On the optimal frequency of the central bank’s operations in the reserve market

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Abstract

In this paper we analyze the optimal frequency of open market operations carried out by a central bank with the objective of steering the overnight interest rate and its relationship with required reserves and standing facilities. For this purpose, we construct a simple model of the reserve market, assuming an error-correction mechanism for the commercial banks’ reserve management. The results provide not only a theoretical underpinning to some stylized facts and assertions made in the literature but also an opportunity of making precise quantitative comparisons of interest rates’ volatility for various frequencies of central banks’ operations.

1 Introduction

This paper addresses an issue in the central bank’s policy-making which has been taken for granted in the literature so far - the frequency of interventions in the reserve market. We develop a simple framework, which allows us to explain the number and timing of operations which the central bank carries out. Some of the questions that the results of our analysis will help to answer are: can a continuous participation of the central bank in the reserve market reduce the overnight interest rates’ volatility to zero; why interest rates’ volatility levels do not vary as much among countries as differences in the central banks’ operating frameworks would suggest; is it possible to limit the number of open market operations carried out without boosting the volatility of interest rates, etc.

We begin with a short review of issues fundamental for the analysis: a concept of the operational framework and its relation towards the monetary policy strategy, a review of major differences between operational frameworks

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applied in reality and a glance at stylized facts on the volatility of short-term interest rates (in the following section).

The operational framework is a set of instruments and procedures applied by the central bank to implement monetary policy. Most commonly used instruments include open market operations (OMOs), standing facilities and minimum reserve requirements. Open market operations imply an exchange of financial assets between the central bank and financial institutions, executed on the initiative of the central bank. Applied procedures vary depending first of all on the type of operations (reverse or outright transactions), but also on the frequency, maturity, assets traded, etc. The term ‘standing facilities’ is used to describe both the lending and the deposit facility. The lending facility can be used by financial institutions to obtain funds from the central bank against eligible assets. The deposit facility, on the other hand, allows them to deposit funds at the central bank. Minimum reserve requirements mean that financial institutions are obliged to hold certain reserves at the central bank for a specified period of time (the reserve maintenance period). Applied procedures vary across the central banks with regard to rules for calculating the level of required reserves, length of the maintenance period, averaging provisions, remuneration, etc.

The instruments described above are designed to allow the central bank to achieve a certain (operating) target. This target represents the desired level of an economic variable which is effective in achieving the ultimate objective of monetary policy. There are two candidates to be considered as operating targets: a short-term interest rate and the monetary base, which represent a price and a quantity of funds traded in the reserve market. This choice of price versus quantity targeting was first analyzed by Poole (1970) and is nowadays a standard part of the textbooks on monetary economics (e.g. Friedman and Hahn (1990)). In a nutshell, the optimal choice of operating target depends on the parameters describing the economy (e.g. the elasticity of money demand with respect to output and the interest rate or elasticity of output itself with respect to the interest rate) and relative magnitudes of different sources of uncertainty in the economy (aggregate demand disturbances, money demand disturbances, etc.).

The table below, adapted from Bofinger (2001), summarizes theoretical considerations on the suitability of various instruments for the two choices of the operating target.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Monetary base targeting</th>
<th>Interest rate targeting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lending facility</td>
<td>Incompatible</td>
<td>Suitable</td>
</tr>
<tr>
<td>Deposit facility</td>
<td>Incompatible</td>
<td>Suitable</td>
</tr>
<tr>
<td>Outright OMOs</td>
<td>Suitable</td>
<td>Possible</td>
</tr>
<tr>
<td>Reverse OMOs¹</td>
<td>Variable/fixed rate tender: suitable/incompatible</td>
<td>Variable/fixed rate tender: incompatible/suitable</td>
</tr>
<tr>
<td>Required reserves</td>
<td>Useful</td>
<td>Not required</td>
</tr>
</tbody>
</table>

¹ In original: ‘repo’
The second table presents the actual frameworks applied in four countries (monetary areas): the Eurozone, United States, Japan and United Kingdom. All central banks except the Bank of Japan\(^2\) use the interest rate as the operating target. The implemented setups are rather varied\(^3\), although they all fit the theoretical framework presented above.

<table>
<thead>
<tr>
<th>Country</th>
<th>Eurozone</th>
<th>UK</th>
<th>USA</th>
<th>Japan(^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final target</td>
<td>Price stability(^5)</td>
<td>Price stability(^6)</td>
<td>Price stability and full employment</td>
<td>Inflation(?)(^7)</td>
</tr>
<tr>
<td>Operating target</td>
<td>Interest rate</td>
<td>Interest rate</td>
<td>Interest rate</td>
<td>Current Accounts</td>
</tr>
<tr>
<td>Lending facility</td>
<td>Applied</td>
<td>Applied(^8)</td>
<td>Applied but not important</td>
<td>Applied but not important</td>
</tr>
<tr>
<td>Deposit facility</td>
<td>Applied</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Outright OMOs</td>
<td>Applied</td>
<td>Applied</td>
<td>Applied</td>
<td>Applied</td>
</tr>
<tr>
<td>Reverse OMOs</td>
<td>Applied</td>
<td>Applied</td>
<td>Applied</td>
<td>Applied</td>
</tr>
<tr>
<td>Required reserves</td>
<td>Applied</td>
<td>-(^9)</td>
<td>Applied but not important</td>
<td>Applied</td>
</tr>
</tbody>
</table>

2 Stylized facts

We want to investigate the effectiveness of various monetary policy instruments in achieving the operating target. To this end, we will start by looking at actual overnight interest rate developments in years 1999-2001 in two monetary areas: Europe and the United States. The operational frameworks employed by both central banks represent limiting cases: the European Central Bank (the ECB) has a full set of instruments at its disposal, whereas the Federal Reserve relies mainly on open market operations. Let us see how these differences translate into the stability of the overnight interest rate around its target value.

The data presented in figures 1 through 6 is organized as follows:

- horizontal lines depict the development of the overnight interest rate dur-

\(^2\) Actually, faced with liquidity trap, deflation and shrinking economy, Bank of Japan changed its operating target in March 2001 from the overnight call rate to the amount outstanding of financial institutions’ current accounts (henceforth, reserves).
\(^3\) For an exemplary recent discussion of European Monetary Union, United States and Japan see BIS (2001).
\(^4\) The framework as described in Miyanoya (2000).
\(^5\) A positive inflation rate not exceeding 2%
\(^6\) Inflation target of 2.5% (±1%)
\(^7\) Bank of Japan has committed itself to maintain its current policy framework until the consumer price index will reach a stable non-negative level (See e.g. Ueda (2001))
\(^8\) “Late lending” partly aimed at limiting the rise in the overnight rate.
\(^9\) Not used for monetary policy purposes.
Figure 1: Eurozone overnight rate, EONIA, 1999

Figure 2: USA overnight rate, Federal funds rate, 1999
Figure 3: Eurozone overnight rate, EONIA, 2000

Figure 4: USA overnight rate, Federal funds rate, 2000
Figure 5: Eurozone overnight rate, EONIA, 2001

Figure 6: USA overnight rate, Federal funds rate, 2001
ing each reserve maintenance period\textsuperscript{10}

- vertical lines in the Eurozone graphs denote Tuesdays - the days when open market operations (the so-called main refinancing operations) are carried out (the Fed conducts OMOs more or less continuously)

- the date-points indicate changes in the main policy rates: the target rate (USA) or the tender rate (Eurozone).

