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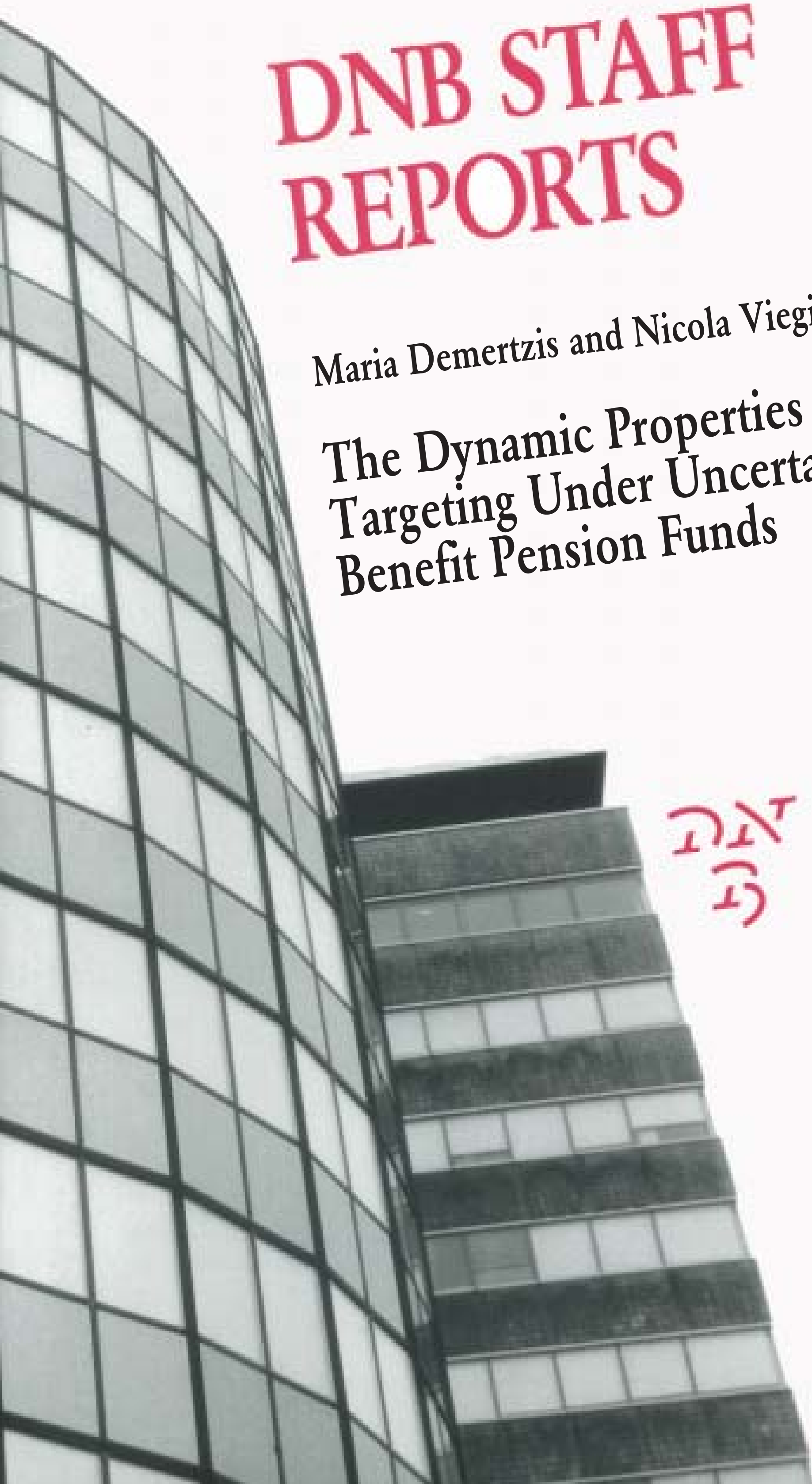

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The Dynamic Properties of Inflation Targeting Under Uncertainty*

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Abstract

We study the implications of uncertainty for inflation targeting in a dynamic set-up. Using Svensson's inflation forecast targeting model, we compare the Brainard conservative principle to a more active monetary policy rule, derived from a two-step optimisation procedure. Our analysis points to a trade-off between the ability to control expectations and the introduction of greater variability in the system. We show that Brainard´s attenuation principle is optimal only in a backward looking set-up where there is no role for expectations in the determination of inflation equilibrium. On the other hand, we show that in a forward looking model, Brainard's conservative principle may produce instability, because of its inability to control expectations. A more aggressive rule, like the one derived in the paper, can instead provide greater stability because it provides for a better and more direct management of expectations, despite the uncertainty in the transmission parameters. In that respect, we show that there are conditions under which the benefits of tying down expectations, more than compensate the costs of having to overuse the instrument.

J.E.L. Classification: E42, E52
Keywords: Inflation Targeting, Parameter Uncertainty, Two-Step Target, Dynamic Models

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1 Introduction

The objective of inflation targeting is to provide a more efficient monetary policy framework because it is more transparent and therefore more predictable to the private sector. Demertzis and Viegi (2002) argue that the ability of a policy rule to provide such an effective framework is compromised in an uncertain environment. If monetary policy operates under conditions of parameter uncertainty, any *ex ante* target announced that is not hit also *ex post*, reduces the credibility of monetary policy and therefore the efficiency of the regime itself. The only way to make the target relevant is to adjust the rule such that the inflation target is hit the majority of times. And it is important that such an adjustment is done within an optimising and hence efficient framework. Demertzis and Viegi (2002) develop such an algorithm which helps attain the objectives set by pinning down expectations. In this paper we apply this framework to an explicitly dynamic framework. We will show that such an approach will produce gains only if expectations play a role in the economy and therefore, draw a distinction between a backward and a forward-looking model.
Previous attempts to analyse the issue in a dynamic setting have confined themselves to a backward-looking framework for two main reasons: forward-looking models do not appear to replicate real data patterns accurately, and more importantly, it is not clear how to optimise forward-looking models with multiplicative uncertainty. On the first point, the issue is not one of realism but of plausibility. Modern economic policy is developed around the idea that the private sector "reacts" to economic policy actions, and this reaction must be taken into consideration when determining the optimal policy rule. This is particularly evident when looking at the debate about inflation targeting. The proponents of inflation targeting argue that its main advantage is the fact that it gives the private sector a credible and transparent nominal anchor to tie down expectations of inflation with (see Johnson, 2002 and Levin *et al*, 2003 for an empirical confirmation of this). If this is the case, it is important to analyse the effects of uncertainty on the relation between the Central Bank (CB) announcement on the one hand, and the actual policy outcomes, which are subject to expectations feedback, on the other.
The paper is organised as follows: Section 2 will describe a standard backward-looking model and section 3 will apply the two-step algorithm found in Demertzis and Viegi (2002). Section 4 will demonstrate numerically, why an algorithm that ties down expectations is irrelevant in a backward-looking model. By contrast, Section 5 will demonstrate (but not prove) that the ability to tie down expectations generates greater stability when the economy is forward-looking.

2 The Model

2.1 Certainty in the Parameters

We assume a dynamic IS-AS model, that we borrow from Svensson (1999a and 1999b) (see also Söderström, 2002). The model has been used extensively in the inflation forecast targeting literature and has the characteristic that monetary policy affects the level of inflation with a two-period time lag. It is represented by the following two equations:

$$\pi_{t+1} = \pi_t + a\,(y_t) + \varepsilon_{t+1} \tag{1}$$

$$y_{t+1} = b\,(y_t) - c\left(i_t - \pi^e_{t+1|t}\right) + \eta_{t+1} \tag{2}$$

where y_t is the log of aggregate output in deviation from potential output $(Y_t - Y^*)$, π_t is the inflation rate, $i_t - \pi^e_{t+1|t}$ is the policy maker's intended deviation of the real interest from its neutral level, and ε_t and η_t are white noise random shocks. Furthermore, the coefficients satisfy, $a > 0$, $0 < b < 1$, $c > 0$. We consider a standard sequential game between the Central Bank and the private sector. The latter forms expectations first; these expectations form the basis which to base wage negotiations on, such that $w = \pi^e$. A shock occurs next and the CB reacts by choosing that interest rate which optimises the conditional expectation of its loss function, expressed in terms of deviations of inflation and output from their respective targets.

