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A game-theoretic perspective

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Abstract

In this study a formal game-theoretic framework is presented to analyse monetary-fiscal interactions within EMU. The main focus is on the strategic role of debt and the effects of fiscal spillovers on the determination of Union-wide inflation. Within the framework of a static three-player policy game it is shown that unduly lax fiscal behaviour indeed increases Union-wide inflation. Further, in a dynamic two-player game the introduction of debt causes inflation to rise over time. Implementing the Stability Pact relieves the ECB in conducting monetary policy since the fiscal policymaker optimally responds by cutting spending admitting lower inflation rates.

Keywords: monetary-fiscal interaction, fiscal spillovers, optimal debt
JEL Codes: C70, E58
1 INTRODUCTION

Fiscal restraints as stipulated by the Maastricht Treaty and the degree of independence of the European Central Bank (ECB) have become topical issues and main subjects of intense academic and policy debates. As recent theoretical and empirical research has indicated, it is by now widely assumed that increased central bank independence lowers inflation by increasing the credibility of commitments to price stability (see e.g. Alesina and Summers (1993), and Cukierman (1992)). However, in a monetary union this relation becomes blurred and much more complex, since preferences of all participating individual national fiscal authorities now also affect union-wide inflation and output. In the context of the EMU, fiscal free riding could create inflationary pressures that the ECB may find difficult to resist, thereby jeopardizing its functional independence and ultimately its credibility. This offers a justification for fiscal rules in the Union by restraining unduly lax fiscal behavior by Member States to avoid Union-wide adverse economic activity. It is in this context that the Pact for Stability and Growth must be regarded: it seeks to supplement the common monetary policy framework within EMU with sound fiscal policies by the Member States so as to relieve the burden on the ECB’s monetary policy and to leave room for the operation of the automatic stabilizers.

In this study a formal game-theoretic framework is presented to analyse the monetary-fiscal interactions within EMU. Obviously, since many diverse aspects of political, institutional and economic nature play an important role in the process of European monetary integration, any model in this context can be accused of being too special. Here, in this report, the main focus is on the strategic role of debt and the effects of fiscal spillovers on the determination of Union-wide inflation. In addition, the implications of implementing the Stability Pact are studied in a dynamic context.

In the first part of this report a static one-period, three-player policy game between the ECB and two fiscal players is proposed to study the effects of fiscal free riding behaviour on Union-wide inflation. In the model, fiscal policy affects the conduct of monetary policy through the distortionary effects of taxation on output and through the

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1 Only recently the discussion was on the forefront of news again when both the Italian and German governments put pressure on the ECB to lower interest rates.

2 Alesina and Grilli (1992) distinguish between ‘legal independence’ and ‘functional independence’. Legal independence can be described as the ability of a central bank to autonomously choose its policy objectives, whereas functional independence refers to the ability to use monetary policy instruments to pursue monetary policy goals without any restrictions.

3 See Houben (1997) for a discussion on the merits of the Stability Pact.
government budget constraint. On the one hand, higher tax rates cause output losses and therefore create an extra incentive to generate inflation, but on the other hand, reduce the burden on the monetary authority to finance part of government spending. Since the two fiscal players act non-cooperatively and do not take into account the consequences of their fiscal policies on their opponent’s behaviour, fiscal spillovers may emerge. It is often argued that the main purpose of imposing the Stability Pact is to avoid this fiscal free riding by restraining fiscal behaviour of Member States. Excessive deficits and high debt ratios endanger sound monetary policy, causing high inflation and interest rates. Moreover, in a monetary union where the exchange rate adjustment mechanism no longer exists, all countries face the economic consequences of one country’s irresponsible fiscal behaviour in terms of higher inflation and interest rates in the whole Union. This moral hazard behaviour may impose a heavy burden on a credible common monetary policy. To model the external effects of fiscal policy the interest rate is endogenized serving as the transmission channel through which the negative effects of fiscal behaviour are spread throughout the EMU4. As experienced in 1970s and 1980s, the idea is that excessive budget deficits cause upward tension in interest rates and inflationary pressures (see, for instance, Barrell and Pain (1997), Knot and de Haan (1995), and De Bondt, van Els, and Stokman (1997) for recent empirical studies).

In the second part of the analysis the focus is on the explicit role of debt in a dynamic framework. Obviously, by continuously running a budget deficit government debt starts to accumulate. This might in the end lead to a ‘debt crisis’ putting pressure on both fiscal and monetary authorities to bail out the troubled Member State (see Von Hagen and Eichengreen (1996)). The consequences of optimal debt policy on Union-wide equilibrium inflation are studied in a dynamic two-period, two-player policy game between the ECB and one (coordinated) fiscal player. In the game debt serves as a substitute for tax revenues. Issuing debt may be used as a means to avoid output losses due to tax distortions. However, by issuing too much debt in the first period a heavy burden is placed on both policy authorities in the second period to repay the debt. The question is how non-cooperative strategic behaviour between the authorities shifts this debt burden and how this affects Union-wide inflation during the game. Moreover, it is also investigated what role the Stability Pact plays in this dynamic policy game. Indeed, under certain conditions on the parameters of the game, imposing

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4 Various other contributions on fiscal policy coordination within EMU recently appeared in the literature. Levine and Pearlman (1992), for instance, study fiscal externalities through the real interest rate and terms of trade mechanism within a two-country, two goods framework. However, they impose ad hoc forms of government behaviour which is explicitly modelled here; see also Jensen (1996).
the Stability Pact has beneficial effects for inflation by shifting the financing burden onto the fiscal policymaker. These underlying mechanisms are clarified by means of a numerical example.

Summarizing, this report investigates two different aspects of Union-wide inflation determination are investigated using two different models (although heavily linked to each other): fiscal spillovers in a static three-player game and the role of debt in a dynamic two-player game. The model heavily draws on Alesina and Tabellini (1987) and Debelle (1996) but is extended to allow for two fiscal authorities, an endogenous gross real interest rate, and the incorporation of the Stability Pact. The report is organized as follows. Section 2 introduces the static policy game allowing an analysis of the effects of monetary-fiscal interactions and fiscal spillovers on Union-wide inflation. In section 3 the dynamic game is analysed focusing on the strategic role of debt and how the financing burden is split between the two policymakers. The implications of the Stability Pact are analysed in this dynamic game. Finally, some concluding remarks and comments are made in section 4.

2 EQUILIBRIUM INFLATION AND FISCAL SPILLOVERS IN THE EMU

In this section the focus is on the monetary-fiscal interactions which influence the Union-wide inflation rate. Broadly stated, in our static policy game two main effects drive the equilibrium inflation rate. First, both monetary and fiscal preference parameters together with economy-wide target levels directly affect inflation. Second, and more indirectly, the endogenous interest rate which transmits the fiscal spillovers has an effect on inflation. To isolate these effects we distinguish between the exogenous and endogenous interest rate case in two separate subsections. In subsection 2.1 the game is introduced in terms of the players, their actions and utilities. Subsection 2.2 provides a short discussion on the chosen equilibrium concept. In this report we focus on Stackelberg equilibria reflecting timing asymmetries across players when choosing their optimal actions in the game. Subsection 2.3 discusses the results for an exogenous interest rate, whereas the effects of fiscal spillovers with an endogenous interest

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5 Ideally, to study possible strategic interactions between fiscal free riding behaviour and debt accumulation one would have to exploit a two-period, three-player game endogenizing the interest rate. This analysis is much more complicated and cumbersome, basically due to non-linear dynamics. Non-linear dynamics may cause all sorts of trouble: non-existence of equilibria, multiplicity of equilibria or equilibria which cannot be solved analytically; see also Tabellini (1986) and Beetsma and Bovenberg (1997).
rate are discussed in subsection 2.4.

2.1 Description of the static policy game

The strategic interaction within EMU is modelled by means of a (non-cooperative) one-period monetary-fiscal policy game consisting of three players, including the ECB and two Member States, the fiscal authorities ($F_1$ and $F_2$) \(^6\). Given inflation expectations, the ECB chooses the level of Union-wide inflation, and both fiscal authorities choose distortionary taxes and government spending. In addition, the fiscal authorities start the game with an endowment of debt which must be fully repaid at the end of the period. The interdependence between spending, taxes, inflation and debt follows from the government budget constraint, which will be discussed more in detail below. But before elaborating on the players’ actions and utilities, we first turn to output and its relation to inflation and taxation.

