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\* Views expressed are those of the author and do not necessarily reflect official positions of De Nederlandsche Bank.

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# Are banks too big to fail?

## Measuring systemic importance of financial institutions

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**Abstract** We consider three measures on the systemic importance of a financial institution within a interconnected financial system. Based on the measures, we study the relation between the size of a financial institution and its systemic importance. From both theoretical model and empirical analysis, we find that in analyzing the systemic risk posed by one financial institution to the system, size should not be considered as a proxy of systemic importance. In other words, the "too big to fail" argument is not always valid, and alternative measures on systemic importance should be considered. We provide the estimation methodology of systemic importance measures under the multivariate Extreme Value Theory (EVT) framework.

**Keywords:** Too big to fail; systemic risk; systemic importance; multivariate extreme value theory.

**JEL Classification Numbers:** G21; C14.

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# 1 Introduction

During financial crises, authorities have an incentive to prevent the failure of a financial institution because such a failure would pose a significant risk to the financial system and consequently the broader economy. A bailout is usually supported by the argument that a financial firm is "too big to fail", i.e. larger banks exhibit higher systemic importance. A natural question arising from the debate on bailing out large financial firms is why particular large banks should be favored by authorities, i.e. are banks really too big to fail? The question can be rephrased as "does the size of a bank really matter for its systemic impact if it fails?" The major difficulty in answering such a question is to design measures on the systemic importance of financial institutions. More specifically, we need to measure how much the failure of a particular bank will "contribute" to the systemic risk.

This paper deals with the problem in four steps. Firstly, we discuss the potential drawbacks of existing measures of systemic importance and propose a new one that overcomes these drawbacks. Secondly, we construct a theoretical model to assess whether larger banks correspond to higher systemic importance. Thirdly, we employ statistical method in estimating such measures within a specific system consisting 27 US banks. Lastly, we use the estimated systemic importance measures and the size measures to empirically test the "too big to fail" statement.

Although the term "too big to fail" appears frequently in supporting bailout activities, its downside is well acknowledged in literature. Besides the distortion of the market discipline, the preference on large financial firms encourages excessive risk-taking behavior which potentially imposes more risk. Therefore, using "too big to fail" to support intervention will result in a moral hazard problem: large firms that the government is compelled to support were the greatest risk-takers during the boom period. Furthermore, such a moral hazard problem will provide incentive for firms to grow in order to be perceived as "too big to fail". We refer to Stern and Feldman (2004) for more discussions on the moral hazard problem.

Recently, both policy makers and academia start to distinguish size from the systemic

importance by introducing new terms emphasizing on the potential systemic impact if a particular bank fails. For example, Bernanke (2009) addressed the problem of financial institutions that are deemed "too interconnected to fail"; Rajan (2009) used the term "too systemic to fail" to set the central focus of new regulation development. This urges the design of alternative measures on systemic importance. Measuring the systemic importance of financial institutions is particularly important for policy maker. It is the key issue in both financial stability assessment and macro-prudential supervision. On one hand, during crises, it is necessary to have such measures in order to justify bailout actions. On the other hand, it is crucial to supervise and monitor banks with higher systemic importance during regular period. Policy proposals for stabilizing financial system always rely on such measures. For instance, liquidity insurance is an alternative for defending systemic risk proposed by, e.g. Acharya et al. (2009) and Perotti and Suarez (2009). Systemic importance measures can serve as an indicator for pricing the corresponding insurance premium or taxation.

A few applicable measures on systemic importance appear in recent empirical studies. Adrian and Brunnermeier (2008) proposed the conditional Value-at-Risk (CoVaR) measure to measure risk spillover. Similar to the Value-at-Risk measure which quantifies the unconditional tail risk of a financial institution, the CoVaR quantifies how financial stress of one institution can increase the tail risk of the others. This measure provides a clear view on the bilateral relation between the tail risks of two financial institutions. From the setup of the CoVaR measure, it is designed for bilateral risk spillover. To assess the systemic importance of one financial institution to the system, it is necessary to construct a system indicator on the status of the system. Due to the complexity of the financial system, a general indicator of the system is usually difficult to construct. When treating the system as a whole, the CoVaR measure is difficult to be generalized into such a systemic context. As an alternative, Segoviano and Goodhart (2009) introduced the "probability that at least one bank becomes distressed" (PAO). Comparing these two measures, we observe that the CoVaR is a measure of conditional quantile, while PAO is a measure of conditional probability which acts as the

counterpart of conditional quantile. Within a probability setup, generalization from bivariate to multivariate is possible. However, the PAO measure focuses on the probability of having a systemic impact—there exists at least one extra crisis—without specifying how severe the systemic impact is, for example, how many banks are influenced, when a particular bank fails. Therefore, it may not provide sufficient information on the systemic importance of a financial institution. Our empirical results partially reflect the less informative feature: the PAO measure remains in a constant level across different financial institutions.

Extending the PAO measure while staying with the multivariate context, we propose the systemic importance index (SII) which measures the expected number of bank failures in the banking system given one particular bank fails. Clearly, the SII measure emphasizes more on the systemic impact. We also consider a reversed measure: the probability of a particular bank failure given that there exists at least one another failure in the system. We call it the vulnerability index (VI).

To test "too big to fail", we first consider a theoretical model from which both size and systemic importance measures—PAO, SII, VI, can be explicitly calculated. The model stems from the literature on crises contagion. In literature, two categories of models consider crises contagion and systemic risk in banking system: banks are systemically linked via either direct channels such as interbank market or indirect channels such as similar portfolio holdings in bank balance-sheets. For the first category on modeling interbank market, we refer to Allen and Gale (2000), Freixas et al. (2000), and Dasgupta (2004). Cifuentes et al. (2005) consider both two channels: similar portfolio holdings and mutual credit exposure. They show that contagion is mainly driven by changes in asset prices, i.e. the indirect channel. For models focusing on the indirect channel, Lagunoff and Schreft (2001) assume that the return of one bank's portfolio depends on other banks' portfolio allocation and show that crises can either spread from one to another, or happen simultaneously due to forward-looking. de Vries (2005) starts from the fat tail property of the underlying assets shared by banks and argues that this creates the potential systemic breakdown. For an overview of

systemic risk modeling, we refer to de Bandt and Hartmann (2001) and Allen et al. (2009).

Notice that the contagion literature on systemic risk mainly focuses on explaining the existence of a contagion effect, i.e. how a crisis in one financial institution leads to a crisis in the other. Thus, the models usually consider the risk spillover between only two banks. To address the financial system as a complex, network models combined with bilateral spillover are considered, e.g. Allen and Gale (2000). That explains why empirical analysis on systemic importance such as the CoVaR measure only focus on measuring bilateral relations.

We address the systemic importance issue within a systemic context by considering a multi-bank approach. The fundamental intuition is that banks are interconnected due to the common exposures on their bank balance sheets. Thus, the systemic importance of a particularly bank is closely associated with how many risky banking activities the bank is participating. This, in turn, may not be directly associated with the bank's size. Our model shows that banks concentrating on few specific activities can grow large without increasing their systemic importance.

This finding links the "too big to fail" problem to policy discussions on micro-level risk management and macro-level banking supervision. Since diversification is the usual tool in micro-level risk control. Financial institutions, particularly the large one, intend to take part in more banking activities in order to diversify away their individual risk. This may, on the other hand, increase its systemic importance. It is important to acknowledge the tradeoff between managing individual risk and keeping independency from the entire banking system. Reduction in individual risk can transfer the risk to systemic linkage and thus the systemic risk. Therefore, regulations on individual risk taking, such as Basel II, is not sufficient for maintaining stability of financial system. A macro-prudential supervision is thus necessary for financial stability. In such a supervision scheme, it is necessary to consider proper systemic importance measures.

Nextly, we demonstrate how to empirically estimate our systemic importance measures. We adopt the multivariate Extreme Value Theory (EVT) framework for empirical estimation.

When investigating crises, or rare events, the major difficulty is the scarcity of observations. Since we intend to address the interconnection of banking crises, the difficulty is further enhanced. A modern statistical instrument—EVT, fills the gap. The essential idea of EVT is to model the the intermediate level observations which are close to extreme, and extrapolate the observed properties into an extreme level. Univariate EVT has been applied in Value-at-Risk assessment for individual risks, see e.g. Embrechts et al. (1997). Recent development on multivariate EVT provides the opportunity to investigate extreme co-movements which serves our purpose. For instance, it has been applied to measure risk contagions across different financial markets in Poon et al. (2004) and Hartmann et al. (2004). Although most applications of multivariate EVT in existing empirical literature are limited to measure bivariate risk spillover, the Global Financial Stability Report by IMF in April, 2009 (IMF (2009)) demonstrates the interconnection of financial distresses within a system consisting of three banks. In our empirical estimation of the systemic importance measures, we use multivariate EVT without restricting the number of banks under consideration.

We provide an empirical methodology on estimating the systemic importance measures—PAO, SII, and VI—under multivariate EVT framework. An exercise for a specific system formed by 27 US banks is conducted. Then we test the correlation between the systemic importance measures and different measures on size. We find that all systemic importance measures are not correlated with all bank size measures. Hence, the size of the bank does not reflect its systemic importance. This agrees with our theoretical model. Overall, we conclude that size should not be considered as a proxy of systemic importance while alternative measures should take the position.

## 2 Systemic importance measures

We consider a banking system containing  $d$  banks with their status indicated by  $(X_1, \dots, X_d)$ : an extreme high value of  $X_i$  indicates a distress or crisis in bank  $i$ . Potential



candidate for such a indicator can be the loss of equity returns, loss returns on balance sheet or Credit Default Swap (CDS) rates.

