Liquidity Effects and the Welfare Costs of Inflation in an Endogenous Growth Model

C.K. Folkertsma
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ABSTRACT

The paper has two subjects. The first subject is the development of a monetary general equilibrium model with endogenous growth. By combining the two-sector endogenous growth model and the limited participation approach, the model is able to explain the empirically observed liquidity effect of an expansionary monetary policy. The second subject is the effect of inflation on growth and economic welfare. It is shown that the traditional approach to measure the welfare costs of inflation may be misleading: It ignores the costs or benefits of the transition to the new steady state. This omission may bias estimates of the optimal degree and of the total benefits of disinflation. It is also argued that, once the transition is taken into account, the welfare gains of lowering inflation depend on the monetary policy rule and the fiscal response to disinflation. The two themes of the paper are related. Given that the welfare costs of inflation depend on the transitional dynamics, then simulating disinflation processes requires models with sensible short run properties.

Keywords: Monetary general equilibrium model, endogenous growth, welfare costs of inflation, monetary transmission, liquidity effects

JEL Codes: D58, E31, E52, O41

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1 INTRODUCTION

The number of publications on the effects and the welfare costs of inflation is huge and still growing. The paper makes three contributions to this ongoing discussion. The first aim of the paper is to develop a monetary general equilibrium model in which a variety of inflationary distortions can be analysed. This model should be general enough to allow for a possible effect of inflation on economic growth, an aspect that has often been neglected in the general equilibrium literature. Moreover, since the explanation of the so-called liquidity effect of an expansionary monetary policy is the most basic test a general equilibrium model should pass in order to qualify for monetary policy analysis, the model should possess reasonable short run properties. The model developed in the paper is a combination of a two-sector endogenous growth model and the limited participation framework. It will be shown that this model can explain the liquidity effect. Therefore, the model described here, is the first endogenous growth model that is especially suited for the simulation and analysis of, for example, disinflation processes and their interaction with economic growth.

The second aim of the paper is to clarify some issues with regard to the measurement of the costs of inflation. The common approach to calculate these costs is to determine the equivalent variation of switching directly to the steady state corresponding to a reference inflation rate. As reference situation one usually choose either a situation of price stability or the Pareto-optimal inflation rate. The equivalent variation is calculated as the annuity value of consumption which would make households indifferent between remaining in the current steady state and being in the steady state with the lower inflation rate. It is argued that this approach might be misleading for two reasons. Firstly, the inflation rate with the highest steady state welfare level, is not necessarily the welfare maximising inflation rate. Secondly, politicians are not faced with the choice between different steady states, but they have to take the costs or benefits of the transition to the new steady state into account. The reason why the traditional approach may be misleading is that it ignores transitional costs or benefits. This omission may bias not only estimates of the total welfare gains to be achieved by reducing inflation but also of the optimal degree of disinflation.

The insight that the traditional approach yields a biased estimate of the welfare gains of disinflation has been discussed before (see e.g. Cooley and Hansen (1991), Einarsson and Marquis (1999), and Moran (1999)). With the exception of Einarsson and Marquis, however, these authors provide evidence that the transition to a lower inflation rate is costly. Similar to Einarsson and Marquis, this paper shows that disinflating the economy is not necessarily costly and may even generate net benefits. Related to the fact that the transition to a lower inflation rate is not necessarily costly, our second contribution is that we

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1 See O’Reilly (1998) for a recent comprehensive survey of the literature.
show that the welfare optimal rate of inflation, taking the costs and benefits of disinflation into account, may even be lower than the inflation rate which yields the highest welfare along the steady state. In other words, the benefits of the transition may overcompensate a lower welfare level in the steady state.

Once one accounts for the transitional dynamics, it will be shown that the optimal degree of disinflation depends on the monetary policy rule. We provide realistic examples in which the welfare maximising inflation rate given that monetary policy is constraint to a simple money growth rule is higher than the optimal inflation rate a benevolent and unconstrained social planner would implement. Moreover, in these examples a simple monetary growth rule fails to attain the upper bound on potential welfare gains associated with lowering inflation.

Since the transitional dynamics are, as we argue, essential for the analysis of the welfare costs of inflation, the model used for the analysis should imply sensible short run dynamics, especially in response to monetary policy changes. This is the main motivation for the development of an endogenous growth model that is capable of explaining the liquidity effect.

The third aim of the paper is to provide an estimate of the welfare costs of inflation for the European Monetary Union. Based on the limited participation model with endogenous growth it is estimated that reducing inflation from 4.7% to 0% by a simple money growth rule would lead to a permanent annual welfare gain amounting to 0.026% of GDP. The present value of these welfare gains is roughly 1% of current annual European GDP. Applying more sophisticated monetary policy rules might increase these welfare gains. The upper bound on the present value of the welfare gains a monetary policy rule can realise when lowering inflation from 4.7% to price stability is roughly 1.3% of current European GDP. Although these gains may seem small, they are economically significant.

The rest of the paper is organised as follows. The next section describes the monetary endogenous growth model and defines the equilibrium concept. Section 3 discusses in general terms how inflation distorts various economic decisions and shows the theoretical possibility of a liquidity effect in this framework. Section 4 introduces a parameterisation of the utility and production functions and calibrates their parameter to the European Monetary Union. Using impulse response functions, the short run properties are investigated for the calibrated model in Section 5. Section 6 discusses questions relating to the measurement of the welfare costs of inflation and presents estimates of these costs for the European Monetary Union. In the final section, we draw the conclusions.
2 A MONETARY GENERAL EQUILIBRIUM MODEL WITH ENDOGENOUS GROWTH

The model outlined in this section is a combination of an endogenous growth with a limited participation model. Endogenous growth is modelled here by a two sector structure along the lines suggested by Uzawa (1965) and Lucas (1988). In the first sector, consumer and investment goods are produced, whereas the second sector enhances human capital through the provision of schooling, training and research. Although the production in both sectors is characterised by decreasing returns to each of the two input factors, labour and physical capital, and the population is assumed to be constant, unlimited growth is possible because investment in human capital raises labour supply in terms of efficiency units. Accumulating human capital increases effective labour supply at the same pace. The modelling of the human capital sector deviates here from the Lucas-Uzawa approach because we assume the more realistic case that as in the production sector, human and physical capital are combined to generate additional human capital. Clearly, education and research and development require physical capital such as real estate, libraries, computers, laboratories and so on. The human capital sector is, however, more human capital intensive than the production sector.

Money is introduced in this model economy through a cash-in-advance constraint, that is to say that certain transactions have to be settled in cash. We assume that expenditure on consumer goods and at least part of the firm’s wage bill have to be paid in cash. The money for these transactions is brought into circulation by the central bank according to a simple money growth rule.

The most basic test a general equilibrium model should pass in order to qualify for monetary policy analysis is whether it is able to explain the liquidity effect of monetary policy. The liquidity effect is the empirically observed decline of the nominal interest rate and the rise of output and employment after an unexpected monetary expansion.

The limited participation framework as developed by Fuerst (1992) and Christiano and Eichenbaum (1992) has proven to be a promising approach to the explanation of the liquidity effect in general equilibrium models. The limited participation approach entails three assumptions. The first is that households split their accumulated cash balances at the end of a period into cash to be kept for transaction purposes in the next period and deposits at a financial intermediary. Households need cash in order to buy con-

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2 E.g. Hall (1992) documented that investment in physical capital accounts for a significant fraction of expenditure in the human capital sector. See for a similar modeling of the human capital sector e.g. De Guevara, Ortigueira, and Santos (1997) and the references therein.

3 Barro and Sala-i-Martin (1995, p. 179–180) argued also that a human capital sector which is relatively less physical capital intensive than the production sector is the empirically relevant case.

sumer goods, but they want to hold as little as possible because currency, contrary to deposits at the intermediary, does not earn interest. During each period, households cannot revise their portfolio composition. The second assumption is that firms demand credit from the financial intermediary in order to finance part of their wage bill. The final assumption is that new money is injected into the economy by the central bank through the financial intermediary. Because households have no access to the financial intermediary during the period, newly created funds are available only to the firms. This set-up creates the possibility that the nominal interest rate declines after an unanticipated increase of the money supply, because the higher money stock and thus inflation expectations do not only create an upward pressure but also a downward pressure on the interest rate because firms have to be induced to absorb the excess liquidity in the credit market.

