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On the Emergence of a Binding Zero Lower Bound

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Abstract

Overoptimism regarding productivity developments is often seen as the root of the worldwide cyclical downturn at the beginning of the 21st century. This paper develops a New Keynesian model with credit market imperfections that shows how overly positive productivity expectations initially drive the interest rate risk premium below its equilibrium value, boosting real economic growth. When a downward correction of the outlook appears inevitable, collateral value shrinks and the interest rate risk premium rises sharply. Given this risk premium, a substantially lower (risk free) policy rate is required in order to reduce market rates. In such a situation a binding zero lower bound can emerge.

JEL codes: E31, E32, E52

Key words: zero lower bound, financial accelerator, technology shocks

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1 Introduction

The zero bound on nominal interest rates has increasingly attracted attention from economists in recent years. In particular, Japan hitting the zero bound has triggered an intense debate on monetary policy effectiveness at low inflation and interest rate levels. Most literature in this field focuses on ways to escape from a situation in which the monetary authority is no longer able to stimulate the economy by reducing official interest rates. [Krugman (1998)] and [Svensson (2001)], for instance, show that an escape is costly though conceivable. [Krugman (1998)] emphasises the importance of lifting expected inflation in order to reduce real market interest rates. [Svensson (2001)] suggests unlimited foreign exchange intervention to initially depreciate the home currency, so as to create inflationary expectations. The more fundamental issue of how a binding zero lower bound can arise has attracted far less attention in the recent literature.¹ This is surprising, since one might suspect that knowledge on the origins of a binding zero lower bound can be helpful in its prevention.

The focus of this paper is on the causes of a binding zero lower bound. On the basis of shocks similar in magnitude to those observed historically, [Orphanides and Wieland (1998)] and [Reifschneider and Williams (2000)] conclude that hitting the zero bound is unlikely to occur in the US. [Viñals (2001)] draws the same conclusion for the euro area. Their models do not take into account structural factors that can undermine growth expectations and depress total demand. [Ullersma (2002)] argues that these factors can reinforce shocks, making a binding zero lower bound more likely. In particular, mechanisms that work through the balance sheets of private households seem capable of exacerbating cyclical fluctuations in the economy [Borio et al. (2001)]. Balance sheet effects appear to have become more important over recent years, following the increasing leverage of households on financial markets. This paper examines these issues in a New Keynesian framework along the lines of [Clarida et al. (1999)], extended with a financial accelerator mechanism. Through this mechanism, financial market imperfections can cause pronounced amplifications of shocks. While the model presented can be regarded as a stylised representation of the [Bernanke et al. (1999)] model, the dynamics are fully described by the New Keynesian Phillips curve, an IS type equation, the external finance premium and a specification of monetary policy. This allows for a transparent evaluation of the interaction of

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¹There exists a related literature on the sources of the Great Depression, with seminal contributions from [Fisher (1933)] and [Friedman and Schwartz (1963)].
economic shocks and financial market imperfections.

The paper’s main contribution to the existing literature is that it shows analytically how a binding zero lower bound can emerge. Financial accelerator effects, in conjunction with misperceptions regarding future productivity growth, appear pivotal. This is noteworthy, since uncertainty about productivity prospects has caused an intense debate ever since US productivity growth moved to unprecedented rates in the second half of the 1990s [Roberts (2000), e.g.]. In the model, a wave of overoptimism regarding future productivity growth, generated by favourable, but temporary, developments in the real economy, tends to prolong cyclical upturns. The underlying mechanism is that higher expected future (productivity) growth reduces the cost of borrowing by raising the value of collateral\(^2\), thereby (further) stimulating current activity. With expectations of productivity growth readjusting gradually to sustainable levels, the value of collateral may eventually decline to such an extent that real activity is depressed significantly. This process can result in interest rates reaching a binding zero lower bound.

The remainder of the paper is organised as follows. Section 2 presents the model. Section 3 shows how the interaction of misperceptions and financial accelerator effects can bring about a binding zero lower bound. Section 4 draws policy conclusions.

2 Model

The model used in this paper is a New Keynesian general equilibrium model that allows for credit market imperfections in the vein of [Barro (1976)] and [Jaffee and Russell (1976)]. The representative household maximises its utility under certain constraints. Its intertemporal decision problem can be considered a project involving planned consumption, planned labour supply and planned deposit holdings over an infinite horizon. Project financing is sought from income streams, initial wealth and bank loans. At the end of each time period (i.e. ex post), the household may be ‘unlucky’ in that it is not able to meet its loan obligations because of unanticipated economic shocks. Otherwise, it will be ‘lucky’ (the typology is due to [Jaffee and Russell (1976)]). In case of default, the household has limited liability, so that the bank bears the shortfall. As a consequence, the bank imposes a risk premium on the

\(^2\)The emphasis is on the balance sheet channel, working through the financial position of households. The bank lending channel is represented by the sensitivity of the cost of borrowing to private sector net wealth. Both types of credit channels are surveyed in [Walsh (2003)], Chapter 7.
household, on top of the risk-free rate of interest on deposits. This external finance premium, which varies inversely with the household’s net worth, tends to amplify economic shocks. This phenomenon is usually referred to as the financial accelerator mechanism.

Consumption goods are supplied under perfect competition by final-goods producing firms, which use inputs purchased from firms producing intermediate goods. These last firms use labour as input and act as monopolistic competitors and staggered-price setters in the sense of [Calvo (1983)], which gives rise to real effects of monetary policy. From these assumptions, a New Keynesian Phillips Curve is derived, relating actual consumer price inflation to expected future inflation and expected future marginal costs.

Monetary policy is modelled through an instrument rule, in line with other New Keynesian models.

2.1 IS curve

The aggregate behavioural equations follow from explicit optimisation by households and firms, as shown in the appendices. The representative household is infinitely-lived. In line with [Ireland (2001), e.g.] an isoelastic utility function is assumed. The household’s objective is to maximise:

$$E_t \left\{ \sum_{i=0}^{\infty} \omega^{t+i} \left( \frac{Y_{t+i}^{1-\sigma}}{1-\sigma} \frac{(D_{t+i}/P_{t+i})^{1-\eta}}{1-\eta} - \phi_2 \frac{L_{t+i}^{1+\phi}}{1+\phi} \right) \right\},$$

subject to a flow budget and a solvency constraint, respectively:

$$D_{t+1} - \ell_{t+1} = \max[0, D_t - \ell_t + W_t L_t - P_t Y_t + i_{D,t} D_{t+1} - i_{L,t} \ell_{t+1}],$$

$$\lim_{T \to \infty} \left( \frac{1}{1+i_{L,T}} \right)^T (D_T - \ell_T) \geq 0,$$

where $Y$ is the level of consumption, $D$ deposits, $P$ the consumption-based price level, $L$ the amount of labour supply, $\ell$ loans, $W$ the nominal wage

3Capital accumulation is ignored, as the analysis focuses on developments at the business cycle frequency.

4The derivation of the model’s equilibrium conditions follows the methods described in [King and Wolman (1996)], [Woodford (2003)] and [Yun (1996)]. Appendix A shows an overall picture of the current transactions among the different sectors in the economy.