We can identify the following interest rate pattern: throughout the sample years EONIA rate was relatively stable at the beginning and more volatile in the second half of the maintenance period.\textsuperscript{11} Its volatility was very much boosted during the strategic bidding episodes (underbidding on 7 April 1999, 14 February 2001, 11 April 2001, 10 October 2001 and 7 November 2001; and overbidding on 31 May and 7 June 2000). The usual pattern for the federal funds (FF) rate - a slight fall over the first days of the maintenance period and a significant rise over the last three days\textsuperscript{12} - is not clearly visible. Overall the FF rate was rather volatile throughout 1999 and relatively stable in the following two years.

The stylized facts show that frequent interventions in the reserve market do not automatically translate into more stable overnight rates. The Federal Reserve, which intervenes daily, seems not to be able to stabilize the FF rate to a significantly larger extent than the ECB, which conducts OMOs only once a week. This conclusion is corroborated by the results in the table below, which presents the average-volatility-around-target figures for both interest rates. Over three years the difference in volatility corrected for strategic reserve management episodes was only 0.006.

<table>
<thead>
<tr>
<th>Average volatility</th>
<th>EONIA</th>
<th>FF rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>0.046831</td>
<td>0.032104</td>
</tr>
<tr>
<td>1: Without under- and overbidding\textsuperscript{13}</td>
<td>0.042320</td>
<td>-</td>
</tr>
<tr>
<td>2: Without (1) and end-of-maintenance days\textsuperscript{14}</td>
<td>0.034246</td>
<td>0.028127</td>
</tr>
</tbody>
</table>

Since the performance of both central banks seems comparable in terms of the overnight interest rates’ control, the observed differences in the frequency of open market operations must be compensated for by the design of other instruments. This has already been noted in the literature. In the already-mentioned BIS paper Borio observes that “...The frequency of market operations is much higher in the United States and Japan. In addition to mirroring the

\textsuperscript{10}In a single year there are twelve reserve maintenance periods of one month in Europe, and 25-26 2-week maintenance periods in the United States.

\textsuperscript{11}Which is in line with the results from a more comprehensive research by Hartmann et al. (2001).

\textsuperscript{12}See Hamilton (1996) and Bartolini et al. (2001)

\textsuperscript{13}Without one observation, with the largest squared error around the target rate, on the very day or directly after the (under/over)bidding episode.

\textsuperscript{14}Without one observation, with the largest squared error around the target rate, on the last day of the reserve maintenance period or directly preceding it.
different use of standing facilities, this appears to derive primarily from the
class characteristics of reserve requirements in the United States (low level and short
averaging period) and from the high volatility of autonomous factors [net foreign
assets, net lending to the government, cash in circulation, etc.] in Japan...”.
However, this issue has not been investigated analytically until now - and this
is the fundamental contribution of this paper.

In the following sections we will present a simple model of the reserve mar-
et, which is managed by the central bank with the objective of reducing the
interest rate volatility. We will consider two operating frameworks: the contin-
uous intervention framework (the Fed case) and the less frequent intervention
framework (the ECB case). The results will hopefully provide an explanation
of the minor differences in the volatility of EONIA and FF rate.

3 Analytical setup

We will focus on the (short-term) interest rate targeting approach, since it is
the strategy applied in most industrial countries. We will assume that the
performance of the central bank’s operational framework can be assessed in
terms of the variability of the overnight interest rate \(i_t\) around its target (or
desired) rate \(i\) throughout the reserve maintenance period of length \(T\):

\[
L = \sum_{t=1}^{T} E((i_t - i)^2)
\]

The simple form of the loss function \(L\) captures the idea, that in order to achieve
its ultimate goals, the central bank must be able to exercise a close control over
the overnight interest rate and keep it as close as possible to the operating
target.

The model of the reserve market used here assumes that the central bank
may employ all instruments described in the introduction: standing facilities,
open market operations and required reserves (to be held over the maintenance
period). It consists of two equations\(^\text{15}\):

- a simplified central bank’s balance sheet identity for each sub-period \(t = 1, 2, ..., T\):

\[
r_t + a_t + df_t = m_t + lf_t
\]

"Liquidity-absorbing" liabilities consist of \(r_t \geq 0\) - reserves held/demanded
by a banking sector, \(a_t \geq 0\) - net autonomous factors (if \(a_t \leq 0\) then autonomous
factors provide net liquidity) and \(df_t \geq 0\) - deposit facility. "Liquidity-providing"
assets are a sum of \(m_t \geq 0\) - open market operations\(^\text{16}\) (if \(m_t \leq 0\) then open

\(^{15}\)The model builds on the work developed in the European Monetary Institute in the
preparatory phase for the Stage Three of the Economic and Monetary Union.

\(^{16}\)The regular main refinancing operations in case of the ECB. We will ignore ad hoc op-
erations (structural and fine-tuning) and the longer term refinancing operations, which (by
construction) are carried out only once per maintenance period.
market operations absorb liquidity from the market) - and \( l_f t \geq 0 \) - lending facility.\(^{17}\) The components of the central bank’s balance sheet have the following properties:

Autonomous factors comprise aggregates which are not controlled by the central bank (or commercial banks): net foreign assets, net lending to the government, other net assets, cash in circulation, etc. They constitute an exogenous stochastic element in the model. In each sub-period \( t \), their average \( E(\alpha_t) = \alpha_t \) is assumed to be equal to the central bank’s forecast. This figure is available to market participants, therefore expectations, that financial institutions form about autonomous factors, coincide with this forecast (A1). Autonomous factors are also assumed to be homoscedastic (A2), that is their volatility is constant throughout the maintenance period \( (\sigma^2_a(t) = \sigma^2_a \text{ for all } t = 1, 2, ..., T) \).

The size of open market operations is decided upon by the central bank. The central bank is assumed to react in a pre-determined way to the publicly available information on the developments of the items on its balance sheet (see the next section). Therefore the actual size of OMOs is not stochastic and \( E(m_t) = m_t \) for all \( t = 1, 2, ..., T \) (A3).

The recourse to standing facilities of the central bank is assumed to represent errors made by commercial banks in the reserve funds’ management. As we assume that banks do not make systematic errors, the expected value of the future use of the facilities is set to zero: \( E(df_t) = E(lf_t) = 0 \) (A4).

- The size of reserves held by commercial banks is assumed to evolve within the reserve maintenance period according to the following mechanism:

\[
\begin{align*}
    r_t &= -\alpha t r_{t-1} - \beta t r_{t+1}^e + \gamma t R - \delta t i_t \\
\end{align*}
\]

where \( r_{t+1}^e \) denotes the expected reserves to be held in the sub-period \( t+1 \), \( R \geq 0 \) is the level of required reserves and \( i_t \) is the overnight rate. This mechanism has two important characteristics. First of all, we assume that banks manage their reserve holdings based on the cost of obtaining the funds \( (i_t) \) on the one hand and the compulsory level of reserves imposed by the central bank \( (R) \) on the other hand. Furthermore, the specification emphasizes the intertemporal character of funds’ management. Commercial banks are supposed to analyze their reserve position in the context of several sub-periods within the reserve maintenance period (sub-periods \( t - 1 \), \( t \) and \( t + 1 \)).