To define crisis, it is necessary to consider proper high threshold. In our approach, we take Value-at-Risk as such a threshold. VaR at a tail probability level  $p$  is defined by

$$P(X > VaR(p)) = p.$$

Regulators consider the  $p$ -level as 1% or 0.1% in order to evaluate risk-taking behavior of a particular bank. We call a bank in crisis if  $X > VaR(p)$  with a extremely low  $p$ . Here we do not specify the level  $p$  explicitly. Instead, we restrict that the  $p$ -level in the definition of banking crises is constant across all banks. Notice that banks may differ in their risk profiles which results in different durability on risk-taking, i.e. different  $VaR(p)$  level. Thus a unified level of loss for crises definition may not fit the diversified situation of different financial institutions. Instead, an extreme event  $X > VaR(p)$  corresponds to a return frequency as  $1/p$ . Fixing such a frequency for crises definition takes the diversity of bank risk profiles into consideration. Furthermore, such a definition is aligned with the usual crisis description, for examples, with yearly data, a  $p$  equals to  $1/50$  corresponds to "a crisis once per fifty year".

The systemic importance measures consider the impact on other financial institutions when a particular one falls into crisis. We start from the measure proposed by Segoviano and Goodhart (2009): the conditional probability of having at least one extra bank failure given a particular bank fails (PAO). From our model, it considers the following probability

$$PAO_i(p) := P(\{\exists j \neq i, \text{ s.t. } X_j > VaR_j(p)\} | X_i > VaR_i(p)). \quad (1)$$

We argue that the PAO measure may not provide sufficient information in identifying the systemically important banks by considering the following example. Suppose we have a banking system with banks categorized into two separate groups. Banks within each group

are strongly linked, while crises do not spillover between the two groups. One group contains only two banks  $X_1$  and  $X_2$ , while the other segment contains more banks  $X_3, \dots, X_d$ ,  $d > 4$ . In other words,  $X_1$  and  $X_2$  are highly related,  $X_3, \dots, X_d$  are highly related, while  $X_i$  and  $X_j$  are independent for any  $1 \leq i \leq 2$  and  $3 \leq j \leq d$ .

Then the PAO measure for  $X_1$ ,  $PAO_1$  will be close to 1 since a crisis of  $X_1$  will be accompanied by a crisis of  $X_2$ . On the other hand, the PAO measure for  $X_3$ ,  $PAO_3$  will also be close to 1 because of similar reasoning. When  $d$  is high, for example,  $d = 10$ , it is clear that  $X_3$  is more systemically important than  $X_1$  because it is associated with a larger fraction of the entire banking system. However, this will not be reflected by the comparison between  $PAO_1$  and  $PAO_3$ . In this example,  $PAO_1$  and  $PAO_3$  should be at a high, comparable level.

Generally speaking, the PAO measure only provides the probability of having a systemic impact—having an extra crisis in other financial institutions, without specifying the size of such an impact—the number of extra crises in the entire system, when one particular bank fails. Hence, in the case that every institution in the system is connected to a certain fraction of the system, their PAO measures should all stay at a high, comparable level. With indistinguishable PAO measures, it is not sufficient to identify the systemically important financial institutions.

A natural extension of the PAO measure is to consider the expected number of failures in the system given a particular bank fails. This is defined as our systemic impact index (SII). Using the notation above, it can be written as

$$SII_i(p) := E\left(\sum_{j=1}^d 1_{X_j > VaR_j(p)} \mid X_i > VaR_i(p)\right), \quad (2)$$

where  $1_A$  is the indicator function which equals to 1 when  $A$  holds, 0 otherwise.

Since the PAO and SII measures characterize the outlook of the financial system when a particular bank fails, a reverse question is what is the probability of a particular bank fails when the system exhibits some distress. To characterize the system distress, we use the same

term as in the PAO measure: there exists at least one extra bank failure. Hence, we define a vulnerability index (VI) by swapping the two items in the PAO definition as follows.

$$VI_i(p) := P(X_i > VaR_i(p) | \{\exists j \neq i, \text{ s.t. } X_j > VaR_j(p)\}). \quad (3)$$

From the definitions, all three measures summarize specific information on the risk spillover in the banking system. It is necessary to consider all of them when assessing the systemic importance of financial institutions.

### 3 Extreme Value Theory and systemic importance measures

#### 3.1 The setup of Extreme Value Theory

Consider our  $d$ -bank setup. Modeling crisis of a particular financial institution  $i$  corresponds to modeling the tail distribution of  $X_i$ . Moreover, modeling the systemic risk, i.e. the extreme co-movements among  $(X_1, \dots, X_d)$ , corresponds to modeling the tail dependence structure of  $(X_1, \dots, X_d)$ . Extreme Value Theory provides models for such a purpose.

To assess VaR with a low probability level  $p$ , univariate EVT was applied in modeling the tail behavior of the loss. Since we focus on systemic risk, we omit the details on univariate risk modeling (see e.g. Embrechts et al. (1997)). Multivariate EVT models considers not only the tail behavior of individual  $X_i$ , but also the extreme co-movements among them.

The fundamental setup of multivariate EVT is as follows. For any  $x_1, x_2, \dots, x_d > 0$ , as  $p \rightarrow 0$ , we assume that

$$\frac{P(X_1 > VaR_1(x_1 p) \text{ or } \dots \text{ or } X_d > VaR_d(x_d p))}{p} \rightarrow L(x_1, x_2, \dots, x_d). \quad (4)$$

where  $VaR_i$  denotes the Value-at-Risk of  $X_i$ , and  $L$  is a finite positive function.<sup>1</sup> The  $L$  function characterizes the co-movement of extreme events, i.e.  $X_i$  exceeds a high threshold  $VaR_i(x_i p)$ .  $(x_1, \dots, x_d)$  controls the level of high threshold, which in turn controls the direction of extreme co-movement. For the property on  $L$  function, see de Haan and Ferreira (2006).

The value of  $L$  at a specific point,  $L(1, 1, \dots, 1)$ , is a measure on the systemic linkage of banking crises among the  $d$  banks. From the definition in (4), we have that

$$L(1, 1, \dots, 1) = \lim_{p \rightarrow 0} \frac{P(X_1 > VaR_1(p) \text{ or } \dots \text{ or } X_d > VaR_d(p))}{p}. \quad (5)$$

In our banking system context, when  $p$  is at a low level, it approximates the quotient ratio between the probability that there exists at least one bank in crises and the tail probability  $p$  used in the definition of individual crisis. For bivariate case, this was considered by Hartmann et al. (2004) in measuring risk spillover.

We remark that  $L$  function is connected with the modern instrument on dependence modeling—copula. Denote the joint distribution function of  $(X_1, \dots, X_d)$  as  $F(x_1, \dots, x_d)$  while the marginal distributions are denoted as  $F_i(x_i)$  for  $i = 1, \dots, d$ . Then there exists a unique distribution function  $C(x_1, \dots, x_d)$  on  $[0, 1]^d$  such that

$$F(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d)),$$

where all marginal distributions of  $C$  are standard uniform distributions.  $C$  is called the *copula*. By decomposing  $F$  into marginal distributions and copula, we separate the marginal information from the dependence structure summarized in the copular  $C$ . Condition (4) is

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<sup>1</sup>Notice that considering the union of the events, i.e. using "or" in (4) is simply due to the definition of distribution function. Define  $F(x_1, \dots, x_d) = P(X_1 \leq x_1, \dots, X_d \leq x_d)$  as the distribution function of  $(X_1, \dots, X_d)$ . In order to consider the tail property, the assumption is made on the tail part  $1 - F$ , which is the probability of the union of extremal events as in relation (4).

equivalent to the following relation. For any  $x_1, x_2, \dots, x_d > 0$ , as  $p \rightarrow 0$ ,

$$\frac{1 - C(1 - px_1, \dots, 1 - px_d)}{p} \rightarrow L(x_1, x_2, \dots, x_d)$$

Hence  $L$  function characterize the limit behavior of the copula  $C$  at the corner point  $(1, \dots, 1) \in [0, 1]^d$ . In other words,  $L$  function captures the tail behavior of the copula  $C$ .

Linking  $L$  function to the tail behavior of copula yields the two following views. Firstly, since it is connected to copula,  $L$  function does not contain any marginal information. Thus in modeling banking crises,  $L$  function is irrelevant to the individual bank risk profiles. Secondly, with characterizing the tail behavior of copula,  $L$  function does not contain dependence information at a moderate level as in the copula  $C$ . Instead,  $L$  only contains tail dependence information. To summarize,  $L$  function contains the minimal required information in modeling extreme co-movements. Therefore, models on  $L$  is flexible to accommodate all potential marginal risk profiles and potential moderate level dependence structures. Compared to Segoviano and Goodhart (2009) which consider the CIMDO approach on modeling the copula  $C$ , we argue that models on the copula  $C$  consider the interconnection of banking system in regular time. Thus, estimating such a copula model may miss-specify the tail dependence structure. Since our interest concentrates on modeling the interconnection of banking crises, considering the  $L$  function only is sufficient and less restrictive.

### 3.2 Systemic importance measures under multivariate EVT

Under multivariate EVT setup, all three systemic importance measures—PAO, SII, VI, can be directly calculated from the  $L$  function. Notice that in the definitions of the systemic importance measures, the probability level  $p$  for defining crisis is considered. However, we prove that, as  $p \rightarrow 0$ , the systemic importance measures can be well approximated by their limits.

The following proposition shows the limit of the PAO measure. The proof is in Appendix A.

**Proposition 1** *Suppose  $(X_1, X_2, \dots, X_d)$  follows the multivariate EVT setup. With the definition of PAO in (1), we have that*

$$PAO_i := \lim_{p \rightarrow 0} PAO_i(p) = L_{\neq i}(1, 1, \dots, 1) + 1 - L(1, 1, \dots, 1), \quad (6)$$

where  $L$  is the  $L$  function characterizing the tail dependence of  $(X_1, \dots, X_d)$  and  $L_{\neq i}(1, 1, \dots, 1)$  is the  $L$  function characterizing the tail dependence of  $(X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_d)$ .

Notice that  $L$  is defined on  $\mathbb{R}^d$  while  $L_{\neq i}$  is defined on  $\mathbb{R}^{d-1}$ . Moreover,

$$L_{\neq i}(1, 1, \dots, 1) = L(1, 1, \dots, 1, 0, 1, \dots, 1),$$

where 0 appears at the  $i$ -th dimension.

