An Operational Measure of Riskiness:
A Comment

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1 Introduction

In a very clear and innovative article, Foster and Hart (2009) derive a measure of riskiness. This measure relies on linking the choice of any given gamble to a level of critical wealth, below which it becomes risky to accept it. Otherwise put, if the level of wealth available to the decision-maker is below this critical threshold, then bankruptcy is possible and the decision-maker should reject the gamble. This critical wealth is then the measure of riskiness. The authors argue that as this measure depends only on the outcomes and the probabilities (the ‘distribution’), and not on the gamble itself or the decision-maker’s preferences, this measure is ‘objective’. In this paper we will argue that the set-up assumed implies a very specific set of preferences on the part of the decision-maker, and that it is therefore not ‘objective’ in the sense intended. Indeed, other strands of the literature have very similar characteristics of ‘subjectiveness’. However, what we believe sets this measure apart from other attempts is the way a social planner could operationalize it. In particular, we will argue that both the fact that the measure is a tool for rejecting a gamble (not accepting it), as well as the fact that there is an explicit wealth measure that can be made state-contingent, are potentially useful to a social planner. By means of an example, we discuss this in the context of bank regulation.

2 A measure of riskiness

A gamble \( g \) has finitely many values, say \( x_1, x_2, \ldots, x_m \), with respective probabilities, \( p_1, p_2, \ldots, p_m \) (where \( p_i > 0 \) and \( \sum_{i=1}^{m} p_i = 1 \)). Let \( \mathcal{G} \) then denote a collection of such gambles. The principle behind this measure is that the attractiveness of a given gamble \( g \) is dependent on the level of initial wealth available
to the decision-maker. It derives then the minimal wealth (reserve) required for a gamble \( g \in \mathcal{G} \) to guarantee no bankruptcy. That minimal level of wealth, \( R(g) \), is the implied measure of riskiness. The authors make two important assumptions in deriving this critical threshold. First, process \( \mathcal{G} \) is generated by a finite set of gambles, i.e.:

\[
\mathcal{G}_0 = (g^{(1)}, g^{(2)}, ..., g^{(m)}) \subset \mathcal{G}.
\]  

This in turn implies that the gambles considered, and outcomes \( x_1, x_2, ..., x_m \) in each gamble, are well defined. Second, the authors emphasize that the underlying probability distributions that characterize the gambles are not known. The set-up is therefore non-Bayesian in nature and applies the principle of the ‘worst-case’. For any given gamble \( g \), the decision-maker will be viewing the \((l_\infty)\) norm of \( g \), captured by \( \|g\| \). This will include the returns of a set that contains the maximum losses as well as the maximum gain for that gamble, thus defining the range of possibilities. This assumption refers to the sequence of gambles one will face and not the realization of any one specific. Over an unlimited amount of periods, the measure is designed to reject a gamble when the initial wealth available to the decision-maker is below the critical wealth \( R(g) \). More specifically, a simple strategy, \( s_Q \), guarantees no-bankruptcy, if:

\[
Q(g) > R(g) \quad \forall \; g \in \mathcal{G},
\]  

where \( Q(\cdot) \) is the critical function that associates each gamble \( g \) to a level of wealth. Equivalently, a gamble \( g \) is rejected at all levels of wealth below \( R(g) \).

The authors go on to provide an explicit form for determining \( R(g) \) uniquely.

### 2.1 An objective measure?

The authors define the measure as ‘objective’, if it “...depends on the gamble itself and not on the decision-maker; that is, only the outcomes and the probabilities of the gamble should matter”. In what follows we argue that the decision-maker’s preferences are still imposed on this set-up and that the term ‘objective’ may therefore be misleading. We base our argument on two observations.

- Broadly speaking, the non-Bayesian literature, of which this paper is a part, has adopted the ‘worst-case’ approach out of concerns for stability.\(^1\) These concerns for stability translate into concerns for no-bankruptcy in the Foster-Hart approach. Instability maybe equated with infinite loss (infinity norm) in modelling terms, so optimizing under the assumption of ‘worst-case’ outcomes insures stability, or no bankruptcy in this case. But as has been shown before, seeking to insure against ‘worst-case’ scenarios is equivalent to the decision-maker being averse to ‘ambiguity’, or to disliking consequences with unknown odds.\(^2\) Such a characterization is therefore

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based on a very specific type of preferences and any decision that follows depends on them. By implication, a decision-maker who is not ambiguity averse would not rely on such a measure, and would seek to exploit the benefits of Bayesian approaches to decision making, instead.

- Second, the authors claim that assumption (1) is one of convenience. But as the authors show, the threshold derived is not independent of the space of events considered, or what constitutes the ‘worst-case’. If the intention is to have a measure of riskiness that is ‘operational’, then the sequence of outcomes \( x_1, x_2, ..., x_m \) in each gamble \( g \) needs to be defined. Only then can one meaningfully derive a minimal level of wealth, \( R(g) \), below which bankruptcy is not guaranteed. But if the set (sequence) of outcomes in the gambles considered is well defined, then the measure is not objective in the sense intended, because it represents somebody’s evaluation of what constitutes a feasible set (and therefore belongs in \( G_0 \)). It is of course possible that it is not the decision-maker himself that defines the set of outcomes, but it is still defined based on someone’s judgement and is therefore subjective. In which sense can one define the string of outcomes (and subsequently \( G_0 \) as well) objectively?