The above mechanism is empirically supported by vector-error-correction regressions based on weekly data on the ECB operations, which have satisfactory explanatory power (see the Appendix). The regressions moreover corroborate our prior expectations on the sign of the parameters in (3), i.e. the regression coefficients corresponding to \( \alpha_t, \beta_t, \gamma_t, \delta_t \) are (generally) positive for all the

\(^{17}\) Data on the ECB’s balance sheet indicate that reserves held by the banking sector have, on average, constituted around 54.7% and net autonomous factors - 45.0% of liabilities. On the assets’ side, 73.6% of liquidity was provided via main refinancing operations and 26.1% via longer term operations. The deposit and lending facilities accounted for around 0.3% of assets and liabilities, respectively. (Source: ECB Monthly Bulletin, May 2002)
weeks of the maintenance period (i.e. \( t = 1, 2, 3, 4 \)). These results are in line with the behavior of reserves driven by interest rate expectations, described in the literature. In order to minimize the cost of holding reserves, commercial banks try to front- or back-load reserves if they expect interest rates to increase or decrease later on in the maintenance period.\(^\text{18}\) Moreover, they suggest that in the Eurozone, where required reserves are implemented, the weighted average of reserves held by commercial banks over three consecutive sub-periods \((\bar{r}_t + \alpha_1 r_{t-1} + \beta_2 r_{t+1})\) is positively related to the level of required reserves\(^\text{19}\) and negatively related to the overnight interest rate, which seems plausible.

We will furthermore assume that the magnitude of the parameters \(\alpha_t, \beta_t, \gamma_t\) and \(\delta_t\) in equation (3) vary over time. This assumption will capture changing motives for holding reserves within the maintenance period: as time passes and approaches the end of the maintenance period, commercial banks are more concerned about fulfilling reserve requirements\(^\text{20}\) and less about avoiding opportunity cost of holding excess reserves\(^\text{21}\).

In the remainder of the paper we will solve the model for a 2-day reserve maintenance period \((T = 2)\). This case is of interest, as it is both simple and rich enough to address policy-relevant issues. Specifically, it captures an (intertemporal) dependency between liquidity and interest rates and does not represent interest rates as weighted averages of the rates on standing facilities\(^\text{22}\).

### 4 Interest rates over a 2-day reserve maintenance period

We solve for interest rates on both days of the maintenance period using equation (3) written for \( t = 1, 2 \):

\[
\begin{align*}
    r_1 &= -\alpha_1 r_0 - \beta_1 r_2^\circ + \gamma_1 R - \delta_1 i_1 \\
    r_2 &= -\alpha_2 r_1 - \beta_2 r_3^\circ + \gamma_2 R - \delta_2 i_2
\end{align*}
\]

The first equality implies

\[
i_1 = \frac{1}{\delta_1} (\gamma_1 R - \beta_1 r_2^\circ - r_1 - \alpha_1 r_0)
\]

\(^\text{18}\) See e.g. Swank (1995), Bindseil (2000) and BIS (2001). In our model \(\frac{\partial r_t}{\partial r_{t+1}} = -\beta_t (\bar{r}_{t+1} - \epsilon_t) > 0\) if \(\beta_t, \delta_{t+1} > 0\).

\(^\text{19}\) This characteristic seems to be in line with the averaging provision, which allows commercial banks to fulfill reserve requirements on average over the maintenance period.

\(^\text{20}\) This implies an increasing sensitivity of reserves with respect to the size of reserves accumulated so far and the size of required reserves \((\frac{\partial r_t}{\partial r_t} \geq 0\) and \(\frac{\partial r_t}{\partial R} \geq 0\)).

\(^\text{21}\) Which means a decreasing sensitivity to the interest rate and expected future reserve holdings \((\frac{\partial r_t}{\partial i_t} \leq 0\) and \(\frac{\partial r_t}{\partial R} \leq 0\)).

\(^\text{22}\) Which is justified in the case of the Eurozone (see e.g. the model used by Bindseil (2000) to study central bank’s liquidity management or the model used in Quirós and Mendizábal (2001) to analyze the daily market for funds in Europe) but problematic in the case of the United States.
and the second
\[ i_2 = \frac{1}{\delta_2} (\gamma_2 R - r_2 - \alpha_2 r_1 - \beta_2 r_3^0) \] (7)

Both expressions remain positive for \( \gamma_1 \) and \( \gamma_2 \) sufficiently large or, in other words, for higher sensitivity of reserves held by commercial banks with respect to the level of required reserves. Interest rates are relatively lower if commercial banks hold more reserves and if required reserves decline.

Using the balance sheet identity we can write the above expressions in terms of OMOs, autonomous factors and standing facilities:

\[ i_1 = \frac{1}{\delta_1} (\gamma_1 R - \beta_1 (m_2 - a_2^0) - (m_1 - a_1 + l f_1 - d f_1) - \alpha_1 r_0) \] (8)

\[ i_2 = \frac{1}{\delta_2} (\gamma_2 R - (m_2 - a_2 + l f_2 - d f_2) + \alpha_2 (m_1 - a_1 + l f_1 - d f_1) - \beta_2 r_3^0) \] (9)

where we have made use of the assumptions regarding the non-stochastic character of open market operations (A3) and the zero expected value of standing facilities (A4) made in the previous section.

Overnight interest rates are lower if there is more liquidity available in the market due to OMOs, net liquidity-providing autonomous factors (negative \( a_1 \)'s) and lending facility. Lower required reserves also reduce market interest rates.

In subsequent sections we will calculate the size of open market operations necessary to keep interest rates given by expressions (8) and (9) as close as possible to the target rate \( i \). We will do this first under the assumption that the central bank intervenes in the reserve market twice - that will be our multiple (or continuous) intervention benchmark. Secondly, we will explore the consequences of intervening only once within the maintenance period.

5 Multiple open market operations

The central bank, that wants to use open market operations to minimize volatility of interest rates around the operating target, has to solve the following stochastic optimization problem:

\[ \min_{m_1, m_2} L = E[(i_1 - i)^2 + (i_2 - i)^2] \]

subject to conditions (8), (9) and \( r_0 = r_3^0 = 0 \) (11)

The last constraint (assumption (A5)) is added to improve the transparency of the analysis and is justified if we restrict our attention to the relationship between interest rates and reserves held within a single maintenance period.\(^{23}\)

The sequence of events is as follows: before the beginning of the maintenance period the central bank calculates its forecasts of autonomous factors for both

\(^{23}\)\( r_0 \) is the level of reserves held in the previous maintenance period and \( r_3^0 \) is the expected level of reserves on the first day of the following maintenance period.
days and makes them public. The level of required reserves as well as the target interest rate are also known to market participants. The central bank has to decide on its operations before it observes the stochastics of the autonomous factors on the first day of the maintenance period.

