International business cycle indicators, measurement and forecasting

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INTERNATIONAL BUSINESS CYCLE INDICATORS, MEASUREMENT AND FORECASTING

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ABSTRACT

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In this paper the business cycles of nine OECD-countries are identified by applying the Christiano-Fitzgerald bandpass filter. Turning points, recession and expansion phases and other descriptive statistics are derived from these business cycle indicators. Moreover, the international linkage between two economies is investigated by measuring the degree of synchronism of the two corresponding cyclical motions. In addition to measuring the cyclical fluctuation, a composite leading indicator is constructed which replicates and predicts the business cycle. This leading indicator is a single index composed of economic, financial and expectation variables possessing leading properties. The real-time performance of the constructed business cycle indicator for the Netherlands is compared to the performance of a similar indicator based on the Hodrick-Prescott filter methodology.

Key words: business cycles, turning points, leading indicators, bandpass filter, Forecasting
JEL codes: C82, E32, E37

SAMENVATTING

Internationale conjunctuurindicatoren, meting en voorspelling
A.H.J. den Reijer

In deze studie worden de conjunctuurcyci voor negen OESO-landen geïdentificeerd door toepassing van het Christiano-Fitzgerald bandpass filter. Van deze conjunctuurindicatoren worden omslagpunten, recessie- en expansieperioden en beschrijvende statistieken afgeleid. Bovendien wordt de internationale koppeling tussen twee economiën onderzocht door het bepalen van de mate van synchroniciteit tussen de bijbehorende cyclische bewegingen. Aanvullend op het meten van de cyclische fluctuatie wordt een samengestelde leidende indicator geconstrueerd, die de conjunctuurcyclus replicateert en voorspelt. Deze leidende indicator is een enkelvoudige index en is opgebouwd uit economische, financiële en verwachtingsvariabelen met leidende eigenschappen. De real-time prestaties van de geconstrueerde conjunctuurindicator voor Nederland worden vergeleken met de prestaties van een soortgelijk indicator die is gebaseerd op de Hodrick-Prescott filter methodologie.

Trefwoorden: business cycles, turning points, leading indicators, bandpass filter, Forecasting
JEL codes: C82, E32, E37
1 INTRODUCTION

Under the second pillar of the ECB’s monetary policy strategy information on the cyclical position of the economy is an important element in the assessment of the outlook for future price developments. The cyclical instability of aggregate economic behaviour will sooner or later be reflected in similar patterns across various macroeconomic time series. The cyclical motions of the variables are mostly simultaneous or in a rapid succession of one another. This phenomenon of leading and lagging cyclical behaviour over time across macroeconomic variables can be exploited for the measurement and forecasting of the conjunctural position of the economy. The economic state of affairs is normally identified as the cyclical state of GDP. The study of business cycles, that is of cyclical fluctuations in economic activity, goes back to the seminal contribution of Burns and Mitchell (1946), one of the earliest writings on the business cycle. As a starting point, it seems worthwhile to recall their definition:

‘Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.’

This classical analysis describes movements in actual economic time series, in particular the identification of expansions and contractions in the absolute level of aggregate economic activity. Recent research in this spirit is performed by Harding & Pagan (2002b). In contrast, mainstream modern literature investigates deviation cycles, focussing on deviations of economic activity from a long run trend. These are also called growth cycles in the literature, so that growth expansions (growth contractions) are described when the growth rate is above (below) the long-term trend rate of growth in aggregate economic activity, see for instance Canova (1998) and Hodrick & Prescott (1980). As the business cycle is defined in terms of deviations from trend, we will elaborate more on the nature and the properties of the filter used to estimate the long run trend.

This research is originated from the periodic revision of the currently operational business cycle indicators at de Nederlandsche Bank. This study ultimately aims firstly at identifying growth cycles for 9 OECD-countries: the Netherlands, Belgium, Germany, France, Italy, Spain, the United Kingdom, the United States and Japan. Secondly, we aim at forecasting these business cycles by exploiting the early signals provided by leading indicators. After measuring business cycles as deviation cycles in GDP and industrial production, the cyclical movement of a broad set of macroeconomic variables will be determined. The resulting cyclical series are evaluated on economic and statistical grounds and the accordingly selected set of series forms the basis for the construction of a composite leading indicator.
The selection criteria are amongst others a sufficient degree of similarity both in terms of the motion of the business cycle and in terms of its turning points and, secondly, leading properties with respect to the business cycle. This composite leading indicator provides therefore predictions of the business cycle. This procedure implies the investigation of many potential time series on leading properties. This research is not performed on a Euro area aggregate level for practical reasons of data collection. The construction of such composite leading indicators builds on the tradition of business cycle indicator research at the Nederlandsche Bank, for instance Fase & Bikker (1985), Bikker & De Haan (1988), Berk & Bikker (1995) and De Haan & Vijselaar (1998).

We will construct indicators along these lines for nine OECD-countries with special attention to the Netherlands. Moreover, we will provide for each country descriptive statistics like the dating of business cycle turning points, moment properties like the amplitude describing the severity of the recessions and the strength of the booms. These statistics are derived for each country separately. In addition, the international linkages of economic motions among countries can be measured by the fraction of time two economies are simultaneously in an economic upturn and/or a downturn. Finally, after having measured the business cycle and constructed a leading indicator for the Netherlands, the performance in real time of this indicator is investigated. For that purpose we execute an out-of-sample backtest for both the newly, in this study constructed, indicator and the currently operational one as constructed by De Haan & Vijselaar (1998).
2 MEASURING THE BUSINESS CYCLE

The classical decomposition of a time series into the components trend, cyclical, season and erratic, is in fact determined by the nature of the method of decomposition used. The trend estimation is not only specifying the long run motion of the time series variable, but implicitly the remaining part of the series consisting of the other three components. There is a strand of literature dealing with the question of how one should measure business cycles using different trend estimation techniques. For an overview of the comparative properties of cyclical components obtained by using different trend estimators on a couple of common macroeconomic time series, see for instance Canova (1998). He concludes that the existence of a set of business cycle stylized facts that is robust to varying trend estimators is misleading, because different concepts of a business cycle, and correspondingly different trend estimators, generate different economic objects which not necessarily have similar properties. However, although the second order properties, that is the amplitude of the cycle, of the estimated cyclical components vary widely across detrending procedures only minor differential effects emerge in higher moments.

The conceptual framework underlying business cycle analysis basically boils down to the question whether an economic or a statistical based decomposition should be used. The real business cycle literature is an example of the economics based decomposition point of view. Structural time series models are set up explicitly in terms of components that have a direct interpretation, see Harvey & Jaeger (1993). Consider for instance a structural model where the unobserved permanent component is a major cause of the business cycle. For example, a productivity shock determines both long-run economic growth and the fluctuations around that growth trend. So, removing this component by filtering out a trend implies that we are no longer examining business cycles, at least according to the originally modeled business cycle concept. In fact, the trend-cycle dichotomy is only justified if the factors determining long-run growth and those determining cyclical fluctuations are largely distinct. However, there is no consensus on the precise sources of fluctuations. Even if that would be the case, the questions remain what model appropriately incorporates them and properly describes the propagation mechanism turning them into actual business cycle patterns. The adherents of the statistical based decomposition propose a so called ‘measurement without theory’ approach. The underlying conceptual framework consists of a set of statistical facts, such as the additivity property that a time series is the sum of its constituent components and the condition of orthogonality of these

1 For instance the components trend, cyclical, seasonal and irregular. Consider the earlier mentioned example of a productivity shock, for instance the ICT-revolution allegedly shaping the new economy. Whether this shock is a trend- or a cyclical one is an important question to be solved for a structural model. Even if it is a trend shock, is the propagation mechanism such that the shock results in cyclical impulse responses? In the statistical approach the source and the propagation of the shock are not an issue and the shock will only be a cyclical feature to the extent that the shock shows cyclical behaviour.
components to one another. So, the total separation of trend and cyclical behaviour in the observed time series is statistically required.