Output in Member State $j$, $j = 1, 2$, is modelled via the usual Lucas supply curve, which in this case relates surprise inflation and the level of distortionary taxes to output. Let us assume that a representative trade union in country $j$ sets the nominal wage $w_j$ (in logarithms) to achieve a target real wage $\bar{w}_j$. It acts as a Stackelberg leader vis-à-vis the policymakers, that is, the nominal wage is chosen in advance of the actions of the policymakers, but taking their optimal responses into account. The trade union’s preferences are represented by the following loss function:

$$L_{TU_j} = \frac{1}{2}E(\ln w_j - \ln p_j - \ln \bar{w}_j)^2,$$

(1)

where $E$ denotes the expectation operator and $p_j$ the (log) price level. This formulation implies that the trade union sets a wage $w_j = p^*_j + \bar{w}_j$, with $p^*_j$ the expected (log) price level. Output $Y_j$ is produced by labour $L_j$ using production technology (capital letters denote antilogs):

$$Y_j = L_j^\nu.$$

(2)

A representative firm in country $j$ maximizes after tax profit $P_jY_j(1 - \tau_j) - W_jL_j$, where $P_j$ denotes the (nominal) price level, $W_j$ the (nominal) wage level, and $\tau_j$ the tax rate, representing a tax levied on output. Solving for the firm’s labour demand

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\(^6\) Strictly speaking, a representative trade union determining inflation expectations by setting the nominal contract wage can be viewed as a fourth player in the game. Here, this player is implicitly modelled via surprise inflation and rational expectations.
curve, leads to (normalized) log output in country $j$:

$$y_j = \pi - \pi^e - \tau_j - \tilde{w}_j,$$

where $(\pi^e)\pi$ denotes (expected) Union-wide inflation.$^7$

The fiscal authorities control the instruments of fiscal policy, \textit{i.e.}, they determine taxes and spending. More specifically, in this static game the fiscal authorities choose distortionary taxes which represent the only source of revenue. Given the existing stock of debt, government spending follows from the government budget constraint. In choosing their optimal tax rates, the fiscal authorities take into account their budget constraints. Since it assumed that the existing stock of debt $b_j$ must be fully repaid at the end of the game, fiscal authority $j$’s budget constraint, expressed as a fraction of output, becomes

$$g_j + Rb_j = \tau_j + \pi, \quad j = 1, 2,$$

where $R = 1 + r$ denotes the (gross) real interest rate. Correspondingly, the primary budget deficit follows from the government budget constraint and is denoted by $\theta_j$, that is, for $j = 1, 2$,

$$\theta_j = g_j - \tau_j = \pi - Rb_j,$$

where $\theta_j > 0$ denotes a deficit and $\theta_j < 0$ a surplus. The budget constraint shows that government spending and repayment of debt is financed either by distortionary taxes or seigniorage. Both countries share equally in the seigniorage revenue raised by the ECB.$^8$ The seigniorage term in (4) reflects in a simplified linear way all the means by which the ECB can raise additional revenue through inflation, such as lightening the debt-service burden by keeping (\textit{ex post}) real interest rates low or monetizing the stock of debt. The common inflation rate $\pi$ drives the strategic interaction between the two fiscal authorities in conducting their policies. Ceteris paribus, excessive spending through distortionary taxation in one country affects the utility of the other country because of a higher common inflation rate. In addition, to model the external effects

$^7$ Here, we normalized $\nu = 1/2$, set $\tilde{w}_j = \tilde{w}_j^* + \log 2$, and approximated $\log(1 - \tau_j) \approx -\tau_j$.

$^8$ See Beetsma and Bovenberg (1997) for a derivation of the government budget constraint in a monetary union.
of excessive fiscal behaviour, the interest rate $R$ will be endogenized by specifying a relation between the interest rate and the deficits, or rather, the tax rates of both countries. The formal specification is left to subsection 2.4. The fiscal authorities’ preferences are formally represented by the following loss function: 

$$L^F_j = \frac{1}{2} \left( \pi^2 + \delta^j_y(y_j - \bar{y}_j)^2 + \delta^j_g(g_j - \bar{g}_j)^2 \right), \quad j = 1, 2. \quad (6)$$

This loss function shows that a fiscal authority not only tries to minimize the deviations of output and government spending from their target levels $\bar{y}_j$ and $\bar{g}_j$, but also cares for price stability. These target levels represent first-best levels or ‘bliss points’ referring to an economy without any distortions; $\bar{y}_j$ corresponds to the level of full employment output and $\bar{g}_j$ to the desired level of spending on public goods in terms of output. Preference parameters $\delta^j_y$ and $\delta^j_g$ correspond to the weights attached to the output and government spending objectives, respectively, relative to the price stability objective.

The ECB operates the common monetary policy within the Union by setting the Union-wide inflation rate. In our framework the ECB actually controls the money stock which is assumed to map directly into the inflation rate. Its preferences are formally expressed by:

$$L^{ECB} = \frac{1}{2} \pi^2 + \frac{\mu}{2} \sum_{j=1,2} \frac{1}{2}(y_j - \bar{y}_j)^2. \quad (7)$$

In spirit of Barro and Gordon (1983), this loss function expresses that the ECB not only cares about deviations from price stability, but also assigns weight to individual Member States achieving an output target. The formulation of (7) implies that the ECB is concerned with output of each Member State separately. For simplicity they are all equally weighted as measured by parameter $\mu$. The inverse of this parameter $\mu$ 

9 Because the game in this section is static, time indices are left out. 

10 Given a very simple money demand function, $m_t = p_t + \bar{x}$, where $m_t$ denotes log money supply and $\bar{x}$ is some measure of aggregate output, independent from taxes $\tau_j$, then it immediately follows that inflation is given by $\pi_t = m_t - m_{t-1}$, see e.g. Alesina and Tabellini (1987). However, directly modelling ECB’s behaviour via the interest rate instrument would in principle not qualitatively alter the analysis. 

11 Alternatively one may consider a loss function which takes the deviation of average Union-wide output into account. This would not alter the results or conclusions in any qualitatively way.
is often in the literature regarded as the degree of central bank independence. However, this concept seems too narrow if fiscal authorities are modelled. As it turns out and shown in subsection 2.3, fiscal preference parameters $\delta_j^y$ and $\delta_j^g$ also influence the inflation bias and therefore the ECB’s functional independence in controlling the inflation. This even holds if government spending does not enter the ECB’s loss function. Although formally, as specified in the Maastricht Treaty, the ECB is only concerned with achieving price stability in the Union, loss function (7) admits a more general framework reflecting the possible trade-off between the pure monetary price stability objective and the macro-economic goal of achieving a certain level of output and employment. The pure case of only caring for price stability is retrieved by setting $\mu = 0$. However, for $\mu = 0$ the game essentially degenerates since the conflict of interest between the fiscal authorities and the ECB disappears. Then, in any period, the ECB is always able to achieve price stability $\pi = 0$, which is obviously also counterfactual $^{12}$. It is noted that in choosing its optimal inflation rate, the ECB does not take the government budget constraints of the fiscal authorities into account $^{13}$. Furthermore, we assume that $\mu < \delta_j^y$, indicating that the ECB attaches less weight to output relative to the fiscal authorities.

Society’s loss function sums up over the preferences of individual fiscal authorities but with different weights reflecting either political business cycle considerations or the different weights of the different political parties. That is,

$$L^* = \frac{1}{2} \left( \pi^2 + \frac{1}{2} \sum_{j=1,2} \beta_j^y (y_j - \bar{y}_j)^2 + \frac{1}{2} \sum_{j=1,2} \beta_j^g (g_j - \bar{g}_j)^2 \right).$$  \hspace{1cm} (8)

In this case, society simultaneously optimizes over inflation and taxes, taking into account both government budget constraints and ex ante imposing rational expectations.