It can be shown that the optimal solution to this problem reads as follows:

\[
m_1^* = a_1^* + \alpha_1^* \bar{\gamma} - \bar{\gamma} R + \beta_1 \bar{\delta} \frac{\bar{\gamma} R}{\beta_1 \bar{\alpha}} i \tag{12}
\]

\[
m_2^* = a_2^* + \alpha_2^* \bar{\gamma} - \bar{\gamma} R + \beta_2 \bar{\delta} \frac{\bar{\gamma} R}{\beta_2 \bar{\alpha}} i \tag{13}
\]

Under the assumptions \(\beta_1 \alpha_2 - 1 > 0, \beta_1 \gamma_2 - \gamma_1 > 0, \alpha_2 \gamma_1 - \gamma_2 > 0, \beta_1 - \beta_2 \beta_1 < 0, \) \(\) and \(\delta_2 - \alpha_2 \delta_1 < 0 (or \beta_1 \alpha_2 - 1 < 0, \beta_1 \gamma_2 - \gamma_1 < 0, \alpha_2 \gamma_1 - \gamma_2 < 0, \delta_1 - \beta_2 \beta_1 > 0 \) and \(\delta_2 - \alpha_2 \delta_1 > 0) \) the size of OMOs increases with required reserves and decreases with the target interest rate. These conditions are satisfied for the Eurozone, where \(\gamma's \) and \(\alpha's \) are small in relation to \(\gamma's \), in other words, where the level of required reserves has a much stronger impact on day-t reserves’ holdings than the size of reserves held on other days of the maintenance period (see equations (4) and (5)). Furthermore, if it happens that \(\delta_1 = \delta_2 \beta_1 \) and \(\beta_1 \alpha_2 - 1 = \beta_1 \gamma_2 - \gamma_1 = \alpha_2 \gamma_1 - \gamma_2 = 1 \) then the amount of liquidity supplied by the central bank actually corresponds to what the ECB terms as ‘estimated liquidity needs’.

Substituting results (12) and (13) into equations (8) and (9) we can calculate resulting market interest rates:

\[
i_1(m_1^*, m_2^*) = i + \frac{1}{\delta_1}(a_1 - a_1^* + df_1 - l f_1) \tag{14}
\]

\[
i_2(m_1^*, m_2^*) = i + \frac{1}{\delta_2}(a_2 - a_2^* + df_2 - l f_2 + \alpha_2 (a_1 - a_1^* + df_1 - l f_1)) \tag{15}
\]

If the central bank implements the optimal-size operations, market interest rates will be equal to the target rate on average. They might deviate from the target if the central bank cannot forecast autonomous factors accurately and financial institutions cannot use standing facilities to offset unexpected liquidity fluctuations, or if financial institutions manage their reserves badly and must use standing facilities. Deviations from the target rate are reduced the higher the interest rate sensitivity of reserves held by commercial banks.

We can also calculate the actual loss due to the volatility of autonomous factors, conditional on OMOs being implemented optimally. This value will be useful for further comparisons. For homoscedastic autonomous factors (assumption \(A2\)):

\[
L(m_1^*, m_2^*) = E(i_1(m_1^*, m_2^*) - i)^2 + E(i_2(m_1^*, m_2^*) - i)^2
= \frac{\delta_1^2 + \delta_2^2 (1 + \alpha_2)}{\delta_1^2 \delta_2} \sigma_a^2 + \frac{\delta_1^2 + \alpha_2^2 \delta_2^2}{\delta_1^2 \delta_2^2} E(df_1 - l f_1)^2 + \frac{1}{\delta_2^2} E(df_2 - l f_2)^2 \tag{16}
\]

The loss function increases with the variance of autonomous factors and the unpredictable recourse to standing facilities (unless the net recourse is zero\(^{24}\)).

\(^{24}\)As is in fact the case in the Eurozone (see footnote 21).
We will now move to the case of non-continuous interventions, implemented on day 1 and day 2 of the reserve maintenance period, respectively.

6 Single open market operations

6.1 OMOs on the first day of the reserve maintenance period

If the operational framework of the central bank presumes open market operations only on the first day of the maintenance period, that implies $m_2 = 0$, which is assumed to be known to all market participants. In this case, the interest rate equations (8) and (9) under assumption (A5) are as follows:

$$i_1 = \frac{1}{\delta_1}(\beta_1 a_2^e - (m_1 - a_1 + lf_1 - df_1) + \gamma R) \quad (17)$$

$$i_2 = \frac{1}{\delta_2}(-(-a_2 + lf_2 - df_2) - \alpha_2(m_1 - a_1 + lf_1 - df_1) + \gamma R) \quad (18)$$

Offsetting interest rates fluctuations on both days by intervening only once is feasible since both interest rates depend on the liquidity supplied through $m_1$.

The optimal size of day 1 open market operations is:

$$m_1^{**} = a_1^e + \frac{\alpha_2 \delta_1 + \alpha_2 \delta_2}{\delta_2 + \alpha_2 \delta_1} a_2^e + \frac{\delta_1 \gamma + \alpha_2 \delta_2}{\delta_2 + \alpha_2 \delta_1} R - \delta_1 \delta_2 \frac{\delta_2 + \alpha_2 \delta_1}{\delta_2 + \alpha_2 \delta_1} R \quad (19)$$

In comparison to equation (12), the effect of autonomous factors, required reserves and the target interest rate on OMOs is now more clear-cut: the necessary provision of funds is higher for a higher expected liquidity withdrawal due to autonomous factors (on both days!), a higher level of required reserves and a lower target interest rate. The optimal size of open market operations depends now more heavily on interest rate elasticities $\beta_1$ and $\beta_2$.

If the central bank implements the optimal-size operations, interest rates in the reserve market will be:

$$i_1(m_1^{**}) = \frac{\delta_1 \gamma + \alpha_2 \delta_2}{\delta_2 + \alpha_2 \delta_1} i + \frac{\alpha_2 \delta_1 (\beta_1 a_2^e - (m_1 - a_1 + df_1 - df_1)) + \alpha_2 \delta_2 (\gamma a_2 - \gamma_1)}{\delta_2 + \alpha_2 \delta_1} R + \frac{1}{\delta_1} (a_1 - a_1^e + df_1 - df_1) \quad (20)$$

$$i_2(m_1^{**}) = \frac{\delta_2 \gamma + \alpha_2 \delta_1}{\delta_2 + \alpha_2 \delta_1} i - \frac{\delta_2 (\beta_2 a_2^e - (m_1 - a_1 + df_1 - df_1)) - \delta_2 (\gamma a_2 - \gamma_2)}{\delta_2 + \alpha_2 \delta_1} R + \frac{1}{\delta_2} (a_2 - a_2^e + df_2 - df_2 + \alpha_2 (a_1 - a_1^e + df_1 - df_1)) \quad (21)$$

Comparing to the equilibrium interest rates in the previous section, given by equations (14) and (15), interest rates resulting from single operations are not

