An Upper Bound of the Sum of Risks: two Applications of Comonotonicity
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* Views expressed are those of the individual author and do not necessarily reflect official positions of De Nederlandsche Bank.
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Abstract

This paper discusses the method of comonotonicity to estimate the sum of risks. Two applications are presented. First, we estimate a property insurer’s exposure to claims after a severe storm. Second, we apply our approach to a pension fund’s investment risk to estimate the prospective total assets and the conditional prospective funding rate. Both applications show that comonotonicity can be a useful tool to assess the upper bound for the risk exposure of financial institutions.

JEL Codes: C13, G22, G23

Keywords: estimate, sum of risks, investment

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1. Introduction

Insurers and pension funds are exposed to a variety of risks. Hence, it is important to have an idea of the total impact of the several risks. However, calculating the sum of risks is often hard or even impossible. In particular, it is impossible when the joint probability distribution function of the risks or the mutual dependencies is unknown. We apply a straightforward method to estimate the sum of risks with an upper bound. The method assures that the total risk in the actual sum is at most equal to the total risk in the estimation, but the expectations of the actual and the estimated sum are the same. Two applications are discussed here: an insurer’s exposure to claims caused by a storm and the pension fund’s investment risk for different portfolio mixes and the corresponding funding rate.

An exact assessment of an insurer’s total exposure requires detailed information on all individual insurance products and the correlations. The total claim of insurance portfolios or the total investments of a pension fund are all stochastic variables with an uncertain outcome, but can be described by the probability that the claim or asset will have at most a certain value. Simulations can be used to approximate the total claim or total asset plus the accompanied uncertainties. Running a simulation has disadvantages; it is time consuming while the underlying model is based on particular assumptions. Another approach is to apply a simplification and to estimate the sum with an upper bound in the associated risks. In order to determine an upper bound for the total risk, comonotonicity is a straightforward way to do this because only the marginal cumulative distribution function of each the stochastic variables are needed, see Dhaene et al. (2002a, 2002b, 2005), Goovaerts et al. (2005), Vanduffel et al. (2005) or Vanduffel (2005).

The outline of this paper is as follows. The following section briefly discusses the theoretical background of comonotonicity and its limitations. Section 3 will describe how the concept is applied to estimate the maximum loss and the probability due to a storm catastrophe in northwest Europe. Section 4 presents how the concept can be used to estimate the total assets and the corresponding probability of a pension fund and the (conditional) funding rate in the future. The last section summarizes the results and conclusions, and gives suggestions for future work.
2. The background of comonotonicity and the implications

Risk means uncertainty in the value of a claim, asset, loan and etcetera. Therefore, such a value can be described as a stochastic variable with the corresponding risk giving the uncertainty of this value. The method of comonotonicity forces that the separate stochastic variables which are making up a sum, are interdependent as much as possible. From the viewpoint of risk management this enforcement is prudent, because the uncertainty in the actual sum of the stochastic variables will be at most equal to the uncertainty in the comonotonic sum in any circumstance. However, the fact that the interdependencies are maximized does not mean that these interdependencies are corresponding to correlation coefficients equal to one.

The uncertainty in a continuous stochastic variable $X$ (like the value of a claim, asset etc.) is described by its cumulative distribution function (cdf) $F_X(x)$. Such a function represents the probability $p$ that the stochastic variable $X$ has at most the value $x$: $p = \Pr[X = x] = F_X(x)$. This implies that the stochastic variable $X$ is more uncertain than $Y$ when for different values of $p$ the corresponding $x$ is more volatile than $y$. The sum of $n$ stochastic variables is equal to:

$$S = X_1 + X_2 + X_3 \ldots + X_n$$  \[1\]

When the inverse cdf of $X$ is written as: $x = F_X^{-1}(p)$, the comonotonic sum of the same $n$ stochastic variables is defined as:

$$S^C = F_X^{-1}(U) = X_1^C + X_2^C + \ldots + X_n^C = \sum_{j=1}^{n} F_X^{-1}(U)$$ \[2\]

