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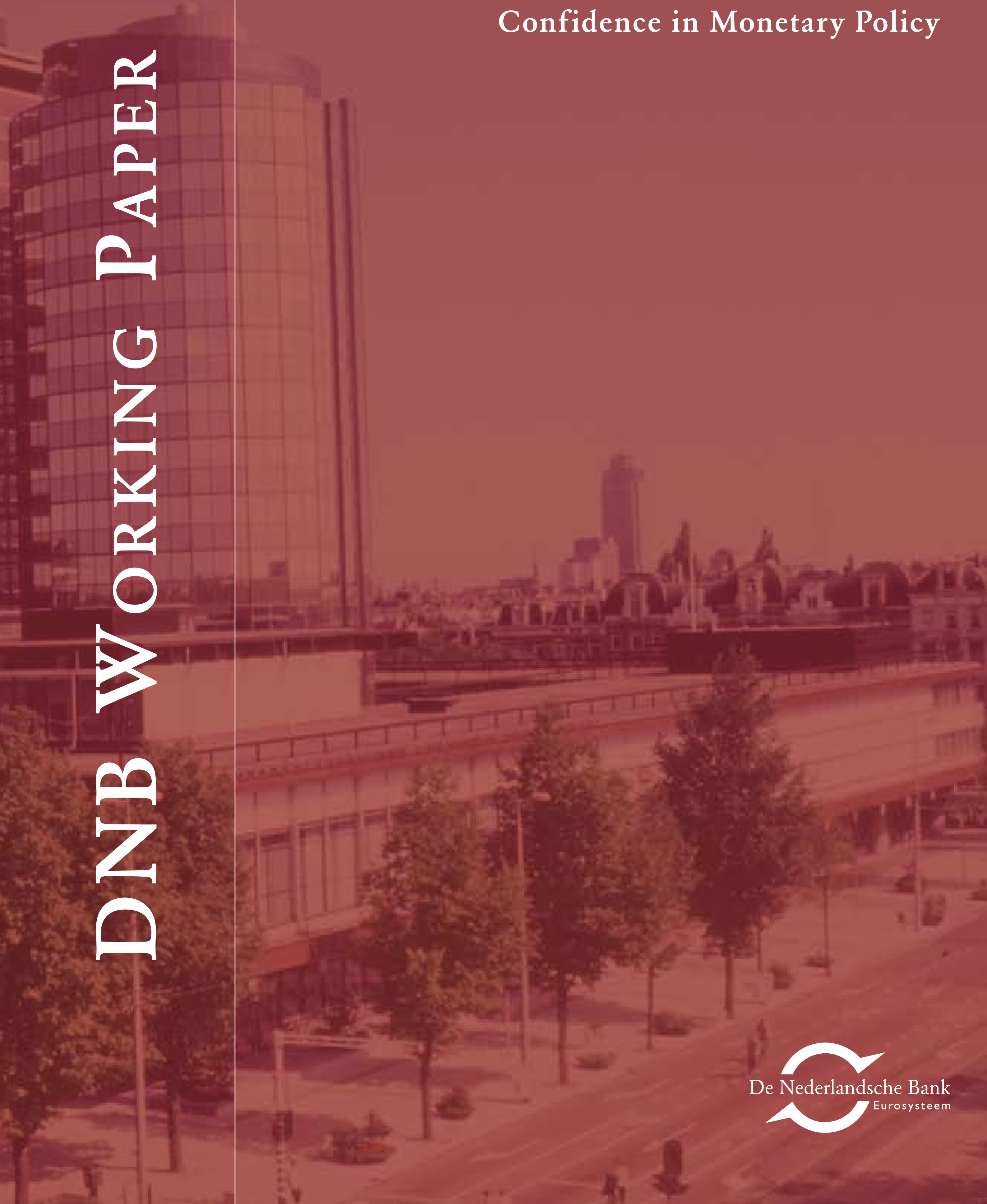

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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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Confidence in Monetary Policy*

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Abstract

In situations of relative calm and certainty, policy makers have confidence in the mechanisms at work and feel capable of attaining precise and ambitious results. As the environment becomes less and less certain, policy makers are confronted with the fact that there is a trade-off between the quality of a certain outcome and the confidence (robustness) with which it can be attained. Added to that, in the presence of Knightian uncertainty, confidence itself can no longer be represented in probabilistic terms (because probabilities are unknown). We adopt the technique of *Info-Gap Robust Satisficing* to first define confidence under Knightian uncertainty, and second quantify the trade-off between quality and robustness explicitly. We apply this to a standard monetary policy example and provide Central Banks with a framework to rank policies in a way that will allow them to pick the one that either maximizes confidence given an acceptable level of performance, or alternatively, optimizes performance for a given level of confidence.

Keywords: Knightian Uncertainty, Satisficing, Bounded Rationality, Minmax
J.E.L Codes: D81, E52, E58

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1 Introduction

"Optimization works in theory but risk management is better in practice. There is no scientific way to compute an optimal path for monetary policy".

Alan Greenspan, Financial Times, 27-28 August 2005.

Policy makers are concerned with achieving outcomes that are both fundamentally good (high quality) as well as predictable (reliable). In situations of relative calm and certainty, policy makers have confidence in the mechanisms at work and feel capable of attaining precise and ambitious results. They know what is feasible and therefore concern themselves with simply choosing the best possible, from a well understood set of feasible outcomes. By contrast, decisions in a world characterized by uncertainty are by definition based on models which are faulty and on data which are inaccurate and incomplete. This implies that in such circumstances, it is not as easy to achieve outcomes that are both good as well as reliable. In fact the existence of uncertainty generates a trade-off between the two, such that good outcomes come at the cost of predictability, or the other way round, predictable outcomes come at the cost of quality. We argue that good decision strategies designed to deal with uncertainties must allow for this dual nature of policy makers' concerns. The main objective of this paper is to provide a framework that explicitly addresses the trade-off between confidence/robustness and good outcomes.

Before we quantify the trade-off however, we need to define how confidence is represented under uncertainty. To do so, we first draw a distinction between risk and uncertainty as defined by Knight ([1921], 1965), who referred to *risk* when the underlying probabilities are known and to (Knightian) *uncertainty* when they are not. Typically in the presence of risk, confidence is expressed in terms of probabilities of certain events occurring. Forecasts produced by Central Banks (CBs), for example, are usually accompanied by confidence intervals around them, which reflect the likelihood, or indeed confidence, with which the forecaster believes they will materialize. However, in the presence of Knightian uncertainty when no priors are known, it is no longer possible to express confidence in similar probabilistic terms.
One thing that one can do then is engage in 'what if' scenarios and choose between policies that produce the best outcome given the set-ups considered. One such approach often used is that of min-max, in which policy is formulated under the assumption that the 'worst-case' will occur[1]. The policy maker decides first what shape and form the worst outcome gets and then chooses a policy that will deliver the best possible under that worst case. The advantage of such an approach is that, within the confines of the worst-case definition, it provides policy makers with full-insurance, as it caters for - the most - extreme events. This feature is intuitively appealing but is not without problems. Often critics argue that by designing policy to cater for extreme events, min-max effectively

[1] See Hansen and Sargent, (2008) and Williams, (2007) for comprehensive overviews of the literature.

assigns very high likelihoods to those rather extreme (and actually unlikely) events (Sims, 2001). Moreover, by nature of their extremity, such events are the ones that are the least known and understood and it is therefore paradoxical that policy makers would base their decisions on conditions they know the least about.

Beyond these issues however, min-max does not address the dual nature of policy makers' concerns about confidence and quality raised above. With this in mind, Ben-Haim (2006) puts forward an alternative method for addressing Knightian uncertainty. The method of *Info-Gap Robust Satisficing* is designed based on the well known principle of satisficing (Simon, 1957), which is distinct from that of optimizing. The idea is that rather than seek to attain the best possible outcome, decision makers should identify what type of outcomes they are prepared to live with. By comparison to min-max, the important difference is that the policy makers have a better view about what they are prepared to accept, than about what the worst possible outcome may be. *Confidence* is then defined as the maximum error that policy makers can make in their estimates of the uncertain parameters, before their satisficing criterion is violated. An important feature of such an approach is that as aspirations are relaxed, the number of policy alternatives available to decision makers increases. And as the number of options increases, so does the level of confidence with which they will achieve the less ambitious objective. *Info-gap robust satisficing* puts then forward a tool that explicitly captures the trade-off between the quality of the outcome and the confidence (robustness) with which this outcome is attained. Ambitious Central Banks who aim at a relatively precise specification of inflation, will see their degree of confidence of achieving it decline, when faced with Knightian uncertainty. Central Banks who want instead the comfort of knowing what the outcome will be, will also have to settle for less tightly specified monetary policy outcomes. Most importantly, the trade-off between robustness and quality is quantitatively evaluated.