The main characteristic of this model is that monetary policy can only operate on the basis of the conditional inflation forecast of two periods ahead, which is given by:

$$\pi_{t+2} = \pi_{t+1} + a\,(y_{t+1}) + \varepsilon_{t+2} \tag{3}$$

Substituting (1) and (2) in (3) we have:

$$\pi_{t+2} = \pi_t + a\,(y_t) + \varepsilon_{t+1} + a\left[b\,(y_t) - c\left(i_t - \pi_{t+1|t}\right) + \eta_{t+1}\right] + \varepsilon_{t+2} \tag{4}$$

We observe that expected inflation at time $t+1$ is predetermined and following (1) equal to:

$$E_t\pi_{t+1} = \pi_t + a\,(y_t) \tag{5}$$

We can therefore, re-write (4), as

$$\pi_{t+2} = \pi_t + a\,(1+b)\,y_t - ac\,(i_t - \pi_t - ay_t) + a\eta_{t+1} + \varepsilon_{t+1} + \varepsilon_{t+2} \tag{6}$$

$$\pi_{t+2} = (1+ac)\,\pi_t + a\,(1+b+ac)\,y_t - aci_t + a\eta_{t+1} + \varepsilon_{t+1} + \varepsilon_{t+2} \tag{7}$$

The Central Bank aims to stabilise inflation and the output gap. In other words, it aims to minimise the following intertemporal loss function:

$$\min_{\{i_\tau\}_{\tau=t}^{\infty}} E_t \sum_{\tau=t}^{\infty} \delta^{\tau} L(\pi_\tau, y_\tau) \tag{8}$$

We will assume for simplicity that the Central Bank is following a strict inflation targeting objective and given the backward looking nature of the model, minimises the period-by-period problem. The objective function (8), thus simplifies to:

$$\min_{i_t} E(L) = \delta^2 E_t \left[\frac{1}{2}(\pi_{t+2} - \pi^*)^2\right] \tag{9}$$

The solution to this linear quadratic problem[1] produces the following rules:

$$i_t = \pi_t + a y_t + \frac{1}{ac}(\pi_t - \pi^*) + \frac{(1+b)}{c} y_t \tag{10}$$

or

$$i_t = \pi_{t+1|t} + \frac{1}{ac}(\pi_t - \pi^*) + \frac{(1+b)}{c} y_t \tag{11}$$

and

$$\pi^e_{t+2|t} = \pi^* \tag{12}$$

2.2 Multiplicative Uncertainty

Following Brainard's (1967) analysis, we consider next a simple form of parameter uncertainty (see Svensson 1999a). Parameter c in equation (2) is assumed now to be drawn randomly from $c_t \to N(\bar{c}, \sigma_c^2)$. This implies that there is uncertainty in year t when the instrument is chosen, about its effect (via the multiplier) on the policy variable. The CB's objective is now modified as follows:

$$\min_{i_t} E(L) = \delta^2 E_t \left[\frac{1}{2}(\pi_{t+2} - \pi^*)^2\right] \tag{13}$$

$$= \delta^2 \left[\frac{1}{2}(\bar{\pi}_{t+2} - \pi^*)^2 + \sigma^2_{\pi_{t+2}}\right] \tag{14}$$

assuming zero covariances between the shocks and the uncertain parameter. Based on equations (6) and (7), we can re-write the first and second moments of the distribution of inflation at time $t+2$ as follows:

[1] Note that, as argued in Demertzis and Viegi (2002), (9) also needs to have a term $\left|\pi^e_{t+2|t} - \pi^*\right|$ in it to allow for the loss in credibility as a result of missing the target. As this does not affect the optimal rule derived and we are only interested in simulating the rules, not analysing their welfare implications, we will ignore it here.

$$
\begin{aligned}
E(\pi_{t+2}) &= (1+a\bar{c})\,\pi_t + a\,(1+b+a\bar{c})\,y_t - a\bar{c}i_t \\
var(\pi_{t+2}) &= a^2\,(i_t - \pi_t - ay_t)^2\,\sigma_c^2 + 2\sigma_\varepsilon^2 + a^2\sigma_\eta^2
\end{aligned}
\tag{15}
$$

This allows us to re-write (14) in terms of the instrument:

$$
\min_{i_t} E\,(L) = \frac{\delta^2}{2}\left[(\bar{\pi}_{t+2} - \pi^*)^2 + a^2\,(i_t - \pi_t - ay_t)^2\,\sigma_c^2 + 2\sigma_\varepsilon^2 + a^2\sigma_\eta^2\right] \tag{16}
$$

Solving the objective function subject to the economic set-up produces the following monetary policy reaction function and expected inflation, respectively:

$$
i_{t,BR} = \pi_{t+1|t} + \frac{\bar{c}}{a\left(\bar{c}^2 + \sigma_c^2\right)}\,(\pi_t - \pi^*) + \frac{\bar{c}}{\left(\bar{c}^2 + \sigma_c^2\right)}(1+b)y_t \tag{17}
$$

$$
\pi^e_{t+2|t} = \frac{\sigma_c^2}{\left(\bar{c}^2 + \sigma_c^2\right)}\pi_t + \frac{\bar{c}^2}{\left(\bar{c}^2 + \sigma_c^2\right)}\,\pi^* + \frac{\sigma_c^2}{\bar{c}^2 + \sigma_c^2}a(1+b)y_t \tag{18}
$$

Solutions (17) and (18) confirm the traditional properties of Brainard's analysis. Uncertainty in the transmission mechanism increases the penalty associated with using the instrument. Therefore, an increase in uncertainty immobilises the instrument, as well as reducing the relevance of the inflation target in determining inflation. This in turn, makes the value added of an inflation targeting regime subject to the level of uncertainty present in the economy. The higher the level of uncertainty, the more distant expected inflation will be from the desired target. In that respect, the private sector discounts the ability of the monetary authority to get to its intended level of inflation, thus making its task more difficult to attain. The approach put forward in Demertzis and Viegi (2002), looks for a policy rule that allows for private expectations to match the intentions of the authority, thereby reactivating the relevance of the inflation target.

3 Two-Step Inflation Targeting

The CB can now improve on the previous result by using its information advantage, namely the knowledge of the shock that has hit the economy[2]. Demertzis and Viegi (2002) put forward a two-step optimisation procedure according to which the Central Bank, first identifies the optimal rule as a function of $\pi^* + \theta$, where θ is a deviation from the target decided after the shock is observed, and second, it optimises with respect to this deviation, aiming to close the gap from its objectives which arise due to the existence of uncertainty[3]. The following two sections describe the two-step procedure in greater detail.

[2] As already mentioned, this presumes that the private sector forms expectations first, a shock occurs next and the CB reacts by choosing that interest rate which optimises the conditional expectation of its loss function.

[3] In effect this amounts to giving the Central Bank an extra instrument while the number of targets remains the same.

3.1 Step 1

In the first step, and after the shock has occurred, the monetary policy authorities identify the optimal policy rule as a function of an auxiliary target $(\pi^* + \theta)$. Formally this means optimising the following objective function

$$\min_{i_t} E(L) = \delta^2 \frac{1}{2} \left\{ \left[\bar{\pi}_{t+2} - (\pi^* + \theta)\right]^2 + \sigma^2_{\pi_{t+2}} \right\} \tag{19}$$

subject to the system of equations (1) and (2). In other words it optimises the expected value of the following auxiliary objective function[4]:

$$\begin{aligned} \min_{i} E(L) &= \delta^2 \frac{1}{2} \left\{ \left[\pi_t + aby_t - a\bar{c}i_t - (\pi^* + \theta)\right]^2 \right. \\ &\quad \left. + \sigma_c^2 a^2 (i_t - \pi_{t+1|t}) + 2\sigma_\varepsilon^2 + a^2 \sigma_\eta^2 \right\} \end{aligned} \tag{20}$$

Optimising (20) produces an optimal rule as a function of θ. The monetary policy reaction function and the resulting inflation forecast for period $t+2$ are:

$$i_t = \pi_{t+1|t} + \frac{\bar{c}}{a\left(\bar{c}^2 + \sigma_c^2\right)} \left[\pi_t - (\pi^* + \theta)\right] + \frac{\bar{c}}{\left(\bar{c}^2 + \sigma_c^2\right)} (1+b) y_t \tag{21}$$

$$\pi^e_{t+2} = \frac{\sigma_c^2}{\bar{c}^2 + \sigma_c^2} \pi_t + \frac{\bar{c}^2}{\bar{c}^2 + \sigma_c^2} (\pi^* + \theta) + \frac{a(1+b)\sigma_c^2}{\bar{c}^2 + \sigma_c^2} y_t \tag{22}$$

The above two equations imply that for a given level of uncertainty, the CB will choose to deviate, at first instance, from its ultimate target π^* by a parameter θ.