Before showing that the net effect of these two opposing forces on the nominal interest rate does indeed generate a liquidity effect, a more detailed description of the model economy and the definition of the equilibrium concept will be given.

The model economy consists of the central bank, a continuum of identical households with an infinite planning horizon, a firm and a financial intermediary and five markets, an output market for consumer and investment goods, a labour, a credit and a money market and a market for capital goods. As already mentioned in the introductory paragraph, it is assumed that the central bank applies a simple monetary growth rule. If $\zeta_t$ denotes the stochastic money growth rate in period $t$ which follows an AR(1) process with normally distributed innovations $\epsilon_{M_t}^M$, then the per capita money stock $M_t$ evolves according to

$$M_t = e^{\bar{\zeta}} M_{t-1},$$

$$\zeta_t = (1 - \rho_M \bar{\zeta}) + \rho_M \zeta_{t-1} + \epsilon_t^M, \quad \epsilon_t^M \sim N(0, \sigma_M^2),$$

where $\bar{\zeta}$ is a policy parameter and corresponds to the long run money growth rate. The new currency $M_t - M_{t-1}$ is transferred to the financial intermediary.

2.1 The representative household

A household in this economy owns the physical capital stocks used as input in the production sector and the human capital sector, respectively. By hiring out the stock of physical capital to the firm, he receives a rent\(^5\. A household, therefore, takes not only decisions on consumption, portfolio composition and

\(^5\) Closer to reality is the assumption that the representative firm owns the capital stock and households hold its shares. Both assumptions, however, are equivalent, if the firm behaves in the interest of its shareholders. That is to say if the firm discounts future profits at the same rate as the shareholders their dividends.
the allocation of time over labour, leisure and training in the human capital sector, but also with regard to investment. He takes these decisions such that the expected utility over an infinite planning horizon would be maximised. The utility $U$ in each period depends on consumption $c$ and leisure $l$ of that period. The utility of future periods is discounted at a rate $\beta$. Thus, a household maximises

$$E_t \sum_{i} \beta^i U(c_i, l_i),$$

(2)

where $E_t$ denotes the expectation conditional on the information in period $t$. This information set contains the realisations of all variables with time index $t$ and earlier.

Except for the usual monotonicity and concavity assumption, the period utility function has to satisfy an additional restriction, which entails that the utility function can be written as $U(c_t, l_t) = v_1(c_t)v_2(l_t)$ with $v_1$ a function homogenous of degree $1 - \sigma$ for $\sigma \neq 1$. For $\sigma = 1$ the utility function would be of the form $U(c_t, l_t) = \ln c_t + v_2(l_t)$. The parameter $\frac{1}{\sigma}$ has an economic interpretation and corresponds to the intertemporal elasticity of substitution in consumption. In order to keep the exposition as simple as possible, it will be assumed that the intertemporal elasticity of substitution is smaller than one, which is the empirically more relevant case (see Mehra and Prescott (1985)). This restriction on the utility function is required for the existence of a balanced growth path, that is a long run equilibrium path in which aggregates such as output and consumption grow at constant positive rate, while labour supply remains stationary (see King, Plosser, and Rebelo (1988)).

When deciding on consumption, investment and his allocation of time, a household considers certain restrictions. The first restriction is that time allocated to labour $n_t$, leisure $l_t$ and learning cannot exceed total available time per period, which is normalised to unity. Thus, the time used for the formation of human capital is $1 - l_t - u_t$. The second restriction is that current expenditure on consumption cannot exceed cash balances $mc_{t-1}$ carried over from the previous period. Thus, if $P_t$ is the price level in period $t$, the cash-in-advance constraint is

$$P_t c_t \leq mc_{t-1}.$$  

(3)

The final restriction is the flow constraint according to which total outlays less total income equal the change of money holdings

$$mc_t - mc_{t-1} + md_t - md_{t-1} = F_t + md_{t-1}R_t + W_t h_{t-1} n_t + PK_t k_{y,t-1} - P_t c_t - P_t (invh_t + invy_t).$$

(4)

A household receives income from different sources. He obtains rent on his physical capital stock $k_{y,t-1}$ hired out to the production sector, which amounts to $PK_t k_{y,t-1}$, where $PK_t$ is the rental rate of capital. Moreover, he earns labour income $W_t h_{t-1} n_t$, with $W_t$ the nominal wage rate, and $h_{t-1}$ the human capital stock of a household. Finally, he receives dividends $F_t$ from the intermediary and interest on his
deposits \( R_t m_{t-1} \), where \( m_{t-1} \) denotes the deposits and \( R_t \) the interest rate. Household’s outlays are consumer expenditures \( P_t c_t \) and expenditure on capital goods \( P_t (invh_t + invy_t) \), where \( invh_t \) and \( invy_t \) are the investment in the physical capital stock used in the production and human capital sector, respectively.

The stocks of physical capital of a household increase due to investment and decline through depreciation at a rate \( \delta \). Consequently, the motion equations for the capital stocks are

\[
\begin{align*}
ky_t &= invy_t + (1 - \delta)ky_{t-1} \\
kh_t &= invh_t + (1 - \delta)kh_{t-1},
\end{align*}
\]

with \( kh_t \) the physical capital stock in the human capital sector.

A household can accumulate human capital by allocating time (measured in efficiency units) \((1 - n_t - l_t)h_{t-1}\) and physical capital to education such that the production of new human capital exceeds its depreciation. Depreciation of human capital takes place at the rate \( \delta_H \)

\[
h_t = (1 - \delta_H)h_{t-1} + G(kh_{t-1}, (1 - n_t - l_t)h_{t-1}).
\]

The production function for human capital \( G \) is assumed to have constant returns to scale and to satisfy the Inada conditions.

The decision problem of a household is thus to maximise his expected utility (2) under the sequence of restrictions (3), (4), (5), (6) and (7), given his initial stocks \( kh_0, ky_0, mc_0 \) and \( md_0 \), sequence of prices \( P_t, W_t, PK_t \) and interest rate \( R_t \). When a household takes his decisions he takes into account all available information at that moment which includes the realisation of all variables up to that date.

### 2.2 The representative firm

The representative firm maximises profits by combining labour \( LY_t \), measured in efficiency units and physical capital \( KY_{t-1} \) to produce output by means of a constant returns to scale production function \( e^{\eta_t} F(KY_{t-1}, LY_t) \). The production function is subject to technological shocks \( \eta_t \) which follow an AR(1) process with normally distributed innovations \( \varepsilon_t^Y \).

\[
\eta_t = \rho_Y \eta_{t-1} + \varepsilon_t^Y, \quad \varepsilon_t^Y \sim N(0, \sigma_Y^2).
\]

\( \eta_t \) follows a normal distribution with mean 0 and variance \( \sigma_Y^2 \). The innovations to the technological and money growth shocks are assumed to be independent at all leads and lags.

\( \rho_Y \) is the autoregressive coefficient for the technological shocks, and \( \sigma_Y^2 \) is the variance of the innovations. The shocks are assumed to be independent of one another at all leads and lags.
It is assumed that the firm has to pay at least a fraction $\tau$ of the wage bill $W_tLY_t$ with cash, which it has to borrow from the financial intermediary. The credit from the intermediary has to be repaid with interest at the end of the period. Thus, the profit the firm maximises is

$$P_t e^h F(KY_{t-1}, LY_t) - (1 + \tau R_t)LY_t W_t - PK_t KY_{t-1}. \tag{9}$$

Note that in equilibrium the firm’s profits are zero, due to the assumption of constant returns to scale.

2.3 The financial intermediary

The financial intermediary collects deposits $MD_{t-1}$ at the end of each period from the households, receives the monetary injection $M_t - M_{t-1}$ from the central bank at the beginning of a period and lends these funds to the firm. At the end of the period, the firm redeems its loans and the intermediary returns the deposits including interest to the households. The intermediary behaves competitively implying that the interest on loans equals the interest paid on deposits. Because of the monetary injection, the intermediary makes a profit amounting to $(1 + R_t)(M_t - M_{t-1})$ which it pays as dividend to the households.