The symbol $U$ stands for a standard uniform random variable: $U \sim \text{Uniform}[0, 1)$. Because all stochastic variables depend on the same stochastic variable $U$, this is the sum that is maximal uncertain and is called: comonotonic sum. In other words, when $X_i$ increases due to an increase of $U$, this cannot be compensated by any other stochastic variable because of the same dependency on $U$. The marginal distributions of $X_i$, $X_2$, ... $X_n$ and $X_1'$, $X_2'$, ..., $X_n'$ are equal, so the expected value of the comonotonic sum $S'$ and of the sum $S$ are the same. When the comonotonic sum $S'$ of functions $f_i$ of $n$ stochastic variables must be calculated, this sum is equal to:

$$S^C = F_{X}^{-1}(U) = f_1(X_1^C) + f_2(X_2^C) + \ldots + f_n(X_n^C) = \sum_{i=1}^{n} f_i(F_X^{-1}(U))$$  \[3\]
Note that the comonotonic sum $S^c$ will only return the most risky sum when all the $f_i$ are non-decreasing and continuous\(^2\) in the same stochastic variable $U$. The proof that the risk in the actual sum is at most equal to the comonotonic sum can be found in Dhaene et al. (2002a). The cdf of the comonotonic sum $S^c$ can be derived by inversion of expression [3].

The complexity of the actual sum $S$ is reduced by incorporating the standard uniform distributed variable $U$ in the comonotonic sum $S^c$. As a consequence only the marginal cumulative distributions of the stochastic variables $X_i$ are needed to derive the cumulative distribution function of the comonotonic sum, the expectation of the real sum and the comonotonic sum are equal, but the uncertainty of the comonotonic sum is at least as large as the uncertainty in the real sum.

The comonotonic sum gives an upper bound in risk for the actual sum of stochastic variables. The difference between the risk in the actual and the comonotonic sum depends on the correlations between the terms in the actual sum. The larger these correlations are, the smaller the difference between the comonotonic and the actual sum will be. If we assume that the terms in the sum are independent, this sum gives a lower bound (in risk) for the actual sum when the actual correlations between the terms are at least zero. To illustrate the accuracy of the comonotonic sum (upper bound in risk), it is compared to the sum of independent terms (lower bound in risk). The smaller the gap between these two bounds, the better the accuracy of the prudent comonotonic sum is.

3. Maximum loss due to a storm catastrophe in northwest Europe

Storm catastrophes in northwest Europe mostly take place in the autumn and winter. Typically, the severities of these so-called winter storms in northwest Europe differ across countries, which also lead to differences in insurance claims. Data of property claims caused by winter storms are publicly available on a country level. It is obvious that these claims on country level are correlated, but this correlation is unknown and will differ across storms and between countries.

In this paper, a winter storm in a certain country is modeled as a function of one stochastic variable: the storm’s intensity. Intensity is defined as the expected value of the period (in years) that such a storm shows up once. Typically, severe winter storms in northwest Europe have an intensity of 5 to 15 years. The property claim as a function of intensity is derived for Belgium, France, Germany, the Netherlands and the UK by a regression. Other northwest European countries are not included, because of a lack of data. The cdf of the actual sum of property claims

\(^2\) If the functions $f_i$ are discrete and/or all non-increasing in the stochastic variables $X_i$, and the comonotonic sum must be calculated, see Dhaene et al. (2002a).
in the five listed countries cannot be determined analytically. The main reason is that the joint cdf is unknown and varies per storm. A short description of the model applied in this section can be found in the Appendix I.

Beside the upper bound in risk of the sum of claims, we also have calculated the sum of claims assuming they are independent. We know the sum of independent terms is a lower bound in risk for the actual sum (assuming the correlation between the claims in a country is at least zero). It appears that the comonotonic and independent sum are reasonable close to each other (see Graph 1), meaning the comonotonic sum is a close but prudent estimate for the actual sum.

