Are Asset Returns Predictable from the National Accounts?
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Pierre Lafourcade *

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De Nederlandsche Bank

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Abstract

Rational expectations and the logic of budget constraints imply that the predictability of asset returns hinges on the stability and persistence of households’ ratio of saving to asset wealth, not of consumption to total wealth. This misalignment undermines the rationale for Lettau and Ludvigson’s estimated $cay$, a stationary but unduly loose approximation to the true but non-mean-reverting $cay$. Definitional considerations on saving, assets and returns suggest rehabilitating money in the households’ flow of funds identity. Accounting for money balances in $cay$ restores stationarity, but at the cost of drastically lower persistence and predictive potential.

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*Address: De Nederlandsche Bank. Westeinde 1, 1000 AB Amsterdam, The Netherlands. E-mail:P.M.Lafourcade@dnb.nl. Tel: (+31) 20524 3410.
1 Introduction

Rational expectations and the logic of household budget constraints imply that the predictability of returns hinges on the cointegration properties of households’ consumption-to-wealth ratio. The power of this insight, formalized by Lettau and Ludvigson (2001, 2004), lies in the fact that it holds true regardless of preferences. Indeed, in simple micro parlance, the dynamic stability of the offer curve—the observed budget constraint—may be sufficient to forecast marginal rates of transformation—returns—without reference to parameterized marginal rates of substitution. Lettau and Ludvigson’s work has been highly influential because of its applicability to the many dynamic accounting identities that pepper the macroeconomic landscape. Their analysis differs from the benchmark treatment of return predictability in the finance literature (Cochrane, 2001) in that the dynamic constraint in the latter—the arbitrage condition—is an Euler equation, not an accounting identity.

Abstracting from preferences, however, must come at the price of greater scrutiny of accounting relationships, data definitions and approximations. In this paper, I focus on two issues underlying the cointegrated-present-value meme: the econometrics behind the log-linearization of an additive identity, and the joint definitions of consumption, wealth and return imposed by the accounting framework. Upon closer look, the case for return predictability from national accounts data is significantly weakened.

The first, strictly arithmetic, point deals with the fact that the original budget constraint is non-linear in wealth and returns. Non-linearity requires either an unfamiliar treatment or a linear approximation that justifies bringing in standard analytical tools. In the latter case, however, the cointegration space underpinning the log-linearized budget constraint is fully observable—that is, the cointegrating vector need not be estimated since the linearization constants and the restrictions they must satisfy are known. Building on a recent paper by Whelan (2008), I show that a priori knowledge of linearization constants and restrictions implies that Lettau and Ludvigson’s estimated cointegrating vector is a distorted approximation of the true linear combination of \( c, y, \) and \( a, \) which happens to be non-stationary. The return predictability result therefore relies on the persistence of a stationary but poor approximation of a known but non-stationary vector (for which the concept of persistence has little meaning). Consequently, predictability is likely to be simply fortuitous, and a fortiori tenuous, since the underlying econometrics are misspecified.

The second, data-related point provides a solution to the stationarity problem. The predictability result rests implicitly on the fact that income flows and returns derive from
the same asset classes. However, most empirical work focuses on the CRSP return, which is not defined over the same assets as those recorded in the national accounts. Increasing the definitional overlap between CRSP- and NIPA-based returns requires pairing stocks and flows differently in the budget constraint. Rudd and Whelan (2006) moved in this direction by excluding durable goods from household assets, on the grounds that official statistics report expenditures, not service flows. I argue for a similar treatment of money balances: they have no corresponding income flow in the national accounts or in the definition of the CRSP return. Furthermore, money deserves special consideration in the households’ portfolio of assets because it is held for liquidity purposes—not deferred consumption—and it earns essentially no return. I re-interpret money as a durable good whose user cost is the nominal return on alternative assets. Re-allocating money in the national accounts from assets to expenditures in the budget constraint restores stationarity of $cay$. To wit, with money holdings moving from denominator to numerator, the dynamics of the saving-to-wealth ratio—and by implication, its predictive power for future returns—are significantly altered.

An added benefit of rehabilitating money holdings in the budget constraint is to set up a quasi-money demand equation, in the sense that the stationarity of growth-adjusted returns implies a cointegrating relationship involving consumption, income, assets, returns and money balances—the ingredients in a standard money demand equation—albeit not in a fully log-linearized form. ‘Quasi-’ because, mirroring the consumption-wealth relationship studied by Lettau and Ludvigson, this money-based relationship is not restricted by household preferences. Fundamentally, the only modification to their framework is to redefine the consumption bundle to include the rental flow of money balances.

In what follows, I address the analytical and empirical issues mentioned above and introduce money in Lettau and Ludvigson’s and Whelan’s framework. The upshot is that a more coherent accounting of households’ sources and uses of funds significantly weakens the case for return predictability via national accounts data. The reason is that predictability rests on the persistence of the ratio around which the budget constraint is linearized, and the ratio that involves money balances is indeed stationary—indicating cointegration—but poorly autocorrelated—meaning little predictability.
2 Cointegration in log-linearized budget constraints

2.1 Saving-to-asset wealth, or consumption-to-total wealth?

The centerpiece of the predictability result is the accounting identity. It is worth recalling briefly how the relevant data is reported in the national accounts. The household flow of funds states that the period flow of saving goes into asset accumulation,

\[ Y_t + I_t - C_t = \Delta A_{t+1}. \]  

(1)

\( Y \) and \( I \) are after-tax labor and asset income, respectively, \( C \) is consumer expenditures and \( \Delta A \) is net investment in assets. Roughly, the Bureau of Economic Analysis’ national income and product accounts (NIPA) record the left-hand side, while the Federal Reserve Board’s flow of funds accounts (FOF) record the right-hand side.\(^1\)

This flow identity consists of four components. For a variety of reasons, the literature has found it useful to reduce the system rank to three, by pairing flows. The natural step would be to define saving \( S = Y - C \) and the gross return on asset \( R^a_{t+1} = \left(1 - \frac{I_t}{A_{t+1}}\right)^{-1} \), so that the constraint may be written as

\[ A_{t+1} = R^a_{t+1} (A_t + S_t). \]  

(2)

Note that for the return \( R \) to be meaningful, income \( I \) should flow from the same assets that compose the stock variable \( A \)—not an innocuous requirement, given that stock and flow come from different accounts. Nevertheless, the advantage of the pairing above is that \( A \) and \( S \) are observable, so \( R^a \) is computable—a point recently stressed by Whelan (2008).\(^2\)

However, this is not the road followed in the bulk of the literature.