2.4 The recursive equilibrium

An equilibrium of this economy is defined by the first order conditions of the households and firms, market clearing conditions for the commodity, labour, credit and money market

$$e^h F(KY_{t-1}, LY_t) = INVY_t + INVH_t + C_t, \tag{10}$$

$$LY_t = H_{t-1}N_t, \tag{11}$$

$$\tau W_tLY_t = MD_{t-1} + M_t - M_{t-1}, \tag{12}$$

$$M_t = MC_t + MD_t, \tag{13}$$

and the condition that decisions of the households are consistent. The last condition entails that individual and aggregate per capita decision rules are identical $(n_t, l_t, mc_t, md_t, invy_t, invh_t, ky_t, kh_t, h_t) = (N_t, L_t, MC_t, MD_t, INVY_t, INVH_t, KY_t, KH_t, H_t)$ where uppercase variables denote aggregate per capita variables.

In order to simplify solving for the equilibrium the solutions for the prices $P_t, PK_t, W_t$ and investment
INVY_t, INVH_t are substituted and real and nominal variables are standardised as $\hat{KY}_t = \frac{KY_t}{KY_{t-1}}$, $\hat{KH}_t = \frac{KH_t}{KY_{t-1}}$, $\hat{C}_t = \frac{C_t}{KY_{t-1}}$, $\hat{MD}_t = \frac{MD_t}{M_t}$. The recursive equilibrium of this economy is then a set of stationary decision rules $\hat{KY}_t$, $\hat{KH}_t$, $\hat{C}_t$, $\hat{MD}_t$, $L_t$, $N_t$ and $R_t$ satisfying the following Euler conditions

$$
\frac{(1 + \tau R_t) U_2(t)}{\hat{H}_{t-1} e^{\nu H} F_2(t)} = \beta E_t \left[ \frac{\hat{KY}_t^{1-\sigma} \left( \left(1 - \delta \right)(1 + \tau R_{t+1}) + G(t+1) \right)}{\hat{H}_t \hat{KY}_{t-1}} U_2(t+1) \right]
$$

(14)

$$
\frac{(1 + \tau R_t) U_2(t)}{\hat{H}_{t-1} e^{\nu H} F_2(t)} = \beta E_t \left[ \frac{\left(1 + \tau R_{t+1}\right) \hat{KY}_t^{1-\sigma} \left(1 - \delta + e^{\nu H} F_1(t+1)\right)}{\hat{H}_t \hat{KY}_{t-1} e^{\nu H} F_2(t+1)} U_2(t+1) \right]
$$

(15)

$$
\frac{U_2(t)}{\hat{H}_{t-1} G_2(t)} = \beta E_t \left[ \frac{\hat{KY}_t^{1-\sigma} \left(1 - \delta_H + (1 - L_{t+1}) G_2(t+1)\right)}{\hat{H}_t \hat{KY}_{t-1} G_2(t+1)} U_2(t+1) \right]
$$

(16)

$$
\frac{\hat{C}_t (1 + \tau R_t) \hat{KY}_{t-1} U_2(t)}{\hat{H}_{t-1} \hat{MC}_t e^{\nu H} F_2(t)} = \beta E_t \left[ \frac{\hat{C}_{t+1} \hat{KY}_t^{1-\sigma} U_1(t+1)}{e^{\nu MC_t}} \right]
$$

(17)

$$
\frac{\hat{C}_t (1 + \tau R_t) \hat{KY}_{t-1} U_2(t)}{\hat{H}_{t-1} \hat{MC}_t e^{\nu H} F_2(t)} = \beta E_t \left[ \frac{\hat{C}_{t+1} (1 + \tau R_{t+1}) \left(1 + \tau R_{t+1}\right) \hat{KY}_t^{2-\sigma} U_2(t+1)}{e^{\nu + \nu H} \hat{H}_t \hat{MC}_t F_2(t+1)} \right]
$$

(18)

$$
\hat{H}_t \hat{KY}_{t-1} = G(t) + (1 - \delta_H) \hat{H}_{t-1}
$$

(19)

$$
e^{\nu H} F(t) = (\hat{C}_t + \hat{KH}_t + \hat{KY}_t) \hat{KY}_{t-1} - (1 - \delta) (\hat{KH}_{t-1} + \hat{KY}_{t-1})
$$

(20)

$$\hat{MC}_t + \hat{MD}_t = 1
$$

(21)

$$
e^{\nu H} - 1 + \hat{MD}_{t-1} = \frac{\tau \hat{H}_{t-1} \hat{MC}_{t-1} N_t e^{\nu H} F_2(t)}{\hat{C}_t (1 + \tau R_t) \hat{KY}_{t-1}}
$$

(22)

where $F(t)$, $G(t)$ and $U(t)$ denote the functions $F(\hat{KY}_{t-1}, \hat{H}_{t-1} N_t)$, $G(\hat{KH}_{t-1}, \hat{H}_{t-1} (1 - L_t - N_t))$, and $U(\hat{C}_t, L_t)$, respectively, and $f_i(t)$ the partial derivative of $f$ with respect to its $i$-th argument.

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7 For the derivation of the standardised equilibrium conditions, the following property of homogenous functions has been used. If $f$ is homogenous of degree $\lambda$, than the partial derivatives of $f$ are homogenous of degree $\lambda - 1$. 
3 NON-NEUTRALITIES OF INFLATION AND THE LIQUIDITY EFFECT

The equilibrium conditions show how inflation distorts the decisions with respect to the factor allocation across sectors, with respect to the consumption-leisure choice and with respect to the accumulation of capital in this model economy.

The distortion of the factor allocation across sectors becomes apparent when comparing the condition derived from equation (14) and (15)

\[ E_t \left[ U_2(t+1) \left( (1 + \tau R_{t+1}) \frac{F_1(t+1)}{F_2(t+1)} - \frac{G_1(t+1)}{G_2(t+1)} \right) \right] = 0 \] (23)

and the condition for a Pareto-efficient allocation in this economy\(^8\)

\[ E_t \left[ U_2(t+1) \left( \frac{F_1(t+1)}{F_2(t+1)} - \frac{G_1(t+1)}{G_2(t+1)} \right) \right] = 0. \] (24)

In market equilibrium, the marginal rate of substitution between physical capital and labour differs systematically in the two sectors, because of the term \((1 + \tau R_{t+1})\) in (23). The requirement that the firms have to finance part of their wage bill implies that the wage rate from the perspective of the producers is higher in the production sector than in the human capital sector. This difference increases with the nominal interest rate and thus with inflation. Inflation drives up the labour to capital ratio \(H_t - \frac{1}{N_t - L_t} \frac{1}{K_{H_t - L_t}}\) in the human capital sector compared to that of the production sector \(H_t - \frac{1}{N_t} \frac{1}{K_{H_t}}\). An efficient allocation of labour would require that the rate of substitution between labour and physical capital is the same in the production and training sector. Note that the capital stock of both sectors is predetermined, so that even in the Pareto-efficient allocation, the relation holds only on average. Note as well that a monetary policy, which succeeds in setting the nominal interest rate \(R_t\) equal to zero, would remove this distortion with respect to the factor allocation. Friedman (1969) argued already that a monetary policy rule, which would fix the nominal interest rate at zero is optimal.

Inflation in this economy distorts also the choice between leisure and consumption. This can be seen from (17) and (18)

\[ E_t \left[ (1 + R_{t+1}) U_2(t+1) \hat{C}_{t+1} \left( \frac{U_1(t+1)}{(1 + R_{t+1}) K_{Y_t} U_2(t+1)} - \frac{(1 + \tau R_{t+1})}{H_t e^{\eta_{t+1}} F_2(t+1)} \right) \right] = 0. \] (25)

This equation shows that not even on average the households’ marginal rate of substitution between leisure and consumption equals the marginal product of labour. It is the cash-in-advance constraint and the financing of the wage bill which are responsible for this distortionary effect of inflation. The return of a household of supplying one unit of extra labour in terms of consumption is given by \(\frac{R_{t+1} U_{t+1}}{K_{Y_t} R_{t+1} (1 + R_{t+1}) U_2(t+1)}\).

\(^8\) See the Appendix for the derivation of necessary conditions for a Pareto-efficient allocation.
Reducing leisure by one unit reduces his utility by \( U_2(t + 1) \). Borrowing on his future income \( \hat{H}_{t+1} \hat{W}_{t+1} \) at the nominal interest rate \( R_{t+1} \) and buying consumer goods for the price \( \hat{P}_{t+1} \) increases his utility by \( \hat{H}_t \hat{W}_t \hat{P}_t \hat{Y}_t(t+1) \), which is the marginal value product of labour, whereas the increase in costs equals the nominal wage rate times the cost of financing the payment: \( \hat{H}_t \hat{W}_t \hat{P}_t \hat{Y}_t(t+1) \). Thus, because of the cash-in-advance constraint, the relative price of leisure in terms of scaled consumption is not, as in the Pareto-efficient case, \( \hat{H}_t \hat{W}_t \hat{P}_t \hat{Y}_t(t+1) \) but \( \hat{H}_t \hat{W}_t \hat{P}_t \hat{Y}_t(t+1) \). The likely effect of this distortion is that inflation induces households to substitute cash requiring consumption by leisure and firms to decrease the labour to capital ratio. Note that this distortion can be eliminated completely as well, if monetary policy sets the interest rate equal to zero in each period.