The difference between both sums above the expected value seems very small, but this is due to the logarithmic scale of the horizontal axis. The graph shows that the comonotonic sum is more risky than the independent sum. The graph of the comonotonic sum is less concentrated around the expected value, showing a higher volatility. Here the focus is on the large claims and the corresponding probabilities (see Graph 2), which leads to higher risks and default probabilities for the insurer.
With a probability of 97.5% the independent sum is at most 15.58 [billion euros] and the comonotonic sum is at most 17.98 [billion euros]. With a probability of 99.5% the independent sum is at most 54.60 [billion euros] and the comonotonic sum is at most 68.01 [billion euros]. The actual sum will be between the lower and upper bounds.

*) The applicable countries are: Belgium, France, Germany, The Netherlands and the UK.
4. Prudent estimate of a pension fund’s prospective total assets

In this section, a second example of the comonotonic sum is discussed. It is shown that a prudent estimate of the sum of risks can be calculated quickly by the comonotonic method: we estimate the total assets of a pension fund next year. For pension funds it is attractive to invest in products with high returns. Because high returns are accompanied with a high risk, a pension fund usually invests in both equities and bonds to diversify their investment risk but retain part of the return. Calculating the prospective value of the investment portfolio implies that the sum of (functions of) stochastic variables needs to be accumulated. The comonotonic approach is used to estimate the cumulative distribution function (cdf) of this prospective value of the investment portfolio.

We assume that the pension fund invests in 2 equity indices and in 2 different bonds. In stead of looking for the probabilities of claims (as in the preceding section), here we focus on the portfolio’s lower bound of the portfolio and the corresponding probabilities. A low portfolio value manifests itself when the returns of all the products are negative which is in line with the comonotonic concept. Therefore, the portfolio value can be estimated prudently with the comonotonic sum of each investment product.

We assume that the prospective quotation $X_i$ of each investment $i$ is lognormally distributed: $\ln(X_i) \sim \mathcal{N}(\mu_i, \sigma_i^2)$. The value of the actual total assets, a four asset portfolio, is equal to: $S = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_4$. The $\alpha_1, \ldots, \alpha_4$ are each the weighting factor to the corresponding stochastic variable. Again, an analytical expression for the distribution function of the comonotonic total assets can be derived, but not for the actual total assets (see Dhaene et al. (2002a)). To calculate the comonotonic sum, the standard deviation and the expected value of the lognormal distributions must be known. These parameters are estimated from the historical data, see Table 1.
Note that the German zero rate in Table 1 corresponds to a zero bond with duration of 10 years. This zero rate is used as proxy for an investment in German bonds with constant duration of 10 years. The remaining duration of bonds reduces in time, but the duration of the 10 year zero rate is constant. So the problem of a varying duration which we would run into when using the quotation of real German bonds is removed. The German 10 year zero rate is used to calculate an indexed quotation of an artificial German bond. From these quotations the mean and standard deviation are estimated and it is assumed this quotation is lognormal distributed. The same approach is applied for an investment in short term bonds. The Dutch lending rate with duration of 6 months is used to calculate the quotation of an artificial Dutch zero bond with duration of 6 months, which is assumed to be lognormal distributed with a corresponding estimated mean and standard deviation.

