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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.
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Abstract

We develop a model of two-sided markets that illustrates the role of bargaining power between the two sides of the market. We are interested in the profit maximizing usage fees set by identical duopolistic platforms which engage in homogeneous, Bertrand-type competition. We find that for a sufficiently low marginal cost duopolistic two-sided competition reduces to a “grab-the-dollar” game with two asymmetric (pure) Nash equilibria. These equilibria are characterized by highly skewed prices, in which the side with all the bargaining power pays a minimum price. The other side of the market is used for cross-subsidization and is charged a high price. Compared to the monopoly outcome, competition lowers the total price charged to both sides, although the seller’s equilibrium price may exceed the monopoly price. Both platforms enjoy excess profits.

Key Words: platform competition, bargaining power, asymmetric equilibria, skewed pricing

JEL Classification Code: L10, L13

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1 Introduction

Two-sided markets are markets where platforms provide a joint service to two distinct groups of customers, such that the total usage of the service is a function of both the total price and the relative price (i.e., price structure) between the two sides of the market. For the price structure to have a real effect, it is necessary that frictions in the market prevent the sides from reaching a “Coasian bargain” that renders the relative price between the two sides neutral (see e.g., Weyl, 2008).\(^1\) Many important branches of the economy can be characterized as two-sided, including many financial services (trading platforms, payment services), online mediated networks (eBay, Amazon WebStore) and the media (newspapers, commercial TV). Two-sided platforms differ from “one-sided” merchants in the sense that merchants act as intermediaries that buy from sellers and resell to buyers, whereas two-sided platforms act as intermediaries that enable affiliated sellers to sell directly to affiliated buyers (see Hagiu, 2007).

Recently a burgeoning literature has come to front investigating the impact of competition on price level and price structure in two-sided markets. The models used for the study of two-sided markets can broadly be categorized as usage models (e.g., Chakravorti and Roson, 2006; Guthrie and Wright, 2007, Rochet and Tirole, 2003), membership models (e.g., Armstrong, 2006; Armstrong and Wright, 2007) and combinations thereof (e.g., Caillaud and Jullien, 2003; Rochet and Tirole, 2006). In usage models, both sides of the market pay individually for each transaction (or a fixed fee for an ex ante known number of transactions), while in membership models they pay a fixed fee for an uncertain number of future transactions. Although a complete picture of two-sided competition is not drawn yet, some “conventional wisdom” has emerged.

Conventional wisdom has it that while two-sided competition will generally reduce the price level, it is not yet clear what its effect will be on the price structure or even whether competition will necessarily reduce prices on both sides of the market. In a recent paper, Weyl (2008) obtains a formal demonstration of these conjectures. Moreover, he shows that competitive outcomes very much depend on the rate at which platforms pass through cross-subsidies from

\(^1\)In card payments for example, legislation and contractual agreements between card providers and merchants (“no-surcharge rules”) prevent merchants from charging consumers extra for card payments. Hence, payment prices cannot be bargained over, and poses a friction in the market. Only the choice of payment instrument remains “negotiable” in the store, which dimension of bargaining we will analyze here.
one side of the market to the other. Our analysis further sharpens the two-sided wisdom by showing how homogeneous Bertrand competition and multihoming in two-sided markets affect both the price level and structure.

We develop a usage model of two-sided markets with a focus on the role of bargaining power between the two sides of the market. In our model, bargaining is not over end-user prices, but rather over the choice of platform that is used for transactions when both platforms render net benefits to both sides. We analyze profit maximizing usage fees set by duopolistic homogeneous platforms. As there are no fixed cost in our model, it best reflects real life markets such as mature debit and credit cards, securities brokerage or mature telecoms.

Our model is close to Rochet and Tirole (2003). There are, however, some key differences that drive our results. First, as we are interested in the consequences of bargaining power, neither sellers nor buyers act strategically in our model. When platforms compete on the market, the “homing” behaviour of sellers and buyers becomes important. Each side can choose to singlehome (i.e. connect to only one platform to transact on) or multihome (i.e. connect to several platforms). In Rochet and Tirole (2003) sellers decide whether to single or multihome on the basis of expected buyer demand, given that buyers choose the actual platform if both sides multihome.\(^2\) In our model the decision is symmetric for both sides of the market and we assume that both sides will multihome whenever benefits exceed prices of both platforms. The side that gets to choose the platform in case of two-sided multihoming is given exogenously and enjoys full bargaining power. Second, in contrast to Rochet and Tirole (2003), the platforms in our model are identical and offer homogeneous services. This homogeneity assumption may better capture the competitive properties in some markets. For example, VISA and MasterCard offer a very similar product for executing payments. Moreover, while existing two-sided literature has mainly focused on heterogeneous competition, no comprehensive study has yet been carried out—to our knowledge—for analyzing homogeneous two-sided competition. We attempt to start filling this gap.

Caillaud and Jullien (2003) also develop a competitive usage model of two sided markets. However, they are only interested in the total level of usage fees as they assume that the two sides of the market engage in efficient bargaining to share their total surplus. Effectively

\(^2\) However, an issue of credibility may arise when a seller tries to convince consumers that he singlehomes, especially when there are no fixed costs of connecting, and dealing with the other platform still renders net benefits.
this would neutralize the two-sidedness of the market. Instead, we assume bargaining over
the choice of platform and are particularly interested in the effects of bargaining power on
pricing. Guthrie and Wright (2007) examine a usage model of two sided markets in the context
of competing payment cards. They assume that perfect intrasystem competition drives the
total price to the total cost level for each platform, and competition between platforms take
place at a given price level by varying the price structure. In our model both the price
structure and price level are an outcome of competition.

Bargaining power plays a crucial role when both sides of the market can use more than
one platform. While each side might prefer a different platform, they would still be willing
to execute the transaction on a, for at least one side, less preferred platform—instead of
foregoing it completely. Imagine the following situation: suppose there are two different
credit card networks. Besides cash, consumers carry both cards in their wallets, and retailers
accept all payment instruments, cash and both credit cards. A consumer who wants to buy
a good or service from a retailer prefers to use the credit card that gives him the highest
net benefits relative to the other card or cash. These benefits would also include rebates in
the form of bonus points, frequent flyer miles, insurance services or cash-backs. If this card
happens to be the preferred card for the retailer as well, then the choice is clear, i.e. the card
is preferred by both sides. However, if the retailer has a strong preference for the other credit
card (possibly because the card offering the customer the best benefits is more expensive to
the merchant), then “bargaining power” decides the choice of the credit card network. It is
often assumed that consumers make the final choice of payment instrument in the shop. In
our model, this would then correspond to full bargaining power by buyers. If on the other
hand, retailers can credibly steer consumers to using their less preferred card, then effectively
sellers have full bargaining power. Hermalin and Katz (2006) consider this scenario in the
framework of a strategic game of routing rules, that endogenously determines the party who
gets to choose the platform. In our paper we take the outcome of this bargaining game as
given and investigate the effect of bargaining power on pricing equilibria.