Related to the distortion of capital accumulation, which will be discussed below, and a necessary condition for the existence of a liquidity effect is that the current nominal interest rate in this economy is not fully determined by the Fisherian fundamentals, that is the current real interest and inflation rate. Instead, the Fisher relation holds only on average,

\[
E_{t-1} \left[ \frac{1}{1 + \pi_t} \left( \frac{1 + R_t}{1 + \pi_{t+1}} \beta \hat{Y}_t \hat{Y}_{t+1} - U_t(t+1) - U_t(t) \right) \right] = 0,
\]

(26)

where \( \pi_t \) denotes the inflation rate in period \( t \). This corresponds to the usual Fisher relation in expectations and follows from (17), (18), and the cash-in-advance constraint. The actual nominal interest rate during a specific period, however, follows from equation (22)

\[
R_t = \frac{\hat{M}C_{t-1}e^{\eta_t} \hat{H}_{t-1} \hat{F}_2(t)}{(\hat{M} \hat{D}_{t-1} + e^{\eta_t} - 1) \hat{K} \hat{Y}_{t-1} \hat{C}_t} - \frac{\hat{N}_t}{\hat{C}_t} \cdot \frac{1}{R_{t+1}} \quad (27)
\]

Since the cash holdings, human capital and physical capital are predetermined, it is clear that the deviation of the actual nominal interest rate in each period from the Fisherian relation (26) depends on the money shock \( \zeta_t \), the technological shock \( \eta_t \), and the households’ response to these shocks with regard to their labour-consumption decision \( \frac{\hat{N}_t}{\hat{C}_t} \). Therefore, a monetary shock generates a liquidity effect in this model if households do not respond by significantly increasing their labour supply in relation to consumption.

The fact that in this model the Fisher relation holds only on average, has also implication for the capital accumulation. The condition governing in this economy the accumulation of physical capital for the production sector is

\[
E_t \left[ \left( 1 - \delta + e^{\eta_t+1} F_1(t+1) \right) \frac{\hat{B} \hat{K} \hat{Y}_t \hat{Y}_{t+1} - U_t(t+1) \hat{Y}_t \hat{Y}_{t+1}}{1 + \pi_{t+2}} \right] = E_t \left[ \frac{U_t(t+1)}{1 + \pi_{t+1}} \right],
\]

(28)

which follows from (15) and (17). Comparing this equation with the condition for the Pareto-efficient
capital accumulation

\[ E_t \left[ (1 - \delta + e^{\pi_{t+1}}F_1(t+1))\beta \dot{K}Y_t^{-\sigma}U_1(t+1) \right] = U_1(t), \quad (29) \]

reveals that inflation affects the growth rate in this economy. The reason for this distortion is, again, the cash-in-advance constraint. Increasing the capital stock by one unit in the market economy requires investing \( P_t \), which reduces available cash in \( t+1 \) by the same amount and consumption in \( t+1 \) by \( \frac{P_{t+1}}{R} \). Thus, investing one unit in \( t \) reduces utility in \( t+1 \) by \( \frac{U(t+1)}{1+\pi_{t+1}} \). The investment, however, which adds to the capital stock \( \dot{K}Y_t \) can be hired out to the firm only in \( t+1 \) and the rent \( \dot{PK}_{t+1} = P_{t+1}e^{\pi_{t+1}}F_1(t+1) \) can be used for consumption – due to the cash-in-advance constraint – only in \( t+2 \). The increase in utility due to this additional income is given by the term on the left-hand side.

These three inflationary distortions have not only a transitory impact, but affect the long-run equilibrium of the economy, too. Indeed, evaluating equations (23), (25) and (28) at the deterministic steady state of this economy or the deterministic balanced growth path, yields

\[ (1 + \tau R) \frac{F_1}{F_2} = \frac{G_1}{G_2} \quad (30) \]

\[ \frac{U_1}{(1+R)\dot{K}YU_2} = \frac{(1 + \tau R)}{\dot{H}F_2} \quad (31) \]

\[ (1 - \delta + F_1)\beta \dot{K}Y^{-\sigma} = 1. \quad (32) \]

The first and second of these equations show that the rate of substitution between labour and capital in the two sectors is not equalised and that the rate of substitution between leisure and consumption does not equal the marginal product of labour in the production sector. Although the final equation is identical to the Pareto-efficient growth rule, the distortion of the factor allocation (the first equation) implies that the growth rate in this economy is not generally the same as in the Pareto-efficient case. However, if the central bank would choose a policy such that the nominal interest rate is zero in the steady state, the balanced growth path of this economy satisfies the conditions of the Pareto-efficient path.

\[ ^9 \text{On the possibility of a monetary policy rule which removes even this distortion, see Carlstrom and Fuerst (1995).} \]
4 PARAMETERISATION AND CALIBRATION

From the discussion in the previous section, it became clear that inflation distorts economic decisions. In this section the production and utility functions will be specified and their parameters will be calibrated to the 8 largest countries participating in the European Monetary Union in order to show numerically that these distortions lead to a liquidity effect of a monetary shock. The fully specified model will also be used to calculate the size of the welfare costs of these distortions and to investigate whether these costs can be avoided by reducing the money growth rate.

It is assumed that the period utility function of the representative household has the form

\[ U(c, l) = \frac{1}{1 - \gamma} \left( c^{\xi} l^{1 - \xi} \right)^{1 - \gamma}. \]  

(33)

This specification satisfies the necessary condition for the existence of a balanced growth path (see Section 2.1) which was also imposed during the discussion in the previous section. The intertemporal elasticity of substitution \( \frac{1}{\sigma} \) equals here \( \frac{1}{\xi(1-\gamma)} \).

The production functions for output and human capital are assumed to belong to the Cobb-Douglas variety

\[ F(K, L) = K^\alpha L^{1-\alpha} \]  

(34)

\[ G(K, L) = \chi K^\theta L^{1-\theta}. \]  

(35)

For the numerical exercise, the structural parameters have been calibrated to (per capita) long-run averages from the eight largest countries of the European Monetary Union (EMU8), viz. Austria, Belgium, Finland, France, Germany, Italy, the Netherlands, and Spain. These countries represent 96.3% of total GDP of the EMU. These parameter values are summarised in Table 1. The area-wide data are aggregated national quarterly data for the period 1979:II–1999:IV, beginning with the start of the EMS. Nominal variables, for example GDP, were first converted to Deutsche Mark using the 1990 purchasing power parities. The area-wide nominal interest rate has been constructed as a weighted average of the national 3-month rates using 1990 GDP shares as weights, where national GDP figures were converted to Deutsche Mark using 1990 purchasing power parities. The same weights have been used to aggregate national GDP deflators for the calculation of the EMU8 inflation rates10.

The average nominal interest rate and average per capita real growth rate of GNP, which were used to calibrate \( \gamma, \xi \) and \( \beta \), are 2.28% and 0.47% on a quarterly basis. For the calibration of these three parameters, it was also assumed that the intertemporal elasticity of substitution is \( \frac{2}{3} \). This value is well

10 See Winder (1997) on the aggregation of area-wide data.
Table 1  Calibrated parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.330</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.28</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.016</td>
</tr>
<tr>
<td>$\delta_H$</td>
<td>0.008</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.02</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.4</td>
</tr>
<tr>
<td>Utility</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.879</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.26</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.996</td>
</tr>
<tr>
<td>Shock processes</td>
<td></td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.016</td>
</tr>
<tr>
<td>$\rho_\xi$</td>
<td>0.86</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
<td>0.0053</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.000</td>
</tr>
<tr>
<td>$\rho_\eta$</td>
<td>0.79</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>0.0040</td>
</tr>
</tbody>
</table>

Within the range found in microeconomic studies as reported in Mehra and Prescott (1985)\(^{11}\). Finally, it was used that the fraction of total available time, i.e. of the time not required for sleeping and eating, allocated by households to working in the production sector is roughly 23%. This fraction is based on microeconomic evidence for working age men and women in Netherlands for 1975–1995\(^ {12}\). Microeconomic evidence from other European countries (Finland, Denmark, Sweden) reported by Juster and Stafford (1991) seems to indicate that the data for the Netherlands may not be representative. However, the results presented in Juster and Stafford are obtained from time allocation surveys that do not cover unemployed men, male students or men unable to participate in the labour market. Therefore, these statistics are likely to overestimate the fraction of time that the working age population spent in working in the production sector.