Proposition 1 shows that when considering a low level  $p$ , the measure  $PAO_i(p)$  is close to its limit denoted by  $PAO_i$ . For calculating  $PAO_i$ , it is sufficient if the  $L$  function is known. Therefore, we could have a proxy of the PAO measure with low level  $p$  by estimating the  $L$  function. In a theoretical model, the  $L$  function may have an explicit formula. For an empirical analysis,  $L$  function can be estimated from historical data. We provide a practical guide for estimating the  $L$  function in Appendix B. For more discussions, see de Haan and Ferreira (2006).

Analogous to that of  $PAO$ , the limit of  $VI(p)$  exists under multivariate EVT setup. We present the result in the following proposition but omit the proof.

**Proposition 2** *Suppose  $(X_1, X_2, \dots, X_d)$  follows the multivariate-EVT setup. With the def-*

inition of  $VI$  in (3), we have that

$$VI_i := \lim_{p \rightarrow 0} VI_i(p) = \frac{L_{\neq i}(1, 1, \dots, 1) + 1 - L(1, 1, \dots, 1)}{L_{\neq i}(1, 1, \dots, 1)}, \quad (7)$$

with the same notation defined in Proposition 1.

From Proposition 1 and 2, we get the following corollary.

**Corollary 3**  $PAO_i > PAO_j$  holds if and only if  $VI_i > VI_j$ .

Corollary 3 implies that when considering the ranking instead of the absolute level, the  $VI$  measure is in fact as informative as the PAO measure.

The following proposition shows how to calculate the limit of SII under multivariate EVT setup. The proof is again postponed to Appendix A.

**Proposition 4** Suppose  $(X_1, X_2, \dots, X_d)$  follows the multivariate-EVT setup. With the definition of SII in (2), we have that

$$SII_i := \lim_{p \rightarrow 0} SII_i(p) = \sum_{j=1}^d (2 - L_{i,j}(1, 1)), \quad (8)$$

where  $L_{i,j}$  is the  $L$  function characterizing the tail dependence of  $(X_i, X_j)$ .

Notice that

$$L_{i,j}(1, 1) = L(0, \dots, 0, 1, 0, \dots, 0, 1, 0, \dots, 0),$$

where 1 appears only at the  $i$ -th and  $j$ -th dimensions. Again, Proposition 4 shows that  $SII_i$  is a good approximation of  $SII_i(p)$  when  $p$  is at a low level. And the estimation of  $SII_i$  is only based on the estimation of the  $L$  function. From the calculation of PAO and SII, they are providing different information. A ranking based on PAO does not necessarily imply the same ranking on SII. Thus, it is still necessary to look at both two measures in order to obtain a full picture on the systemic importance of a bank.

To summarize, the multivariate EVT setup provides the opportunity to evaluate all three systemic importance measures –PAO, SII, and VI when the  $L$  function is known. Since the  $L$  function characterizes the tail dependence structure in  $(X_1, \dots, X_d)$ , all the systemic importance measures can be viewed as characterization of the tail dependence among banking crises.

## 4 Are banks "too big to fail"? A theoretical model

We construct a simple model showing that large banks might have a lower level of systemic importance compared to small banks, i.e. banks are not necessarily too big to fail.

We start by reviewing a simple model in de Vries (2005) which explains the contagion effect within a two-bank system.

Consider two banks  $(X_1, X_2)$  holding exposures on two independent projects  $(Y_1, Y_2)$  as in the following affine portfolio model.

$$\begin{cases} X_1 = (1 - \gamma)Y_1 + \gamma Y_2 \\ X_2 = \gamma Y_1 + (1 - \gamma)Y_2, \end{cases} \quad (9)$$

where  $0 < \gamma < 1$ ,  $(Y_1, Y_2)$  indicates the loss returns of the two projects, for example, two syndicated loans. To measure the contagion effect, de Vries (2005) considers the following measure,

$$\lim_{s \rightarrow \infty} E(\kappa | \kappa \geq 1) := \lim_{s \rightarrow \infty} \frac{P(X_1 > s) + P(X_2 > s)}{P(X_1 > s \text{ or } X_2 > s)}. \quad (10)$$

Intuitively,  $E(\kappa | \kappa \geq 1)$  is the expected number of bank crises in the two-bank system given that at least one bank is in crisis. Here, the crisis of  $X_i$  is defined as  $X_i > s$ . It is proved that when  $Y_i$ ,  $i = 1, 2$ , are normally distributed,  $\lim_{s \rightarrow \infty} E(\kappa | \kappa \geq 1) = 1$ . Thus, given that there exists at least one bank in crisis, the expected total number of crises is 1, i.e. there is no extra crisis except the existing one. This is called a weak fragility case. Hence, the contagion effect does not exist. On the contrary, suppose  $Y_i$ ,  $i = 1, 2$  follow a heavy-tailed



distribution on the right tail. The result differs. The heavy-tail distribution is defined as

$$\begin{cases} P(Y_i > s) = s^{-\alpha}K(s), & i = 1, 2, \\ P(Y_i < -s) = o(P(Y_i > s)), \end{cases} \quad (11)$$

where  $\alpha > 0$  is called the *tail index* and  $K(s)$  is a *slowly varying function* satisfying that

$$\lim_{t \rightarrow \infty} \frac{K(ts)}{K(s)} = 1,$$

for all  $s > 0$ . de Vries (2005) proved that for  $\gamma \in [1/2, 1]$ ,

$$\lim_{s \rightarrow \infty} E(\kappa | \kappa \geq 1) = 1 + (1/\gamma - 1)^\alpha > 1.$$

This is called the strong fragility case because one existing crisis will be accompanied by potential extra crises. It has been extensively documented that the losses of asset returns follow heavy-tailed distributions. Therefore, the latter model based on heavy-tailed distributions reflects the empirical observations and explains the contagion effects existing in financial crises.

We remark that when assuming the heavy-tailness of  $(Y_1, Y_2)$ , and the affine portfolio model in (9), it is a direct consequence that  $(X_1, X_2)$  follows a two-dimensional EVT setup.<sup>2</sup> Moreover, if  $Y_1$  and  $Y_2$  are identically distributed, for a fixed tail probability  $p$ , the VaRs of  $X_1$  and  $X_2$  are equal, i.e.  $VaR_1(p) = VaR_2(p)$ . Replacing  $s$  by  $VaR_i(p)$  in the definition of the contagion effect measure (10), and asking  $p \rightarrow 0$ , we get that

$$\lim_{p \rightarrow 0} E(\kappa | \kappa \geq 1) := \lim_{p \rightarrow 0} \frac{P(X_1 > VaR_1(p)) + P(X_2 > VaR_2(p))}{P(X_1 > VaR_1(p) \text{ or } X_2 > VaR_2(p))} = \frac{2}{L(1, 1)}.$$

Therefore, the setup in de Vries (2005) imposes a multivariate-EVT setup, and the measure on contagion effect is essentially based on  $L(1, 1)$ .

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<sup>2</sup>For a formal proof, see Zhou (2008, Chapter 5).

We point out that within such a two-bank model, it is not possible to compare the systemic impact measures between banks. From the model and Proposition 1, we get that

$$SII_i = 3 - L(1, 1), \quad i = 1, 2.$$

Hence the systemic importance of the two banks measured by SII are equal. Similar results hold for the other two measures, PAO and VI. Intuitively, within a two-bank setup, the linkage of crises is a mutual bilateral relation. Hence, one could not distinguish the systemic importance of the two banks. In order to construct a model in which it is possible to compare the systemic importance at different levels, it is necessary to generalize de Vries (2005) model to a system consisting of at least three banks.

Next, we consider the size issue. In de Vries (2005) two-bank model, suppose that both bank  $X_1$  and  $X_2$  have total capital 1. Then according to the affine portfolio model (9), the two projects  $Y_1$  and  $Y_2$  receive both capital 1. The market is clear. In this case the two banks have the same size. In order to differentiate the sizes of the banks, a more complex affine portfolio model is necessary.

Addressing the above two points, we consider a model with three banks ( $X_1, X_2, X_3$ ) and three independent projects ( $Y_1, Y_2, Y_3$ ). Suppose  $X_1$  holds capital 2 for investment, while  $X_2$  and  $X_3$  hold capital 1 each. Moreover, suppose the project  $Y_1$  demands an investment 2, while  $Y_2$  and  $Y_3$  have a capital demand 1 each. Then the market is clear with the following affine portfolio model.

$$\begin{cases} X_1 &= (2 - 2\gamma)Y_1 + \gamma Y_2 + \gamma Y_3 \\ X_2 &= \gamma Y_1 + (1 - \gamma - \mu)Y_2 + \mu Y_3, \\ X_3 &= \gamma Y_1 + \mu Y_2 + (1 - \gamma - \mu)Y_3, \end{cases} \quad (12)$$

where  $0 < \gamma, \mu < 1$  and  $\gamma + \mu < 1$ . Clearly, this is not the only possibility for a clear market. Nevertheless, this setup is sufficient to demonstrate our argument on "too big to

fail” problem. Notice that  $X_1$  is a larger bank compared to  $X_2$  and  $X_3$ . We intend to compare the systemic importance of  $X_1$  with those of  $X_2$  and  $X_3$ .

The two parameters  $\gamma$  and  $\mu$  are interpreted as the control of similarity in portfolio holdings across three banks. The parameter  $\gamma$  controls the similarity between the large bank and the small banks. When  $\gamma$  is close to 1, the strategy of the large bank is different from those of the two small banks, while the two small banks hold similar portfolios. When  $\gamma = 1/2$ , the large bank has exposures on all three projects proportional to their capital demands. Hence the large banks are well involved in all projects. When  $\gamma$  is close to 0, the large bank is again different from the two small banks. In the latest case, the similarity of the two small banks is controlled by the parameter  $\mu$ : a  $\mu$  lying in the middle of  $(0, 1 - \gamma)$  shows that the two small banks are similar in portfolio holding, while a  $\mu$  lying close to the two corners of  $(0, 1 - \gamma)$  corresponds to different strategies between the two small banks.