### Table 1 Cdf parameters of the normal distributed returns

<table>
<thead>
<tr>
<th>investment product $i$</th>
<th>description</th>
<th>period</th>
<th>mean $\mu_i$, annualized</th>
<th>std.dev. $\sigma_i$, annualized</th>
<th>weight $a_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Netherlands 6M lending rate</td>
<td>1-1-1990 until 5-9-2005</td>
<td>1.97E-1 %</td>
<td>4.24E-3</td>
<td>10%</td>
</tr>
<tr>
<td>2</td>
<td>Germany 10Y zero curve rate</td>
<td>17-2-1997 until 5-9-2005</td>
<td>3.26 %</td>
<td>6.47E-3</td>
<td>40%</td>
</tr>
<tr>
<td>3</td>
<td>DJ Euro Stoxx 50</td>
<td>until 5-9-2005</td>
<td>7.24 %</td>
<td>0.205</td>
<td>25%</td>
</tr>
<tr>
<td>4</td>
<td>Amsterdam MidKap</td>
<td>until 5-9-2005</td>
<td>5.79 %</td>
<td>0.205</td>
<td>25%</td>
</tr>
</tbody>
</table>

Source: Datastream.

The number of observations of the German 10 year zero curve are less than of the other dataseries. The estimations of the mean and standard deviation of the German zero rate are therefore less accurate (with weekly frequency of the data, approximately by a factor 1.35).
When the initial weights \((a_i)\) are kept constant and a buy-and-hold strategy is carried out, the expression for the comonotonic sum is, see also Dhaene (2002a):

\[
S_n^c = F_{S_n}^{-1}(U) = \sum_{i=1}^{4} F_{\mu_i, X_i}^{-1}(U) = S_0 \sum_{i=1}^{4} \alpha_i e^{\sum_{j=1}^{\mu_i,j} + \sum_{j=1}^{\sigma_i,j} b^{-1}(U)}
\]

The \(\mu_{ij}\) and \(s_{ij}\) are the estimates of the expected value and the actual standard deviation of each investment product \(i\). The variable \(j\) stands for the prospective time in years. Setting the total assets of the pension fund at \(t = 0\) to 1, the cdf of the comonotonic assets next year (so \(j = 1\)) can be calculated. Graph 3 presents the cumulative distribution function of the comonotonic sum and the sum of mutual independent investment products.

Note that here it is not reasonable to assume that the correlation between all the products is at least zero as in the preceding section, because investment products can have any mutual correlation coefficient between \(-1\) and \(1\). Therefore, we cannot assume the actual prospective value of the portfolio is between the comonotonic sum and the independent sum. However, when the correlation between some of the products is negative, than the prospective value of the portfolio might be even larger and reflects less risk than the independent sum. For risk management, the comonotonic sum remains valuable because it allows us to calculate quickly an estimate of the actual sum which has an equal expectation and contains at least the same risk.

**Graph 3 Cdf of the estimated total assets next year**

![Graph 3 Cdf of the estimated total assets next year](image)
As a risk measure is taken the loss in total assets that will occur at most with a probability of 97.5%. This risk measure is called “Value at Risk with a probability of 97.5%” and is abbreviated as: 97.5% VaR. The 97.5% VaR of the comonotonic total assets is 0.170. The independent portfolio has a 97.5% VaR of 0.081. The 97.5% VaR of the actual prospective total assets will be at most equal, but presumably smaller than the comonotonic one.

For a less risky asset mix (i.e. 80% bonds, 20% equities), the cdf of the comonotonic sum will be more concentrated around the expected value. For this portfolio, the 97.5% VaRs of the comonotonic and independent sum of the prospective assets are 0.115 and 0.058. If the pension fund chooses for a more risky asset mix (i.e. 25% bonds, 75% equities), the cdf will deviate more from the expected value. For this portfolio, the 97.5% VaRs are: 0.219 and 0.123 for the comonotonic and the independent sum.

Table 2 lists the composition of the portfolio mixes. Graph 4 shows the cumulative distribution functions for the three investment mixes.