5Rather than following the money-in-the-utility function approach, deposit holdings enter the utility function directly.
rate, \( \omega \) the discount factor, \( t \) the time index, and \( \sigma, \eta \) and \( \phi \) are elasticities. All coefficients (mostly in Greek) are positive, unless specified otherwise. At time \( t \), \( D_t \) and \( L_t \) are given and the household decides on \( D_{t+1}, L_{t+1}, Y_t \) and \( \ell_t \). \( \mathcal{E}_T \) denotes \( E_T \), i.e. the expected value of \( \ell \) at \( T \), expected at \( t \). \( i_D \) is the risk-free nominal interest rate on \( D \), while \( i_L \) is the risk-bearing nominal interest rate on \( \ell \). Therefore, \( i_{L,t} \geq i_{D,t} \). Note that negative values for end of period net financial wealth (\( V_{t+1} = D_{t+1} - \ell_{t+1} \)) are ruled out by eq. 2. This is because the household can choose to default on its loan obligations, resulting in a zero net financial wealth position at the end of the period (see Section 2.2). In equilibrium, households assume that these interest rates are constant over time. Utility maximisation by the household leads to a familiar IS curve, in so far that the risk bearing nominal interest rate \( i_L \) replaces the risk-free rate \( i_D \) (see appendix B):

\[
x_t = x_{t+1|t} - \frac{1}{\sigma} (i_{L,t} - \pi_{t+1|t} - \Xi_{L,t} + \Xi),
\]

where \( x_t = \log Y_t - \log Y_{t+1|t} \) is the output gap, and \( \pi_t = \log P_t - \log P_{t-1} \) is the inflation rate.8,9

### 2.2 External finance premium

The risk-bearing interest rate \( i_L \) is determined in the credit market, where all borrowers are treated symmetrically by the banks. This follows from the representative household framework, which implies that there is no role for idiosyncratic risks or distributional issues among households.

Consider a standard single period loan of amount \( \ell \) provided by a bank to a household at the beginning of period \( t \). The full principal plus interest is due at the end of the period, with the loan secured by collateral \( \mathcal{C} \). Collateral consists of all means to fulfill the household’s loan obligations, i.e. the sum of his assets and income minus his expenditure at the end of the period:

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8In equilibrium, the representative household can borrow and lend at the same moment. Such behaviour is rational, since deposits not only yield interest income but also direct utility.

9An upper bar denotes the value that is consistent with a balanced growth path.

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\[ C_{t+1}^{ep} = (1 + i_{D,t}) D_{t+1}^{ep} + W_{t}^{ep} L_{t}^{ep} - P_{t}^{ep} Y_{t}^{ep}, \]  
where the superscript \( ep \) denotes ex-post values. \( C_{t+1} \) is stochastic at the time of loan negotiations, since the ex-post value of \( W_{t} L_{t} \) may deviate from its ex-ante value. This follows from the assumption that changes in firms’ turnover translate one-for-one into labour income changes (see appendix A). It is assumed that \( C_{t+1} \) is characterised by a uniform distribution around a positive mean.\(^\text{10}\) The lower bound of the distribution is determined by \( C_{t+1} = 0 \), since negative collateral has no economic content. If the ex-post collateral value \( (C_{t+1}^{ep}) \) appears equal or higher than principal plus interest, the household will comply with its loan obligations, and financial wealth \( (V) \) at the end of the period is:

\[ V_{t+1} = C_{t+1}^{ep} - (1 + i_{L,t}) L_{t+1}. \]  
If the ex-post collateral value turns out to be lower than its loan obligations, the household will default, implying that \( V_{t+1} = 0 \). The probability of default is:

\[ \Pr_{t} \left[ C_{t+1}^{ep} < (1 + i_{L,t}) L_{t+1} \right] = \frac{1}{2} \frac{(1 + i_{L,t}) L_{t+1}}{C_{t+1}^{ep}}, \]  
where the expected amount paid in case of default (\( \tilde{C} \)) is:

\[ \tilde{C}_{t+1} = E_{t} \left[ C_{t+1} | C_{t+1}^{ep} < (1 + i_{L,t}) L_{t+1} \right] = \frac{1}{2} (1 + i_{L,t}) L_{t+1}. \]  
Assume that banks obtain their funds at the constant risk-free interest rate \( i_{D} \), which is set by the central bank, and that they have no other costs. The banking sector is characterised by constant returns to scale, implying that banks serve as many borrowing households as are forthcoming. They are assumed to be risk-neutral and to maximise the expected value of their profits (\( \Pi_{\text{bank}} \)):

\[ \Pi_{\text{bank}}^{t+1 | t} = \left( 1 - \Pr_{t} [..] \right) (1 + i_{L,t}) L_{t+1} + \Pr_{t} [..] \tilde{C}_{t+1} - (1 + i_{D,t}) L_{t+1}. \] 
\(^\text{10}\)The assumption on the specific probability distribution is arbitrary, since the probability distribution of technology shocks is unknown (see below). For convenience, a uniform distribution is assumed. Assuming a normal distribution would yield similar results, compare [Barro (1976)].
With a competitive loan market, a zero expected profit condition must hold, while \( C_{t+1|t} = (1 + i_{D,t}) D_{t+1} \), since \( W_t L_t \) and \( P_t Y_t \) will cancel out in eq. 5 (see appendix A). Taking also into account that \( 0 \leq \text{Pr}_t[.] \leq 1 \), it follows that:

\[
\text{Pr}_t[.] = 1 - \sqrt{1 - \chi},
\]

(10)

\[
\frac{1 + i_{L,t}}{1 + i_{D,t}} = \frac{2}{1 + \sqrt{1 - \chi}},
\]

(11)

where \( \chi = \frac{D_t}{D_{t+1}} \) is the initial debt burden and \( 0 \leq \chi \leq 1 \). This leads to:

**Proposition 1** The external finance premium, being the wedge between the interest rates on loans and (risk-free) deposits, is convex and strictly increasing in \( \chi \).

**Proof.** See appendix B.

Intuitively, the external finance premium determines the financial accelerator effects included in the IS curve eq. 4. This premium rises progressively as the debt burden increases, since the risk of default rises more than proportionally.

### 2.3 Role of technological progress

The level of the nominal external finance rate that is consistent with a balanced growth path \( \bar{r}_L \) is determined by technological progress (see appendix C and D for a derivation):

\[
\bar{i}_{L,t} = \bar{r}_{L,t} + \bar{r} = \rho + \mu (a_{t+1} - a_t) + \bar{r},
\]

(12)

where \( \rho \) is the time discount rate, \( a = \log A \), and \( A \) is labour productivity per hour in units of consumption goods. The corresponding production function reads (see appendix C):

\[
y_t = a_t + l_t.
\]

(13)

A rise in productivity growth will drive up the steady state real interest rate as households try to borrow at initial steady state rates to bring higher future consumption forward in time. Technological progress is represented by:

\[
a_t = a_{t-1} + \alpha_1 + \alpha_2 x_t + \varepsilon_t^a,
\]

(14)
where \( \varepsilon_t^a \) is a technology shock, of which the moments of the probability distribution are unknown. The \( a_{t-1} \)-component expresses the assumption that new techniques, once introduced, will be available forever. This is in line with the real business cycle theory [Cogley and Nason (1995), e.g.]. The output gap component is included following [Chang et al. (2002)]. This allows for systematic changes in productivity, associated with skill accumulation through work experience (learning-by-doing). Model consistent expectations are:

\[
a_{t+1|t} = a_t + \alpha_1 + \alpha_2 x_{t+1|t} + \varepsilon^a_{t+1|t}.
\]

Since the moments of the probability distribution of \( \varepsilon^a_t \) are unknown, the value of \( \varepsilon^a_{t+1|t} \) is an open issue. I assume \( \varepsilon^a_{t+1|t} = \varepsilon^a_t \).

