td>
</tr>
<tr>
<td>ES</td>
<td>Spain</td>
<td>IT</td>
<td>Italy</td>
</tr>
<tr>
<td>FI</td>
<td>Finland</td>
<td>NE</td>
<td>Netherlands</td>
</tr>
<tr>
<td>FR</td>
<td>France</td>
<td>PT</td>
<td>Portugal</td>
</tr>
<tr>
<td>GE</td>
<td>Germany</td>
<td>LU</td>
<td>Luxembourg</td>
</tr>
</tbody>
</table>

Data Sources

Nominal and real GDP

Table A2 Nominal and real GDP, data sources

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>Source</th>
<th>Seasonally Adjusted (SA) / Non-Seasonally Adjusted (NSA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1979.1 – 1979.4</td>
<td>ECB</td>
<td></td>
</tr>
<tr>
<td>FI</td>
<td>1979.1 – 2002.4</td>
<td>OECD, QNA 2003.1</td>
<td>SA</td>
</tr>
<tr>
<td>GE</td>
<td>1979.1 – 2002.4</td>
<td>ECB</td>
<td>SA, adjusted for unification</td>
</tr>
<tr>
<td>GR</td>
<td>1979.1 – 2002.4</td>
<td>ECB</td>
<td>SA</td>
</tr>
<tr>
<td></td>
<td>1979.1 – 1996.4</td>
<td>Economic Outlook 69</td>
<td></td>
</tr>
<tr>
<td>IT</td>
<td>1979.1 – 2002.4</td>
<td>OECD, QNA 2003.1</td>
<td>SA</td>
</tr>
<tr>
<td>NE</td>
<td>1979.1 – 2002.4</td>
<td>OECD, QNA 2003.1</td>
<td>SA</td>
</tr>
<tr>
<td>PT</td>
<td>1986.1 – 2002.4</td>
<td>OECD, QNA 2003.1</td>
<td>SA</td>
</tr>
<tr>
<td></td>
<td>1979.1 – 1985.4</td>
<td>Economic Outlook 69</td>
<td></td>
</tr>
<tr>
<td>LU</td>
<td>1985 – 2002</td>
<td>ECB</td>
<td>SA</td>
</tr>
<tr>
<td></td>
<td>1979 – 1984</td>
<td>Economic Outlook 72</td>
<td></td>
</tr>
</tbody>
</table>

Quarterly nominal and real GDP series in levels for each of the twelve Euro-countries. All series are in millions of euro using the irrevocable conversion rates as set by the ECB on December 31st 1998. Whenever two sources are given, the older series is multiplicatively added to the newer series, using the quarterly growth rates in the older series to 'update' the newer series, e.g. $x_{1985.4, new} = x_{1986.4, new} \times x_{1985.4, old} / x_{1985.4, old}$. As most available series were seasonally adjusted (SA), the series for Ireland and Austria, which were only available non seasonally adjusted (NSA), are seasonally adjusted using the multiplicative X12-ARIMA method. The multiplicative X12-ARIMA method is chosen as the ECB also uses this method for its published data. 1995 is taken as the base year for the GDP series. For
Luxembourg only yearly data are available so these data are made into quarterly data using the Lisman-method. The data obtained from Economic Outlook are semi-annual and are also transformed into quarterly data.

**Short term interest rates**
Quarterly short term interest rates expressed in yearly return for each of the twelve Euro-countries for the period 1979.1 – 2002.4. For 1979.1 – 2000.4 3-month interest rates for all countries are obtained from the BIS (Bank of International Settlements) database. For the period 2001.1 – 2002.4, the 3-month euro-deposit rate is used for all countries.

**Long term bond yields**
Quarterly long term bond yields expressed in yearly return for each of the twelve Euro-countries for the period 1979.1 – 2002.4. 10-year interest rates on government bonds (or a proxy) for 1979.1 – 2000.4 are obtained from the BIS database and for 2001.1 – 2002.4 these are taken from Datastream.