3.2 Step 2

The degree of deviation θ is chosen optimally. In other words, the CB chooses θ in full knowledge of the extent of uncertainty and the size of the shock, and aims to maximise the probability of achieving its true objectives. In other words, since inflation expectations move away from the target as uncertainty increases, the deviation term θ will move to close that gap. Similarly, the instrument will In that respect θ is therefore, an auxiliary step, necessary in order to make full use of the information available to the bank. The derived rules from Step 1 (21) and (22) are now substituted into the objective function of the Central Bank:

$$\min_{\theta} E(L) = \delta^2 E_t \left[\frac{1}{2} \left(\pi_{t+2} - \pi^*\right)^2 \right] \tag{23}$$

to produce

$$\min_{\theta} E(L) = f(\theta, \sigma_c^2, y_t, \pi_t) \tag{24}$$

[4] The expected value of the objective function is conditional on the shocks, omitted here for simplicity.

Given the rules, the aim of the CB is to find the optimal value for θ, contingent on the economy's past history and the perceived uncertainty of the transmission of policies, i.e.:

$$\theta(\sigma_c^2, y_t, \pi_t) = \arg\min_\theta L$$

which in its analytical form is

$$\theta = -\frac{\sigma_c^2}{\overline{c}^2}\left[\pi_t - \pi^* + a\,(1+b)\,y_t\right] \tag{25}$$

As uncertainty decreases, the deviations from π^* decrease as well, such that at the limit they become zero, i.e.

$$\lim_{\sigma_a^2 \to 0} (\theta) = 0$$

Substituting the analytical solutions for θ into (21) - (22) produces the following interest rate rule and inflation forecast:

$$i_{t,TS} = \pi_{t+1|t} + \frac{1}{a\overline{c}}\,(\pi_t - \pi^*) + \frac{(1+b)}{\overline{c}}\,y_t \tag{26}$$

$$\pi^e_{t+2|t} = \pi^* \tag{27}$$

The rules achieved are similar to those attained with no uncertainty (with c replaced by $\overline{c}$). This demonstrates that by varying the target optimally, uncertainty in the transmission process is neutralised. This however, is an *ex ante* result. As we will show next, this happens at the expense of using i_t more actively, thereby introducing greater variability in the system. It follows that the obvious *ex ante* benefits of the two-step procedure do not necessarily carry also *ex post*. To evaluate these, we turn to an analysis of the stability properties of the system.

4 Ex Post Sensitivity Analysis

The above results seem to confirm that it is theoretically possible under Brainard uncertainty to use a "two-step inflation target" to stabilise inflation forecast around a preferred inflation path. But this only arises at the expense of greater variability in the system. We perform a Monte Carlo study of the property of the system under the two rules.

The following table shows the results of 30,000 stochastic simulations of our model under the two regimes , à la Brainard, and *two-step* inflation targeting. A random shock ε is drawn from a $N(0,1)$ distribution, while parameter c is drawn from a $N\left(0.5, 0.5^2\right)$ distribution. The inflation target π^* is assumed to be 2. As this simulation is performed for illustrative purposes, we have chosen for values of the parameters which present a clearer picture, without affecting the results: the sensitivity of inflation to output is thus $a = 0.8$ and the income inertia parameter $b = 0.4$. We present the mean, standard deviation and the maximum and minimum values of a ten period average value of the variables i, π and y derived. The first three columns show the results for the Brainard case and the last three, those under the *two-step* inflation targeting procedure.

TABLE 2 : Monte Carlo Simulations:

	i_{BR}	π_{BR}	y_{BR}	i_{TS}	π_{TS}	y_{TS}
Mean	-0.27	1.23	0.23	1.84	2.11	1.15
St. Dev.	0.31	0.37	0.01	16.79	5.48	6.83
Max	1.36	3.20	0.67	271.07	91.48	107
Min.	-6.82	-6.19	-0.36	-183	-62.59	-106

It is clear that a two-step strategy produces instability. This is confirmed by looking at one of those realisations. In the context of a backward-looking model, the Brainard rule seems quite a successful one, which achieve both stability as well as realisation of the target, at least in the long run. The two-step rule, although designed to reach the target on average, it introduces instability in the system with an excessive use of the instrument. The two-step rule always underperforms, regardless of the parameterisation applied.

[Insert Figure 1]

This raises an important issue about the context in which monetary policy rules are derived. The two-step targeting rule is derived in order to hit the inflation target with regularity. The main advantage of this strategy is that monetary policy is more transparent and therefore more credible because it is more likely to reach the announced target. In contrast to that, a monetary policy rule which emphasises the risk over the achievement of the target (as in Brainard), should pay a price in terms of losses in credibility, transparency and control. However, this argument is only valid if there is an explicit role for expectations in the economic system. We expect therefore, the merits of the two-step rule to manifest themselves only in a forward-looking system. We examine this next.

5 A Forward Looking Model

Most of the attempts to examine the effects of uncertainty in a dynamic framework rely on a backward looking set-up (Söderstöm, 2002, Srour, 1999). The somehow surprising result, from the point of view of inflation targeting proponents, is that uncertainty in the structure of the economy implies that achieving the target is not optimal as it may lead to instability in the system. This seems at odds with the general perception that the main advantage of inflation targeting is that it stabilises expectations (and the evidence which verifies that, see Johnson, 2002). But this is relevant, as already mentioned, only if expectations are an important determinant in the economic system. The importance of this distinction is evident if we apply our rules to a standard New Keynesian, but this time forward-looking model, where expectations do indeed play an active role (Woodford 2000, 2001). Our economy is thus described by a pair of log-linear relations:

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \tag{28}$$
$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1}\right) + \eta_t \tag{29}$$

where (28) is an expectations-augmented "AS" relation in which present inflation is a function of the private sector expectations of inflation one period ahead, and (29) is an intertemporal "IS" relation. The coefficients satisfy, $\alpha, \gamma > 0$, and $0 < \beta < 1$. The CB's instrument is the nominal interest rate. Ideally, what we would need to do to check the value of our two-step procedure is to obtain the optimal interest rate rule under the two alternative assumptions as in sections 2.2 and 3. However, it is not immediately obvious how to solve forward-looking models analytically in the presence of uncertainty. Indeed previous attempts to do so, have resorted to numerical simulations (see, *inter alia,* Wieland, 2000, Giannoni, 2002) and do not provide therefore, analytical solutions.
In the absence of a solution technique, we check instead how the two rules derived in the previous section perform in a forward-looking model, aware of the fact that neither of the two are the optimal rules for the given model. The monetary policy rules derived in the context of a backward-looking model above can however, both be formulated in a general Taylor rule format of the following kind:

$$i_t = \bar{i}_t + \phi_\pi \left(\pi_t - \pi^*\right) + \phi_y \left(y_t\right) \tag{30}$$

We restrict our attention to sets of rules that have $\phi_\pi,\ \phi_y \geq 0$. For given values of $\phi_\pi,\ \phi_y$ we can therefore identify which of the rules, the Brainard rule or the two-step procedure, is more likely to be stable in the current model. Substituting the general (30) now in (28) and (29), the above system can be written in state-space form:

$$E_t \mathbf{z}_{t+1} = \widetilde{\mathbf{A}} * \mathbf{z}_t + \widetilde{\mathbf{B}} * \pi^* + \mathbf{K}\mathbf{u}_{t+1}$$

where, $\mathbf{z}_t' = \begin{bmatrix} \pi_t & y_t \end{bmatrix}$, $\mathbf{u}_t' = \begin{bmatrix} \varepsilon_t & \eta_t & \bar{i}_t \end{bmatrix}$ and

$$\widetilde{\mathbf{A}} = \begin{pmatrix} \frac{1}{\beta} & -\frac{\alpha}{\beta} \\ -\frac{\gamma}{\beta} + \gamma\phi_\pi & \frac{\alpha\gamma}{\beta} + 1 + \gamma\phi_y \end{pmatrix}, \widetilde{\mathbf{B}} = \begin{bmatrix} 0 \\ -\gamma\phi_\pi \end{bmatrix}, \mathbf{K} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\gamma & -c \end{bmatrix}$$