The authors argue that the measure does not depend on the decision-maker. We argue that the set-up considered maps to a very specific sets of preferences, namely ambiguity aversion, and requires that the set of feasible gambles is well defined. But in this respect, the Foster-Hart approach is not different from other contributions to non-Bayesian decision theory, as we show next. The main difference is in that while these alternative strands of the literature contribute a methodology for accepting a given gamble (policy), the Foster-Hart approach contributes a measure for rejecting one. We briefly describe the two alternative contributions and how they differ in this respect.

The Robust Control literature (Hansen and Sargent, 2008) also invokes the ‘worst-case’ principle and applies mini-max, or maximizing returns under the worst scenario. The decision-maker is explicit about putting some structure in the uncertainty set (Williams, 2007), equivalent to defining the ‘worst-case’. Based on the explicit definition of the ‘worst-case’ the decision-maker accepts a gamble. Following the same notation as above, the decision-maker picks strategy \( s_Q \) that maximizes wealth, \( Q(g) \), for the worst of the possible outcomes \( x_i \) in gambles \( g \) in the \( G \) set.

\[
\max_{s_Q} \min_{x_i \in G} \{ Q(g) \}. \tag{3}
\]

Applying this strategy, the decision-maker is effectively ‘insured’ against that ‘worst’ event. The Info-Gap approach to non-Bayesian uncertainty (Ben-Haim, 2006, 2010) invokes the principle of satisficing, rather than mini-max. But again, the decision-maker is led to the choice of a particular gamble. He starts by making an assessment of the most likely outcomes. At the same time he defines the minimum set of objectives that he wants to achieve. He then asks the
following question: how wrong can the assessment of the most likely outcome be, and still ensure that the strategy chosen meets the minimum objectives required? The metric $f$ that answers this question generates preferences on the options between alternative strategies. The decision-maker therefore picks strategy $s_Q$ as follows:

$$f(s_Q) = \max_{s_Q} \left\{ f : \left( \min_{g \in Q} Q(g) > W \right) \right\},$$

where $W$ is the minimum satisficing wealth level acceptable to the decision-maker. In other words, the decision-maker picks strategy $s_Q$, such that minimum wealth generated is above $W$, for the greatest deviation around his best assessment. No probabilistic information is required for such a metric (i.e. how likely the best outcome is) and the set-up remains non-Bayesian in nature. Here again, the decision is not independent of the decision-maker’s preferences. The decision-maker can rationalize his decision either in terms of minimum aspirations $W$, or in terms of maximum permissible error, say $\tilde{f}$. Either way, the decision-maker has a tool for ranking the options available to him, given his preferences.

We argue that all three approaches have a subjective way of defining thresholds for ‘acceptable’ outcomes. As mentioned above, the main difference is in that while these alternative strands of the literature contribute a methodology for accepting a given gamble (policy) $s_Q$, the Foster-Hart approach contributes a measure for rejecting one. We will argue next how this feature can be of interest in bank regulation.

3 An operational measure

Consider a bank regulator in charge of imposing and maintaining minimum capital requirements for financial institutions. The purpose behind such restrictions is stability in terms of meeting liquidity demands. Typically, such regulations (see Basel Accord II\(^3\)) set up requirements designed to ensure that financial institutions hold capital reserves appropriate to their risk exposure. But the assessment of the demand for liquidity at any given time is subject to economic conditions and their likelihood for prevailing. In the absence of probability distributions that describe the likelihood of outcomes, the Foster-Hart measure provides a minimum requirement, below which meeting those demands is not guaranteed. We see three advantages in such a measure:

1. unlike other policy interventions, it does not prescribe the actions that agents should take, but rather the actions that in fact they should not take. This is in line with the prudential role of a financial regulator, who aims to contain risks in the system without dictating actions;

\(^3\)http://www.bis.org/publ/bcbsca.htm
2. it provides an explicit measure of \( R(g) \) that emphasizes that the riskiness of any gamble is not independent of the wealth position of the risk-taker. Analogously, the liquidity requirement should not be independent of the financial health of the bank at any point in time;

3. last, it is often said that capital requirements amplify the fluctuations of the business cycle (Ayuso et 2004). During a downturn they dictate a reduction of loans that squeezes credit and adds further to the downturn. The opposite holds in the upturn. It is desirable therefore to construct rules that dampen any excess cyclicality of the minimum capital requirement. The Foster-Hart measure offers the opportunity of defining \( \mathcal{G} \) differently at different positions of the business cycle. As long as the minimal wealth available to banks is above the prescribed threshold \( (R(g) < W) \), the measure does not dictate what level of capital to be held and therefore breaks the link to the business cycle. Naturally, if this condition is no longer met (and is more likely not to be met at the downturn), banks need to find ways of increasing their wealth position accordingly. However, we feel that this approach helps keep the excess cyclicality of the measure to a minimum.

We have argued that the Foster-Hart measure is no less subjective than other attempts in the non-Bayesian literature. In our view presenting a measure as being ‘objective’ runs contrary to operationalizing it, in ways that are either realistic or easy to communicate. But it is in the way this measure can be operationalized where we find its main contribution. In particular, it is designed to help prevent adverse outcomes without dictating actions. On the one hand therefore, it helps fulfill the prudential role of a social planner without, on the other hand, preventing decision-makers from identifying themselves what constitutes an optimal action.

References


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