The capital share parameter in the production sector has been set to $\alpha = 0.33$ which is the long run average capital share of area-wide GDP. Based on available data on investment and the capital stock, the empirical depreciation rate has been estimated at $\delta = 1.56\%$ per quarter\(^ {13}\). Because the driving force of growth in this model is human capital, one cannot use the Solow residuals to calibrate the variance of technology shocks. Although the autocorrelation coefficient for the technological shocks was derived from an analysis of the Solow residuals, the variance was chosen such that the variance of observed GDP equals the variance of output in the model\(^ {14}\). The final parameter relevant for the decision problem of the

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\(^{11}\) The intertemporal elasticity of substitution that the risk premium on European assets implies, would even be lower. See Fase (1997, Table 3).


\(^{13}\) Reliable data on the capital stock are scarce. For some EMU countries, i.e. Belgium, Finland, France, Germany, and Italy annual capital stock data exist (OECD 1997). The annual depreciation rate of the capital stock (exclusive dwellings and government sector) according to OECD data for Belgium, Finland, France and Germany for the sample period 1979–1995 is 6.5\% per year. We did not use Italian data for our estimation because the depreciation rate for Italy is implausibly low (1.7\% per year), which is probably due to the fact that the capital stock data cover also dwellings which have a much lower depreciation rate than equipment.

\(^{14}\) See Gomme (1993) for the same approach.
firm is the share of the wage bill that has to be financed by cash. For the sake of the numerical exercise a value of $\tau = 40\%$ was chosen.

Since there are no reliable data on the human capital production function, we choose to set $\theta$, the capital share in the human capital sector to 0.28. The only argument for this choice is that the capital intensity in the human capital sector is lower than that in the production sector. Lacking reliable data, there is no guideline to choose the depreciation rate of human capital, either. Since knowledge and skills are not consumed in the production process as equipment is, it seems reasonable to assume that the depreciation rate of human capital is lower than that of physical capital. Without further evidence on this issue, a depreciation rate half as large as that of physical capital seems reasonable. The scale parameter $\chi$ of the human capital production function was set to 0.02316, such that the model’s rate of growth on the balanced growth path equals the average annual per capita real growth rate in the EMU8 countries of 1.9% for the period 1979:I–1999:IV.

The parameters governing money growth have been derived from area-wide data on M1 $^{15}$. On estimating an AR(1) model for per capita M1 growth, we found an autocorrelation coefficient of 0.856 and a standard deviation of 0.0053.

$^{15}$ Time series on area-wide M1 have been published by the ECB, see European Central Bank (1999).
5 SHORT RUN DYNAMICS AND THE LIQUIDITY EFFECT

In this section, the dynamic properties of the model economy are investigated numerically. We begin discussing the effects of an unexpected rise of the money growth rate by 1 percentage point. Then the effects of a shock to total factor productivity are described. Finally, we confront the second moments implied by the model with the stylised facts of the European business cycle. In all exercises, we solved for the equilibrium path using a log-linear approximation around the steady state.

The response of the economy to an unexpected monetary expansion is depicted in Figure 1. The effect of the shock on key variables is shown as deviation from the balanced growth path along which the economy would have evolved in the absence of the shock. The deviation of those variables which are stationary without prior transformation (see Section 2.4), such as the nominal interest rate and the fraction of time allocated to working in the production sector and to education are expressed in percentage points.

Figure 1 shows that the model calibrated to the European Monetary Union exhibits a liquidity effect. The unexpected rise of the money stock in period 1, lowers immediately the nominal interest rate and boosts employment and output. The downward pressure on the nominal interest rate, which is caused by the reluctance of firms to absorb the excess liquidity in the credit market, dominates the effect of increased inflation expectations. A lower interest rate decreases the costs to finance the wage bill and reduces ceteris paribus the real wage rate as perceived by the firm. Thus, labour demand and after bidding up the real wage rate, equilibrium employment rises. With the physical capital stock predetermined, higher employment implies higher output. A consequence of the higher real wage rate is that the household’s return to working in the production sector increases relative to the return of education. This leads households to reallocate time from training to working.

Clearly, the monetary expansion implies higher inflation, which reduces the real value of households’ cash holdings and, at least in the period when the shock occurs, reduces consumption. Therefore, the additional output produced by the firms in the initial period is not demanded for consumption purposes but for investment. The households invest or save because consumption in periods with high inflation is relatively more expensive than in periods with lower inflation. This intertemporal substitution is achieved through investment in the capital stock. Although not visible in Figure 1, the change in composition of the aggregate capital stock does not remain unchanged. The expected high interest rate for the second and following periods leads to investment in the physical capital stock of the human capital sector and disinvestment in the production sector (cf. equation (23)).

In the second and following periods, the money stock expands further, because of the persistence of the money shock. The households, however, will anticipate this further expansion and adjust their cash
Figure 1  Impulse response to a monetary shock

Explanatory notes: The vertical axis measures percent respectively percentage point deviations from the pre-shock balanced growth path. Physical capital and investment are aggregated over both sectors.

holdings accordingly. This adjustment has the consequence that after the first period no excess liquidity remains in the credit market. Without excess liquidity, only expected inflation will have an impact on the nominal interest rate. Only when compensated for the expected inflation, households are willing to hold part of their money as deposits. Since inflation remains above its steady state level, the nominal interest rate will jump above its steady state level as well. The rise of the nominal interest rates also bids up the firm’s cost to finance the wage bill, with adverse effects on employment and the real wage rate. The ensuing fall of the return to labour in the production relative to that in the human capital sector, reverses the initial reallocation of time towards the production sector.

With inflation above its steady state level, households tend to substitute intratemporally leisure for consumption, since inflation increases the price of cash requiring consumption vis-à-vis leisure. Moreover, consumption is substituted intertemporally towards periods with lower inflation. This behaviour explains the response pattern for the periods following the shock: time allocated to working and education declines, consumption is persistently below its steady state level.
To our knowledge, this model is the first endogenous growth model that is able to explain a liquidity effect of expansionary monetary policy. The limited participation assumption is essential for this property. This can be seen from Figure 2, depicting the impulse response to a monetary shock in the model of Gomme (1993). Gomme’s model is an endogenous growth model in which money is introduced through a cash-in-advance constraint. It is very similar to our model but does not include the limited participation assumption. The figure clearly shows, that no liquidity effect occurs, if households can adjust their portfolios immediately. From the first period onwards, the higher expected inflation leads to an increase of the nominal interest rate, and, since inflation raises the relative price of cash requiring consumption with respect to leisure, decreases consumption, employment and output. This pattern is exactly the opposite to the effect of a monetary expansion which common sense and empirical research suggest.\footnote{The figure shows the impulse response to an unexpected increase of the money growth rate by 1 percentage point. In order to make the models comparable, Gomme’s model has been calibrated for the European Union using, as far as applicable, the same data as described in Section 4. Note that the absence of a liquidity effect in an endogenous growth model without the limited participation assumption has also been shown by Thijssen (1999).}

The consequences of a technological shock in our model are depicted in Figure 3. An increase of the pro-
Figure 3  Impulse response to a technological shock

Explanatory notes: See Figure 1.

...ductivity of both input factors boosts ceteris paribus output and lowers inflation. A higher productivity of both factors makes it profitable to invest and to increase employment. The increase of employment, however, is curbed in the impact period of the shock by the rise of the nominal interest rate. More employment implies an excess demand for credit to finance the wage bill. Since credit supply is predetermined the excess demand bids up the nominal interest rate.

The productivity shock leads to an expected increase of the relative return to labour and physical capital in the production vis-à-vis the human capital sector. In the first period households reallocate their time from education to production in order to finance further investment. This diversion is strong enough to reduce the real wage rate with an adverse effect on total time allocated to working and education. The mechanism here is that households work more when (and where) productivity is high and smooth consumption by saving (investing) and dissaving. Nevertheless, the decline of the real wage rate following a productivity shock is a relatively implausible implication of this model.

