Note on zero lower bound worries
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Abstract

Although some authors have suggested that monetary expansion is still possible when the monetary policy interest rate cannot be reduced further, central banks tend to avoid interest rates close to the zero lower bound. Taking into account central banks’ aversion to very low interest rates, we investigate optimal monetary policy in a New-Keynesian macro-economic model. Our analysis shows that appointing a central banker with a high aversion to low interest rate levels can mitigate the zero bound risk at the cost of persistent inflation deviations from target. If fear of the zero lower bound is unfounded, it is better to appoint a central banker with no aversion to the zero lower bound, who will not shy away from unorthodox policies when the policy interest rate cannot be reduced further.

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1 Introduction

Central banks tend to avoid reducing policy interest rates to very low levels. They prefer a safety margin above the zero lower bound, where further monetary policy expansion by reducing interest rates is ruled out. Nevertheless, the Bank of Japan has kept the policy interest rate at or very close at the zero lower bound since 1999. Also the central banks of the US, the euro area and Switzerland lowered policy interest rates to unprecedented levels in the early 2000s. These developments have triggered a debate on whether monetary policy really is impotent at the zero bound, as most stylized macro-economic models suggest. Bernanke and Reinhart (2004) argue that unorthodox policies will allow further monetary expansion when policy interest rates cannot be reduced further, for instance through unconditional commitment to zero policy interest rates over long horizons.

Central banks’ aversion to very low interest rates may influence the choice of the inflation target and other targets of monetary policy. We investigate how a central bank should set its policy targets under the assumption that hitting the zero lower bound exhausts the monetary policy expansion possibilities. In addition, we investigate the costs of zero lower bound fears if this bound does not pose real obstacles to further monetary expansion. The analysis is done in a New-Keynesian framework along the lines of Clarida et al. (1999), which is today’s workhorse model for examining monetary policy issues. We incorporate the policy maker’s aversion to the zero lower bound in the loss function. We solve for the optimal monetary policy, both under discretion and under commitment from a time-less perspective.

This introduction is followed by a presentation of the model in Section 2. Section 3 shows how persistent deviations from the inflation target may arise. Section 4 concludes.

2 Model

The central bank minimizes a loss function conditional on the specification of the economy, represented by a standard New-Keynesian expectational IS and Phillips curve. The loss function is modified in order to reflect the central bank’s aversion to interest rate levels where the room for manoeuvre is limited because of the zero lower bound.

We take the monetary policy maker’s aversion to the zero lower bound
explicitly into account in the central bank loss function $L$ at time $t$:

$$
L_t = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[ \alpha x_{t+j}^2 + (\pi_{t+j} - \pi^*)^2 + \gamma (i_{t+j} - i^*)^2 \right] \right\},
$$

where $x$ is the output gap, $\pi$ is the inflation rate, $i$ is the nominal interest rate, $\alpha (\geq 0)$ is the weight on the output gap, $\beta$ is the discount rate, $\gamma$ is the weight on interest rate deviations from target, and $^*$ refers to a non-negative policy target level. We add the $\gamma (i_{t+j} - i^*)^2$ term, where $\gamma \geq 0$, to an otherwise standard loss function.\(^1\) The central bank tries to close the output gap ($x^* = 0$) and to stabilize inflation around $\pi^*$. The level of the inflation target is determined exogenously. For instance, it can be the inflation rate that equals the social marginal cost of producing money (close to zero) to the private opportunity cost of holding money ($i_t$), as suggested by Friedman (1969). Or it can be based on a desire to allow for real wage declines without nominal wage cuts, as emphasized by Akerlof et al. (1996). In line with Woodford (2003), the $\gamma (i_{t+j} - i^*)^2$ term indicates that the monetary policy maker wants to avoid large deviations from the positive interest rate target. In order to preserve room for manoeuvre in the interest rate, the policy maker is also assumed to prefer a nominal interest rate clearly above the zero lower bound. This is taken into account in determining $i^*$.

The $\gamma (i_{t+j} - i^*)^2$ term is included to keep the monetary policy interest rate at safe distance from the zero lower bound, where the monetary policymaker can no longer boost the economy by reducing the policy interest rate. From a model builder’s perspective taking away the zero lower bound as a relevant restriction simplifies the analysis of optimal policy considerably.

