Liquidity effects and market frictions

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LIQUIDITY EFFECTS AND MARKET FRICTIONS

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Abstract

The goal of this paper is to tackle two problems in standard limited-participation models: (1) an interest rate liquidity effect that is not as persistent as in the data; and (2) nominal variables that are unrealistically volatile. To address these problems, we introduce nominal wage and price rigidities, as well as portfolio adjustment costs and monopolistically competitive firms, to better understand how each of these costs affects the model economy.

Quantitative analysis shows that different adjustment costs have very different effects on the responses of the nominal interest rate, inflation and output. The model’s impulse response functions are more realistic in the case with all three adjustment costs than any other case. The main findings are: (1) limited-participation models generate too much liquidity effect compared with the data; (2) portfolio adjustment costs are solely responsible for the persistence of the liquidity effect following a monetary policy action; (3) wage adjustment costs can help to reduce inflation volatility and increase output and inflation persistence; (4) price adjustment costs are unable to significantly reduce inflation volatility; (5) there can be important interactions between the adjustment costs; (6) limited-participation models, even with additional adjustment costs, are still not successful in replicating the stylized facts for nominal variables.

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1 INTRODUCTION

Some economists argue that money plays a role in the economy due to its asymmetric distribution to economic agents. That is, money is first distributed to financial intermediaries and then to firms before it finally reaches consumers’ hands. This is the basic idea embedded in a standard limited-participation model. However, there are still some limitations with the basic version of this model. First, the liquidity effect is not as persistent as that observed in the data. Most empirical estimates find that the interest rate should fall for a few quarters following an expansionary money-growth shock. [See Figure 1.] Second, stochastic simulations of limited-participation models generally find too much volatility of inflation and other nominal variables.

As we know, monetary policy shocks are transmitted through agents’ decision-making processes via dynamic mechanisms, such as adjustment costs. If markets operated without any frictions, monetary policies would have no (persistent) effect on interest rates or any real variables. In addition, some economists have conjectured that price and wage rigidities may be a primary cause of the persistent liquidity effect of a monetary shock. 2

In this vein, Christiano, Eichenbaum, and Evans (1996) compare sticky-price models with limited-participation models. They conclude that any model equipped with only one type of friction cannot successfully account for the basic stylized facts unless unrealistic parameter values are assumed. Aiyagari and Braun (1997) speculate that a combination of the limited-participation model and a price-adjustment cost will lead to useful insights into business fluctuations. In this paper, we pursue our research along this avenue. More precisely, we attempt to determine the relative importance of three major frictions -- price, wage and portfolio adjustment costs -- in understanding the monetary transmission mechanism. Our model is based on the basic limited-participation model developed by Christiano and Eichenbaum (1992b).

This paper introduces different types of adjustment costs to investigate whether they improve the model’s ability to replicate some of the major

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2 Aiyagari (1997) points out that a modelling approach that considers frictions can be expected to have a significant impact on answers to questions of interest to macroeconomists and policymakers. See also Williamson (1996).
stylized facts of empirical impulse response functions and higher moments. First, portfolio-adjustment costs are introduced to prolong the interest-rate effects of a monetary policy shock. Second, nominal price and wage adjustment costs are also added to the model to dampen the volatility of the nominal side of the economy.\(^3\)

In general, the adjustment costs we introduce do improve the model, to some extent at least, in the expected manner. Portfolio adjustment costs are able to lengthen the interest rate liquidity effect while price costs and wage costs, in particular, are able to reduce inflation volatility following a monetary policy action.

Wage adjustment costs, which can be thought of as wage negotiation costs or possibly information accumulation costs, are particularly effective in lowering the volatility of inflation by slowing wage increases and, consequently reducing any wage cost-push incentives for firms to increase their prices. With no price costs in the model, the wage cost does not change the liquidity effect. However, if any price costs are present, wage costs will deepen the liquidity effect (i.e., lower the loan demand) by making it easier for firms to avoid the price costs. This last result is in contrast to results generally found in other sticky wage models.

Portfolio adjustment costs reduce the incentives for households to change the level of their cash holdings thereby limiting the adjustment of deposits and creating a persistent liquidity effect. The extended deviation of the interest rate from steady state induces a persistent deviation of inflation and output as well.

In contrast to the other costs, price adjustment costs (that is, marketing costs, advertising costs, or information accumulation costs) are less effective, having only a minimal impact on inflation and output when wage and portfolio adjustment costs have already been introduced into the model. In particular, models with portfolio costs made price costs much less effective at reducing inflation volatility. In examples with only price adjustment costs, there were reductions of inflation volatility, but mostly in expected future volatility not the contemporary response. For small

\(^3\) Dow (1995) has looked at the liquidity effects of monetary shocks by considering frictions in both commodity and credit markets. Unfortunately, his model does not lead to more persistent liquidity effects.
price adjustment costs there is a deepening of the interest-rate liquidity effect. However, as the costs are increased, the liquidity effect is reversed as firms try to avoid the costs by increasing hours and output and hence loan demand and the interest rate. In general, attempts by the firms to avoid the price adjustment costs lead to excessive interest-rate volatility in the model.

The rest of this paper is organized as follows. Section 2 provides a detailed description of the economic environment and the dynamic general equilibrium problem of money. Section 3 calibrates the model. The quantitative analysis is presented in Section 4. Finally, Section 5 summarizes the findings in this paper, and points to the direction of our future research.

2 THE MODEL

2.1 Economic environment

2.1.1 Households

The preferences for a typical household, \( i = 1, \ldots, I \), are given by:

\[
E_{it}\left\{ \sum_{t=0}^{\infty} \beta^t U(C_{it}^1 - L_{it}^s - AC_{it}^Q) \right\}
\]

(1)

where \( 0<\beta<1 \) and,

\[
U(C_{it}^1 - L_{it}^s - AC_{it}^Q) = \frac{1}{\phi} [C_{it}^1(1 - L_{it}^s - AC_{it}^Q)^{\gamma}]^\phi
\]

(2)

where \( C_{it}, L_{it}^s, \) and \( AC_{it}^Q \) are the period \( t \) consumption, labour supply, and time cost of portfolio adjustment, respectively. We assume that households must take time to adjust their financial portfolios between periods. This assumption is different from the intra-period adjustment costs assumption employed in Dotsey and Ireland (1995). As with the standard limited-participation models, households cannot adjust their portfolios within a period. The portfolio adjustment cost is assumed to be a convex function of the following form:
where $\varphi_q \geq 0$ and $Q_i$ is the amount of cash that household $i$ has at the beginning of period $t$. In deterministic steady state, the adjustment cost will be zero since all nominal variables will grow at the money growth rate, $1 + x$.\(^4\) As the growth rate of cash holdings deviates from its steady state level, the adjustment cost increases quadratically.