In Figure 4 the stylised facts of the European business cycle are confronted with the properties of the
model. As common in the real business cycle literature, these stylised facts are summarised by the standard deviations, autocorrelations and the cross-correlations of detrended time series of key variables. The trend in the observed and simulated time series have been removed with the aid of a Hodrick-Prescott filter. This figure illustrates that the model does not succeed in going a long way in explaining observed business cycle facts. The model is relatively successful in replicating observed standard deviations. The exception is that observed inflation shows little variation, whereas the model implies the opposite. The persistence of deviations from the trend (the autocorrelation) as predicted by the model is generally lower than the observed persistence. Finally, the comovement of output with other aggregates is measured by the cross-correlations at different leads and lags. All aggregates, except real wages, included in Figure 4 behave procyclically in Europe: if current output deviates from its trend, then consumption, investment etc. tend to deviate from their trend in the same direction. Except for inflation, the model replicates this behaviour. Furthermore, none of the observe time series which are considered here are significantly leading or lagging the trend deviations of output. This regularity is shared by the model-simulated series.

Clearly, the model succeeds only to explain the observed moments qualitatively, but it is too stylised to do so quantitatively. An important point, however, which the model fails to explain is the procyclical behaviour of inflation. Although it is difficult to draw definite conclusions from the simulated moments, it is likely that the response to a technological shock accounts for the countercyclical behaviour. Indeed, as discussed above, a productivity shock moves output and inflation in opposite directions.
Figure 4 Second moments

Explanatory notes: Solid white bars correspond to the moments calculated from observed time series, solid black bars correspond to moments calculated from simulated time series (3000 periods). The upper seven panels show the cross-correlations of current output with the indicated aggregates at different leads and lags.
6 WELFARE COSTS OF INFLATION

This section explores the implications of the model with respect to the welfare costs of inflation. More specifically, the question to be answered is what are the welfare gains of achieving a lower inflation rate than the average European rate of 4.7% between 1979 and 1999? The common approach to answer this question is to determine the equivalent variation of jumping directly to the steady state corresponding to the lower inflation rate. The equivalent variation is calculated as the annuity value of consumption which would make households indifferent between remaining in the current steady state and being in the steady state with the lower inflation rate. Thus, if \((L(\pi), C(\pi), Y(\pi))\) denote leisure, per capita consumption and output in the steady state corresponding to the inflation rate \(\pi\), then \(\lambda\) solving

\[
\sum_t \beta^t U(C(\pi) + \lambda Y(\pi), L(\pi)) = \sum_t \beta^t U(C(\pi'), L(\pi'))
\]

or, equivalently

\[
U(C(\pi) + \lambda Y(\pi), L(\pi)) = U(C(\pi'), L(\pi'))
\]

is the additional consumption as a fraction of GDP a household would require in order to be indifferent between remaining in the steady state corresponding to the inflation rate \(\pi\) and switching to the steady state with inflation rate \(\pi'\). The welfare costs of the inflation rate \(\pi\) as such is the equivalent variation (37) with respect to the Pareto-optimal inflation rate \(\pi'\), that rate where inflation ceases to distort economic decisions.

This approach to calculate the welfare costs of inflation, however, might be misleading for two reasons. Firstly, the inflation rate with the highest steady state welfare level, is not necessarily the welfare maximising inflation rate. Consequently, the welfare costs of inflation evaluated at the steady state might be biased. Secondly, politicians are not faced with the choice between different steady states, but they have to take the costs or benefits into account of the transition from one to the other.

In order to illustrate the first argument against the traditional approach, Table 2 shows the equivalent variation according to definition (37) of different inflation rates in our endogenous growth model. For the purpose of illustration, however, a slightly different set of parameters than that presented in Table 1 was chosen. We set \(\delta = 2\%, \delta_H = 1\%, \chi = 0.026, \gamma = 2.8, \xi = 0.28, \beta = 0.995\) and \(\bar{\zeta} = 1.8\%\). The first column in this table contains the annual money growth rate for which the welfare costs have been calculated, the second column shows the corresponding annual inflation rate and the third column the change of the economy’s growth rate with respect to the benchmark situation. The benchmark corresponds with a situation with an annual real growth rate of 1.2% and an annual inflation rate of 5.7%. The annuity value of consumption as percentage of GDP that would compensate the households for remaining in the
The costs or benefits of the transition from the initial situation to the new situation into account. In this context, we are interested in the welfare costs of inflation. Politicians cannot choose between different steady states, but have to take the welfare costs of inflation into account.

**Table 2: Equivalent variation as percentage of GDP**

<table>
<thead>
<tr>
<th>Money growth rate (%)</th>
<th>Inflation growth rate (%)</th>
<th>Change of growth rate (%)</th>
<th>Steady state comparison</th>
<th>Comparison including transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>102.26</td>
<td>100.00</td>
<td>−0.64</td>
<td>−2.251</td>
<td>−2.510</td>
</tr>
<tr>
<td>27.01</td>
<td>25.00</td>
<td>−0.17</td>
<td>−0.238</td>
<td>−0.283</td>
</tr>
<tr>
<td>11.91</td>
<td>10.00</td>
<td>−0.04</td>
<td>−0.033</td>
<td>−0.043</td>
</tr>
<tr>
<td>10.90</td>
<td>9.00</td>
<td>−0.03</td>
<td>−0.024</td>
<td>−0.032</td>
</tr>
<tr>
<td>9.89</td>
<td>8.00</td>
<td>−0.02</td>
<td>−0.016</td>
<td>−0.021</td>
</tr>
<tr>
<td>8.89</td>
<td>7.00</td>
<td>−0.01</td>
<td>−0.009</td>
<td>−0.012</td>
</tr>
<tr>
<td>7.88</td>
<td>6.00</td>
<td>0.00</td>
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<td>−0.003</td>
</tr>
<tr>
<td>7.57</td>
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<td>0.00</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>6.87</td>
<td>5.00</td>
<td>0.01</td>
<td>0.004</td>
<td>0.006</td>
</tr>
<tr>
<td>5.86</td>
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<td>0.02</td>
<td>0.009</td>
<td>0.013</td>
</tr>
<tr>
<td>4.85</td>
<td>3.00</td>
<td>0.03</td>
<td>0.014</td>
<td>0.019</td>
</tr>
<tr>
<td>3.84</td>
<td>2.00</td>
<td>0.04</td>
<td>0.017</td>
<td>0.025</td>
</tr>
<tr>
<td>2.84</td>
<td>1.00</td>
<td>0.05</td>
<td>0.020</td>
<td>0.030</td>
</tr>
<tr>
<td>1.83</td>
<td>0.00</td>
<td>0.06</td>
<td>0.022</td>
<td>0.034</td>
</tr>
<tr>
<td>0.82</td>
<td>−1.00</td>
<td>0.07</td>
<td>0.023</td>
<td>0.037</td>
</tr>
<tr>
<td>0.31</td>
<td>−1.50</td>
<td>0.07</td>
<td>0.345</td>
<td>0.357</td>
</tr>
<tr>
<td>−0.19</td>
<td>−2.00</td>
<td>0.08</td>
<td>0.023</td>
<td>0.039</td>
</tr>
<tr>
<td>−0.69</td>
<td>−2.50</td>
<td>0.08</td>
<td>0.023</td>
<td>0.039</td>
</tr>
<tr>
<td>−1.20</td>
<td>−3.00</td>
<td>0.09</td>
<td>0.022</td>
<td>0.040</td>
</tr>
<tr>
<td>−1.70</td>
<td>−3.50</td>
<td>0.09</td>
<td>0.022</td>
<td>0.040</td>
</tr>
<tr>
<td>−2.21</td>
<td>−4.00</td>
<td>0.10</td>
<td>0.021</td>
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</tr>
<tr>
<td>−2.71</td>
<td>−4.50</td>
<td>0.10</td>
<td>0.019</td>
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</tr>
<tr>
<td>−2.83</td>
<td>−4.62</td>
<td>0.10</td>
<td>0.019</td>
<td>0.040</td>
</tr>
</tbody>
</table>

**Explanatory notes:** The money growth and inflation rate are measured on an annual basis. The change of the growth rate is measured with respect to the benchmark situation corresponding to an annual inflation rate of 5.69%. The equivalent variation (columns 4–9) is the transfer (as percentage of quarterly GDP) to the households which would make them indifferent between remaining in the benchmark situation or being in, respectively moving towards, a steady state corresponding to a higher or lower inflation rate.