We find that for a sufficiently low marginal cost duopolistic competition reduces to a
“grab-the-dollar” that yields two asymmetric (pure) Nash equilibria. Both equilibria are char-
acterized by highly skewed prices, in which the side with all the bargaining power is charged
a minimum (zero) price. This result of highly skewed prices in two-sided markets resembles
real life pricing strategies where in practise many platforms treat one side as a “profit center” and the other side as a “loss leader” (see Evans, 2003). Moreover, the market is segmented according to the seller’s price: one platform charges a high price and services a small market (with low total profits), the other platform sets a lower price and has a larger market share (with high total profits). Compared to the monopoly outcome, platform competition always lowers the total price charged to both sides. However, depending on the cost level, all sellers may end up paying more than their monopoly price. Interestingly, the equilibrium price levels allow excess profits for both platforms. For too high cost levels, no equilibrium exists, and “best reply dynamics” end up in a cycle.

The paper is organized as follows. In section 2 we introduce the model. Section 3 analyzes the conditions and results for competitive equilibrium pricing. We discern two cases: a situation where rebates are not allowed so that prices are effectively restricted to be non-negative, and a situation where rebates can be granted allowing negative prices. Section 4 gives some numerical examples and discusses the robustness of the findings. Section 5 concludes.

2 The model

There are two types of agents, buyers and sellers, and there are two identical platforms, 1 and 2, which enable the buyers and sellers to transact. Surplus is created when buyers and sellers transact on a platform. We assume that platforms only charge a usage fee per transaction. There are no membership fees or transaction costs to sign up for a platform.

The notation introduced here applies to both the monopolistic and the duopolistic model. Platform $k$ charges buyers (subscript “$b$”) and sellers (subscript “$s$”) a per-transaction usage fee, denoted by $p^k_b$ and $p^k_s$. Platforms incur a marginal cost $c \geq 0$ per transaction. It is convenient to introduce a distinction between the price level, defined as the the total price $p^k_T = p^k_b + p^k_s$ charged by a platform to the two sides, and the price structure, referring to

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3 For historical, practical or regulatory reasons, rebates to end-users may not be feasible. For example, the use of credit card rebates is fairly recent. While credit cards date to the late 60s, rebates (like frequent-flyer miles and other loyalty rewards) became not common until the beginning of the 90s, see Schwartz and Vincent (2006).

the allocation between $p_b^k$ and $p_s^k$ of the total price to buyers and sellers.\textsuperscript{5} Buyers and sellers enjoy benefits when transacting on a platform. We assume that both buyers and sellers are heterogeneous in the benefits $u_i$, $u_i \in [\underline{u}_i, \bar{u}_i]$, $i = b, s$, that they receive from a transaction. These benefits are described by probability density functions $f_i(\cdot)$ and cumulative density functions $F_i(\cdot)$.

To analytically track the model we assume that benefits from using the service are uniformly distributed across the buyers and sellers. Without loss of generality we additionally assume that benefits are non-negative, i.e. $\underline{u}_i = 0$. It is convenient to define $\alpha = \bar{u}_b / \bar{u}_s$ as a measure of asymmetry of benefits between buyers and sellers. Robustness of our uniformity assumption is discussed in section 4.

Assumption 2.1. \textit{[Uniform distribution]}

Buyer’s and seller’s heterogeneity is described by uniform distributions $u_i \sim U[0, \bar{u}_i]$, $i = b, s$.

Naturally, for platform services to attract any positive demand, marginal costs cannot exceed the sum of both maximum benefits.

Assumption 2.2. \textit{[Maximum cost]}

Marginal costs do not exceed $c_{\text{max}} = \bar{u}_b + \bar{u}_s$, that is: $0 \leq c \leq c_{\text{max}}$.

Finally, as we are interested only in the usage fees, we assume that joining the platforms is not costly.

Assumption 2.3. \textit{[No membership costs]}

Platforms do not charge fixed membership fees and there are no transaction costs in joining the platform.

2.1 Monopolistic model

To derive demand for platform services, observe that only buyers with benefits $u_b$ larger than incurred usage fee $p_b$ will transact on the monopolistic platform. Formally, the fraction of buyers connecting to the platform is given by

$$D_b(p_b) = \Pr(u_b \geq p_b) = 1 - F_b(p_b). \quad (1)$$

\textsuperscript{5}As Evans (2003) argues, in two-sided industries the product may not exist at all if the business does not get the price structure right.
Analogously, the fraction of sellers which connects to the platform is equal to

$$D_s(p_s) = \Pr(u_s \geq p_s) = 1 - F_s(p_s).$$

(2)

Under uniform distributions, the demand functions and corresponding price elasticities of demand reduce to

$$D_i(p_i) = 1 - \frac{p_i}{u_i} \text{ and } \epsilon_i(p_i) = \frac{p_i}{u_i - p_i}, \ i = 1, 2.$$

Assuming independence between $u_b$ and $u_s$, the total expected fraction of transactions processed by the platform amounts to\footnote{Multiplicative equation (3) was first used by Schmalensee (2002). This “non-rivalry” condition is not crucial, and will be made here for convenience as well.}

$$D(p_b, p_s) = D_b(p_b) D_s(p_s).$$

(3)

For simplicity, we normalize the total number of transactions, both on and off the platform, to one. So, total demand for network services on the platform is given by $D(p_b, p_s)$. In setting the fees, the monopolistic platform tries to maximize its profits, that are are given by

$$\Pi(p_b, p_s) = (p_b + p_s - c)D(p_b, p_s).$$

(4)
2.2 Duopolistic model

Competing platforms set prices $p^k = (p^b_k, p^s_k)$, $i = 1, 2$, to maximize profits. Formally, buyers prefer the platform with the lowest price as long as the benefits of using it exceed its cost, $u_b \geq \min\{p^1_b, p^2_b\}$. If $u_b < \min\{p^1_b, p^2_b\}$, neither platform is used as both prices are too high. For equal prices $u_b \geq p^1_b = p^2_b$, the buyers are indifferent and both platforms are equally likely to be chosen. The same reasoning holds for sellers. Note that in the model the benefits do not depend on the choice of the platform, hence the platforms are homogeneous. The duopolistic model is schematically depicted in figure 2.

In case both platforms are acceptable to both sides, i.e. $u_b \geq \max\{p^1_b, p^2_b\}$ and $u_s \geq \max\{p^1_s, p^2_s\}$, the distribution of bargaining power determines the choice of platform. In our model, bargaining power is characterized by $\tau \in \{0, 1\}$. Full bargaining power by buyers is identified by $\tau = 1$, and implies that the transaction is executed on the buyer’s preferred platform in case both sides multihome. The reverse holds for full bargaining power by sellers when $\tau = 0$. In essence, our bargaining power parameter $\tau$ incorporates any strategic behaviour between buyers and sellers when they bargain over the choice of platform. Effectively, we take the outcome of this bargaining process as given.