Decision making then is explicitly described in two dimensions: robustness to uncertainty and quality of outcome. In the context of monetary policy one can therefore formulate the issue in the following way: what is the maximum level of permissible inflation? Robust-satisficing will then quantify the confidence of achieving it or in other words, the extent by which our best guess of the uncertain parameters can be wrong, before inflation exceeds the permissible level. Alternatively, if guaranteeing a certain level of confidence is policy maker's objective, this technique will then show which levels of inflation they would have to endure. Confidence in policies is naturally a feature which is desirable in its own right. However, beyond that, we show that the notion of confidence described above can be equivalent to the probability of 'success', or of attaining pre-defined aspirations. We will formally derive and discuss the conditions for which info-gap robustness is equivalent to the probability of success. When this holds, it is then correct to say that when one is prepared to accept a modest outcome, one will also see that outcome materialize more often. This is of course intuitively appealing as modest aspirations are easier to achieve; it is great ambitions that are not so easily attainable. However, in the context of monetary policy this is crucial because 'success', in other words demonstrating that one can achieve predefined targets, is tantamount to credibility. So a Central Bank that has demonstrated it can achieve acceptable levels of inflation, earns credibility in the eyes of the private sector, which in turn enhances its ability to achieve possibly even lower inflation in the future (Demertzis and Viegi, 2007). The contrary is also true: bad performance reduces its future ability to achieve good results through loss of credibility, in a vicious circle which Central Banks fear to get caught in. Among policy rules that provide similar outcomes, we will argue that a set-up that allows policy makers to rank policies according to their info-gap robustness, will also allow them to rank policies according to the probability of success, and is therefore additionally attractive.

The paper is organized as follows. Section 2 provides an intuitive discussion of info-gap methodology and revisits the concept of satisficing. Section 3 then applies the technique to a standard monetary policy model and derives the trade-off between robustness and performance, in the context of parameter uncertainty. In section 4 we elaborate on specific features of info-gap theory and present a discussion on how it fits in the current literature on modelling under Knightian uncertainty. Sections 5 and 6 are dedicated to the issue of robustness and probability of success. We first present why being 'successful' is of particular relevance to monetary policy and then derive the conditions for which info-gap robustness is a proxy for probability of success. Section 7 concludes.

2 What is Info-Gap Theory?

Info-gap theory is motivated by the ideas of bounded rationality and Knightian uncertainty. Info-gap theory is a methodology for supporting model-based decisions under severe uncertainty (Ben-Haim, 2006). An info-gap is a disparity between what *is known,* and what *needs to be known* in order to make a comprehensive and reliable decision. An info-gap is resolved when a surprise occurs, or a new fact is uncovered, or when our knowledge and understanding change. We know very little about the substance of an info-gap. For instance, we rarely know what unusual event will delay the completion of a task. Even more strongly, we *cannot* know what is not yet discovered, such as tomorrow's news, or future scientific theories or technological inventions. The ignorance of these things are info-gaps. An info-gap is a Knightian uncertainty (Knight, 1921) since it is not characterized by a probability distribution.

Info-gap decision theory is based on three elements. The first element is an *info-gap model of uncertainty,* which is a non-probabilistic quantification of uncertainty. The uncertainty may be in the value of a parameter, such as the slope of the Phillips curve, or in all the parameters of a given model. An info-gap may be in the shape of a function, such as demand vs. price, or the shape of the tail of the probability distribution function (pdf) of extreme financial loss. An info-gap may be in the size and shape of a set of such entities, such as the set of possible pdf's or the set of possible Phillips curves. For all examples of info-gap models of uncertainty, an info-gap model is an unbounded family of nested sets of possible realizations. For instance, if the uncertain entity is a function then the info-gap model is an unbounded family of nested sets of realizations of this function. An info-gap model does not posit a worst case or most extreme uncertainty.[2]

The second element of an info-gap analysis is a *model of the system,* such as a macroeconomic model of the form that we will describe in the next section. The model expresses our knowledge about the system, and may also depend on uncertain elements whose uncertainty is represented by an info-gap model of uncertainty. The system model also depends on the decisions to be made, and quantifies the outcomes of those decisions given specific realizations of the uncertainties. For instance, the model may express macroeconomic outcomes such as inflation, unemployment, growth of the GDP, and so on.

The third element of an info-gap analysis is a set of *performance requirements.* These specify values of the outcomes, which the decision maker requires or aspires to achieve. These values may constitute success of the decision, or at least minimally acceptable values. For instance, inflation targeting is sometimes formulated as a range of inflation values which are acceptable. Performance requirements can embody the concept of satisficing: doing good enough or meeting critical requirements.

These three components—uncertainty model, system model, and performance requirements—are combined in formulating a *decision function* which supports the choice of a course of action. The *robustness function* assesses the greatest tolerable horizon of uncertainty and is a quantitative answer to the question: how wrong can we be in our data, models and understanding, and the action we are considering will still lead to an acceptable outcome. Crucial to our original motivation, this function captures the trade-off between confidence and aspiration, and it does so without the use of probabilities, which under Knightian uncertainty are unknown. Similarly, this function allows us to rank alternative policies in a way that allows us to pick that policy that guarantees a given acceptable outcome for the greatest robustness. The robustness function is based on a satisficing performance requirement. When operating under severe uncertainty, a decision which achieves an acceptable outcome over a large range of uncertain realizations is preferable to a decision which fails to achieve an acceptable outcome even under small error. In this way the robustness function generates preferences on available decisions[3].

[2] Sometimes the family of sets is bounded by virtue of the definition of the uncertain entity. For instance, a probability must be between zero and one, so the family of nested sets of possible probability values is bounded. However, this bound does not derive from knowledge about the event whose probability is uncertain, but only from the mathematical definition of probability. Such an info-gap model is unbounded in the universe of probability values.

[3] Info-gap theory originated in engineering (Ben-Haim, 1996, 2006) and has since been

2.1 Satisficing: a criterion for decision making

The behavioral principle of satisficing, as distinct from optimizing, was introduced by Simon (1957). The notion of optimization as a paradigm of actual behavior was challenged by Simon (1955) in light of the bounded rationality of decision makers, referring to cognitive limitations and computational complexity that prevent individuals from finding optimal solutions to problems. Limitations may stem from imperfect information, imperfect information-processing capabilities, and environmental variability in space and time. Knight ([1921] 1965) stressed that information is sometimes so deficient and conditions so variable that probabilistic models are inaccessible; under severe uncertainty, one simply cannot choose a probability distribution (Carmel and Ben-Haim 2005). Simon (1997) consistently argued that the goal of maximization is virtually always unrealizable in real life, and that satisficing is more successful in managing both the complexity of the human environment as well as the limitations of human information processing. Alchian (1950, 1977) has made a very similar point regarding the impossibility of reliably planning the maximization of profit in dynamic environments. Conlisk (1996) summarizes much evidence in this direction. It has also been shown that satisficing is more robust to the decision-maker's lack of knowledge than optimizing (Ben-Haim, 2006). Furthermore, in some situations satisficing is more likely than optimizing to achieve specified goals (Ben-Haim, 2006, section 11.4; Ben-Haim, 2008), a point we will return to further down[4].

3 Info-Gap: a Monetary Policy Application

We define and operationalize the concept of robust-satisficing in the context of a standard monetary policy model. We use a simple AS-AD model, estimated for the US during the period 1961Q1-1996Q2, presented by Rudebusch and Svensson (1999). Based on the estimated parameters and the historical data, we apply the info-gap robust satisficing methodology to choose between alternative Taylor rules in the context of parameter uncertainty.

The model is represented by (1) and (2).

$$\pi_{t+1} = a_0\pi_t + a_1\pi_{t-1} + a_2\pi_{t-2} + a_3\pi_{t-3} + by_t + \varepsilon_{\pi,t+1} \quad (1)$$

$$y_{t+1} = c_0 y_t + c_1 y_{t-1} + d\left(\bar{\imath}_t - \bar{\pi}_t\right) + \varepsilon_{y,t+1} \quad (2)$$

π_t is the deviation of the inflation from a target value in the tth quarter. y_t is the output gap at time t, measured as 100 times the log ratio of the actual real

applied to a wide range of disciplines such as biological conservations, environmental management (Burgman, 2005), zoology (Regan *et al*, 2005), harvesting and water management (McCarthy and Lindenmayer, 2007), management science (Ben-Haim and Laufer, 1998 and Regev *et al,* 2006) and conflict resolution (Ben-Haim and Hipel, 2002).