Benchmark situation with an inflation rate of 5.7% is shown in the fourth column of the table. The next column contains essentially the same information as the fourth, except that the compensation has been calculated as a once and for all transfer in the first period (quarter). These figures can be interpreted as the present value of the annuity. By comparing equivalent variations derived from steady state comparisons, it seems as if an annual inflation rate close to −1.5% would be the choice in this example if one wishes to maximise welfare. However, the true welfare maximising balanced growth path is attained at the much lower inflation rate of −4.6%, where the nominal interest rate equals zero. This is the Pareto-optimal path a social planner would choose in order to maximise the welfare of the representative household.

Why would a benevolent social planner choose a policy, which does not maximise welfare *at the steady state*? The explanation also illustrates our second argument against the traditional method to calculate the welfare costs of inflation. Politicians cannot choose between different steady states, but have to take the costs or benefits of the transition from the initial situation to the new situation into account. In this model it turns out that the adjustment path chosen by the social planner, if the initial situation is on the

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17 See the discussion in Section 3 for an explanation why a nominal interest rate of zero would be Pareto-optimal.
balanced growth path belonging to the current inflation rate, generates *benefits* which overcompensate the present value of the lower welfare level attained in the new balanced growth path. This can be seen in the last two columns of Table 2. These columns show the annuity and present value of consumption, which would make households indifferent between remaining in the current balanced growth path or moving to the new steady state on the Pareto-optimal adjustment path. It is calculated as the fraction of GDP $\lambda$, which solves

$$\sum_t \beta^t U(C_t(\pi) + \lambda Y_t(\pi), L_t(\pi)) = \sum_t \beta^t U(C_t(\pi'), L_t(\pi'))$$ \hspace{1cm} (38)

with $(C_t(\pi), Y_t(\pi), L_t(\pi))_{t=0}^{\infty}$ being leisure, per capita consumption and output on the balanced growth path corresponding to the inflation rate $\pi$, and $(C_t(\pi'), L_t(\pi'))_{t=0}^{\infty}$ leisure and consumption during the transition to and on the balanced growth path corresponding to the inflation rate $\pi'$. This equivalent variation includes, therefore, the benefits or costs of the transition. It can be seen that once the transition is accounted for, the welfare maximising inflation rate does indeed correspond to the situation where inflation ceases to distort any economic decision 18.

Since, in economic terms, no politician in a decentralised economy can do better than the benevolent social planner, the welfare gains realised by the planner is the upper bound for the benefits the best policy can attain. Note that in this model the maximal welfare gains from reducing inflation is more than twice as high as suggested by the comparison of steady state welfare levels!

It has already been said that politicians in a decentralised economy cannot achieve larger welfare gains than those associated with the Pareto-optimal adjustment. What are then the welfare gains of reducing inflation if the central bank would follow a simple money growth rule? Assuming that the central bank can credibly commit itself to a permanently lower money growth rate, the answer to this question is provided in column six and seven of Table 2. By simply lowering the long run money growth rate, the welfare gains of the transition to the new steady state would fall short of the maximal attainable gains 19. Indeed, if politicians were constrained to a simple money growth rule, the optimal inflation rate they should aim at is close to $-3.5\%$ which is higher as well than the Pareto-optimal inflation rate of $-4.6\%$. Attempting to reduce inflation further by simply lowering the money growth rate would decrease welfare due to an inefficient adjustment path. Indeed, as shown in Figure 5, the transition of the decentralised economy under a monetary growth rule from the benchmark balanced growth path to the balanced growth path with an inflation rate of $-4.6\%$ differs significantly from the Pareto-optimal adjustment path. The most important difference is that households in the market economy substitute higher consumption and leisure in the period of the policy change by lower consumption in the future, whereas optimal adjustment would

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18 The transition has been calculated from the log-linearised equilibrium conditions.

19 The policy change is anticipated by the household, that is the new policy for the next period is announced when households decide on their portfolio composition.
Table 3 shows the results of the same calculations for our model calibrated to the EMU. Again, one

smooth consumption at a level which is, from the period after the policy change onwards, permanently
higher than in the market economy. The explanation for this difference is probably the hike in the nom-
inal interest rate following the reduction in money supply. One might therefore conjecture is that the
transitional path would be much closer or even identical to the Pareto-optimal path if the central bank
would implement an interest rate rule, which fixes the nominal interest rate directly at its long run level
of 0%.

Returning to the welfare results in Table 2, note that even under the simple money growth rule, the
implementation of the inflation rate which is optimal under this policy rule yields welfare gains that
are nearly twice as high as the maximal welfare gains estimated by comparing the welfare levels at the
steady states. Therefore, neglecting the transitional dynamics may significantly affect the estimated total
welfare gains which can be achieved by lowering inflation and it may bias the estimate of the welfare
maximising inflation rate.

Table 3 shows the results of the same calculations for our model calibrated to the EMU. Again, one
finds that the traditional method gives a biased estimate of the optimal inflation rate if monetary policy is restricted to a simple money growth rule. If monetary policy is restricted to a money growth rule and if transitional welfare effects are taken into account, the optimal inflation rate would be one percentage point higher than suggested by comparing steady state allocations. The table shows that the upper bound on the welfare gains achievable by reducing the current level of inflation to the Pareto-optimal level is more than 1.3% of current annual GDP. Note that for this calibration, steady state comparisons identify the Pareto-optimal inflation rate as welfare maximising. This would be correct if monetary policy could implement a policy rule leading the economy along the Pareto-optimal path to the new steady state. The simulations also show that reducing inflation from its current long run level of 4.7% to price stability by adjusting the growth of the money stock would yield permanent welfare gains amounting to 0.26% of European GDP. The present value of these transfers is roughly 1% of current annual European GDP.

How do the welfare costs of inflation calculated with our endogenous growth model calibrated for the EMU compare with the costs estimated in related studies? In our model of the EMU, a reduction of the inflation from 10% to zero increases the annual growth rate by roughly 0.1 percentage point. The welfare benefits of this reduction seems small: A transfer of consumer goods in each quarter amounting to merely 0.08% of GDP would make households indifferent between remaining in a situation with 10% inflation and a reduction of money growth which would ultimately lead to price stability. However, although this welfare effect is small, it is economically significant, the present value of the annuity of 0.08% GDP equals 2.9% of annual GDP. Gomme (1993) used a similar endogenous growth model to estimate the welfare costs of inflation. He found that reducing inflation from 10% to price stability yields welfare gains of 0.028% in terms of an annuity, less than half of our estimate. The reason for this difference is twofold. On the one hand, Gomme did not use the limited participation framework and, on the other hand, he ignored the transition to the new steady state and the costs and benefits associated with it.

In Section 5, especially Figure 2, it was shown that Gomme’s model did not explain the liquidity effect of a monetary expansion. Clearly, when comparing only steady states, neglecting short run properties of the model is of no consequence. If, however, as we argued in this section, the transition matters for assessing the welfare costs of inflation, the short run response to a change of monetary policy cannot be neglected. Without taking proper account of the transition and the short run dynamics of the model, the calculated welfare costs of inflation are likely to be biased.
Figure 5  Transitional dynamics

- Output
- Consumption
- Human capital
- Investment
- Investment in production sector
- Investment in training sector
- Leisure
- Working fraction
- Training
Figure 5   Transitional dynamics (continued)

Explanatory notes: Dotted lines depict the Pareto-optimal adjustment, solid lines the transition to the new balanced growth path in the decentralised economy under a simple monetary growth rule. Before the announced policy change, the economy is on the balanced growth path corresponding to an inflation rate of 5.7%. The new long run inflation rate is −4.6%.
The low welfare costs of inflation calculated may increase, if other distortions in the economy are included in the analysis. Decisions in the economy may be distorted due to, for example, taxation (see Cooley and Hansen (1991)) or capital externalities arising in other types of endogenous growth models (see Wu and Zhang (1998)). If the economy is subject to other distortions the welfare losses due to inflation are not of second- but of first order. The intuition behind this result can be illustrated with a partial equilibrium example. In the left panel of Figure 6 the hatched triangle under the money demand function is the dead-weight loss due to inflation in an otherwise undistorted economy. If other distortions are present, the dead-weight loss due to inflation is not a triangle, but a trapezium. This is shown in the right hand panel of Figure 6, where the black triangle under the money demand function is the dead-weight loss due to e.g. taxation, whereas the hatched trapezium is the welfare loss due to inflation. The welfare costs of inflation increase in an already distorted economy since inflation erodes part of the tax base. However, the presence of taxes does not necessarily raise the welfare costs of inflation. If the reduction of inflation decreases government revenue or leads to higher government expenditure and the budget deficit is financed by increasing taxes, the welfare gains of reducing inflation may turn out to be lower than estimated in our model or may even be negative. From the analysis of Cooley and Hansen (1991) and Love and Wen (1999) on the interaction of inflation and taxation, it emerges that the specification of the fiscal response to the disinflation is crucial for the welfare analysis of lowering inflation.