**M3**
M3 data for each of the twelve Euro-countries for the period 1979.1 – 2002.4. A monthly series for each country is obtained from the European Central Bank (ECB) database. The series are in millions of euro using the irrevocable conversion rates of December 31st 1998. The monthly series are converted into quarterly series taking the average of the three corresponding months for each quarter. Then the quarterly series are seasonally adjusted using the multiplicative X12-ARIMA method (see Findley et al. 1998). The German series is corrected for the effect of the unification by first regressing the growth rates on a dummy for 1990.3 (see Fagan and Henry, 1998; Beyer et al., 2001; Wesche, 1997). The effect was found to be 8.1% with a standard error of 0.98. Using this estimated effect the growth rate of 1990.3 is corrected by subtracting the unification effect. These corrected growth rates are then used to calculate back the series in loglevels (and levels).

**Exchange rates**
Exchange rates of each of the twelve Euro-countries against the ECU/EURO for the period 1979.1 – 2002.4. A monthly series is obtained from the Dutch Central Bank – FM database for each country. These series are converted into quarterly series taking the average of the three corresponding months for each respective quarter. For the period 1999.1 – 2002.4 the irrevocable conversion rates as set by the ECB are used (for Greece from 2001.1 on).
Figure A1.1 Quarterly real M3 ($m^d-p$) series in loglevels

Figure A1.2 Quarterly real GDP ($y$) in loglevels

Note: going from upper left to lower right the graphs represent the series for country AU, BE, ES, FI, FR, GE, GR, IER; IT, LU, NE, PT
Figure A1.3 Quarterly inflation $\Delta p$ in yearly returns (growth rate GDP deflator)

Note: going from upper left to lower right the graphs represent the series for country
AU, BE, ES, FI, FR, GE, GR, IER; IT, LU, NE, PT

Figure A1.4 Quarterly short term interest rate in yearly returns

Note: going from upper left to lower right the graphs represent the series for country
AU, BE, ES, FI, FR, GE, GR, IER; IT, LU, NE, PT
Figure A1.5 Quarterly long term bond yield in yearly returns

Note: going from upper left to lower right the graphs represent the series for country AU, BE, ES, FI, FR, GE, GR, IER; IT, LU, NE, PT

Figure A1.6 Quarterly exchange rates in local currency per euro

Note: going from upper left to lower right the graphs represent the series for country AU, BE, GE, FI, FR, ES, GR, IER; IT, LU, NE, PT
APPENDIX B  MAXIMUM LIKELIHOOD PROCEDURES

To be able to calculate the various test statistics that can be looked at in a panel VECM framework, the procedure used by Groen and Kleibergen (2002) to calculate the maximum likelihood in case \( \Theta \in \{A, B, C\} \), see pp.10-15 of their paper, is adapted. Case B.1. – B.3 below describe how this procedure has to be adapted in order to obtain the maximum likelihood in those cases.

B.1. Homogeneous cointegrating space for predefined groups of panel members

The log-likelihood function for any of the panel VECM specification, \( \Phi_\Theta, \Theta \in \{A, B, C\} \), is:

\[
I(\Phi_\Theta, \Omega) = -\frac{NkT}{2} \ln(2\pi) - \frac{T}{2} \ln|\Omega| - \frac{1}{2} \text{tr}(\Omega^{-1}(\Delta Y - Y_{\cdot i}\Phi_\Theta')(\Delta Y - Y_{\cdot i}\Phi_\Theta))
\]

where \( \Delta Y = \begin{pmatrix} \Delta Y'_1 \\ \vdots \\ \Delta Y'_{N_{\cdot i}} \end{pmatrix} \) and \( Y_{\cdot i} = \begin{pmatrix} Y'_0 \\ \vdots \\ Y'_{T-1} \end{pmatrix} \) are \((T \times Nm)\) matrices. In their paper, Groen and Kleibergen (2002) show that the log-likelihood function in (b.1) can be rewritten as