Indeed, the more popular pairing is to consider both asset and labor income as flow returns on total wealth, including unobservable human wealth \( H \). Adding \( \Delta H \) on both sides of (1),

\[ \Delta W_{t+1} \equiv \Delta A_{t+1} + \Delta H_{t+1} = Y_t + I_t - C_t + \Delta H_{t+1}, \]  

(3)

\(^1\)The FOF also include holding gains on assets. When these are incorporated in \( I \), so that \( I \) includes dividends and capital gains, \( \Delta A \) is redefined as net worth. Appendix A.2 provides a full treatment of the reconciliation of NIPA and FOF saving.

\(^2\)Whelan writes the constraint in terms of ‘excess consumption’ \( X = C - Y = -S \) because \( S \) is negative in the US postwar NIPA data. However, ‘saving’ is easier on the ear than ‘excess consumption’ when combined with ‘assets’ and ‘returns’. Furthermore, this semantic point is harmless when I move to log-linearization later on because, as long as \( A > S \), I can always write \( \ln (A + S) \) as \( \ln (A - (-S)) \).
one can always write the constraint in the form

\[ W_{t+1} = R^w_{t+1} (W_t - C_t), \]  

(4)

if returns on total and human wealth are defined appropriately:

\[ R^w_{t+1} \equiv R^a_{t+1} \frac{A_t + Y_t - C_t}{A_t + H_t - C_t} + R^h_{t+1} \frac{H_t - Y_t}{A_t + H_t - C_t}, \]

\[ R^h_{t+1} = \frac{H_{t+1}}{H_t - Y_t}. \]

Total return \( R^w \) is then an average of asset and human wealth returns, with time-varying weights.

The choice of versions of budget constraints should be dictated by the modeler’s narrative angle. If the focus is asset returns, Whelan’s decomposition is the more useful because returns are directly observable. However, version (4), defined over total wealth, has traditionally prevailed because of the natural emphasis in the literature on consumption as the main argument of a utility function. Indeed, equation (4) is useful for pedagogical purposes—to explain, for example, the inner workings of a model of consumption with a single endowment, interpreting consumption as the implicit dividend on total wealth. However, it is only a manipulation of the original flow of funds identity. In practice, the restrictions required on \( H \) and \( R^h \) to re-identify \( Y \) necessarily reduce the wealth-based budget constraint back to the asset-based, observable version (2). In other words, when taken to the data, both versions of the accounting identity should contain the same information. This sounds trivial, but as I show below, this fact has been overlooked in the literature, with stark consequences in empirical applications.

2.2 Log-linearization around a fixed point

Equations (2) and (4) are of the same form, so the following manipulations apply to both. The logarithmic version of equation (2), with logs denoted in small letters, is:

\[ \Delta a_{t+1} - r^a_{t+1} = \ln \left( 1 + \frac{S_t}{A_t} \right). \]  

(5)

Taylor-expanding around a possibly time-varying ratio \( \frac{S_t}{A_t} \) yields:

\[ \Delta \tilde{a}_{t+1} - \tilde{r}^a_{t+1} = \frac{S_t}{S_t^* + A_t^*} (\tilde{s}_t - \tilde{a}_t) \]  

(6)
where $\tilde{x} \equiv x - x^*$ indicates log-trend deviation. This first-order approximation is always feasible but it is non-linear in the log-variables when the intercept $\ln \left(1 + \frac{S_t}{A_t}\right)$ and slope $\frac{S_t}{S_t + A_t}$ depend on time. I will show an example of this below.

Consider in the meantime the case where $\frac{S_t}{A_t} = \phi < 0$ is indeed time-invariant. Rolling forward the constraint, applying the relevant transversality condition and conditioning on time $t$ information yields the expected present value identity:

$$s_t - a_t = E_t \sum_{k=1}^{\infty} (1 + \phi)^k \left(r_{t+k}^a - \Delta s_{t+k}\right) + \kappa,$$

with $\kappa$ capturing the constants in the Taylor expansion. This is the canonical equation taken to the data to verify (i) whether $s - a$ is indeed stationary around the fixed point $\phi$ (and justifies the linearization in the first place), (ii) how much of $s$ and $a$ does the adjusting to maintain the stationarity, and (iii) whether the cointegrating vector forecasts future $r^a$, $\Delta s$ or both.3

### 2.3 Log-linearizing sums

As equation (7) shows, the two-variable system in $S$ and $A$ is sufficient to study the predictability of returns. This is the crux of Whelan’s (2008) analysis. However, Whelan stops short of making the link from the two- to the three-variable system in $A$, $Y$, and $C$. Yet, this is Lettau and Ludvigson’s (2001) object of interest. They back it out from the wealth-based constraint (4), but an equivalent way is to unearth labor income in the asset-based constraint (2). First, expand log-saving,

$$s_t \equiv \ln S_t = \ln (Y_t - C_t) = \ln Y_t + \ln \left(1 - \frac{C_t}{Y_t}\right),$$

then log-linearize the consumption-output ratio around a fixed point,

$$\ln \left(1 - \frac{C_t}{Y_t}\right) \simeq \kappa_c - \gamma (c_t - y_t),$$

and replace in equation (7) to obtain

$$(1 + \gamma) y_t - a_t - \gamma c_t = E_t \sum_{k=1}^{\infty} (1 + \phi)^k \left(r_{t+k}^a - (1 + \gamma) \Delta y_{t+k} + \gamma \Delta c_{t+k}\right) + \kappa.$$  

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This is exactly the same cointegration relationship as the one derived by Lettau and Ludvigson, but with their cointegrating vector—dubbed \( cay \)—scaled by \( \gamma \). Importantly, the linearization of log-saving into a weighted average of log-consumption and log-income restricts the coefficients in \( cay \) to sum to zero, regardless of the value of the expansion point: returns and growth rates should be invariant to scale.\(^4\)

The extra log-linearization also implies that equation (9) captures two cointegrating vectors. Indeed, the left-hand side can be written as

\[
(1 + \gamma) y_t - a_t - \gamma c_t = (y_t - a_t) - \gamma (c_t - y_t)
= (c_t - a_t) - (1 + \gamma) (c_t - y_t),
\]

a linear combination of the vectors representing deviations from the fixed points around which the constraint was log-linearized.

The null hypothesis of two underlying cointegrating vectors reflects the fact that the logarithm of a sum of variables can be approximated by the weighted average of log-variables only if these variables are pairwise stationary, so that the weights are time-invariant. Indeed, one could have noted directly by taking logs of (1) that, if

\[
\ln (A_t + Y_t - C_t) = \alpha_t a_t + \beta_t y_t + (1 - \alpha_t - \beta_t) c_t + k^*_t,
\]

where

\[
\alpha_t = \frac{A^*_t}{A^*_t + Y^*_t - C^*_t}, \quad \beta_t = \frac{Y^*_t}{A^*_t + Y^*_t - C^*_t},
\]

then \( \alpha, \beta \) and \( k^* \) are constant only if \( A^*/Y^* \) and \( A^*/C^* \) (and thus \( Y^*/C^* \)) are constants. The full log-linearization of the budget constraint expressed in terms of the three variables \( A, C \) and \( Y \) rests on two assumptions: the initial null that growth-adjusted returns are stationary (reflected in (7)) and the null of balanced growth, which justifies the expansion in (8).