### 2.4 Phillips curve

Staggered nominal price setting in the spirit of [Calvo (1983)] is assumed. The output gap varies proportionally with marginal costs, except for cost-push shocks (\( \varepsilon^\pi \)). This results in a New Keynesian Phillips curve, as is shown in appendix C:

\[
\pi_t = \omega \pi_{t+1|t} + \kappa x_t + \varepsilon^\pi_t,
\]

where the discount factor is \( \omega (= 1 - \rho) \).

### 2.5 Monetary policy

In line with other New Keynesian models, the model is closed by a monetary policy instrument rule [Svensson (1999), in terms of]:

\[
i_{D,t} = \rho_i i_{D,t-1} + (1 - \rho_i) \left( \rho_\pi \left( \pi_{t+1|t} - \bar{\pi} \right) + \rho_x x_t + E_t \bar{\pi}_{D,t} + \bar{\pi} \right) + \varepsilon^i_t.
\]

where \( \rho_i \) is an indicator for the degree of interest rate smoothing, \( r_D \) is the real policy interest rate and \( \varepsilon^i_t \) covers zero-mean policy interest rate shocks. This specification is based on [Clarida et al. (1998)]. They emphasise that this rule provides a good description of interest rate setting by major central banks around the world. This policy rule generally yields a stable equilibrium. Combining insights on technological misperceptions, the external finance premium and the policy rule results in:
**Proposition 2** If technological progress turns out to be lower than expected \((\Delta a_{t+1} < \Delta a_{t+1}^E)\), the external finance premium \((i_L - i_D)\) will increase and the balanced growth level of the nominal interest rate on loans \((\tilde{i}_L)\) will decrease. Both effects tend to lower \(i_D\).

**Proof.** See appendix D. ■

The intuition behind Proposition 2 is that a downward correction of overoptimistic expectations on technological growth lowers households’ earning capacity, which reduces wealth and lifts the external finance premium. In addition, the fall in growth prospects lowers the steady state level of the interest rate that drives intertemporal decisions \((\tilde{i}_L)\). Then, policy rule eq. 17 calls for a reduction of \(i_D\), in order to avoid a monetary policy contraction.

### 3 Transmission of shocks

The model allows for the analysis of a wide variety of shocks. Here, I present the effects of several shocks to technology and one to net worth in order to shed light on the interactions between misperceptions regarding technological progress and the fragility of private sector balance sheets. To highlight the specific impact of financial accelerator effects, each shock is considered in an environment with no \((\chi = 0)\) and strong \((\chi = 0.9)\) accelerator effects. Table 1 summarizes the discussion below.

**TABLE 1 ABOUT HERE**

#### 3.1 Calibration

The calibration of the quarterly model is largely based on [Galí (2002)]. The time discount factor \(\omega\) is 0.99, which is consistent with a time discount rate \(\rho\) of about 4 percent. As to the elasticities in the utility function it is assumed that \(\sigma = 0.85, \eta = 1, \) and \(\phi = 1\). The scaling parameters are \(\phi_1 = \phi_2 = 1\). Trend productivity growth \(\alpha_1\) is 1 percent, and the learning-by-doing parameter \(\alpha_2\) is set at 0.3. In the New Keynesian Phillips curve (eq. 16), \(\kappa\) equals 0.17, which follows from combining the other parameter values with the assumption that prices are fixed on average for a year. The parameter values in the monetary policy rule are \(\rho_i = 0.9, \rho_\pi = 1.1, \rho_x = 0\) and \(\pi = 0\); these values are in line with a mild reaction to current inflation divergences from zero and significant inertia in nominal interest rates.
3.2 Misperceptions without financial accelerator

As a benchmark case, consider a technology shock in an environment with $\chi = 0$, where the interest rate on loans equals the monetary policy rate (or interest rate on deposits) ($i_L = i_D$). The details of the dynamics of the model following this and other shocks are provided in appendix E. Figures 1-5 show the impulse responses of key variables, illustrating percentage deviations from the initial steady state values. In the benchmark case (Figures 1 and 2) economic agents perceive the technological growth shock as persistent ($\varepsilon^a_{t+1} | t = \varepsilon^a_t$). However, it may also turn out to be a transitory shock that only lifts the level of technology ($\varepsilon^a_{t+1} = 0$). Either way, output rises initially, since households bring planned higher future consumption forward in time. If the shock is transitory, the real interest rate will be above its balanced growth level due to the increase in the nominal policy interest rate against the background of overestimation of future technological progress. An output gap and negative inflation emerge. If the shock is persistent, the real interest rate is below its balanced growth level, which has increased. This follows from the sluggish adjustment of the nominal interest rates. Then, there will be excess demand and inflation. In both cases economic agents adjust their expectations and variables converge gradually towards the new steady state.

3.3 Misperceptions in the presence of a financial accelerator

Consider now a technology shock in a financial accelerator environment (Figures 3 and 4). For the same reasons as in the case without a financial accelerator, an output gap emerges if the shock is only transitory ($\varepsilon^a_{t+1} = 0$), whereas excess demand will result if the shock is persistent ($\varepsilon^a_{t+1} = \varepsilon^a_t$). In the former case, the initial increase in the nominal policy interest rate follows from overoptimistic technological growth expectations. When expectations do not materialise, the nominal policy interest rate will have to be reduced considerably in order to lower the interest rate on loans, which steers the

11 During the shock, the unstable eigenvalues of the system of equations are moving $i_D (= i_L)$ away from their equilibrium values. The stable eigenvalues determine the adjustment speed back to steady state (see appendix E).

12 Inflation and the output gap are jump variables. Their initial levels are determined by the predetermined variables $i_D$ and $i_L$, the model parameters and the intensities of the shocks.

13 If the shock is transitory, the steady state levels $\bar{T}_L, \bar{T}_D, \bar{\pi}$ and $\bar{\pi}$ are unchanged.
The sharp reduction in the nominal policy interest rate is necessary since the adjustment of overoptimistic technological prospects lowers net worth, increasing the default risk. As a result, the external finance premium will rise in line with Proposition 1, putting upward pressure on the interest rate on loans. The path for the policy interest rate follows from its sluggish adjustment and the new balanced growth level of the loan rate. The substantial reduction in the policy interest rate is required in order to lower market rates, boosting the economy in the direction of the new balanced growth path in line with Proposition 2. In such a situation the policy interest rate can reach the zero lower bound.

If the shock turns out to be persistent, the nominal interest rate on loans will have to rise further after its initial increase. However, net worth effects lower significantly the external finance premium. The net additional change in the policy rate is ambiguous: with substantial financial accelerator effects as shown in the example, a decrease in the nominal policy interest rate is consistent with an increase in the nominal interest rate on loans. The new lower balanced growth value of the nominal policy interest rate is not associated with a binding zero lower bound, since the economy is not in need of monetary stimuli. Instead, this example shows that permanent higher technology growth can go hand in hand with a lower steady state level of the monetary policy interest rate.

The persistence of the technology shock may also turn out to be somewhere in between a transitory and a fully persistent shock. Intermediate cases are reported in table 1. More persistent misperceptions regarding the increase in the technology growth level lift the risk of a binding zero lower bound.