To illustrate how the demand for a platform changes when it reacts to its competitor’s prices, let us first denote platform $k$'s demand function by $D^k(p^k, p^l)$, $k, l = 1, 2$, $k \neq j$, and its corresponding profit function by

$$\Pi^k(p^k, p^l) = (p^k_b + p^k_s - c)D^k(p^k, p^l).$$

Initially both platforms charge $p^0 = (p^0_b, p^0_s)$, and the left panel of figure 3 shows how total demand is split between the two platforms. Under the uniform distribution assumption, initial demand is given by

$$D^1(p^0, p^0) = D^2(p^0, p^0) = \frac{1}{2} \frac{(u_b - p^0_b)(u_s - p^0_s)}{u_b u_s}.$$
Suppose now that platform 1 deviates by setting a price $\tilde{p} = (\tilde{p}_b, \tilde{p}_s)$ with $\tilde{p}_b < p^0_b$ and $\tilde{p}_s > p^0_s$ (see figure 3). Naturally, buyers with benefits $u_b \in [\tilde{p}_b, p^0_b]$ want to transact on platform 1, and sellers with benefit $u_s \in [\tilde{p}_s, \bar{u}_s]$ as well (area A). Platform 2 holds on to sellers with $u_s \in [p^0_s, \tilde{p}_s]$, and buyers with $u_b \in [\tilde{p}_b, \bar{u}_b]$ (area B). The question arises which platform attracts demand from buyers and sellers with utilities in $[p^0_b, \bar{u}_b] \times [\tilde{p}_s, \bar{u}_s]$ (area C). With full bargaining power on the buyers side $\tau = 1$, platform 1 (that offers the lowest buyers price) attracts demand area C. Obviously, with full bargaining power on the seller’s side $\tau = 0$, this demand is attracted by platform 2 as its seller’s price is lower.9

Without loss of generality, in subsequent analysis we will assume full bargaining power by buyers.

**Assumption 2.4. [Buyer’s Bargaining Power]**

*Buyers have full bargaining power, that is: $\tau = 1$.*

In figure 3, with $\tau = 1$, demand for platform 1 totals the area A and the area C, which is

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9Naturally, values of $\tau \in (0, 1)$ would correspond to a proportional split of area C between both platforms.
equal to\(^{10}\)

\[
D^1(\tilde{p}, p^0) = \frac{(\bar{u}_b - \tilde{p}_s)(\tilde{p}_b - \tilde{p}_b) + (\bar{u}_b - \tilde{p}_b)(\tilde{u}_s - \tilde{p}_s)}{\bar{u}_b \bar{u}_s} = \frac{(\tilde{u}_b - \tilde{p}_s)(\tilde{u}_b - \tilde{p}_b)}{\bar{u}_b \bar{u}_s}
\]

Platform 2 that charges \(p^0\) is left with area \(B\), equal to

\[
D^2(p^0, \tilde{p}) = \frac{(\bar{u}_b - p^0_b)(\tilde{p}_s - p^0_s)}{\bar{u}_b \bar{u}_s}
\]

As is shown in the appendix, given platform 2’s prices \((p^2_b, p^2_s)\), platform 1 must evaluate nine different “price regions” to calculate her own demand (see Appendix).

Given platform \(l\)’s prices \(p^l = (p^l_b, p^l_s) \in \mathbb{R}^2\), platform \(k\) sets her price \(p^k = (p^k_b, p^k_s) \in \mathbb{R}^2\) so as to maximize her own profits. We define the best reply function, \(BR^k(\cdot): \mathbb{R}^2 \rightarrow \mathbb{R}^2\), such that

\[
BR^k(p^l) = \arg \max_{p^k} \left\{ \Pi^k(p^k, p^l) \right\}
\]

A maximum always exists if we assume a smallest unit of account \(\epsilon > 0\), requiring that platforms cannot undercut by a smaller amount than \(\epsilon\). Since the platforms are identical and the game is symmetric, the best reply functions are the same, i.e. \(BR^k(\cdot) = BR^l(\cdot) = BR(\cdot)\).

Regarding feasible price strategies, platforms are not restricted in what prices to set. However, in a static model, since platforms can always decide not to produce, they will

\(^{10}\)Note that the demand areas in figure 3 need to be scaled with the normality constant \(\bar{u}_b \bar{u}_s\).
choose prices such that $p^k_b + p^k_s \geq c$, for $k = 1, 2$. Moreover, setting prices higher than $\bar{u}_i$ will lead to zero demand for side $i$ and thus zero profits. Hence, prices $p_b$ and $p_s$ are effectively bounded by $[c - \bar{u}_s, \bar{u}_b]$, and $[c - \bar{u}_b, \bar{u}_s]$ respectively. Note that negative prices $p^k_i \leq 0$, will induce full demand on market side $i$, i.e. the fraction of agents $i$ that connect to platform $k$ is 100 percent. Finally, in our one-shot game, we assume that both platforms set prices simultaneously.

3 Equilibrium pricing

In this section we study competitive pricing in two-sided markets. But before we proceed we briefly examine monopolistic pricing and its main properties as a benchmark.

3.1 Monopolistic platform pricing

In a two-sided market the monopoly platform must make sure that both sides of the market optimally use the platform by appropriately setting the transaction fees. Two-sidedness has strong implications for the pricing strategy of the platform, which will depend on both sides’ price elasticities of demand.

A monopoly platform sets its transaction fees to maximize total profits

$$\Pi(p_b, p_s) = (p_b + p_s - c)D_b(p_b)D_s(p_s).$$  \hfill (6)

Under log-concavity of the demand functions $D_b$ and $D_s$ (which is satisfied under uniform distributions), the maximum is determined by the first-order conditions

$$\frac{p_b}{p_b + p_s - c} + \epsilon_b(p_b) = 0 \quad \text{and} \quad \frac{p_s}{p_b + p_s - c} + \epsilon_s(p_s) = 0.$$  \hfill (7)

Consider the following proposition, which was first proved by Rochet and Tirole (2003) for log-concave distributions.