Suppose all  $Y_i$  follow a heavy-tail distribution defined in (11) for  $i = 1, 2, 3$ . Then similar to the two-bank case,  $(X_1, X_2, X_3)$  follows a three-dimensional EVT setup. Instead of discussing all possible values on the parameters  $(\gamma, \mu)$ , we focus on three cases:  $\gamma$  is close to 1,  $\gamma = 1/2$  and  $\gamma$  is close to 0. The results on comparing the SII measures are in the following theorem. The proof is again in Appendix A.

**Theorem 5** *Consider a three-bank three-project model with the affine portfolio given in (12). Suppose the losses of the three projects exhibit heavy-tail feature as in (11) with  $\alpha > 1$ . We have the following relations.*

**Case I:**  $\frac{2}{3} \leq \gamma < 1$

$$SSI_1 < 1 = SSI_2 = SSI_3.$$

**Case II:**  $\gamma = \frac{1}{2}$

$$SSI_1 \geq SSI_2 = SSI_3.$$

*The equality holds if and only if  $\mu = 1/4$ .*

**Case III:**  $0 < \gamma < \frac{1}{3}$

There exists a  $\mu^* < \frac{1-\gamma}{2}$ , such that for any  $\mu$  satisfying  $\mu^* < \mu < 1 - \gamma - \mu^*$ ,

$$SSI_1 < SSI_2 = SSI_3.$$

On the other hand, for any  $\mu$  satisfying  $0 < \mu < \mu^*$  or  $1 - \gamma - \mu^* < \mu < 1 - \gamma$ , we have

$$SSI_1 > SSI_2 = SSI_3.$$

When  $\mu = \mu^*$  or  $\mu = 1 - \gamma - \mu^*$ ,

$$SII_1 = SII_2 = SII_3.$$

The following lemma shows that the comparison among the PAO measures follows the comparison among the SSI measures in the three-bank model.

**Lemma 6** *With the assumptions in Theorem 5, the order of PAO follows the order of SSI, i.e. for any  $i \neq j$ ,  $PAO_i > PAO_j$  holds if and only if  $SII_i > SII_j$ .*

Combine Lemma 6 and Corollary 3, we have that the order of VI also follows the order of SII in the simple three-bank model. Notice that the three-bank model is a very specific and simple case. The results in Lemma 6 does not hold in a general context when the number of banks is more than three. Therefore, for empirical study within a complex financial system, it is still necessary to estimate both the SII measure and the PAO measure. The three measures may provide different views.

We interpret the results in Theorem 5 as follows.

In case  $\gamma$  is close to 1, the large bank  $X_1$  focuses on the two smaller project  $Y_2$  and  $Y_3$ , while small banks  $X_2$  and  $X_3$  focus on the large project  $Y_1$ . In this case, the balance sheet of the large bank is quite different from the small banks, while the two small banks are holding similar portfolio. Therefore, the large bank has less systemic linkage to the other two small banks. We observed that it is less systemically important compared to the others, i.e. the

large bank is not "too big to fail".

In case  $\gamma = 1/2$ , the large bank  $X_1$  invests  $(1, 1/2, 1/2)$  at three projects. Hence it is involved in all three projects which creates the linkage to the other two small banks. In this case, it is "too big to fail". The inequality turns to be an equation if and only if  $\mu = 1/4$ . For  $\mu = 1/4$ , the three banks all invest in three projects proportional to their capital demands. They have exactly the same strategy in managing their portfolios. A crisis in any of the three banks will be accompanied by the crises in the other two. Therefore, they are equally systemically important. Excluding  $\mu = 1/4$ , the large bank will be the most systemically important bank.

In case  $\gamma$  is close to 0, the large bank  $X_1$  focuses on the large project  $Y_1$ , while still has exposures on  $Y_2$  and  $Y_3$ . The small banks  $X_2, X_3$  focuses on the two small projects  $Y_2$  and  $Y_3$ . Now it matters how similar their portfolios are. In case  $\mu$  is in the middle ( $\mu^* < \mu < 1 - \gamma - \mu^*$ ), the two small banks have relatively similar composition of balance sheet. Hence, they are more systemically important compared to the large bank. In case  $\mu$  is close to the corner ( $0 < \mu < \mu^*$  or  $1 - \gamma - \mu^* < \mu < 1 - \gamma$ ), the two small banks differ in their balance sheet. Since the large bank still has exposures on  $Y_2$  and  $Y_3$  equally, it is the most systemically important bank.

All in all, we observe that "too big" is not necessarily the reason for being "too systemically important". Instead, having a bank balance sheet that expose to more risky projects would turn a bank to be more systemically important. Here, we regard  $Y_i, i = 1, 2, 3$  as different risky projects. One may also regard them as different risky banking activities. Therefore, a bank that is more diversified in banking activities may turn to be "too big to fail".

Notice that having a diversified bank balance sheet is the usual way of managing individual risk. To obtain the diversification effect encourages banks, particularly large banks, to take part in more banking activities. This will result in a "too big to fail" problem. On the contrary, a large bank specified in a limited number of banking activities, might be risky

as an individual, but being less systemically important at the same time. There is a tradeoff between managing individual risk and keeping independency from the entire banking system. For maintaining financial stability, it is necessary to control both individual risk taking and systemic importance.

## 5 Empirical results

### 5.1 Empirical setup and data

We apply the three measures on systemic importance to an artificial banking system constructed from 27 US financial institutions. After estimating the three measures, we calculate the correlation coefficient between these measures and the measures of the size of the banks. From the test on correlation coefficients, we can empirically test whether larger banks exhibit larger systemic importance, i.e. testing the "too big to fail" argument.

The dataset for conducting the systemic importance measures consists of daily equity returns of 27 US banks listed on New York Stock Exchange from 1987 to 2008 (22 years). The chosen banks are listed in Table 1 with the descriptive statistics on their stock returns. From the descriptive statistics, we observe that all daily returns except TOMPKINS FINANCIAL exhibit high kurtosis compared to the kurtosis from normal distribution, 3. It indicate that the daily stock returns may follow heavy-tailed distributions. Moreover, most of the skewness measures are negative or close to 0. This indicates that the heavy-tailness is on the left side of the tail, i.e. the losses. Hence, our heavy-tail assumption on the tail of losses are valid for the employed dataset.

We remark that using the stock returns is only one option in indicating distress or crisis of a bank. Other indicators such as Credit Default Swap (CDS) rates are also possible. The only restriction imposed by the methodology of estimating the  $L$  function is the sample size, see Appendix B. Daily or higher frequency is necessary for a full non-parametric approach. This limits us to using financial market data. To apply the methodology with low frequency data,

such as bank balance sheet data, further modeling on the  $L$  function should be considered. Since we intend to illustrate the methodology without modeling the  $L$  function, we stick to the equity return data.

The dataset regarding the size of the banks consists of various measures. We consider total assets, total equity, total debt, equity asset ratio, debt equity ratio for the 27 banks. They are in a yearly frequency.

## 5.2 Results

By estimating the  $L$  function, we obtain the estimates of three systemic importance measures—SII, PAO, and VI across the full sample period as shown in Table 2. We start with the PAO measure proposed by Segoviano and Goodhart (2009). Except TOMPKINS FINANCIAL which corresponds to a PAO at 60% level, all the other banks have a PAO around 80% level. This agrees with our prediction that the PAO measures of all banks in a system lie in a relatively high level, and do not differ much from each other. Since the PAO measure is closely connected to the VI measure as shown in Corollary 3, similar feature is observed for the VI measures. In fact, the order of the VI measures follow the order of the PAO measures as shown in Corollary 3. To name a few with the highest PAO and VI measures, FIRST HORIZON NATIONAL reports the highest PAO at 86.4% followed by REGIONS FINL. NEW with 85.4% and IRWIN FINL. with 85.0%. The VI measure corresponding to these three institutions are 7.37%, 7.29% and 7.26% respectively.

The SII measure introduced in this paper gives a somewhat different outlook compared to the PAO measure. TOMPKINS FINANCIAL still reports the lowest value 2.57. Hence when TOMPKINS FINANCIAL fails, it will be accompanied by on average 1.57 extra failures in this system. Compared to the size of the system, 27 banks, this is a trivial systemic impact. Excluding TOMPKINS FINANCIAL, the other 26 banks report different SII measure ranging from 3.44 to 6.26 which shows the variation of the SII measures across banks. A SII measure at 6 is interpreted as that the failure of the concerned bank will be accompanied

by other 5 bank failures, which is about one fifth of the entire system. To name a few with the highest SII measures, WASHINGTON MUTUAL reports 6.26 as the highest, followed by IRWIN FINL. with 6.06 and then FIRST HORIZON NATIONAL and WACHOVIA with 6.00. Only IRWIN FINL. appears in the top-three systemically important banks according to both the PAO and the SII analysis. In general, banks in the top of the PAO ranked list are different from those in the top of the SII ranked list. For example, WACHOVIA reports a PAO measure at 80.1% which is ranked at the middle range among all banks, however, exhibiting a high SII measure.

To summarize, the comparison between the three measures shows that although having different economic background, the PAO measure and the VI measure are equally informative in terms of measuring the systemic importance. On the contrary, the SII measure provides information on the size of the systemic impact corresponding to the failure of a particular bank. Across different banks, the SII measure varies while the PAO measures remain in a comparable level. This agrees with our theoretical prediction. Therefore, the SII measure is more informative in distinguishing the systemic importance of financial institutions.

Next, we check the correlation between the size measures and the three systemic importance measure. For each measure of size, to have a unified value for each individual bank, we use the average across the full sample period, i.e. from 1987 to 2008. Then, we calculate the Pearson correlation coefficient between each pair of size measure and systemic importance measure across 27 banks. Moreover, we test whether the correlation coefficient is significantly different from zero. The results are shown in Table 3. None of the estimated correlation coefficients is significantly different from zero under 5% confidence level. The only one which is significantly different from zero under 10% level is the correlation between the SII measure and the debt asset ratio. We conclude that the systemic importance measures are not correlated with the size measures. Therefore, "too big to fail" is not valid, at least for the constructed banking system. The observation on the positive correlation between the SII measure and the debt asset ratio points to the direction that a high leverage ratio may



corresponds to a high systemic importance, however, with weak empirical evidence.