Given the described features of the model, it is straightforward to show that the general problem of time-inconsistency of monetary policy can be avoided if a sufficient level of non-distortionary taxes exists. Assuming without any loss of generality that $b_j = 0$, this result can be deduced as follows. If government spending does not enter the loss function $L^F_j$, so $\delta_j^g = 0$, then both fiscal authorities could provide a subsidy of $\tau_j = -(\bar{w}_j + \bar{y}_j)$ to achieve their output targets $\bar{y}_j$. Then, from supply curve (3), there

$^{12}$ Moreover, the recent resignation of the German Minister of Finance, Lafontaine, in March 1999, shows that there is still political disagreement on how monetary policy should be conducted in the EMU in relation to the ECB’s credibility. To a certain extent, the preference weights in the loss function express the impact of this trade-off.

$^{13}$ This is in sharp contrast with Bolt (1997) where the ECB did attach some weight to a fiscal goal, incorporating the government budget constraint; see also Dornbusch (1997) and Tabellini (1986).
would be no incentive for the ECB to inflate, and thus $\pi = \pi^e = 0$. These subsidies are financed by means of a non-distortionary tax on output via a (lump sum) transfer scheme, such that $g_j = \tau_j = -(\bar{w}_j + \bar{y}_j) < 0$. This is the first-best outcome with zero inflation and output and spending at their desired levels. As already pointed out by Fisher (1980) and Alesina and Tabellini (1987) this confirms the notion that time-inconsistency is ultimately a question of lacking non-distortionary taxes.

In this game the strategic interaction arises since all players act non-cooperatively, attaching different weights to achieve their targets. Both the ECB and the fiscal authorities have an incentive to create surprise inflation so as to raise output towards its target and finance part of government spending. There is a conflict of interest, however, about the optimal amount of surprise inflation and the optimal mix of financing. Part of the conflict is due to the fact that the ECB does not internalize the government budget constraints and therefore neglects its role as a source of seigniorage revenue. In addition, since taxes affect the gross interest rate, and thus the debt-service burden, this may well represent a negative fiscal externality putting extra pressure on the monetary authority.

2.2 Commitment technology and equilibrium concept

Often in the economic literature on modelling monetary-fiscal interactions, the corresponding policy game is solved for the Nash equilibrium, indicating that all players optimize their own actions given the actions of their opponents. The resulting Nash equilibrium describes a more or less stable situation in which no player can benefit unilaterally from its strategy. In fact, the Nash concept presupposes that all players move simultaneously. As Debelle (1996) points out, this structure may be appropriate if the policy game has evolved over a long time so that the players have gained full knowledge over each other’s actions and reaction functions. Another, but related concept, is Stackelberg equilibrium which reflects timing asymmetries when choosing actions in the game. In a Stackelberg game one player, or a set of players, act first -the ‘leaders’- to which the other players react -the ‘followers’. However, unlike the Nash solution, the first-movers anticipate the optimal actions of the second-movers, that is, more specifically, they take into account the so-called best reply functions of the followers. So, leaders do not act given any choice of the followers, but act given the best choice of the followers. In this sense, one may conjecture that a first-mover is able to exploit this timing asymmetry to his own advantage, since by credibly committing
to his first action, in a way the first-mover is able to reduce his set of choices. Alternatively, it is said that the first-mover has a better 'technology' to credibly commit himself to a certain strategy from which he cannot pull out easily.

It is argued that that the Stackelberg structure is a more appropriate way to describe the monetary-fiscal relations (see Debelle (1996)). In this report we will focus on Stackelberg equilibria with the fiscal authorities being the two leaders of the game, and the ECB the follower. It is assumed that they choose taxes before the ECB determines inflation, and therefore, may influence the ECB. This Stackelberg structure may be defended on several grounds. First, as part of the agreement on the Stability Pact, Member States have to include projections of their fiscal stance every year in the so-called Stability Program, to which they are heavily committed. Obviously, in conducting proper monetary policy the ECB will take notice of these policy programs. Second, fiscal authorities may not be as flexible in changing publicly announced government spending and taxes as a central bank is in, say, changing short term interest rates. This indicates that the ECB is more likely to 'accommodate' to the fiscal authorities' actions, than the other way round.

Applying the Stackelberg concept to the monetary-fiscal policy game will have an effect on central bank independence and on Union-wide inflation. The Stackelberg structure implies that the fiscal authorities move first by setting the tax rates and thereby determining government spending. The ECB reacts by choosing the Unionwide inflation. But, in contrast to the Nash solution, the fiscal authorities anticipate this choice of inflation by taking the ECB's best reply to the chosen level of taxes into account. The timing of events in the policy game is now as follows. First, fiscal authorities $F_1$ and $F_2$ simultaneously choose tax rates $\tau_1$ and $\tau_2$. Then, given these tax rates $\tau_1$ and $\tau_2$ and inflation expectations $\pi^e$, the ECB forms its best reply function $\pi_R^e = \pi_R^e(\pi^e, \tau_1, \tau_2)$. That is, the ECB is faced with the following optimization program

\[
\pi_R^e(\pi^e, \tau_1, \tau_2) = \arg \min_{\pi} L_{ECB} \tag{9}
\]

subject to

\[
y_1 = \pi - \pi^e - \tau_1 - \bar{w}_1, \]
\[
y_2 = \pi - \pi^e - \tau_2 - \bar{w}_2.
\]

The fiscal authorities act simultaneously taking into account $\pi_R^e(\pi^e, \tau_1, \tau_2)$, resulting in best reply functions $\tau_j^R = \tau_j^R(\pi^e, \pi_R^e, \tau_i) = \tau_j^R(\pi^e, \tau_i), \ i, j = 1, 2, \ i \neq j$. In fact, the fiscal
authority $F_j$, $j = 1, 2$, faces

$$
\tau_j^R(\pi^e, \tau_j) = \arg \min_{\tau_j} L^{F_j}_j
$$

subject to

$$
\pi = \pi^R, \\
\gamma_j = \pi - \pi^e - \tau_j - \bar{w}_j, \\
g_j = \tau_j + \pi - Rb_j.
$$

Subsequently, rational expectations are realized, i.e., $\pi^e = \pi$. In sum, the Stackelberg equilibrium solves

$$
\pi = \pi^R(\pi^e, \tau_1, \tau_2), \quad \tau_1 = \tau_1^R(\pi^e, \tau_2), \quad \tau_2 = \tau_2^R(\pi^e, \tau_1), \quad \pi^e = \pi.
$$

This leads to a Stackelberg equilibrium $(\pi^S, \tau_1^S, \tau_2^S)$ which only depends on the parameters of the game. Correspondingly, $\theta_j^S = g_j - \tau_j^S = \pi^S - Rb_j$ denotes the Stackelberg equilibrium primary deficit for country $j$.

### 2.3 Equilibrium inflation without fiscal spillovers

In this section we will concentrate on Stackelberg inflation $\pi^S$ and equilibrium budget deficit $\theta_j^S$ under the assumption of an exogenous interest rate $R = R_0$, neglecting the role of fiscal spillovers. In order to simplify the calculations, without loss of generality, symmetry conditions are imposed on the two fiscal authorities. That is, we impose

$$
\delta_1^y = \delta_2^y = \delta_y, \quad \delta_1^g = \delta_2^g = \delta_g, \quad \bar{w}_1 = \bar{w}_2 = \bar{w}, \quad \bar{y}_1 = \bar{y}_2 = \bar{y}, \quad \bar{g}_1 = \bar{g}_2 = \bar{g}, \quad b_1 = b_2 = b.
$$

These conditions can be interpreted by saying that the two countries are equal-sized and/or have sufficiently economically and politically converged.