---

25 An in-depth analysis of the results is presented in the following section.
equal to the target rate even on average:

\[
E(i_1(m_1^{**}) - i) = \frac{\alpha_2 \delta_2 (\delta_2 - \alpha_2 \delta_1)}{\delta_2^2 + \alpha_2^2 \delta_1^2} i + \frac{\alpha_2 \delta_1 (\beta_1 \alpha_2 - 1)}{\delta_2^2 + \alpha_2^2 \delta_1^2} a_2^e + \frac{\alpha_2 \delta_1 (\gamma_1 \alpha_2 - \gamma_2)}{\delta_2^2 + \alpha_2^2 \delta_1^2} R
\]

\[
= \frac{\alpha_2 \delta_1}{\delta_2^2 + \alpha_2^2 \delta_1^2} \left( (\delta_2 - \alpha_2 \delta_1) i + (\beta_1 \alpha_2 - 1) a_2^e + (\gamma_1 \alpha_2 - \gamma_2) R \right)
\]

(22)

\[
E(i_2(m_1^{**}) - i) = -\frac{\delta_2}{\delta_2^2 + \alpha_2^2 \delta_1^2} \left( (\delta_2 - \alpha_2 \delta_1) i + (\beta_1 \alpha_2 - 1) a_2^e + (\gamma_1 \alpha_2 - \gamma_2) R \right)
\]

(23)

Average control errors are non-zero and have the opposite direction: if the central bank overshoots its target on the first day, it will undershoot the target on the second day of the maintenance period and vice versa.\(^{26}\) The magnitude of errors depends on relations between parameters of the model. In particular, control errors vanish if \(\delta_2 - \alpha_2 \delta_1 = \beta_1 \alpha_2 - 1 = \gamma_1 \alpha_2 - \gamma_2 = 0\) or \(\tilde{R}(m_1^{**}) = \frac{\delta_2 - \alpha_2 \delta_1 + (\beta_1 \alpha_2 - 1) a_2^e}{\gamma_1 \alpha_2 - \gamma_2}\). Adjusting the level of required reserves towards \(\tilde{R}(m_1^{**})\) reduces control errors. For \(\beta_1 \alpha_2 < 1\) and \(\gamma_1 \alpha_2 - \gamma_2 < 0\) the level of required reserves \(\tilde{R}(m_1^{**})\) increases in the expected liquidity withdrawal due to autonomous factors on the second day of the maintenance period (on the day, when there are no OMOs).

The mean square error loss does no longer arise exclusively from unforeseeable factors; it is also due to control errors:

\[
L(m_1^{**}) = L(m_1^*, m_2^*) + \frac{1}{\delta_2^2 + \alpha_2^2 \delta_1^2} \left( (\delta_2 - \alpha_2 \delta_1) i + (\beta_1 \alpha_2 - 1) a_2^e + (\gamma_1 \alpha_2 - \gamma_2) R \right)^2
\]

(24)

The loss due to foregoing the second-day OMOs is larger than the loss under multiple operations unless control errors cancel out. Therefore, in some circumstances, it is not possible to limit the frequency of interventions in the reserve market without major increase in the volatility of interest rates. However, the difference between the interest rate volatility levels \(L(m_1^{**})\) and \(L(m_1^*, m_2^*)\) decreases with the interest rate sensitivity of reserves held by commercial banks.

6.2 OMOs on the second day of the reserve maintenance period

Similarly, if open market operations are carried out solely on the final day of the maintenance period, interest rates in the overnight market are given by:

\[
i_1 = \frac{1}{\delta_1} (\gamma_1 R - \beta_1 (m_2 - a_2^e) - (a_1 + l f_1 - d f_1))
\]

(25)

\[
i_2 = \frac{1}{\delta_2} (\gamma_2 R - (m_2 - a_2 + l f_2 - d f_2) - a_2 (a_1 + l f_1 - d f_1))
\]

(26)

\(^{26}\)At first sight this result might seem surprising, but its origin is simple. The FOC condition for minimising the loss function given by (10) is \(\frac{\partial E}{\partial m_1} = 2E(i_1 - i) \frac{\partial i_1}{\partial m_1} + 2E(i_2 - i) \frac{\partial i_2}{\partial m_1} = 0\). Since the derivatives \(\frac{\partial i_1}{\partial m_1}\) and \(\frac{\partial i_2}{\partial m_1}\) have the same sign, the average error terms must be of opposite signs to satisfy the FOC condition.

14
If the informational structure of the problem remains the same as in the previous sections (the central bank has to pre-commit to the size of OMOs before the start of the maintenance period or data on the actual realization of autonomous factors is available with considerable lags), the optimal size of single open market operations executed on the final day of the maintenance period is:

\[ m_{2}^{**} (a_i^m) = a_{2}^e + \frac{\alpha_2^{\gamma_2} + \beta_2^{\gamma_2} \gamma_2 + \delta_1^{\gamma_2}}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, R - \frac{\delta_1^{\gamma_2} \alpha_2}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, i \]  

(27)

This formula mirrors the result from the previous subsection, except that the central bank now fully neutralizes forecasted liquidity absorbing factors on day 2. If, on the other hand, the bank acts after having observed autonomous factors on the first day of the maintenance period, it can take this information into account and adjust the liquidity in the reserve market according to the formula:

\[ m_{2}^{**} (a_1) = a_{2}^e + \frac{\alpha_2^{\gamma_2} + \beta_2^{\gamma_2} \gamma_2 + \delta_1^{\gamma_2}}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, a_1 + \frac{\delta_1^{\gamma_2} \alpha_2}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, R - \frac{\delta_1^{\gamma_2} \alpha_2}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, i \]

where the expected value \( a_1^m \) was replaced by the observed value of autonomous factors.

Differences arising from the timing of the central bank’s decision on the size of OMOs have a more profound effect on interest rates in the overnight market. If the bank’s strategy is to pre-commit before the maintenance period begins, interest rates will settle at:

\[
\begin{align*}
    i_1 (m_2^{**}; a_1^m) &= \frac{\beta_2^{\gamma_2} + \beta_2^{\gamma_2} \gamma_2 + \delta_1^{\gamma_2}}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, i + \frac{\delta_1^{\gamma_2} (\gamma_2 - \gamma_2)}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, R - \frac{\delta_1^{\gamma_2} \alpha_2}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, a_1^m + \\
    &+ \frac{1}{\alpha} \left( a_1 - a_1^m - l f_1 + d f_1 \right) \\
    i_2 (m_2^{**}; a_1^m) &= \frac{\beta_2^{\gamma_2} + \beta_2^{\gamma_2} \gamma_2 + \delta_1^{\gamma_2}}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, i + \frac{\delta_1^{\gamma_2} (\gamma_2 - \gamma_2)}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, R + \frac{\delta_1^{\gamma_2} \alpha_2}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, a_1^m + \\
    &+ \frac{1}{\alpha} \left( a_2 - a_1^m - 2f_2 + 2f_2 + \alpha_2 \left( (a_1 - a_1^m) - l f_1 + d f_1 \right) \right)
\end{align*}
\]

(28)