### Table 2 Composition of the portfolio mix

<table>
<thead>
<tr>
<th>investment product $i$</th>
<th>description</th>
<th>weight $a_i$ in portfolio mix with</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>low risk</td>
</tr>
<tr>
<td>1</td>
<td>Netherlands 6M lending rate</td>
<td>10%</td>
</tr>
<tr>
<td>2</td>
<td>Germany 10Y zero curve rate</td>
<td>70%</td>
</tr>
<tr>
<td>3</td>
<td>DJ Euro Stoxx 50</td>
<td>10%</td>
</tr>
<tr>
<td>4</td>
<td>Amsterdam Midkap</td>
<td>10%</td>
</tr>
</tbody>
</table>
The results show that the maximum risk, given by the comonotonic sum, depends on the composition of the portfolio. The comonotonic method guarantees that the risk of the actual prospective total asset will be smaller than the risk of the comonotonic total assets, and that the expected value of the actual and the comonotonic sum are equal. The cdf of the actual total assets consisting of normal distributed returns cannot be derived analytically, but the comonotonic total assets can easily be calculated and gives an upper bound for the risk. All are useful tools for risk management to get a first and prudent impression of the prospective total assets. The quality of the upper bound depends on how close the correlation coefficients between the corresponding terms are to the maximal possible correlation coefficients. The more close they are, the better the cdf of the comonotonic sum will estimate the cdf of the actual sum. Note that an investor usually aims to a diversified portfolio, i.e. by investing in a hedge fund. Diversification means reducing the correlation coefficients, and therefore the risk in the comonotonic sum will exaggerate more the risk in the actual sum.

For a pension fund, we are usually interested in the prospective funding rate rather than the value of the total assets. The funding rate cannot be estimated via comonotonicity, because it is a ratio and therefore not non-decreasing in all components. However, when the sum of assets in the expression for the funding rate is replaced with the comonotonic sum, here we can derive the cdf of the prospective funding rate conditionally on the correlation between the comonotonic total assets and the liabilities. In addition it is assumed that the return on liabilities has also a normal distribution.
As an example, we will derive the cdf of the funding rate for a normalized pension fund with the medium risk portfolio for two correlation coefficients. To set up the initial size of the liabilities, we apply the “standard model”\(^3\). This results in the balance sheet of our normalized pension fund in Table 3. The funding rate for a pension fund is defined as the ratio between total assets and liabilities. As a consequence of the applied “standard model”, the funding rate today is larger than one: 1.2015. In the calculations, it is assumed the return on the surplus will be zero.

<table>
<thead>
<tr>
<th>total assets</th>
<th>liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>bonds 0.5</td>
<td>liabilities 0.8323</td>
</tr>
<tr>
<td>equities 0.5</td>
<td>surplus 0.1677</td>
</tr>
<tr>
<td>total 1</td>
<td>total 1</td>
</tr>
</tbody>
</table>

The duration of the liabilities is assumed to be 16 years. Through a linear approximation between the 15 and 20 year swap rate in the Netherlands, a 16 year swap rate is estimated. With this estimated swap rate which is a proxy for the return on the liabilities, the value of the liabilities can be calculated. It is assumed the liabilities are lognormal distributed, so with these calculated quotations the annualized expected value and standard deviation of the distribution can be estimated, see Table 4.

<table>
<thead>
<tr>
<th>liabilities product i</th>
<th>description</th>
<th>period</th>
<th>mean (\mu_i), annualized</th>
<th>std.dev. (\sigma_i), annualized</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Netherlands 16Y swap rate*</td>
<td>7-7-1997 until 5-9-2005</td>
<td>4.31 %</td>
<td>0.100</td>
</tr>
</tbody>
</table>

\(^*\): Linear approximation between the 15 and 20 year swap rate in the Netherlands.
Source: Datastream.