The stability of the system requires that the eigenvalues of matrix $\mathbf{A}$ (given by the solutions of the characteristic equation) are outside the unit circle (Blanchard and Kahn, 1980). In other words,

$$\left| \frac{tr(\widetilde{\mathbf{A}}) \pm \sqrt{tr(\widetilde{\mathbf{A}})^2 - 4\det(\widetilde{\mathbf{A}})}}{2} \right| > 1$$

which, given our parameter restrictions and the nature of the problem, reduces to the following three conditions (see Appendix B for a detailed derivation):

$$\det(\breve{\mathbf{A}}) > 0, \qquad \det(\breve{\mathbf{A}}) - tr(\breve{\mathbf{A}}) > -1, \qquad \det(\breve{\mathbf{A}}) + tr(\breve{\mathbf{A}}) > -1$$

This in turns requires:

$$\left(\frac{1-\beta}{\alpha}\right)\phi_y + \phi_\pi > 1 \tag{31}$$

Stability condition (31) provides a general condition for selecting monetary policy rules. The specific monetary policy rules derived in a backward-looking set-up (summarised in Appendix A) are:

$$\begin{array}{lll} i_{t,BR}: & \phi_{\pi,BR} = \frac{\bar{c}}{a(\bar{c}^2+\sigma^2)}, & \phi_{y,BR} = \frac{\bar{c}}{\bar{c}^2+\sigma^2}(1+b) \\ i_{t,TS}: & \phi_{\pi,TS} = \frac{1}{a\bar{c}}, & \phi_{y,TS} = \frac{(1+b)}{\bar{c}} \end{array}$$

When applying the two-step inflation targeting rule, condition (31) is always respected, since $\frac{1}{a\bar{c}} \geq 1$, independently of the level of uncertainty faced by the CB. On the other hand, using a more cautious rule which makes the response of the instrument to a deviation from the target decreasing in uncertainty, the system is stable if and only if: .

$$\sigma_c^2 < \frac{1}{ac} + \left(\frac{1-\beta}{\alpha}\right)\left(\frac{1+b}{c}\right) \tag{32}$$

The importance of condition (32) is that it shows the limits of Brainard's analysis when applied in a forward-looking framework. The Brainard result is thus, shown to be specific to the backward-looking nature of the model it has been analysed in. In a forward-looking model, Brainard's recommendation of being cautious could well lead to losing control of the system, as expectations of the private sector become independent of the policy rule followed by the CB.
The main implication of our analysis is the emphasis on identifying the role of expectations. If expectations happen to play a role in the way the economic system operates, then it pays for policy makers to try and tie them down to something which is clear and transparent. This will help them achieve their objectives, quicker and more effectively.

6 Conclusions

We have studied the implications of uncertainty for inflation targeting in a dynamic set-up. Using a standard inflation forecast targeting set-up, we have compared Brainard's conservative principle with a more active monetary policy rule derived from a two-step maximisation procedure. The analysis shows that the Brainard attenuation effect is optimal only because of the backward-looking framework in which this principle is usually analysed. In fact, if expectations do not play any role in determining equilibrium, any rule which tries to control expectations is by definition, inefficient. On the other hand, we show that the Brainard conservative principle could produce instability in a forward-looking model, because of its failure to control expectations. A more aggressive rule, like the one suggested in the paper, can instead increase stability because of its direct targeting of expectations, despite the uncertainty in the transmission parameters.

References

[1] **Bernanke, Ben and Frederic Mishkin**, 1997, 'Inflation Targeting: A New Framework for Monetary Policy?', *Journal of Economic Perspectives*, Vol 11, No 2, 97-116.

[2] **Bernanke, Ben, Thomas Lauback, Frederic Mishkin and Adam Posen**, 1999, *Inflation Targeting: Lessons from the International Experience*, Princeton University Press.

[3] **Blanchard, Olivier and Charles Kahn**, 1980, 'The Solution of Linear Difference Models Under Rational Expectations', *Econometrica*, Vol. 48, No. 5, July.

[4] **Brainard, William**, 1967, 'Uncertainty and the Effectiveness of Policy', *American Economic Review*, 57, 411-25.

[5] **Craine, Roger**, 1979, 'Optimal Monetary Policy with Uncertainty', *Journal of Economic Dynamics and Control*, 1, 59-83.

[6] **Demertzis, Maria and Nicola Viegi**, 2002, 'Aiming for the Bull's Eye: Inflation Targeting Under Uncertainty', De Nederlandsche Bank, Mimeo.

[7] **Ellison, Martin and Natacha Valla**, 2001, 'Learning, uncertainty and central bank activism in an economy with strategic interactions', *Journal of Monetary Economics*, 48, 153-171.

[8] **Faust, Joh and Lars Svensson**, 2001, 'Transparency and Credibility: Monetary Policy with Unobservable Goals', *International Economic Review*, May, 42 (2), 369-397.

[9] **Giannoni, Marc P.**, 2002, 'Does Model Uncertainty Justify Caution? Robust Optimal Monetary Policy in a Forward-looking Model', *Macroeconomic Dynamics*, 6, 111-144.

[10] **Holly, Sean and Andrew Hughes Hallett**, 1989, *Optimal Control, Expectations and Uncertainty*, Cambridge University Press.

[11] **Johnson, David R.**, 2002, 'The Effect of Inflation Targeting on the Behavior of Expected Inflation: Evidence from an 11 Country Panel', *Journal of Monetary Economics*, 49, 1521-1538.

[12] **Levin, Andrew, Fabio M. Natalucci and Jeremy M. Piger**, 2003 'The Macroeconomic Effects of Inflation Targeting', October, *Mimeo*, Federal Reserve Board of Governors.

[13] **Södeström**, **Ulf**, 2002, 'Monetary Policy with Uncertain Parameters' Working Paper, Sveriges Riskbank, *Scandinavian Journal of Economics*, 104, 125-145.

[14] **Srour, Gabriel**, 1999, "Inflation Targeting under Uncertainty" Technical Report 85, Bank of Canada, April.

[15] **Svensson, Lars**, 1997, 'Inflation Forecast Targeting: implementing and monitoring inflation targeting', *European Economic Review*, 41, 1111-1146.

[16] **Svensson, Lars**, 1999a, 'Inflation Targeting:some extensions' *Scandinavian Journal of Economics*, 101, 337-361.

[17] **Svensson, Lars**, 1999b, 'Inflation Targeting as a Monetary Policy Rule', *Journal of Monetary Economics*, 43, 607-654.

[18] **Wieland, Volker, 2000**, 'Monetary Policy, Parameter Uncertainty and Optimal Learning', *Journal of Monetary Economic*s, August.

[19] **Woodford, Michael**, 2002a, 'The Taylor Rule and Optimal Monetary Policy', *American Economic Review*, 91, No.2, 232-237.

[20] **Woodford, Michael**, 2002b, *Interest and Prices*, Manuscript, Princeton University ch.4. (www.princeton.edu/˜woodford).

APPENDICES

A Analysing the stability of the systems

The distinction between *ex ante* and *ex post* that we have emphasised in the analysis undertaken implies that although the two-step procedure does actually deliver the forecast target intended, it is not necessarily the case that the system does better overall, *ex post*. Our simulations show this very clearly for a backward-looking model. In fact what we see is that the two-step scenario generates instability in the system. This is what we analyse next by looking at the stability properties of the system. For completeness sake, we examine the properties for all three cases shown in the main text.