The 3.4 Net worth shock

Finally, consider a permanent shock to net worth in the financial accelerator case. Recall from section 2 that when agents adjust downward their expectations about future technological progress, (perceived) net worth falls. Figure 5 shows that a downward shock to net worth initially lowers demand, the output gap and inflation, since the nominal interest rate on loans - which drives the real economy - rises in line with Proposition 1 due to the sharp

14 Again, the stable eigenvalues determine the adjustment speed towards the new steady state after the shock is over. However, with high levels of \( \chi \), eigenvalues are complex and a cyclical pattern emerges. The eigenvalues are such that after the shock is over \( i_L \) continues to move away from the new steady state until convergence takes place.
increase in the external finance premium. The central bank will try to counterweight this increase, which requires a permanent lower nominal policy interest rate. This illustrates an additional manner in which a binding zero lower bound can emerge.

FIGURE 5 ABOUT HERE

4 Concluding remarks

It follows from the analysis that a binding zero lower bound on nominal policy interest rates is most likely to emerge when overoptimistic expectations about technological progress are revised downward in an environment with fragile private sector balance sheets. At times, these factors have been driving forces behind strong reductions in nominal policy interest rates towards the zero lower bound. [Calomiris (1993)] draws attention to the collapse of the ‘new-age’ optimism of the 1920s during the early phase of the Great Depression and its long-run effects through the structure of credit markets and the balance sheet of borrowers. From 1932 to the mid 1940s the US nominal monetary policy interest rate was close to zero. The Japanese slump of the 1990s was also preceded by a period of overoptimism on future economic growth. The subsequent downscaling of economic prospects, which was reflected in asset price declines, has lowered perceived future earning capacity and reduced private sector balance sheets [Borio et al. (2001), e.g.]. This has resulted in a surge in non-performing loans, higher default risks and a rise in risk premia, putting downward pressure on the policy interest rate. From 1999 onwards the Japanese nominal monetary policy interest rate was at or very close to the zero lower bound. Similar, but more modest, patterns as in Japan have been observed in the US and Europe in the early 2000s, after the downward revision of ‘new economy’ optimism. Nominal monetary policy interest rates in both the US and Europe have fallen to levels not seen since the early 1960s, but they have not reached the zero lower bound. More optimistic views on future economic growth than in Japan might be an important factor, in particular in the US. On the other hand, in Europe household indebtedness in relation to production has remained substantially below that in the US.

Against this background, future work could focus on the empirical validation of the model. In addition, future research could shed light on how the emergence of a binding zero lower bound can be avoided. This paper suggests that the first-best policy option is removing the causes of the problems. The effects of the financial accelerator can be mitigated through a
reduction in the level of $\chi$, possibly through prudential supervision and regulation. However, misperceptions on technological progress and a financial accelerator effect are very much facts of life. This might call for a change in monetary policy. One way forward would be choosing an inflation target substantially higher than zero, increasing the room for manoeuvre for nominal interest rates. Unfortunately, such a policy action would not reduce the magnitude of the cyclical fluctuations in output, inflation and interest rates and also brings with it the costs of higher inflation. Choosing a different policy rule is less likely to be helpful as misperceptions hit the economy irrespective of the weights in the policy interest rate rule. Only if the central bank has at its disposal information on the sustainability of a boom can it avoid a binding zero lower bound. In such a case it is possible to counter the build-up of economic and financial fragilities, rather than continue to follow a monetary policy instrument rule. Generally, this will require information outside the scope of the stylised model presented.
References


<table>
<thead>
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<th>Reference</th>
<th>Description</th>
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| Meijdam and Verhoeven (1998)                | Meijdam, A.C. and M.J. Verhoeven (1998), 'Comparative Dynamics in...


5 Appendix A

Social accounts

There are three sectors in the economy: (consumer) households, firms, and banks. Within the firm sector, a distinction is made between intermediate and final goods producing firms. The banking sector also consists of two subsectors: the central bank and the commercial banks that are the financial intermediaries. The behavioural relations of the sectors are studied in the main text and in appendices B and C. This appendix sketches the complete macroeconomic representation of the current operations in the model economy. To do so, the social accounts are presented in the tradition of [Malinvaud (1985)].

The equality between resources and uses of goods requires that production in value terms \( P_t Y^*_t \) equals nominal household demand \( P_t Y^d_t \) plus stock accumulation \( \Delta \text{Stock} \). Labour income is paid by firms to households: \( W_t L^d_t = W_t L^*_t \). The model features fully flexible labour markets, so that the labour market clears continuously (balanced market for labour). Similarly, households’ interest income on deposits equals banks’ costs \( i_{D,t} D_{t+1}^* = i_{D,t} D_{t+1}^d \). The central bank steers the (risk-free) interest rate on deposits. On the other hand, households’ interest costs on loans are a source of income for the banks \( i_{L,t} L_{t+1}^d = i_{L,t} L_{t+1}^* \). For households, the excess of income over costs results in an increase in deposits or a decrease in loans. For banks, this implies higher cash holdings, while firms’ cash holdings decline and their stock levels rise. This is presented in table A1.

Note that savings by different sectors \( (S_{\text{holds}}, S_{\text{firms}}, S_{\text{banks}}) \) can not be measured on their own. Savings are notional in the sense of [Clower (1965)]. Note also that the social accounts involve ex-post amounts, whereas supply and demand decisions involve ex-ante amounts. In this respect, it is assumed that firms earn zero ex-ante profits, implying that stock accumulation is not planned, nor is a change in cash holdings. Ex-post, unanticipated real shocks can cause a rise or fall in \( P_t Y^*_t \). If a rise occurs, the balanced labour market implies that there is an instantaneous distribution of additional labour income to households. Hence, there is excess supply of goods, a buyers’ market for goods cf. [Malinvaud (1985)]: \( P_t Y^d_t < P_t Y^*_t (= W_t L_t) \). If \( P_t Y^*_t \) falls, there is excess demand for goods, a sellers’ market for goods in terms of Malinvaud: \( P_t Y^d_t > P_t Y^*_t (= W_t L_t) \). With excess demand for goods, there are two possibilities. Either the households will still be able to comply with their loan obligations, or they will default. In the former case, there will be a deposit/cash transfer from households via the banking sector to
firms, which will lower their stocks. In the latter case, collateral will be transferred from the insolvent households to the banks, which will replenish the households’ payments. This default case is presented in table A2.

TABLE A1 AND A2 ABOUT HERE

Table A1: Social accounts, no default

<table>
<thead>
<tr>
<th>Households resources uses</th>
<th>Firms resources uses</th>
<th>Banks resources uses</th>
</tr>
</thead>
<tbody>
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<td>$W_t L_t^s$</td>
<td>$P_t Y_t^d$</td>
<td>$P_t Y_t^s$</td>
</tr>
<tr>
<td>$i_{D,t} D_{t+1}^s$</td>
<td></td>
<td>$\Delta Stocks_t$</td>
</tr>
<tr>
<td>$S_t^{holds}$</td>
<td></td>
<td>$W_t L_t^d$</td>
</tr>
<tr>
<td>$L_{t+1} - L_t$</td>
<td>$D_{t+1} - D_t$</td>
<td></td>
</tr>
<tr>
<td>$S_t^{firms}$</td>
<td></td>
<td>$i_{L,t} L_{t+1}^s$</td>
</tr>
<tr>
<td>$S_t^{banks}$</td>
<td>$D_{t+1} - D_t$</td>
<td>$\Delta Cash_t^{firms}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta Cash_t^{banks}$</td>
</tr>
</tbody>
</table>