First, before the Stackelberg equilibrium inflation rate is calculated, it is interesting to see what optimal inflation rate results if society minimizes its loss function (8) subject to the economy-wide constraints and *ex ante* imposing rational expectations. Formally, society minimizes
\[
\min_{\pi, \tau_1, \tau_2} \frac{1}{2} \left( \pi^2 + \frac{1}{2} \sum_{j=1,2} \beta^y_j (y_j - \bar{y})^2 + \frac{1}{2} \sum_{j=1,2} \beta^g_j (g_j - \bar{g})^2 \right) 
\]

s.t.
\[
\begin{align*}
\pi &= \pi', \\
y_j &= \pi - \pi' - \tau_j - \bar{w}, \quad (j = 1, 2), \\
g_j &= \tau_j + \pi - R_0 b, \quad (j = 1, 2).
\end{align*}
\]

Assuming that society’s preferences are equal to those of the fiscal authorities, i.e., \(\beta^1_y = \beta^2_y = \delta_y\) and \(\beta^1_g = \beta^2_g = \delta_g\), optimal inflation \(\pi^*\) is given by

\[
\pi^* = \frac{(\bar{C} + R_0 b) \delta_g \delta_x}{\delta_x + \delta_y + \delta_g \delta_y}
\]

where \(\bar{C} = \bar{y} + \bar{w} + \bar{g}\) is a constant, depending on all Union-wide targets but independent from any policy weights. The optimal inflation rate increase both with society’s weight attached to spending and society’s weight attached to output (relative to price stability). This reflects that more weight on spending will push up government expenditures and increases the financing burden. This will lead to higher tax rates and inflation. More weight on output on the other hand will reduce distortionary taxes which in turn causes higher inflation to cover for the financing of government spending by taking the budget constraint into account.

Now turning to the derivation of the non-cooperative Stackelberg inflation rate, note again that the ECB is optimizing over inflation without taking the government budget constraints into account. The fiscal authorities being the leaders of the game incorporate the ECB’s best reply function with respect to inflation. Solving the system of equations \(11\), the Union-wide Stackelberg inflation \(\pi^S\) is given by

\[
\pi^S = \frac{(\bar{C} + R_0 b) \delta_g \mu (2 + 3 \mu)}{\mu^2 + \delta_y (2 + \mu) + \delta_g (1 + \mu) (2 + 3 \mu)} > 0.
\]

This Stackelberg inflation \(\pi^S > 0\) immediately illustrates the standard time-inconsistency problem of monetary policy (see Barro and Gordon (1983)). Since trade unions correctly anticipate that the ECB has an incentive to generate surprise inflation to boost
Union-wide output, an inflationary bias results. The more weight is attached to output (that is, the larger $\mu$), the more incentive the ECB has to generate surprise inflation, and the larger the inflationary bias. Formally stated, for $\delta_y$ large enough relative to $\delta_g$ and $\mu$, we have 14

$$\frac{\partial \pi^S}{\partial \mu} > 0.$$  \hspace{1cm} (15)

In the end, no direct gain in output is achieved because inflation is completely expected. However, increased inflation does indirectly have beneficial effects on output through the induced seigniorage revenues in the government budget constraint. A higher inflation rate reduces the need for distortionary taxes, and for that matter reduces output losses. Obviously, when the ECB attaches no weight to output, that is, $\mu = 0$, no time-inconstistency problem occurs, thus inducing the ECB to set the desired target inflation $\pi^S = \pi^e = 0$.

More insight in $\pi^S$ can be derived from the following derivatives. Some algebraic manipulations show that

$$i) \frac{\partial \pi^S}{\partial \delta_g} > 0, \hspace{0.5cm} ii) \frac{\partial \pi^S}{\partial \delta_y} > 0, \hspace{0.5cm} iii) \frac{\partial \pi^S}{\partial \delta_g} > 0, \hspace{0.5cm} iv) \frac{\partial \pi^S}{\partial \delta_y} < 0.$$  \hspace{1cm} (16)

Not surprisingly, the first derivative $i)$ reflects that higher intial debt ratios increase inflation. Higher debt ratios increase the repayment burden which is partly financed by seigniorage, causing higher inflation. The second derivative $ii)$ states that inflation rises with higher economy-wide targets $\bar{y}$, $\bar{w}$, and $\bar{g}$. Obviously, a higher output and/or wage target increases the incentive for surprise inflation, which -being rationally anticipated- causes higher equilibrium inflation rates. A higher government spending target leads to higher taxes, and therefore to lower output, which induces the ECB to generate inflation. The third derivative $iii)$ is based on the same reasoning: a larger weight attached to government spending also induces a rise in the level of distortionary taxes and therefore higher inflation to compensate for output losses. The last derivative $iv)$ indicates that a higher weight attached to output by the fiscal authorities leads to a lower inflation rate. By putting more weight on achieving the output target, spending and distortionary taxes are reduced, and therefore the need to generate surprise inflation. These properties of $\pi^S$ show that, given a positive ECB’s output weight $\mu > 0$, the

14 For $\delta_y > (2\mu^2 - \delta_g(2 + 3\mu)^2)/(4 + 3\mu(4 + \mu))$ the inflation rate is rising in $\mu$. 

equilibrium inflation set by the ECB is also determined by the fiscal preference parameters. This puts the concept of central bank independence within a monetary union in a broader context, implying that in empirical analysis fiscal settings and institutions also matter for an appropriate measurement of central bank independence (see also Debelle (1996) on this argument). Further, compared to society’s optimal inflation rate $\pi^*$, one can generally say that $\pi^S > \pi^*$, since in the Stackelberg game policymakers act non-cooperatively, not taking into account that their own actions affect payoffs and actions of their opponents. Obviously, $L^*(\pi^*) < L^*(\pi^S)$.

Given country $j$’s equilibrium primary budget deficit $\theta^S_j = g_j - \tau^S_j = \pi^S - R_0 b$, it is immediately clear that deficits are equal across countries, that is, $\theta^S_1 = \theta^S_2 = \theta^S$, since initial debt ratios are assumed equal. Further, in an economy without debt (i.e., $b = 0$), the primary deficit is equal to the inflation rate. Hence, $\theta^S > 0$ and all abovementioned properties apply. However, if there is an initial endowment of debt ($b > 0$) it may be the case that a primary budget surplus is attained, depending on the values of the monetary and fiscal preference parameters and the amount of initial debt. In particular, if $\mu = 0$ implying zero inflation $\pi^S = 0$, then taxation is the only instrument to finance both spending and debt repayments. In this case a primary budget surplus $\theta^S = -R_0 b$ results. More generally stated, going from high inflation to low inflation, the primary deficit changes from a positive sign to a negative. The reason is that at low inflation, taxation must also cover the (exogenous) debt repayments, inducing a lower level of spending, resulting in a budget surplus. In particular, other things being equal it can be shown that a balanced budget can be achieved for a specific value of the fiscal preference parameter $\delta_y = \bar{\delta}_y$. For values $\delta_y > \bar{\delta}_y$ equilibrium inflation is low enough to guarantee a primary budget surplus$^{15}$.

### 2.4 Fiscal spillovers

To model the external effects of excessive fiscal behaviour the interest rate $R$ is endogenized. This interest rate mechanism serves as the transmission channel through which the negative effects of irresponsible fiscal behaviour are spread throughout the EMU. It reflects the idea that excessive deficits create inflationary pressures and up-

$^{15}$ This lower bound is given by

$$\bar{\delta}_y = \frac{\bar{C}\delta_g\mu(2 + 3\mu) - R_0 b(\mu^2 + \delta_g(2 + 3\mu))}{R_0 b(2 + \mu)}.$$
ward tension on interest rates (see also Van Els and Vlaar (1996), or Correira-Nunes and Stenitsiotis (1995) for an empirical analysis). However, since deficits are no direct choice variable in the game, we endogenize \( R \) in terms of tax rates\(^{16}\). The reasoning here is that, ceteris paribus, low tax rates imply relatively high primary budget deficits. That is, we specify

\[
R = R(\tau_1, \tau_2) = R_0 - \alpha(\tau_1 + \tau_2), \quad \alpha > 0.
\]

Parameter \( \alpha \) measures the impact of fiscal spillovers: the larger \( \alpha \), the more effect fiscal spillovers will have on Union-wide inflation. In choosing its optimal tax rate a fiscal authority does not take into account the impact his choice will have on the optimal choice of tax rates of foreign fiscal authorities. This negligence on part of the fiscal authorities in the game may well lead to excessive fiscal behaviour causing upward pressure on interest rates. The exogenous interest rate case without spillover effects is retrieved for spillover parameter \( \alpha = 0 \), that is, \( R = R_0 \).