A.1 The System with no Parameter Uncertainty

A system with no parameter uncertainty is stable due to full controllability of the system. Substituting (10) in (2) we have:

$$y_{t+1} = b\left(y_t\right) - c\left(\frac{1}{ac}\left(\pi_t - \pi^*\right) + \frac{1+b}{c} y_t\right) + \eta_{t+1} \tag{A1}$$

The system of equations (1) and (A1) can be summarised as:

$$\pi_{t+1} = \pi_t + a\left(y_t\right) + \varepsilon_{t+1} \tag{A2}$$

$$y_{t+1} = -\frac{1}{a}\pi_t - \left(y_t\right) + \frac{1}{a}\pi^* + \eta_{t+1} \tag{A3}$$

or in a matrix form, this can be written as

$$\begin{pmatrix} \pi_{t+1} \\ y_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & a \\ -\frac{1}{a} & -1 \end{pmatrix} \begin{pmatrix} \pi_t \\ y_t \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{a} \end{pmatrix} \pi^* + \begin{pmatrix} \varepsilon_{t+1} \\ \eta_{t+1} \end{pmatrix}$$

which has the following state-space representation form, $\mathbf{z}_{t+1} = \mathbf{A}_{CE} * \mathbf{z}_t + \mathbf{B}_{CE} * \pi^* + \mathbf{u}_{t+1}$. Analysing the stability of the system implies looking at the eigenvalues of matrix $\mathbf{A}_{CE}$. The determinant of the characteristic matrix $|\mathbf{D}_{CE}|$ is

$\mathbf{D}_{CE} = \begin{vmatrix} 1-\lambda & a \\ -\frac{1}{a} & -1-\lambda \end{vmatrix} = (1-\lambda)(-1-\lambda) - a\left(-\frac{1}{a}\right) = -(1-\lambda)(1+\lambda) + 1 = -\left(1-\lambda^2\right) + 1 = \lambda^2 = 0$

and the two eigenvalues are therefore identical and equal to zero.

A.2 The system with Brainard Uncertainty

Introducing now multiplicative uncertainty in parameter c implies that the optimal rule is (17) which we can substitute in (2) to get:

$$y_{t+1} = b(y_t) - c_i \left(\frac{\bar{c}}{a\left(\bar{c}^2 + \sigma_c^2\right)} (\pi_t - \pi^*) + \frac{\bar{c}}{\bar{c}^2 + \sigma_c^2} (1+b) y_t \right) + \eta_{t+1} \tag{A4}$$

where $\bar{c}$ is the average transmission effect of the instrument expected and c_i is the random effect drawn each time. We re-specify c_i as a deviation from its mean such that $c_i = \bar{c} + \varphi_i$. For simplicity we also call $k = \bar{c}^2 + \sigma_c^2$. The system of equations (6) and (2) can be re-written as

$$\begin{aligned}
\pi_{t+1} &= \pi_t + a(y_t) + \varepsilon_{t+1} && \text{(A5)} \\
y_{t+1} &= \frac{-(\bar{c} + \varphi_i)\bar{c}}{ak}\pi_t + \left[b - \frac{(\bar{c} + \varphi_i)\bar{c}}{a}(1+b) \right](y_t) + \frac{(\bar{c} + \varphi_i)\bar{c}}{ak}\pi^* + \eta_{t+1} && \text{or} \\
y_{t+1} &= \frac{-(\bar{c} + \varphi_i)\bar{c}}{ak}\pi_t + \left[\frac{kb - (\bar{c} + \varphi_i)\bar{c}}{k}(1+b) \right](y_t) + \frac{(\bar{c} + \varphi_i)\bar{c}}{ak}\pi^* + \eta_{t+1} && \text{(A6)}
\end{aligned}$$

In a matrix form this can be written as

$$\begin{aligned}
\begin{pmatrix} \pi_{t+1} \\ y_{t+1} \end{pmatrix} &= \begin{pmatrix} 1 & a \\ \frac{-(\bar{c}+\varphi_i)\bar{c}}{a(\bar{c}^2+\sigma_c^2)} & b - \frac{(\bar{c}+\varphi_i)\bar{c}}{(\bar{c}^2+\sigma_c^2)}(1+b) \end{pmatrix} \begin{pmatrix} \pi_t \\ y_t \end{pmatrix} \\
&+ \begin{pmatrix} 0 \\ \frac{(\bar{c}+\varphi_i)\bar{c}}{a(\bar{c}^2+\sigma_c^2)} \end{pmatrix} \pi^* + \begin{pmatrix} \varepsilon_{t+1} \\ \eta_{t+1} \end{pmatrix}
\end{aligned}$$

which has the following state-space representation form: $\mathbf{z}_{t+1} = \mathbf{A}_{BR} * \mathbf{z}_t + \mathbf{B}_{BR} * \pi^* + \mathbf{u}_{t+1}$. Again, to analyse the stability of the system implies looking at the eigenvalues of matrix $\mathbf{A}_{BR}$.

A.3 Two-Step Inflation Targeting

Analysing now the stability of the system derived with the two-step inflation targeting procedure, implies that we can substitute the Certainty rule, (10) in (2) but this time replacing parameter c with $\bar{c}$.

$$y_{t+1} = b(y_t) - c_i \left(\frac{1}{a\bar{c}} (\pi_t - \pi^*) + \frac{1+b}{\bar{c}} y_t \right) + \eta_{t+1} \tag{A7}$$

Similarly to above we specify c_i as a deviation from its mean such that $c_i = \bar{c} + \varphi_i$. For a given draw of parameter c_i the system is thus:

$$\begin{aligned}
\pi_{t+1} &= \pi_t + a(y_t) + \varepsilon_{t+1} \\
y_{t+1} &= b(y_t) - (\bar{c} + \varphi_i) \left(\frac{1}{a\bar{c}} (\pi_t - \pi^*) + \frac{1+b}{\bar{c}} y_t \right) + \eta_{t+1}
\end{aligned}$$

or

$$\begin{aligned}
\pi_{t+1} &= \pi_t + a\,(y_t) + \varepsilon_{t+1} && \text{(A8)}\\
y_{t+1} &= -\left[\frac{\bar{c}+\varphi_i}{a\bar{c}}\right]\pi_t - \left[\frac{\bar{c}+\varphi_i\,(1+b)}{\bar{c}}\right](y_t) + \left(\frac{\bar{c}+\varphi_i}{a\bar{c}}\right)\pi^* + \eta_{t+1} && \text{(A9)}
\end{aligned}$$

Or in matrix form:

$$\begin{aligned}
\begin{pmatrix} \pi_{t+1} \\ y_{t+1} \end{pmatrix} &= \begin{pmatrix} 1 & a \\ -\frac{\bar{c}+\varphi_i}{a\bar{c}} & -\frac{\bar{c}+\varphi_i(1+b)}{\bar{c}} \end{pmatrix} \begin{pmatrix} \pi_t \\ y_t \end{pmatrix} \\
&+ \begin{pmatrix} 0 \\ \frac{\bar{c}+\varphi_i}{a\bar{c}} \end{pmatrix} \pi^* + \begin{pmatrix} \varepsilon_{t+1} \\ \eta_{t+1} \end{pmatrix}
\end{aligned}$$

and in simplified notation: $\mathbf{z}_{t+1} = \mathbf{A}_{TS} * \mathbf{z}_t + \mathbf{B}_{TS} * \pi^* + \mathbf{u}_{t+1}$.