Past research on the construction of De Nederlandsche Bank’s business cycle indicator belongs in this category by using a centered moving average as trend estimation method and later on a Hodrick-Prescott filter. This HP-filter has commonly been used in applied research, but is since recently suffering from severe criticism. For instance Cogley & Nason (1995) criticise the HP-filter for inducing spurious cycles in filtered time series and argue that the resulting ‘stylized facts’ are therefore rather ‘stylized artifacts’. Moreover, the HP-filter has been criticized for restricting by construction the focus on particular cycles with an average duration of 4 to 6 years, which is quite restrictive compared to conventional definitions of business cycle periodicities. The approach taken in this study elaborates on this notion of defining business cycles as those parts of a time series consisting of cycles within a particular frequency band. The notion builds on the theory of spectral time series, which in the Spectral Representation Theorem 2 states that any time series within a broad class can be decomposed into different frequency components. Moreover, the parts of a time series belonging to disjunct frequency intervals are mutually orthogonal. We can isolate the component of a time series associated with a frequency interval by applying a linear transformation of the data, which eliminates all components outside this interval. This procedure basically isolates business cycle components by applying weighted moving averages to macroeconomic data. However, this moving average is of infinite order and an approximation to this filter is necessary to be able to apply it on finite time series. One such approximation is given by Baxter & King (1999) and consists of an optimisation under side constraints. The optimization consists of minimising the squared difference of the exact and the approximate filter under the constraints, amongst others, that firstly the application of the filter results in a stationary time series even when applied to trending data. Secondly, the filter induces no phase shift which means that it does not alter the timing relationships between series at any frequency by shifting the corresponding cycles forward or backward in time. In the filter approximation, there is a general trade-off involved: the ideal band-pass filter can be better approximated with longer moving averages at the cost of dropping more observations at the beginning and the end of the series, leaving fewer observations for analysis. So, thirdly, the filter is required to be operational in a practical sense.

An alternative approximation to the ideal band pass filter is derived by Christiano & Fitzgerald (1999). They perform the derivation relaxing the constraints and using a different weighting scheme 3 focussing on a pre-determined frequency range. This results in a linear filter which uses all the data

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3 A more detailed technical description of both the Baxter-King filter and the Christiano-Fitzgerald filter can be found in the appendix.
and which is more efficient in the sense that it better approaches the ideal band-pass filter. Moreover, they demonstrate that this efficiency gain comes at little costs in terms of the filter rendering non-stationary filtered series and inducing phase shifts. For these reasons in this study we will use the Christiano-Fitzgerald band-pass filter to extract business cycles from time series.

The Baxter-King band-pass filter (BKBP-filter) has been applied to business cycle research by, amongst others, Stock & Watson (2000) and Agresti & Mojon (2001). As stated before the cyclical component can be thought of as those movements in the series associated with periodicities within a certain range of business cycle durations. A starting point in defining this range of business cycle periodicities is given in the introduction’s quote saying between one year and ten or twelve years. Stock & Watson and Baxter & King both use the convention of business cycles as variations in GDP between 6 and 32 quarters. This is based on the NBER business cycle reference dates whereby the shortest cycle since 1858 lasted 6 quarters and the longest ones are mostly shorter than 8 years. Agresti & Mojon choose for the construction of an Euro Area business cycle a periodicity band of 1.5 years and 10 years. They arrive at this higher upper bound by stating that the euro area saw only three recessions over the last thirty years. In addition, the latest two US-cycles lasted more than eight years. In this study we will adopt the Agresti&Mojon convention of business cycle frequencies.

While GDP gets most attention from forecasters and provides a broader coverage of the economy we will use, like in Berk & Bikker (1995) the manufacturing output volume as the reference series from which the business cycle indicator is filtered. The production of manufacturing industry represents roughly only a quarter of GDP. However, some expenditure components of GDP are typically non-cyclical or even counter-cyclical possibly with different leads and lags. Government expenditure, for example, does not follow the general economic cycle. The social security system acts as an ‘automatic stabilizer’ and tends to be counter-cyclical and lagging the cycle. From a practical point of view, GDP is only available on a quarterly basis, the publication lag is considerably longer than for manufacturing output data and the revision of GDP numbers after the first publication is considerably larger. As predicting the business cycle is one of the main purposes, an up to date business cycle figure is deemed essential.

The construction of the business cycle indicator consists of applying the Christiano-Fitzgerald filter to the manufacturing output series and standardising the resulting cyclical pattern. The resulting business cycle indicator for the Netherlands is graphed in figure 1. The first figure shows the seasonally adjusted GDP-time series, the Christiano-Fitzgerald trend and the resulting cyclical pattern. The second picture shows the standardised cyclical pattern which is called the business cycle indicator. The turning points are derived from this indicator according to a procedure outlined below. The periods between a peak and a trough are the recession periods and are indicated in all three graphs by the shaded area. The third graph shows the resemblance of recession periods defined by our methods
and the more commonly used year-on-year GDP-growth numbers. Recession periods correspond to declining growth rates and *vice versa*. The beginning and one period after the ending of the recession.

**Figure 1** Business Cycle Indicators and GDP-growth for the Netherlands

**Industrial production, trend and cycle**

- Industrial production
- Cyclical
- Trend

**Business cycle indicator Netherlands**

- Recession period
- Business cycle

**GDP growth rates and cycle dating**

- Recession period
- Annual GDP growth
period are the business cycle turning points. At the heart of dating cyclical turning points is the standard calculus rule for finding extreme values, i.e. setting the first derivative of the time series variable $y$ with respect to time equal to zero: $\frac{dy}{dt}=0$. A modified rule is needed in a discrete setting that is typical for macroeconomic time series. A local peak in industrial production, $y_t$, occurs at time $t$ if $y_t$, exceeds values $y_s$ for $t-k<s<t$ and $t+k>s>t$, where $k$ delineates some symmetric window in time around $t$. This simple idea is the basis of the NBER procedures summarized in the Bry & Boschan (1971) dating algorithm. In practice, the Bry-Boschan algorithm with $k=5$ for monthly data also applies some censoring rules ensuring a minimum cycle duration of 15 months and alternation of peaks and troughs. This algorithm has been applied on time series data in levels and so produces the dating of classical business cycles. Now consider the growth cycle time series variable $BCI_t = y_t - g_t$. The variable $BCI_t$ is now the deviation of industrial production, $y_t$, from its long run Christiano-Fitzgerald-trend, $g_t$, and a similar turning point algorithm can be applied on this deviation series. The Bry-Boschan algorithm has been adapted by Dungey & Pagan (2000) to date growth cycles. We perform our dating in this spirit and apply an algorithm that follows the following rules:

- peaks and troughs must alternate,
- a peak (trough) must represent above (below) trend growth and so the corresponding $z_t > (<) 0$,
- the turning point must be a local optimum

\[ (BCI_{t-p},\ldots,BCI_{t},BCI_{t+1},\ldots,BCI_{t+q}), BCI_t > 0 \text{ for } t \in [-p,\ldots,q], \]
\[ (BCI_{t-p},\ldots,BCI_{t-1},BCI_{t+1},\ldots,BCI_{t+q}), BCI_t < 0 \text{ for } t \in [-p,\ldots,q]. \]  

The business cycle indicators of the nine countries are calculated by applying the Christiano-Fitzgerald filter and standardising the resulting cyclical series. Subsequently, the business cycle dating is performed by applying the abovementioned dating rules to the business cycle indicators. The dating procedure provides the collection of dates at which the business cycle indicator reaches a peak or a trough. A contraction (expansion) period is straightforwardly defined as the period between a peak (trough) and a trough (peak) indicating declining (increasing) growth rates. The business cycle indicators together with the contraction periods for the nine countries are shown over a subperiod in the graphs of appendix B. The business cycles are represented by the thick lines and the contraction periods by the shaded areas. The resulting two lines will be explained in the next section.