[4] Schwartz (2004) and Schwartz et al (2002) apply this idea in the context of 'happiness' and argue that while *maximizers* might do better in some objective sense, *satisficers* do a lot better subjectively. And they argue, that as what matters for happiness is how well individuals feel about the decisions they make, and not necessarily how good their decisions are in some objective metric, satisficers tend to be overall happier.

output to the potential output. i_t is the Federal funds rate at an annual rate, and $\bar{\imath}_t$ is the 4-quarter average Federal funds rate:

$$\bar{\imath}_t = 0.25(i_t + i_{t-1} + i_{t-2} + i_{t-3}) \tag{3}$$

Likewise, $\bar{\pi}_t$ is the 4-quarter average of the inflation variable:

$$\bar{\pi}_t = 0.25(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}) \tag{4}$$

The Federal funds rate is regulated by a Taylor rule:

$$i_t = g_\pi \bar{\pi}_t + g_y y_t \tag{5}$$

where g_π and g_y are decision variables to be chosen by the policy maker. The estimates for the eight coefficients in (1) and (2) are reported in table 1.[5]

Table 1. Estimated Parameters of (1) and (2).

	a_0	a_1	a_2	a_3	b	c_0	c_1	d
Mean	0.70	−0.10	0.28	0.12	0.14	1.16	−0.25	−0.10
Standard Error	0.08	0.10	0.10	0.08	0.03	0.08	0.08	0.03

Source: Rudebusch and Svensson (1999).

We adopt the following notation for time-sequences of variables:

$$\pi_{t(k)} = (\pi_t, ..., \pi_{t-k})^T \tag{6}$$

$$y_{t(k)} = (y_t, ..., y_{t-k})^T \tag{7}$$

Furthermore, let $F = (a_0, ..., a_3, b, c_0, c_1, d)$ denote the 8 model coefficients (which we will subsequently consider to be uncertain). Eqs.(1)-(5) constitute the economic model. Examination of these equations shows that, given values of $\pi_{t(6)}$ and $y_{t(3)}$, the timing of the model predicts the deviation of the inflation, π_{t+2} , and output gap y_{t+1}. In other words, policies at time t affect output at $t+1$ and inflation at $t+2$. We denote these k-step average predictions by $\pi_{t+k}\left[F|\pi_{t(6)}, y_{t(3)}\right]$ and $y_{t+k}\left[F|\pi_{t(6)}, y_{t(3)}\right]$ respectively.

3.1 An Info-Gap Model of Uncertainty

We formulate next an info-gap model for uncertainty in the coefficients F of the economic model (1) and (2). The quantitative information which we have about these 8 coefficients is their empirically estimated means and standard deviations, shown in table 1, which we denote as $\tilde{F}_k$ and s_k, $k = 1, ..., 8$. While $\tilde{F}_k$ may be the "best" estimate of the current economy, at the same time, decision makers worry that this model may not necessarily be the best representation of

[5] Note that the mean estimate represents the modeler's best estimate of the unknown parameters and the standard errors provide a convenient metric with which to define deviations from that estimate. The existence of Knightian uncertainty prevents us from making any inference about the underlying probabilities.

future economic conditions. Furthermore, the basic Knightian intuition is that unknown future surprises cannot always be modelled probabilistically (Knight [1921] 1965). The method of info-gap robust satisficing assumes the following. We anticipate that more accurate values F_k will deviate from the estimates $\tilde{F}_k$, and that the relative tendencies for deviation of the coefficients are expressed by the standard errors s_k.

We define an unbounded-interval-uncertainty info-gap model as follows:

Definition 1 *For any given value of h, the set $\mathcal{U}(h, \tilde{F}_k)$ defines a range of variation of the model coefficients. However, the 'horizon of uncertainty' h is unknown, so the info-gap model contains an unbounded family of nested sets of possible realizations of the coefficients F_k*[6].

$$\mathcal{U}(h, \tilde{F}_k) = \left\{ F = \frac{\left|F_k - \tilde{F}_k\right|}{s_k} \leq h,\ k = 1, ..., 8 \right\}, \quad h \geq 0 \tag{8}$$

All info-gap models of uncertainty obey the following two axioms (Ben-Haim, 2006).

Axiom 1 - *Contraction:* $\mathcal{U}(0, \tilde{F}_k) = \left\{\tilde{F}_k\right\}$.

Axiom 2 - *Nesting:* $h < h' \implies \mathcal{U}(h, \tilde{F}_k) \subseteq \mathcal{U}(h', \tilde{F}_k)$.

The contraction axiom states that, in the absence of uncertainty, $\tilde{F}_k$ is the only possible model. The nesting axiom states that increasing robustness ($h < h'$), implies an increasing range of variation of the model coefficients. Next we discuss the decision maker's aspirations.

3.2 Aspirations

The policy maker selects the coefficients of the Taylor rule, $g = (g_\pi, g_y)$, to achieve acceptably small inflation-target $|\pi - \pi^*|$ and output gap $|y|$ deviations, over a specified time horizon $t = 1, ..., T$. For simplicity, we assume that the inflation target $\pi^* = 0$ and can therefore discuss inflation aspirations in terms of $|\pi|$. Since the coefficients of the model are highly uncertain and subject to

[6] It is important to emphasize that an info-gap model is *not* a realization of "bounded uncertainty". This is important for two reasons, one methodological and one epistemic. First, there is (usually) no worst case in an info-gap model, since the horizon of uncertainty is unknown and unbounded. Methodologically, this means that an info-gap analysis is fundamentally different from 'worst case' or 'min-max' analysis (which we will discuss later). Second, the assertion of a sharp boundary which delineates 'possible' from 'impossible' is, epistemically, a very strong assertion. Sharp bounds are usually difficult to verify unless they are in effect not meaningful. (An illustration of a true but meaningless sharp bound: 'The log-ratio of the output gap is bounded by 10^8'.) The verification of a meaningful sharp cutoff of a highly uncertain quantity is empirically difficult, and should not be done incautiously. If such an assertion *is* verified empirically, then quite possibly a much more informative uncertainty model (e.g., a probability model) can be verified and should be used.

unanticipated variations, it is not possible to reliably predict the outcome of any specific choice of the Taylor coefficients g. However, it is possible to evaluate a proposed g in terms of how robust, to uncertainty in the model, the resulting behavior is.

Small values of $|\pi|$, and $|y|$, over the time horizon $t = 1, ..., T$, are desired. It is unlikely that these quantities will become identically zero. Hence the policy maker specifies tolerances on the achievement of these targets. The policy, g, is considered acceptable if:

$$|\pi| \le r_{\pi,t} \qquad t = 1, ..., T \tag{9}$$

$$|y| \le r_{y,t} \qquad t = 1, ..., T \tag{10}$$

Let r_π and r_y denote the vectors of tolerances[7]. By specifying tolerances such as these, the policy maker aspires to **satisfice** the dynamic variables of the system, rather than to minimize or optimize them. We begin by constructing robustness functions for both variables, π and y. We then examine the performance-vs.-aspiration trade-off expressed by the robustness functions for a number of alternative policies, g. Our goal is to choose that policy rule that maximizes robustness.

3.3 Robust Satisficing: a definition

Given a satisficing requirement, we define then robustness as follows. First, consider the robustness of output gap aspirations, for a given Taylor rule g.