\(^3\) In this “standard model”, as described in the nFTK (2004), a pension fund is obliged to increase the liabilities with a solvency surplus. This solvency surplus follows from the composition of the asset mix and is aimed to set the probability of insolvency next year to at most 2.5%. Here, the liabilities are set equal to the total assets minus the required solvency surplus.
Given the assumptions, the cdf of the funding rate $FR_t$ can be derived analytically when the actual prospective total assets is replaced with the comonotonic sum of the total assets.

$$FR_{t=1}^{c} \leq \frac{S_{t=1}^{c}}{L_{t=1}} = \frac{F_{S_{t=1}}^{-1}(U)}{F_{L_{t=1}}^{-1}(V)} = F_{FR_{t=1}}^{-1}(U, V) =$$

$$= FR_{t=0} \left \{ \sum_{i=1}^{4} \alpha_i e^{\mu_i - \mu s + \sigma_s \Phi^{-1}(U) - \sigma_s \Phi^{-1}(V)} \right \}$$ [5]

The symbol $A \leq B$ means that the stochastic variable $A$ (the actual funding rate) is less risky than variable $B$ (the funding rate based on the comonotonic sum of assets). For the derivation why the actual funding rate is less risky than the comonotonic funding rate, see the Appendix II. $U$ and $V$ are stochastic variables with a standard uniform distribution, but can be correlated. From the expression, it is clear that the funding rate is less risky when the correlation coefficient between the returns of liabilities and the comonotonic sum of assets gets closer to 1. The risk in the funding rate is at its maximum when the correlation is -1. Graph 5 displays the cdfs of the funding rate conditional on both the correlation coefficients.

**Graph 5 Cdf of funding rate next year, conditional on correlation coefficient $\gamma$**

Medium risk portfolio

- correlation coefficient = -1
- correlation coefficient = +1
- expected value, correlation coefficient = -1
- expected value, correlation coefficient = +1

*: Correlation coefficient $\gamma$ between the returns of the liabilities and the comonotonic sum of assets.
With a probability of 2.5%, the funding rate is 1.0325 if the correlation coefficient is 1. If the correlation coefficient is –1, with a probability of 2.5% the funding rate is 0.7360. Note that a correlation coefficient of –1 corresponds to the situation in which the value of each portfolio asset goes down, while the value of the liabilities moves up (or vice versa). This may occur in a scenario of an increasing medium and short term interest rates and a downturn in the equities, while the interest rate for the long term bonds decreases. The correlation of 1 and negative investment returns means that the value of the liabilities also decreases, so all interest rates move upward and the equity returns move downward.

5. Conclusions

This paper shows an easy way to calculate an upper bound in risk of the sum of (functions of) stochastic variables, even when the variables are interdependent with unknown and/or varying correlations like in stress scenarios. The reason is that the comonotonic sum only requires the marginal distributions of the stochastic variables in the corresponding sum. The method guarantees that the expected value of the upper bounded sum and the actual sum are equal, while the variance (and the higher moments) of the upper bounded sum is at least as large as the actual sum. Therefore, it is an elegant way to get quickly a first impression of the total risk in the sum of stochastic variables.

The method is applied to derive the cdf of the total claims due to a severe winter storm in several European countries, and to derive the prospective value of an investment portfolio for different mixes of a pension fund. The comonotonic method enables to calculate the cdf of the prospective funding rate of the pension fund (conditional on the correlation between the assets and liabilities), assuming that the assets and liabilities of the fund have a lognormal distribution.

Further work needs to be done to apply the method to derive the funding rate if the assets and liabilities are not lognormal distributed. Also, in this paper only two kinds of investment products are considered: bonds and equities. The portfolio of a pension fund sometimes contains other investment products (like put options). A product like a put option violates the conditions to assure the method returns the riskiest sum, and research is needed to extend the method. The accuracy of the comonotonic sum may be improved by conditioning the terms in that sum, see Dhaene et al. (2002a), and through minimizing the variance of the conditional comonotonic sum, see Vyncke et al. (2004). All these opportunities can be addressed to future work.
References


http://www.dnb.nl/dnb/pagina.jsp?pid=tcm:13-40591-64 and


Appendix

I. Modelling insured claim due to a winter storm catastrophe in Western Europe

The model for the total insured claim caused by a winter storm in the five European countries: Belgium, France, Germany, the Netherlands and the United Kingdom, see Mout (2005). For convenience, the model is repeated here.