We assume that money is introduced into the economy by a cash-in-advance (CIA) constraint. That is, households make their consumption and investment purchases out of the sum of the nominal cash balance transferred from the last period and the labour income earned in the current period. This constraint is described in (4). The periodic budget constraint is given by (5), which says that cash and deposits carried forward to period $t+1$ is the sum of interest payments, dividends, capital income, and unspent cash from the goods market.\(^5\)

$$AC^{Q}_{it} = \frac{\varphi_q}{2} \left( \frac{Q_{i,t+1}}{Q_{i,t}} - (1 + x) \right)^2 \quad (3)$$

where $\varphi_q \geq 0$ and $Q_i$ is the amount of cash that household $i$ has at the beginning of period $t$. In deterministic steady state, the adjustment cost will be zero since all nominal variables will grow at the money growth rate, $1 + x$.\(^4\) As the growth rate of cash holdings deviates from its steady state level, the adjustment cost increases quadratically.

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$$P_t C_{i,t} + P_t I_{i,t} + P_t AC^{W}_{i,t} \leq Q_{i,t} + W_{i,t} I_{i,t} \quad (4)$$

$$Q_{i,t+1} + N_{i,t+1} = R_{kt}^t N_{i,t} + D_{it} + F_{it} + R_{kt}^t K_{i,t} + [Q_{i,t} + W_{i,t} I_{i,t} - P_t C_{i,t} - P_t I_{i,t} - P_t AC^{W}_{i,t}] \quad (5)$$

where $P_t$ is the price level; $W_t$ is the wage rate; $I_{i,t}$ is period $t$ investment; $N_{i,t}$ is the household’s level of demand deposits for period $t$ determined in period $t-1$;\(^6\) $D_{it}$ and $F_{it}$ are the dividends received from the firm and the financial intermediaries, respectively, at the end of period $t$; and $AC^{W}_{i,t}$ is the wage adjustment cost. The rental price of capital is given by $R_{kt}^t$ so that households earn $R_{kt}^t K_{i,t}$ from their capital stock in period $t$.

We assume that a worker has to bear a real cost to propose and realize a change in their nominal wages. We can think of this as a negotiating and information gathering costs. Even though the magnitude of the cost can

\(^4\) This functional form is similar to that in Christiano and Eichenbaum (1992).

\(^5\) Our model treats money primarily as a medium of exchange, even though other factors, such as variation in velocity, might have important roles in the monetary transmission mechanism. In addition, a binding CIA constraint is not imposed in this model.

\(^6\) The sum of cash and deposit holdings is equal to the money supply so that $\sum (Q_i + N_i) = M_c$. 

be small, the impact of the cost on aggregate labour supply might be significant. This wage adjustment cost is very different from the nominal wage rigidities reflected in the long-term contracts between workers and firms. In this model, the wage can be changed at any time but only after a cost is paid.

During period $t$, households take as given, from period $t-1$, their capital stock holdings ($K_{it}$) and their distribution of money holdings between deposits ($N_{it}$) and cash ($Q_{it}$). Assume that households must choose their period $t+1$ split of financial assets between cash ($Q_{it+1}$) and deposits ($N_{it+1}$) before the end of period $t$. This is the standard assumption of the limited-participation models.

The law of motion for the physical capital stock is given in equation (6).

$$K_{it+1} = (1 - \delta)K_{it} + I_{it}$$

(6)

where $\delta$ is the capital depreciation rate.

Wage adjustment costs are assumed to have the following functional form:

$$AC_{it}^w = \frac{W_t \phi_w}{P_t} \left( \frac{W_{it}}{W_{it-1}} - (1 + x) \right)^2$$

(7)

where $\phi_w \geq 0$. The inclusion of the real wage ensures the real value of the costs will grow with the economy. Since households have monopoly power over their differentiated labour, they recognize that their wage rate, $W_{it}$, is a function of their labour supply as shown below in (11) and account for this when maximizing their utility. Consequently, the wage adjustment cost function can be written as a function of $L_{it}$ by combining (7) and (11) below.

A household’s decision problem is to maximize utility given by (1) and (2) by selecting decision variables $Q_{it+1}$, $N_{it+1}$, $C_{it}$, $K_{it+1}$, and $L^2_{it}$ subject to the constraints in (3) to (7) and (11).
2.1.2 The final goods firm

The final goods producer in this economy is simply an aggregator which buys inputs from the intermediate goods producers and combines them into the final good. Also, the aggregator hires labour from the differentiated households to generate a composite labour commodity which is used by intermediate goods firms in their production process. The final good can either be consumed or invested. The production technology for the final good is summarized by the following constant return to scale function

\[ Y_t = J^{1-\theta} \left( \sum_{j=1}^{J} Y_{jt} \right)^{\frac{\theta}{\theta-1}} \]  

(8)

where \(Y_t\) is the output of the final good, \(Y_{jt}\) is the amount of input acquired from intermediate producer \(j\), and \(J\) is the number of intermediate firms. The parameter \(\theta\) is the elasticity of substitution between any two different intermediate goods.

The final goods producer’s profit maximization problem is given as

\[
\max \Pi_t = P_t Y_t - \sum_{j=1}^{J} P_{jt} Y_{jt}
\]  

(9)

From its first order condition for \(Y_{jt}\), we can easily derive the following simple relation between the price, \(P_t\), for the final good and the price, \(P_{jt}\), charged by the producer of the intermediate good \(j\).

\[
P_{jt} = P_t \left( \frac{Y_{jt}}{Y_t} \right)^{-\frac{1}{\theta}}
\]

(10)

Following the same logic, we assume this final good producer also buys heterogeneous labour, \(L_{it}\), from household \(i\) and sells a composite labour input to the intermediate goods producers. Therefore, the relationship between the competitive wage rate, \(W_t\), faced by the firms and the individual wage rate, \(W_{it}\), set by household \(i\) is given by
where $\theta_L$ is the elasticity of substitution between any two types of labour.\footnote{The composite labour is produced by the aggregator with constant return to scale technology,}

\begin{equation}
W_{it} = W_i \left( \frac{IL_{it}}{L_i} \right)^{\frac{1}{\theta_i}}
\end{equation}

\subsection{The intermediate goods firms}

Each intermediate good producer $j$ ($j=1,2,\ldots,J$) hires the aggregate labour commodity available from the aggregator, $L_{jt}$, in a perfectly competitive market at wage rate $W_t$. The firm also rents capital goods ($K_{jt}$) from the households, in a competitive market, at a rental price of $R_{kt}$. The firm must borrow cash from the financial intermediary each period, at interest rate $R_t$, in order to have the funds to hire labour and begin production. The entire wage bill of $W_tL_{jt}$ must be borrowed before production occurs. The capital rental cost is assumed to be financed through internally generated revenue. Capital and labour inputs are combined in an increasing returns to scale production function to produce a differentiated product, $Y_{jt}$.