To explicitly indicate that the Stackelberg inflation will generally depends on the spillover parameter \( \alpha \) we will use notation \( \pi^S(\alpha) \). The Union-wide Stackelberg inflation rate \( \pi^S(\alpha) \) can be calculated from (11), which gives

\[
\pi^S(\alpha) = \frac{(1 + 2\alpha b)\bar{C} + R_0 b)\delta_y\mu(\mu + 2(1 + \mu)(1 + \alpha b))}{\mu^2 + \delta_y(2 + \mu) + \delta_y(2 + (1 + 2\alpha b))(\mu + 2(1 + \mu)(1 + \alpha b))} > 0.
\]

It can be shown that for large enough weight attached to output by the fiscal authorities relative to the output weight by the ECB, it is found that equilibrium inflation rises with \( \alpha \). Formally, for \( \delta_y > \delta^2_y \) it holds that\(^{17}\)

\[
\frac{\partial \pi^S(\alpha)}{\partial \alpha} > 0.
\]

Thus, the larger \( \alpha \), the stronger the spillover effects, and the larger the Union-wide equilibrium inflation. A large \( \alpha \) magnifies the impact of lax fiscal behaviour in the

\(^{16}\) Alternatively, the game could be modelled in terms of deficits being the choice variable, see Bolt (1997). This would not in any way qualitatively affect the results.

\(^{17}\) This lower bound \( \delta^2_y \) is a somewhat complex function of all other parameters of the game, but not very restrictive for given plausible parameter values.
sense that it increases the primary deficit, reflecting the spillover effects of excessive fiscal policies. This increased financing burden is partly shifted onto the ECB causing higher inflation rates. Hence, this result gives a rationale to impose fiscal restraints, thus from this point of view offering a justification for imposing the Stability Pact. However, in a static game one cannot expect to get clear results on the effects of such a Pact. Therefore, we will explore the impact of imposing the Stability Pact in the next section within the framework of a dynamic two-period game\textsuperscript{18}.

3 THE ROLE OF DEBT

In a static one-period game such as investigated in the previous section the effects of debt accumulation on inflation cannot be properly analysed. Still, debt accumulation is an important issue in the discussion on ECB’s independence and the merits of the Stability Pact. Indeed, high debt ratios in some of the Member States might in the end cause a ‘debt-crisis’ forcing the policy authorities to bail out the imperiled governments. This would have strong adverse effects on the credibility of the ECB’s monetary policy in and outside the Union.

In this section the policy game is extended to two periods in order to study the role of government debt in the determination of Union-wide inflation. However, because of analytical complexity due to non-linear dynamics, the game of this section consists of only two players, that is, the ECB and one fiscal authority, in a model with an exogenous interest rate. Hence, fiscal spillovers between Member States are neglected in this section. The next subsection shortly describes the extended two-period model in terms of players, actions, (discounted) utilities and the derivation of the two-period Stackelberg equilibrium. As a useful intermezzo, in subsection 3.2 brief attention is paid to the simple related static game. This turns out to be helpful in characterizing the full dynamic game in terms of equilibrium inflation. The results are derived in subsection 3.3. Finally, subsection 3.4 introduces the Stability Pact in a formal way and offers a discussion on its efficacy in a dynamic context.

\textsuperscript{18} A first tentative attempt to explore the efficacy of the Stability Pact within a static game led to somewhat ambiguous results, see Bolt (1999).
3.1 Description of the two-period policy game

The two-period game consists of two players: the ECB and one fiscal authority (FA). This fiscal authority may be interpreted as a player which coordinates fiscal policies among the Member States internalizing all country-specific constraints\(^\text{19}\). In this dynamic game the use of debt serves as a substitute for distortionary taxation. It enables the fiscal authority to finance part of its spending by issuing debt. It is assumed that all debt must be repaid in the second period of the game. In essence, the availability of debt introduces another trade-off in the game: issuing debt in the first period reduces the need for distortionary taxes and therefore reduces output losses, but increases the financing burden on the policy authorities to repay the debt in the second period. Introducing debt in the game means another decision variable for the fiscal authority.

The timing for the fiscal authority is now as follows. Note that in this section a subscript denotes a time index, whereas in the previous paragraph it indicated a country index. In the first period, given inflation expectations \(\pi^e_1\), the fiscal authority chooses the amount of debt \(b\) and the first-period tax rate \(\tau_1\), taking into account -still being the leader of the game- the best reply function \(\pi^R_1\) of the ECB. In the second period, it only chooses the second-period tax rate \(\tau_2\), which now obviously depends on the amount of debt issued in the first period. Formally, its discounted utility is now given by

\[
L_{FA} = l_{FA}^1 + \rho l_{FA}^2, \tag{20}
\]

where

\[
l_{FA}^t = \frac{1}{2} \pi_t^2 + \frac{\delta_e}{2} (y_t - \bar{y})^2 + \delta_{\pi} (g_t - \bar{g})^2, \quad t = 1, 2, \tag{21}
\]

denotes the period-\(t\) utility, with \(\rho < 1\) the discount factor of the fiscal authority. The fiscal authority dynamically optimizes over \(b\), \(\tau_1\), and \(\tau_2\) taking its two budget constraints into account. The first-period government budget constraint is given by

\[
g_1 = \tau_1 + \pi_1 + b, \tag{22}
\]

and the second-period constraint by

\[
g_2 + Rb = \tau_2 + \pi_2, \tag{23}
\]

again noticing that the interest rate \(R\) is exogenous\(^\text{20}\). In addition, throughout the remainder of this study it is assumed that the fiscal authority has a relatively high time

\(^{19}\) See Bolt (1997) for an elaborate analysis of the strategic interaction between the ECB and a coordinated fiscal authority, using a static model.

\(^{20}\) In a general equilibrium type of model one could in principle simulate the effects of endogenous interest rate, see e.g. Levine and Pearlman (1992).
preference due to myopic behaviour reflecting electoral motives. In particular, it is assumed that $\rho < 1/R$.

The ECB determines its Union-wide inflation $\pi_t$ in each period $t = 1, 2$, given inflation expectations $\pi^e_t$, the level of taxes $\tau_t$ and the amount of debt $b$. Its discounted utility is now represented by

$$L^{ECB} = l_1^{ECB} + \omega l_2^{ECB},$$  \hspace{1cm} (24)

where

$$l_t^{ECB} = \frac{1}{2} \pi_t^2 + \mu (y_t - \bar{y})^2, \quad t = 1, 2,$$  \hspace{1cm} (25)

with $\rho \leq \omega < 1$ the discount factor of the ECB, indicating that the ECB is less myopic than the fiscal authority, enhancing its credibility.

Again, we focus on solving for the Stackelberg equilibrium of the dynamic game. As usual in a dynamic game with finite horizon, the equilibrium is found by working backwards. However, its derivation is somewhat complex, since government debt acts as a so-called state-variable in the game. In particular, second-period utilities depend on the amount of debt chosen in the first period, also affecting the optimal choice of second-period tax rates and inflation$^{21}$. Conditional on $b$, starting at time $t = 2$, the ECB faces the following problem

$$\pi^R_2(\pi^e, \tau_2) = \arg\min_{\pi_2} l_2^{ECB}$$  \hspace{1cm} (26)

s.t.

$$y_2 = \pi_2 - \pi^e_2 - \tau_2 - \bar{w},$$

whereas the fiscal authority optimizes

$$\tau^R_2(\pi^e) = \arg\min_{\tau_2} l_2^{FA}$$  \hspace{1cm} (27)

s.t.