\[
I(\Phi_\Theta, \Omega) = -\frac{NkT}{2} \ln(2\pi) - \frac{T}{2} \ln|\Omega| - \frac{1}{2} \text{tr}(\Omega^{-1}\Delta Y'M_{\cdot i}\Delta Y) - \frac{1}{2} G(\Phi_\Theta, \Omega),
\]

where \( M_{\cdot i} = I - Y_{\cdot i}(Y'_{\cdot i}Y_{\cdot i})^{-1}Y'_{\cdot i} \) and

\[
G(\Phi_\Theta, \Omega) = \text{vec}(Y'_{\cdot i}e)'(\Omega \otimes Y'_{\cdot i}Y_{\cdot i})^{-1}\text{vec}(Y'_{\cdot i}e)
\]

which can be interpreted as a GMM objective function with all variables in \( Y_{\cdot i} \) acting as instruments. Now in (b.2) \( \Omega \) can be estimated conditional on \( \Phi_\Theta \) using the conditional maximum likelihood estimator of \( \Omega \) given \( \Phi_\Theta \), i.e.

\[
\hat{\Omega}(\Phi_\Theta) = \frac{1}{T}(\Delta Y - Y_{\cdot i}\Phi_\Theta')(\Delta Y - Y_{\cdot i}\Phi_\Theta)
\]

and \( \Phi_\Theta \) can be estimated given \( \Omega \) using GMM objective function (b.3). By sequentially applying this estimation procedure of \( \Omega \) and \( \Phi_\Theta \) until convergence, the log-likelihood function (b.1) is maximised.
As the cases for which $\Theta = A$, B or C are explained in detail in Groen and Kleibergen (2002), p.10-15, here only the case $\Theta = G$ is explained in more detail. The group specification (15) implies that the GMM objective function (b.3) can be written as:

$$G(\Phi_G, \Omega) = \text{vec}(Y_{11}'(\Delta Y - Y_{11}\Phi_G'))(\Omega \otimes Y_{11}'Y_{11})^{-1}\text{vec}(Y_{11}'(\Delta Y - Y_{11}\Phi_G'))$$ (b.5)

Rewrite part of this objective function (b.5) as,

$$\text{vec}(Y_{11}'(\Delta Y - Y_{11}\Phi_G')) = \text{vec}(Y_{11}'\Delta Y) - \text{vec}(Y_{11}'A_1B_1'\ldots Y_{11}'A_iB_i'\ldots Y_{11}'A_nB_n'\ldots, Y_{11}'A_iB_i'\ldots, Y_{11}'Y_{11}A_iB_i'\ldots, Y_{11}'Y_{11}A_iB_i'\ldots)$$

$$= \text{vec}(Y_{11}'\Delta Y) - R \left( \begin{array}{c} \text{vec}(A_1) \\ \text{vec}(A_2) \\ \vdots \\ \text{vec}(A_n) \end{array} \right)$$ (b.6)

where

$$R = (I_{Nm} \otimes Y_{11}'Y_{11})R_{\text{sure}}$$ (b.7)

and

$$R_{\text{sure}} = \begin{pmatrix}
B_1 \otimes (e_1 \otimes I_m) & 0 & 0 \\
\vdots & \ddots & \vdots \\
B_n \otimes (e_n \otimes I_m) & 0 & 0 \\
0 & B_{n+1} \otimes (e_{n+1} \otimes I_m) & 0 \\
0 & B_{n+n_2} \otimes (e_{n+n_2} \otimes I_m) & \ddots \\
\vdots & \ddots & \ddots \\
0 & 0 & B_{n+n+n_{11}} \otimes (e_{n+n+n_{11}} \otimes I_m) \\
0 & 0 & B_{n+n+n_{11}} \otimes (e_{n+n+n_{11}} \otimes I_m)
\end{pmatrix}$$ (b.8)

where $e_i$ is the $i^{th}$ N-dimensional unity vector and $I_m$ a $(m \times m)$ identity matrix.
In Groen and Kleibergen (2002) it is shown that under the null the common cointegrating rank is fixed at \( r \) and the estimate of the covariance matrix in (b.4) yields a consistent estimate of \( \Omega \). Furthermore under normalisation of the cointegrating spaces \( A_1, \ldots, A_s \), a consistent estimator of \( B_i \) is:

\[
\hat{B}_i = \text{the first } r \text{ columns of } \Phi_i \text{ from (15) for } i = 1, \ldots, N \tag{b.9}
\]