We assume that the intensity of a winter storm determines the insured claim size. Furthermore, the higher the intensity of the storm, the lower the probability that such a storm appears in one year. If the intensity is doubled, the probability of a storm with at most such intensity will be reduced by a half. An inverted uniform distributed stochastic variable (here the intensity) exactly has such a cdf. Via empirical data, the relation between the intensity of the storm and the insured claim is derived. It appears that the claim size also corresponds to in which country the claim occurs. Summarizing, we end up with the following model describing the claim size $S_Z$ in country $Z$ as function of the uniform distributed intensity $\Omega$ of a winter storm.

$$S_Z = e^{C(1)+C(2)\cdot UKGE_D} \cdot (FIX_Z)^{C(3)} \cdot (1/\Omega)^{C(4)}$$ \hspace{1cm} [A1]$$

The $FIX_Z$ is a country specific constant, see Table A1.

<table>
<thead>
<tr>
<th>Country</th>
<th>FIXZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>2,007</td>
</tr>
<tr>
<td>France</td>
<td>9,250</td>
</tr>
<tr>
<td>Germany</td>
<td>5,103</td>
</tr>
<tr>
<td>the Netherlands</td>
<td>3,969</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>15,000</td>
</tr>
</tbody>
</table>

$UKGE_D$ is a dummy variable equal to 1 for Germany and the United Kingdom, and otherwise 0. The stochastic $\Omega$ is uniform distributed on the interval $[0, a)$. Here, a is chosen as 1, meaning only one winter storm per year can occur. The value of C(1) until C(4), see Table A2.

<table>
<thead>
<tr>
<th>Table A2. Constants of the claimsize model</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
</tr>
<tr>
<td>C(2)</td>
</tr>
<tr>
<td>C(3)</td>
</tr>
<tr>
<td>C(4)</td>
</tr>
</tbody>
</table>
Because the stochastic variable $\Omega$ is uniform distributed on the interval $[0, 1)$, the cdf of the claim size $S_Z$ can be derived:

$$ p = F_{S_Z}(s) = \Pr[S_Z \leq s] = 1 - \left(\frac{e^{C(1)+C(2)*UKGE\_D} \ast (FIX\_z)^{C(3)}}{s}\right)^{1/C(4)} \quad [A2] $$

This cdf can be inverted analytically, resulting:

$$ s = F_{S_Z}^{-1}(p) = \frac{e^{C(1)+C(2)*UKGE\_D} \ast (FIX\_zz)^{C(3)}}{(1 - p)^{C(4)}} \quad [A3] $$

The valid interval for $s$ is: $e^{C(1)+C(2)*UKGE\_D} \ast (FIX\_zz)^{C(3)} \leq s \leq \infty$

The inverse cdf is needed for the comonotonic sum $S_{EU}^{C}$ of the claim size in the five European countries:

$$ s = F_{S_{EU}^{C}}^{-1}(p) = \sum_{Z = Be, Fr, Ge, Ni, UK} F_{S_Z}^{-1}(p) = \sum_{Z = Be, Fr, Ge, Ni, UK} \left(\frac{e^{C(1)+C(2)*UKGE\_D} \ast (FIX\_z)^{C(3)}}{(1 - p)^{C(4)}}\right) = \left(\frac{1}{1 - p}\right)^{C(4)} \left(\sum_{Z = Be, Fr, Ni} FIX\_Z^{C(3)} + \sum_{Z = Ge, UK} FIX\_Z^{C(3)}\right) = \left(\frac{1}{1 - p}\right)^{C(4)} \text{const}_{EU} \quad [A4] $$

The valid interval for $s$: $\text{const}_{EU} \leq s \leq \infty$

The inverted cdf of the comonotonic sum $S_{EU}^{C}$ is used to produce Graph 1 and 2.
II. Convex order of the actual funding rate $FR$ and the comonotonic funding rate $FR^C$