A.4 Analysing the stability of the systems

The two policy rules derived above are in the Taylor-type of rules, in which the current nominal interest rate reacts to deviations of output and inflation from some target (for output this is its long run growth path, and for inflation the target decided by the CB herself) i.e.:

$$i_t = \bar{i}_t + \phi_\pi\,(\pi_t - \pi^*) + \phi_y\,(y_t) \tag{A10}$$

where $\bar{i}_t$ is an exogenous and possibly time varying intercept[5]. Parameters ϕ_π and ϕ_y take the following values for the two rules under consideration, respectively:

$$\begin{array}{lll}
i_{t,BR}: & \phi_{\pi,BR} = \frac{\bar{c}}{a(\bar{c}^2+\sigma^2)}, & \phi_{y,BR} = \frac{\bar{c}}{\bar{c}^2+\sigma^2}\,(1+b) \\
i_{t,TS}: & \phi_{\pi,TS} = \frac{1}{a\bar{c}}, & \phi_{y,TS} = \frac{(1+b)}{\bar{c}}
\end{array}$$

For any (unknown) realization of c at time t, the system can then be written as:

$$\begin{aligned}
\pi_{t+1} &= \pi_t + a\,(y_t) + \varepsilon_{t+1} && \text{(A11)}\\
y_{t+1} &= b\,(y_t) - c\left(\bar{i}_t + \phi_\pi\,(\pi_t - \pi^*) + \phi_y\,(y_t) - \pi_{t+1|t}\right) + \eta_{t+1} && \text{or,}\\
y_{t+1} &= b\,(y_t) - c\left(\bar{i}_t + \phi_\pi\,(\pi_t - \pi^*) + \phi_y\,(y_t) - \pi_t - a\,(y_t)\right) + \eta_{t+1} && \text{(A12)}
\end{aligned}$$

In matrix form this is represented as:

$$\mathbf{z}_{t+1} = \mathbf{A} * \mathbf{z}_t + \mathbf{B} * \pi^* + \mathbf{u}_{t+1} \tag{A13}$$

where $\mathbf{z}'_t = \begin{bmatrix} \pi_t & y_t \end{bmatrix}$, $\mathbf{u}'_t = \begin{bmatrix} \varepsilon_t & \eta_t \end{bmatrix}$ and

$$\mathbf{A} = \begin{bmatrix} 1 & a \\ c\,(1-\phi_\pi) & b + c\left(a - \phi_y\right) \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ c\phi_\pi \end{bmatrix} \tag{A14}$$

[5] Svensson (1999), considers that the average federal funds rate.

A stable backward-looking system requires both eigenvalues of $\mathbf{A}$ to be inside the unit circle (see Holly and Hughes Hallett, 1989). For a 2×2 matrix, and given our parameter restrictions, this requires:

$$\left| \frac{tr(\mathbf{A}) \pm \sqrt{tr(\mathbf{A})^2 - 4\det(\mathbf{A})}}{2} \right| < 1 \tag{A15}$$

Relation (A15) gives a set of sufficient conditions for the stability of the system. A necessary condition for stability is:

$$\begin{aligned} \frac{tr(\mathbf{A}) + \sqrt{tr(\mathbf{A})^2 - 4\det(\mathbf{A})}}{2} &< 1 \\ tr(\mathbf{A}) + \sqrt{tr(\mathbf{A})^2 - 4\det(\mathbf{A})} &< 2 \\ +\sqrt{tr(\mathbf{A})^2 - 4\det(\mathbf{A})} &< 2 - tr(\mathbf{A}) \\ tr(\mathbf{A})^2 - 4\det(\mathbf{A}) &< [2 - tr(\mathbf{A})]^2 \\ tr(\mathbf{A})^2 - 4\det(\mathbf{A}) &< 4 - 4tr(\mathbf{A}) + tr(\mathbf{A})^2 \\ -\det(\mathbf{A}) &< 1 - tr(\mathbf{A}) \\ tr(\mathbf{A}) - \det(\mathbf{A}) &< 1 \end{aligned} \tag{A16}$$

and

$$\begin{aligned} \frac{tr(\mathbf{A}) - \sqrt{tr(\mathbf{A})^2 - 4\det(\mathbf{A})}}{2} &< 1 \\ tr(\mathbf{A}) - \sqrt{tr(\mathbf{A})^2 - 4\det(\mathbf{A})} &< 2 \\ -\sqrt{tr(\mathbf{A})^2 - 4\det(\mathbf{A})} &< 2 - tr(\mathbf{A}) \end{aligned}$$

But if $0 < \sqrt{tr(\mathbf{A})^2 - 4\det(\mathbf{A})} < 2 - tr(\mathbf{A})$ then $-\sqrt{tr(\mathbf{A})^2 - 4\det(\mathbf{A})} < 2 - tr(\mathbf{A})$ will also be true. So we only need (33) to hold. This implies that (33) is rewritten as

$$1 + a\bar{c} - \frac{\bar{c}^2}{\bar{c}^2 + \sigma_c^2} < 1 \tag{A17}$$

From (A17) we can study the behaviour of the LHS for a change in the variance parameter, which is the one that differentiates the Brainard policy rule from out two-step procedure. The LHS is clearly an increasing function of σ_c^2, for any realisation of $c > 0$. Moreover, if $\frac{\bar{c}^2}{\bar{c}^2+\sigma_c^2} > a\bar{c}$, then the inequality is not satisfied, and uncertainty increases the likelihood of monetary policy producing instability. Further numerical analysis of condition (A15) confirms this observation[6].

[6] Maple workbook available from the Authors

B A Forward Looking Model

We now consider a forward-looking model:

$$
\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \\
y_t &= E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) + \eta_t
\end{aligned}
$$

The CB's instrument is the nominal interest rate but the private sector forms expectations at time t for inflation at time $t+1$. This can be re-written as

$$
\begin{aligned}
E_t \pi_{t+1} &= \frac{1}{\beta} \left(\pi_t - \alpha y_t - \varepsilon_t \right) \\
\gamma E_t \pi_{t+1} + E_t y_{t+1} &= y_t + \gamma i_t - \eta_t
\end{aligned}
$$

where parameters α, β, γ are all positive. We can then examine the stability of the system for any interest rate rule, i. The monetary policy rules derived in the context of a backward-looking model above can both be formulated in a general Taylor rule format of the kind:

$$
i = \phi_\pi \left(\pi_t - \pi^* \right) + \phi_y \left(y_t \right) \tag{B1}
$$

We restrict our attention to sets of rules that have ϕ_π, $\phi_y \geq 0$. For given values of ϕ_π, ϕ_y we can therefore identify which of the rules, that identified by Brainard or the two-step target, is more likely to be stable.
Substituting the general (B1) now in the above system can be written in matrix form:

$$
E_t \pi_{t+1} = \frac{1}{\beta} \pi_t - \frac{\alpha}{\beta} y_t - \frac{1}{\beta} \varepsilon_t
$$

$$
\begin{aligned}
\gamma E_t \pi_{t+1} + E_t y_{t+1} &= y_t + \gamma \left[\phi_\pi \left(\pi_t - \pi^* \right) + \phi_y \left(y_t \right) \right] - \eta_t \quad \Leftrightarrow \\
\gamma E_t \pi_{t+1} + E_t y_{t+1} &= \gamma \phi_\pi \pi_t + \left(1 + \gamma \phi_y \right) y_t - \gamma \phi_\pi \pi^* - \eta_t
\end{aligned}
$$

And in matrix terms:

$$
\begin{aligned}
\begin{pmatrix} 1 & 0 \\ \gamma & 1 \end{pmatrix} \begin{pmatrix} E_t \pi_{t+1} \\ E_t y_{t+1} \end{pmatrix} &= \begin{pmatrix} \frac{1}{\beta} & -\frac{\alpha}{\beta} \\ \gamma \phi_\pi & 1 + \gamma \phi_y \end{pmatrix} \begin{pmatrix} \pi_t \\ y_t \end{pmatrix} \\
&+ \begin{pmatrix} 0 \\ -\gamma \phi_\pi \end{pmatrix} \pi^* + \begin{pmatrix} -\frac{1}{\beta} & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \varepsilon_{t+1} \\ \eta_{t+1} \end{pmatrix}
\end{aligned}
$$

or in simplified form

$$
\begin{aligned}
\mathbf{C} E_t \mathbf{z}_{t+1} &= \mathbf{A} * \mathbf{z}_t + \mathbf{B} * \pi^* + \mathbf{K} \mathbf{u}_{t+1} \qquad \text{and} \\
E_t \mathbf{z}_{t+1} &= \breve{\mathbf{A}} * \mathbf{z}_t + \breve{\mathbf{B}} * \pi^* + \breve{\mathbf{K}} \mathbf{u}_{t+1}
\end{aligned}
$$

where $\breve{\mathbf{A}} = \mathbf{C}^{-1}\mathbf{A}$, $\breve{\mathbf{B}} = \mathbf{C}^{-1}\mathbf{B}$ and $\breve{\mathbf{K}} = \mathbf{C}^{-1}\mathbf{K}$. The stability of the system requires that the eigenvalues of matrix $\breve{\mathbf{A}} = \mathbf{C}^{-1}\mathbf{A}$ are outside the unit circle. Matrix $\breve{\mathbf{A}}$is

$$\breve{\mathbf{A}} \equiv \begin{pmatrix} 1 & 0 \\ -\gamma & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\beta} & -\frac{\alpha}{\beta} \\ \gamma\phi_\pi & 1+\gamma\phi_y \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta} & -\frac{\alpha}{\beta} \\ -\frac{\gamma}{\beta}+\gamma\phi_\pi & \frac{\alpha\gamma}{\beta}+1+\gamma\phi_y \end{pmatrix}$$

where

$$\mathbf{C}^{-1} = \begin{pmatrix} 1 & 0 \\ \gamma & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ -\gamma & 1 \end{pmatrix}$$