The firm sells its output in a monopolistically competitive market in which it has the power to set its own price, $P_{jt}$. However, we assume it is costly for a firm to change the price of its product. These real costs can be thought of as menu, advertising, or marketing costs for the setting of new prices. Firm $j$'s objective is to maximize the present value of its lifetime stream of dividend payments to its shareholders, $D_{jt}$. These dividends are discounted by a factor, $\beta$, as well as by a term representing the marginal utility value of the dividends, $\lambda_{2t}$, for the households, which own the firms.\footnote{For simplicity, we will assume there is a unit mass of both households and firms.}
where

\[ D_{jt} = P_{jt} \cdot Y_{jt} - R_t \cdot W_t \cdot \frac{L_{jt}^d}{L_{jt}^d - R_{jt}} \cdot K_{jt} - P_t \cdot AC_{jt}^p \]

Let \( AC_{jt}^p \) represent a price adjustment cost which the firm must pay whenever it changes its price level at a rate different from the steady state growth rate. The price-adjustment cost function is described in equation (14).

\[ AC_{jt}^p = Y_t \cdot \left\{ \frac{\phi_p}{2} \frac{P_{jt}}{P_{jt-1}} \left( 1 + \frac{1}{\exp(\mu)} \right)^2 \right\} \]

where \( \phi_p \geq 0 \). The presence of the \( Y_t \) term will ensure the costs will grow with rest of the economy.

The production technology exhibits increasing return to scale with labour-augmented technological progress, that is,

\[ Y_{jt} = F(K_{jt}, L_{jt}) = (K_{jt}^{\alpha}(z_t L_{jt})^{1 - \alpha})^\Phi \]

where \( 0 < \alpha < 1 \) and \( \Phi \geq 1 \). The variable \( z_t \) represents labour-augmenting technological progress following the process

\[ z_t = \exp(\eta_t + \theta_t) \]

The parameter \( \eta \) is the steady state growth rate of the technology level in the economy and the technology shock, \( \theta_t \), is assumed to follow the random process,

\[ \theta_t = (1 - \rho_\theta)\theta + \rho_\theta \theta_{t-1} + \epsilon_{\theta_t} \]

This is a simple AR(1) process for which \( 0 < \rho_\theta < 1 \) and \( \epsilon_{\theta_t} \) is i.i.d. with zero mean and standard deviation \( \sigma_\theta \).
Firm $j$’s dividend-maximization problem can be characterized by a set of marginal conditions for labour and capital as given in Appendix II.

2.1.4 Financial intermediary

At the beginning of period $t$, the financial intermediary has demand deposits of $N_t$ that it received from the households in the previous period. The financial intermediary uses these funds, along with any transfer from the government, $X_t$, to make loans to the intermediate firms, $B_t^f$.

$$B_t^f = N_t + X_t$$  \hspace{1cm} (18)

At the end of each period, the financial intermediary pays back the household deposits, with interest, using the debt repayments collected from firms.\footnote{We assume that there is no risk of default in this model.} The objective of the financial intermediary is to choose the optimal amount of loans made to firms and the optimal level of demand deposits to maximize the expected present value of its dividend:

$$\max \left\{ E_0 \left[ \sum_{t=0}^{\infty} \beta^t \lambda_2 T_t \right] \right\}$$  \hspace{1cm} (19)

where the dividend is given by,

$$F_t = (1 + R^d_t)B_t^f - (1 + R^d_t)N_t$$  \hspace{1cm} (20)

Financial intermediation is assumed to be a costless activity. With no barriers to entry, competitive forces will ensure that the equilibrium interest rate on loans equals the rate paid on deposits, that is $R^l_t = R^d_t = R_t$. Consequently, in equilibrium, the financial intermediary will pay $F_t = (1 + R_t)X_t$ in dividends to the households.

2.1.5 The central bank and government

We assume that the government neither collects taxes nor issues debt. However, the government makes transfer, $X_t = M_{t+1} - M_t$, to the financial intermediaries through the central bank. In this case, new money is dis-
tributed directly by the central bank to only financial institutions. This restriction on how money is distributed provides the motivation for the non-neutrality of money in the short run in this model.

Assume that the central bank follows a money growth rate rule as described in equation (21)

\[ x_t = (1 - \rho_s)x + \rho_s x_{t-1} + \varepsilon_{xt} \]  

where \( x_t \) is the money growth rate, \( X_t/M_t \).

An exogenous positive monetary shock, \( \varepsilon_{xt} > 0 \), increases the funds available to the financial intermediary in period \( t \). Consequently, the financial intermediary responds by lending more to firms for the employment of labour. To ensure that firms will borrow these excess funds, the banks lower the equilibrium interest rate thereby creating the liquidity effect.

2.1.6 Market clearing

Assuming a unit mass of households and firms, the following equations describe the market-clearing conditions which must hold for an equilibrium to exist:

1. Goods:

\[ C_t + I_t + AC_t^w + AC_t^p = Y_t \]  

2. Loans:

\[ W_tL_t = N_t + X_t \]  

3. Money:

\[ M_{t+1} = M_t + X_t \]  

4. Labour:

\[ L_t^s = L_t^d \]  

10 In Canada, monetary expansions and contractions are carried out by the central bank’s cash management: drawdowns and redeposits of federal government deposits with direct clearers.
2.2 Equilibrium

A stationary competitive equilibrium is defined to be a sequence of allocations \(\{C_{it}, L^s_{it}, N_{it+1}, Q_{it+1}, I_{it}\}\) for households, \(\{K_{jt}, L^d_{it}\}\) for firms, a set of prices \(\{P_{it}, P^j_t, W^j_t, W_t, R_t, R_{kt}\}\), and a central bank reaction function \(\{x_t\}\) such that, given the equilibrium prices and other parties’ actions,

1. households choose \(\{C_{it}, L^s_{it}, N_{it+1}, Q_{it+1}, I_{it}, W_{it}\}\) to maximize (1) subject to constraints (2) to (7) and (11);

2. firms choose \(\{K_{jt}, L^d_{it}, P^j_t\}\) to maximize (12) subject to constraints (13) to (17);

3. financial intermediaries solve the maximization problem in (19);

4. prices adjust to ensure that market-clearing conditions hold: \(P_t\) clears the final-goods market, \(W_t\) clears the composite labour market, \(R_t\) clears the loan (or credit) market.

3 CALIBRATION

The balanced growth path of the model is calibrated to quarterly Canadian data for the 1955 to 1996 period. The mean annual growth rate of per capita output during this period was 1.83%, so we set \(\mu = 0.004563\) (1.83% = (exp(\(\mu\))-1)*4). The discount factor \(\beta\) was assumed to be 0.993, so the annual real rate of return on investment is about 2.8%. The annualized depreciation rate, \(\delta\), was set at 10% to approximately match the capital-output ratio observed in the Canadian data.

The parameters \(\theta_y\) and \(\theta_l\) represent the market power that a firm and a household have to adjust their price and wage, respectively. Smaller \(\theta\) values represent stronger market power. A constant-return-to-scale (CRS) economy would set these \(\theta\) values to infinity so that firms and households would be price and wage takers. However, with an increasing-return-to-scale (IRS) economy we choose values closer to those used in Kim (1996)
and set \( \theta_y = 5.0 \) and \( \theta_l = 10.0 \).\(^{11}\) The increasing return to scale parameter, \( \Phi \), is set at a moderate value of only 1.25. Finn (1995) has suggested that \( \Phi \) should not be too large because the volatility of real variables is decreasing in the degree of IRS. The value of \( \alpha = 0.3369 \) is chosen such that the steady state labour share of income is 0.65, which is found in the data.