Table A2: Social accounts, default

<table>
<thead>
<tr>
<th>Households resources uses</th>
<th>Firms resources uses</th>
<th>Banks resources uses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_t L_t^s$</td>
<td>$P_t Y_t^d$</td>
<td>$P_t Y_t^s$</td>
</tr>
<tr>
<td>$i_{D,t} D_{t+1}^s$</td>
<td></td>
<td>$\Delta Stocks_t$</td>
</tr>
<tr>
<td>$S_t^{holds}$</td>
<td></td>
<td>$W_t L_t^d$</td>
</tr>
<tr>
<td>$S_t^{firms}$</td>
<td></td>
<td>$i_{D,t} D_{t+1}^d$</td>
</tr>
<tr>
<td>$S_t^{banks}$</td>
<td>$D_{t+1} - D_t$</td>
<td>$\Delta Cash_t^{firms}$</td>
</tr>
<tr>
<td>$-D_t + C_{t+1}^{15}$</td>
<td></td>
<td>$\Delta Cash_t^{firms}$</td>
</tr>
<tr>
<td>$S_t^{firms}$</td>
<td></td>
<td>$S_t^{banks}$</td>
</tr>
<tr>
<td>$S_t^{holds}$</td>
<td></td>
<td>$D_{t+1} + C_{t+1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta Cash_t^{banks}$</td>
</tr>
</tbody>
</table>

$^{15}L_{t+1} - D_{t+1} = 0, C_{t+1} = (1 + i_{D,t}) D_{t+1} + W_t L_t - P_t Y_t$. 

20
6 Appendix B

Derivation of the IS curve

Taken together, the flow budget constraint eq. 2 and the solvency constraint eq. 3 can be rewritten as an intertemporal budget constraint:

$$D_t - L_t \geq \frac{1}{1 - Pr_t[\ldots]} E_t \sum_{i=0}^{\infty} \frac{1}{(1 + i_{L,t})^{t+i+1}} \left\{ P_{t+i}Y_{t+i} - W_{t+i}L_{t+i} + (i_{L,t} - i_{D,t}) D_{t+i+1} \right\},$$

where $Pr_t[\ldots]$ is the default probability. The Lagrangian is as follows:

$$Z_t = E_t \left\{ \sum_{i=0}^{\infty} \omega^{t+i} \left( \frac{Y_{t+i}^{1-\sigma}}{\sigma} + \phi_1 \frac{(D_{t+i}^{1-\sigma_i} / P_{t+i}^{1-\sigma_i})^{1-\eta}}{(1 - \eta)} - \phi_2 \frac{L_{t+i}^{1+\phi}}{i_{L,t} - i_{D,t}} \right) + \right\},$$

where $z_t$ is the Lagrange multiplier and initial financial wealth $V_t = D_t - L_t$ is predetermined.

The Kuhn-Tucker maximum conditions are:

$$\frac{\delta Z_t}{\delta Y_{t+i|t}} = \omega^{t+i} Y_{t+i|t} - \frac{z_t}{1 - Pr_t[\ldots]} (1 + i_{L,t})^{t+i+1} \leq 0,$$

$$\frac{\delta Z_t}{\delta (L_{t+i|t} / P_{t+i|t})} = \omega^{t+i} \phi_1 \left( \frac{D_{t+i}^{1+\phi}}{P_{t+i}^{1+\phi}} \right)^{-\eta} - \frac{z_t}{1 - Pr_t[\ldots]} (1 + i_{L,t})^{t+i+1} \leq 0,$$

$$\frac{\delta Z_t}{\delta L_{t+i|t}} = -\omega^{t+i} \phi_2 L_{t+i|t}^{1+\phi} + \frac{z_t}{1 - Pr_t[\ldots]} (1 + i_{L,t})^{t+i+1} \leq 0,$$

$$\frac{\delta Z_t}{\delta z_t} \leq 0, Y_{t+i|t} \geq 0, \frac{D_{t+i+1|t}}{P_{t+i|t}} \geq 0, L_{t+i|t} \geq 0, z_t \geq 0,$$

$$\frac{\delta Z_t}{\delta Y_{t+i|t}} = 0, \frac{\delta Z_t}{\delta (L_{t+i|t} / P_{t+i|t})} = 0,$$

$$\frac{\delta Z_t}{\delta L_{t+i|t}} = 0, z_t \frac{\delta Z_t}{\delta z_t} = 0.$$

By combining the equations for $Y_t$ with their $t+1$ counterpart, one obtains the Euler equation, since $\frac{z_t}{1 - Pr_t[\ldots]}$ is the same in both expressions:
\[
\omega \left( \frac{Y_{t+1|t}}{Y_t} \right)^{-\sigma} = \frac{1}{1+i_{L,t}} \left( \frac{P_{t+1|t}}{P_t} \right).
\]  

(26)

Approximating eq. 26 around its balanced growth path results in the IS curve:

\[
x_t = x_{t+1|t} - \frac{1}{\sigma} (i_{L,t} - \pi_{t+1|t} - \bar{\gamma}_{L,t} + \bar{\pi}).
\]  

(27)

**Proof of Proposition 1**

It follows from eq. 11 that:

\[
i_{L,t} - i_{D,t} = f(\chi) \approx -\ln \frac{1}{2} - \sqrt{1 - \chi},
\]  

(28)

\[
\frac{\partial f(\chi)}{\partial \chi} \approx \frac{\partial (-\ln \frac{1}{2} - \sqrt{1 - \chi})}{\partial \chi} = \frac{1}{2\sqrt{(1 - \chi)}} \geq \frac{1}{2},
\]  

(29)

\[
\frac{\partial^2 f(\chi)}{\partial \chi^2} \approx \frac{\partial \left( \frac{1}{2\sqrt{(1 - \chi)}} \right)}{\partial \chi} = -\frac{1}{4(-1 + \chi) \sqrt{(1 - \chi)}} \geq \frac{1}{4}.
\]  

(30)

This proves Proposition 1.
7 Appendix C

Firms

In this appendix the labour demand function, the zero-profit condition for firms under monopolistic competition, and the New Keynesian Phillips curve are derived.

Real aspects

Final output is produced using inputs from a continuum of intermediate goods producers. The model features perfect competition among final goods producers. The production function for final output is given by the following CES aggregator (in the tradition of [Dixit and Stiglitz (1977)]):

\[ Y_t = \left( \int_0^h Y(j)^{\frac{\theta - 1}{\theta}} dj \right)^{\frac{\theta}{\theta - 1}} \quad j \in [0, h], \theta > 1, \]  \hspace{1cm} (31)

where \( \theta \) is the elasticity of substitution between inputs. As \( \theta \) increases, inputs become closer substitutes; alternatively, market power of intermediate goods producers is decreasing in \( \theta \). \( Y_t \) is the period-\( t \) final output of consumption goods and \( Y_t(j) \) is the input of intermediate good \( j \) in period \( t \). There are \( h \) intermediate goods producing firms.\(^{16}\) Final goods producers minimise the costs of producing \( Y_t \):

\[ \int_0^h P_t(j) Y_t(j) dj \]  \hspace{1cm} (32)

subject to the production function of final output eq. 31. \( P_t(j) \) is the price of input \( j \) in period \( t \). The first order condition is:\(^{17}\)

\[ -P_t(j) + \Lambda_1 \frac{\theta}{\theta - 1} \left( \int_0^h Y_t(j)^{\frac{\theta - 1}{\theta}} dj \right)^{\frac{1}{\theta - 1}} \frac{1 - \frac{1}{\theta}}{\theta} Y_t(j)^{\frac{1}{\theta}} = 0, \]  \hspace{1cm} (33)

where marginal costs (=lagrangian multiplier (\( \Lambda_1 \))) equal the price of the final output, since the final goods producer operates in a competitive market. Therefore, the demand for a single input \( j \) is a decreasing function of its relative price:

\(^{16}\)A limited number of intermediate goods producing firms gives rise to a discrete production function for final output. This discrete function is approximated by its linear representation in eq. 31.