Besides estimating the three systemic importance measures from the full sample period, we consider subsamples for the estimation. By moving the subsample window, we could obtain time varying estimation on the systemic importance measures. A first attempt is to consider the estimation window as 10-year, then move the estimation window year by year. In other words, the first estimation considers data from 1987 to 1996, and the second estimation considers data from 1988 to 1997, etc. By moving the estimation window, we observe 13 estimates at the end of each year during 1996-2008 as shown in Figure 1. The upper panel shows the moving window SII and the bottom panel shows the moving window PAO. For simplicity we only plot the results for selected banks.

From the moving window SII, we observe that the SII measures are relatively stable before 2007, with a sharp rising in 2008. On the contrary, the PAO measures are stable across time. This again confirms our theoretical prediction that the PAO measures always stay at a high, comparable level, while the SII measure varies across time and across institutions. The sharp rising of the SII measure addresses the crisis from 2008, hence, is more informative in analyzing the systemic risk in a financial system. However, we do not emphasize that the SII measure is a predictor of the crisis. The sharp rising of SII might either be a predictor before the crisis, or an ex-post consequence caused by the crisis. The intuition for the latter possibility is as follows. Banks intend to take similar strategies during a crisis, which results in similar portfolio holdings across all banks. According to the theoretical model in Section 4, that leads to an increase of the SII measures on all banks. Hence, it is still an open question on the timing of the sharp rising of the SII measures. A more careful analysis involving narrower window size and short moving steps will help to clarify this question. However, it is beyond the scope of this paper.

There are a few other observations from the moving window analysis. Notice that the constructed financial system contains 27 banks. A SII measure at 8 means that given a certain bank fails, there will be on average extra 7 bank failures simultaneously. This is

more than a quarter of the entire system, which must be considered as a severe risk. Hence, the observed SII measures in 2008 indicate that the banking system suffers severe systemic risk during the crisis.

It is remarkable that TOMPKINS FINANCIAL, the least systemically important bank from the full sample analysis, also shows the least systemic importance during the crisis. In fact, in most of the time, its SII measure is the lowest one, and during the crisis, it only rises at a level below 6. On the other hand, WACHOVIA shows a SII measure higher than 9 in 2008 as the most systemically important bank.

## 6 Conclusion

In this paper, we consider three measures on the systemic importance of financial institutions in a financial system. Since we regard the system as the combination of individuals, it is a multivariate relation rather than bilateral. We consider the PAO measure proposed by Segoviano and Goodhart (2009) as well as new measures: the SII measure which measures the size of the systemic impact if one bank fails, and the VI measure which measures the impact on a particular bank when the other part of the system is in distress.

We use a theoretical model based on affine portfolio holdings to show that a large bank is not necessarily more systemically important in terms of exhibiting high levels of the three proposed systemic importance measures. Only with diversified banking activities, a large bank may become systemically important. On the contrary, an isolated large bank will not be harmful to the system. The discussion can be extended to regulation issues. Acknowledging the tradeoff between micro-level risk management and systemic importance demonstrates the necessity of establishing macro-prudential supervision scheme. Moreover, measuring systemic importance is the key to such a scheme.

Besides the theoretical model, we conduct an empirical analysis estimating the systemic importance measures and test whether they are correlated with the measures on the bank

size. Multivariate EVT is employed in calculating the systemic importance measures. There is no empirical evidence supporting the "too big to fail" arguments.

The empirical observation confirms that the PAO measure is not as informative as the SII measure in terms of distinguishing the systemically important banks. A moving window analysis shows similar results. Moreover, the VI measure is shown to be as informative as the PAO measure in terms of identifying systemically important banks.

The empirical analysis in this paper is based on an artificial bank system, which is restricted by the data availability on equity returns. Such a restriction may impose sample selection bias: banks standing in the equity market for such a long period may have established a relatively large size. Therefore, the evidence from the empirical analysis should not be regarded as a strong disproof on the "too big to fail" argument. Instead, we emphasize that it is possible to have a banking system in which the size measures are not good proxy of the systemic importance. Size should not be automatically regarded as a proxy of systemic importance. Therefore, for financial stability analysis or macro-prudential supervision, it is necessary to apply systemic importance measures in order to identify systemically important financial institutions.

Although in the current empirical analysis, our proposed SII measure is shown to be more informative than the PAO measure proposed by Segoviano and Goodhart (2009), we address one potential drawback of the SII measure: it is a simple counting measure without taking into account the differences between potential losses when different financial institutions fall in crisis. In other words, when calculating the expected number of failures in the system, whether it is a failure of a big bank or a small bank is not distinguished in the SII measure. This can be improved by considering the expected total loss in the system given one bank fails. That requires calculating expect shortfall conditional on a certain bank failure and incorporating with the sizes of the banks. Although more sophisticated, a measure addressing the two above issues can still be established and estimated under the multivariate EVT framework. This is left for future research.

## References

- V.V. Acharya, J.A.C. Santos, and T. Yorulmazer. Systemic Risk and Deposit Insurance Premiums. *Economic Policy Review, Federal Reserve Bank of New York*, forthcoming, 2009.
- T. Adrian and M. Brunnermeier. CoVaR. *Federal Reserve Bank of New York Staff Report*, 348, 2008.
- F. Allen and D. Gale. Financial contagion. *Journal of Political Economy*, 108:1–33, 2000.
- F. Allen, E. Carletti, and A. Babus. Financial Crises: Theory and Evidence. *Annual Review of Financial Economics*, 1(1), 2009.
- B. Bernanke. Financial Reform to Address Systemic Risk. *Speech at the Council on Foreign Relations*, March 10, 2009.
- R. Cifuentes, H.S. Shin, and G. Ferrucci. Liquidity risk and contagion. *Journal of the European Economic Association*, 3(2-3):556–566, 2005.
- A. Dasgupta. Financial contagion through capital connections: a model of the origin and spread of bank panics. *Journal of the European Economic Association*, 2(6):1049–1084, 2004.
- O. de Bandt and P. Hartmann. Systemic risk in banking: a survey. In C. Goodhart and G. Illing, editors, *Financial Crisis, Contagion, and the Lender of Last Resort: A Reader*, pages 249–298. Oxford University Press, 2001.
- L. de Haan and A. Ferreira. *Extreme Value Theory: An Introduction*. Springer, 2006.
- C.G. de Vries. The simple economics of bank fragility. *Journal of Banking and Finance*, 29(4):803–825, 2005.

- P. Embrechts, C. Klüppelberg, and T. Mikosch. *Modelling extremal events: for insurance and finance*. Springer, 1997.
- X. Freixas, B.M. Parigi, and J.C. Rochet. Systemic risk, interbank relations, and liquidity provision by the central bank. *Journal of Money, Credit and Banking*, 32:611–638, 2000.
- P. Hartmann, S. Straetmans, and C.G. de Vries. Asset market linkages in crisis periods. *Review of Economics and Statistics*, 86:313–326, 2004.
- IMF. Global financial stability report. *International Monetary Fund*, April, 2009.
- R. Lagunoff and S.L. Schreft. A model of financial fragility. *Journal of Economic Theory*, 99(1):220–264, 2001.
- E. Perotti and J. Suarez. Liquidity Insurance for Systemic Crises. *CEPR Policy Insight*, 31, 2009.
- S. Poon, M. Rockinger, and J. Tawn. Extreme value dependence in financial markets: Diagnostics, models, and financial implications. *The Review of Financial Studies*, 17, No.2:581–610, 2004.
- R.G. Rajan. Too systemic to fail: consequences, causes and potential remedies. *Written statement to the Senate Banking Committee Hearings*, May 6, 2009.
- M. Segoviano and C. Goodhart. Banking Stability Measures. Technical report, IMF Working Paper 09/04 (Washington: International Monetary Fund), 2009.
- G.H. Stern and R.J. Feldman. *Too big to fail: The hazards of bank bailouts*. Brookings Institution Press, 2004.
- C. Zhou. *On Extreme Value Statistics*. PhD thesis, Tinbergen Institute, 2008.

# Appendix A Proofs

## Proof of Proposition 1.

Recall the definition of the PAO measure in (1). We have that

$$\begin{aligned}
 PAO_i(p) &= \frac{P(\{\exists j \neq i, \text{ s.t. } X_j > VaR_j(p)\} \cap \{X_i > VaR_i(p)\})}{P(X_i > VaR_i(p))} \\
 &= \frac{1}{p}P(\{\exists j \neq i, \text{ s.t. } X_j > VaR_j(p)\}) + 1 \\
 &\quad - \frac{1}{p}P(\{\exists j \neq i, \text{ s.t. } X_j > VaR_j(p)\} \cup \{X_i > VaR_i(p)\}) \\
 &= \frac{1}{p}P(\{\exists j \neq i, \text{ s.t. } X_j > VaR_j(p)\}) + 1 - \frac{1}{p}P(\{\exists j, \text{ s.t. } X_j > VaR_j(p)\}) \\
 &=: I_1 + 1 - I_2
 \end{aligned}$$

From the definition of the  $L$  function in (4), as  $p \rightarrow 0$ ,  $I_1 \rightarrow L_{\neq i}(1, 1, \dots, 1)$  and  $I_2 \rightarrow L(1, 1, \dots, 1)$ , which implies (6). ■

## Proof of Proposition 4.

Recall the definition of the SII measure in (2). We have that

$$\begin{aligned}
 SII_i(p) &= \sum_{j=1}^d E(1_{X_j > VaR_j(p)} | X_i > VaR_i(p)) \\
 &= \sum_{j=1}^d P(X_j > VaR_j(p) | X_i > VaR_i(p)) \\
 &= \sum_{j=1}^d \frac{P(X_j > VaR_j(p), X_i > VaR_i(p))}{P(X_i > VaR_i(p))} \\
 &= \sum_{j=1}^d \frac{P(X_j > VaR_j(p)) + P(X_i > VaR_i(p)) - P(X_j > VaR_j(p) \text{ or } X_i > VaR_i(p))}{p} \\
 &= \sum_{j=1}^d 2 - \frac{P(X_j > VaR_j(p) \text{ or } X_i > VaR_i(p))}{p}
 \end{aligned}$$

From the definition of the  $L$  function in (4), as  $p \rightarrow 0$ ,

$$\frac{P(X_j > VaR_j(p) \text{ or } X_i > VaR_i(p))}{p} \rightarrow L_{i,j}(1, 1).$$

The relation (8) is thus proved. ■

**Proof of Corollary 3.**

Since  $L(1, 1, \dots, 1) - 1 < 0$ , the relation (7) implies that a higher value of the VI measure corresponds to a higher level of  $L_{\neq i}(1, 1, \dots, 1)$ . Together with (6). The corollary follows.