The preference parameters \( \gamma = 3.8068 \) and \( \phi = -0.4213 \) are calibrated such that the steady state employment and consumption-output ratio are equal to 0.171 and 0.745, respectively, as observed in the Canadian data.\(^{12}\)

As described in equation (17), the technology shock is assumed to follow an AR(1) process. The autocorrelation parameter, \( \rho_\theta \), is set to be 0.95 based on the assumption that the technology shock is fairly persistent. The standard deviation of the technology shock is set so that the standard deviation of output from the model is close to that from the data (0.0166).

The monetary policy parameters describing the money growth rate process from equation (21) are calibrated to match the quarterly mean growth rate and autocorrelation coefficient of the Canadian money base. This implies \( \alpha = 0.011 \) and \( \rho_x = 0.18 \). The standard deviation of the monetary policy shock is set such that the variance of money growth rate from the model is close to that from the data (0.0096).

For the adjustment-cost parameters, which do not affect the steady state, we assume \( \phi_p = 1, \phi_w = 10, \) and \( \phi_q = 1 \). If the model’s time endowment of one unit per period represents 1460 hours per quarter in real time about 16 hours per day, then \( \phi_q = 1 \) indicates that a household spends an extra 4.38 minutes on portfolio adjustment when the desired change in cash holdings deviates by 1% from the steady state change. A value of \( \phi_w = 10 \) implies that a worker pays 0.05% of his/her quarterly real wage rate to change the nominal wage rate by 1% different than the steady state change. Similarly, if a firm wants to adjust its price level by 1% more than the steady inflation rate, it has to pay a cost equivalent to 0.005% of its quarterly output. These values were chosen to yield reasonably persistent impulse responses for a monetary policy action.

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\(^{11}\) Our model structure is fairly similar to that in Kim (1996).

\(^{12}\) The consumption-output ratio may seem high since government and private consumption have been lumped together for simplicity.
4 RESULTS

4.1 Some stylized facts on liquidity effects in Canada

We present a vector autoregressive analysis of the Canadian data (1955Q1 to 1996Q4) in Figure 1. These basic results summarize the more detailed analysis from Fung and Kasumovich (1998). Following a one standard deviation shock to M1 growth, the short-term nominal interest rate falls immediately by about 30 basis points. In the subsequent quarters, this liquidity effect becomes dominated by the expected inflation effect and the interest rate starts to increase gradually. After about eight quarters, the interest rate moves above steady state. The various policy shock identification techniques usually estimate liquidity effects that last between three and six quarters for Canada.

The low interest rate leads to an extended expansion of output lasting for three to four years (following an initial perverse decline). The policy shock has an immediate impact on inflation in the first quarter. The inflation responds immediately then falls back toward steady state before jumping up again to another peak about six quarters following the shock. The deviation of inflation from steady state persists for much longer than output. Other VAR identification techniques can find no initial response of inflation followed by an extended increase above steady state that does not start until as many as four quarters after the shock.

4.2 Expansionary policy action: impulse response functions

This section analyses the model through a discussion of the model’s impulse responses to a one period monetary policy action. The innovation, $\varepsilon_{xt}$, in equation (21) is set to 0.01, representing a 4 per cent annualized money growth rate shock. The impulse response functions for the interest rate, inflation rate, and output are plotted in each of Figures 2 to 7 for different adjustment cost assumptions. Figures 2, 3, and 4 show the impulse responses when portfolio, wage, and price adjustment costs, respectively, are added to the basic limited-participation model. Figure 5 shows the model responses when all of the costs are combined. To illustrate that the model can respond differently depending on the combination of costs assumed, Figure 6 shows the effects of adding wage costs when
there are also price costs present while Figure 7 adds price costs to a model with portfolio costs.

Portfolio adjustment costs:

The base case in Figure 2 is a model with just the limited-participation assumption. This model generates too much liquidity effect compared with the Canadian data. In addition, the effects of a policy action on all variables are short lived.

However, once the inter-period portfolio adjustment cost is added all of the responses become much more persistent. The liquidity effect becomes very persistent because financial institutions stay flush with cash to lend for a much longer period. By holding the funds in the banking sector longer, the portfolio costs delay the demand side price pressure from the expansionary shock. Consequently, the inflation response is much lower and more persistent. The output response is also dampened and made much more persistent. The effects of this type of portfolio cost have been illustrated by other authors as well. The contribution of this paper lies in the addition of other nominal rigidities to the model.

Wage adjustment costs:

Figure 3 illustrates the effects of introducing the wage adjustment cost into the model when no other costs are present. The base case is the same as in Figure 2. The wage adjustment cost has no effect on the first period response of the interest rate but it does raise the rate in subsequent periods. The initial response of inflation is reduced by about one percentage point while the second period response is reduced by about 50 basis points. Subsequently, the inflation response is larger so that the overall effect of the wage cost is to delay the inflation response.

The output response is strengthened substantially and becomes much more persistent. The wage cost makes households more willing to supply labour. Therefore, for a given shock, there will be a larger increase in labour and output and less of an increase in wages and prices.

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13 All of the models discussed maintain the assumption that households cannot adjust their portfolios within a period. The case without portfolio costs assumes no inter-period adjustment cost ($\phi_q=0$) but still maintains the infinite intra-period cost.
Price adjustment costs:

Figure 4 adds price adjustment costs to the base case of no adjustment costs. These price costs deepen the first period liquidity effect but have little impact on the initial responses of output or inflation. The second period inflation response is reduced by about 50 basis points while subsequent periods are increased slightly. The output response is slightly larger and more persistent than the base case. The price adjustment cost makes labour demand more elastic and the firms’ loan demand slightly more inelastic. The cost also causes an overall increase in labour and loan demand as firms try to hire more labour to increase output and avoid paying the price cost. For small price costs, as illustrated, the net effect is a deepening of the initial period liquidity effect. However, as the cost becomes larger, the firms increased demand for labour and loans dominates and the interest rate increases sharply in the impact period. A sensitivity analysis revealed that the interest rate became unrealistically volatile long before the inflation response was dampened in any substantial manner as observed in the data.

Adjustment costs interactions:

Figure 5 shows the combined effect of all three adjustment costs on the model. The liquidity effect is deepened such that the interest rate is at least one percentage point below steady state for three quarters. The inflation response is reduced substantially over the first two periods and subsequently becomes much more persistent. The output response is much larger and more persistent than in the base case.

To illustrate that there can be important interactions between the adjustment costs, Figure 6 shows the effects of adding a wage cost to a model that already has a price adjustment cost. In comparison to the results shown in Figure 3, we see that the wage cost now deepens slightly the initial liquidity effect. This will only occur if there is a price cost present. The larger is the price cost, the more effective is the wage cost at deepening the initial period liquidity effect. The wage cost reduces the firms need to work to avoid paying the price cost. Consequently, there is a relative reduction in loan demand so that banks must reduce the initial interest rate by more to get firms to borrow the extra funds made available by the monetary injection.
Figure 7 shows the effects of adding a price adjustment cost when a portfolio cost is also present. Here, we see that the price cost has virtually no ability to reduce inflation volatility in contrast to the results shown in Figure 4. Also, the liquidity effect is now reduced in the first period while the previous analysis showed that the price cost could deepen the liquidity effect. In general, greater portfolio costs reduced the ability of the price costs to lower inflation volatility.