### 3 Observability and cointegration space in \( cay \)

The equivalence of the log-linearized expected present discounted value relationships highlights some key restrictions in the \( cay \) framework that have not been taken into account in the literature.

\(^4\)A proportional increase in \( A, C \) and \( Y \) (\( \Delta a = \Delta c = \Delta y \) in the LHS of (9)) should leave the RHS unchanged.
3.1 Observable returns

First, every variable in (9) is observable, including returns. This contrasts with the view that Lettau and Ludvigson adopt as a consequence of their derivations. Indeed, their approach, which starts from the consumption to total wealth ratio, led them to the equation in the following form\(^5\),

\[(1 + \gamma)y_t - a_t - \gamma c_t = E_t \sum_{k=1}^{\infty} (1 + \phi)^k \left( r^a_{t+k} - (1 + \gamma) r^h_{t+k} + \gamma \Delta c_{t+k} \right) + (1 + \gamma) z_t,\]

where \(z\) is implicitly defined as

\[z_t = E_t \sum_{k=1}^{\infty} (1 + \phi)^k \left( \Delta y_{t+k} + r^h_{t+k} \right).\]

Since \(r^h\) is assumed to be unobservable, one cannot fully identify \(r^a\) in the expected discounted sum. Whelan (2008) notes furthermore that since \(z\) is also unobservable, \(cay\) need not exclusively reflect expected changes in returns or consumption growth. However, \(z\) collapses back into the present discounted value term to eliminate \(r^h\) and yield the fully observable version (9). Indeed, \(cay\) is just the log-linearized and forwarded version of the manipulation performed in equations (3).

3.2 Observable coefficients

Second, the coefficients of (9) are known \(a\ priori\): they reflect the values around which the linearization is performed, typically the sample average of the consumption-to-labor income ratio, or equivalently, the asset-to-total wealth ratio. To see this equivalence, consider the fact that along a balanced growth path, labor income is the annuity value of human wealth,

\[Y = \left(1 - \frac{G^h}{R^h}\right) H,\]

while excess consumption is the annuity value of asset wealth,

\[C - Y = \left(1 - \frac{G^a}{R^a}\right) A.\]

\(^5\)See equation (9) in Lettau and Ludvigson (2001).
Then, if returns are equalized in the long-run, \( R^a = R^b \), it follows that

\[
\gamma = \frac{C}{Y - C} = \frac{A - H}{A}.
\]

Lettau and Ludvigson argue that, since \( H \) is not observable, they can only infer \( \gamma \) via estimation of the cointegrating vector \( \hat{cay} \). However, the restrictions required to re-identify \( Y \) from the wealth-based budget constraint impose that \( \gamma \) is observable by the ratio of consumption to labor income.

### 3.3 Quality of the log-approximation

Taylor-expanding the identity (5) around a fixed point \( \frac{S^*}{A^*} = \phi < 0 \), and omitting constants yields the approximate identity:

\[
\Delta a_{t+1} - r^a_{t+1} = \frac{\phi}{1 + \phi} (s_t - a_t).
\]

As Whelan (2008) notes, the accuracy of the approximation can be easily verified because all variables and coefficients are observable. However, he stops at the two-variable vector \( s - a \). Pushing his intuition further and linearizing log-saving \( s \) around a fixed point \( \gamma = C^* (Y^* - C^*)^{-1} \) yields

\[
\Delta a_{t+1} - r^a_{t+1} = \frac{\phi}{1 + \phi} ((1 + \gamma) y_t - a_t - \gamma c_t).
\]

That is, \( cay \) is also an approximation of the log growth-adjusted return (the LHS of 10), reflecting two underlying linearizations.

Knowing the LHS of (10) enables us to evaluate the quality of the various approximations. The upper chart of figure 1 plots several normalized ratios over the post-war sample: the observable growth-adjusted return \( \Delta a - r \), its approximation \( s - a \equiv cay \) conditional

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6On page 819 of their 2001 paper, the authors state that they do not impose long-run equalization of returns. However, their derivations imply otherwise. Indeed, since by definition \( W_t - C_t = (A_t + S_t) + (H_t + Y_t) \), returns are linked as follows

\[
\frac{W_{t+1}}{R_{t+1}^w} = \frac{A_{t+1}}{R_{t+1}^a} + \frac{H_{t+1}}{R_{t+1}^h}.
\]

Log-linearizing wealth and returns yields the approximations:

\[
w_t = \varphi a_t + (1 - \varphi) h_t
\]

\[
w_t - r^w_t = \psi (a_t - r^w_t) + (1 - \psi) (h_t - r^h_t),
\]

where \( \varphi = A/W \) and \( \psi = (A/W)(R^w/R^a) \). In their equations (6) and (7), Lettau and Ludvigson assume that \( \varphi = \psi \), implying that \( R^w = R^a = R^h \).
on the known constant $\gamma$ (its sample average), and two estimated ratios $\hat{s} - a$ and $\hat{cay}$. The lower left-hand chart is the corresponding scatterplot of these ratios against $\Delta a - r$. The data (in particular the treatment of durables) is discussed in section 4 and in appendix A.2.

Note first how $cay$ tracks $\Delta a - r$ to a tee; the scatterplot shows the residual curvature of the first-order expansion of the logarithmic function. Next, I plot $\hat{s} - a$ computed from dynamic least squares to highlight how the approximation worsens when estimated without parameter restrictions.\footnote{That is, I use the fitted value from the DLS regression $s_t = c + \beta a_t + \sum_{i=1}^{k} \gamma_i \Delta a_{t+1}$ without the restriction that $\beta = 1$.} The approximation does not fit as well because the regression attempts to flatten the inverted v-shape of the true ratio $s - a$ in the outer parts of the sample. The fitted value $\hat{cay}$ from a similar DLS regression—Lettau and Ludvigson’s choice variable—matches the true growth-adjusted return poorly, for essentially the same reason: the DLS estimation attempts to correct for the break and trends in $s - a$ clearly visible in the chart, distorting the ratio towards a stationary process. However, it never fully manages to get there, as the Johansen test (not reported) does not reject the null of no cointegration.
Inspection of the upper chart reveals that the borderline stationarity of \( \hat{cay} \) is achieved by deflating \( cay \) in the period 1975-1980 and inflating it in 1995-2000, creating spurious mean-reversion roughly every four to five years at the cost of a poor approximation—visible in the scatterplot. The lower right-hand chart in figure 1 plots \( cay \) against its two components, \( c - a \) and \( c - y \). The scatter shows how \( cay \) inherits its properties mostly from the consumption-to-labor income ratio (one minus the saving rate), which is well-known to be unstable over the post-war sample\(^8\).