Definition 2 *The robustness, to model uncertainty, of Taylor coefficients g, given output-gap aspirations r_y, is the greatest horizon of uncertainty h at which the aspirations are not violated by any economic model F in $\mathcal{U}(h, \tilde{F}_k)$, at the relevant time horizon T:*

$$\hat{h}_y(g, r_y, T) = \max \left\{ h : \left(\max_{F \in \mathcal{U}(h, \tilde{F})} |y_t| \right) \le \ r_{y,t}, \forall \quad t = 1, ..., T \right\} \tag{11}$$

The robust-optimal Taylor coefficients for the output gap are:

$$\hat{g}_y\left(r_y\right) = \arg\max_g \hat{h}_y(g, r_y, T)$$

Similarly, the robustness for the inflation-gap variable is:

Definition 3 *The robustness, to model uncertainty, of Taylor coefficients g, given inflation-aspirations r_π, is the greatest horizon of uncertainty h at which*

[7] We can redefine aspirations in terms of losses in which inflation and output are simultaneously considered, to capture a Central Banker's desire to satisfy both. However, we find that it is more intuitive to talk about inflation and output aspirations in their own units and will therefore, limit our presentation to separate robustness functions.

the aspirations are not violated by any economic model F in $\mathcal{U}(h,\tilde{F}_k)$, at the relevant time horizon T:

$$\hat{h}_\pi(g,r_\pi,T) = \max\left\{h : \left(\max_{F\in\mathcal{U}(h,\tilde{F})} |\pi_t|\right) \leq\ r_{\pi,t}, \forall \quad t=1,...,T\right\} \tag{12}$$

A large value of $\hat{h}_\pi(g,r_\pi,T)$ is desirable, and means that Taylor coefficients g can be relied upon to keep the inflation-deviations $|\pi_t|$ within the specified tolerances r_π. A small value of the robustness implies that policy g cannot be relied upon to achieve these aspirations. "Bigger is better" for robustness, so the robust-optimal Taylor coefficients, regarding inflation, are those which maximize the robustness while satisficing the inflation performance:

$$\hat{g}_\pi\left(r_\pi\right) = \arg\max_g \hat{h}_\pi(g,r_\pi,T)$$

Given the timing of the model, the relevant robustness is $\hat{h}_\pi(g,r_\pi,2)$. The robustness for satisficing the aspirations regarding both variables is the smallest of the two robustness:

$$\hat{h}(g,r_\pi,r_y,T) = \min\left\{\hat{h}_\pi(g,r_\pi,T),\hat{h}_y(g,r_y,T)\right\} \tag{13}$$

The corresponding robust-optimal Taylor coefficients maximize this robustness:

$$\hat{g}\left(r_\pi,r_y\right) = \arg\max_g \hat{h}(g,r_\pi,r_y,T)$$

Summary *For a particular choice of policy g, how wrong can our best estimate of the model F be, before the policy objective (r_π,r_y) is violated? The robustnesses in eqs.(11) and (12) is the answer to this question. In other words, robustnesses, $\hat{h}_y(g,r_y,T)$ and $\hat{h}_\pi(g,r_\pi,T)$, of policy choice g, are the greatest error in F, up to which the policy objectives (r_π,r_y) are guaranteed.*

Policies that produce higher robustness are preferred. The policy maker will choose that policy, which gives the highest robustness for a given satisficing requirement. In other words, the robust-satisficing preference relation between two alternative policies y and y' is:

$$g \succ_g g' \quad if \quad \hat{h}(g,L_c) >\ \hat{h}(g',L_c) \tag{14}$$

We will now evaluate the robustness functions in order to study the implications of this preference relation.

3.4 Robustness Curves

For a given choice of g_π and g_y, we derive next the one-step projection of the output gap $y_{t+1}\left[F|_{\pi_{t(6)},y_{t(3)}}\right]$, and the two-step projection of the inflation gap $\pi_{t+2}\left[F|\pi_{t(6)},y_{t(3)}\right]$ and the respective robustness functions, $\hat{h}_y(g,r_y,1)$ and

$\hat{h}_\pi(g, r_\pi, 2)$. Note that inflation one year ahead (π_{t+1}) is also subject to uncertainty through the transmission in the AS curve but it is not a function of policy. We do not therefore derive its robustness function. The purpose is to illustrate in detail the procedure for evaluating the info-gap robustness functions, and to demonstrate their meaning. We concentrate on the info-gap uncertainty in the model coefficients and explore the implications for choice of the Taylor coefficients. We use the realizations for inflation and the output gap used in Rudebusch and Svensson (1999) to predict inflation in the second out-of-sample quarter (i.e. 96Q4) and in the first out-of-sample quarter for the output gap (i.e. 96Q3). Figure 1 plots the relevant data from 1993 to 2003.

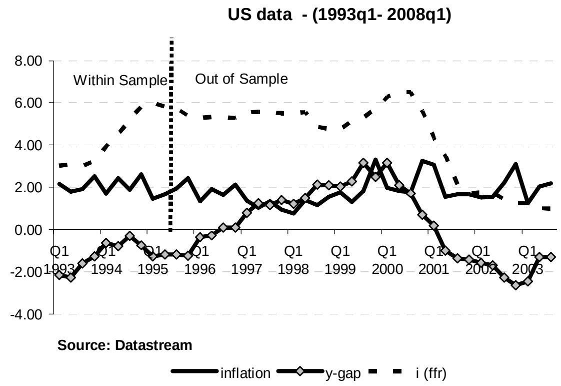

Figure 1:

The robustness functions will be affected by both the data as well as the choice of policy parameters g_π and g_y. Given the transmission, both the output as well as the inflation gap will be subject to uncertainty. We derive the robustness curves as follows:

Output gap. The one-step projection of the output gap is derived by substituting (3)-(5) into (2):

$$y_{t+1}\left[F|_{\pi_{t(6)},y_{t(3)}}\right] = c_0 y_t + c_1 y_{t-1} + d\underbrace{\left(\frac{1}{4}\sum_{k=0}^{3}\left(\frac{g_\pi}{4}-1\right)\pi_{t-k} + \frac{g_y}{4}y_t + \frac{1}{4}\sum_{k=1}^{3} i_{t-k}\right)}_{A} \tag{15}$$

The maximum absolute output gap, up to uncertainty h, is:

$$\max_{F \in \mathcal{U}(h,\tilde{F})} \left| y_{t+1} \left[F | \pi_{t(6)}, y_{t(3)} \right] \right| = \underbrace{\left| \tilde{c}_0 y_t + \tilde{c}_1 y_{t-1} + \tilde{d} A \right|}_{|\tilde{y}_{t+1}|} + h \underbrace{\left[s_{c_0} |y_t| + s_{c_1} |y_{t-1}| + s_d |A| \right]}_{\theta} \tag{16}$$

where $\tilde{c}_0, \tilde{c}_1$ and $\tilde{d}$ are the best-estimates, $\tilde{y}_{t+1}$ is the anticipated projected output gap, and θ results from the info-gap model and depends only on past state variables. The robustness of the output gap is evaluated by equating (16) to the critical output gap r_y and solving for h:

$$\hat{h}_y(g, r_y, 1) = \begin{cases} 0 & \text{if} \quad |\tilde{y}_{t+1}| \geq r_y \\ \frac{r_y - |\tilde{y}_{t+1}|}{\theta} & \text{else} \end{cases} \tag{17}$$

Inflation gap. The projected inflation-deviation, two quarters ahead, is:

$$\begin{aligned} \pi_{t+2} \left[F | \pi_{t(6)}, y_{t(3)} \right] &= b y_{t+1} + \sum_{k=0}^{3} a_k \pi_{t+1-k} \\ &= \underbrace{b \sum_{k=0}^{1} c_k y_{t-k} + b d A}_{b y_{t+1}} + \underbrace{a_0 \left(b y_t + \sum_{k=0}^{3} a_k \pi_{t-k} \right)}_{a_0 \pi_{t+1}} + \sum_{k=1}^{3} a_k \pi_{t+1-k} \end{aligned} \tag{18}$$

A plot of $\max_h \pi_{t+2}$ horizontally vs. h vertically is equivalent to a plot r_π horizontally and $\hat{h}_\pi(g, r_\pi, 2)$ vertically. Note that we do not derive the equivalent of (17) analytically but pursue a numerical solution[8]. Dependence on the Taylor coefficients arises only through the quantity A. The quantity bd is generally small ($\tilde{b} = 0.14$, $\tilde{d} = -0.10$), so the dependence on Taylor rule will be weak. Equation (18) is linear or bilinear in all the model coefficients except a_0 for which there is a self-quadratic term, $a_0^2 \pi_t$. For each of the 7 strictly linear or bilinear terms, the maximum of π_{t+2} will occur at one or the other of their extreme values of horizon of uncertainty h. We must simply check each combination, of which there are $2^7 = 128$ cases for the 7 model parameters excluding a_0. This term appears in the parabola $b y_t a_0 + \pi_t a_0^2$. The location of the maximum depends on whether the parabola is concave or convex. Thus, $\max_h \pi_{t+2}$ occurs at one or the other extreme value of a_0 if $\pi_t \geq 0$ (upward parabola), while $\max_h \pi_{t+2}$ occurs at $a_0 = b y_t / 2\pi_t$ otherwise. We thus evaluate π_{t+2} for each of the 128 combinations of extreme values of the 7 model coefficients other than a_0, with a_0 chosen according to the sign of π_t. The greatest of these 128 values is the $\max_h \pi_{t+2}$. Similarly to before, a plot of $\max_h \pi_{t+2}$ horizontally versus h vertically is equivalent to a plot of r_π horizontally and $\hat{h}_\pi(g, r_\pi, 2)$ vertically.