Overall, the adjustment costs improve the model’s impulse responses in the intended manner. However, the results were much more promising for the portfolio and wage costs than for the price costs. The former two costs reduce nominal volatility and make the responses of real and nominal variables much more persistent while the price costs have only a marginal effect on inflation and a much larger impact on the interest rate. The model also reveals some important interactions between the wage and price costs. Wage costs can deepen the liquidity effect but only if price costs are present.

4.3 Higher moments

This section analyses the higher moments of a number of different versions of the model to illustrate some of the relative contributions of each of the adjustment costs. Table 1 contains some summary statistics for Canada from 1955 to 1996 for the primary real and nominal variables. Some of the main stylized facts are: (1) interest rate is quite smooth; (2) price is as volatile as output; (3) inflation is more variable than output.

A 42-year sample was replicated 50 times assuming \( \sigma_\theta = 0.01 \) and \( \sigma_x = 0.0096 \). The model’s trend was reintroduced into the simulated data before it was logged and HP detrended. The summary statistics for certain variables of interest are given in Table 2. The first set of columns in Table 2 shows the results of the model when there are no adjustment costs included. The next two columns add price adjustment costs, followed by wage adjustment costs, and finally, in the last column, portfolio adjustment costs are added.

On the real side, comparing Tables 1 and 2 shows that the standard deviations of output and consumption are quite close to that found in the Canadian data although the simulated investment series are not as volatile as
they should be. In general, our model, like the other dynamic general equilibrium models, can mimic reasonably well the cyclical behavior of the real economy.

On the nominal side, all nominal variables are more volatile compared with their counterparts in the data. This is caused partially by the monopolistic-competition structure, where workers and intermediate producers are not price takers. All three adjustment costs reduce the volatility of inflation but not by enough to replicate the variance observed in the Canadian data. The price adjustment costs greatly increase the volatility of the nominal interest rate. This implies that the real interest rate also becomes more volatile given that price adjustment costs reduce the inflation volatility. This is one reason the real variables also fluctuate more with the addition of price adjustment costs, as shown in the second column of Table 2. The last two columns of the table present the effect of the wage and portfolio adjustment costs. It is clear that the wage and portfolio adjustment costs offset some of the volatility of the interest rate, but not enough to allow the price adjustment costs to be increased until the model’s inflation volatility matched the Canadian data.

All versions of the model exhibit negative correlations of the interest rate, price level, and inflation rate with output while the actual data has a negative correlation between only output and the price level. This is determined by the basic feature of a limited-participation model, that is, the strong negative correlation between interest rate and output. The inclusion of the adjustment costs strengthens this negative relation, as we can see in Table 2. As well, the wage adjustment costs are most useful for trying to reverse the negative correlations of the inflation rate and interest rate with output.\textsuperscript{14} The basic version of the model also has a negatively autocorrelated interest rate, contrary to the Canadian data, which the portfolio adjustment cost is capable of reversing. However, the actual interest rate was still much more persistent than could be generated by the model. The inflation rate in the model was also negatively autocorrelated until portfolio adjustment costs were added, but, again, none of the costs could really create the desired degree of inflation persistence. Our findings imply that the market adjustment costs can help generate the persistence in output.

\textsuperscript{14} In a version of the model in which firms owned capital instead of households, the inclusion of the wage adjustment cost generates results with a positive correlation between inflation and output and a negative correlation between the price level and output, as in the data.
fluctuation, but cannot successfully replicate the variation of the nominal side of the Canadian economy.

5 CONCLUDING REMARKS

This paper examines how different types of market frictions can affect the transmission of monetary policy. It is shown that different types of real and nominal side adjustment costs can be used to improve the impulse response functions and higher moment characteristics of limited-participation models.

The price adjustment cost could not effectively reduce inflation volatility without first introducing excessive interest rate volatility. In contrast, wage and portfolio adjustment costs are relatively more effective than the price rigidity at reducing the nominal volatility in the economy. For modest levels of price adjustment costs, this relative inability to lower inflation occurs because the changes in the firm’s behaviour caused by the price cost create subsequent effects that offset the firm’s desire to minimize the price cost. For instance, higher price adjustment costs increase labour demand which pushes up wages and hence consumer demand and prices. These offsetting effects limit the price cost’s ability to reduce inflation volatility in this model.

The model also reveals some important interactions between the adjustment costs. Price adjustment costs were even less effective at reducing inflation volatility when there were portfolio costs present in the model. Also, in contrast to results found in the sticky wage literature, the wage adjustment costs could deepen the liquidity effect in this model but only if there were price adjustment costs already present.

Future research will extend the current model by considering various specifications of the monetary policy reaction function, including money growth rules, interest rate rules, as well as inflation or price level targeting. In particular, the policy rules would be ranked to determine whether an inflation-targeting rule dominates the others according to a central banks objective function. Also it would be interesting to completely
endogenize the money growth rule and determine the globally optimal policy reaction function when faced with a particular set of shocks.

Another related extension would be to incorporate a more finely articulated banking sector, which would inform our understanding of the role of financial intermediation, and the interaction of money and credit, in the transmission of monetary policy.
Figure 1: Impulse response functions for the Canadian data (1955 - 1996)
Figure 2: Impulse responses (portfolio adjustment costs only)

Solid: No adjustment costs
Dashed: Portfolio adjustment costs only
Figure 3: Impulse responses (wage adjustment costs only)

- Money Growth (% dev.)
- Nom. Interest Rate (% dev.)
- Inflation (% dev.)
- Output (% dev.)

Solid: No adjustment costs
Dashed: Wage adjustment costs only
Figure 4: Impulse responses (price adjustment costs only)
Figure 5: Impulse responses (all adjustment costs)

Solid: No adjustment costs
Dashed: All adjustment costs
Figure 6: Impulse responses (wage and price adjustment costs)

Solid: Price adjustment costs only
Dashed: Wage and price adjustment costs
Figure 7: Impulse responses (price and portfolio adjustment costs)
Table 1  Cyclical behaviour of the Canadian economy: 1955:01 to 1996:04