### 3.4 Estimation bias

Why is the DLS estimation producing such a different process for the cointegrating relationship? Knowing the linearization constants helps gauge whether the expansion point implied by the estimation procedure is reasonable. To see this, re-write the approximation of log-saving (8) as

\[
c_t = \xi y_t + (1 - \xi) s_t,
\]

where \( \xi = 1 + \gamma^{-1} \) is a suitable expansion point of the ratio \( \frac{Y}{C} \) (typically, the sample mean). Scale the variables by assets \( a \),

\[
c_t - a_t = \xi (y_t - a_t) + (1 - \xi) (s_t - a_t).
\]

Recall that everything is known in this identity. Projecting both sides on \( y - a \) yields the estimate of the cointegration parameter

\[
\hat{\xi} = \xi + (1 - \xi) \frac{E(s - a)(y - a)}{E(y - a)^2}.
\]

Clearly, \( \hat{\xi} \) is unlikely to be a consistent estimate of \( \xi \), since \( s - a \) and \( y - a \) are unlikely to be orthogonal. In fact, over the sample 1952Q4-2008Q1, the projection of \( s - a \) on \( y - a \) yields \( \frac{E(s - a)(y - a)}{E(y - a)^2} \simeq -1 \), so that the bias is

\[
\hat{\xi} - \xi \simeq 1.
\]

Dynamic LS provides the following fit over the same sample,

\[
c_t \simeq -0.8 + \frac{2}{3} y_t + \frac{1}{3} a_t,
\]

\(^8\)Nominal and real ratios are equivalent because the budget constraint is deflated by the same price index.
where $\hat{\xi}$ is roughly the value obtained by Lettau and Ludvigson. Figure 2 shows the time series (in levels) of $Y/C$ over the postwar period. Observe that the bias relationship is satisfied, as $\frac{\hat{\xi} + 1}{2} \simeq 0.8$, roughly the sample mean of the ratio. The expansion point implied by least squares estimation (the red line) lies far beyond any reasonable neighborhood suitable for first order approximation. This provides another sense in which estimated $cay$ is an inadequate approximation of true $cay$.

### 3.5 Predictive content of the saving-to-asset wealth ratio

What are the implications of this loose approximation for return predictability? Consider again the system in asset wealth and saving $X = [a \ s]'$. Under the correct null, the standard VECM set-up would be

$$
\Delta X_{t+1} = A(L) \Delta X_t + \begin{pmatrix} \gamma_a \\ \gamma_s \end{pmatrix} (a_t - s_t) + u_{t+1}.
$$

(11)

The cointegration vector is restricted to $[1 - 1]$ from the underlying log-linearization. Projecting the approximate identity (5) on time $t$ information yields

$$
E_t \Delta a_{t+1} = E_t r_{t+1}^a + \frac{\phi}{1 + \phi} (s_t - a_t).
$$

Then, the estimated VECM imposes that

$$
E_t r_{t+1}^a = [1 \ 0] A(L) \Delta X_t + \begin{pmatrix} \gamma_a - \frac{\phi}{1 + \phi} \end{pmatrix} (s_t - a_t).
$$
Thus, a test of whether returns are predictable by the saving-wealth ratio is a test of \( \gamma_a = \frac{\phi}{1+\phi} \), where the linearization constant \( \phi \) is known. A test whether returns are predictable at all is a joint test that \( [1 \ 0] A(L) = [0 \ 0] \) and \( \gamma_a = \frac{\phi}{1+\phi} \). Predictability of returns is fully contained in the VECM equation for wealth in the \([a \ s]\) system.

This point is worth emphasizing because expanding the system to \([a \ c \ y]'\) while maintaining the parameter restrictions imposed by the linearization in (8) will not alter the predictability of returns. Indeed, since \( s = (1+\gamma)y - \gamma c \) and \( \gamma \) is known, the vector \([a \ c \ y]'\) contains the same information as \([a \ s \ y]'\) and the return \( r \) is a function of \( a \) and \( s \) only. Lettau and Ludvigson’s results based on \( \hat{cay} \) differ from those that the set-up above would produce only to the extent that their estimation is unrestricted.

Although the theoretical implications of VECM for predictability are clear, the empirics are irrelevant because the null is incorrect. A simple glance at figure 1 should convince that \( s - a \) is not stationary, so that (11) is not a balanced system of equations. Therefore, estimation results will suffer from mis-specification bias and likely overstate significance. Predictability from \( \hat{cay} \) rests only on the fact that the linearization restrictions are not satisfied. There stands the trade-off: return predictability versus theoretical consistency.

### 3.6 Linear trend-fitting in \( cay \)

If \( s - a \) does not appear to mean-revert over the US postwar period, why not explicitly account for breaks and trends in the VECM above? This is precisely what Hoffmann (2006) attempts, in order to rehabilitate \( cay \) as a cointegrating vector.\(^9\) However, doing so breaks the link between return predictability and intertemporal accounting because linear trend-fitting is not compatible with the original budget constraint in level form.

To see this, return to equation (6) and suppose that the saving-to-wealth ratio fluctuates around a possibly non-linear trend, \( \frac{S_t}{A_t} = \phi(t) \). The appropriate present value relationship involving the trending cointegration vector is then

\[
\tilde{s}_t - \tilde{a}_t \equiv s_t - a_t - \ln \phi(t) = E_t \sum_{k=1}^{\infty} \Phi_{t,t+k} \left( \tilde{r}_{t+k}^a - \Delta \tilde{s}_{t+k} \right),
\]

where \( \Phi_{t,t+k} = \prod_{i=1}^{k} (1 + \phi(t+i)) \). The cointegrating vector now forecasts trend deviations of returns and growth of saving, with a time-varying discount coefficient.

\(^9\)Hoffmann also established that \( cay \) was a linear combination of the non-stationary ‘great ratios’ \( c - a \) and \( c - y \). I was made aware of this paper only after having independently derived the same conclusion on the faulty null of the cointegration space in \( cay \).
Trends, linear or not, complicate the econometrics of return forecastability. They require keeping track of the $\phi(t)$ in the trend term of the cointegrating vector and in the compounding discount factor. This matters for out-of-sample forecasts from long-horizon regressions—of the type $\sum_{k=1}^{n} r_{t+k}^a = \alpha + \beta (s_t - a_t) + \varepsilon_t$—because approximation errors stack up quickly with $n$.