[8] Matlab codes for both output gap and inflation gap robustness functions available from the authors.

3.5 Results and Discussion

We can now plot (17) and the transform of (18) for different values of the satisficing criteria r_π and r_y and for given values of $g = (g_\pi, g_y)$, to illustrate the trade-off between aspirations and robustness under model uncertainty. We use the realizations for inflation and the output gap (figure 1) used in Rudebusch and Svensson (1999) to predict the first out-of-sample quarter for the output gap (i.e. 96Q3) in figure 2 and inflation in the second out-of-sample quarter (i.e. 96Q4) in figure 3.

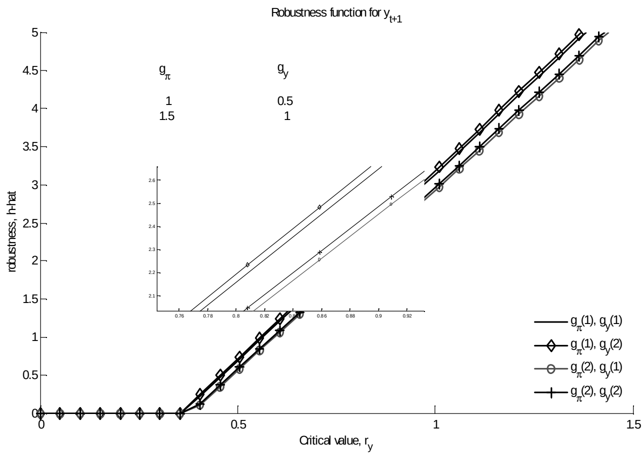

Figure 2:

Similarly we derive the robustness function $\hat{h}_\pi(g, r_\pi, 2)$ for inflation in figure 3 (same legend as figure2).

Both figures plot 4 different robustness curves for different coefficient values in the Taylor rule. As indicated in the graph, $g_\pi(1)$ and $g_y(1)$ correspond to the first values (1 and 0.5 respectively, the values originally suggested by Taylor, see for example Taylor 1999). The points $(h = 0, r_y)$ and $(h = 0, r_\pi)$ constitute respectively the forecasts for the output gap (≈0.4%) and inflation (≈0.75%) produced by the model. As h increases, figures 2 and 3 plot the highest level of output gap and inflation that will be satisficed for corresponding robustness. Looking at the figures closer now, figure 2 shows that there is no crossing of the curves and that values of $g_\pi = 1$ and $g_y = 1$ correspond to a robustness function that is always preferred to any other policy rule examined. Inflation robustness on the other hand (figure 3) is not visibly affected by the choice of rule (differences arise at the 3rd decimal). As explained in the previous section this is the result of the fact that dependence on the Taylor rule comes from the

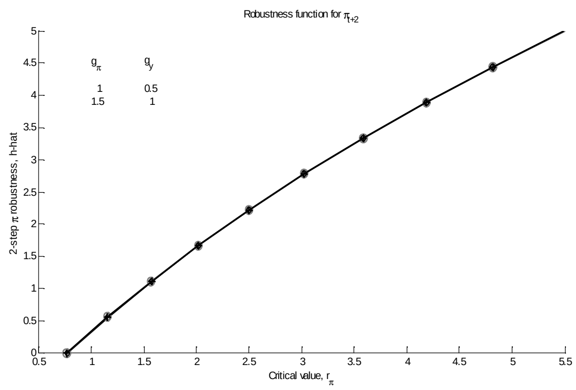

Figure 3:

term bd which is very small. For robustness equal to one, then $r_y \approx 0.5\%$ and $r_\pi \approx 1.5\%$. This means that each model coefficient F_k can vary by $\pm 1 s_k$ around its best estimate, $\tilde{F}_k$, without causing the 1-step output gap to deviate from the target value of zero by more than 0.5% percentage points and inflation from its optimal value by 1.5%.

We repeat the same exercise next but for a different time period. Current period t is now 2000q1 and therefore forecasts for the output gap will refer to 2000q2 and for inflation 2000q3. Looking at figure 1, this period is different to the in-sample as we observe a positive output gap. Our intention is then to show how robustness curves are affected by models that are not fitted to the data. Figures 4 and 5 now plots $\hat{h}_\pi(g, r_\pi, 2)$ and $\hat{h}_y(g, r_y, 1)$ against various satisficing requirements for r_π, r_y and similarly various policy rules (g_π, g_y).
Similarly for $\hat{h}_\pi(g, r_\pi, 2)$.

As figure 5 shows inflation robustness is not much affected. However, output gap robustness of is significantly displaced to the right, which implies deterioration. The output gap forecast is about 1.5%. At the same time as the inserts in figure 4 show there is crossing of the curves at robustness ≈ 3. For small levels of robustness (or ambitious aspirations), the best policy to apply is $g_\pi = 1.5$ and $g_y = 1$. However, as policy makers require greater robustness (and are therefore prepared to put up with greater output gap deviations), then it is preferable to apply a Taylor rule with weights $g_\pi = 1$ and $g_y = 0.5$.
Deviation of the inflation target by 2.5 percentage points is quite significant but much within the 'tradition' of forecast surprises as discussed by Poole (2004). The info-gap robustness predicts that such large deviations can occur even though the model coefficients err by no more than one standard deviation. The point is that the info-gap robustness function $\hat{h}_\pi(g, r_\pi, 2)$ is providing a fairly realistic assessment of the vulnerability to surprises. Furthermore, $\hat{h}_\pi(g, r_\pi, 2)$ incorporates a specific policy option into this surprise-modelling.

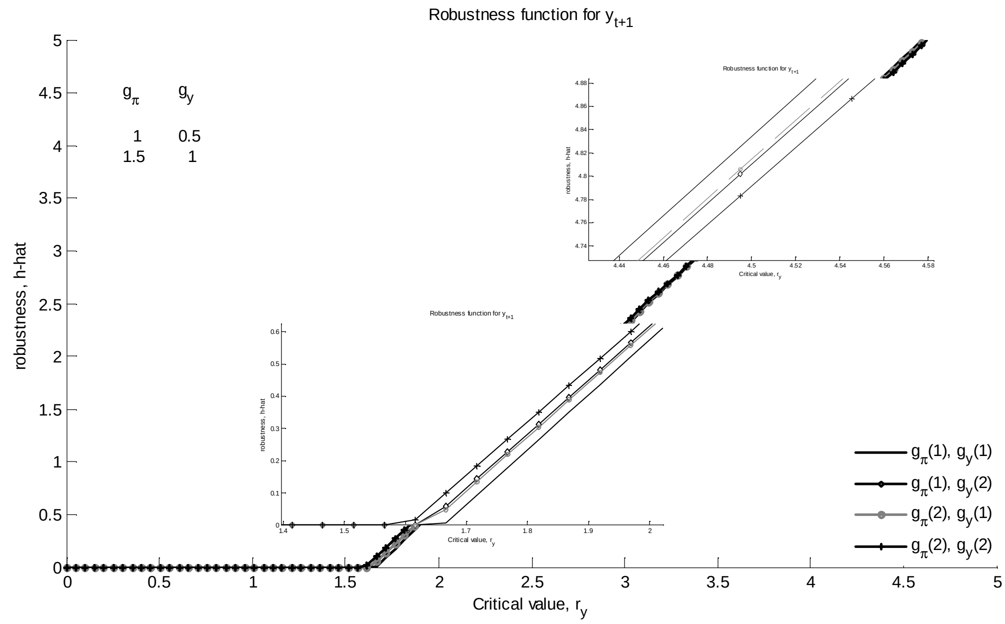

Figure 4:

4 Info-Gap: A Closer Look

We consider next two aspects relating to the robustness functions: i) we examine when robustness curves cross and ii) we discuss how info-gap compares to the standard min-max/robust control methodology. As we have seen inflation robustness is not sensitive to the choice of Taylor rule considered. We will thus continue the analysis in terms of the output gap and the corresponding robustness function.

4.1 Crossing of Robustness Curves

The following proposition asserts that robustness curves, $\hat{h}_y$ vs. r_y, cross one another for different choice of the Taylor coefficients.