<table>
<thead>
<tr>
<th>Variables</th>
<th>Standard Deviation (%)</th>
<th>Correlation with real GDP</th>
<th>Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>1.66</td>
<td>1.00</td>
<td>0.82</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.75</td>
<td>0.61</td>
<td>0.60</td>
</tr>
<tr>
<td>Investment</td>
<td>5.38</td>
<td>0.88</td>
<td>0.77</td>
</tr>
<tr>
<td>Hours</td>
<td>1.85</td>
<td>0.85</td>
<td>0.93</td>
</tr>
<tr>
<td>Nominal:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Money base growth rate</td>
<td>0.96</td>
<td>0.07</td>
<td>0.18</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.38</td>
<td>0.32</td>
<td>0.75</td>
</tr>
<tr>
<td>Price</td>
<td>1.40</td>
<td>-0.45</td>
<td>0.94</td>
</tr>
<tr>
<td>Inflation</td>
<td>1.93</td>
<td>0.29</td>
<td>0.42</td>
</tr>
</tbody>
</table>

a. All the real variables and the price level are logged and then HP detrended. Money growth, the interest rate and the inflation rate are HP detrended. The real variables are in per capita terms. The data on hours worked spans the 1976 to 1996 period only. We also apply this same procedure to the time series generated from the model.
Table 2  Cyclical behavior of the simulated monetary economies (50 Replications)\textsuperscript{a}

<table>
<thead>
<tr>
<th>Variables</th>
<th>No adjustment cost</th>
<th>Add price adjustment costs ($\phi_p=1; \phi_w=0; \phi_q=0$)</th>
<th>Add wage adjustment costs ($\phi_p=1; \phi_w=10; \phi_q=0$)</th>
<th>Add portfolio adj. costs ($\phi_p=1; \phi_w=10; \phi_q=1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>1.71</td>
<td>1.0</td>
<td>0.54</td>
<td>1.73</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.75</td>
<td>0.97</td>
<td>0.63</td>
<td>0.76</td>
</tr>
<tr>
<td>Investment</td>
<td>4.60</td>
<td>0.99</td>
<td>0.50</td>
<td>4.69</td>
</tr>
<tr>
<td>Hours</td>
<td>1.03</td>
<td>0.88</td>
<td>0.20</td>
<td>1.05</td>
</tr>
<tr>
<td>Nominal:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Money growth</td>
<td>0.96</td>
<td>0.33</td>
<td>0.18</td>
<td>0.96</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.51</td>
<td>-0.23</td>
<td>-0.07</td>
<td>1.30</td>
</tr>
<tr>
<td>Price</td>
<td>2.07</td>
<td>-0.72</td>
<td>0.71</td>
<td>2.03</td>
</tr>
<tr>
<td>Inflation</td>
<td>6.16</td>
<td>-0.30</td>
<td>-0.07</td>
<td>5.92</td>
</tr>
</tbody>
</table>

\textsuperscript{a} All the real variables and the price level are logged and then HP detrended. Money growth, the interest rate, and the inflation rate are HP detrended. The real variables are in per capita terms.
APPENDIX I: MODEL SOLUTION TECHNIQUE

The model is solved using a technique which “stacks” the first order conditions and market clearing conditions, one for each endogenous variable at each period of a proposed horizon, and then solves the complete system simultaneously using a Newton procedure. A more complete description of this ‘stacked time’ methodology and a comparison to similar techniques can be found in Armstrong et al. (1995). Two of the primary benefits of this technique are that it does not involve a linear approximation so that the model’s important nonlinearities are not lost, and that it converges on a solution relatively easily and quickly.

APPENDIX II: THE STATIONARY REPRESENTATION OF THE MODEL

We assume that along the balanced growth path all the real variables grow at the rate of \( \mu \) and nominal variables at the rate of \( x \). To solve the model we need to find the stationary representation for the economy. All stationary variables, except \( L_t \) and \( R_t \), along the balanced growth path are denoted by lower case letters. Notice that there is no population growth in the model economy.

\[
c_{it} = \frac{C_{it}}{\exp(\mu t)}, \quad k_{it+1} = \frac{K_{it+1}}{\exp(\mu t)}, \quad i_{it} = \frac{I_{it}}{\exp(\mu t)}
\]

\[
n_{it} = \frac{N_{it}}{M_t}, \quad q_{it} = \frac{Q_{it}}{M_t}, \quad p_{jt} = \frac{P_{jt}}{M_t}, \quad r_{kt} = \frac{R_{kt}}{M_t}, \quad w_{jt} = \frac{W_{jt}}{M_t}
\]

Define the stationary marginal utilities as follows for the case in which \( \phi < 0 \),

\[
u_{1t} = \frac{U_{1t}}{\exp(\mu (\phi - 1) t)} = c_{it}^{\theta - 1} (1 - L_{it}^x - ac_{it}^q)^{\gamma - \theta}
\]

\[
u_{2t} = \frac{U_{2t}}{\exp(\mu \phi t)} = \gamma c_{it}^{\phi} (1 - L_{it}^x - ac_{it}^q)^{\gamma - \theta - 1}
\]
Now define the following stationary variables

\[ \hat{\lambda}_{2t} = \frac{M_{t+1}}{\exp(\mu \phi t)} \cdot \hat{\lambda}_{2t} \]  
(A3)

\[ \beta^* = \beta \exp(\mu \phi) \]  
(A4)

\[ 1 - \delta^* = \frac{1 - \delta}{\exp(\mu)} \]  
(A5)

The stationary representation of the adjustment costs and their derivatives are given in the following equations.

\[ ac^q_{it} = AC^Q_{it} = \frac{\Phi_q(q_{it+1} - (1 + x_t) - (1 + x))}{2} \]  
(A6)

\[ ac^w_{it} = AC^W_{it} = \frac{Ac^w_{it}}{\exp(\mu t)} = \frac{w_i(t)}{p_1} \left\{ \frac{\Phi_w(w_{it+1} - (1 + x_{t+1} - (1 + x))}{2} \right\} \]  
(A7)

\[ \frac{\partial (ac^q_{it+1})}{\partial q_{it+1}} = M_t \cdot \frac{\partial (AC^q_{it+1})}{\partial Q_{it+1}} = \frac{1}{q_{it}} \left\{ \Phi_q(q_{it+1} - (1 + x_t) - (1 + x)) \right\} \]  
(A8)

\[ \frac{\partial (ac^q_{it+1})}{\partial q_{it+1}} = M_{t+1} \cdot \frac{\partial (AC^q_{it+1})}{\partial Q_{it+1}} = \frac{-1}{q_{it+1}} \left\{ \Phi_q(q_{it+1} + 2(1 + x_{t+1}) - (1 + x)) \right\} \]  
(A9)

\[ \frac{\partial (ac^w_{it})}{\partial w_{it}} = \frac{M_{t-1}}{\exp(\mu(t+1))} \cdot \frac{\partial (AC^w_{it})}{\partial W_{it}} = \frac{w_i}{p_1 w_{it-1}} \left\{ \Phi_w(w_{it-1} - (1 + x_{t-1}) - (1 + x)) \right\} \]  
(A10)

\[ \frac{\partial (ac^w_{it+1})}{\partial w_{it}} = \frac{M_t}{\exp(\mu(t+1))} \cdot \frac{\partial (AC^w_{it+1})}{\partial W_{it}} = \frac{-1}{w_{it}} \left\{ \Phi_w(w_{it+1} - (1 + x_t) - (1 + x)) \right\} \]  
(A11)
The next set of equations represent the household’s first order conditions,

\[
\frac{\partial w_{it}}{\partial L_{it}} = \frac{1}{M} \cdot \frac{\partial w_{it}}{\partial L_{it}} = \frac{1}{\theta} \cdot \frac{w_{it}}{L_{it}} 
\]