**Proposition 3.1.** (Monopolistic pricing)

1) Under uniform distributions $U[0, \bar{u}_b]$ and $U[0, \bar{u}_s]$, the monopoly platform earns maximum
profits if it sets usage fees

\[(p^M_b, p^M_s) = \left( \frac{c \epsilon^M_b}{\epsilon^M_T - 1}, \frac{c \epsilon^M_s}{\epsilon^M_T - 1} \right) = \left( \frac{1}{3}(2\bar{u}_b - \bar{u}_s + c), \frac{1}{3}(2\bar{u}_s - \bar{u}_b + c) \right), \quad (8)\]

where \(\epsilon^M_i = \epsilon_i(p^M_i), i = b, s, \) and \(\epsilon^M_T = \epsilon^M_b + \epsilon^M_s.\)

ii) The total fee \(p^M_T = p^M_b + p^M_s\) is determined by the ‘inverse elasticity rule’ for total elasticity \(\epsilon^M_T = \epsilon^M_b + \epsilon^M_s.\) That is,

\[
\frac{p^M_T - c}{p^M_T} = \frac{1}{\epsilon^M_T},
\]

\[(9)\]

iii) The corresponding price structure is characterized by

\[
\frac{p^M_b}{p^M_s} = \frac{\epsilon^M_b}{\epsilon^M_s}.
\]

\[(10)\]

Equation (9) illustrates that the profit-maximizing price level in a two-sided market is governed by the well-known monopolistic Lerner pricing rule when demand is log-concave. However, the Lerner rule applies to the total price \(p_T\) and total elasticity \(\epsilon_T:\) The joint price mark-up is inversely related to the total elasticity (defined by the sum of the two individual elasticities). Hence, if the market as a whole gets less elastic, the total price will rise. For the two separate sides of the market a different but related result holds. Rewriting (10) using (8)-(9) yields

\[
\frac{p^M_i - (c - p^M_j)}{p^M_i} = \frac{1}{\epsilon^M_i}, \quad i, j = b, s, i \neq j,
\]

\[(11)\]

which can be seen as a modified Lerner rule, where the individual price mark-up needs to be corrected for the contribution in terms of revenues from the opposite side. The question arises how these findings in the monopolistic case carry over to the duopolistic case.

Finally, note that for \(c \leq c_{\text{max}}\) (see assumption 2.2), monopolistic prices will never exceed maximum benefit levels, but we need an additional assumption with respect to the asymmetry of benefits to make sure that monopolistic prices are non-negative for all admissible cost levels.

**Assumption 3.2.** \([\text{Bounded asymmetry}]\)

Asymmetry of benefits is bounded, that is: \(\frac{1}{2} \leq \alpha \leq 2.\)
Under assumptions 3.2, monopolistic prices are interior, i.e. $0 \leq p_i^M \leq \bar{u}_i$, $i = b, s$, for all $0 \leq c \leq c_{max}$.

### 3.2 Duopolistic platform pricing

To derive the Nash equilibrium in a two-sided market between two homogeneous platforms we will study the “best-reply price dynamics” in our competition game. Given an arbitrary starting point, we analyze the sequence of best replies to each other prices set by the two platforms. If this best-reply process converges to a fixed point, then the resulting prices constitute a Nash equilibrium.

We first study the competitive equilibrium where we restrict buyer’s and seller’s prices to the carriers of the benefit distributions, i.e. $p_i^k \in [0, \bar{u}_i]$. This describes a situation with non-negative prices so that rebates to either side of the market are not allowed nor possible. Later on, we will relax this restriction by also allowing rebates—i.e. negative prices. This “out-of-carrier pricing” slightly changes the game in terms of demand and profit when competing platforms undercut each other below zero.

#### 3.2.1 Equilibrium without rebates

We first restrict prices to $p_i^k \in [0, \bar{u}_i]$, $i = b, s, k = 1, 2$. To describe the sequence of best replies, let us assume that both platforms initially charge $p^0 = (p_b^0, p_s^0)$ and equally split the market. The monopolistic price is of crucial importance here, and we distinguish two cases: $p_b^0 > p_b^M$ and $p_b^0 \leq p_b^M$. Recall that $(p_b^M, p_s^M) = ((2\bar{u}_b - \bar{u}_s + c)/3, (2\bar{u}_s - \bar{u}_b + c)/3)$. First we study the case with $p_b^0 > p_b^M$. Consider the following lemma (see Appendix for a proof).

**Lemma 3.3.** [Monopolistic Best Reply]

The best reply to a price $p^0 \in R^2$, such that $p_b^0 > p_b^M$, is the monopolistic price $p^M$. That is,

$$BR(p^0) = p^M, \quad \text{if} \quad p_b^0 > p_b^M.$$  

Because buyers determine the choice of platform in case of multihoming, a platform can always optimally set $p_b^M < p_b^0$ so that all buyers with benefits in $[p_b^M, \bar{u}_b]$ are captured. Regardless of $p_s^0$, the platform is then able to secure total demand $(\bar{u}_b - p_b^M)(\bar{u}_s - p_s^M)/(\bar{u}_b \bar{u}_s)$ and therefore able to obtain monopoly profits. Naturally, the monopolistic outcome $p^M$ is not
an equilibrium outcome. The other platform has an incentive to deviate from it by playing its best reply to $p^M$ (see lemma 3.4).

The other case with $p^0_b \leq p^M_b$ triggers a phase of "\(\epsilon\)-undercutting" prices on the buyer’s side. The intuition is clear. Since buyers have full bargaining power, the best strategy is to marginally undercut on that side of the market. This boosts total demand, while the total price margin only slightly changes. Given that a platform undercuts, it must decide on an optimal seller’s price. This triggers a tradeoff between price margin and demand, which is a simple quadratic problem under uniform distributions. That is, given initial price $p^0$, platform 1 maximizes:

$$\max_h \ (p_b + h - c)(\bar{u}_b - p_b)(\bar{u}_s - h)/(\bar{u}_b \bar{u}_s), \ \text{s.t.} \ p_b < p^0_b.$$  

This yields:

$$h(p_b) = (\bar{u}_s - p_b + c)/2. \quad (12)$$

Since profits are increasing in the buyer’s price, it is optimal to set $p_b = p^0_b - \epsilon$. Interestingly, undercutting one side of the market may be accompanied by higher prices on the other side, so that the price margin could even increase under competition.