■

**Proof of Theorem 5.**

Firstly, since the heavy-tail feature in (11) assumes that the right tail of  $Y_i$  dominates its left tail, it is sufficient to assume that  $Y_i$  are all positive random variables for  $i = 1, 2, 3$ , i.e. without left tail. We adopt this assumption in the rest of the proof.

We use the Feller convolution theorem to deal with the sum of independently heavy-tailed distributed random variables as in the following lemma.

**Lemma 7** *Suppose positive random variables  $U$  and  $V$  are independent. Assume that they are both heavy-tailed distributed with the same tail index  $\alpha$ . Then, as  $s \rightarrow \infty$ ,*

$$P(U + V > s) \sim P(U > s) + P(V > s).$$

Notice that the heavy-tailed feature implies  $P(U > s)P(V > s) = o(P(U > s) + P(V > s))$ , as  $s \rightarrow \infty$ . Hence, the Feller convolution theorem is equivalent to

$$P(U + V > s) \sim P(\max(U, V) > s),$$

i.e., the sum and the maximum of two independently heavy-tailed distributed random variables are tail equivalent. A proof using sets manipulation can be found in Embrechts et al. (1997). With an analogous proof under multivariate framework, a multivariate version of

the Feller theorem can be obtained. In multivariate case, the tail equivalence between two random vectors is defined as the combination of having tail equivalence for each marginal distribution and having the same  $L$  function for the tail dependence structure. We present the result in a 2-d context in the following lemma without providing the proof.

**Lemma 8** *Suppose positive random variables  $U_1$  and  $U_2$  are independent. Assume that they are both heavy-tailed distributed with the same tail index  $\alpha$ . Then for any positive constants  $m_{ij}$ ,  $1 \leq i, j \leq 2$ , we have that the distribution functions of  $(m_{11}U_1 + m_{12}U_2, m_{21}U_1 + m_{22}U_2)$  and  $(m_{11}U_1 \vee m_{12}U_2, m_{21}U_1 \vee m_{22}U_2)$  are tail equivalent.*

To prove Theorem 5, it is necessary to calculate the  $L$  function of  $(X_1, X_2, X_3)$  on points  $(1, 1, 0)$ ,  $(1, 0, 1)$  and  $(0, 1, 1)$ . The main instrument in the calculation is Lemma 8. We start by comparing the individual risks taken by the three banks.

**Proposition 9** *For the three-bank model on  $(X_1, X_2, X_3)$ , as  $p \rightarrow 0$ ,*

$$VaR_2(p) = VaR_3(p) \sim cVaR_1(p), \quad (13)$$

where

$$c := \left( \frac{(2 - 2\gamma)^\alpha + 2\gamma^\alpha}{\gamma^\alpha + \mu^\alpha + (1 - \gamma - \mu)^\alpha} \right)^{-1/\alpha}.$$

**Proof of Proposition 9.**

From Lemma 7, we get that, as  $s \rightarrow \infty$ ,

$$\begin{aligned} P(X_1 > s) &\sim P((2 - 2\gamma)Y_1 > s) + P(\gamma Y_2 > s) + P(\gamma Y_3 > s) \\ &= ((2 - 2\gamma)^\alpha + 2\gamma^\alpha) s^{-\alpha} K(s) \end{aligned} \quad (14)$$

Similarly, we get that

$$P(X_2 > s) = P(X_3 > s) \sim (\gamma^\alpha + \mu^\alpha + (1 - \gamma - \mu)^\alpha) s^{-\alpha} K(s) \quad (15)$$



By comparing (14) and (15), the relation (13) is a direct consequence. ■

We remark that  $c < 1$ . This agrees with the fact that the large bank  $X_1$  can take more risks than the small banks. On the other hand, when  $\gamma > 1/2$ , we get  $c > 1/2$ . In this case, the comparison between bank risk taking are not proportional to their sizes. The small banks are taking relatively more risks. In other words, the large bank  $X_1$  benefits from diversification.

Next, we calculate  $L(1, 1, 0)$ . Denote  $v(p) := VaR_1(p)$ . From Lemma 8, we get that, as  $p \rightarrow 0$

$$\begin{aligned}
& P(X_1 > VaR_1(p) \text{ or } X_2 > VaR_2(p)) \\
& \sim P((2 - 2\gamma)Y_1 \vee \gamma Y_2 \vee \gamma Y_3 > v(p) \text{ or } \gamma Y_1 \vee (1 - \gamma - \mu)Y_2 \vee \mu Y_3 > cv(p)) \\
& = P\left(Y_1 > \frac{v(p)}{(2 - 2\gamma) \vee \frac{\gamma}{c}} \text{ or } Y_2 > \frac{v(p)}{\gamma \vee \frac{1 - \gamma - \mu}{c}} \text{ or } Y_3 > \frac{v(p)}{\gamma \vee \frac{\mu}{c}}\right) \\
& \sim P\left(Y_1 > \frac{v(p)}{(2 - 2\gamma) \vee \frac{\gamma}{c}}\right) + P\left(Y_2 > \frac{v(p)}{\gamma \vee \frac{1 - \gamma - \mu}{c}}\right) + P\left(Y_3 > \frac{v(p)}{\gamma \vee \frac{\mu}{c}}\right) \\
& \sim \left[ \left((2 - 2\gamma) \vee \frac{\gamma}{c}\right)^\alpha + \left(\gamma \vee \frac{\mu}{c}\right)^\alpha + \left(\gamma \vee \frac{1 - \gamma - \mu}{c}\right)^\alpha \right] P(Y_1 > v(p)) \tag{16}
\end{aligned}$$

From (14), we get that  $p \sim ((2 - 2\gamma)^\alpha + 2\gamma^\alpha)P(Y_1 > v(p))$  as  $p \rightarrow 0$ . Together with (16) and (5), we have that

$$\begin{aligned}
L(1, 1, 0) &= \lim_{p \rightarrow 0} \frac{P(X_1 > VaR_1(p) \text{ or } X_2 > VaR_2(p))}{p} \\
&= \frac{\left((2 - 2\gamma) \vee \frac{\gamma}{c}\right)^\alpha + \left(\gamma \vee \frac{\mu}{c}\right)^\alpha + \left(\gamma \vee \frac{1 - \gamma - \mu}{c}\right)^\alpha}{(2 - 2\gamma)^\alpha + 2\gamma^\alpha}. \tag{17}
\end{aligned}$$

Due to symmetry, we have that  $L(1, 0, 1) = L(1, 1, 0)$ . Following similar calculation, it is obtained that

$$L(0, 1, 1) = \frac{\left(\frac{\gamma}{c}\right)^\alpha + 2\left(\frac{\mu \vee (1 - \gamma - \mu)}{c}\right)^\alpha}{(2 - 2\gamma)^\alpha + 2\gamma^\alpha}. \tag{18}$$

Proposition 4 implies that  $SII_2 = SII_3$ . Moreover,  $SII_1 > SII_2$  if and only if  $L(1, 1, 0) <$

$L(0, 1, 1)$ . Hence it is only necessary to compare the two values of  $L$  function.

Denote

$$Q := ((2 - 2\gamma)^\alpha + 2\gamma^\alpha)(L(1, 1, 0) - L(0, 1, 1)).$$

We study the sign of  $Q$  in order to compare the systemic importance measures  $SII_i$ ,  $i = 1, 2, 3$ .

**Case I:**  $\frac{2}{3} \leq \gamma < 1$

Since  $\gamma > 1/2$ , we have  $c > 1/2$ . Thus,  $\frac{\gamma}{1-\gamma} > 2 > \frac{1}{c}$ , which implies that

$$\gamma > \frac{1-\gamma}{c} > \max\left(\frac{1-\gamma-\mu}{c}, \frac{\mu}{c}\right).$$

Therefore

$$Q = \left( \left( (2 - 2\gamma) \vee \frac{\gamma}{c} \right)^\alpha + 2\gamma^\alpha \right) - \left( \left( \frac{\gamma}{c} \right)^\alpha + 2 \left( \frac{\mu \vee (1 - \gamma - \mu)}{c} \right)^\alpha \right) > 0.$$

Hence,  $SII_1 < SII_2 = SII_3$ .

**Case II:**  $\gamma = 1/2$

In this case, we still have  $c \geq 1/2$ . The equality holds if and only if  $\mu = 1/4$ . Due to the symmetric position between  $X_2$  and  $X_3$ , without loss of generality, we assume that  $\mu \leq 1/4$ . Then we get  $\mu \leq 1 - \gamma - \mu$ . Moreover, the inequality  $\mu/\gamma \leq 1/2 \leq c$  implies that  $\gamma \geq \frac{\mu}{c}$  and

it is not difficult to obtain that  $\gamma \leq \frac{1-\gamma-\mu}{c}$ . Hence,

$$\begin{aligned}
Q &= \left(1 + 2^{-\alpha} + \left(\frac{1/2 - \mu}{c}\right)^\alpha\right) - \left(\left(\frac{\gamma}{c}\right)^\alpha + 2\left(\frac{1/2 - \mu}{c}\right)^\alpha\right) \\
&= 1 + 2^{-\alpha} - \left(\frac{1}{2^\alpha} + \left(\frac{1}{2} - \mu\right)^\alpha\right) c^{-\alpha} \\
&= 1 + \frac{1}{2^\alpha} - \frac{1 + (1 - 2\mu)^\alpha}{2^\alpha} \frac{2^\alpha + 2}{1 + (2\mu)^\alpha + (1 - 2\mu)^\alpha} \\
&= 1 + \frac{1}{2^\alpha} - \left(1 + \frac{2}{2^\alpha}\right) \left(1 - \frac{(2\mu)^\alpha}{1 + (2\mu)^\alpha + (1 - 2\mu)^\alpha}\right) \\
&\leq 1 + \frac{1}{2^\alpha} - \left(1 + \frac{2}{2^\alpha}\right) \left(1 - \frac{(1/2)^\alpha}{1 + (1/2)^\alpha + (1/2)^\alpha}\right) \\
&= 0
\end{aligned}$$

Here we used the facts that  $\frac{(2\mu)^\alpha}{1+(2\mu)^\alpha+(1-2\mu)^\alpha}$  is an increasing function with respect to  $\mu$  and  $\mu \leq 1/4$ . The equality holds if and only if  $\mu = 1/4$ . Hence, in case  $\gamma = 1/2$ , we conclude that  $SII_1 \geq SII_2 = SII_3$ , with the equality holds if and only if  $\mu = 1/4$ .