The inclusion of the $\gamma (i_{t+j} - i^*)^2$ term can also be justified when the zero lower bound does not pose a serious obstacle to monetary policy expansion. Even in the very stylized model presented below it is imaginable that the central bank commits to maintaining a zero policy interest rate over long horizons irrespective of output and inflation outcomes. In the real world, the central bank has alternative means of providing monetary stimulus when the zero bound is binding, such as increasing the size of the central bank balance sheet beyond what is needed to set the nominal interest rate at zero (Bernanke and Reinhart, 2004). Even in such an environment monetary policymakers may want to avoid very low interest rates, since this

\(^1\)Note that our loss function nests the commonly used loss function, which is an approximation of the socially optimal loss function (Woodford, 2003), as a special case ($\gamma = 0$).
would bring them in uncharted waters. The calibration and communication of policy actions under these circumstances would be very complicated. For instance, large-scale policy expansion resulting from a long-term commitment to zero interest rates will restrict the central bank’s flexibility, and a liquidity overhang may emerge with possible long-term inflationary consequences. In this context, Eggertsson (2003) assumes that the central bank is too risk averse to accept strong monetary policy intervention. Finally, coordination problems between monetary and political authorities may prevent powerful monetary policy action. Svensson’s (2001) foolproof way to escape from a liquidity trap is a case in point, since it requires substantial foreign exchange intervention, which is usually the responsibility of the political authorities.

The demand side is represented by a linear approximation of the household’s Euler equation for optimal consumption. This expectational IS curve is:

\[ x_t = E_t x_{t+1} - \varphi (i_t - E_t \pi_{t+1} - \bar{\pi}) + g_t, \quad (2) \]

where \( \varphi > 0 \), \( r \) is the real interest rate \( (r_t = i_t - E_t \pi_{t+1}) \), \( g \) is a disturbance term with zero mean and variance \( \sigma_g^2 \), and an upper bar refers to a steady state level. \( \bar{\pi} \) is determined exogenously and assumed to be non-negative. The supply side is derived under the assumption of monopolistic competition with staggered price setting, which results in a Phillips curve:

\[ \pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t, \quad (3) \]

where \( \lambda > 0 \) and \( u \) is a disturbance term with zero mean and variance \( \sigma_u^2 \).

### 3 Bias

The central bank chooses a time path for the instrument \( i \) to steer \( x \) and \( \pi \) in order to minimize the loss function eq. 1, subject to the constraints implied by eqs. 2 and 3. We solve this problem both under discretion and under commitment from a time-less perspective. Solving under discretion is common in the inflation bias literature. Solving under commitment from a time-less perspective puts aside the problems related to time-inconsistency. It offers a good approximation of current best practices in central banking.

Under discretion, it is assumed that the central bank takes private sector expectations as given. The central bank and the public play a one-shot
game without private information. The first order condition to this problem defines the central bank’s reaction function:

\[ \pi_t - \pi^* = -\frac{\alpha}{\lambda} x_t + \frac{\gamma}{\varphi\lambda} (i_t - i^*) . \] (4)

Inserting this condition into eqs. 2 and 3 yields:

\[ \pi = 0, \] (5)

\[ \pi = -\frac{\gamma}{\varphi\lambda - \gamma} (i^* - \bar{\pi}) + \frac{\varphi\lambda}{\varphi\lambda - \gamma} \pi^* , \] (6)

where \((i^* - \bar{\pi})\) is the inflation target that would be consistent with the desired safety margin for the interest rate \((\pi^{**} = i^* - \bar{\pi})\). Since in the long run there is no trade-off between the real target \((x^* = 0)\) and the nominal targets \((\pi^* \text{ and } i^*)\), output remains at potential irrespective of the level of \(\gamma\). As expectations materialize in the steady state, the private sector uses eq. 6 to forecast inflation. It follows that \(\bar{\pi} < \pi^* \text{ if } \pi^* < \pi^{**}\), provided that \(\gamma < \varphi\lambda\).

Under commitment from a time-less perspective, it is assumed that the central bank can credibly commit itself to policy targets. The time-less perspective implies that such a policy has been chosen from the distant past onwards. The first order condition to this problem is:

\[ \pi_t - \pi^* = -\frac{\gamma}{\beta} (i_t - i^*) . \] (7)

This results in the following steady state values for output and inflation:

\[ \bar{\pi} = 0, \] (8)

\[ \bar{\pi} = \frac{\gamma}{\beta + \gamma} (i^* - \bar{\pi}) + \frac{\beta}{\beta + \gamma} \pi^* . \] (9)

This implies that \(\pi^* < \bar{\pi} < \pi^{**}\) if \(\pi^* < \pi^{**}\).

In words, assuming modest emphasis on room for manoeuvre for the interest rate \((\gamma < \varphi\lambda)\), inflation will persistently undershoot the level consistent with the desired safety margin for the interest rate (where the inflation target equals \(i^* - \bar{\pi}\)) if the inflation target is set below this level. This is true both under discretion and under commitment from a time-less perspective. The low level of the inflation target increases the risk of hitting the zero lower bound. Similarly, inflation will systematically miss the target if this target

\(^2\text{See the Appendix for derivations of eqs. 4 and 7.}\)
deviates from the inflation rate consistent with zero lower bound aversion. Note that an inflation/deflation bias emerges not only under discretion (as is the case with the inflation bias in the time-inconsistency literature), but also under commitment from a time-less perspective. This is because the driver of the deflation bias is mutually inconsistent targets for inflation and the nominal interest rate, given $r$. Since there is just one policy instrument, the central bank is not able to meet its mutual targets simultaneously, and tries to mitigate inevitable deviations from target levels.