In order to derive descriptive statistics for the business cycle indicators we construct a state variable $S_t$, which equals one in contraction periods, the period from one month after the peak date to the date of the trough, and zero otherwise. The information $\{BCI_t, S_t\}$ provides a picture of the characteristics of the cyclical motion and can be used to produce descriptive statistical measures like duration, amplitude, steepness and sharpness of expansion and contraction phases. We will use the descriptive statistics for the business cycle indicators of Harding & Pagan (2002b). We need to keep in mind that the statistics describe what the indicators measure. By construction, the business cycle indicators are the deviation of the log of industrial production series from its long run Christiano-Fitzgerald trend.
Because the data is used in logs the indicator represents percentage deviations from trend. So, the statistics describe the cyclical behaviour of industrial production and only serve as an approximation to cyclical behaviour of GDP (-growth) to the extent that the cycles in industrial production correspond to cycles in GDP.

The most commonly used descriptive statistics are the standard deviation, skewness and kurtosis. The standard deviation measures the variability of the indicator. Skewness measures the one-sided fat-tailedness of the probability distribution of the indicator. So, it measures whether expansions are relatively more exuberant than that contractions are severe. Kurtosis measures the two-sided fat-tailedness and indicates whether large cyclical movements are relatively more common.

The state variable \( S_t \) separates expansion and contraction phases. A peak occurs at time \( t \) when \( S_t=1 \) and \( S_{t+1}=0 \). An expansion (contraction) phase consists of the periods beginning with the month after the date of a trough (peak) and ending at the date of a peak (trough). Expansions and TP (trough-to-peak) and contractions and PT (peak-to-trough) will be used interchangeably. The average duration of expansions and contractions are:

\[
DUR_{PT} = \frac{\sum_{t=1}^{T} S_t}{\sum_{t=1}^{T-1} (1 - S_{t+1}) S_t} , \tag{2}
\]

\[
DUR_{TP} = \frac{\sum_{t=1}^{T} (1 - S_t)}{\sum_{t=1}^{T-1} (1 - S_t) S_{t+1}} . \tag{3}
\]

The numerator in (2) measures the total time spent in contractions and the denominator counts the number of peaks. The average amplitude straightforwardly equals:

\[
AMP_{PT} = \frac{\sum_{t=1}^{T} S_t \Delta BC_t}{\sum_{t=1}^{T-1} [1 - S_{t+1}] S_t} , \tag{4}
\]

\(^4\) In calculating average of expansion and contraction phases one has to deal with uncompleted phases at both end of the sample. In this study we base the statistics on completed phases and then the summation runs from the beginning of the first completed phase until the end of the last one rather than over 1,…,T.
Thinking of a phase as a triangle with as base its duration and as height its amplitude, then the steepness of a phase is naturally measured by the slope of the triangle:

\[
STEEP = \frac{AMP}{DUR}.
\]  

In addition to looking at the steepness over the phases one might want to compare the shape of the cycle in the early part of the cycle with that in the latter part. A simple way of capturing this shape is to divide a phase into two parts and to compare the average growth rates in the first part to that in the second part. This statistic is called sharpness and essentially measures a degree of convexity of the indicator, that is the relative steepness of the early stage in the business cycle. Let \(i\) be the \(i^{th}\) phase and \(d_i\) be the duration of this phase, starting in time \(t_i\). The measure of the shape of a single phase can then be defined as:

\[
v_i = \frac{1}{\lfloor d_i/2 \rfloor} \left[ \sum_{k=1}^{\lfloor d_i/2 \rfloor} BCI_{k+i} - BCI_{k+i-1} \right] - \sum_{k=i+1}^{d_i} \left( BCI_{k+i} - BCI_{k+i-1} \right) .
\]  

Thus, if in an expansion phase, growth is faster in the first half of the phases’ duration than in the second half, then this measure will be positive. If in a contraction phase the decline is slower in the first half of the phases’ duration than in the second half, then this measure will be positive. If a cyclical indicator shows these two positive measurements it implies that the peaks are relatively long lasting and rounded and the troughs are relatively short lasting and pointed.

Averaging (7) over all similar phases can be defined as:

\[
SHARP_{PT} = \sum \left( \frac{d_j}{\sum_j S_j} \right) v_j ,
\]  

\[
SHARP_{TP} = \sum \left( \frac{d_j}{\sum_j (1-S_j)} \right) v_j .
\]  

Equations (8) and (9) are in effect weighted moving averages of the appropriate form of (7). The weight for a particular expansion is the duration of that expansion divided by the total sum of the
indicator being in an expansion phase. So, this weighting scheme gives more weight to longer phases and the sum of the weights equals one.