\( i_{jt} = k_{jt+1} - (1 - \delta^*) k_{jt} \) \hspace{1cm} (A13)

The next set of equations represent the household’s first order conditions,

\[
U_{2t} + U_{1t} \cdot \frac{\partial (AC_{it}^{w})}{\partial W_{it}} = U_{1t} \left( \frac{W_{it} + L_{it}^s}{\partial L_{it}^s} \right) - \beta E_i \left[ U_{1t+1} \cdot \frac{\partial (AC_{it+1}^{w})}{\partial W_{it}} \right] \hspace{1cm} (A14)
\]

\[
\lambda_{2t} = -U_{2t} \frac{\partial AC_{it}^{Q}}{\partial Q_{it+1}} + \beta E_i \left[ \frac{U_{1t+1}}{P_{t+1}} - U_{2t+1} \frac{\partial (AC_{it+1}^{Q})}{\partial Q_{it+1}} \right] \hspace{1cm} (A15)
\]

\[
\beta E_i (\lambda_{2t+1} R_{t+1}^e) = \lambda_{2t} \hspace{1cm} (A16)
\]

\[
U_{1t} = \beta E_i [U_{1t+1}(1 - \delta) + R_{t+1} \lambda_{2t+1}] \hspace{1cm} (A17)
\]

We can then get the following stationary representation of the above conditions,

\[
u_{2t} + u_{1t} \cdot (1 + x_{t-1}) \cdot \frac{\partial (AC_{it}^{w})}{\partial w_{it}} = 
\]

\[
\frac{u_{1t}}{P_{t}} \left( \frac{w_{it} + L_{it}^s}{\partial L_{it}^s} \right) - \beta^* E_i \left[ \frac{u_{1t+1}}{P_{t+1}} - u_{2t+1} \frac{\partial (AC_{it+1}^{w})}{\partial w_{it}} \right] \hspace{1cm} (A18)
\]

\[
\hat{\lambda}_{2t} = -(1 + x_t) u_{2t} \frac{\partial ac_{it}^q}{\partial q_{it+1}} + \beta^* E_i \left[ \frac{u_{1t+1}}{P_{t+1}} - u_{2t+1} \frac{\partial (ac_{it+1}^q)}{\partial q_{it+1}} \right] \hspace{1cm} (A19)
\]
The binding CIA constraint becomes

\[ p_i \cdot (c_{it} + i_{it} + ac_{it}^w) = q_{it} + w_{it}L_{it} \]  

(A22)

For the firm’s problem, let

\[ at_{jt}^p = \frac{AC_{jt}^p}{\exp(\mu t)} = y_{jt}\exp(-2\mu)\frac{\phi_{jt}^p}{2} \left( \frac{P_{jt}}{P_{jt-1}} \left( 1 + x_{t-1} \right) - (1 + x_t) \right)^2 \]  

(A23)

The stationary representation of the production function is given by the following:

\[ y_{jt} = \frac{Y_{jt}}{\exp[(\alpha \mu + \eta(1 - \alpha))\Phi t]} \]  

(A24)

where capital grows at rate \( \mu \) and productivity at rate \( \eta \) in steady state. Along a balanced growth path with all real variables growing at the same rate, the following restriction must be satisfied.\(^1\)

\[ \Phi = \frac{\mu}{\alpha \mu + \eta(1 - \alpha)} \]  

(A25)

Under these conditions, output can be written as

\[ y_{jt} = \frac{Y_{jt}}{\exp(\mu t)} = \exp(-\alpha \mu \Phi)(k_{jt}^\alpha(\exp(\theta)L_{jt}^d)^{1 - \alpha} \Phi) \]  

(A26)

---

\(1\) Given \( \Phi \geq 1 \), then \( \mu \geq \eta \).
The resulting stationary representations for the marginal productivities of labour and capital are

\[ f_{L_j t} = \frac{F_{L_j t}}{\exp(\mu)} = (1 - \alpha) \left( \frac{\nu_{jt}}{L_{jt}} \right) \tag{A27} \]

and

\[ f_{K_j t} = \frac{F_{K_j t}}{\exp(\mu)} = \alpha \left( \frac{\nu_{jt}}{k_{jt}} \right) \tag{A28} \]

The stationary representations of the derivatives of firm prices are given by

\[ \frac{\partial p_{jt}}{\partial L_{jt}^d} = \frac{\exp(\mu)}{M_t} \left( \frac{\partial P_{jt}}{\partial L_{jt}^d} \right) = \frac{1}{\theta_y} \frac{p_{jt}}{P_{jt}} f_{L_j t} \tag{A29} \]

\[ \frac{\partial p_{jt}}{\partial K_{jt}^d} = \frac{\exp(2\mu)}{M_t \exp(\mu)} \left( \frac{\partial P_{jt}}{\partial K_{jt}^d} \right) = \frac{1}{\theta_y} \frac{p_{jt}}{P_{jt}} f_{K_j t} \tag{A30} \]

Similarly, the marginal adjustment costs are given by

\[ \frac{\partial (ac_{jt}^p)}{\partial L_{jt}^d} = \frac{1}{\exp(\mu)} \left( \frac{\partial (AC_{jt}^p)}{\partial L_{jt}^d} \right) = \frac{1 + x_{jt-1}}{\exp(\mu)} \frac{y_{jt}}{p_{jt-1}} \frac{\partial p_{jt}}{\partial L_{jt}^d} \tag{A31} \]

\[ \left\{ \Phi_p \exp(-\mu) \left( \frac{p_{jt}}{p_{jt-1}} \left( 1 + x_{jt-1} \right) \right) \left( 1 + x_{jt-1} \right) \right\} \]

\[ \frac{\partial (ac_{jt+1}^p)}{\partial L_{jt}^d} = \frac{1}{\exp(\mu(t + 1))} \left( \frac{\partial (AC_{jt+1}^p)}{\partial L_{jt}^d} \right) = \frac{(1 + x_{jt})}{\exp(\mu)} \frac{y_{jt+1}}{p_{jt}^2} \frac{\partial p_{jt}}{\partial L_{jt}^d} \tag{A32} \]

\[ \left\{ \Phi_p \exp(-\mu) \left( \frac{p_{jt+1}}{p_{jt}} \left( 1 + x_{jt} \right) \right) \left( 1 + x_{jt} \right) \right\} \]
The firm’s first order conditions can be written as

\[ \frac{\partial (ac^p_{jt})}{\partial k_{jt}} = \frac{1}{(1+x_{jt}) \partial K_{jt} + 1} \frac{\partial (AC^p_{jt})}{\partial k_{jt}} = \frac{y_t}{p_{jt-1}} \frac{\partial p_{jt}}{\partial k_{jt}}. \]  