4 Concluding remarks

It follows from the above that zero lower bound considerations should determine the level of the inflation target if the zero lower bound poses a serious impediment to monetary policy effectiveness. In particular, choosing an excessively low inflation target would result in a deflation bias and a higher risk of ending up in a zero lower bound situation. Under these circumstances, the central bank may be only one downward shock away from a binding zero lower bound. Appointing a central banker with strong aversion to the zero lower bound can mitigate the bias. On the other hand, if the zero lower bound poses no real obstacle to monetary policy effectiveness, the best solution is to appoint a central banker who has no preference for a particular interest rate level and who will not shy away from unorthodox monetary policy measures if necessary. In case the central bank would want to preserve room for manoeuvre above the zero lower bound even when monetary expansion is still possible in this situation, deviations from the inflation target can also be avoided by choosing an inflation target in line with the desire to preserve room for manoeuvre above the zero lower bound.
5 Appendix

This Appendix shows the derivations of eqs. 4 and 7 along the lines of Woodford (2003).

Under discretion the central bank is not in a position to influence the private sector’s expectations, and optimization is a single-period problem. The Lagrangian can be written as:

$$Z_t = \frac{1}{2} \left( \alpha x_t^2 + (\pi_t - \pi^*)^2 + \gamma (i_t - i^*)^2 \right)$$  \hspace{1cm} (10)

$$\quad + \mu_1 (x_t - E_t x_{t+1} + \varphi (i_t - E_t \pi_{t+1} - \pi))$$  \hspace{1cm} (11)

$$\quad + \mu_2 (\pi_t - \lambda x_t - \beta E_t \pi_{t+1}),$$  \hspace{1cm} (12)

where $\mu_1$ and $\mu_2$ are Lagrange multipliers. It follows that:

$$\frac{\partial Z_t}{\partial x_t} = \alpha x_t + \mu_1 - \mu_2 \lambda = 0;$$  \hspace{1cm} (13)

$$\frac{\partial Z_t}{\partial \pi_t} = (\pi_t - \pi^*) + \mu_2 = 0;$$  \hspace{1cm} (14)

$$\frac{\partial Z_t}{\partial i_t} = \gamma (i_t - i^*) + \mu_1 \varphi = 0.$$  \hspace{1cm} (15)

Combining eqs. 13, 14 and 15 yields eq. 4.

The baseline case, with $\gamma = 0$, is:

$$\pi = \pi^*.$$  \hspace{1cm} (16)

With $\gamma > 0$ eqs. 5 and 6 result.

Under commitment from a time-less perspective the Lagrangian can be written as:

$$Z_t = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left( \frac{1}{2} \left[ \alpha x_{t+j}^2 + (\pi_{t+j} - \pi^*)^2 + \gamma (i_{t+j} - i^*)^2 \right] \right) + \mu_{1,t+j} (x_{t+j} - x_{t+j+1} + \varphi (i_{t+j} - \pi_{t+j+1} - \pi) \right)$$

$$\quad + \mu_{2,t+j} (\pi_{t+j} - \lambda x_{t+j} - \beta \pi_{t+j+1}) - \beta^{-1} \mu_{1,t-1} (x_0 + \varphi \pi_0) - \mu_{2,t-1} \pi_0 \right) \right\},$$  \hspace{1cm} (17)

It follows that:

$$\frac{\partial Z_t}{\partial x_t} = \alpha x_t + \mu_{1,t} - \beta^{-1} \mu_{1,t-1} - \mu_2 \lambda = 0;$$  \hspace{1cm} (18)
\[
\frac{\partial Z_t}{\partial \pi_t} = (\pi_t - \pi^*) - \beta^{-1}\varphi \mu_{1,t-1} + \mu_2 - \mu_2, t-1 = 0, \tag{19}
\]

\[
\frac{\partial Z_t}{\partial i_t} = \gamma (i_t - i^*) + \mu_4 \varphi = 0. \tag{20}
\]

There is only a solution with the output gap and the interest rate being constant over time if the Lagrange multipliers are also constant over time. Taking this into account, combining eqs. 18, 19 and 20 yields eq. 7.

The baseline case, with \( \gamma = 0 \), is:

\[
\bar{\pi} = \pi^*. \tag{21}
\]

With \( \gamma > 0 \) eqs. 8 and 9 result.
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