The descriptive statistics for the nine business cycle indicators are presented in Table 1. Almost all countries display kurtosis indicating that large cyclical movements are relatively common. The Netherlands and to a lesser extent Belgium stand out on stability, but not on moderateness. Stability is revealed by the volatility of the cyclical behaviour around the trend and by the amplitude. This is the absolute deepness of a cycle decomposed in contraction and expansion phases. Both the Netherlands and Belgium show the lowest numbers for all nine countries on the standard deviation and the amplitude. Moderateness is revealed by the steepness statistic. Steepness can be interpreted as the motion deepness of a contraction or expansion phase like amplitude can be seen as the absolute deepness of a contraction or expansion phase. For both phase types, the Netherlands and to a lesser extent Belgium show among the highest numbers for the steepness statistic indicating that both economies move quickly from one turning point to another. Japan acts as the mirror image of these two countries by showing the highest cyclical volatility and the highest cyclical severity both in terms of amplitude and steepness for both contraction and to a lesser extent expansion periods. The United States also display high cyclical volatility and severity. The negative skewness indicates contraction phase outliers indicating at relatively severe recessions. The large contraction sharpness and the large expansion sharpness for the U.S. reveals a pattern of smooth rounded peaks and pointed deep troughs. Italy demonstrates the largest positive skewness and among the highest peaks, that is the average amplitude of the expansion phase. Both statistics indicate large upward cyclical outliers. The quite moderate sharpness statistic reveals that the Italian cyclical pattern surrounding turning points is particularly smooth. Germany shows a relatively long expansion phase compared to the contraction phase. On the other hand the contraction phases’ motion is relatively fast according to the steepness statistic and relatively deep according to the negative skewness. The sharpness statistic reveals that the German turning points are the most sharply pointed ones of the nine countries. Therefore, the German indicator looks somewhat like a saw tooth.
<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Duration</th>
<th>Amplitude</th>
<th>Steepness</th>
<th>Sharpness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PT TP</td>
<td>PT TP</td>
<td>PT TP</td>
<td>PT TP</td>
<td>PT TP</td>
<td>PT TP</td>
<td>PT TP</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1.67 -0.38 3.51</td>
<td>26.00 22.00</td>
<td>2.87 2.42</td>
<td>0.11 0.11</td>
<td>0.015 0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td>2.16 -0.0091 4.08</td>
<td>22.00 28.13</td>
<td>2.67 2.38</td>
<td>0.12 0.085</td>
<td>-0.017 -0.012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>2.50 -0.56 4.78</td>
<td>24.83 22.33</td>
<td>2.44 2.59</td>
<td>0.098 0.12</td>
<td>-0.038 -0.042</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>2.64 -0.25 2.68</td>
<td>24.90 28.00</td>
<td>2.93 2.31</td>
<td>0.12 0.083</td>
<td>-0.023 -0.017</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>2.67 0.21 3.25</td>
<td>33.40 33.50</td>
<td>3.40 2.83</td>
<td>0.10 0.085</td>
<td>-0.66 -0.55</td>
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<td></td>
</tr>
<tr>
<td>Spain</td>
<td>2.33 -0.68 3.69</td>
<td>38.83 30.67</td>
<td>3.02 2.97</td>
<td>0.078 0.097</td>
<td>0.053 0.067</td>
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<tr>
<td>Japan</td>
<td>3.38 0.16 2.67</td>
<td>26.00 39.00</td>
<td>3.39 3.12</td>
<td>0.13 0.080</td>
<td>-0.14 -0.097</td>
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<tr>
<td>United Kingdom</td>
<td>2.47 -0.13 3.14</td>
<td>36.75 33.40</td>
<td>3.00 2.39</td>
<td>0.098 0.072</td>
<td>0.14 0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>2.82 -0.35 3.55</td>
<td>27.00 36.50</td>
<td>2.74 2.51</td>
<td>0.10 0.069</td>
<td>0.083 0.061</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: the statistics represent the deviations from trend series which are not standardised. The mean of the deviation cycle is by construction approximately zero. So, standardising comes down to dividing the deviation time series by its standard deviation. PT means the phase Peak-to-Trough and is therefore synonym to contraction phase. Even so is TP synonym to expansion phase. The skewness measure is $\mu_3/\mu_2^{1.5}$ and the kurtosis measure is $\mu_4/\mu_2^2$, where $\mu_i$ is the $i^{th}$ central moment. The skewness of a symmetrical distribution, such as the normal, is zero. Similarly, the kurtosis of the normal distribution is 3.

1) the numbers are presented with opposite signs for the sake of comparison with expansion periods.
2) these numbers are multiplied by 100.
Now that the Euro area is composed of a number of countries for which individual country cycles are available it is interesting to go beyond examining individual cycle characteristics and analyse the relationships between these business cycles. More precisely, we want to examine how closely the cycles of individual countries are synchronized to one another. A popular index that has been proposed for this task, Harding & Pagan (2001a), is the index of concordance, which measures the fraction of time that two business cycles spend in the same phase. Let $S_i$ and $S_j$ denote the state variables of the business cycles of country $i$ and $j$. The index of concordance is defined as:

$$IC_{ij} = \frac{1}{T} \sum_{t=1}^{T} \{S_i^t S_j^t + (1 - S_i^t)(1 - S_j^t)\}.$$  

For all nine countries the results for the index of concordance are presented in the upper triangle of Table 2. The italic numbers in the lower triangle are the cross correlation coefficients of the two concerning business cycles.

Six of the nine countries examined in this study are part of the Euro area. These six countries can form 15 unique couples 5 which provides 15 indices of concordance (ICs) measuring Euro area business cycle synchronisation. The average of these 15 indices is 0.74 and the average correlation is 0.66. As can be calculated from table 2, two Euro area countries are on average roughly three quarters of the time in the same conjunctural state. Both the correlation and the IC of the big Euro area countries Germany and France are just 0.68 and those between Germany and Italy only 0.32 respectively 0.65. The average IC and correlation of the potential Euro area member the United Kingdom and the five Euro area countries equal 0.64 respectively 0.56, which is even worse than the average IC of 0.67 between the United States and the Euro area countries and slightly better than the average correlation of 0.54 between these two economic blocks. Moreover, the IC and the correlation between the United Kingdom and the United States equal 0.75 and 0.71 respectively. This indicates that the United Kingdom’s business cycle is more synchronised with the United States’ one than with the cycle of the Euro area.

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5 Of course, one can construct an index of concordance which only measures the simultaneity of more than two countries together. It then measures the fraction of time that all countries are in the same state. For example, the concordance of Germany, France and Italy together is 0.57
Table 2  Business cycle indicators’ correlation and index of concordance

<table>
<thead>
<tr>
<th></th>
<th>NL</th>
<th>BE</th>
<th>FR</th>
<th>GE</th>
<th>IT</th>
<th>SP</th>
<th>JP</th>
<th>UK</th>
<th>US</th>
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</thead>
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<td>NL</td>
<td>0.81</td>
<td>0.68</td>
<td>0.81</td>
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<td>0.57</td>
<td>0.58</td>
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</table>

Note: The sample period is 1965:1-2001:9. The indices of concordance are represented in the upper triangle of the table. The correlation coefficients are represented in italic in the lower triangle of the table. The country abbreviations are as follows: Netherlands (NL), Belgium (BE), France (FR), Germany (GE), Italy (IT), Spain (SP), Japan (JP), United Kingdom (UK) and United States (US).
3 HOW TO REPLICATE AND PREDICT THE BUSINESS CYCLE

Cyclical motion in the economy is reflected in varying macroeconomic time series. In the previous section the business cycle is identified as the cyclicality of industrial production. This provides a signal for the current state of the economy. By nature of the macroeconomic system, variables can be classified as leading, coincident or lagging dependent on the timing behaviour of their cyclicality vis-à-vis the business cycle indicator. Leading variables give by nature an early signal on the conjunctural position of the economy. We will exploit this feature by selecting a set of leading variables and transforming them into a single composite leading indicator, which replicates and predicts the business cycle indicator. Potential leading variables which, from an economic point of view, can be expected to lead and predict the general business climate are gathered and will be called basic indicators. Moreover, candidate variables must fulfill practical criteria like availability of long time series with as few interruptions and definition alterations as possible and availability of data with minimum delay. The cyclical parts of all basic indicators are compared to the business cycle indicator in order to determine the numbers of months by which the basic indicators lead the business cycle, i.e. the timing. The first measure of this timing is the lead \(-l\) in number of months corresponding to the maximum cross correlation coefficient between the business cycle indicator (BCI) and the lagged basic indicator (BI):

\[
\rho(l) = \sum (BCI_t - \bar{BCI})(BI_{t+l} - \bar{BI}) / \left( \sum (BCI_t - \bar{BCI})^2 \sum (BI_{t+l} - \bar{BI})^2 \right)^{1/2}
\]  

\[
l = \arg \max_l \rho(l)
\]

In formula (11) \(\bar{BCI}\) and \(\bar{BI}\) are the averages of \(BCI\) and \(BI_{t+l}\) respectively, over \(t\). The magnitude of the correlation coefficient (11) measures the degree of cyclical similarity of the business cycle indicator and the basic indicator. Moreover, the corresponding lead of \(l\) is measured in (12) and represents the optimal lead of the basic indicator. The two indicators resemble the most similar motion at this lead because of the maximisation procedure and the definition of the correlation coefficient.