\(^{17}\)Note that eq. 31 is continuously differentiable and satisfies the Inada conditions.
\[
\frac{Y_t(j)}{Y_t} = \left( \frac{P_t(j)}{P_t} \right)^{-\theta}. \tag{34}
\]

This can be rewritten in log-linear terms as:

\[
y_t(j) = -\theta (p_t(j) - p_t) + y_t. \tag{35}\]

\(P_t\) represents the amount the final goods producer obtains for one unit of the composite good \(C_t\). Substitution of eq. 34 in eq. 31 gives:

\[
Y_t = \left( \int_0^h \left( \frac{P_t(j)}{P_t(j)} \right)^{\theta - 1} \frac{1}{P_t(j)} dj \right)^{\frac{1}{1-\theta}}, \tag{36}\]

\[
P_t = \left( \int_0^h P_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \tag{37}\]

The intermediate goods market is monopolistically competitive, so that suppliers behave as price setters. Cournot behaviour is assumed. In other words, all firms behave as if they are completely independent of each other. Each firm produces a differentiated good with technology:

\[
Y_t(j) = A_t L_t(j), \tag{38}\]

where \(A_t\) is an aggregate technology index. Eq. 38 can be expressed in log-linear form as:

\[
y_t(j) = a_t + l_t(j). \tag{39}\]

Given the representative agent character of the model, this eq. can be written as eq. 13 in the body of the text. Technological progress is characterised by:

\[
\log \left( \frac{A_t}{A_{t-1}} \right) = \alpha_1 + \alpha_2 \log \left( \frac{Y_t}{Y_t} \right) + \varepsilon^a_t, \tag{40}\]

where \(\varepsilon^a_t\) is a technology shock. In line with the real business cycle theory [Cogley and Nason (1995), e.g.] it is assumed that new techniques, once introduced, will be available forever. The output gap component is included following [Chang et al. (2002)]. This allows for systematic changes in productivity, associated with skill accumulation through work experience (learning-by-doing). Eq. 40 can be rewritten as in eq 14 in the body of the text.
The intermediate goods producing firm’s objective is to maximise profits (\(\Pi (j)\)):

\[
\Pi_t (j) = P_t (j) Y_t (j) - W_t L_t (j)
\]

subject to the demand function eq. 34 and the production function for intermediate goods eq. 38. The wage rate is assumed exogenous to the firm. The first order conditions are:

\[
Y_t (j) - \theta \Lambda_2 P_t (j)^{-1} \left( \frac{P_t}{P_t (j)} \right)^\theta Y_t = 0,
\]

\[
-W_t + \Lambda_3 A_t = 0.
\]

It follows that:

\[
\Lambda_2 = \frac{P_t (j)}{\theta},
\]

\[
\Lambda_3 = \frac{W_t}{A_t},
\]

\[
P_t (j) = \Lambda_2 + \Lambda_3 = \frac{\theta}{\theta - 1} \frac{W_t}{A_t},
\]

where \(\frac{\theta}{\theta - 1}\) is the mark-up per unit \(Y_t (j)\). Real marginal costs are the inverse of the mark-up: \(\frac{\theta - 1}{\theta}\). Combining eq. 38 and eq. 46 results in the labour demand function:

\[
L^d_t (j) = \frac{Y_t (j)}{A_t (j)} = \frac{\theta - 1}{\theta} \frac{Y_t (j)}{P_t (j)},
\]

which can be rewritten in log-linear form as:

\[
L^d_t (j) = y_t (j) - a_t = y_t (j) - w_t + p_t + \log \frac{\theta - 1}{\theta}.
\]

It is assumed that positive economic profits will instantaneously attract new firms into the intermediate goods industry, competing the profits away. Similarly, negative profits will drive away firms from the industry. This assumption is more common in long-term than in short-term models such as the one under consideration. It simplifies the analysis without changing it significantly. In line with this entry/exit assumption, a zero-profit restriction is introduced:
\[ P_t (j) = W_t L_t (j). \]

Since all firms behave in exactly the same way, it follows from eq. 38 and 46 that the price set by the representative firm can be written as:

\[
P_t (j) = \frac{1}{h} \left( \int_0^h P_t (i)^{1-\theta} \, di \right)^{-\frac{1}{\theta}} = \frac{\frac{\theta}{\vartheta - 1} W_t^\theta \int_0^h L_t (i) \, di}{\frac{1}{\vartheta} \left( \int_0^h Y_t (i)^{\frac{\theta}{\vartheta} - 1} \, di \right)^{\frac{1}{\theta - 1}}}.
\]

(50)

For the economy as a whole, this eq. can be rewritten as follows:

\[
P_t (j) = \frac{h^{\frac{\theta}{\vartheta - 1}} W_t L_t}{Y_t}.
\]

(51)

With representative firms, \( L_t (j) Y_t (j) = L_t Y_t \). It is clear from eq. 49 and 51 that 
\[ h^{\frac{\theta}{\vartheta - 1}} = 1; \] the number of intermediate producing firms equals \( \frac{\vartheta - 1}{\theta} \).

**Price setting**

Some sort of price stickiness is critical to generating significant real effects of monetary policy. Intermediate goods producing firms do not adjust their product prices flexibly to maintain a constant, profit-maximising mark-up. Instead, firms balance over time the one-time cost of changing prices against the benefit of staying close to the profit-maximising level of the mark-up.  

This is a common assumption in the New Keynesian literature. Staggered nominal price setting along the lines of [Calvo (1983)] is assumed. The intermediate goods producing firm is in a position to set a new price for her good at time \( t \). The price will apply at time \( t \) with certainty, with probability \( \vartheta \) at time \( t + 1 \), with probability \( \vartheta^2 \) at time \( t + 2 \), and so on. Provided a profit maximising solution exists, such a solution is characterised by the minimisation of production costs, which are conditional on the frequency of future price adjustments:

\[
\min_{P_t^*} \sum_{k=0}^{\infty} E_t \left\{ \left( \frac{Y_{t+k}}{Y_t} \right)^{-\sigma} \left( \omega \theta \right)^k \frac{Y_{t+k} (j)}{Y_{t+k}} \right\}^{1+\theta}
\]

subject to:

\[ \text{[For a complete treatment of the derivation of the New Keynesian Phillips Curve, see [Goodfriend and King (1997)], [King and Wolman (1996)] and [Woodford (2003)].] \]

\[ \text{[Deviations from the optimal price level trigger the entry or exit of new firms.]} \]
\[
\sum_{k=0}^{\infty} \vartheta^k E_t \left\{ \left( \frac{1}{1 + i L, t} \right)^k P^*_{t+k} (j) \right\} \leq V_t, \quad (53)
\]

\[
\frac{Y_{t+k} (j)}{Y_{t+k}} = \left( \frac{P^*_t}{P_{t+k}} \right)^{-\theta}.
\quad (54)
\]

This results in a solution for the optimal price \( P^*_t \). Since the optimal price is the same for all goods, the actual price at time \( T \) will either equal the price at \( T - 1 \), or be at the optimal level. The price index eq. 37 can be simplified as follows:

\[
P_t = \left( \vartheta P_{t-1}^{1-\vartheta} + (1 - \vartheta) (P^*_t)^{1-\vartheta} \right)^{\frac{1}{1-\vartheta}}.
\quad (55)
\]