However, there is one caveat. If the initial seller’s price is high, and profits close to zero due to low demand, a platform may have an incentive to “flip prices” and to jump to $p^J$, which lies to the “south-east” of the monopoly price with $p^J_b > p^M_b$ and $p^J_s < p^M_s$. Although it loses many buyers in doing so, this loss can be fully compensated by setting a sufficiently lower seller’s price (if initial profits are not too high). That is, given $p^0$, an optimal jump to $p^J$ induces a quadratic maximization problem:

$$\max_{p^J_b, p^J_s} \ (p^J_b + p^J_s - c)(\bar{u}_b - p^J_b)(p^0_s - p^J_s)/(\bar{u}_b \bar{u}_s), \ \text{s.t.} \ p^J_b > p^0_b \ \text{and} \ p^J_s < p^0_s.$$  

This yields

$$p^J(p^0_s) = (p^J_b(p^0_s), p^J_s(p^0_s)) = \left(\frac{2\bar{u}_b - p^0_s + c}{3}, \frac{2p^0_s - \bar{u}_b + c}{3}\right). \quad (13)$$

Note that the optimal jump price $p^J(p^0_s)$ is very similar to the monopoly price, but now
adjusted for a different maximum benefit level $\bar{u}_s = p^0_s$. The indifference curve

$$g(p_b) = 3\sqrt[3]{\frac{1}{4}(\bar{u}_b - p_b)(\bar{u}_s - c + p_b)^2 - \bar{u}_b + c},$$

(14)

describes where profits from undercutting are equal to profits from (optimal) jumping. Consider the following lemma (see Appendix for a proof).\(^{11}\)

**Lemma 3.4. [Undercutting Phase]**

*Given $\epsilon > 0$, the best reply to a price $p^0 \in R^2$, such that $0 < p^0_b \leq p^M_b$, is*

$$BR(p^0) = \begin{cases} 
(p^0_b - \epsilon, h(p^0_b - \epsilon)), & \text{if } p_s^0 \leq g(p^0_b), \\
p^J(p^0_s), & \text{if } p_s^0 > g(p^0_b), 
\end{cases}$$

where $h(p_b)$ is given by

$$h(p_b) = \frac{1}{2}(\bar{u}_s - p_b + c).$$

As seen from the previous lemma 3.3, the best-reply to jump price $p^J(p^0_s)$ will be the monopoly price $p^M_s$, which is then followed by a sequence of undercutting, described in lemma 3.4. This process of $\epsilon$-undercutting continues along the path $p_s = h(p_b)$ until the buyer’s price hits the zero bound, while the sellers price converges to a corner price $h(0) = (\bar{u}_s + c)/2$.

Without rebates (so that negative prices are not possible), the question arises what is the best reply to $(0, h(0))$? Does the competing platform charge the same prices and split the market equally? Or does it only charge a zero price for buyers, but set another seller’s price? Or does it want to jump and “flip prices” altogether, thereby increasing the buyer’s price and lowering the seller’s price? The outcome will depend on the cost level and the parameters of the buyer’s and seller’s uniform distributions. It turns out that for sufficiently low marginal cost, we can support an asymmetric equilibrium where one of the platform sets a high seller’s price and the other platform a low seller’s price. In an asymmetric equilibrium, both platforms will set the buyer’s price equal to zero. Essentially, the two-sided competition problem is reduced to a “grab the dollar game”.

More precise, given $p^2 = (0, h(0))$, platform 1 may choose to lower its seller’s price, wile

\(^{11}\)Note that $p^J(\bar{u}_s) = p^M_s$ and $g(p^M_b) = \bar{u}_s$. 

15
keeping a zero buyer’s price. This will increase its demand, which need not be shared, but at a lower margin. That is, given \((0, h(0))\), platform 1’s best choice for a lower seller’s price, keeping \(p_b^1 = 0\) fixed, is given by:

\[
\max_{l_s} (l_s - c) \left( \frac{\bar{u}_s - h(0)}{2\bar{u}_s} + \frac{h(0) - l_s}{\bar{u}_s} \right), \text{ s.t. } l_s < h(0).
\]

This yields \(l_s^* = (\bar{u}_s + h(0) + 2c)/4 = (3\bar{u}_s + 5c)/8\).

Similarly, given \(p^2 = (0, l_s^*)\), platform 1 may have an incentive to increase the seller’s price, while keeping the buyer’s price fixed at zero.\(^{12}\) This would improve its margin, but lower its demand. Given \((0, l_s^*)\), the optimal trade off is characterized by:

\[
\max_{h_s} (h_s - c) \frac{\bar{u}_s - h_s}{2\bar{u}_s}, \text{ s.t. } h_s \geq l_s^*.
\]

This yields \(h_s^* = (\bar{u}_s + c)/2 = h(0)\). Further, we can show that for sufficiently small cost \(c\) it holds that \(l_s^* < h_s^*\), and that jumping to \(P^J(l_s^*)\) or \(P^J(h_s^*)\) will not lead to higher profits than playing \(p^H\) or playing \(p^L\) respectively. Therefore, \(BR((0, l_s^*)) = (0, h_s^*)\) and \(BR((0, h_s^*)) = (0, l_s^*)\).

Before we present a full characterization of the Nash equilibrium, let us first denote equilibrium prices

\[
p^H = (0, h_s^*) = \left(0, \frac{\bar{u}_s + c}{2}\right) \text{ and } p^L = (0, l_s^*) = \left(0, \frac{3\bar{u}_s + 5c}{8}\right).
\] (15)

Consider the following proposition (see Appendix for a proof).

**Proposition 3.5.** /Nash Equilibrium without rebates/

There exists \(0 < c^* < \bar{u}_s\) such that for all \(0 \leq c < c^*\), competition between two homogeneous platforms yields two asymmetric (pure) Nash equilibria \((p^H, p^L)\) and \((p^L, p^H)\).

The threshold level \(c^*\) is determined by equating profits from deviating (i.e. jumping) to adhering to equilibrium actions, which boils down to solving a third degree polynomial. An easier and sufficient condition for existence of equilibrium is \(c \leq \bar{u}_s - \bar{u}_b\) which is admissible if \(\alpha < 1\) (i.e. \(\bar{u}_s > \bar{u}_b\) if \(\alpha < 1\)).

\(^{12}\)It is straightforward to show that setting a higher seller’s price in response to \((0, h(0))\), or setting a lower seller’s price replying to \((0, l_s^*)\) is not profitable.
Figure 4: Platform competition, best reply dynamics, and Nash equilibrium

Note: Left panel shows the low cost case with convergence to an asymmetric Nash equilibrium, right panel show the high cost case with no price convergence.

Figure 4 shows the best-reply dynamics and the topology of equilibrium. The left panel depicts the low cost case, where the best reply dynamics converges to an asymmetric Nash equilibrium with prices \( p^H \) and \( p^L \). In the high-cost case depicted in the right panel, there is no convergence. When the “corner” is hit at \((0, p^H_s)\), prices will flip and jump to \( p^I(p^H_s) \), and the best-reply price adjustment process starts again. Initial prices \( p^0 \) located in the grey area will first jump to the south-east of the monopoly price (to \( p^I(p^0_s) \)), and then start converging to \((0, p^H_s)\).

Effectively, for \( c < c^* \), competition between two homogeneous platforms in a two-sided market reduces to a so-called “grab-the-dollar” game. In this game, both platforms have two strategies; setting a low price \( p^L = (0, p^L_s) \) (i.e. “grab”), or setting a high price \( p^H = (0, p^H_s) \) (i.e. “concede”). Grabbing the dollar by playing the low price \( p^L \) is the best thing to do if your competitor concedes and plays the high price \( p^H \). However, when both platforms reach for the dollar at the same time by playing the low price \( p^L \), this yields the worst possible outcome since the dollar will be destroyed in the struggle and both platforms “feel bitter”.