A second restriction concerning the lead is the matching of turning points of the two series. The ultimate composite leading indicator is not only supposed to reflect the motion of the business cycle indicator, but also to provide an early signal of upcoming turning points. The optimal lead obtained from (12) will therefore be fine-tuned by matching peaks and troughs of both series more closely. The algorithm executing this task consists of identifying turning points as stated in (1), matching the peaks and troughs of the business cycle indicator to nearest ones of the basic indicator, which is shifted by the lead \(l\), and finally fine-tuning this lead by adjusting it in order to minimize a distance metric. This
metric is straightforwardly the sum of the distances measured in months between all peaks and troughs of the business cycle indicator and the corresponding peaks and troughs of the basic indicator.

Both the cross correlation (11) and the fine-tuned version of (12) act as statistical criteria for the selection of the leading indicators that together constitute the basis of the composite leading indicator. Analogous to Bikker & Berk (1995), a potential leading indicator should show a cross correlation (11) of at least 0.5 and a minimum fine-tuned lead (12) of 6 months in order to be selected. These boundaries are chosen as high as possible under the restriction that a sufficient number of leading indicators be selected.

The aim is to transform the selected basic leading indicators into a single composite index which replicates and predicts the business cycle indicator. The replication property is ensured by (11) and the procedure of matching turning points. The prediction property is ensured by the criterion of minimum lead. A single composite index possessing both properties is then constructed as an optimal linear combination of normalised and synchronised selected basic leading indicators. Normalisation means transforming the basic indicator such that an indicator showing zero mean and unit standard deviation results. This normalisation procedure makes indicators exhibiting weak cyclical variation, for instance wages, comparable to indicators exhibiting strong cyclical variation, for instance confidence indicators and prevents the latter ones from dominating the former ones in a single composite index. The selected leading indicators are synchronised with the business cycle indicator by shifting them forward by their respective leads. This synchronisation procedure aligns the leading indicators’ cyclical phases with the business cycle indicator’s one and causes their turning points to coincide on average. Shifting a leading indicator variable forward by a number of months equal to its lead means that the shifted leading indicator variable no longer leads, but instead coincides with the cyclical phase of the total economy. Because of the shifting however, the series extends beyond the observation period and provides a predictive signal for the near future cyclical development. After applying normalisation and synchronisation each selected leading indicator provides its own early signal. A one-dimensional single signal is obtained by constructing an optimal linear combination of the individual normalised and synchronised leading indicators by means of principal component analysis. The first principal component is the common element in the set of selected leading indicators and is supposed to be a reflection of the business cycle indicator. Principal component analysis ensures that there is no other linear combination of the leading indicators which will account for more of the intrinsic total variance of the set of leading indicators than the first principal component. So this component represents the most common motion across the set of leading indicators.
4 EMPIRICAL RESULTS ON THE LEADING INDICATORS

To perform the procedures of the previous section a collection of potential leading variables is compiled consisting of about 40 monthly time series for each country. These variables are tested by calculating the optimal values for (11) and (12). The results are presented in Table 3. The first number in each cell is the optimal lead and the second number is the maximum cross correlation for the concerning leading indicator and country. The selected leading indicators are of an economic, financial and monetary nature and confidence and expectations surveys.

Of the economic flow variables consumption, investment, exports and imports only in Japan consumption shows a leading cyclical pattern. Consumption can straightforwardly be seen as a demand-pull trigger for economic activity. Anti-cyclical government consumption and influencing private consumer behaviour by tax inducements are part of the Keynesian recipe. Inventories are a result of non-matching economic flows and for most countries the storage of final products provides an early signal for production activity. According to De Haan & Vijselaar (1995) there exists a one-way Granger-causality from the storage of final products towards industrial production for the Netherlands. A second type of inventories are those at the start of the production process, like the level order positions and issued building permits. Both variables procyclically lead the business cycle.

Prices and wages are a fundamental part in every economic model describing cyclical fluctuation. Following Stock & Watson (2000), a general pattern emerges of leading, countercyclical price levels and lagging, procyclical rates of inflation. The nominal wage index exhibits a pattern quite similar to consumer prices, probably due to the contractual indexing of wages. Consumer prices, sales prices, total wages and hourly wages emerge in this study as business cycle leaders. Moreover, labour costs per unit product is a combination of wages and productivity and on average leads the cycle countercyclically. The same pattern shows up for the world market prices of commodities, which basically is a cost variable for production.

A dominant class of leading variables concerning business cycles are those specifically related to the future. These variables are the result of surveys and the following are identified to procyclically lead the general cycle: consumer and producer confidence, expected future industrial production, prospects total economy, judgement of order arrivals and the IFO-indicator for Germany.

Financial variables deal with the purchasing and investment possibilities and therefore also with the choices made by private and public agents. As the sum of discounted expected future cash flows share prices are, like surveys, future linked variables. They partly determine the budget constraint and can

6 Think of the appeal of the Bush administration to the American people following the September 11-attacks to consume the U.S. out of a recession.

7 We choose a subindex of the IFO-indicator which refers to future developments.
therefore act, unlike surveys, as a catalyst for a mechanism of self-fulfilling prophecy. Financial markets expecting an upturn create the possibilities for an upturn. Interest rates are a cost of capital and therefore also partly determine the budget constraint. Both short term and long term interest rates lead the business cycle with positive values of interest rates associated with cyclical declines in output. The lead of the short term interest rate is approximately 1.5 years for most countries to over two years for Japan, see table 3. The connection between short term interest rate movements and the output gap is established in the Taylor rule. The spread between the short and long term interest rates has long been recognised as a leading indicator. An inverted yield curve, that is short rates exceeding long rates, anticipates substantial declines in economic activity. However, given it’s widely acknowledged predictive potential we prohibit taking this variable into the model together with both the short and the long term interest rates.

Monetary aggregates play an important role in the determination of price levels and can result in movements of real quantities because of nominal frictions in the economy. Following the time inconsistency literature, in the short run a monetary expansion creates a boost in employment and output and in the long run only causes a rise in the price level. We basically consider three monetary aggregates, namely if available M1, M2 and M3 and both test them in nominal and real terms and in levels and growth rates. We however find only for Spain a monetary aggregate functioning as a leading indicator. Stock & Watson (1998) note that the contemporaneous cross correlation between the cyclical components of nominal M2 and output has drastically declined since the early eighties for the United States.