**Case III:**  $0 < \gamma \leq 1/3$

In this case, it is not difficult to verify that  $2 - 2\gamma > \frac{\gamma}{c}$  and  $\gamma < \frac{1-\gamma}{2c}$ . Due to symmetry, we only consider the case  $0 < \mu \leq \frac{1-\gamma}{2}$ . Then we have that  $\mu \leq 1 - \gamma - \mu$  and  $\gamma < \frac{1-\gamma-\mu}{c}$ . Therefore

$$Q = (2 - 2\gamma)^\alpha + \left(\gamma \vee \frac{\mu}{c}\right)^\alpha - \frac{\gamma^\alpha + (1 - \gamma - \mu)^\alpha}{c^\alpha}. \quad (19)$$

For any fixed  $\gamma$ ,  $c$  is a function of  $\mu$  denoted by  $c(\mu)$ . For  $0 < \mu < \frac{1-\gamma}{2}$ ,  $c(\mu)$  is a strictly decreasing function. Thus  $g(\mu) := \frac{\mu}{c(\mu)}$  is a continuous, strictly increasing function. Notice that  $g(0) = 0 < \gamma$  and  $g((1-\gamma)/2) > \gamma$ . There must exist a unique  $\mu^* \in (0, (1-\gamma)/2)$  such that  $g(\mu^*) = \gamma$ .

Denote  $c^* := c(\mu^*)$ . From  $\frac{\mu^*}{c^*} = \gamma$ , we get that

$$\frac{\gamma^\alpha}{\mu^{*\alpha}} = (c^*)^{-\alpha} = \frac{(2 - 2\gamma)^\alpha + 2\gamma^\alpha}{\gamma^\alpha + \mu^{*\alpha} + (1 - \gamma - \mu^*)^\alpha}.$$

It implies that

$$\frac{\gamma^\alpha}{\mu^{*\alpha}} = (c^*)^{-\alpha} = \frac{(2-2\gamma)^\alpha + \gamma^\alpha}{\gamma^\alpha + (1-\gamma-\mu^*)^\alpha}.$$

Continuing from equation (19), we get that

$$Q(\mu^*) = (2-2\gamma)^\alpha + \gamma^\alpha - (\gamma^\alpha + (1-\gamma-\mu^*)^\alpha) (c^*)^{-\alpha} = 0.$$

Hence, we conclude that for  $\mu = \mu^*$ ,  $SII_1 = SII_2 = SII_3$ .

For  $0 < \mu < \mu^*$ , it is clear that

$$\begin{aligned} Q(\mu) &= (2-2\gamma)^\alpha + \gamma^\alpha - (\gamma^\alpha + (1-\gamma-\mu)^\alpha) c^{-\alpha} \\ &= (2-2\gamma)^\alpha + \gamma^\alpha - (\gamma^\alpha + (1-\gamma-\mu)^\alpha) \frac{(2-2\gamma)^\alpha + 2\gamma^\alpha}{\gamma^\alpha + \mu^\alpha + (1-\gamma-\mu)^\alpha} \\ &= \frac{((2-2\gamma)^\alpha + \gamma^\alpha) \mu^\alpha - \gamma^\alpha (\gamma^\alpha + (1-\gamma-\mu)^\alpha)}{\gamma^\alpha + \mu^\alpha + (1-\gamma-\mu)^\alpha} \end{aligned}$$

is an strictly increasing function with respect to  $\mu$ . Moreover, for  $\mu^* < \mu < \frac{1-\gamma}{2}$ ,  $Q$  is calculated as

$$\begin{aligned} Q(\mu) &= (2-2\gamma)^\alpha + (\mu^\alpha - \gamma^\alpha - (1-\gamma-\mu)^\alpha) c^{-\alpha} \\ &= (2-2\gamma)^\alpha + (\mu^\alpha - \gamma^\alpha - (1-\gamma-\mu)^\alpha) \frac{(2-2\gamma)^\alpha + 2\gamma^\alpha}{\gamma^\alpha + \mu^\alpha + (1-\gamma-\mu)^\alpha} \\ &= \frac{2(2-2\gamma)^\alpha \mu^\alpha + 2\gamma^\alpha (\mu^\alpha - \gamma^\alpha - (1-\gamma-\mu)^\alpha)}{\gamma^\alpha + \mu^\alpha + (1-\gamma-\mu)^\alpha} \end{aligned}$$

which is still an strictly increasing function with respect to  $\mu$ . Therefore, for  $0 < \mu < \mu^*$ ,  $Q < 0$  while for  $\mu^* < \mu < \frac{1-\gamma}{2}$ ,  $Q > 0$ . Correspondingly, we have that  $SII_1 > SII_2 = SII_3$  in the former case, and  $SII_1 < SII_2 = SII_3$  in the latter case.

Due to symmetry, similar result holds when  $\frac{1-\gamma}{2} < \mu < 1-\gamma$ , and the switch point is then  $1-\gamma-\mu^*$ . More specifically, for  $\frac{1-\gamma}{2} < \mu < 1-\gamma-\mu^*$ ,  $SII_1 < SII_2 = SII_3$ ; for  $\mu = 1-\gamma-\mu^*$ ,  $SII_1 = SII_2 = SII_3$ ; for  $1-\gamma-\mu^* < \mu < 1-\gamma$ ,  $SII_1 > SII_2 = SII_3$ . The theorem is thus proved for Case III. ■

### Proof of Lemma 6.

Along the lines of the proof of Theorem 5, we obtained that in a three-bank model.

$$\begin{aligned} SII_1 - SII_2 &= \{1 + (2 - L(1, 1, 0)) + (2 - L(1, 0, 1))\} - \{(2 - L(1, 1, 0)) + 1 + (2 - L(0, 1, 1))\} \\ &= L(0, 1, 1) - L(1, 0, 1). \end{aligned}$$

Therefore,  $SII_i > SII_j$  if and only if  $L_{\neq i}(1, 1, \dots, 1) > L_{\neq j}(1, 1, \dots, 1)$ . Together with (6), the lemma is proved.

We remark that the above calculation is not based on the affine portfolio model. Hence, the result is valid in a general three-bank model. However, one can not obtain similar relations when the dimensions exceed three. Hence, Lemma 6 is not valid for assessing a banking system with more than three banks. ■

## Appendix B Statistical Estimation on the $L$ function

Consider independently and identically distributed (i.i.d.) observations from the random vector  $(X_1, \dots, X_d)$  denoted by

$$\{(X_{1s}, X_{2s}, \dots, X_{ds}) | 1 \leq s \leq n\}.$$

The sample size is  $n$ . The non-parametric approach of estimating the  $L$  function starts from the assumption (4). Roughly speaking, the estimation takes a certain  $p$  value for which the VaR for each dimension can be estimated by the order statistics. Then the probability in the numerator of (4) is estimated by a counting measure. To ensure that  $p \rightarrow 0$ , theoretically we take a sequence  $k := k(n)$ , such that  $k(n) \rightarrow \infty$  and  $k(n)/n \rightarrow 0$  as  $n \rightarrow \infty$ . We get an empirical estimation of the  $L$  function from replacing  $p$  by  $k/n$  and using the empirical estimation on the distribution function of  $(X_1, X_2, \dots, X_d)$ . The explicit estimator is given

as

$$\hat{L}(x_1, \dots, x_d) := \frac{1}{k} \sum_{s=1}^n 1_{\exists 1 \leq i \leq d, \text{ s.t. } X_{is} > X_{i, n - [kx_i]}}$$

where  $X_{i,1} \leq X_{i,2} \leq \dots \leq X_{i,n}$  are the order statistics of the  $i$ -th dimension of the sample,  $X_{i1}, \dots, X_{in}$ , for  $1 \leq i \leq d$ . Particularly,  $L(1, 1, \dots, 1)$  is estimated by

$$\hat{L}(1, 1, \dots, 1) := \frac{1}{k} \sum_{s=1}^n 1_{\exists 1 \leq i \leq d, \text{ s.t. } X_{is} > X_{i, n - k}}$$

For the estimator of the  $L$  function, usual statistical properties, such as consistency and asymptotic normality, have been proved, see, e.g. de Haan and Ferreira (2006).

Practically, since  $n$  is always finite, the theoretical conditions on  $k$  is not relevant for a finite sample analysis. Thus it is a major issue on how to choose a proper  $k$  in the estimator. Instead of taking an arbitrary  $k$ , a usual procedure is to calculate the estimator of  $L(1, 1, \dots, 1)$  under different  $k$  values and draw a line plot against the  $k$  values. With a low  $k$  value, the estimation exhibits a large variance, while for a high  $k$  value, since the estimation uses too many observations in the moderate level, it bears a potential bias. Therefore,  $k$  is usually chosen by picking the first stable part of the line plot which balances the tradeoff between the variance and the bias. The estimates follow from the  $k$  choice. Such a procedure has been applied in univariate EVT for tail index estimation.