The be able to order stochastic variables in risk, convex ordering is a useful approach, see Kaas et al. (2001). If two stochastic variables have the same expected value, the variable with the highest variability is the riskiest one. A comonotonic sum and actual sum are convex ordered: their expected values are equal, but the comonotonic sum has at least the same volatility as the actual one. In case of the actual and the comonotonic funding rate, the condition of equal expectations is not fulfilled. This is due to the different correlation coefficients between the liabilities and the actual sum of assets, and between the liabilities and the comonotonic sum of assets. However, in this appendix is shown that the difference between both the expected values of the actual and the comonotonic funding rate are very small, and therefore a statement about the convex ordering can be made.

The expression [5] for the actual funding rate can be rewritten as:

$$FR = \frac{S}{L} = FR_0 e^{\mu_s - \mu_L + \sigma_s \sigma_L \Phi^{-1}(U)} \quad [A\ 5]$$

$$\sigma_t = \sqrt{\sigma_s^2 + \sigma_L^2 - 2 \rho_{SL} \sigma_s \sigma_L} \quad [A\ 6]$$

$$E[FR] = e^{\mu_s - \mu_L + \frac{1}{2} \sigma_s^2} \quad [A\ 7]$$

$\mu_s$: expected value of the return of actual sum of assets

$\mu_L$: expected value of the return of liabilities

$\sigma$: std.dev. of actual sum of assets

$\sigma_L$: std.dev. of liabilities

$\sigma_t$: std.dev. of actual funding rate

$\rho_{SL}$: correlation coefficient between the liabilities and actual sum of assets
The expression [5] for the comonotonic funding rate can be rewritten as:

\[
FR^C = \frac{S^C}{L} = FR_0 e^{\mu_s - \mu_L + \sigma_{tc} \Phi^{-1}(U)} \quad [A8]
\]

\[
\sigma_{tc} = \sqrt{\sigma_C^2 + \sigma_L^2 - 2 \rho_{LC} \sigma_C \sigma_L} \quad [A9]
\]

\[
E[FR^C] = e^{\mu_s - \mu_L + \frac{1}{2} \sigma_{tc}^2} \quad [A10]
\]

**σ_c**: std. dev. of comonotonic sum of assets, note that \( σ_c = σ \)

**σ_{tc}**: std. dev. of comonotonic funding rate

**ρ_{LC}**: correlation coefficient between the liabilities and comonotonic sum of assets

It should hold: \( E[FR] = E[FR^C] \) to state a convex order between \( FR \) and \( FR^C \) =>

For the correlation coefficient between the liabilities and the actual sum of assets (\( ρ_L \)) can be written:

\[
e^{\mu_s - \mu_L + \frac{1}{2} \sigma_{tc}^2} = e^{\mu_s - \mu_L + \frac{1}{2} \sigma_L^2} \iff \sigma_{tc}^2 = \sigma_L^2 \iff \rho_L = \rho_{LC} \frac{\sigma_C}{\sigma} + \frac{1}{2\sigma \sigma_L} (\sigma^2 - \sigma_C^2)
\]

When the std. dev. of the comonotonic sum of assets is expressed as a first order approximation of the std. dev. of the actual sum of assets: \( σ_c \approx σ + Δ \), with Δ small compared to σ. For \( ρ_L \) can be written:

\[
ρ_L \approx ρ_{LC} (1 + \frac{Δ}{σ}) + \frac{1}{2σ σ_L} (-2σ Δ - Δ^2) \approx
\]

\[
ρ_{LC} (1 + \frac{Δ}{σ}) - \frac{Δ}{σ_L} \approx ρ_{LC} \quad [A11]
\]

When \( ρ_L \approx ρ_{LC} \), then also holds: \( E[FR] \approx E[FR^C] \), and therefore: \( FR \leq FR^C \).
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