In the three-variable system $[a \ c \ y]'$, the relevant LHS of the present value identity is

$$(1 + \gamma_t) y_t - a_t - \gamma_t c_t + \phi(t),$$

where the weight $\gamma_t = C^*_t (Y^*_t - C^*_t)^{-1}$ is time-dependent. Note furthermore that $\gamma_t$ and $\phi(t)$ cannot both be linear in $t$. Thus, a correct log-linearization of the original accounting identity around time-trends yields a cointegrating vector with time-varying parameters. Reversing the argument, if one simply tacks on a trend to an already log-linearized weighted average of $c$, $a$, and $y$, the resulting $cay$ cannot be an appropriate approximation of the present value identity. One must then drop the pretense that $cay$ is derived solely from a well-specified aggregate budget constraint.

Another important issue is that trend-fitting affects the overall predictability of returns: by accounting identity, any filter applied to the cointegrating vector must be applied to
growth-adjusted returns. The inverted v-shape of \( s - a \) easily lends itself to retrofitting with a break and two trends. This is roughly what Hoffmann does. To illustrate the implications, figure 3 plots such a fit and the resulting cycle for two different break dates. The normalized cycles are obviously stationary with markedly weaker persistence than the original series—they cross the zero line much more often. I show different breaks only to highlight how their choice can affect the resulting persistence: the choice of a later date, for example, induces longer cycles around the earlier trend (bottom two charts). The upshot is that trend-adjusted returns are now forecastable at a much shorter horizon. Predictability is shifted into the unexplained trends, with much of the intrinsic dynamics swept under the rug. The RHS of figure 3 show how similar the cycles are to \( \hat{c}_{ay} \), which essentially uses an implicit expansion point several standard deviations away from a suitable neighborhood (recall figure 2) to fit the break and trends.

3.7 Permanent and transitory shocks under the wrong null

A third implication of the wrong null arises when analyzing the nature of the shocks hitting the system, as in Lettau and Ludvigson (2004). One cointegrating vector implies one temporary and two permanent shocks, while two vectors imply two temporary and one permanent. Thus impulse response analysis and variance decomposition exercises will be misleading if the null is incorrect. The arithmetic of this point is involved and therefore relegated to Appendix A.1.

4 Consistent accounting of saving and wealth

The analysis up to this point shows that, if the goal is to uncover a stationary combination of elements of the budget constraint that is useful for predicting returns, saving and wealth in the canonical treatment of \( c_{ay} \) are not defined optimally. To wit, the accounting identity (1)

\[
Y_t + I_t - C_t = A_{t+1} - A_t,
\]

does not appear to yield stationary log-linear combinations of variables, given their definitions. The alternative I explore below is to split money holdings out of the asset measure \( A \). I do this on the statistical grounds that money does not yield any asset income recorded in the NIPA (the \( I \) above), and on the implicit normative grounds that they do not serve the same function as other types of assets in agents’ intertemporal choice—namely, they
are held for liquidity purposes, not deferred consumption.

In what follows, I briefly review the debate on durable goods accounting in cay and re-interpret money holdings as a durable good. I then explore the consequences or correctly accounting for money holdings for the cointegrated present value relationship and return predictability.

4.1 Durable goods accounting

Appendix A.2 reports the reconciliation of NIPA and FOF-based definitions of household saving in the national accounts. These accounts record the data to which the cay framework is taken, so the pairings of flows must be consistent with this reconciliation. This point was highlighted by Palumbo, Rudd and Whelan (2006). I re-iterate it below and expand.

The flow concept $C$ in the NIPA-based budget constraint is expenditure, not consumption. The treatment of durable goods $D$—whose relative price is $p$ and depreciation rate $\delta$—yields two equivalent forms of the budget constraint,

$$A_{t+1} = R^a_{t+1} \left( A_t + Y_t - C_{t}^{nd} - p_t (D_t - (1 - \delta) D_{t-1}) \right)$$

and

$$W_{t+1} = R^a_{t+1} \left( W_t + Y_t - C_{t}^{nd} - u_t D_t \right),$$

where $W_t \equiv A_t + p_t (1 - \delta) D_{t-1}$ now denotes wealth including the stock of durables. In the second form, expenditures include the implicit rental flow of services of durable goods, with the user cost defined as $u_t \equiv p_t \left( 1 - \frac{p_{t+1}(1 - \delta)}{p_t R^a_{t+1}} \right) \simeq p_t (r + \delta - \pi)$.

National accounts report durable goods as expenditures, not service flows. Thus, the first form of the constraint is appropriate and such items should be excluded from the definition of wealth. In the case of housing, however, the national accounts attribute a flow of services (including rental cost) to owning a home—recorded as owner’s equivalent rent. Therefore, the housing stock should be included in wealth. If consistency is maintained, the return is the same in both formulations, and applies to asset wealth $A$ only.

Lettau and Ludvigson argue in favor of the second version of the constraint (in $W$) on the grounds that nondurable goods would be the argument of a utility function in a fully-fledged model to be determined. Yet, because they do not include the implicit service
flow of durables in expenditure, it ends up in the rate of return

\[ W_{t+1} = R_{t+1}^{ll} \left( W_t + Y_t - C_{t}^{nd} \right) \implies R_{t+1}^{ll-1} = R_{t+1}^{ll-1} + \frac{u_t D_t}{W_{t+1}}. \]

This choice would be perfectly acceptable if their narrative emphasized, say, the normative consequences of policies affecting non-durable consumption. It is also logical insofar as the return is based on the rental flow of wealth including durables. However, if the purpose of their work was to immunize the econometrics of return forecastability from considerations about preferences, this argument does not hold. Furthermore, if the true focus of the analysis is market-based returns, this version of the budget constraint is inappropriate because durables are not traded on financial markets. More specifically, Lettau and Ludvigson’s choice variable for the return is not \( r^a \), the return defined by the accounting identity, but \( r^{crsp} \), the CRSP return, which is computed from financial data and excludes implicit service flows from durables. The whole of section 3 therefore followed Palumbo et al. (2006) in imputing durables to expenditures rather than net worth.

### 4.2 Money balances as durables

Many other asset classes besides durables cloud the mapping from \( r^a \) to \( r^{crsp} \). I focus here on households’ deposits because they are means of payment and generate no income flow.\(^{10}\) The analogy to durables is straightforward (see Obstfeld and Rogoff (1996, Chapter 8)): money balances can be interpreted as a durable good whose rental cost is a nominal rate. Consider that consumers must allocate part of their initial endowment (assets, labor income and initial money holdings) to liquid balances which earn no return (or a small one, see below) before investing the remainder:

\[ A_{t+1} = R_{t+1}^{a} \left( A_t + Y_t - C_t - \frac{M_t - M_{t-1}}{P_t} \right). \] (13)

The timing convention is that \( M_t \) is the quantity of nominal balances accumulated over \( t \) and carried over to period \( t + 1 \). The constraint is now written in real terms: \( P \) is the PCE deflator and the real rate \( R^a \) is defined accordingly.\(^{11}\) With wealth now including real

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\(^{10}\)The appendix suggests the possibility of splitting out housing wealth as well.