Proposition 1: *Given robustness curves for the 1-step output-gap in the presence of uncertainty, $\hat{h}_y(g, r_y, 1)$ and $\hat{h}_y(g', r_y, 1)$, for two different vectors of Taylor coefficients, g and g'. These robustness curves cross at a single positive value of r_y if and only if:*

$$\left[\theta\left(g\right)-\theta\left(g^{'}\right)\right]\left(\left|\tilde{y}_{t+1}\left(g\right)\right|-\left|\tilde{y}_{t+1}\left(g^{'}\right)\right|\right)\leq 0 \quad \text{and} \quad \theta\left(g\right)\neq\theta\left(g^{'}\right) \tag{19}$$

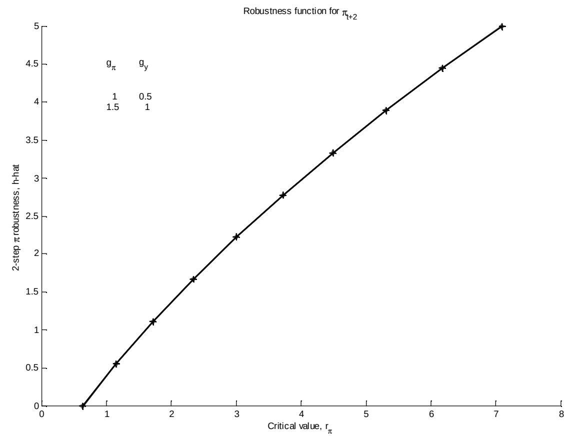

Figure 5:

They cross at positive robustness if and only if the inequality '$\leq$' is strict '$<$'.

Proof. The proof derives directly from the fact that the robustness curve, $\hat{h}_y(g, r_y, 1)$ vs. r_y, is a straight line with slope $1/\theta(g)$ and intercept $|\tilde{y}_{t+1}(g)|$.
Q.E.D

The crossing of robustness curves has important implications. First of all, crossing of robustness curves implies the potential for reversal of preference between policies for a robust-satisficing decision maker. This is because the robust-satisficing analyst will prefer the most robust option. Next, the robustness curves of quite different policies can cross. There are myriad values of the Taylor coefficients which satisfy eq.(19). For instance, two specific realizations of these conditions are:

$$A(g) < -A(g') < 0, \quad \text{if } \tilde{d\gamma} > 0 \qquad (20)$$

$$-A(g) < A(g') < 0, \quad \text{if } \tilde{d\gamma} < 0 \qquad (21)$$

where $\tilde{\gamma} = \tilde{c}_0 y_t + \tilde{c}_1 y_{t-1}$ and A is defined in (15). Relations (20) and (21) require $A(g)$ and $A(g')$ to differ in sign. Thus there are Taylor-coefficient vectors g and g' which are quite different and whose robustness curves cross. This means that the preference reversal will sometimes be between substantially different policy options, one of which is likely to be substantially more aggressive than the other. That is, as the critical loss, r_y, varies, there can be large and sudden change or bifurcation in policy preference. Sometimes the more robust policy will be more aggressive, and sometimes not.

4.2 Min-Max, Robust Control and Robust-Satisficing

We take a brief intermezzo to compare the robust satisficing strategy with a class of alternatives. The terms 'min-max', 'robust control' and 'worst-case' refer to a collection of decision strategies which attempt to ameliorate a maximally adverse outcome. This can of course be formulated in a variety of ways. In one form or another, either explicitly or implicitly, a greatest level of uncertainty or a worst possible outcome is posited. Then a strategy is sought which maximally diminishes the impact of this worst outcome.

Info-gap robust satisficing is motivated by the same perception of uncertainty, which motivates the min-max class of strategies: lack of reliable probability distributions and the potential for severe and extreme events. We will see that the robust satisficing decision will sometimes coincide with a min-max decision. On the other hand we will identify some fundamental distinctions between the min-max and the robust satisficing strategies and we will see that they do not always lead to the same decision.

First of all, if a worst case or maximal uncertainty is unknown, then the min-max strategy cannot be implemented. That is, the min-max approach requires a specific piece of knowledge about the real world: "What is the greatest possible error of the analyst's model?". This is an *ontological* question relating to the state of the real world. In contrast, the robust satisficing strategy does not require knowledge of the greatest possible error of the analyst's model. The robust satisficing strategy centers on the vulnerability of the analyst's knowledge by asking: "How wrong can the analyst be, and the decision still yields acceptable outcomes?" The answer to this question reveals nothing about how wrong the analyst in fact is. The answer to this question is the info-gap robust satisficing function, while the true maximal error may or may not exceed the info-gap robust satisficing. This is an *epistemic* question, relating to the analyst's knowledge, positing nothing about how good that knowledge actually is. The epistemic question relates to the analyst's knowledge, while the ontological question relates to the relation between that knowledge and the state of the world. In summary, knowledge of a worst case is necessary for the min-max approach, but not necessary for the robust satisficing approach.

The second consideration is that the min-max approaches depend on what tends to be the least reliable part of our knowledge about the uncertainty. Under Knightian uncertainty we do not know the probability distribution of the uncertain entities. We may be unsure what are typical occurrences, and the systematics of extreme events are even less clear. Nonetheless the min-max decision hinges on ameliorating what is supposed to be a worst case. This supposition may be substantially wrong, so the min-max strategy may be mis-directed.

A third point of comparison is that min-max aims to ameliorate a worst case, without worrying about whether an adequate or required outcome is achieved. This strategy is motivated by severe uncertainty which suggests that catastrophic outcomes are possible, in conjunction with a precautionary attitude, which stresses preventing disaster. The robust satisficing strategy acknowledges unbounded uncertainty, but also incorporates the outcome requirements of the analyst. The choice between the two strategies—min-max and robust satisficing—hinges on the priorities and preferences of the analyst.

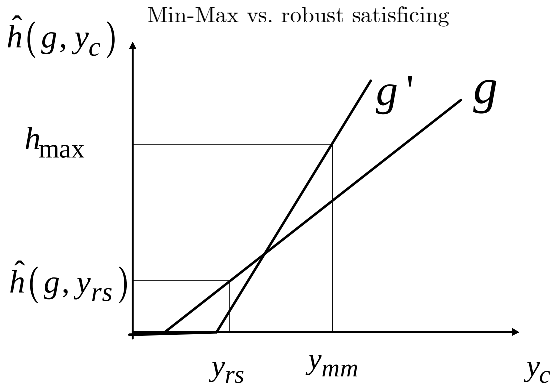

Figure 6:

The fourth distinction between the min-max and robust satisficing approaches is that they need not lead to the same decision, even starting with the same information. Consider the robust satisficing curves in figure 6. We must choose between two alternative decisions, g and g', whose robust satisficing curves are shown in the figure. Suppose that the greatest possible horizon of uncertainty in the analyst's models is known to be h_{max}. This knowledge allows us to implement the min-max strategy. The min-max choice is g' rather than g since g' has a less adverse worst possible outcome, y_{mm}. Now suppose that the analyst requires that the outcome be no worse than the value y_{rs} where y_{rs} is not only less than y_{mm} but also less than the value at which the robust satisficing curves cross, as shown in figure 6. The robust satisficing choice, given this performance requirement, is g rather than g' since g guarantees that the outcome will be less than y_{rs} for a wider range of uncertainty than does g'. The robust satisficing analyst acknowledges that the outcome could be as bad as y_{mm}, and could be worse with g than with g' for large uncertainty. The robust-satisficer chooses g since it guarantees the required outcome for a wider range of contingencies than does g'.

Fifth and finally, when a proxy theorem holds (as will be shown next to be the case for this particular problem) then the robust satisficing choice g is in fact more likely to achieve the requirement, $y \leq y_{rs}$, than is the min-max choice g'.

5 Robust-Satisficing in Monetary Policy

In the absence of probability distributions, the robustness function allows us to evaluate a measure of confidence for alternative policies. This is the first advantage of such a technique. However, and with relevance to monetary policy in particular, if robustness is synonymous to the probability of success, then policies that maximize robustness also help increase credibility between periods. This is because Central Banks that have previously demonstrated an ability to achieve pre-defined objectives, earn credibility in the eyes of the private sector. Greater levels of credibility in turn, allow Central Banks to attain their objectives more easily in subsequent periods. Thus, when robustness is equivalent to the probability of success, it reinforces the success-credibility loop, applicable to monetary policy. Demertzis and Viegi (2007) show explicitly how success and credibility in monetary policy are linked in this re-enforcing loop, such that policies that are successful help Central Banks build up credibility (Moscarini, 2007), and more credibility helps achieve further success (Bomfin and Rudebusch 2000). The opposite is also true: Central Banks that cannot achieve predefined objectives will see their credibility tumble and as credibility falls, it becomes increasingly more difficult to remain successful. In what follows we discuss the conditions for which robustness is tantamount to the probability of achieving explicit performance requirements.