Notice that from the estimation, we in fact consider a crisis as the loss return exceeds a VaR with tail probability level  $k/n$ . Since the  $k$  level is ad hoc chosen from the estimation, we have a objective definition on crises. Moreover, because  $k$  is chosen from a stable part of the line plot, a small variation of the  $k$  value does not change the estimates. Thus, the exact  $k$  value is not sensitive for the estimation on the  $L$  function.

In our empirical application, the chosen  $k$  value is always around 5% of the total sample size, even though we have different sample sizes when performing the analysis for the full sample period and the moving windows.

## Appendix C Tables and Figures

Table 1: Descriptive statistics on daily stock returns of 27 US banks

BANKS	Mean	Std. Dev.	Min	Max	Skewness	Kurtosis
BANK OF AMERICA	0.031	2.24	-30.42	24.06	-0.51	24.87
BANK OF HAWAII	0.045	1.69	-25.51	12.95	-0.79	22.89
BB&T	0.043	1.92	-26.61	21.20	0.02	21.30
CITY NATIONAL	0.031	2.18	-18.92	20.21	0.07	11.22
COMERICA	0.033	1.89	-22.69	18.81	-0.30	19.99
CULLEN FO.BANKERS	0.056	2.13	-21.46	19.78	0.12	14.30
FIRST HORIZON NATIONAL	0.029	2.24	-44.11	25.54	-1.46	55.45
IRWIN FINL.	0.009	2.87	-39.47	60.68	2.10	68.37
KEYCORP	0.019	2.23	-40.55	43.34	-0.63	72.75
MARSHALL & ILSLEY	0.033	1.96	-26.45	25.54	-0.26	38.38
M&T BK.	0.053	1.58	-17.59	22.83	0.17	28.08
JP MORGAN CHASE & CO.	0.030	2.39	-32.46	19.38	-0.39	16.72
PNC FINL.SVS.GP.	0.031	1.88	-17.56	14.95	-0.11	11.84
REGIONS FINL.NEW	0.008	2.31	-52.88	30.47	-1.42	75.10
SYNOVUS FINL.	0.042	2.28	-19.77	16.44	0.06	11.15
TOMPKINS FINANCIAL	0.052	2.82	-12.26	17.33	0.10	5.89
STERLING BANC.	0.028	2.13	-21.26	19.39	0.37	12.58
VALLEY NATIONAL BANCORP	0.039	2.11	-17.48	21.71	0.29	11.77
WILMINGTON TRUST	0.037	1.81	-20.07	18.74	-0.12	15.05
WELLS FARGO & CO	0.064	1.98	-21.03	28.34	0.36	19.95
CITIGROUP	0.033	2.54	-30.66	45.63	0.22	37.34
MORGAN STANLEY	0.037	2.78	-29.97	62.58	1.24	62.40
SUNTRUST BANKS	0.030	1.97	-26.44	23.19	-0.22	24.46
NORTHERN TRUST	0.055	1.97	-20.84	17.41	0.06	13.93
WACHOVIA	0.002	3.48	-169.28	64.30	-18.16	1012.25
WASHINGTON MUTUAL	0.075-	5.16	-235.47	100.53	-21.19	981.82
MERRILL LYNCH & CO.	0.025	2.75	-33.43	32.39	-0.35	24.54

Note: The sample period is from Jan 1 1987 to Dec 31 2008. All values except the skewness and kurtosis are in percentage.

Table 2: Systemic importance measures (full sample)

BANKS	SII	PAO	VI
BANK OF AMERICA	5.18	77.4%	6.65%
BANK OF HAWAII	4.25	72.8%	6.29%
BB&T	5.08	81.5%	6.99%
CITY NATIONAL	3.91	77.7%	6.68%
COMERICA	5.78	82.9%	7.10%
CULLEN FO.BANKERS	3.44	68.6%	5.95%
FIRST HORIZON NATIONAL	6.00	86.4%	7.37%
IRWIN FINL.	6.06	85.0%	7.26%
KEYCORP	5.71	83.3%	7.13%
MARSHALL & ILSLEY	5.61	77.7%	6.68%
M&T BK.	5.26	82.6%	7.07%
JP MORGAN CHASE & CO.	4.43	75.6%	6.51%
PNC FINL.SVS.GP.	4.40	73.5%	6.34%
REGIONS FINL.NEW	5.92	85.4%	7.29%
SYNOVUS FINL.	4.38	77.7%	6.68%
TOMPKINS FINANCIAL	2.57	60.6%	5.29%
STERLING BANC.	4.72	79.8%	6.85%
VALLEY NATIONAL BANCORP	3.88	73.2%	6.32%
WILMINGTON TRUST	4.66	79.1%	6.79%
WELLS FARGO & CO	4.97	75.6%	6.51%
CITIGROUP	5.00	76.7%	6.60%
MORGAN STANLEY	4.61	77.4%	6.65%
SUNTRUST BANKS	5.61	80.5%	6.90%
NORTHERN TRUST	4.43	74.9%	6.46%
WACHOVIA	6.00	80.1%	6.88%
WASHINGTON MUTUAL	6.26	84.0%	7.18%
MERRILL LYNCH & CO.	5.10	75.6%	6.51%

Note: SII is the systemic importance index defined as the number of expected bank failures given a particular bank fails, see (2); PAO is the probability of causing at least one extra bank failure when a particular bank fails defined in (1); VI is the vulnerability index defined as the probability of failure given there exists at least one other bank failure in the system, see (3).



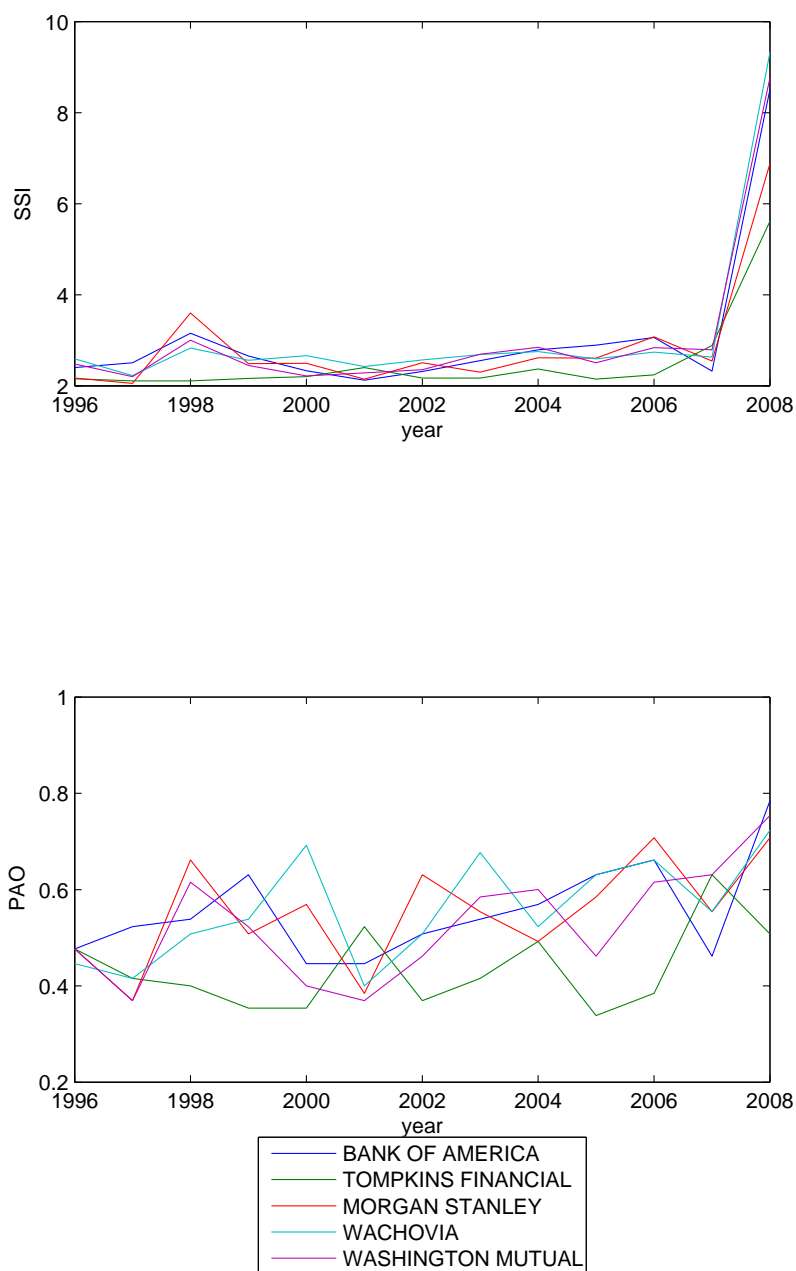
Table 3: Correlation coefficients between size and systemic importance measures

	SII	PAO	VI
Total asset	0.0781 (0.6986)	-0.0686 (0.7337)	-0.066 (0.7434)
Total equity	0.1091 (0.5881)	-0.0466 (0.8176)	-0.0441 (0.8272)
Total debt	0.073 (0.7176)	-0.0785 (0.6972)	-0.0759 (0.7068)
Equity asset ratio	0.0766 (0.7043)	0.0449 (0.8240)	0.043 (0.8315)
Debt asset ratio	0.3296* (0.0931)	0.1322 (0.5109)	0.1337 (0.5061)

Note: SII is the systemic importance index defined as the number of expected bank failures given a particular bank fails, see (2); PAO is the probability of causing at least one extra bank failure when a particular bank fails defined in (1); VI is the vulnerability index defined as the probability of failure given there exists at least one other bank failure in the system, see (3). The numbers in parentheses are the p-value for testing whether the correlation coefficient is significantly different from zero.

Significance level: 1%<sup>\*\*\*</sup>, 5%<sup>\*\*</sup>, 10%<sup>\*</sup>.

Figure 1: Moving window results on systemic importance measures



Note: The moving window measures are estimated from 10-year subsample ending at the end of the corresponding year. The upper panel presents the results for SII, which is the systemic importance index defined as the number of expected bank failures given a particular bank fails in (2), while the bottom panel presents the results for PAO which is the probability of causing at least one extra bank failure when a particular bank fails defined in (1).

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