The selected basic leading indicators are composed into one single composite leading indicator by means of principal component analysis as outlined in the previous section. In figure A1 in the appendix the business cycle indicator together with the composite leading indicator are graphed for each country. By selection the basic leading variables are by selection leading the business cycle by at least 5 months and the composite leading indicator is therefore by construction also leading the business cycle with 5 months. The correlation coefficient between the business cycle indicator and the composite leading indicator are displayed for each country in the last row of Table 3. In figure A2 in the appendix the selected basic variables used to construct the composite leading indicator are graphed for the Netherlands. In each single graph the raw times series and the trend are plotted together with the resulting standardised cyclical part.
Table 3  Results of selection leading indicators

<table>
<thead>
<tr>
<th></th>
<th>Netherlands</th>
<th>Belgium</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
<th>Spain</th>
<th>Japan</th>
<th>United Kingdom</th>
<th>United States</th>
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<td>9</td>
<td>12</td>
<td></td>
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<td></td>
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<td>M1*</td>
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<td>18</td>
<td>18</td>
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<td>0.93</td>
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<td>0.80</td>
<td>0.64</td>
<td>0.88</td>
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<td>Long term interest</td>
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<td>Expected business</td>
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<td>Storage final products*</td>
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<td>5</td>
<td>6</td>
<td>13</td>
<td>10</td>
<td>9</td>
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<td>Prospects total economy</td>
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<td>0.61</td>
<td>0.90</td>
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<td>0.70</td>
<td>0.92</td>
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<td>Judgement of order</td>
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<td>0.76</td>
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<td>Hourly wage industry*</td>
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<td>11</td>
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<td>Domestic sales prices*</td>
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<td>confidence</td>
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<tr>
<td>Total wages*</td>
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<td></td>
<td></td>
<td>0.77</td>
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<tr>
<td>Labour costs per unit product*</td>
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<td>12</td>
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<td></td>
<td></td>
<td></td>
<td>0.92</td>
<td>0.63</td>
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</table>

Note: The first number in each cell is the optimal fine-tuned lead (12) and the second number is the maximum correlation (11) of the corresponding variable. The variables marked with a '*' countercyclically lead the business cycle instead of procyclically. The statistics presented in this table are, except for the Netherlands, based on the cyclical motions of the variables filtered with a lowerbound of 36 instead of 18 months and an upper bound of 120 months. All countries possess a minimum lead of 6 months, except for the Netherlands, France and Germany. These three countries possess a lead of 5 months. De correlation coefficient between the business cycle and the leading indicator should be read with caution, because the leading indicators are also constructed on the basis of turning point congruency.
5 COMPARING BUSINESS CYCLE INDICATORS FOR THE NETHERLANDS

In this study we have outlined a methodology and constructed accordingly business cycle indicators and composite leading indicators for nine countries. In this section we will provide a comparison between the constructed indicator for the Netherlands and the indicator as constructed by De Haan & Vlijmelaar (1998). We will refer to this indicator as Hodrick-Prescott, because this filter methodology was used for trend estimation. Additional to a different methodology, the Hodrick-Prescott leading indicator is partly based on different basic leading indicators. It is based on the real monetary aggregate M1 and the yield curve, which are both not used by the newly constructed indicator. The IFO-indicator for Germany, expected business activity and the short-term interest rate are basic leading indicators used by both the newly constructed and the Hodrick-Prescott indicators. The two business cycle indicators and the two leading indicators are graphed in figure 2.

Figure 2 Two business cycle indicators for the Netherlands

Indicators based on bandpass filter

Indicators based on Hodrick-Prescott
Both methodologies track a quite similar picture of the past conjunctural position of the Dutch economy. Although either indicator produces broadly an equal cyclical shape, detail differences remain. Table 4 provides the dating according to (1) of the business cycle turning points for both methodologies. The biggest turning point date difference between both indicators is 13 months, namely between the trough dating of October 1984 and November 1985 for Hodrick-Prescott and Christiano-Fitzgerald respectively.

Table 4  Turning point dating for the Netherlands

<table>
<thead>
<tr>
<th>Hodrick-Prescott</th>
<th>Christiano-Fitzgerald</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 1967</td>
<td>June 1967</td>
</tr>
<tr>
<td>August 1970</td>
<td>December 1969</td>
</tr>
<tr>
<td>May 1972</td>
<td>January 1972</td>
</tr>
<tr>
<td>March 1974</td>
<td>March 1974</td>
</tr>
<tr>
<td>June 1975</td>
<td>July 1975</td>
</tr>
<tr>
<td>October 1976</td>
<td>August 1976</td>
</tr>
<tr>
<td>January 1978</td>
<td>July 1977</td>
</tr>
<tr>
<td>October 1979</td>
<td>November 1979</td>
</tr>
<tr>
<td>April 1983</td>
<td>December 1982</td>
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<tr>
<td>November 1985</td>
<td>October 1984</td>
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<tr>
<td>December 1987</td>
<td>November 1987</td>
</tr>
<tr>
<td>April 1990</td>
<td>August 1990</td>
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<tr>
<td>July 1993</td>
<td>July 1993</td>
</tr>
<tr>
<td>June 1995</td>
<td>February 1995</td>
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<tr>
<td>August 1996</td>
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<td>February 1998</td>
<td>January 1998</td>
</tr>
<tr>
<td>November 1998</td>
<td>February 1999</td>
</tr>
<tr>
<td>July 2000</td>
<td>October 2000</td>
</tr>
</tbody>
</table>

Figure 3  Annual change of trend component
In % mutation

[Graph showing the annual change of trend component with trend lines for Hodrick-Prescott and Christiano-Fitzgerald methods]
The measurement accuracy of the cyclical motion mainly depends on the filter used for the trend component. The difference between the trend estimations of the newly constructed indicator based on the Christiano-Fitzgerald bandpass filter and the existing indicator based on the Hodrick-Prescott filter is graphed in figure 3. This figure plots a monthly time series of the year-on-year growth rates of both trend methodologies. It follows that trend growth is about 5% in the sixties and 2.5% to 3% in the late seventies, eighties and nineties. Moreover, it clearly shows that the Hodrick-Prescott filter takes up much less of the cyclical motion than the Christiano-Fitzgerald one. In terms of bandpass filtering, applying the Hodrick-Prescott filter results in a cyclical component incorporating cycles lasting longer than 10 years.

All filters face the end point problem. This means that they cannot provide accurate estimates of the real-time trend and the real-time cyclical position unless data is available for the entire current cycle. A real-time estimate is an estimate on the date for which the latest observation is available. An important feature from a forecasting perspective is the behaviour of the indicator in real-time. More specifically, what is the difference between the indicator at time \( t \) given data until time \( t \), \( BCI_{t|t} \) and the indicator at time \( t \) given data until time \( T (T>t) \), \( BCI_{t|T} \). This difference is called the revision error and is purely caused by the inevitable end point problem of the filter methodology. The mean absolute revision error is defined by:

\[
MARE_n = \frac{1}{(T-120-n)} \sum_{i=61}^{T-60-n} \left| BCI_{t+i+n|t} - BCI_{t+i|T} \right|
\]  

(13)

In (13) the first and last 60 observations of the sample are disregarded in order to avoid a bias. The underlying assumption is that the cyclical motion has reached its definite, never changing shape after having additional data available for a half cycle, that is 5 years. The validity of this assumption stems from Table 5, where mean absolute revision errors are displayed for different values of \( n \). This parameter represents the number of additional available data at time \( t \) for the calculation of the business cycle at that time. The table clearly shows the expected decline of the revision error as more data becomes available for the Christiano-Fitzgerald filter. The Hodrick-Prescott methodology shows a deterioration after a few additional data and subsequently a much more definite cyclical shape. The initial deterioration is probably due to the elimination procedure of the seasonal component. The more definite cyclical shape is shown by the lesser MARE when more than a year of extra data is available. This phenomenon can be explained by the lesser cyclicality of the Hodrick-Prescott filter due to the initial parameter setting as shown in figure 3. Incorporating cyclical fluctuations of duration of more than 10 years in the trend estimation causes much more volatility of this trend and therefore for the resulting cyclical motion when more and more data becomes available.
A second feature of the end point problem is the turning point date instability. This refers to the change in the dating of a turning point when more and more data becomes available. The initial date of a turning point is the first identified date corresponding to a specific turning point. The initial dating turns out to be robust if a turning point is defined by (1) with $p=q=3$. Only if turning points are relatively rounded, like the one in 1990, the turning point date is quite fluctuating until the exact one is established. Moreover, it holds for both algorithms that false signals are absent if one disregards the subcycle of 1982.