5.1 Goals, Models, Uncertainties

Before we proceed to identify the conditions for which robustness is a proxy for probability of success, we introduce a number of definitions. The output gap in eq. (15) can be represented succintly as:

$$y_{t+1} = F^T v, \quad F^T = (c_0, c_1, d), \quad v^T = (y_t, y_{t-1}, A) \tag{22}$$

Our choice of a Taylor rule, g, determines the value of $A(g)$. $A(g)$ is an affine function of the Taylor coefficients g, so g can be chosen so that $A(g)$ takes any real value. The values of y_t and y_{t-1} are known, and F is uncertain.
The goal is to control the range of variation of y_{t+1}, and we consider three different specifications for the decision maker's aspirations:

$$y_{t+1} \leq r_y, \quad y_{t+1} \geq -r_y, \quad |y_{t+1}| \leq r_y \tag{23}$$

Intuitively, the decision maker would be interested in the first condition when concerned about over-shoot of the output gap. Likewise, concern about under-performance would focus attention on the second condition, while the third condition treats both types of errors equivalently. We can immediately appreciate one of the challenges facing decision makers, by noting that the goals in eq.(23) tend to conflict with one another. A strategy which moves y_{t+1} below and away from r_y, brings y_{t+1} closer to $-r_y$. As long as the analyst is concerned only with either the first or the second goal, but not both, this conflict is not problematic. However, this conflict of goals in inherent in the third strategy by itself, which we will see to have important repercussions when focussing on the third goal.
Based on our info-gap uncertainty model (8), each of the three definitions of aspirations in eq.(23) generates a robustness function, which is the greatest

horizon of uncertainty, h, up to which the corresponding condition is satisfied for all realizations of the model parameters:

$$\hat{h}_+(g, r_y, 1) = \max\left\{h: \left(\max_{F\in\mathcal{U}(h)} y_{t+1}\right) \le r_y\right\} \tag{24}$$

$$\hat{h}_-(g, r_y, 1) = \max\left\{h: \left(\min_{F\in\mathcal{U}(h)} y_{t+1}\right) \ge -r_y\right\} \tag{25}$$

$$\hat{h}_y(g, r_y, 1) = \max\left\{h: \left(\max_{F\in\mathcal{U}(\alpha)} |y_{t+1}|\right) \le r_y\right\} \tag{26}$$

Since both the upper and lower inequalities must be satisfied in the overall robustness, it is clear that:

$$\hat{h}_y(g, r_y, 1) = \min\{\hat{h}_-,\ \hat{h}_+\} \tag{27}$$

We repeat the definitions for the predicted value of y_{t+1}, and an error of this estimate:

$$\tilde{y}_{t+1} = \tilde{c}_0 y_t + \tilde{c}_1 y_{t-1} + \tilde{d}A \tag{28}$$

$$\theta = s_{c_0}|y_t| + s_{c_1}|y_{t-1}| + s_d|A|, \tag{29}$$

where θ is necessarily non-negative. One can readily show that:

$$\hat{h}_+ = \frac{r_y - \tilde{y}_{t+1}}{\theta} \tag{30}$$

$$\hat{h}_- = \frac{r_y + \tilde{y}_{t+1}}{\theta} \tag{31}$$

$$\hat{h}_y = \frac{r_y - |\tilde{y}_{t+1}|}{\theta} \tag{32}$$

where $\hat{h}_+$, $\hat{h}_-$ or $\hat{h}_y$ is defined to equal zero if the corresponding expression, eq.(30), (31) or (32), is negative. Eqs.(27)–(31) are equivalent to eq.(32).

5.2 Alternative Definitions for Probability of Success

Each of the three aspirations in eq. (24), generating a robustness function, corresponds to a probability of success.

Upper Robustness

Define the following set of all F-values which do not violate the condition $y_{t+1} \le r_y$ in the upper robustness $\hat{h}_+$:

$$\begin{aligned}\Lambda_+ &= \{F:\ y_{t+1} \le r_y\} \\ &= \left\{F:\ \frac{y_{t+1} - \tilde{y}_{t+1}}{\theta} \le \frac{r_y - \tilde{y}_{t+1}}{\theta}\right\} \\ &= \left\{F:\ \zeta \le \hat{h}_+\right\}\end{aligned} \tag{33}$$

provided that $\hat{h}_+$ is not zero, and where we have defined the "standardized" variable $\zeta = (y_{t+1} - \tilde{y}_{t+1})/\theta$.
If we think of F as a random vector, then y_{t+1} and ζ are both random variables. Let $Z(\zeta)$ denote the cumulative probability distribution (cdf) of ζ, with probability density function (pdf) $z(\zeta)$. The probability that $y_{t+1} \leq r_y$ is the "probability of success" for which $\hat{h}_+$ is the robustness. This probability is:

$$P_{s+} = Z(\hat{h}_+) \tag{34}$$

Lower Robustness

We now do something similar for the lower robustness. Define the set of all F-values which do not violate the condition $y_{t+1} \geq -r_y$ in the lower robustness $\hat{h}_-$:

$$\begin{aligned} \Lambda_- &= \{F : \; y_{t+1} \geq -r_y\} \\ &= \left\{F : \; \frac{y_{t+1} - \tilde{y}_{t+1}}{\theta} \geq -\frac{r_y + \tilde{y}_{t+1}}{\theta}\right\} \\ &= \left\{F : \; \zeta \geq -\hat{h}_-\right\} \end{aligned} \tag{35}$$

provided that $\hat{h}_-$ is not zero. The probability that $y_{t+1} \geq -r_y$ is the probability of success for which $\hat{h}_-$ is the robustness. This probability is:

$$P_{s-} = 1 - Z(-\hat{h}_-) \tag{36}$$

Two-Sided Robustness

Last, define Λ as the set of all F-values which satisfy the requirement $|y_{t+1}| \leq r_y$. Let $\Re$ be the set of all real numbers, and define the complement of Λ_- as $\Lambda_c = \Re - \Lambda_-$. These sets are related as:

$$\begin{aligned} \Lambda &= \{F : \; |y_{t+1}| \leq r_y\} \\ &= \Lambda_+ - \Lambda_c \end{aligned} \tag{37}$$

The probability of success is:

$$\begin{aligned} P_s &= Z(\Lambda) \\ &= Z(\Lambda_+) - Z(\Lambda_c) \\ &= P_{s+} - (1 - P_{s-}) \end{aligned} \tag{38}$$

6 Robustness as a Proxy for Probability of Success

We now use the statistical concept of 'standardization' to explore the relation between robustness and probability of success. We will show that, when the standardization condition holds, each one-sided robustness function is a proxy for the probability of success. This means that any change in the policy augments the robustness if and only if it also augments (or at least does not reduce) the probability of success. We will see that this is not true for the two-sided robustness function, which is related to the inherent conflict between the two sides in the absolute inequality which the two-sided robustness tries to control. When the proxy property holds, (as it does for the one-sided robustness), the policy maker can maximize the probability of success by maximizing the robustness, even if the probability distribution in entirely unknown[9]. The *value* of the probability success will be unknown, but maximal.

6.1 One-Sided Robustness

Definition 4 *Let y be a random variable with distribution $P(y|q)$ which depends on parameters q. The distribution $P(y|q)$ is a class of distributions as q varies over a set of allowed values.*

For instance, q would contain the mean and variance of a normal distribution, or the least and greatest values of a uniform distribution on the interval $[a,\ b]$. Let ζ be a one-to-one transformation of y whose distribution $Z(\zeta)$ does not depend on the parameters q. For instance, $Z(\zeta)$ would be the standard normal distribution, or the uniform distribution on the interval $[0,\ 1]$. $P(y|q)$ is a *standardization class,*[10] and y obeys the *standardization condition,* if $Z(\zeta)$ is the same distribution for all valid choices of q. For instance, the normal distributions form a standardization class since every normal variate, when standardized by its mean and variance, becomes a standard normal variate. Similarly the uniform distributions form a standardization class.