The fraction of firms \( 1 - \vartheta \) that can change their price at \( t \) all choose the price level \( P^* \), since the optimal price level is the same for the differentiated products produced. The index of prices for non-adjusting firms equals the lagged price level due to the law of large numbers. Eq. 55 can be used to derive an expression in terms of inflation:

\[
\pi_t = \omega \pi_{t+1[t]} + \frac{(1 - \vartheta)(1 - \omega \vartheta)}{\vartheta (1 + \theta \phi)} (mc_t - \bar{mc}),
\quad (56)
\]

\[
\pi_t = \omega \pi_{t+1[t]} + \frac{(1 - \vartheta)(1 - \omega \vartheta)}{\vartheta (1 + \theta \phi)} (\phi + \sigma) (y_t - \bar{y}_t) + \varepsilon_t^n,
\quad (57)
\]

\[
\pi_t = \omega \pi_{t+1[t]} + \kappa x_t + \varepsilon_t^n.
\quad (58)
\]

This is the so-called New Keynesian Phillips curve, which is reproduced in eq. 16 in the body of the text. It expresses that firms base their pricing behaviour on expected future marginal costs. [Rotemberg and Woodford (1997)] show that there is an approximate relation between marginal costs and output. The longer prices are fixed on average, the less sensitive is inflation to changes in the output gap. Expected inflation is incorporated, since expected future output gaps influence current inflation. The error term captures deviations from \( (mc_t - \bar{mc}) = \kappa x_t \). It is referred to as the cost-push shock.
8 Appendix D

Balanced growth path
At the balanced growth path, all intermediate goods producing firms will choose a markup $\left( \frac{\theta}{\theta - 1} \right)$ in conformity with eq. 46. This implies that log real marginal costs ($mc$) and the log of the markup ($-\mu$) can be written as:

$$mc = -\mu = \log \left( \frac{\theta - 1}{\theta} \right) = w_t - p_t - a_t. \tag{59}$$

Taking account of the first order conditions that follow from the household’s utility maximisation and eq. 48, the balanced growth equilibrium is characterised as follows:

$$mc = -\log \phi + \frac{(\phi + \sigma) y_t - (1 + \phi) a_t}{\phi + \sigma}, \tag{60}$$

$$y_t = -\log \phi + \frac{1 + \phi}{\phi + \sigma} a_t, \tag{61}$$

$$l_t = y_t - \bar{a}_t = -\log \phi + \frac{1 - \sigma}{\phi + \sigma} a_t. \tag{62}$$

Taking into account eq. 14, it is also true that:

$$r_{L,t} = i_L - \pi = \rho + \sigma \frac{1 + \phi}{\phi + \sigma} \Delta a_{t+1} = \rho + \sigma \frac{1 + \phi}{\phi + \sigma} \alpha_1. \tag{63}$$

Proof of Proposition 2
By definition, $E_t \tau_{D,t} = E_t \tau_{L,t} - (i_L - i_D)$. $E_t \tau_{L,t}$ decreases if $\Delta a_{t+1} < \Delta a_{t+1|t}$, as follows from eq. 63.

$i_L - i_D$ increases if $\Delta a_{t+1} < \Delta a_{t+1|t}$ : It follows from appendix A that $W_{t+1} L_{t+1} = P_{t+1} Y_{t+1}^s < P_{t+1} Y_{t+1}^d$ if $\Delta a_{t+1} < \Delta a_{t+1|t}$. Therefore, $V_{t+1}$ will decline in line with eq. 6. This implies an increase in $\epsilon$ and $i_L - i_D$ in conformity with Proposition 1.

It follows that $E_t \tau_D$ decreases if $\Delta a_{t+1} < \Delta a_{t+1|t}$. Ceteris paribus, this lowers $i_D$ in line with eq. 17.

This proves Proposition 2.
9 Appendix E

System of equations

This appendix rearranges the model’s equations in order to obtain its equilibrium solution. The dynamics of the model are fully described by the IS curve, the New Keynesian Phillips curve, the external finance premium, changes in the debt burden, and the interest rate rule.

The initial debt burden \( \chi \) is a state variable, but changes in \( \chi \) influence the external finance premium. Changes in \( \chi \) are approximated by\[\begin{equation}
\chi_{t+1} = \chi_{t} + \frac{\epsilon_{t+1}}{D_{t+1}} - 1.
\end{equation}\]

In combination with eq. 11, this yields:
\[\begin{equation}
\frac{1 + i_{D,t}}{1 + i_{L,t}} = \frac{1}{2} \left( 1 + \sqrt{1 - \chi (1 + \epsilon_{t+j})} \right)
\approx \frac{1}{2} + \frac{1}{2} \sqrt{1 - \chi} - \frac{1}{4} \frac{\chi}{\sqrt{1 - \chi}} \epsilon_{t+j}.
\end{equation}\]

\( \epsilon_{t+j} \) can be substituted through the budget constraint, which relates the components of \( \epsilon \) to other economic variables. From the budget constraint eq. 2, and taking account of \( i_{L,t+1} = i_{L,t} \) and \( W_{t+1} = \rho_{t+1} Y_{t+1} \), it follows that:
\[\begin{equation}
\log L_{t+2} - \log L_{t+1} = \frac{1}{\chi} \left( d_{t+2} - d_{t+1} \right) + i_{L,t} - \frac{1}{\chi} i_{D,t},
\end{equation}\]

\[\begin{equation}
\epsilon_{t+1} = \frac{1 - \chi}{\chi} \left( d_{t+2} - d_{t+1} \right) + i_{L,t} - \frac{1}{\chi} i_{D,t}.
\end{equation}\]

The term \( \frac{1}{\chi} i_{D,t} \) can be neglected in the latter expression, since \( i_{D} \) is insignificant in relation to \( i_{L} \) for high values of \( \chi \), whereas for low values of \( \chi \) the \( \epsilon \)-term is insignificant in the determination of \( i_{L} \). Combining the first order conditions of the household’s utility maximisation and eq. 64 and 66, it follows that:
\[\begin{equation}
i_{L,t} \approx \frac{\chi}{2(\sqrt{1 - \chi} + (1 - \chi))} \left( \frac{1 - \chi}{\chi \eta} + 1 \right) i_{L,t-1} + i_{D,t} + \frac{1 - \chi}{2(\sqrt{1 - \chi} + (1 - \chi))} \left( 1 - \frac{1}{\eta} \right) \pi_{t} - \frac{1 - \chi}{2(\sqrt{1 - \chi} + (1 - \chi))} \eta \rho_{t},
\end{equation}\]

\[\begin{equation}
i_{L,t} \approx \chi_{0} i_{L,t-1} + i_{D,t} + \chi_{1} \pi_{t} + \chi_{2}.
\end{equation}\]
Using eq. 4, 14, 16, 17, 63, and 68, the equilibrium dynamics of the model can be described by the following system of difference equations:

\[
\begin{bmatrix}
\pi_{t+1} - \pi_t \\
x_{t+1} - x_t \\
i_{D,t} - i_{D,t-1} \\
i_{L,t} - i_{L,t-1}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{\sigma}(\chi_1 - \frac{1}{2}) + \frac{1}{\bar{\sigma}}\mu_1 \\
\chi_1 + \mu_1 \\
\chi_2 + \mu_4 \\
\frac{1}{\sigma}(\chi_1 - \frac{1}{2}) + \frac{1}{\bar{\sigma}}\mu_2
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\sigma} + \frac{\phi}{\sigma \bar{\sigma}} + \frac{1}{\bar{\sigma}}\mu_2 & 0 & 0 & 0 \\
\mu_1 & \mu_2 & \rho_1 - 1 & \mu_3 \\
\mu_4 & \rho_1 & (\chi_0 - 1) + \mu_3 &
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
x_t \\
i_{D,t-1} \\
i_{L,t-1}
\end{bmatrix}
+ \begin{bmatrix}
\frac{\chi_2}{\sigma} - \frac{\rho^2}{\sigma} - \frac{\rho}{\sigma} + \frac{1 + \phi}{\sigma \bar{\sigma}} \alpha_1 + \frac{1}{\bar{\sigma}}\mu_4 \\
\chi_2 + \mu_4 \\
\end{bmatrix},
\]