When both platforms keep still by playing the high price, nothing extra can be gained.\(^\text{13}\)

To verify the specific grab-the-dollar payoff structure, let us first denote \( \Pi_H = \Pi^1(p^H, p^H) = \Pi^2(p^H, p^H) = (\bar{u}_s - c)^2/(8\bar{u}_s) \), when both platforms set the high seller’s price. Given that

\(^\text{13}\)Grab-the-dollar payoff structures (and their dynamic extensions) often arise when analyzing entry and exit decisions by firms, or studying innovation patterns and adoption of new technologies, see e.g. Laraki et al, 2005.
they share the market equally, setting $p^H$ is the best they can do. When both platforms would play $p^L$, this yields the lowest profits for both. We can straightforwardly verify that $\Pi^1(p^L, p^L) = \Pi^2(p^L, p^L) = \Pi_H - \Delta < \Pi_H$, where $\Delta = (\bar{u}_s - c)^2/(128\bar{u}_s) > 0$. The best option is to play $p^L$ when your opponent awaits by playing $p^H$. That is, given price $p^H$, playing $p^L$ increases demand (that need not be shared with the opposing platform) and dominates the negative effect of a smaller price margin. It is easily calculated that $\Pi^1(p^L, p^H) = \Pi^2(p^L, p^H) = \Pi_H + 2\Delta > \Pi_H$. In contrast, given price $p^L$, playing $p^H$ decreases demand but this negative demand effect is now smaller than the positive price margin effect. This is so because demand decreases at only half the pace, since it must be shared with the other platform (that charges the low price). Therefore, $\Pi^1(p^H, p^L) = \Pi^2(p^H, p^L) = \Pi_H$. The diagram below shows the bimatrix game with the corresponding payoff structure. Similar to a grab-the-dollar-game, it is now easy to verify that $(p^H, p^L)$ and $(p^L, p^H)$ are the two (pure) asymmetric Nash equilibria of the “two-sided competition” game (see asterisks in the diagram).

![Diagram](image)

Essentially, the market is segmented based on the seller’s price: one platform charges a high price and services a small market (with low total profits), the other platform sets a lower price and has a larger market share (with high total profits). In both pure equilibria, buyers receive the service for a zero price, whereas the sellers will pay a markedly higher price $p^H$ or $p^L$. In this sense, the prices are heavily skewed to the side without bargaining power.

Interestingly, the properties of these price equilibria are quite different from the usual “one-sided” Bertrand competition findings, where prices are driven to the marginal cost level. Confirming Weyl’s (2008) analysis on “conventional wisdom”, it can readily be shown that

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14 There is also a mixed Nash equilibrium in which both players put $1/3$ probability on playing $p^H$, independent of $\Delta$. The expected profit for both platforms is $\Pi_H$.

15 See Bolt and Tieman (2008) for heavily skewed pricing in the monopolistic case which is driven by benefit distributions that are not log-concave, like a constant elasticity of demand distribution.
platform competition reduces the monopolistic total price level. However, the price structure is maximally skewed, and excess profits remain in competitive equilibrium. Moreover, for sufficiently large cost \( c \), both seller’s equilibrium prices \( p_s^H \) and \( p_s^L \) exceed the seller’s monopolistic price. Denote \( \tilde{c} = \tilde{u}_s - \frac{8}{7} \tilde{u}_b > 0 \), and consider the next proposition (see Appendix for a proof).

**Proposition 3.6. [Equilibrium Properties]**

For all \( 0 \leq c < c^* \) it holds that \( c < p_b^0 + p_s^L < p_b^H + p_s^H < p_b^M + p_s^M \). Moreover, \( p_s^H > p_s^L > p_s^M \) for \( \tilde{c} < c < c^* \).

### 3.2.2 Equilibrium analysis with rebates

Next we consider (out-of-carrier) nonnegative prices so that rebates are allowed. In one-sided markets, undercutting the price below the marginal cost level would never occur since that price strategy would lead to negative profits. In contrast, in two-sided markets, undercutting below cost and even below zero on one market may occur, since it steals business from the competing platform while the other market side can be used to compensate for possible losses.

Naturally, as long as the competing platform charges a price strictly larger than zero, it is never optimal to set your own price strictly smaller than zero. That would be a “money losing” strategy (a smaller margin and no extra demand). However, when both platforms charge a zero buyer’s price, and thus split the buyer’s side of the market, granting rebates becomes valuable, as it attracts full demand instead of half. Mathematically, the undercutting process continues along the line \( p_s = h(p_b) \), where \( p_b < 0 \). Profits diminish along that path, since the seller’s price approaches \( \tilde{u}_s \). Hence, there must exist a cutoff point such that jumping to a lower seller’s price and higher buyer’s price is always beneficial.

This cutoff point is determined by equating the “jump” profit to the “undercutting” profit. That is, the optimal jump price by platform 1 for a given initial price \( p^0 = (p_b^0, p_s^0) \), \( p_b^0 \leq 0 \), is given by \( p^J(p_s^0) \) (see (13) and yields profits \( \Pi^J_1(p_s^0) \). Alternatively, given \( p^0 = (p_b^0, p_s^0) \), undercutting \( p_b^1 = p_b^0 - \epsilon \leq 0 \) along the path \( p_s^1 = h(p_b^1) \) yields profits \( \Pi^U_1(p_b^0) \). Note that buyer’s demand is 100 percent for \( p_b^1 < p_b^0 \leq 0 \). Equating \( \Pi^1_1(p_s^0) = \Pi^U_1(p_b^0) \) yields a non-linear relation \( p_s = \tilde{g}(p_b) \). For \( p_s^0 > \tilde{g}(p_b^0) \) it is optimal to jump to \( p^J(p_s^0) \) rather than to undercut and charge \( (p_b^0 - \epsilon, h(p_b^0 - \epsilon)) \). The intersection of \( p_s = \tilde{g}(p_b) \) with \( p_s = h(p_b) \) determines
Figure 5: Rebates allowed: undercutting below zero

Note: Undercutting below zero continues until buyers’ prices reach \( p^*_b \), asymmetric Nash equilibria exist for \( p^*_b \geq p^*_c \).

a cutoff point \( (p^*_b, p^*_s) \). When competing platforms reach \( p^*_b = p^*_b < 0 \) then the replying platform jumps to \( p^*_b(p^*_s) \), and the “price cycle” starts again.