\(^{11}\)The budget constraint is a purely nominal concept. Consistency requires that all variables be deflated by the same price index to move to reals—as Palumbo et al. (2006) noted—but it need not be the consumption deflator. The choice of this deflator is what links the budget constraint back to preferences: real consumption is the main argument of a standard utility function, and money balances are deflated by the PCE deflator because real consumption is what money is exchanged for.
money balances, $W_t = A_t + \frac{M_{t-1}}{P_t}$, the constraint can be re-written as follows:

$$W_{t+1} = R_t^a \left( W_t + Y_t - \left( C_t + \left( 1 - \frac{1}{I_{t+1}} \right) \frac{M_t}{P_t} \right) \right), \quad (14)$$

where the gross nominal interest rate $I$ is defined as $I_{t+1} = R_t^a \frac{P_{t+1}}{P_t}$. Reading (14) in the light of (2), stationary growth-adjusted returns imply a stationary saving-to-wealth ratio, where the expenditure bundle includes the flow services of money balances, and the return is defined over non-monetary assets. The elements of a cointegrating, ‘money-demand’ relationship are contained in this ratio. I call this a ‘quasi’-money demand equation, because it arises before preferences are defined\(^{12}\).

### 4.3 Revisiting return predictability

The two forms of the budget constraint imply two versions of the cointegrated present value formula:

$$s_{x,t} - x_t = \sum_{k=1}^{\infty} \phi_x^k \left( r_t^a \Delta s_{x,t+k} + \kappa_x \right), \quad x = a, w$$

where the definitions of saving and wealth must be consistent for the return $r^a$ to remain the same:

$$s_{a,t} = \ln \left( Y_t - C_t - \frac{M_t - M_{t-1}}{P_t} \right),$$

$$s_{w,t} = \ln \left( Y_t - C_t - \left( 1 - \frac{1}{I_{t+1}} \right) \frac{M_t}{P_t} \right).$$

When money earns a return $I_m$, saving is modified as follows:

$$s_{a,t} = \ln \left( Y_t - C_t - \frac{I_{m,t+1}^{-1} M_{t+1} - M_t}{P_t} \right)$$

$$s_{w,t} = \ln \left( Y_t - C_t - \left( I_{m,t+1}^{-1} - I_{t+1}^{-1} \right) \frac{M_{t+1}}{P_t} \right).$$

---

\(^{12}\)For example, utility defined over consumption and money balances and maximized subject to (14) would lead to an Euler equation of the form

$$\frac{U_{M/P} (C, \frac{M}{P})}{U_C (C, \frac{M}{P})} = 1 - \frac{1}{I_{t+1}}$$

i.e. the user cost of money balances is equal to the marginal rate of substitution. A parametric version of this equation is an LM curve with consumption capturing the transactions demand for money.
To compute these variables, I use as definition of money the first four components of household deposits reported in the FOF (which rank financial assets roughly by decreasing liquidity): foreign deposits, checkable deposits and currency, time and saving deposits, and money market mutual fund shares. Since the latter two categories represent over 95 percent of total deposits and earn a return which closely tracks the 3-month T-bill rate, I simply define \( I_m \) to be this safe rate and apply it to all deposits, including currency. Table 1 provides a snapshot of the magnitudes involved.

Figure 4 plots the two saving-to-wealth ratios that explicitly account for money balances. These ratios have strikingly different time-series properties from either \( s - a \) or \( \hat{c}ay \) that were described in the previous section: both are noticeably more volatile at short-horizons, and seem to display little of the persistence that would translate into potential return predictability. However, the related spectra show that, while \( s_w - w \) is indeed little different from white noise (its spectrum is basically flat over all frequencies), \( s_a - a \) does display some low frequency swings, as well as a quarterly seasonal factor\(^{13}\).

The lower right-hand chart plots the \( \theta_i \), the autocorrelation coefficients of \( s_a - a \) and \( \hat{c}ay \), with confidence intervals. Restricting equation (7) to the \( k \)-th horizon and conditioning on the relevant \( t \)-period ratio yields

\[
1 - \frac{\phi}{1 + \phi} \sum_{i=0}^{k-1} \theta_i - \theta_k = \beta \left( \sum_{i=1}^{k} \Delta s_{t+i}, s_t - a_t \right) - \beta \left( \sum_{i=1}^{k} r_{t+i}^a, s_t - a_t \right),
\]

where, in Cochrane’s (2006) notation, \( \beta (y, x) \) is the slope of the regression of \( y \) on \( x \). Therefore, the \( k \)-period cumulative autocorrelation function of \( s - a \) (the LHS) splits into

\(^{13}\)The NIPA data is seasonally adjusted, but the FOF is not. The aggregate wealth data that Lettau and Ludvigson use does not display a strong seasonal component, but some of its subcategories do— particularly deposits.
slopes of $k$-horizon regressions of $r^a$ and $\Delta s$ on $s - a$. To the extent that $\Delta s$ is unpredictable, the cumulative autocorrelation function is an upper bound to the predictability of $r^a$. The chart shows that predictability would be much weaker with $s_a - a$ than with $\hat{cay}$, and would extend only to a horizon of maximum 8 quarters (5 to be conservative). There is no predictability with $s_w - w$ since it is white noise. Furthermore, a simple VECM analysis as described in section 3.5 reveals that most of the adjustment in $s_a - a$ is done by $s_a$ rather than $a$ (not shown, but obvious given the different volatilities of the saving variable). These considerations suggest that a coherent treatment of money, assets and returns yields only very short-run evidence of return predictability from the national accounts.

4.4 Any predictability for money?

Does the budget constraint contain other stable relationships that justify further log-linearization? In notation, if
Figure 5. Normalized ratios of components of the budget constraint (4-quarter moving average).

\[ s - a = \ln \left( \sum X_i \right) - a \sim I(0) \]

can the saving-to-wealth ratio be represented as a weighted-average of other stationary ratios?

\[ s - a = \sum \gamma_i x_i - a = \sum \gamma_i (x_i - a), \quad x_i - a \sim I(0) \]

Figure 5 plots the systematic search for such \( x_i - a \) in the money-based budget constraints (13) and (14). The upper left-hand chart reproduces the two already identified in figure 4, in moving average form to increase readability. The two lower charts are similar because money balances are only a small component of net worth, so that the denominators \( A \) and \( W \) are little different. The ratio \( (Y - C)/W \)—the dotted line in the lower RH chart—is just the exponential of \( cay \) from the previous section. These charts show that consumption and income-to-asset ratios as well as their difference are not reverting to well-defined means over the post-war sample. Since \( s - a \) is stationary, however, the ‘complement’ ratio, which includes money balances in either difference or flow form in the numerator, must also suffer from ill-defined mean-reversion. For completeness, the upper right-hand chart plots the remaining money-based ratios implicit in (13), which also appear non-stationary.
The absence of such mean-reversion in money-based ratios suggests that there is little predictability to be exploited for money balances from the budget constraint alone. To be sure, the phases in the velocity measures could be modeled with breaks and trends, but the analysis of section 2.2 warned that such trends cannot be justified by a properly linearized constraint. Extraneous information—such as mediated by preferences and technology—may be brought to bear to try to recoup a stationary relationship between the level of money balances and other aggregates of the flow of funds identity. But this simply highlights the fact that the budget constraint on its own is not ‘stable’ enough to help forecast its subcomponents as they are currently defined.