$$\pi_2 = \pi^R_2,$$

$$y_2 = \pi_2 - \pi^e_2 - \tau_2 - \bar{w},$$

$$g_2 = \tau_2 + \pi_2 - Rb.$$  

$^{21}$ In this sense our dynamic game is not a so-called repeated game, where the one-shot stage game is simply repeated two times, and payoffs in the stage game are independent of the history of the game. In game theoretic language our game represents a so-called difference game.
Solving the system
\[ \pi_2 = \pi^R(\pi_2^e, \tau_2), \quad \tau_2 = \tau_2^R(\pi_2^e), \quad \pi_2^e = \pi_2. \]  
(28)
gives the second-period Union-wide Stackelberg equilibrium \((\pi_2^S(b), \tau_2^S(b))\), which depends on \(b\). By denoting \(l_2^{ECB}(b) = l_2^{ECB}(\pi_2^S(b), \tau_2(b))\) and \(l_2^{FA}(b) = l_2^{FA}(\pi_2^S(b), \tau_2(b))\), the 'contingent' discounted utilities become
\[
L_{ECB}^b = l_1^{ECB} + \omega l_2^{ECB}(b) \\
L_{FA}^b = l_1^{FA} + \rho l_2^{FA}(b),
\]  
(29) 
(30)
In the first period, the ECB now solves
\[
\pi_1^R(\pi_1^e, \tau_1, b) = \arg \min_{\pi_1} L_{ECB}^b \\
\text{s.t.} \\
y_1 = \pi_1 - \pi_1^e - \tau_1 - \bar{\omega},
\]  
(31)
whereas the fiscal authority solves
\[
(\tau_1^R(\pi_1^e), b^R(\pi_1^e)) = \arg \min_{(\tau_1, b)} L_{FA}^b \\
\text{s.t.} \\
\pi_1 = \pi_1^R, \\
y_1 = \pi_1 - \pi_1^e - \tau_1 - \bar{\omega}, \\
g_1 = \tau_1 + \pi_1 + b.
\]  
(32)
Solving the system
\[
\pi_1 = \pi^R(\pi_1^e, \tau_1, b), \quad \tau_1 = \tau_1^R(\pi_1^e), \quad b = b^R(\pi_1^e), \quad \pi_1^e = \pi_1.
\]  
(33)
gives the first-period Union-wide Stackelberg equilibrium \((\pi_1^S, \tau_1^S, b^S)\). Finally, by substituting \(b^S\), the (unconditional) second-period Stackelberg equilibrium \((\pi_2^S, \tau_2^S) = (\pi_2^S(b^S), \tau_2^S(b^S))\) is derived.

### 3.2 Intermezzo: the static one-period, two player policy game

Before we proceed to analyse the dynamic two-player game it is convenient to briefly focus on the related static two player game. It will be shown in the next subsection
that these two games are linked in terms of equilibrium inflation. Here, the static game obviously represents a simplified version of the game under inspection in the previous section, since now it only has one fiscal player. Using the analysis of subsection 2.3 it can now easily be shown that the Stackelberg equilibrium Union-wide inflation in the static game under an exogenous interest rate \( R \) is given by

\[
\pi^S = \frac{(\tilde{C} + Rb)\delta g(1 + 2\mu)}{\mu^2 + \delta_y + \delta_g (1 + \mu)(1 + 2\mu)} > 0.
\] (34)

Notice the similarity to equilibrium inflation (14) of the previous section\(^{22}\). Apart from other normalization constants and an interaction term in the denominator caused by the presence of two fiscal authorities, the two inflation rates are alike\(^ {23} \). Focusing on the game without debt by fixing \( b = 0 \), from (34), the one-shot ’no debt’ inflation rate \( \pi_0 \) is given by

\[
\pi^S_0 = \frac{\tilde{C}\delta_g(1 + 2\mu)}{\mu^2 + \delta_y + \delta_g (1 + \mu)(1 + 2\mu)} > 0.
\] (35)

Alternatively we may write \( \pi^S_0 = \phi_0 \tilde{C} \), with \( 0 < \phi_0 = \frac{(\delta_g(1 + 2\mu))}{(\mu^2 + \delta_y + \delta_g (1 + \mu)(1 + 2\mu))} < 1 \). Notice that \( \phi_0 \) approaches zero for \( \delta_y \) getting large relative to \( \delta_g \) and \( \mu \), so that also the one-shot ’no debt’ inflation rate approaches zero.

### 3.3 Equilibrium behaviour in the dynamic policy game

Solving the dynamic two-period policy game sheds an interesting light on the role of debt. As the next result will show, debt indeed affects the level of Union-wide inflation over time. Intuitively, by issuing debt in the first period some of the burden on the ECB is relieved, since the reduced need for distortionary taxation lowers the incentive to generate inflation. However, the debt repayment requirements in the second period puts extra pressure on the ECB to finance part of the debt service. The following result shows how this burden is shifted onto the ECB over time.

\(^{22}\) Debelle (1996) shows that the Stackelberg inflation rate is higher than the Nash inflation rate under a plausible condition on the preference parameters.

\(^{23}\) Thus, optimizing by \textit{ex ante} imposing coordination gives other results as \textit{ex post} imposing symmetry.
First, define constants $A_1$ and $A_2$ such that

\begin{align*}
A_1 &= \mu^2 + \delta_y + \delta_y (1 + \mu)(1 + 2\mu) > 0 \\
A_2 &= \frac{R(\mu^2 + \delta_y + \delta_y (1 + 2\mu)^2)}{A_1} = R\left(1 + \frac{\delta_y (1 + 2\mu)}{A_1}\right) = R(1 + \phi_0) > 0. \quad (36)
\end{align*}

Now, solving for the equilibrium of the dynamic policy game leads to first-period Union-wide Stackelberg equilibrium inflation

\begin{align*}
\pi_1^S &= \frac{\delta_y \mu (1 + 2\mu)(1 + R)\rho A_2}{A_1 (1 + R \rho A_2)} = \left(\frac{1 + R}{1 + R \rho A_2}\right) \pi_0^S, \quad (38)
\end{align*}

second-period Union-wide Stackelberg inflation

\begin{align*}
\pi_2^S &= \frac{\delta_y \mu (1 + 2\mu)(1 + R)}{A_1 (1 + R \rho A_2)} = \left(\frac{1 + R}{1 + R \rho A_2}\right) \pi_0^S, \quad (39)
\end{align*}

and equilibrium amount of debt

\begin{align*}
b_S^* &= \frac{\delta_y (1 - R \rho^2 + \delta_y + \delta_y (1 + 2\mu)^2)}{A_1 (1 + R \rho A_2)} = \frac{\delta_y (1 - \rho A_2)}{1 + R \rho A_2}. \quad (40)
\end{align*}

The above result gives rise to the following formal equivalence relations, which characterizes equilibrium behaviour in the dynamic game.

\begin{align*}
i) \quad A_2 < \frac{1}{\rho} & \iff \quad b^S > 0 \iff \quad \pi_1^S < \pi_0^S < \pi_2^S. \quad (41)
\end{align*}

This characterization result (41) may clarify the effects of debt on the equilibrium inflation\(^{24}\). Under condition \(i\), which represents a restriction on the parameters of the game, the equilibrium amount of debt is positive, implying that issuing debt is actually an equilibrium action used as a means to finance part of government spending. The condition \(0 < A_2 < 1/\rho\) implies a lower bound on \(\delta_y\), relative to \(\delta_y\) and \(\mu^{25}\). The feasibility of this condition can easily be checked if condition \(i\) is rewritten as

\begin{align*}
0 < A_2 < 1/\rho & \iff \quad 0 < \phi_0 < 1/(R \rho) - 1. \quad (42)
\end{align*}

\(^{24}\)See Debelle (1996) for a similar result, but assuming the Nash structure.

\(^{25}\)Condition \(i\) holds if and only if \(\delta_y > \delta_y^3 = R \rho (\delta_y (1 + 2\mu)/(1 - R \rho)) - (\mu^2 + \delta_y (1 + \mu)(1 + 2\mu))\).
Since, by assumption $R_\rho < 1$, there is an admissible region for $\phi_0$ which must lie between one and zero. Hence, for high enough weight attached to output by the fiscal authority, any positive amount of debt implies a rising tendency of Union-wide equilibrium inflation. In particular, it is shown that first-period period inflation $\pi^S_1$ is lower than the one-shot 'no-debt' equilibrium inflation $\bar{\pi}^S_0$, while the second-period inflation $\pi^S_2$ is higher than $\bar{\pi}^S_0$. This shows that there is an interesting link between the static 'no debt' game and the dynamic game. In addition, this 'rising inflation over time' result reminds of the Sargent-Wallace type of argument of 'unpleasant monetarist arithmetic' in the sense that the unconstrained use of debt may lead to an unstable conduct of monetary policy with inflation getting out of hand (Sargent and Wallace (1981)). Further, it is important to note that none of the above equilibrium outcomes feature the discount factor $\omega$ of the ECB. Since the ECB is not optimizing over the amount of debt and not internalizing the government constraints, debt does not affect the first-period reaction function of the ECB\(^{26}\). The fact that the ECB’s discount factor does not matter may also be interpreted by saying that the ECB follows a more ‘farsighted’ approach in conducting policy than the fiscal authority, who risks the probability of being voted out of office.