The eigenvalues are given by the solution of the characteristic equation and are required to have both values outside the unit circle for stability. In other words,

$$\left| \frac{tr(\breve{\mathbf{A}}) \pm \sqrt{tr(\breve{\mathbf{A}})^2 - 4\det(\breve{\mathbf{A}})}}{2} \right| > 1$$

which is equivalent to two conditions:

$$\frac{tr(\breve{\mathbf{A}}) \pm \sqrt{tr(\breve{\mathbf{A}})^2 - 4\det(\breve{\mathbf{A}})}}{2} > 1 \tag{B2}$$

$$\frac{tr(\breve{\mathbf{A}}) \pm \sqrt{tr(\breve{\mathbf{A}})^2 - 4\det(\breve{\mathbf{A}})}}{2} < -1 \tag{B3}$$

Conditions (B2) and (B3) are mutually exclusive in pairs.

B.1 Solving for (B2)

$$\begin{aligned}
\frac{tr(\breve{\mathbf{A}}) \pm \sqrt{tr(\breve{\mathbf{A}})^2 - 4\det(\breve{\mathbf{A}})}}{2} &> 1 \\
\frac{tr(\breve{\mathbf{A}})}{2} \pm \frac{\sqrt{tr(\breve{\mathbf{A}})^2 - 4\det(\breve{\mathbf{A}})}}{2} &> 1 \\
\frac{tr(\breve{\mathbf{A}})}{2} \pm \sqrt{\frac{tr(\breve{\mathbf{A}})^2 - 4\det(\breve{\mathbf{A}})}{4}} &> 1 \\
\frac{tr(\breve{\mathbf{A}})}{2} \pm \sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}})} &> 1
\end{aligned}$$

We analyse the two solutions separately:

$$
\begin{aligned}
\frac{tr(\breve{\mathbf{A}})}{2} + \sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}})} &> 1 \\
\sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}})} &> 1 - \frac{tr(\breve{\mathbf{A}})}{2} \\
\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}}) &> \left(1 - \frac{tr(\breve{\mathbf{A}})}{2}\right)^2 \\
\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}}) &> 1 - tr(\breve{\mathbf{A}}) + \left(\frac{tr(\breve{\mathbf{A}})}{2}\right)^2 \\
-\det(\breve{\mathbf{A}}) &> 1 - tr(\breve{\mathbf{A}}) \\
\det(\breve{\mathbf{A}}) &< -1 + tr(\breve{\mathbf{A}}) \\
\det(\breve{\mathbf{A}}) - tr(\breve{\mathbf{A}}) &< -1
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{tr(\breve{\mathbf{A}})}{2} - \sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}})} &> 1 \\
-\sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}})} &> 1 - \frac{tr(\breve{\mathbf{A}})}{2} \\
\sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}})} &< \frac{tr(\breve{\mathbf{A}})}{2} - 1 \\
\sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}})} &< \left(\frac{tr(\breve{\mathbf{A}})}{2} - 1\right) \\
\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}}) &< \left(\frac{tr(\breve{\mathbf{A}})}{2} - 1\right)^2 \\
\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}}) &< 1 - tr(\breve{\mathbf{A}}) + \left(\frac{tr(\breve{\mathbf{A}})}{2}\right)^2 \\
-1 &< \det(\breve{\mathbf{A}}) - tr(\breve{\mathbf{A}})
\end{aligned}
$$

B.2 Solving for (B3)

$$\begin{aligned}
\frac{tr(\breve{\mathbf{A}}) \pm \sqrt{tr(\breve{\mathbf{A}})^2 - 4\det(\breve{\mathbf{A}})}}{2} &< -1 \\
\frac{tr(\breve{\mathbf{A}})}{2} \pm \sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}})} &< -1
\end{aligned}$$

Again we analyse the two solutions separately:

$$\begin{aligned}
\frac{tr(\breve{\mathbf{A}})}{2} + \sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}})} &< -1 \\
\sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}})} &< -1 - \frac{tr(\breve{\mathbf{A}})}{2} \\
\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}}) &< \left(-1 - \frac{tr(\breve{\mathbf{A}})}{2}\right)^2 \\
\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}}) &< 1 + tr(\breve{\mathbf{A}}) + \left(\frac{tr(\breve{\mathbf{A}})}{2}\right)^2 \\
-\det(\breve{\mathbf{A}}) &< 1 + tr(\breve{\mathbf{A}}) \\
-1 &< tr(\breve{\mathbf{A}}) + \det(\breve{\mathbf{A}})
\end{aligned}$$

and the minus

$$
\begin{aligned}
\frac{tr(\breve{\mathbf{A}})}{2}-\sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4}-\det(\breve{\mathbf{A}})} &< -1 \\
-\sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4}-\det(\breve{\mathbf{A}})} &< -1-\frac{tr(\breve{\mathbf{A}})}{2} \\
-\sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4}-\det(\breve{\mathbf{A}})} &< -\left(1+\frac{tr(\breve{\mathbf{A}})}{2}\right) \\
\sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4}-\det(\breve{\mathbf{A}})} &> \left(1+\frac{tr(\breve{\mathbf{A}})}{2}\right) \\
\frac{tr(\breve{\mathbf{A}})^2}{4}-\det(\breve{\mathbf{A}}) &> \left(1+\frac{tr(\breve{\mathbf{A}})}{2}\right)^2 \\
\frac{tr(\breve{\mathbf{A}})^2}{4}-\det(\breve{\mathbf{A}}) &> 1+tr(\breve{\mathbf{A}})+\left(\frac{tr(\breve{\mathbf{A}})}{2}\right)^2 \\
-\det(\breve{\mathbf{A}}) &> 1+tr(\breve{\mathbf{A}}) \\
-1 &> tr(\breve{\mathbf{A}})+\det(\breve{\mathbf{A}})
\end{aligned}
$$

B.3 Analysing the stability

The above analysis produces therefore four conditions for stability:

$$
\begin{aligned}
-1 &> \det(\breve{\mathbf{A}})-tr(\breve{\mathbf{A}}) && \text{(B4)} \\
-1 &< \det(\breve{\mathbf{A}})-tr(\breve{\mathbf{A}}) && \text{(B5)} \\
-1 &< \det(\breve{\mathbf{A}})+tr(\breve{\mathbf{A}}) && \text{(B6)} \\
-1 &> \det(\breve{\mathbf{A}})+tr(\breve{\mathbf{A}}) && \text{(B7)}
\end{aligned}
$$

We calculate next the $\det(\breve{\mathbf{A}})$ and $tr(\breve{\mathbf{A}})$.

$$
\begin{aligned}
\det(\breve{\mathbf{A}}) &= \frac{1}{\beta}\left(\frac{\alpha\gamma}{\beta}+1+\gamma\phi_y\right)-\left(-\frac{\gamma}{\beta}+\gamma\phi_\pi\right)\left(-\frac{\alpha}{\beta}\right) \\
&= \frac{\alpha\gamma}{\beta^2}+\frac{1}{\beta}+\frac{\gamma\phi_y}{\beta}-\frac{\alpha\gamma}{\beta^2}+\frac{\alpha\gamma\phi_\pi}{\beta} \\
&= \frac{1}{\beta}+\frac{\gamma\phi_y}{\beta}+\frac{\alpha\gamma\phi_\pi}{\beta} \\
&= \frac{1}{\beta}\left(1+\gamma\phi_y+\alpha\gamma\phi_\pi\right)>0
\end{aligned}
$$

and the trace

$$tr(\breve{\mathbf{A}}) = \frac{1}{\beta} + \frac{\alpha\gamma}{\beta} + 1 + \gamma\phi_y > 0$$