5 Conclusion

Lettau and Ludvigson’s (2001, 2004) insight is simple but powerful: dynamic stability of the offer curve (read, the observed budget constraint) may be enough to forecast marginal rates of transformation (i.e. returns) without reference to marginal rates of substitution (preferences). This approach is well-founded in the sense that it starts from earlier than first principles, what Schelling (2006) describes as one of the few ‘truths’ in economics, namely accounting identities:

The truth of all scientific propositions depends on careful definition; but the truth of the so-called [accounting] identities depends only on careful definition. They are not merely definitions; if they were, they would be obvious. But they can be derived from definitions if the definitions are carefully made consistent.[…] Disparaging them as ”mere identities” at least testifies to their truth.

They are the foundation of any macroeconomics. (Ch.11, p.148)

The rest builds from there: optimality conditions derived from preferences or technology are developed to shed light on the internal stability of these truths, not the other way around. A well-founded research program should begin by investigating systematically this internal stability before launching into modeling.

Although Lettau and Ludvigson likely adhere to this statement of intent, their empirical results rest on unduly loose approximations and on concepts of saving and wealth that are not the most appropriate for dealing with the issue of return predictability. A more coherent accounting of households’ sources and uses of funds—one that acknowledges a distinct role for money—shows little evidence of predictability of returns from the household accounting
identity. Furthermore, instabilities in pairwise ratios—in particular those involving the level of money balances—imply that the budget constraint cannot be fully log-linearized without inducing tenuous patterns of mean-reversion that may be ultimately misleading in forecasting exercises. Modeling strategies that involve preferences and technology should take this into account. In particular, household money demand equations that are expressed solely in logarithmic terms are likely to suffer from functional form misspecification.

Nevertheless, the beauty of accounting ‘truths’ is that, unlike preferences, they can be easily aggregated. Households are not the only holders of cash and market-traded, return-bearing assets. Much of the analysis performed here could be repeated for firms—whose saving is defined as after-tax gross operating surplus net of investment and payouts—without reference to why they should wish to hold such assets. The combined budget constraint of households and firms—the ultimate spending units in the economy who exchange the means of payments against real resources—may bring us closer to mapping national accounts-based returns to the CRSP and reveal greater stability in money-based ratios that would help forecast money and prices. The cointegrated present value meme may have more staying power at this higher level of aggregation.
A Appendix

A.1 Implications of incorrect accounting of cointegrating vectors

Why should the number of error-correction mechanisms matter? The answer comes when analyzing the nature of the shocks hitting the system. One cointegrating vector implies one temporary and two permanent shocks, while two vectors imply two temporary and one permanent. Thus impulse response analysis and variance decomposition exercises will be misleading if the null hypothesis is incorrect.

To understand the mechanics behind this argument, consider the following VECM for $X' = [a, c, y]'$:

$$
\Delta X_t = \phi_0' X_{t-1} + \Gamma \Delta X_{t-1} + \varepsilon_t
$$

which has moving average representation

$$
\Delta X_t = C (L) \varepsilon_t.
$$

Following Gonzalo and Ng (2001), one can rotate the innovations via a matrix $G$, to be defined below, to identify permanent and transitory shocks,

$$
\Delta X_t = C (L) G^{-1} \varepsilon_t = D (L) u_t,
$$

and impulse responses and variance decompositions are performed on the matrix polynomial $D (L)$.

I am interested in contrasting the hypotheses ($H_1$ and $H_2$) of one or two error-correction mechanisms. Lettau and Ludvigson (2004) perform their entire analysis under the null that

$\Omega u = G \Omega_v G'$

then

$$
\Delta X_t = C (L) G^{-1} H H^{-1} G \varepsilon_t = M (L) v_t
$$

with

$$
v_t = H^{-1} u_t = H^{-1} G \varepsilon_t
$$

so that

$$
\Omega_v = H^{-1} G \Omega_v G' H^{-1} = I
$$

Impulse responses and variance decompositions are then run on the $M$ polynomial.
$H_1$ is true, and that the cointegrating vector affects wealth only. Therefore,

$$
\phi\alpha' = \begin{bmatrix} \gamma_w \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \theta & -1 & 1 - \theta \end{bmatrix},
$$

and to identify the shocks $u$,

$$
G_1 = \begin{bmatrix} \phi'_{\perp} \\ \alpha' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \theta & -1 & 1 - \theta \end{bmatrix},
$$

where the orthogonal complement $\phi'_{\perp}$ is such that $\phi'_{\perp} \phi = 0$.

Suppose on the other hand that $H_2$ is true and, without loss of generality, suppose that $c$ error-corrects to $y$. Then,

$$
\phi\alpha' = \begin{bmatrix} \gamma_w \\ \gamma_c \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \theta & 0 & -\theta \\ 0 & -1 & 1 \end{bmatrix}
$$

with

$$
G_1 = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ \theta & 0 & -\theta \end{bmatrix} = AG_2,
$$

where $G_i$ identifies the permanent and temporary shocks under $H_i$. Notice that the system is observationally equivalent under $H_1$ and $H_2$ with regards to the wealth equation (the first element of $X$), but not for consumption and labor income, and not for the implied permanent and temporary shocks. Indeed, reconsider the P-T decomposition:

$$
\Delta X_t = D(L) u_t = P(L) \eta_t
$$

with

$$
A \eta_t = u_t, \quad P(L) = D(L) A
$$

Under $H_2$, the LL innovations are linear combinations of the true permanent and temporary
innovations:

\[
\begin{bmatrix}
u_1^p \\
u_2^p \\
u^t \\
\end{bmatrix}
= 
\begin{bmatrix}
\eta_{i,1}^p - \eta_{i,2}^p \\
\eta_i^p \\
\eta_{i,1}^t + \eta_{i,2}^t \\
\end{bmatrix}.
\]

Thus impulse responses, which assume iid shocks, will be different under $H_1$ and $H_2$.