The question arises what happens to the primary deficit over time. Recall that the first- and second-period primary deficit are defined by

$$\theta_1 = g_1 - \tau_1 = \pi_1 + b, \quad \text{and} \quad \theta_2 = g_2 - \tau_2 = \pi_2 - Rb,$$  

then in equilibrium, the following result holds

$$\theta^S_1 - \theta^S_2 = \frac{\bar{C}(1 - \phi_0)(1 + R)(1 - \rho A_2)}{1 + R \rho A_2} > 0 \quad \text{if and only if} \quad A_2 < 1/\rho.$$  

Hence, under the same condition \(i\), the primary deficit is decreasing over time, while inflation is increasing. In effect, we may write

$$\theta^S_1 - \theta^S_2 = (1 + R)b^S - (\pi^S_2 - \pi^S_1) > 0.$$  

So, (45) implies that the inflation effect is countered by the debt repayment effect, with $1 + R$ serving as a 'modified' interest rate. Since the latter effect is dominant, the primary deficit falls in the second period.

\(^{26}\) So, in fact $\pi^R_1(\pi^*_1, \tau_1, b) = \pi^R_1(\pi^*_1, \tau_1)$.
3.4 Modelling the Stability Pact and implications

This subsection explores the implications of imposing the Stability Pact. Implementing the Stability Pact affects the government budget constraint, since penalty payments are added to the cost side of the budget constraint. The purpose of the Pact is that it sets the fiscal leeway available to the EMU Member States. It mainly seeks to prevent unduly lax fiscal policies in one or more EMU Member States by imposing sanctions which come into operation if deficits become excessive, that is, deficits in excess of the reference value of $\bar{\theta}_{sp}/BP = 3\%$ in terms of nominal gdp. Formally, the sanction mechanism can be described by

$$T(\theta) = \begin{cases} 
\min\{0.5, 0.2 + 0.1(\theta - \bar{\theta}^{sp})\}, & \theta > \bar{\theta}^{sp} \\
0, & \theta < \bar{\theta}^{sp}
\end{cases},$$

where $T(\theta)$, expressed as a fraction of gdp, denotes the sanction to be paid, depending on the level of the deficit $\theta$. As already pointed out in Bolt (1997), this sanction formula is mathematically troublesome, since it is discontinuous in $\theta = \bar{\theta}^{sp}$ and 'kinked' in $\theta = 6$ (in percentage points). This makes the analysis intractable. Therefore, for our purposes the Stability Pact is simply modelled linearly only including the variable part of the sanction and, for convenience, assuming $\bar{\theta}^{sp}/BP = 0$. That is, in any period $t$ we specify the sanction mechanism by

$$T(\theta_t) = \lambda \theta_t, \quad \lambda > 0, \quad t = 1, 2,$$

where the (variable) cost parameter $\lambda$ reflects the severity of the sanction and thus influences the the effectivity of the Stability Pact. In particular, since in this two-period game primary deficits are always positive in equilibrium, imposing $\bar{\theta}^{sp}/BP = 0$ avoids the somewhat troublesome effect of the fiscal authority being subsidized if deficits are smaller than $\bar{\theta}^{sp}/BP = 0$. Hence, sanction rule (47) now states that any primary deficit

27 In our analysis the Stability Pact is represented only by the sanction mechanism. Here, we do not focus on other aspects of the Pact, such as the rationale behind the ‘close to balance or in surplus’ discussion or the possibly weakened disciplinary effects of financial markets and the (perhaps lacking) credibility of the no-bailout clause. See Bolt (1997) for a more elaborate description of the Stability Pact.

28 Moreover the focus here is on primary deficits which do not include interest payments,
triggers a penalty payment. These sanctions must be financed, so they enter the government budget constraint at the expenditure side. Since there is only one fiscal authorities we ignore the issue of rebatements. Instead we assume that the sanction payment implicitly benefits the ECB. The first- and second-period government budget constraints of the fiscal authority now become

\[ \theta_1 + T(\theta_1) = \pi + b, \quad \theta_2 + T(\theta_2) + Rb = \pi, \]  

which leads to

\[ g_1 = \tau_1 + \gamma \pi_1 + \gamma b, \quad g_2 = \tau_2 + \gamma \pi_2 - \gamma Rb. \]  

with

\[ \gamma = 1/(1 + \lambda) \leq 1 \]  

denoting the ‘modified’ cost parameter, measuring the impact of the implementation of the Stability Pact. A more stringent Stability Pact in terms of a higher variable cost parameter \( \lambda \) implies a smaller \( \gamma \). In fact, the modified government budget constraints (49) show that imposing a Stability Pact distorts the degree of substitution between inflation and taxation in financing government spending. Since \( \gamma < 1 \), to maintain a constant level of primary deficit one needs higher inflation or lower initial debt ratios when the Stability Pact is implemented. This reflects an often heard critique against the Stability Pact: trying to encourage greater fiscal discipline by adding sanctions to the expenditure side of a country’s budget is likely to be counterproductive (see e.g. Sims (1998)). However, by rationally anticipating such an implementation by the policymakers as a rule, it still remains to be investigated what the optimal responses of the policymakers will be towards the Stability Pact with respect to inflation and tax rates. Since government spending becomes more ‘costly’ (that is, \( \gamma < 1 \)) the fiscal policymaker may be induced to substitute away from government spending to output by lowering taxes. This reduces the financing burden and may, therefore, lead to lower inflation rates. However, depending on preferences, this may well induce a loss of utility for the fiscal authority.

Calculating the Stackelberg equilibrium gives similar results for inflation rates and debt, only now the constants \( A_1 \) and \( A_2 \) depend on the cost parameter \( \gamma \). More precisely, first-period equilibrium inflation becomes

\[ \text{whereas the 3%-deficit reference value of the Stability Pact does include interest payments. So, as long as } \theta \text{ is near zero this approach should not cause any substantial distortion. In addition, in equilibrium } \theta^{mp} \text{ only affects the Union-wide constant } C. \]

29 In a more extensive model with various Member States, it is possible to model a more realistic sanction mechanism in which the sanction payments are rebated to other Member States that do have a sound budget; see, for instance, Beetsma and Uhlig (1997) and also Bolt (1999) for a preliminary assessment.
\[ \pi_1^\gamma(\gamma) = \frac{\bar{C}\delta\mu(1 + 2\mu)(1 + R)pA_2(\gamma)}{A_1(\gamma)(1 + RpA_2(\gamma))} = \left( \frac{(1 + R)pA_2(\gamma)}{1 + RpA_2(\gamma)} \right) \bar{\pi}_0^\gamma(\gamma), \]  

(51)

second-period equilibrium inflation

\[ \pi_2^\gamma(\gamma) = \frac{\bar{C}\delta\mu(1 + 2\mu)(1 + R)}{A_1(\gamma)(1 + RpA_2(\gamma))} = \left( \frac{1 + R}{1 + RpA_2(\gamma)} \right) \bar{\pi}_0^\gamma(\gamma), \]  

(52)

and equilibrium amount of debt

\[ b^\gamma(\gamma) = \frac{\bar{C}(A_1(\gamma) - Rp(\mu^2 + \delta_y + \delta_y(1 + 2\mu)^2))}{\gamma A_1(\gamma)(1 + RpA_2(\gamma))} = \frac{\bar{C}(1 - pA_2(\gamma))}{\gamma(1 + RpA_2(\gamma))}. \]  

(53)

Given the above result the following equilibrium characterization is not surprising, that is,

\[ i) \ A_2(\gamma) < \frac{1}{\rho} \iff ii) \ b^\gamma(\gamma) > 0 \iff iii) \ \pi_1^\gamma(\gamma) < \pi_2^\gamma(\gamma). \]  

(54)

So, introducing the Stability Pact does not in any way affect the structure of equilibrium outcomes of the dynamic game.