However, if the bank’s strategy is to react flexibly to the information available throughout the maintenance period, interest rates will not depend on the forecast \( a_1^m \) but only on the true value \( a_1 \). The forecast error \((a_1 - a_1^m)\) will also vanish:

\[
\begin{align*}
    i_1 (m_2^{**}; a_1) &= \frac{\beta_2^{\gamma_2} + \beta_2^{\gamma_2} \gamma_2 + \delta_1^{\gamma_2}}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, i + \frac{\delta_1^{\gamma_2} (\gamma_2 - \gamma_2)}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, R - \frac{\delta_1^{\gamma_2} \alpha_2}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, a_1 + \\
    &- \frac{1}{\alpha} (l f_1 - d f_1) \\
    i_2 (m_2^{**}; a_1) &= \frac{\beta_2^{\gamma_2} + \beta_2^{\gamma_2} \gamma_2 + \delta_1^{\gamma_2}}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, i + \frac{\delta_1^{\gamma_2} (\gamma_2 - \gamma_2)}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, R + \frac{\delta_1^{\gamma_2} \alpha_2}{\beta_2^{\gamma_2} + \delta_1^{\gamma_2}} \, a_1 + \\
    &+ \frac{1}{\alpha} \left( a_2 - a_1 - 2 f_2 + 2 f_2 + \alpha_2 (d f_1 - l f_1) \right)
\end{align*}
\]

(30)

As in the previous subsection, overnight interest rates are not equal to the target rate even on average. Average control errors actually do not depend on
the timing of the central bank’s decision and are given as:

\[
E(i_1(m_{2}^{**}) - i) = \frac{\delta_1(\beta_1^2 - \delta_1)}{\rho_1^2 + \zeta_1^2} i + \frac{\delta_1(\gamma_1 - \beta_1^2 \gamma_2)}{\rho_1^2 + \zeta_1^2} R + \frac{\delta_1(1 - \beta_1 \alpha_2)}{\rho_1^2 + \zeta_1^2} a_1^e \\
= \frac{\delta_1(\beta_1^2 - \delta_1)}{\rho_1^2 + \zeta_1^2} ((\beta_1 \delta_2 - \delta_1) i + (\gamma_1 - \beta_1 \gamma_2) R + (1 - \beta_1 \alpha_2) a_1^e) \tag{32}
\]

\[
E(i_2(m_{2}^{**}) - i) = -\frac{\delta_1(\beta_1^2 - \delta_1)}{\rho_2^2 + \zeta_2^2} i - \frac{\delta_1(\gamma_1 - \beta_1 \gamma_2)}{\rho_2^2 + \zeta_2^2} R - \frac{\delta_1(1 - \beta_1 \alpha_2)}{\rho_2^2 + \zeta_2^2} a_1^e \\
= -\frac{\delta_1(\beta_1^2 - \delta_1)}{\rho_2^2 + \zeta_2^2} ((\beta_1 \delta_2 - \delta_1) i + (\gamma_1 - \beta_1 \gamma_2) R + (1 - \beta_1 \alpha_2) a_1^e) \tag{33}
\]

The errors are again opposite in direction and they vanish in two cases: if \(\beta_1 \delta_2 - \delta_1 = \gamma_1 - \beta_1 \gamma_2 = 1 - \beta_1 \alpha_2 = 0\) or if \(\tilde{R}(m_{2}^{**}) = \frac{(\beta_1 \delta_2 - \delta_1) i + (1 - \beta_1 \alpha_2) a_1^e}{\gamma_1 - \beta_1 \gamma_2}\).

Again, by manipulating the level of required reserves, the central bank can limit control errors. \(\tilde{R}(m_{2}^{**})\) is increasing in the expected liquidity withdrawal due to autonomous factors on the day, when the central bank is inactive.

The interest rate volatility due to stochastic factors and control errors is in this case given as:

\[
L(m_{2}^{**}; a_1^e) = L(m_1^1, m_2^2) + \frac{\delta_1(\beta_1^2 - \delta_1)}{\rho_1^2 + \zeta_1^2} ((\beta_1 \delta_2 - \delta_1) i + (\gamma_1 - \beta_1 \gamma_2) R + (1 - \beta_1 \alpha_2) a_1^e)^2 \tag{34}
\]

under the pre-commitment strategy and:

\[
L(m_{2}^{**}; a_1^e) = L(m_2^2; a_1^e) - \frac{s_1^2 + \alpha_2^2 \zeta_2}{\rho_1^2 + \zeta_1^2} \sigma_a^2 \tag{35}
\]

under the flexible strategy.

Although the average deviation from the target interest rate does not depend on the timing of decision-making, the loss due to the volatility of overnight interest rates does. It is lower if the central bank can postpone its decision on the size of the second-day OMOs until it can observe autonomous factors on the previous day. Nevertheless, differences between the interest rate volatility levels, conditional on double or single OMOs, decrease with the interest rate sensitivity of reserves held by commercial banks.

7 Comparative analysis of the results

In this section we will investigate the sensitivity of the results to changes in the interest rate, in order to shed some light on possible implications of averaging provisions on the desired frequency of OMOs. Averaging provisions mean that commercial banks do not have to fulfill reserve requirements exactly every day of the maintenance period but only on average. This implies that the interest rate sensitivity of reserves demanded \(\delta_1\) in equation (3), reproduced below, increases.\(^{28}\)

\[
r_t = -\alpha_r r_{t-1} - \beta_t r_{t+1}^e + \gamma_t R - \delta_t \theta_t
\]

\(^{27}\)Since we have assumed that \(E(a_t) = a_1^e\) for \(t = 1, 2...T\), where \(a_1^e\) is the central bank’s forecast (assumption (A1)).

If $\beta_1$ and $\alpha_2$ are small enough, so that $\beta_1 \alpha_2 < 1$ (as in the case of the Eurozone), the size of single transactions in the multiple operations’ framework increases with the interest rate sensitivity on the day of operations and decreases with the interest rate sensitivity on the other day. The positive relation is due to the fact that both interest rates are negatively related to the size of open market operations and the magnitude of this derivative decreases with the interest rate sensitivity of the same day’s reserves.\footnote{From equations (8) and (9): $\frac{\partial i_1}{\partial m_1} = -\frac{1}{\bar{\beta}_1}$ and $\frac{\partial i_2}{\partial m_2} = -\frac{1}{\bar{\beta}_2}$.} Therefore, for the higher interest rate sensitivity the central bank must provide more liquidity in order to reduce interest rates by the same number of percentage points. The negative relation between the optimal OMOs and the other day’s interest rate sensitivity also stems from the characteristics of interest rates’ derivatives with respect to elasticities $\bar{\gamma}_t$; in this case an increase in interest rate sensitivities reduces the effectiveness of OMOs carried out on one day have in reducing the interest rate on the other day of the maintenance period. If $\alpha_2 < 1$ and $\beta_1 < 1$ the positive effect of the interest rate elasticities on the optimal OMOs outweighs the negative effect and the total liquidity provided through multiple operations increases with the elasticities. Deviations of overnight interest rates from the target rate $i$ under the double OMOs’ regime decrease in interest rate sensitivities (see equations (14) and (15)). Therefore the interest rate volatility around the target rate also decreases with interest rates elasticities (equation (16)). We would expect these results to be fairly general and to remain valid for reserve maintenance periods longer than $T = 2$ (especially if parameters $\alpha_t$ and $\beta_t$ remain much smaller than unity).