Consider also the k-period-ahead forecast error variance:

\[
V^k = \sum_{h=0}^{k-1} C_h \Omega \epsilon C_h' = \sum_{h=0}^{k-1} D_h \Omega D_h' = \sum_{h=0}^{k-1} P_h \Omega P_h'.
\]

The contribution, under $H_i$, of the $j$-th shock to the variance of the $p$-th variable in $\Delta X$ is defined as:

\[
\omega_{p,j}^k (i) = \frac{\sum_{h=0}^{k-1} (e_p^h D_h e_j)^2}{e_p^h V^k e_p} \sigma_j^2 (i)
\]

In matrix form, for $W^k_i = \{ \omega_{pn}^k (i) \}$,

\[
W^k_i . \text{diag}(V_k) = \Omega \sum_{h=0}^{k-1} (D_h \odot D_h)' .
\]

Suppose $H_2$ is true, then

\[
W^k_2 . \text{diag}(V_k) = \sum_{h=0}^{k-1} (D_h A \odot D_h A)',
\]

while

\[
W^k_1 . \text{diag}(V_k) = AA' \sum_{h=0}^{k-1} (D_h \odot D_h)'.
\]

The two are not equal because the dot product (Hadamard) is not distributive.

### A.2 Reconciliation of NIPA and FOF saving

FOF and NIPA saving are reconciled as follows. The symbol $\Delta$ represents net investment (purchases) in the given category.

The reconciliation table R100 of the Flow of Funds Accounts states that the change in household net worth is equal to total net investment plus holding gains on all assets (at market value for financial assets and real estate, current cost for durables and equipment and software):

\[
NW_{t+1} - NW_t = \Delta \text{Total} + \text{HoldingGains}.
\]
Net investment is equal to net physical investment plus net financial investment:

$$\Delta Total = (\text{Capex} - \text{Depreciation}) + \Delta Financial.$$ 

Referring then to table F100 on the flow accounts of households,

$$\text{Saving}^{\text{NIPA}} + \Delta \text{Durables} + \text{NetCapitalTransfers} = \Delta Total + \text{Discrepancy},$$

where discrepancy refers to the gap between NIPA and FOF-based saving measures due to different source data. Substituting back into the net worth identity yields

$$NW_{t+1} - NW_t = \text{Saving}^{\text{NIPA}}$$

$$+ \Delta \text{Durables} + \text{NetCapitalTransfers} + \text{HoldingGains} - \text{Discrepancy}.$$ 

Since holding gains include those on durables $HG = HG^d + HG^{xd}$, and since the change in the value of durable holdings is

$$\text{Durables}_{t+1} - \text{Durables}_t = \Delta \text{Durables} + \text{HoldingGains}^d,$$

we can rewrite the net worth identity as

$$(NW_{t+1} - \text{Durables}_{t+1}) - (NW_t - \text{Durables}_t) = \text{Saving}^{\text{NIPA}} + \text{NetCapitalTransfers}$$

$$+ \text{HoldingGains}^d - \text{Discrepancy}.$$ 

Consider now the definition of NIPA-based saving from NIPA table 2.1:

$$\text{Saving}^{\text{NIPA}} = PDI - \text{Expenditures} - \text{InterestPaid} - \text{Transfers}.$$ 

Personal disposable income is composed of labor income and asset income. Unfortunately, the NIPA do not split these two types of income, and Lettau and Ludvigson (2004) and Rudd and Whelan (2006) use the standard way to break them out, by pro-rating current taxes by the share of wage income in total gross non-transfer income. In the end, one can write PDI as

$$PDI = \text{AftertaxLabourIncome} + \text{AftertaxCapitalIncome}.$$
Substituting back, with net worth defined excluding durables:

\[
NW_{t+1}^{rd} - NW_t^{rd} = \text{AftertaxLabourIncome} - \text{Expenditures} \\
+ (\text{AftertaxCapitalIncome} - \text{InterestPaid}) + \text{HoldingGains}^{rd} \\
+ (\text{NetCapitalTransfers} - \text{Transfers}) \\
- \text{Discrepancy}
\] (15)

In the notation used in the first equation of the paper, this summarizes to

\[
A_{t+1} - A_t = Y - C + I
\]

where ‘asset income’ \(I\) in the text refers to the second, third and fourth lines of (15). The cleanest measure of asset income would be the second line alone. However, two wedges muddy the implicit measure of asset returns. The wedge in transfers is inconsequential, but the discrepancy between NIPA and FOF-based measures of household saving is large relative to saving out of labor income (the first line).

Note in particular that the Flow of Funds accounts exclude durables from net worth and include them in expenditures, and I follow this convention in the paper. Another possible decomposition starts by splitting expenditures into non-durable and durable components:

\[
\text{Expenditures} = C^{nd} + p_t (D_{t+1} - (1 - \delta) D_t) \\
= C^{nd} + p_t D_{t+1} - p_{t-1} D_t + \left(1 - \frac{p_t}{p_{t-1}} \delta \right) p_{t-1} D_t \\
= C^{nd} + \text{Durable}_{t+1} - \text{Durable}_t + u.\text{Durable}_t,
\]

where \(u\) is the implicit rental cost of durables. If one wishes to consider total net worth and non-durable consumption as the relevant variables, as Lettau and Ludvigson (2001, 2004) do, the identity changes to

\[
NW_{t+1} - NW_t = \text{AftertaxLabourIncome} - C^{nd} \\
+ \text{AftertaxNetCapitalIncome} + \text{HoldingGains}^{rd} - u.\text{Durable}_t \\
+ \text{wedges}.
\]

The flow return on assets (line 2) will include the rental cost of durables. The main disadvantage of this version of the budget constraint is that it imputes to the implicit
wealth return a flow return that is not directly observable (as are interest and dividend income, for example) because it applies to assets that are not normally traded on financial markets. That is, durable goods are not assets whose returns are incorporated in market-based returns such as the CRSP.

One could ask, then, what treatment for housing would enable us to get a measure of the wealth return yet closer to a market-based return. Since the change in the value of holdings of residential assets is

\[ H_{t+1} - H_t = \Delta H + \text{HoldingGains}^h, \]

we can substitute back into (15) to obtain

\[
\begin{align*}
    NW_{t+1}^{xdh} - NW_t^{xdh} &= AftertaxLabourIncome - Expenditures - \Delta H \\
    &+ (AftertaxCapitalIncome - InterestPaid) + HoldingGains^{xdh} \\
    &+ \text{wedges},
\end{align*}
\]

so that, by augmenting expenditures by the net investment in housing assets, the implicit return on household net worth net of durables and housing would include only income and holding gains from non-tangible assets, a measure matching more closely the CRSP return.

References


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