Table 5  Revision errors for the indicators of the Netherlands

<table>
<thead>
<tr>
<th></th>
<th>$MARE_0$</th>
<th>$MARE_1$</th>
<th>$MARE_3$</th>
<th>$MARE_6$</th>
<th>$MARE_{12}$</th>
<th>$MARE_{36}$</th>
<th>$MARE_{60}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BCI_{HP}$</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.43</td>
<td>0.36</td>
<td>0.25</td>
<td>0.20</td>
</tr>
<tr>
<td>$BCI_{CF}$</td>
<td>0.48</td>
<td>0.48</td>
<td>0.50</td>
<td>0.52</td>
<td>0.44</td>
<td>0.20</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Note: $MARE_n$ is defined in (13). $BCI_{HP}$ is the business cycle indicator for the Netherlands based on the Hodrick-Prescott methodology. $BCI_{CF}$ is the business cycle indicator for the Netherlands based on the Christiano-Fitzgerald methodology. This latter methodology is slightly modified by taking moving averages up to 60 periods. More precisely, for $i=1\ldots60$ the indicator $BCI_{t-i|t}$ is the average of $BCI_{t-i+j|t-i+j}$ for $j=1\ldots i$. This is done to average out seasonal and end point effects.
6 CONCLUSION

In this study we identified business cycles in deviations from trend for nine OECD-countries. We used a bandpass filter to estimate the trend and the cyclical component of a time series. The motion of a time series within an a priori specified frequency interval can be isolated by applying a bandpass filter. Each bandpass filter requires an optimisation for dealing with finite time series. The Christiano-Fitzgerald bandpass filter uses all available data to calculate the filter weights and therefore provides a more efficient estimate than the well-known Baxter-King bandpass filter. We adopt the convention that a business cycle consists of all cycles with duration longer than 18 months and shorter than 10 years. The business cycle indicator is constructed applying the filter on the production of manufacturing industry series. Business cycle turning points and recession and expansion periods are derived from the indicator. Moreover, a range of summary statistics is provided describing features like amplitude, steepness and duration of the cycle for each country.

The Netherlands and to a lesser extent Belgium stand out compared to the nine countries on stability by showing a relatively modest cyclical spread around the trend. Their cyclical motions are however the least moderate so that both economies move quickly from contraction to expansion phases and vice versa. Japan acts as the mirror image by showing the largest cyclical swings. The U.S. reveals a pattern of smooth rounded peaks and pointed deep troughs. The international linkage between economies is explored by calculating the fraction of time the two economies are either both in the expansion phase or both in the contraction phase. This statistic shows that the United Kingdom is more synchronised with the United States than with the Euro area. The average synchronisation between the United Kingdom and the Euro area countries is lower than the average synchronisation of the Euro area countries with one another.

In addition to measuring cycles we constructed a single composite leading indicator, which replicates and predicts the business cycle. The leading indicator is based on economic, financial and survey variables possessing leading properties. These variables are selected from a set of 40 candidate variables for each country. The variables that have been selected for five or more countries are the short term and long term interest rates, the storage of final products, the hourly wages, the domestic sales prices, the IFO-indicator for Germany and the consumption price index.

The business cycle indicator for the Netherlands shows a similar picture of the past conjunctural positions as the current operational indicator at De Nederlandsche Bank. This indicator is based on the Hodrick-Prescott filter methodology. It possesses a trend which captures less cyclical variation than the Christiano-Fitzgerald trend with a lower bound of 10 years. The performance in real-time of both
indicators are quite similar. The indicator based on the bandpass filter provides however a more
clearcut interpretation. This is due to the filter methodology used which optimally filters out cyclical
variation within a prespecified frequency band, while the Hodrick-Prescott filter is being criticised for
filtering a cycle out of a white noise time series.
REFERENCES


Bry, G. and Boschan, C., 1971, ‘Cyclical analysis of time series: selected procedures and computer programs’, New York, NBER.


APPENDIX

A technical note on the Christiano-Fitzgerald filter

In this appendix a more technical description of the Christiano-Fitzgerald filter is provided with special attention to the differences with the Baxter-King filter. In addition we will investigate the following claim made in the paper of Agresti & Mojon (2001): ‘Christiano and Fitzgerald propose to optimise the weights of the band pass filter with respect to the mean square error of filtered series relative to the ´ideal´ band pass filter of the time series considered. One major drawback of their approach is that the ´ideal´ band pass filtered series is neither feasible nor observable. It is then necessary to make an assumption about the again unobserved data generating process of the time series to be filtered.’

We will show that although the ´ideal´ band pass filter is not feasible, it is observable and moreover, it is used to construct a feasible approximation. Both Christiano-Fitzgerald and Baxter-King are approximations to the ´ideal´ band pass filter, so even if the criticism of feasibility and observability applies to the Christiano-Fitzgerald filter, then it would also apply to the Baxter-King filter. Secondly, calculating approximations to the ideal band pass filter not necessarily implies knowing or estimating the data generating process. As will be shown, this is in fact the case for both filters.

The Spectral Representation Theorem, Priestley (1981), states that any time series within a broad class can be decomposed into different frequency components. Moreover, the parts of a time series belonging to disjunct frequency intervals are mutually orthogonal. The tool for extracting those components is the ideal band pass filter. It is a linear transformation of the data, which leaves intact the components of the data within a specified band of frequencies and eliminates all other components. A band of frequencies in a time series is composed of a continuum of objects, and in general there is no way that a discrete set of observations can pin these down. Literally, application of the ideal band pass filter requires an infinite amount of data. So, an approximation is needed and this exactly makes Baxter-King deviate from Christiano-Fitzgerald.

Let $y_t$ denote the data generated by applying an ideal, though infeasible, band pass filter to the raw data $x_t$. The result is an ideal orthogonal decomposition of the stochastic process, $x_t$:

$$x_t = y_t + \tilde{x}_t.$$
The process, \( y_t \), consists only of components with period of oscillation between \( p_l \) and \( p_u \), where 2 \( \leq p_l < p_u < \infty \) and the process \( \bar{x}_t \) only of the complementary components. It is a well known result \(^8\) that the ideal band pass filter, \( B(L) \), has the following structure:

\[
y_t = B(L)x_t. \tag{A.1}
\]

\[
B(L) = \sum_{j=-\infty}^{\infty} B_j L^j, \quad L^j(x_t) = x_{t-j} \tag{A.2}
\]

\[
B_j = \frac{\sin(jb) - \sin(ja)}{\pi j}, j \geq 1 \tag{A.3}
\]

\[
B_0 = \frac{b-a}{\pi}, a = \frac{2\pi}{p_u}, b = \frac{2\pi}{p_l}.
\]

With this specification of the \( B_j \)'s we have:

\[
B(e^{-i\omega}) = \begin{cases} 
1 & \text{for } \omega \in (a,b) \\
0 & \text{otherwise}
\end{cases}
\]

Together with (A.3) the assumption \( p_u < \infty \) implies that \( B(1)=0 \). Note from (A.2) that calculating the ideal band pass filter requires an infinite amount of observations on \( x_t \). As will be shown, only truncating the \( B_j \)'s will not produce optimal results.