(A33)

\[ \left\{ \phi_p \exp(-\mu) \left( \frac{p_{jt}}{p_{jt-1}} (1 + x_{jt}) - (1 + x) \right) \right\} \]

\[ \frac{\partial (ac^p_{jt+1})}{\partial k_{jt}} = \frac{1}{(1+x_{jt}) \exp(\mu)} \frac{\partial (AC^p_{jt+1})}{\partial k_{jt}} = -y_t \cdot p_{jt+1} \cdot \frac{\partial p_{jt}}{\partial k_{jt}}. \]  

(A34)

\[ \left\{ \phi_p \exp(-\mu) \left( \frac{p_{jt+1}}{p_{jt}} (1 + x_{jt}) - (1 + x) \right) \right\} \]

The stationary representations of the market clearing conditions are:

\[ c_{it} + i_{it} + ac^p_{jt} + ac^w_{jt} = y_t \]  

(A37)

\[ w_t L_{jt} = 1 - q_{jt} + x_t \]  

(A38)

\[ p_t (c_{it} + i_{it} + ac^w_{jt}) = q_{it} + w_t L_{jt} \]  

(A39)
REFERENCES


COMMENTS ON ‘LIQUIDITY EFFECTS AND MARKET FRICTIONS’ BY SCOTT HENDRY AND GUANG-JIA ZHANG

Michiel Keyzer, Free University, Amsterdam

The paper uses a Real Business Cycle model in Euler form to analyze liquidity effects. As usual in this line of research, it maps out the response to shocks, which are in this case due to random variations in money supply, around a steady state. The authors find that the introduction of adjustment costs of various kinds leads to a reduction in inflation and interest rate volatility. This enables them to replicate better the stylized facts which show a longer duration of effects and more stickiness than the models without adjustment costs can produce. It also provides further illustration that a sort of Tobin tax which makes financial transactions more difficult can be useful.

1 AGE VERSUS RBC

Indeed, private banks would presumably appreciate this type of result, since it suggests that they should maintain high interest margins and provide poor service to their customers, so as to raise adjustment costs for the sake of economic stability. On a more serious note, while the paper falls within the research program initiated by Lucas and Sargent, this discussant belongs to a different, though related species, who call themselves applied general equilibrium modellers, or AGE-modellers for short. AGE-modellers follow in the steps of Johanssen, and Adelman-Robinson. What they have in common with RBC-researchers is general equilibrium, and through it, the allegiance to principles of micro-economic optimization. But whereas AGE-modelling is typically Walrasian in its emphasis on multiple agents and commodities and its finite horizon, the RBC-literature builds on the Ramsey-Cass-Koopmans model of the single consumer and the single consumer good and stresses the time consistency of decisions under uncertainty and over an infinite horizon.

Both AGE and RBC models are Pareto, in that they emphasize the equivalence, pointed out by Negishi, between competitive equilibria that are fully decentralized and the maximization of a social welfare objective subject to constraints. Both approaches also have in common
that they are applied and therefore have to make compromises to keep
the model tractable. AGE-modellers are able to depict the social
dimensions and intersectoral relationships in great detail and they can
solve the structural form without approximations or steady state
assumptions. But RBC-research is definitely more powerful in its
analysis of dynamic responses to anticipated random shocks around
steady states.

RBC-research developed from attempts at incorporating uncertainty
within the Ramsey-Cass-Koopmans model. Its early applications were
in projects that sought to prove that observed business cycles not
necessarily due to unanticipated shocks and could be perfectly
compatible with rational expectations. Interestingly, as documented in
Cooley (ed., 1995), this project failed but the various attempts to
“improve the fit” generated a vast and literature that introduced
additional factors such as tax distortions, heterogeneous agents, and
liquidity constraints, and in this respect came closer to the AGE-
tradition.

Yet, these new developments gradually meant a departure from first-
best. While the early generalizations that included flat taxes as only
distortions could still be cast in an optimizing framework, subsequent
models became no more than systems of Euler equations to be
linearized around a steady state. In our view, much was lost in the
process, and since the paper falls within this more recent category, it
may be useful to mention three general issues at stake.

First, though money is no longer a veil in this RBC-model, it definitely
creates a distortion, and does so in all RBC-models with money.
Consumer welfare would rise if money supply was raised until it had a
negligible (but positive) scarcity value, though the resulting volatility
might be high and the empirical fit very poor. Indeed, RBC-modelers
accept that such a distortion even persists at the steady state.

Second, while steady states can be computed and shown to exist,
existence of an equilibrium path is no longer ensured. Whereas macro-
economists often discard this criticism by arguing that parameter
calibration can always be invoked to guarantee existence of an initial
solution, this does not take care of the theoretical problem and it might
even happen that the model has no solution for certain realizations of the
random shock.
Third, there might be more than one steady state. The applied modeller could argue that the calibration can select the appropriate one, while the theoretician will find it difficult to envisage how a model with rational expectations can, before it has been solved, choose this particular steady state as the appropriate one.

Finally, the finding that adjustment costs leads to a reduction inflation and interest rate volatility would seem relatively trivial in a finite horizon AGE-model. In an RBC-model nothing is trivial since the road from assumption to numerical outcome is so long that almost anything can happen. This is a strength as well as a weakness. It is a strength because it illustrates that the model accounts for all indirect multiplier effects. It is a weakness because it becomes very difficult to compare the findings from different models and to assess the robustness of conclusions. The authors assume all sorts of functional forms without testing them and the random shock is one-dimensional and very specific. It would be interesting to investigate the relative advantages of an alternative approach of stochastic optimization from operations research, more specifically the stochastic quasigradient technique pioneered by Ermoliev. This approach can deal with multidimensional randomness for multi-period optimization problems, and does not need any linear approximation. It can handle bankruptcy, fat tails and other extreme events as well as a continuum of agents. And it can also be initialized at a calibrated at a steady state. Its weak point is that it cannot so far accommodate any infinite or even long horizon. It is an empirical matter whether this would be a major shortcoming, in models with frictions where the discounting due to future uncertainty should be so strong that the information loss from finite horizon approximation would be very small. Our own experience with this technique is favourable, and moreover, its algorithms have interesting interpretations as learning devices.

2 SPECIFIC REMARKS

More specifically with respect to the model in the present paper, four points can be noted.

First, it is not clear how liquidity constraints could be active in the long run when the price level has had the time to adjust, and therefore, how a steady state where they are effective could ever be calibrated.

Second, the model even gives up convexity of the individual household model, as the adjustment cost functions (3) for cash is non-convex in the choice variables Q. It therefore becomes unclear how the resulting first-order conditions can define the optimum uniquely. One might argue that around the steady state this non-convex term cannot dominate since the adjustment costs are small, and that calibration will locate this steady state. Yet this could be a local optimum, and at any rate the non-convexity should be discussed.

Third, the model introduces the ratio of cash balances as adjustment costs in the utility function itself. This creates out of steady state a nominal rigidity that is difficult to interpret.

Finally, it was mentioned during the conference that it might seem awkward to treat the random shocks of the money supply of the central bank itself as the only source of uncertainty, since the normative interpretation would then be to eliminate central banks, and clearly this cannot be intention of the authors.