These estimates, \( \hat{B}_i, i = 1, \ldots, N \) and \( \hat{\Omega} = \hat{\Omega}(\hat{\Phi}_G) \), can then be substituted in the GMM objective function (b.5). Minimising this objective function with respect to \( A_i, \ldots, A_s \) gives the following GMM-estimates,

\[
\begin{pmatrix}
\text{vec}(\hat{A}_1) \\
\vdots \\
\text{vec}(\hat{A}_s)
\end{pmatrix} = (\hat{R}'(\hat{\Omega} \otimes Y'_i Y_i)^{-1}\hat{R})^{-1}\hat{R}'(\hat{\Omega} \otimes Y'_i Y_i)^{-1}\text{vec}(Y'_i \Delta Y)
\]

\[
= (\hat{R}'_{\text{sure}}(\hat{\Omega}^{-1} \otimes Y'_i Y_i)^{-1}\hat{R}')^{-1}\hat{R}'_{\text{sure}}(\hat{\Omega}^{-1} \otimes I_T)\text{vec}(Y'_i \Delta Y) \tag{b.10}
\]

The next step is to use the estimates \( \hat{B}_i, i = 1, \ldots, N \) and \( \hat{A}_1, \ldots, \hat{A}_s \) to construct the estimate of \( \hat{\Phi}_G \) and use that in estimating the conditional maximum likelihood estimator (b.4) to obtain \( \hat{\Omega} = \hat{\Omega}(\hat{\Phi}_G) \). Now the estimates of the individual specific loading factors can be made by minimising the GMM objective function (b.5) with respect to \( B_i, i = 1, \ldots, N \) conditional on the estimates \( \hat{A}_1, \ldots, \hat{A}_s \) and \( \hat{\Omega} = \hat{\Omega}(\hat{\Phi}_G) \). This results in the following estimates:

\[
\begin{pmatrix}
\text{vec}(\hat{B}'_i) \\
\vdots \\
\text{vec}(\hat{B}'_s)
\end{pmatrix} = (\hat{\Gamma}'_{\text{R,sure}}(\hat{\Omega}^{-1} \otimes Y'_i Y_i)^{-1}\hat{\Gamma}')^{-1}\hat{\Gamma}'_{\text{R,sure}}(\hat{\Omega}^{-1} \otimes I_T)\text{vec}(Y'_i \Delta Y)
\]

\[
= (\hat{\Gamma}'_{R,\text{sure}}(\hat{\Omega}^{-1} \otimes Y'_i Y_i)^{-1}\hat{\Gamma}'_{R,\text{sure}})(\hat{\Omega}^{-1} \otimes I_T)\text{vec}(Y'_i \Delta Y) \tag{b.11}
\]

where

\[
\Gamma_{R,\text{sure}} = ((e_1 \otimes I_m) \otimes (e_1 \otimes \hat{A}_1) \ldots (e_n \otimes I_m) \otimes (e_n \otimes \hat{A}_1) \ldots (e_n \otimes I_m) \otimes (e_n \otimes \hat{A}_s) \otimes (e_1 \otimes I_m) \otimes (e_1 \otimes \hat{A}_s) \ldots (e_n \otimes \hat{A}_s)) \tag{b.12}
\]
and

$$\Gamma_R = \left( I_{Nm} \otimes Y'_rY_q \right) \Gamma_{R,sure}. \tag{b.13}$$

Now through iteratively applying the estimators (b.10), (b.4) and (b.11), one obtains the maximum likelihood estimates of $B_i, i = 1, \ldots, N$, $\Lambda_1, \ldots, \Lambda_s$ and $\Omega$ and, based on these estimates, the value of the likelihood function (b.1). Each iteration step results in an improvement of the likelihood and after convergence of the likelihood function to its maximum, the maximum likelihood estimates are obtained. Finally, using the maximum value of the likelihood function, the test statistic can be calculated using (16).