Under the single operations’ framework, relations between the optimal OMOs carried out by the central bank and interest rate elasticities depend on autonomous factors, the target interest rate and, which is the most interesting, required reserves. In particular, if $\beta_1$ and $\alpha_2$ are small, the sensitivity of reserves demanded by commercial banks with respect to the level of required reserves $\gamma_t$ is high (and relatively constant, so that $\alpha_2 \gamma_1 - \gamma_2$ and $\gamma_2 \beta_1 - \gamma_1$ are negative) and required reserves are high, then single OMOs behave like the corresponding transactions under the multiple operations’ framework: they increase with the interest rate sensitivity on the day of operations and decrease with the interest rate sensitivity on the other day. These results are generally valid for small values of $\alpha_t$ and $\beta_t$, especially since the derivatives of the optimal OMOs carried out on day $t$ with respect to the interest rate sensitivity $\bar{\gamma}_t$ are positively related to the size of the forecasted liquidity withdrawal due to autonomous factors on days other than $t$.

Furthermore, if $\alpha_2 \gamma_1 - \gamma_2 < 0$ and $\gamma_2 \beta_1 - \gamma_1 < 0$, the derivatives of average deviations of overnight rates from the target (or average control errors) on the second day of the maintenance period are decreasing in required reserves. Therefore if required reserves are high it is very likely that the average deviation of the overnight rate from the target on the second day of the maintenance period will decrease in interest rate elasticities. Under the same circumstances, control...
errors on the first day of the maintenance period are increasing in interest rate elasticities if OMOs are carried out on the last day of the maintenance period; if OMOs are carried out on the first day, the errors are decreasing in $\delta_1$ and increasing in $\delta_2$. If we extend the maintenance period to include more days, the average deviations of overnight rates from the target are likely to decrease in the interest rate sensitivity on the day of operations $t$ and increase with the interest rate sensitivity on days $t-1$ and $t+1^{31}$. provided that required reserves are large enough to steer the derivatives in question.

The behavior of interest rate volatility due to single OMOs relative to the double operations’ benchmark depends on the size and sign of two expressions: $x = (\beta_2 - \alpha_2 \beta_1) i + (\alpha_1 \beta_2 - \gamma_2) R$ and $y = (\beta_1 \delta_2 - \delta_1) i + (\gamma_1 - \beta_1 \gamma_2) R + (1 - \beta_1 \alpha_2) a_1$. For each relative volatility levels there are four different regions that the central bank may operate in, in which the characteristics of the loss function differ. For intermediate interest rate elasticities the central bank is most likely to operate in regions X2, X3, Y2 and Y3, where the effect of elasticities on volatility levels is mixed. However, under the assumption $\alpha_2 \gamma_1 < \gamma_2 \beta_1 < \gamma_1$ and for large required reserves, the central bank may move to the regions X4 and Y1, where relative volatility levels are decreasing in interest rate elasticities. We would expect the excess interest rate volatility due to infrequent OMOs to be decreasing in the interest rate elasticities for longer reserve maintenance periods as well, provided that $\alpha$’s and $\beta$’s are relatively small, whereas $\gamma$’s and required reserves $R$ are large.

<table>
<thead>
<tr>
<th>Region</th>
<th>$\frac{\partial (L(m_1^*) - L(m_1, m_2))}{\partial s_1}$</th>
<th>$\frac{\partial (L(m_1^*) - L(m_1, m_2))}{\partial s_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>$x &gt; \frac{(\delta_2 + \alpha_2 \beta_1)}{\delta_2}$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; x &lt; \frac{(\delta_2 + \alpha_2 \beta_1)}{\delta_2}$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>X2</td>
<td>$-\frac{(\delta_2 + \alpha_2 \beta_1)}{\delta_2} &lt; x &lt; 0$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>X3</td>
<td>$x &lt; \frac{(\delta_2 + \alpha_2 \beta_1)}{\delta_2}$</td>
<td>$&lt; 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Region</th>
<th>$\frac{\partial (L(m_2^* - m_1^<em>) - L(m_1^</em>, m_2^*))}{\partial s_1}$</th>
<th>$\frac{\partial (L(m_2^* - m_1^<em>) - L(m_1^</em>, m_2^*))}{\partial s_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>$y &gt; \frac{(\beta_1 \delta_2 + \delta_1)}{\beta_1}$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; y &lt; \frac{(\beta_1 \delta_2 + \delta_1)}{\beta_1}$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>Y2</td>
<td>$-\frac{(\beta_1 \delta_2 + \delta_1)}{\delta_1} &lt; y &lt; 0$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>Y4</td>
<td>$y &lt; -\frac{(\beta_1 \delta_2 + \delta_1)}{\delta_1}$</td>
<td>$&lt; 0$</td>
</tr>
</tbody>
</table>

Let us now briefly return to the issue of averaging provisions. The analysis above indicates that increased interest rate elasticities, coupled with the appropriate level of required reserves, may reduce differences in the volatility of overnight interest rates between the cases of intervening continuously and less frequently,

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31 The influence of interest rate sensitivities of reserves held on other days ($t-2, t+2, ...T$) will vanish. This result is due to the assumption that the interest rate on day $t$ can be affected only by OMOs carried out on days $t-1$, $t$ and $t+1$.  

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despite the presence of non-zero control errors. Higher interest rate elasticities also reduce the volatility due to the lack of timely information on the realized autonomous factors for the central bank which intervenes on the last day of the maintenance period. However, increased elasticities mean that relatively more liquidity must be provided through the infrequent open market operations.

We can use the above conclusions to explain a relatively small difference in the levels of overnight interest rates’ volatility in Eurozone and America; it might actually be reduced not only by an appropriate use of required reserves, as we have pointed out after Borio in section 2, but also by higher interest rate elasticities of reserves held by commercial banks in Europe.

8 Concluding remarks

In the introductory sections of this paper we have contrasted two issues: the variety of operational frameworks applied by central banks and the likeness in volatility levels of the short-term variable that they are aiming to control (the overnight interest rate). Further examination of two limiting cases - the European Central Bank and the Federal Reserve - revealed that a high frequency of operations in the reserve market does not necessarily reduce interest rate volatility substantially in comparison with a smaller frequency. Our conclusion was therefore that an appropriate design and implementation of other instruments of the operating framework (that is, required reserves and standing facilities) can compensate for the number of open market operations.

In the remainder of the paper we have explored analytically the relationship between the frequency of monetary policy operations and the volatility of overnight interest rates. We have found that continuous operations allow the central bank to keep market interest rates equal to the target rate on average. The average volatility, measured as the sum of squared deviations from the operating target, is positive due to the volatility of autonomous factors and the unpredictable recourse to standing facilities. Henceforth, even in this case the volatility of interest rates could be reduced by improving the quality of autonomous factors’ forecasts and by allowing an unlimited recourse to standing facilities, which alleviates problems arising from unexpected liquidity shocks. Less frequent operations may not keep market interest rates equal to the target rate on average. However, adjusting the level of required reserves so that it compensates the expected liquidity withdrawal due to autonomous factors on the day without interventions can limit these control errors. The interest rate volatility in the case of infrequent operations is larger than in the case of continuous operations if the control errors are nonzero. The effects of intervening only at the end of the maintenance period depend also on the available information. If data on the actual realization of autonomous factors over the first part of the maintenance period is readily available, so that the central bank can observe it before deciding on the size of open market operations\textsuperscript{32}, the volatility level can be reduced.