(69)

where \(\mu_1 = (1 - \rho_1)(\frac{\rho_x}{\sigma} - \chi_1)\), \(\mu_2 = (1 - \rho_1)(-\rho_x \frac{\rho}{\sigma} + \rho_x + \sigma \frac{1 + \phi}{\sigma \bar{\sigma}} \alpha_2)\), \(\mu_3 = -(1 - \rho_1)\chi_0\), \(\mu_4 = (1 - \rho_1)(\rho_x g_t^\pi + (1 - \rho_x) \pi + \rho + \sigma \frac{1 + \phi}{\sigma \bar{\sigma}} (\alpha_1 + \varepsilon_t^\pi) - \chi_2) + \varepsilon_t^\pi\).

[Blanchard and Kahn (1980)] have shown that a difference equation system with \(n\) predetermined and \(m\) jump variables has a unique nonexploding solution if there are exactly \(m\) negative eigenvalues. In system 69 there are two predetermined variables (\(i_D\) and \(i_L\)) and two jump variables (\(\pi\) and \(x\)). Therefore, stability requires two negative eigenvalues. The monetary policy instrument rule is such that the system is stable for reasonable values of the parameters in the model.

A continuous representation of system 69 has been solved by a direct method, involving a solution to its homogeneous version and a particular integral derived by variation of constants (Lagrange’s method), and Laplace transforms. For continuous transitory shocks, this results in:

\[
\begin{bmatrix}
\pi_t \\
x_t \\
i_{D,t} \\
i_{L,t}
\end{bmatrix}
= \begin{bmatrix}
v_{11} \\
v_{12} \\
v_{13} \\
v_{14}
\end{bmatrix} e^{\lambda_1 t} + \begin{bmatrix}
v_{21} \\
v_{22} \\
v_{23} \\
v_{24}
\end{bmatrix} e^{\lambda_2 t} + \Delta(t, T) \begin{bmatrix}
v_{31} \\
v_{32} \\
v_{33} \\
v_{34}
\end{bmatrix} e^{\lambda_3 t} + \begin{bmatrix}
v_{41} \\
v_{42} \\
v_{43} \\
v_{44}
\end{bmatrix} e^{\lambda_4 t},
\]

where \(\lambda_1\) and \(\lambda_2\) are stable eigenvalues; \(\lambda_3\) and \(\lambda_4\) are unstable eigenvalues. The parameter values are determined by the eigenvectors of system 69 and the intensities of the shocks. \(\Delta(t, T)\) is a Heaviside function (1 during the shock, 0 otherwise). For a transitory shock that fades out, the solution is:

\[
\begin{bmatrix}
\pi_t \\
x_t \\
i_{D,t} \\
i_{L,t}
\end{bmatrix}
= \begin{bmatrix}
v_{51} \\
v_{52} \\
v_{53} \\
v_{54}
\end{bmatrix} e^{\lambda_1 t} + \begin{bmatrix}
v_{61} \\
v_{62} \\
v_{63} \\
v_{64}
\end{bmatrix} e^{\lambda_2 t} + \begin{bmatrix}
v_{71} \\
v_{72} \\
v_{73} \\
v_{74}
\end{bmatrix} e^{-\omega t}.
\]
where \( \frac{1}{\tau} \) is the relaxation time (rate of decay) of the shock. For persistent shocks, the solution is:

\[
\begin{bmatrix}
\pi_t \\
x_t \\
i_{D,t} \\
i_{L,t}
\end{bmatrix}
= 
\begin{bmatrix}
v_{81} \\
v_{82} \\
v_{83} \\
v_{84}
\end{bmatrix}
\exp(\lambda_1 t) + 
\begin{bmatrix}
v_{91} \\
v_{92} \\
v_{93} \\
v_{94}
\end{bmatrix}
\exp(\lambda_2 t) + 
\begin{bmatrix}
0 \\
0 \\
\bar{i}_D \\
\bar{i}_L
\end{bmatrix},
\]

where the last vector shows the new steady state levels.
Table 1: Effects on nominal policy interest rates

<table>
<thead>
<tr>
<th>Shock</th>
<th>Character$^a$</th>
<th>$i_D^b$</th>
<th>Risk ZLB$^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>technology:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_t^a &gt; 0$</td>
<td>transitory$^d$</td>
<td>↑/↓</td>
<td>-/+</td>
</tr>
<tr>
<td>$\varepsilon_t^a &gt; 0$</td>
<td>trans. fading out$^d$</td>
<td>↑/↓</td>
<td>-/+</td>
</tr>
<tr>
<td>$\varepsilon_t^a &gt; 0$</td>
<td>trans. longer-lasting$^d$</td>
<td>↑/↓↓</td>
<td>-++</td>
</tr>
<tr>
<td>$\varepsilon_t^a &gt; 0$</td>
<td>persistent$^e$</td>
<td>↑/↓</td>
<td>-/-</td>
</tr>
<tr>
<td>net worth:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon &lt; 0$</td>
<td>persistent</td>
<td>≈ 0/↓↓</td>
<td>-/+++</td>
</tr>
</tbody>
</table>

$^a$Economic agents learn gradually about the character of the technology shock.

$^b$This column shows the medium term reaction of $i_D$ (i.e. after about two years). The first sign refers to the case with no financial accelerator, the second to the strong financial accelerator case.

$^c$The risk of the emergence of a binding zero lower bound is shown. ‘-’ refers to small risks, ‘+’ to some risks and ‘++’ to severe risks. Again, the first sign refers to the case with no financial accelerator, the second to the strong financial accelerator case.

$^d\varepsilon_t^a > 0$, $\varepsilon_{t+1}^a = \varepsilon_t^a$, $\varepsilon_{t+1}^a = 0$.

$^e\varepsilon_t^a > 0$, $\varepsilon_{t+1}^a = \varepsilon_t^a = \varepsilon_{t+1}^a$. 

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Figure 1: $\chi = 0$; transitory shock: $\varepsilon^a_0 > 0, \varepsilon^a_{t+1} = \varepsilon^a_t, \varepsilon^a_{t+1} = 0$

Figure 2: $\chi = 0$; persistent shock: $\varepsilon^a_0 > 0, \varepsilon^a_{t+1} = \varepsilon^a_t = \varepsilon^a_{t+1}$

Figure 3: $\chi = 0.9$; transitory shock: $\varepsilon^a_0 > 0, \varepsilon^a_{t+1} = \varepsilon^a_t, \varepsilon^a_{t+1} = 0$
Figure 4: $\chi = 0.9$; persistent shock: $\varepsilon^a_t > 0, \varepsilon^a_{t+1}|_t = \varepsilon^a_t = \varepsilon^a_{t+1}$

Figure 5: $\chi = 0.9$; net worth shock: $\varepsilon < 0$