B.2. (Over)identifying the cointegrating spaces

The maximum likelihood procedure as described in the previous section can now be used to calculate the maximised value of the likelihood function under the restrictions imposed when (over)identifying the cointegrating spaces and to provide estimates of the coefficients. However some small changes have to be made in the calculation procedure as in section B.1. to take account of the imposed restrictions. The matrices $R$ (b.7) and $R_{sure}$ (b.8) matrices have to be replaced by $E_{restricted}$ and $E_{sure,restricted}$ respectively, where

$$E_{sure,restricted} = \left( (e_1 \otimes B_1) \otimes (e_1 \otimes I_m) \ldots (e_N \otimes B_N) \otimes (e_N \otimes I_m) \right) V \tag{b.14}$$

where

$$V = \begin{pmatrix}
V_{11} & \emptyset & \ldots & \emptyset \\
\emptyset & \ddots & \ldots & \emptyset \\
\emptyset & \ldots & V_{1N} & \emptyset \\
\emptyset & \ldots & \emptyset & V_{rN}
\end{pmatrix} \tag{b.15}$$

with $V_{qi}, i = 1, \ldots, N$, $q = 1, \ldots, r$, are the matrices that contain the restriction(s) on the $q^{th}$ cointegrating vector of the A-matrix of the $i^{th}$ individual, and

$$E_{restricted} = \left( I_{Nm} \otimes Y'_rY_q \right) E_{sure,restricted} \tag{b.16}$$
Using these two matrices, the elements of the restricted cointegrating vectors can be estimated by the following formula, replacing (b.10) in Appendix B.1:

\[
\begin{pmatrix}
\hat{a}_{11} \\
\vdots \\
\hat{a}_{r1} \\
\vdots \\
\hat{a}_{1N} \\
\vdots \\
\hat{a}_{rN}
\end{pmatrix}
= (\hat{E}'_{\text{sure, restricted}} (\hat{\Omega}^{-1} \otimes Y'_{1} Y_{1}^{-1}) \hat{E}'_{\text{sure, restricted}} (\hat{\Omega}^{-1} \otimes I_{N_{m}}) \text{vec}(Y'_{1} \Delta Y))
\]

Having obtained the above estimates, the $\hat{A}$-matrices can be constructed and subsequently used in the rest of the procedure, which remains the same as in Appendix B.1. After convergence of the likelihood function, estimates of the coefficients and the maximised likelihood value are obtained. In the case of testing for overidentifying restriction this maximised likelihood value can be used to calculate the likelihood ratio test statistic as in (19).

**B.3. Testing for the homogeneity of the first cointegrating vector only**

Again the way to calculate the maximum likelihood under the null hypothesis in this case is similar to the procedure described previously in case of the null of homogeneity of the cointegrating space across groups. The only difference being that the matrices $R$ (b.7) and $R_{\text{sure}}$ (b.8) are replaced by $K$ and $K_{\text{sure}}$ respectively, where $K$ and $K_{\text{sure}}$ are defined as follows:

\[
K_{\text{sure}} = \begin{pmatrix}
\tilde{B}_{1,1m} & 0 & \cdots & 0 \\
0 & \ddots & \ddots & 0 \\
\vdots & & \ddots & \ddots \\
0 & 0 & \ddots & \tilde{B}_{n_{1}+1,1m}
\end{pmatrix}
+ \begin{pmatrix}
e_{1} \otimes \tilde{B}_{1,m+1,\ell_{m}} & \cdots & e_{N} \otimes \tilde{B}_{N,m+1,\ell_{m}}
\end{pmatrix}
\]

(b.18)
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