The parameter restrictions imply that condition (B6) is **always** true. Condition (B7) is therefore, **never** true. Furthermore, since $\frac{tr(\breve{\mathbf{A}})}{2} + \sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}})} <$ -1 from original form of (B7), then it follows that $\frac{tr(\breve{\mathbf{A}})}{2} + \sqrt{\frac{tr(\breve{\mathbf{A}})^2}{4} - \det(\breve{\mathbf{A}})} > 1$ cannot be true and therefore (B4) is also never true. Condition (B5) needs therefore to hold. We examine then the parameter restrictions required for this to be true:

$$\begin{aligned}
\det(\breve{\mathbf{A}}) - tr(\breve{\mathbf{A}}) &> -1 \\
\frac{1}{\beta}\left(1 + \gamma\phi_y + \alpha\gamma\phi_\pi\right) - \frac{1}{\beta} - \frac{\alpha\gamma}{\beta} - 1 - \gamma\phi_y &> -1 \\
\frac{1}{\beta} + \frac{1}{\beta}\gamma\phi_y + \frac{\alpha\gamma}{\beta}\phi_\pi - \frac{1}{\beta} - \frac{\alpha\gamma}{\beta} - 1 - \gamma\phi_y &> -1 \\
\frac{1}{\beta}\gamma\phi_y + \frac{\alpha\gamma}{\beta}\phi_\pi - \frac{\alpha\gamma}{\beta} - \gamma\phi_y &> 0 \\
\left(\frac{1}{\beta} - 1\right)\phi_y + \frac{\alpha}{\beta}\left(\phi_\pi - 1\right) &> 0 \\
\left(\frac{1-\beta}{\beta}\right)\phi_y + \frac{\alpha}{\beta}\left(\phi_\pi - 1\right) &> 0 \\
\left(\frac{1-\beta}{\alpha}\right)\phi_y + \phi_\pi &> 1^7
\end{aligned}$$

In summary the conditions are

$$\det(\breve{\mathbf{A}}) > 0, \quad \det(\breve{\mathbf{A}}) - tr(\breve{\mathbf{A}}) > -1, \quad \det(\breve{\mathbf{A}}) + tr(\breve{\mathbf{A}}) > -1 \tag{B8}$$

Our parameter restrictions implies that the second is always true, whereas the first holds when

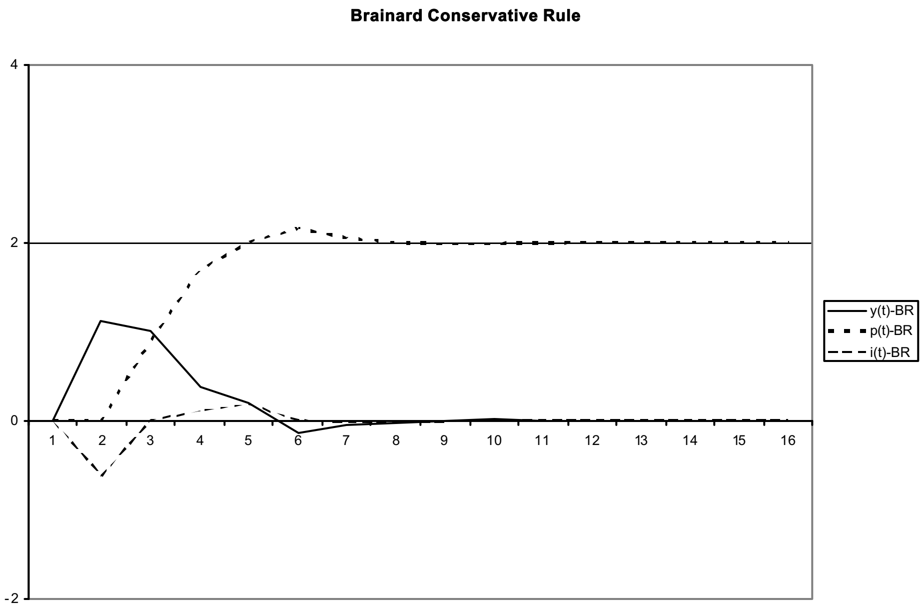
$$\left(\frac{1-\beta}{\alpha}\right)\phi_y + \phi_\pi > 1 \tag{B9}$$

We can therefore compare the likelihood with which the rules $i_{t,BR}$ and $i_{t,TS}$ are unstable. Condition (B9) therefore, is likely to be true for higher values of ϕ_y, ϕ_π which is the case for $i_{t,TS}$.

$$\begin{aligned}
i_{t,BR} &: \qquad \phi_{y,BR} = \frac{-\bar{c}ab}{\bar{c}^2 + \sigma^2}, \quad \phi_{\pi,BR} = \frac{\bar{c}\,(a+b)}{a\,(\bar{c} + \sigma^2)} \\
i_{t,TS} &: \qquad \phi_{y,TS} = \frac{a\bar{c} + 1}{a\bar{c}}, \quad \phi_{\pi,TS} = \frac{b}{\bar{c}}
\end{aligned}$$

and the ratio between them is

$$
\begin{aligned}
\frac{\phi_{y,BR}}{\phi_{\pi,BR}} &= \frac{\frac{-\bar{c}ab}{\bar{c}^2+\sigma^2}}{\frac{\bar{c}(a+b)}{a(\bar{c}+\sigma^2)}} = \frac{-\bar{c}a^2b\left(\bar{c}+\sigma^2\right)}{\left(\bar{c}^2+\sigma^2\right)\bar{c}\,(a+b)} \\
\frac{\phi_{y,TS}}{\phi_{\pi,TS}} &= \frac{\frac{b}{\bar{c}}}{\frac{a\bar{c}+1}{a\bar{c}}} = \frac{ab}{a\bar{c}+1}
\end{aligned}
$$


Brainard Conservative Rule
4
2
0
-2
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
y(t)-BR
p(t)-BR
i(t)-BR

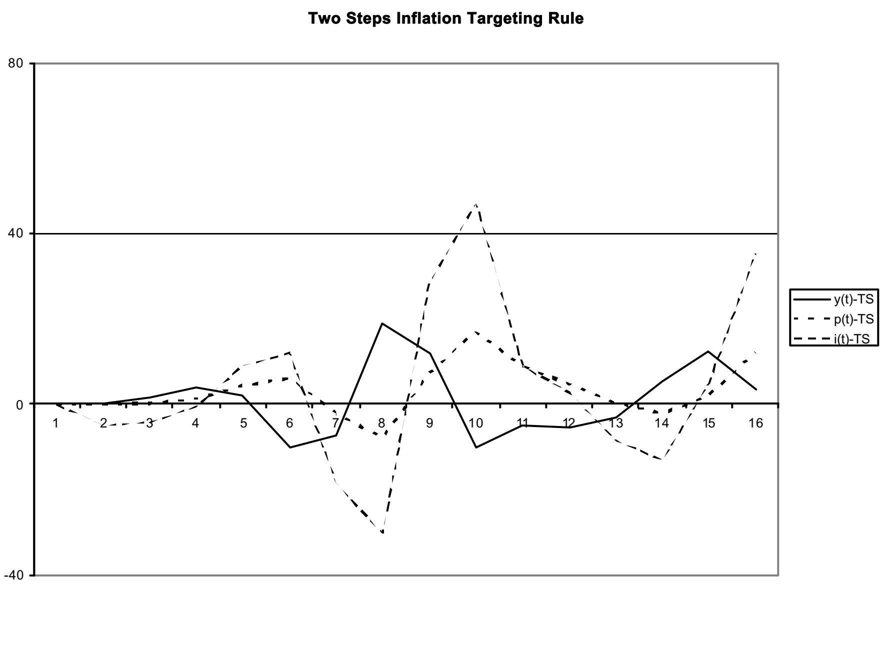

Two Steps Inflation Targeting Rule
80
40
0
-40
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
y(t)-TS
p(t)-TS
i(t)-TS

Figure 1:

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