Hence, when rebates (i.e. out-of-carrier, non-negative prices) are allowed then no equilibrium generally exists. Only if some type of minimum price regulation or “price control” \( p^*_b \) is in place, endorsing \( p_b \geq p^*_b \), then for sufficiently low cost levels there exist two asymmetric equilibria with prices \( p^H(p^*_b) = (p^*_b, h(p^*_b)) \) and \( p^L(p^*_b) = (p^*_b, l(p^*_b)) \), where \( l(p^*_b) = (3\tilde{u}_s + 5c - 5p^*_b)/8 \). As before, \( \Pi^1(p^L(p^*_b), p^H(p^*_b)) > \Pi^1(p^L(p^*_b), p^L(p^*_b)) \) and \( \Pi^1(p^H(p^*_b), p^H(p^*_b)) > \Pi^1(p^L(p^*_b), p^L(p^*_b)) \). Both platforms undercut until they reach the “minimum buyer’s price” \( p^*_b < 0 \). For \( p_b < 0 \), denote \( \tilde{g}_c(p_b) \) the indifference curve derived from equating \( \Pi^1(p^*_s) = \Pi^1(p^L(p_b), p^H(p_b)) \), and similarly denote \( \bar{g}_c(p_b) \) the indifference curve derived from equating \( \Pi^1(p^*_s) = \Pi^1(p^H(p_b), p^L(p_b)) \). The lowest price control \( p^*_b \) that can be supported in equilibrium is derived from the “minimum” of the intersections \( \tilde{g}_c(p_b) = h(p_b) \) and \( \bar{g}_c(p_b) = h(p_b) \), i.e.:

\[
p^*_b = \min\{p_b | \tilde{g}_c(p_b) = h(p_b) \text{ and } \bar{g}_c(p_b) = h(p_b)\}.
\]

For sufficiently low \( c \), this minimum is below zero. In equilibrium, one platform plays “high” by setting \( p^H(p^*_b) \), while the other platform plays “low” by setting \( p^L(p^*_b) \). Figure 5 depicts the analysis.

Summarizing, we conclude that equilibrium characterizations with or without rebates are similar in nature, as long as there is a (feasible) minimum buyer’s price (zero without rebates, or \( p^*_b < 0 \) with rebates) below which the platforms cannot undercut.
4 Examples and robustness

To illustrate our findings, assume $u_b = [0, 1]$, $u_s = [0, 2]$, $\tau = 1$, $c = 0.90$, and $\epsilon = 1/100$ (a “cent”). Our results yield: $p^M = (0.30, 1.30)$, $p^H = (0, 1.45)$, and $p^L = (0, 1.31)$. We can numerically calculate $c^* = 1.11$. Since $c < c^*$, two asymmetric Nash equilibria result: $(p^L, p^H)$ and $(p^H, p^H)$. It is easily verified that $\Pi_H = 1.21/16 = 0.076$ and $\Delta = 1.21/256 = 0.005$.

We observe that both total price levels decreases relative to the monopoly total price level ($1.31 < 1.45 < 1.60$) as a consequence of competition. However, since $c > \tilde{c} = \tilde{u}_s - 8\tilde{u}_b/7 = 0.86$, both seller’s equilibrium prices exceed the seller’s monopoly price, i.e. $p^H_s > p^L_s > p^M_s$. Both platforms enjoy excess profits in equilibrium. The grab-the-dollar game reduces to:

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<th>Platform 2</th>
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<tr>
<td>$p^H$</td>
<td>$(0.076, 0.076)$</td>
</tr>
<tr>
<td>$p^L$</td>
<td>$(0.076, 0.085)^*$</td>
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We can further verify $p^*_b = -0.26$, implying that undercutting can yield negative prices as well when rebates are allowed, but not below $p^*_b = -0.03$ in equilibrium.

With higher cost $c = 1.25 > c^*$, best reply dynamics will not converge, but end up in a cycle. We calculate: $p^M = (0.42, 0.71)$, $p^H = (0, 1.63)$, and $p^J(p^H_s) = (0.54, 1.17)$. Starting with, say, $p^0 = (0.50, 1.00)$ so that $p^0 > p^M$, the price adjustment process will go: $p^0 \rightarrow p^M \rightarrow p^H \rightarrow p^J(p^H_s) \rightarrow p^M \rightarrow p^H \rightarrow \ldots$.

The uniform distribution allows analytical results. To check robustness, we performed numerical analyzes using (rescaled) beta distributions $\tilde{B}_{p,q}(x)$ that allow varying densities on $[0, \tilde{u}_i]$. The numerical results confirm our analytical findings for the uniform distribution. When we e.g. used a beta $\tilde{B}_{2,3}(x), x \in [0, 1]$ for the buyers, and $\tilde{B}_{8,4}(y), y \in [0, 2]$ (see figure 6), we verify $p^M = (0.20, 1.23)$ and two Nash equilibria with prices $p^H = (0, 1.31)$ and $p^L = (0, 1.22)$ for cost level $c = 0.90$. For $c = 1.25$, best reply dynamics ends up in a cycle. Although the distributional characteristics are markedly different (see figure 6), qualitatively the same equilibrium conditions and convergence results hold. Similarly, for a log-normal distribution we find convergence to Nash equilibria for low cost levels with the
buyer’s equilibrium price being zero, but best reply cycles for higher cost levels.

So far, in our analysis we set $\tau = 1$, i.e. full buyer’s bargaining power, making competition for buyers more vigilant. This changes when $\tau$ is set below one. Some first calculations show that a decreasing $\tau$ shifts the “indifference” relation $g(p_b)$ downward, so that Nash equilibria with a low buyer’s price become more unlikely. For sufficiently small $\tau$, the seller is more attractive, and the role of buyers and seller switches. Obviously, for $\tau = 0$, the other polar case, the seller has all the bargaining power and can choose the platform in case of two-sided multihoming.

5 Conclusions

We develop a model of two-sided markets where homogeneous duopolistic platforms compete for buyers and sellers by optimally setting usage fees. The model can be seen as a two-sided analogue of homogeneous Bertrand competition in a regular one-sided market. With Bertrand competition, price undercutting leads to prices that equal costs. In our model undercutting takes place on the side that decides which platform is used for the transactions, while the price for the other side of the market is increased to compensate for the lost margin. Moreover, price undercutting does not lead to marginal cost pricing, and asymmetric equilibria only exists for sufficiently low cost levels. We show that duopolistic competition in two-sided markets effectively reduces to a “grab-the-dollar” game. Moreover, compared to the monopoly outcome, competition always lowers the total price, while excess profits remain for both platforms.
When buyers have full bargaining power, sellers are worst off: in equilibrium they could all end up paying even higher prices than the monopoly price (and some of them will for sure).

As reported in Evans (2003), in practice many two-sided platforms tend to treat one side of the market as a “profit center” with high prices and the other side as a “loss leader” with low prices. Kaiser and Wright (2005) report using data from magazines in Germany that prices for readers are subsidized while magazines make all their money from advertisers. Also in credit card payments the acquirers (the party with the merchant relation) typically transfer funds to issuers (the party with the cardholder relation) via the use of interchange fees, allowing the buyer side to be subsidized by the seller side (Weiner and Wright, 2005). We argue that these heavily skewed pricing strategies may result from differences in bargaining power between the two sides of the market.