In our specific case, $\zeta = (y_{t+1} - \tilde{y}_{t+1})/\theta$, defined following eq.(33), obeys the standardization condition if its cdf does not depend on A. That is, the standardization condition requires that, while y_{t+1} depends on the parameters of the Taylor rule g through the parameter $A(g)$, the distribution of the standardized variable, $Z(\zeta)$, does not. Many distinct classes of distributions obey standardization conditions. We do not know which class $Z(\zeta)$ belongs to. By assuming standardization in our specific case we are assuming two things. First, we assume that the variability of $y_{t+1} = F^T v$ can be represented by a standardizable distribution. Second, we assume that the info-gap model for uncertainty in F contains enough information about the variability of F to properly standardize y_{t+1}. Specifically, we assume that $\tilde{y}_{t+1}$ and θ—which result from the info-gap model—do in fact standardize y_{t+1}.

We can now state the following proposition, which asserts that each one-sided robustness, $\hat{h}_+$ and $\hat{h}_-$, in eqs. (25) and (26), is a proxy for the corresponding probability of success if ζ obeys standardization. That is, any change in A which augments a one-sided robustness, also augments (or at least does not reduce) the probability of satisfying the corresponding requirement. Likewise, any change

[9] Ben-Haim, (2007).
[10] Ben-Haim (2006), section 11.4.

in A which reduces the robustness also reduces (or at least does not augment) probability of success.

Proposition 2. *Each one-sided robustness is a proxy for the probability of one-sided success, if the associated random variable is standardized.*
Given *The random variable* $\zeta = (y_{t+1} - \tilde{y}_{t+1})/\theta$ *obeys the standardization condition, so its pdf is independent of the Taylor coefficients.*
Then *The robustness functions* $\hat{h}_{+}(g, r_y, 1)$ *and* $\hat{h}_{-}(g, r_y, 1)$ *are each a proxy for the probability of corresponding one-sided success:*

$$\left(\frac{\partial P_{s\times}}{\partial A}\right)\left(\frac{\partial \hat{h}_{\times}(g, r_y, 1)}{\partial A}\right) \geq 0 \tag{39}$$

where $\times$ is either "+" or "-".

Proof of Proposition 2. Consider first the upper robustness. The standardization condition allows us to express the variation of P_{s+} in eq.(34) as follows:

$$\begin{aligned}\frac{\partial P_{s+}}{\partial A} &= \left.\frac{\partial Z}{\partial \zeta}\right|_{\widehat{h}_{+}} \frac{\hat{h}_{+}}{\partial A} \\ &= z(\widehat{h}_{+}) \frac{\partial \hat{h}_{+}}{\partial A}\end{aligned} \tag{40}$$

The pdf is non-negative, so eq.(40) is equivalent to eq.(39).

Now consider lower robustness. The standardization condition allows us to express the variation of P_{s-} in eq.(36) as follows:

$$\begin{aligned}\frac{\partial P_{s-}}{\partial A} &= \left.\frac{\partial Z}{\partial \zeta}\right|_{-\widehat{h}_{-}} \frac{\partial \hat{h}_{-}}{\partial A} \\ &= z(-\widehat{h}_{-}) \frac{\partial \hat{h}_{-}}{\partial A}\end{aligned} \tag{41}$$

The pdf is non-negative, so eq.(41) is equivalent to eq.(39). Q.E.D.

Eqs (40) and (41) imply that for one-sided aspirations, ranking policies in terms of their info-gap robustness will also agree with their ranking in terms of the probability of success. Applying policies that maximize success is of particular relevance to monetary policy, as achieving pre-specified objectives improves credibility, which in turn contributes to future successes. In other words, ranking policies according to their level of robustness, not only maximizes the probability of current success but maximizes also the probability of success in subsequent periods, *ceteris paribus*. We emphasize that ranking by robustness does not require knowledge of the probability distributions, and hence can be implemented under Knightian uncertainty.

6.2 Two-Sided Robustness

In this section we will understand why $\hat{h}_y$ is generally *not* a proxy for the probability of two-sided success, and furthermore why analysts should be motivated to choose one of the one-sided robustness functions. This is a continuation of the discussion of conflicting goals introduced in section 5.2.

If we assume that the standardization condition holds, then (38), (40) and (41) imply:

$$\begin{aligned}\frac{\partial P_s}{\partial A} &= \frac{\partial P_{s+}}{\partial A} + \frac{\partial P_{s-}}{\partial A} \\ &= z(\widehat{h}_+)\frac{\partial \widehat{h}_+}{\partial A} + z(-\widehat{h}_-)\frac{\partial \widehat{h}_-}{\partial A} \end{aligned} \tag{42}$$

We see from eq.(27) that $\partial\hat{h}_y/\partial A$ equals either $\partial\hat{h}_+/\partial A$ or $\partial\hat{h}_-/\partial A$. Thus a sufficient (though not necessary) condition for $\hat{h}$ to be a proxy for P_s is that $\partial\hat{h}_+/\partial A$ and $\partial\hat{h}_-/\partial A$ have the same algebraic sign. However, from eqs.(30) and (31) one finds:

$$\frac{\partial \hat{h}_-}{\partial A} = -\frac{\partial \hat{h}_+}{\partial A} - \frac{2r_y}{\theta^2} \tag{43}$$

where we note that $2r_y/\theta^2 \geq 0$. Thus, if either partial derivative is positive, then the other must be negative and of greater absolute value. We conclude that the condition that both derivatives have the same sign will not always hold, and in fact may frequently fail to hold. In fact, we can understand from eqs.(42) and (43) that $\hat{h}_y$ will often *not* be a proxy for the probability of two-sided success. This is because policy-changes which improve success on one side will tend to reduce success on the other side. This motivates the analyst to decide which direction of error is more important, and to use the corresponding one-sided robustness function. Proposition 2 asserts that each one-sided robustness *is* a proxy for the corresponding probability of success.

6.3 Learning

The proxy theorem, proposition 2, depends critically on the assumption of standardization. As explained earlier, this in turn depends on the info-gap model containing enough information about the random variability of F in order for ζ to be correctly standardized. For instance, suppose θ in eq.(29) is the wrong normalization, and the correct normalization is $\theta' = \sqrt{s_{c_0}^2 y_t^2 + s_{c_1}^2 y_{t-1}^2 + s_d^2 A^2}$. This normalization would result from exactly the same analysis, with the following ellipsoid-bound info-gap model:

$$\mathcal{U}(h) = \left\{ F:\ (F-\tilde{F})^T S^{-1}(F-\tilde{F}) \leq h^2 \right\}, \quad h \geq 0 \tag{44}$$

where $S = diag(s_{c_0}^2,\ s_{c_1}^2,\ s_d^2)$. The validity of the proxy theorem is what assures that robust-satisficing policies are most likely to succeed. (This is particularly important since, from proposition 1, we know that robust-satisficing policies may differ from expected-model optimal strategies.) The decision maker who is able to learn how to standardize F, will be able to maximize the probability of success. This does not require learning the correct cdf of F, or even what standardization class it belongs to. It is sufficient for the non-probabilistic info-gap model to capture enough information about the variability of F to enable correct standardization of ζ. This minimal learning is a self-reenforcing process, since correct standardization leads the robust-satisficing decision maker to succeed more than any other decision maker, for a well-defined level of aspirations.

7 Conclusions

In the presence of Knightian uncertainty, it is no longer possible to evaluate confidence in probabilistic terms. The method of info-gap robust satisficing evaluates confidence in terms of degrees of errors around the best estimates of the uncertain parameters. The greater the error one can make around those estimates before violating a well defined performance criterion, the greater the confidence in those estimates. We apply this technique to a standard monetary policy problem and derive the trade-off between confidence and quality of outcomes. As we move away from optimizing towards satisficing, we provide a number of different policy alternatives to decision makers, which we then rank in terms of robustness. Policy makers can then pick those policies that deliver a given outcome with the highest robustness. The framework also grants policy makers the flexibility to decide the other way around: for a given level of robustness, pick that policy that delivers the best outcome. We argue that the flexibility of such a technique makes it desirable to policy makers and is the first contribution of our paper. In the context of monetary policy, the equivalence of robustness to the probability of being successful is of particular relevance. This is the second contribution of our paper. Monetary policy in practice is characterized by a success-credibility loop, which reinforces itself. Central Banks that are able to achieve pre-specified objectives earn credibility, which in turn facilitates success in subsequent periods. And if robustness is a proxy for probability of success, then being able to rank policies according to their robustness maximizes the beneficial effects of this loop. Thus, under Knightian uncertainty, the CB can maximize its probability of success without any knowledge of the probability distributions. We outline the conditions under which this equivalence holds. Our future work will need to address the issue of how policy makers choose their satisficing criteria in a time-consistent manner. This will require linking the criteria themselves to the notion of credibility and monetary policy 'success'.

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