Beside the influence of other distortion on the welfare gains of lowering the inflation rate, there is one additional issue, which has been ignored by the current analysis. In a low inflation environment, monetary policy may be less effective in stabilising output and inflation volatility because of the zero bound on nominal interest rates, as has been argued by Summers (1991) and e.g. Wolman (1998). Note, however, that the discussion of the welfare costs associated with a low inflation regime has not come to any definite conclusion. On one hand, welfare costs are associated with a low inflation regime only if inflation is persistent, implying that the central bank cannot rise expected inflation and thus create a negative real interest rate. Because of lacking empirical evidence, however, it is unclear whether or not inflation would remain persistent under a credible policy regime of price stability. On the other hand, Reifschneider and Williams (1999) show that even with inflation persistence, one can design monetary policy rules, such
as price level targeting, which can restore the ability of the central bank to stabilise output and inflation at low inflation rates. Therefore, only a more detailed analysis of disinflation policies in a general equilibrium framework with realistic monetary policy rules can shed light on the total welfare costs of going from low inflation to, for example, price stability.

A final point we want to stress is the surprisingly small effect of inflation on economic growth in our model. This small effect has also been pointed out in other studies using two-sector endogenous growth models. The size of the growth effect might explain why the existence of a relation between inflation and growth at least for low and moderate rates of inflation is controversial in the empirical literature. Given the limited number of observations for countries and periods with low or even zero inflation, it is difficult to find a significant parameter displaying the effect of inflation on growth of that size.

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20 For example Gomme (1993). In an endogenous growth model with capital externalities Wu and Zhang (1998) calculate comparably large growth effects of inflation, whereas Dotsey and Ireland (1996) reports effects of a similar magnitude as in the two-sector growth model.
7 CONCLUSIONS

In this paper a limited participation model with endogenous growth has been developed. It has been shown that it explains the so-called liquidity effect of an expansionary monetary policy. By explaining the liquidity effect the model passes the most basic test in order to qualify for monetary policy analysis. Arguably, the model is thus better suited to study disinflation processes than other endogenous growth models used for that purpose.

A second contribution of the paper is to clarify the measurement of the welfare costs of inflation. It was shown that the traditional approach of estimating the welfare costs of inflation by comparing the allocations of two long run equilibria, might be misleading. Firstly, the traditional approach neglects the costs or benefits of the transition to the new balanced growth path. Politicians, however, cannot choose between two long run equilibria, but have to take the transition into account when assessing the cost and benefits of reducing inflation. For the model presented here, it has been demonstrated that the welfare costs of inflation according to the traditional method can be biased downward. Secondly, the traditional approach may lead to biased estimates of the welfare maximising inflation rate. For our model, it was shown that the traditional approach can over- as well as underestimate the true welfare maximising inflation rate. The discussion of disinflation processes also clarified that the welfare costs of inflation depends on the total policy package by which the inflation reduction is implemented. It was argued that not only the monetary policy rule affects the assessment of the potential welfare gains of an inflation reduction, but also the fiscal policy in response to possible revenue losses or increases of government expenditure. Finally, it was demonstrated that the disinflation process does not necessarily imply welfare losses, but can even generate welfare gains. This finding is contrary to the widespread belief that reducing inflation is costly. A third contribution of the paper is that the optimal degree of disinflation depends on the monetary policy rule. It has been shown that a simple monetary growth rule does not necessarily attain the upper bound of the welfare gains and that the welfare maximising inflation rate given that monetary policy is constraint to a simple money growth rule is higher than the inflation rate in a Pareto-optimal balanced growth path. It was conjectured that a nominal interest rate rule, which targets the Pareto-optimal nominal interest rate of zero might attain the upper bound on the potential welfare gains.

The final contribution of the paper is an estimate of the welfare costs of inflation for the European Monetary Union. Based on the limited participation model with endogenous growth it was estimated that reducing inflation from its current long run level of 4.7% to price stability by a simple money growth

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21 See e.g. Feldstein (1996), Moran (1999)) as examples. Only Einarsson and Marquis (1999) demonstrated as well that disinflation is not necessarily costly.
rule would lead to permanent annual welfare gains amounting to 0.026% of GDP. The present value of these welfare gains is roughly 1% of current annual European GDP. Applying more sophisticated monetary policy rules might further increase these welfare gains. The upper bound on the welfare gains a monetary policy rule can realise when lowering inflation from 4.7% to price stability is roughly 1.3% of current annual European GDP.

Although economically significant, the estimated welfare costs of inflation are low. If the interaction of inflation with other distortions such as taxation or spill-over effects of capital accumulation as in other models of endogenous growth, are considered as well, the welfare gains associated with lower inflation rates may be higher. It has to be borne in mind, however, that other possible costs of a low inflation regime, for example those that may arise due to the lower bound on the nominal interest rate, have not been taken into account in this study.
A NECESSARY CONDITIONS FOR AN PARETO-OPTIMAL ALLOCATION

The Euler conditions of the social planner’s welfare maximisation problem are derived in this appendix. These conditions are satisfied by any interior Pareto-optimal allocation. In the appendix, the same notation is used as in the text.

The social planner maximises the utility function of the representative household

\[ E_t \sum_i \beta^i U(C_i, L_i), \quad (39) \]

with respect to per capita consumption \( C_t \), physical capital stock in the production sector \( KY_t \), the human capital sector \( KH_t \), the human capital stock \( H_t \), and leisure \( L_t \) and labour supply \( N_t \). The planner takes the resource constraint

\[ C_t + KY_t - (1 - \delta)KY_{t-1} + KH_t - (1 - \delta)KH_{t-1} = e^\eta F(KY_{t-1}, H_{t-1}N_t) \quad (40) \]

and the transition equation for the human capital stock into account

\[ H_t = (1 - \delta_H)H_{t-1} + G(KH_{t-1}, H_{t-1}N_t). \quad (41) \]

Deriving the first order conditions from this maximisation problem and transforming the non-stationary variables by \( \hat{C}_t = \frac{C_t}{KY_t}, \hat{KY}_t = \frac{KY_t}{KY_t}, \hat{KH}_t = \frac{KH_t}{KH_t}, \) and \( \hat{H}_t = \frac{H_t}{KY_t} \) yields the Euler conditions characterising the stationary decision rules of the social planner

\[ \hat{U}_1(t) \quad \text{E} \left[ \frac{1}{\hat{KY}_{t-1}U_2(t)} \right] = \frac{1}{\hat{H}_{t-1}e^\eta F_2(t)} \quad (42) \]

\[ E \left[ \left( 1 - \delta + e^{\eta \cdot 1} F_1(t + 1) \right) \beta KY^{-\sigma}_t U_1(t + 1) \right] = U_1(t) \quad (43) \]

\[ E \left[ \left( \frac{G_1(t + 1)}{G_2(t + 1)} - \frac{F_1(t + 1)}{F_2(t + 1)} \right) U_2(t + 1) \right] = 0 \quad (44) \]

\[ \beta E \left[ \frac{\left( 1 - \delta_H + (1 - L_{t+1})G_2(t + 1) \right) \hat{KY}^{1-\sigma}_t U_2(t + 1)}{H_t G_2(t + 1)} \right] = \frac{\hat{KY}_{t-1}U_2(t)}{H_{t-1}G_2(t)} \quad (45) \]

\[ \hat{C}_t \hat{KY}_{t-1} + \hat{K}Y_t \hat{KY}_{t-1} - (1 - \delta)\hat{KY}_{t-1} + \hat{K}H_t \hat{KY}_{t-1} - (1 - \delta)\hat{KH}_{t-1} = e^\eta F(t) \quad (46) \]

\[ \hat{H}_t \hat{KY}_{t-1} = (1 - \delta_H)\hat{H}_{t-1} + G(t). \quad (47) \]
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