The question now arises how inflation rates are affected by changes in the cost parameter \( \gamma \), measuring the effectiveness of the Stability Pact. One can show that for \( \underline{p} < p < 1/R \) and \( \delta_y > \delta_y^3 \) the following holds\(^{30}\)

\[ i) \ \frac{\partial \pi_1^\gamma(\gamma)}{\partial \gamma} > 0, \quad ii) \ \frac{\partial \pi_2^\gamma(\gamma)}{\partial \gamma} > 0. \]  

(55)

Hence, equilibrium inflation is rising in \( \gamma \) which implies that a less stringent Stability Pact in terms of lower variable costs in the penalty function, drives up inflation. In other words, harsher sanctions have beneficial effects on the level of inflation in

\(^{30}\)This lower bound \( \underline{p} = (2 + 3\mu)/(R(2 + 4\mu)) \) ensures that result (55) holds for all \( 0 < \gamma \leq 1 \) (monotonicity).
this dynamic framework. The rationale behind this finding is that by rationally anticipating the implementation of the Stability Pact government spending becomes more costly and less attractive, due to the changed government budget constraint. As a result, the fiscal policymaker reduces spending and lowers taxes, in a sense substituting spending for output. This admits lower equilibrium inflation rates because the monetary policymakers have less incentive to generate inflation. Lower tax and inflation rates are partly compensated by accumulating more debt. Still, on balance, the primary deficit decreases. In this sense, the Stability Pact helps the proper conduct of monetary policy by the ECB. Interestingly, this result contradicts the common view that imposing a Stability Pact would lead to even bigger problems for both monetary and fiscal authorities given the fact that the countries actually paying the penalties are already in much economic distress (see e.g. Sims (1998)).

A numerical example may clarify matters. Consider Table 1, which summarizes the equilibrium dynamics of the relevant variables in the game with and without the implementation of the Stability Pact. To calibrate the game, we normalize $\bar{C} = 1.1$, assume $R = 1.05$ and $\rho = 0.9$, and fix preference parameters $(\delta_y, \delta_g, \mu) = (1, 0.2, 0.2)$\textsuperscript{31}. The example indicates that although the real interest rate is 5%, the fiscal authority uses a discount rate of about 11%, reflecting its myopic behaviour. Moreover, the fiscal authority attaches relatively much weight to output (and employment) compared to spending and price stability. The ECB mainly cares for price stability and not so much for output, in line with its strive for independence and maintaining credibility. Without the Stability Pact, so that $\gamma = 1$, the calculations produce $b = 0.90\%$, $\pi_1^S = 4.44\%$ and $\pi_2^S = 4.52\%$. This indicates a slightly rising inflation rate and debt accumulation over the period of almost one percent of GDP. The static ‘no debt’ inflation rate would be $\bar{\pi}_0 = 4.48\%$, confirming $\pi_1^S < \bar{\pi}_0 < \pi_2^S$. Equilibrium tax rates are also rising over time, and primary deficits decrease due to the debt repayment in the second period. Imposing the Stability Pact and fixing $\gamma = 0.9$, implying a variable cost of around $\lambda = 10\%$, the table shows that both first- and second-period tax and inflation rates are lower. The fiscal policymaker responds by accumulating more debt. In fact, the fiscal policymaker reduces spending allowing lower tax rates, which diminish the incentive to create inflation. Hence, although the effects are small, the Stability Pact causes equilibrium inflation to be lower in both periods. However, although the ECB somewhat gains in terms of utility, the fiscal policymaker loses. In the end, depending on its preferences, society as a whole might lose in terms of welfare, which will in general depend on its preference weights attached to output and spending relative to price stability\textsuperscript{32}.

\textsuperscript{31} The normalization on $\bar{C}$ ensures non-negative outcomes for inflation and tax rates.

\textsuperscript{32} Most probably, also the penalty payments which are lost in the model would add to these
Table 1  Equilibrium values without and with the Stability Pact

<table>
<thead>
<tr>
<th>γ = 1</th>
<th>t = 1</th>
<th>t = 2</th>
<th>γ = 0.9</th>
<th>t = 1</th>
<th>t = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\pi}^0_0$</td>
<td>4.48</td>
<td></td>
<td>$\bar{\pi}^0_0$</td>
<td>4.44</td>
<td></td>
</tr>
<tr>
<td>$\pi^S_1$</td>
<td>4.44</td>
<td>4.52</td>
<td>$\pi^S_1$</td>
<td>4.41</td>
<td>4.48</td>
</tr>
<tr>
<td>$\tau^S_1$</td>
<td>12.2</td>
<td>12.6</td>
<td>$\tau^S_1$</td>
<td>12.0</td>
<td>12.4</td>
</tr>
<tr>
<td>$b^S$</td>
<td>0.90</td>
<td></td>
<td>$b^S$</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$\theta^S_1$</td>
<td>5.33</td>
<td>3.58</td>
<td>$\theta^S_1$</td>
<td>4.88</td>
<td>3.08</td>
</tr>
<tr>
<td>$y_1 - \bar{y}$</td>
<td>-0.222</td>
<td>$y_2 - \bar{y}$</td>
<td>-0.226</td>
<td>$y_1 - \bar{y}$</td>
<td>-0.220</td>
</tr>
<tr>
<td>$g_1 - \bar{g}$</td>
<td>-0.825</td>
<td>$g_2 - \bar{g}$</td>
<td>-0.838</td>
<td>$g_1 - \bar{g}$</td>
<td>-0.831</td>
</tr>
<tr>
<td>$L^{ECB}$</td>
<td>0.235</td>
<td></td>
<td>$L^{ECB}$</td>
<td>0.231</td>
<td></td>
</tr>
<tr>
<td>$L^{FA}$</td>
<td>0.362</td>
<td></td>
<td>$L^{FA}$</td>
<td>0.364</td>
<td></td>
</tr>
</tbody>
</table>

4  CONCLUSIONS

In this study a formal game-theoretic framework is presented to analyse the monetary-fiscal interactions within EMU. The main focus is on the strategic role of debt and the effects of fiscal spillovers on the determination of Union-wide inflation. In addition, the implications of implementing the Stability Pact are studied in a dynamic context.

The analysis first of all shows that in a monetary union the concept of central bank independence must be placed in a broader perspective. The widely assumed relation that increased central bank independence lowers inflation becomes blurred and more complex in a monetary union, since fiscal preferences of Member States now also affect Union-wide inflation and output. In particular, it is shown that a greater weight attached to output by a Member State drives equilibrium inflation down because it will use less distortionary taxes thereby reducing output losses and hence the incentive to generate inflation. In addition, a smaller weight attached to government spending also decreases inflation, since it relieves the financing burden. In the extreme case, with no weight attached to spending, the first-best outcome with zero inflation and output at its target level can be attained if a sufficient level of non-distortionary taxes exists. The time-inconsistency of monetary policy then disappears.

The effects of fiscal spillovers are analysed in a static three-player policy game including the ECB and two fiscal authorities. To model the external effect of lax fiscal behaviour the interest rate is endogenized in the sense that it depends on the budget deficits of both Member States. In equilibrium, inflation indeed rises if the impact adverse welfare effects.
of fiscal spillovers on the interest rate becomes stronger. This offers a rationale for imposing fiscal restraints as prescribed in the Stability Pact. However, it seems more reasonable to investigate the implications of the Pact in a more dynamic setting where the issue of debt accumulation and its effect on inflation can properly be explored.

The role of debt accumulation in the determination of Union-wide inflation is studied in a dynamic two-period, two-player game between the ECB and a coordinated fiscal policymaker. Issuing debt serves as a substitute for using distortionary taxes. The main result states that, in equilibrium, fiscal policymakers will accumulate debt in the first period creating inflationary pressures over time. This offers a justification for imposing fiscal restraints. Implementing the Stability Pact has interesting consequences in this dynamic framework. It is shown that the Stability Pact has beneficial effects on the level of inflation. Moreover, the more severe the sanctions, the lower the inflation rates. Since government spending becomes more ‘costly’ and therefore less attractive, tax rates are decreased. In this sense, the fiscal policymaker substitutes output for spending, reducing the incentive to generate inflation and hence admitting lower inflation rates. The fiscal authority shifts the financing burden by accumulating more debt to compensate for lower tax and inflation rates. Still, as a result, the primary deficit decreases. In this sense, implementing the Stability Pact is supporting the common monetary policy by the ECB to maintain credibility. However, although the ECB may benefit from imposing the Stability Pact, the national governments are likely to loose in terms of utility. As a consequence, depending on its preferences, society as a whole may suffer welfare losses as a consequence of imposing the Stability Pact.
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