\textsuperscript{32} And so can everybody else in the economy.
The final section explores implications of a higher interest rate sensitivity of the demand for reserves for the model results. Increased interest rate elasticities help to reduce differences in the volatility of overnight interest rates between the cases of intervening more and less frequently. Therefore we have concluded that a relatively small difference in the levels of overnight interest rates’ volatility in Eurozone and America may be due to an appropriate design of required reserves by the ECB and higher interest rate elasticities of reserves held by commercial banks in the Eurozone.

In the introduction to this paper we have posed a couple of policy-related questions. Although we have to acknowledge that our findings are based on certain simplifications vis-a-vis actual practices (e.g. an implicit treatment of averaging provisions or abstracting from the effects of required reserves’ remuneration) we are still able to address them. The volatility of interest rates cannot be eliminated completely (although it can be reduced further) due to the stochastic character of factors affecting the reserve position of financial institutions (autonomous factors). Achieving a similar degree of the interest rates’ volatility for various frequencies of operations in the reserve market actually requires suitable operating frameworks, which should differ in the use of required reserves and standing facilities. It is possible to reduce the frequency of intervening in the reserve market through open market operations but it has to be coupled with the adjustments in required reserves and standing facilities.

9 Appendices. Regression results

The regressions were carried out on weekly data on the ECB’s operations. Therefore the reserve maintenance period is assumed to have four one-week sub-periods (which implies $T = 4$ for our model). The series used comprise current accounts, required reserves and EONIA, as published in the ECB monthly bulletin.

We have performed VEC estimations for two variants of a model, which assumes the cointegrating relation of the form $c_t = r_t - \gamma R_t + \delta i_t + \lambda$ and includes two (exogenous) stationary regressors $x_t$ and $y_t$. For the first variant we define the exogenous variables as $x_t = r_t - r_{t-1}$ and $y_t = r_{t+1} - r_t$, for the second: $x_t = r_{t+1} - R_t$ and $y_t = r_{t-1} - R_t$. Such procedure was chosen in order to emphasize the difference between the long-run relation between reserves held by commercial banks, required reserves and an interest rate and short-term adjustments involving lagged and future reserves.

For each week of the maintenance period separately we have estimated a

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33 I am very grateful to dr Robert Paul Berben for sharing his expertise on time series estimation with me.

34 The procedure assumes perfect foresight: $r_{t+1}^e = r_{t+1}$. 

20
VEC system of the following form\textsuperscript{35}:

\[
\begin{pmatrix}
\Delta r_t \\
\Delta R_t \\
\Delta i_t
\end{pmatrix}
= \begin{pmatrix}
\mu_1 \\
\mu_2 \\
\mu_3
\end{pmatrix}
+ \begin{pmatrix}
\eta_1 \\
\eta_2 \\
\eta_3
\end{pmatrix}
(r_{t-1} - \tilde{\gamma} R_{t-1} + \tilde{\delta} i_{t-1} + \lambda) +
\begin{pmatrix}
c_{rx} & c_{ry} & c_{Rx} & c_{Ry} & c_{ix} & c_{iy} \\
\end{pmatrix}
\begin{pmatrix}
x_{t-1} \\
y_{t-1}
\end{pmatrix}
+ \cdots \text{lagged differences} + \begin{pmatrix}
\epsilon_{rt} \\
\epsilon_{Rt} \\
\epsilon_{it}
\end{pmatrix}
\]

Estimations of type-1 models resulted in the following parameter values\textsuperscript{36}:

<table>
<thead>
<tr>
<th>WEEK 1</th>
<th>Lagged differences included</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>1.13</td>
<td>1.10</td>
<td>1.04</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>*</td>
<td>*</td>
<td>*</td>
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<td>11.26</td>
<td>4.74</td>
<td>7.61</td>
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<td>*</td>
<td>*</td>
<td></td>
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<tr>
<td>$\beta$</td>
<td>*</td>
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<td>6.12</td>
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<td>$\alpha$</td>
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<td>*</td>
<td>*</td>
<td></td>
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<td>$\beta$</td>
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</table>

\textsuperscript{35}As we have looked at 4-week maintenance periods over the years 1999-2002, the sample included at most 26 observations for each week $t = 1, 2, 3, 4$. Some series were shorter due to the unavailability of data during holiday times and the removal of aberrant observations during over- and underbidding episodes (see Section 2).

\textsuperscript{36}* indicates that the estimate in the first equation was not significant and was not used in the computation of the parameters of interest.
### WEEK 4

<table>
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<td>1.20</td>
<td>1.12</td>
<td>1.26</td>
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<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>α</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>β</td>
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<td>*</td>
<td>-0.49</td>
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**Summary Statistics**

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<td>16.61</td>
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<td>6.76</td>
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</table>

Estimations of type-2 models resulted in the parameter values:

### WEEK 1

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<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>1.69</td>
<td>1.20</td>
<td>1.76</td>
</tr>
<tr>
<td>δ</td>
<td>*</td>
<td>*</td>
<td>-0.58</td>
</tr>
<tr>
<td>α</td>
<td>0.24</td>
<td>0.19</td>
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<tr>
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**Summary Statistics**

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<td>4.60</td>
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</tbody>
</table>

### WEEK 2

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<th>2</th>
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<tbody>
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<td>γ</td>
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<tr>
<td>δ</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>α</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>β</td>
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**Summary Statistics**

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<td>6.27</td>
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</tbody>
</table>

### WEEK 3

<table>
<thead>
<tr>
<th>Lagged differences included</th>
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<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>1.52</td>
<td>0.90</td>
<td>*</td>
</tr>
<tr>
<td>δ</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>α</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>β</td>
<td>*</td>
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**Summary Statistics**

<table>
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<th>0.32</th>
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<td>3.82</td>
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In summary, empirical data supports the following numerical ranges for the parameters in equation (3):

<table>
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<tr>
<th>Week (t)</th>
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<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
<tr>
<td>$\alpha_t$</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\beta_t$</td>
<td>0-0.47</td>
<td>0</td>
<td>0</td>
<td>-0.49</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>1.04-1.76</td>
<td>0.90-0.94</td>
<td>0.90-1.52</td>
<td>1.16-1.64</td>
</tr>
<tr>
<td>$\delta_t$</td>
<td>-0.58</td>
<td>0-0.24</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

10 References

- "Comparing monetary policy operating procedures across the United States, Japan and the euro area", BIS Paper No 9, Bank for International Settlements, December 2001


- Quirós G.P. and H.R. Mendizábal "The daily market for funds in Europe: has something changed with the EMU?", ECB Working Paper No. 67, European Central Bank, June 2001


- Ueda K. "Japan’s Liquidity Trap and Monetary Policy", speech transcript, December 2001