The \( \hat{y}_t \)'s are a linear projection of \( y_t \) onto every element in the sample set, \( x_t \), and there is a different projection problem for each date \( t \). The filter weights are selected by minimizing:

\[
\min_{\mathcal{B}} \mathbb{E}[\left( y_t - \hat{y}_t \right)^2 | x = [x_1, ..., x_T]] \tag{A.4}
\]

so that we get:

\[
\hat{y}_t = \sum_{j=-f}^{p} B_{j}^{p,f} x_{t-j}, \quad t=1...T,
\]

where \( f=T-t \) and \( p=t-1 \).

\(^8\) See Priestley (1981) page 275
With help of standard results this optimisation problem can be reformulated in the frequency domain by:

\[
\min_{\theta_1^{(j)} \ldots \theta_p^{(j)}} \int_{-\pi}^{\pi} f_{i}(\omega) \left[ B(\omega) - \hat{B}^{p,j}(\omega) \right]^2 d\omega,
\]

where

\[
\hat{B}^{p,j}(L) = \sum_{j=-f}^{p} B^{p,j}_j L^j,
\]

and \( f_{i}(\omega) \) is the spectral density of \( x_i \).

Equation (A.5) shows that the approximate filter can be made close to the optimal filter over some subintervals at the cost of sacrificing accuracy over other subintervals. The weighting scheme characterising the relative importance of the subintervals is the spectral density. This function describes the time series properties of the data in terms of the relative importance of the frequencies. So, the formulation (A.5) indicates that the solution to the optimisation problem depends on properties of the data being filtered.

The Christiano-Fitzgerald paper analyses a range of time series specifications for a couple of main macroeconomic time series. These series are inflation, GDP, industrial production, the short term interest rate and the unemployment rate on a monthly, quarterly and yearly basis. They use statistics measuring the efficiency of the filter in terms of (A.5), the degree of non-stationarity by the relative variance of the ideal and the approximate filter and, finally, they measure the possible creation of a phase shift between the ideal and the approximate filter. Christiano and Fitzgerald conclude that noticeable efficiency gains are obtained by filters that use all the data and this comes at little cost in terms of nonstationarity and phase shift. Moreover, the approximation under the (most likely false) assumption that the data are generated by a pure random walk performs only slightly worse than the optimal approximation using an estimated time series model. Therefore, a more straightforward approach is to use the weighting scheme of a random walk model causing the optimization to create
nearly optimal results without requiring knowledge and estimation of the true time series representation of the data. The weighting scheme representing a random walk time series, \( x_t = x_{t-1} + \epsilon_t \), is used in (A.5) and reads as follows:

\[
f_x(\omega) = \frac{1}{2(1 - \cos(\omega))}
\]  

(A.6)

Isolating the component of \( x_t \) with period of oscillation between \( p_l \) and \( p_u \), where \( 2 \leq p_l < p_u < \infty \), the approximation of \( y_t, \hat{y}_t \) is computed as the projection of (A.5) using (A.6) and results in the following filter:

\[
\hat{y}_t = B_0x_t + B_1x_{t+1} + ... + B_{T-1}x_{T-1} + \tilde{B}_{T-1}x_T + B_1x_{t-1} + ... + B_{T-2}x_2 + \tilde{B}_{t-1}x_1 \tag{A.7}
\]

for \( t = 3,4,...,T-2 \) and where the \( B_i \)'s follow from (A.3)

Moreover:

\[
\tilde{B}_{T-1} = -\frac{1}{2}B_0 - \sum_{j=1}^{T-1-j} B_j, \text{ for } t = 3,...,T-2.
\]

Also, \( \tilde{B}_{T-1} \) solves \( 0 = B_0 + B_1 + ... + B_{T-1} + \tilde{B}_{T-1} + B_1 + ... + B_{T-2} + \tilde{B} \)

In this terminology, the Baxter-King filter can be described as the solution to (A.5) given the following time series representation \(^{10}\) and accompanying spectral density function:

\[
(1 - L)x_t = (1 - (1 - \eta)L)\epsilon_t, \quad \eta > 0, \eta \text{ small}
\]

\[
\lim_{\omega \to 0} f_x(\omega) = \infty \tag{A.8}
\]

Adding the additional restriction that \( p_f = c \), where the constant \( c < T \), where \( T \) the number of observations. Then the optimisation problem (A.5) using (A.8) and the \( B_i \)'s in (A.3) results in:

\[^9\text{ The approximate filter rewritten in polar form reads as: } \hat{\theta}(\omega) = \theta(\omega, p_f) \text{ in the case there is no phase shift present.} \]

\[^{10}\text{ However, according to Christiano & Fitzgerald the time series representation (A.6) does not suit many macroeconomic variables well. As noted by Granger (1966), the 'typical spectral shape' of macroeconomic time series is one in which there is substantial power in a significant range of low frequencies. In (A.6) the spectrum has only substantial power within a small region around } \omega = 0. \]
\[
\hat{B}_{j}^{p,f} = B_{j} + \frac{\Delta}{1 + 2p}, \quad j = 0, \pm 1, \ldots, \pm p
\]  

where

\[
\Delta = \left[ B_{0} + 2 \sum_{j=1}^{p} B_{j} \right]
\]

In the terminology of Baxter & King, a different optimisation procedure providing the same result is performed. They actually require, apart from \( p = f \), that the approximating filter optimises (A.5) with \( f(x) = 1 \) for all \( x \), subject to the requirement, \( \hat{B}(e^{i\omega}) = \hat{B}(1) = 0 \). This latter condition is a necessary condition for (A.5) to become finite. The filter incorporating this condition renders a time series with a unit root stationary.

So the Baxter-King filter (A.8) truncates the ideal band pass filter and then adjust the weights by a constant to ensure \( \hat{B}_{j}^{p,p}(1) = 0 \). Optimality in case of the Christiano-Fitzgerald filter (A.7) dictates truncating the ideal band pass filter and then only adjusting the highest order terms to ensure \( \hat{B}_{j}^{p,f}(1) = 0 \). Since the filter uses all available data, the truncation is dictated by data availability.

Consequently, since the \( \hat{y}_{t} \)'s are based on variable lag filters, they are non-trivial functions of time inducing potentially non-stationarity and phase-shifts. Using all data improves efficiency at a cost of potentially non-stationarity. Christiano & Fitzgerald conclude however, that noticeable efficiency gains comes at little costs.
Figure A1  International Business Cycle Indicators and Composite Leading Indicators

Netherlands

![Graph showing business cycle indicator and composite leading indicator for Netherlands from 1974 to 2002.]

Germany

![Graph showing business cycle indicator and composite leading indicator for Germany from 1974 to 2002.]

Figure A1 Continued

Belgium

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Spain

---

France

---
Figure A1 Continued

Italy

United Kingdom

United States
Figure A1  Continued

Japan

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- Business Cycle Indicator
- Composite Leading Indicator

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Figure A2  Business Cycle Indicator and leading Indicators for the Netherlands

Industrial production (log)

Consumption price index (log)

Ifo future expectations

Expected business activity
Figure A2 Continued

Short term interest rate

Long term interest rate

Storage final products

Hourly wage (log)

raw series

trend

business cycle component

raw series

trend

business cycle component

raw series

trend

business cycle component