Some two-sided markets have recently been heavily investigated by antitrust and competition authorities (card payments, operating systems). The fact of high prices on one side of the market has been brought forward as evidence for market imperfections. Also, low (or even negative) prices on the other side of the market might be falsely identified as predatory pricing practices. We argue in this paper that one needs to be cautious in such conclusions. We argue that if buyers always make the choice of the platform to transact on, the natural outcome of competition between platforms is a situation where buyers receive rebates and sellers bear all costs.

It is easy to think of natural extensions of the model, some of which could alleviate the “price cycle” found in the case of an unrestricted strategy space for the platforms. These extensions include e.g. a higher number of platforms, social welfare, endogeneity of the bargaining process, and alternative specifications of the platforms profit function. A higher number of platforms would decrease the payoff from sharing a given price. Social welfare considerations may be different in two-sided markets, where externalities play a role. Modeling the bargaining process between buyers and sellers in two-sided markets allows a better understanding of credible “homing” behaviour. Finally, some platforms in two sided markets (e.g. some payment card associations) are jointly owned by one side of the market and may wish to maximize e.g. seller surplus instead of profits.
**Appendix**

**Demand function specification:**
Let the utility for buyer $u_b \sim U[\bar{u}_b, \underline{u}_b]$, and the seller $u_s \sim U[\bar{u}_s, \underline{u}_s]$. The prices set by the two platforms for the buyers and sellers are denoted by $(p^1, p^2) = ((p^1_b, p^1_s), (p^2_b, p^2_s))$. Let $\tau = \{0, 1\}$ denote bargaining power between the buyer and seller. The demand for platform 1 is given by:

$$D^1(p^1, p^2|\tau) = \begin{cases} 
(\bar{u}_b - p^1_b)(\bar{u}_s - p^1_s) & p^1_b < p^2_b \land p^1_s < p^2_s \\
(\underline{u}_b - p^1_b)(\bar{u}_s - p^1_s) + (1-\tau)(\bar{u}_b - p^1_b)(\bar{u}_s - p^2_s) & p^1_b > p^2_b \land p^1_s < p^2_s \\
(p^2_b - p^1_b)(\bar{u}_s - p^1_s) & p^1_b < p^2_b \land p^1_s > p^2_s \\
0.5\tau(\bar{u}_b - p^1_b)(\bar{u}_s - p^2_s) & p^1_b > p^2_b \land p^1_s > p^2_s \\
0.5(1-\tau)(\bar{u}_b - p^1_b)(\bar{u}_s - p^2_s) & p^1_b = p^2_b \land p^1_s = p^2_s \\
0.5\tau(\bar{u}_b - p^1_b)(\bar{u}_s - p^1_s) & p^1_b = p^2_b \land p^1_s = p^2_s \\
0.5(1+\tau)(\underline{u}_b - p^2_b)(\bar{u}_s - p^1_s) + (p^2_b - p^1_b)(\bar{u}_s - p^1_s) & p^1_b < p^2_b \land p^1_s > p^2_s \\
0.5(1-\tau)(\underline{u}_b - p^2_b)(\bar{u}_s - p^1_s) + (p^2_b - p^1_b)(\bar{u}_s - p^1_s) & p^1_b > p^2_b \land p^1_s > p^2_s \\
0 & p^1_b < p^1_b \land p^1_s < p^2_s 
\end{cases}$$

**Proof.** Lemma 3.3: By decreasing the price on the buyer’s side to $p^M_b$, platform 1 will attract the demand of all buyers with utility $u_b \geq p^M_b$. Since buyers have full bargaining power ($\tau = 1$), the seller’s price can be set to $p^M_s$. Although sellers with utility $u_s \geq p^M_s$ prefer to transact on platform 2 when $p^M_s > p^0_s$, they are still willing to do business with platform 1. Evidently, if $p^M_b < p^0_b$ and $p^M_s < p^0_s$ then platform 2 will lose all its demand. Naturally, the optimal tradeoff between attracting buyers and sellers occurs at the monopolistic outcome which yields maximum profits for platform 1.

**Proof.** Lemma 3.4: If $p^0_b \leq p^M_b$, platform 1 cannot obtain monopoly profits by setting $p^1 = p^M$. Sellers with utility $u_s \geq p^0_s$ connect to platform 2, since he attracts all the buyers in case of double multihoming (and in case $p^0_b = p^M$, they share buyers equally when both platforms are acceptable to sellers and buyers). The best platform 1 can do when it increases the buyer’s price is to decrease the seller’s price and jump to $p^f(p^0_s)$, as given in (13). This optimal jump yields profits:

$$\Pi^1_f(p^0_s) = \frac{(\bar{u}_b + p^0_s - c)^3}{27\bar{u}_b\bar{u}_b}.$$  

When platform decreases her buyer’s price ($p^1_b < p^0_b$), then the optimal seller’s price is $h(p^1_b) = (\bar{u}_s - p^1_b + c)/2$, see (12). The resulting profit is equal to:

$$\Pi^1_U(p^1_b) = \frac{(\bar{u}_b - p^1_b)(\bar{u}_s + p^1_b - c)^2}{4\bar{u}_b\bar{u}_b}.$$  

It can readily be shown that $\partial \Pi^1_U(p^1_b) / \partial p^1_b > 0$, so that marginally undercutting $p^1_b = p^0_b - \epsilon$ is optimal. Keeping $p^1_b = p^0_b$ fixed is never optimal, since demand must then be shared, dampening profits. By marginally undercutting, buyers are fully steered to platform 1, which will
Proof. Proposition 3.5: Since undercutting at and solving for \( c = \overline{\text{best reply to } c} \) which is satisfied under assumption 3.2. A similar reasoning holds when contemplating a \( p \) (14). Hence, for \( p_0 > g(p_0) \) it is optimal as a best-reply to jump to \( p^J(p_0^0) \) rather than to undercut and charge \( (p_0^0 - \epsilon, h(p_0^0 - \epsilon)) \). \( \square \)

Proof. Proposition 3.6: Under assumption 3.2 and for \( c < c^* \leq \bar{u}_s \) it is easy to show that \( c < p_b^L + p_s^L < p_b^H + p_s^H < p_b^M + p_s^M \) and that all prices lie in the interior \([0, \bar{u}_b] \times [0, \bar{u}_b] \). If \( c = \bar{u}_s - 7\bar{u}_b/8 \) then \( p_s^L = p_s^M \). \( \square \)

\(^{16}\text{Strictly speaking, the curve } g(p_0) \text{ also depends on } \epsilon, \text{ but this parameter can be taken arbitrarily small.} \)
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