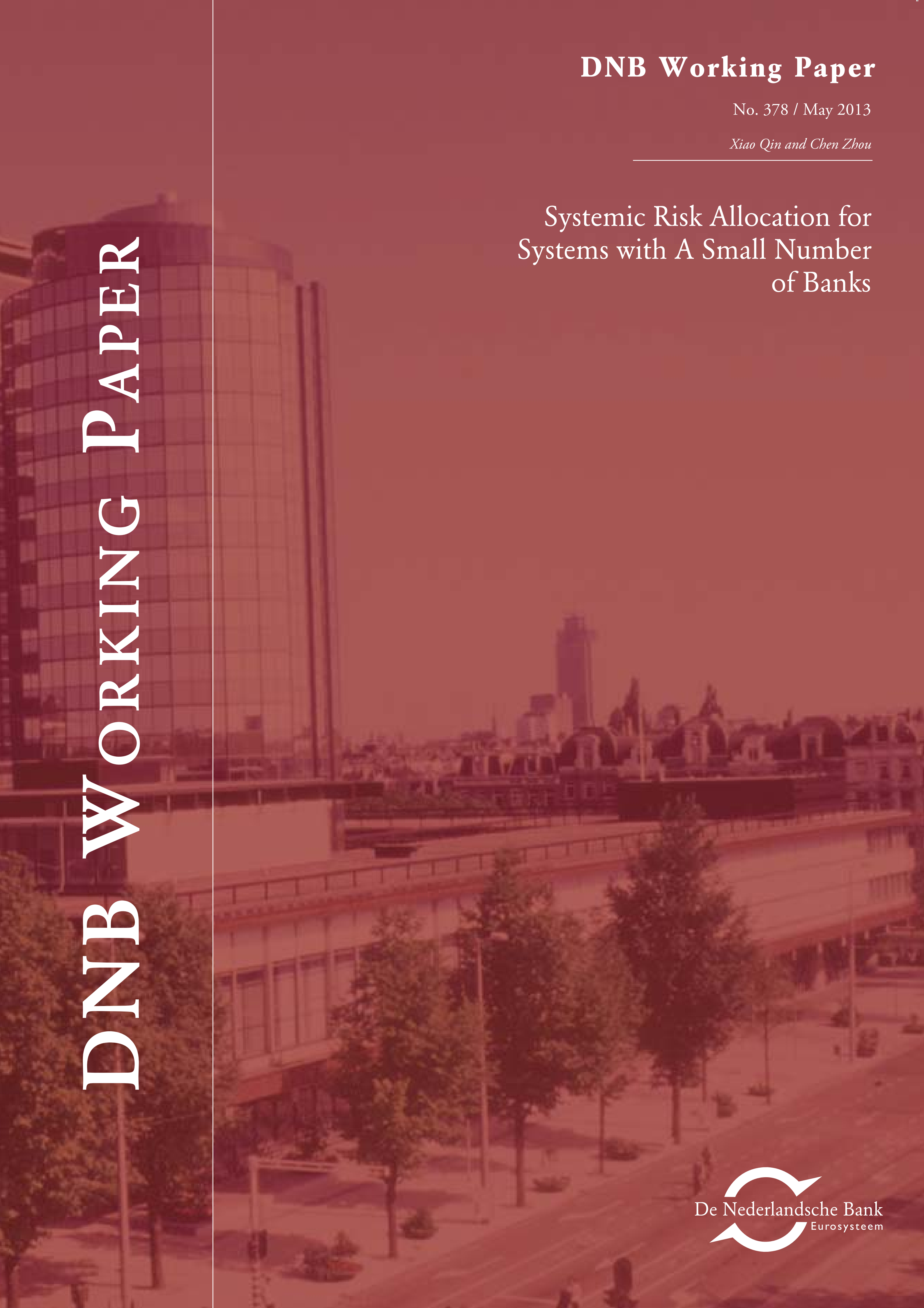

DNB Working Paper

No. 378 / May 2013

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DNB Working Paper

De Nederlandsche Bank
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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

Working Paper No. 378

May 2013

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Systemic Risk Allocation for Systems with A Small Number of Banks *

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April 22, 2013

Abstract This paper provides a new estimation method for the marginal expected shortfall (MES) based on multivariate extreme value theory. In contrast to previous studies, the method does not assume specific dependence structure among bank equity returns and is applicable to both large and small systems. Furthermore, our MES estimator inherits the theoretical additive property. Thus, it serves as a tool to allocate systemic risk. We apply the proposed method to 29 global systemically important financial institutions (G-SIFIs) to evaluate the cross sections and dynamics of the systemic risk allocation. We show that allocating systemic risk according to either size or individual risk is imperfect and can be unfair. Between the allocation with respect to individual risk and that with respect to size, the former is less unfair. On the time dimension, both allocation fairness across all the G-SIFIs has decreased since 2008.

Keywords: Systemic risk allocation; marginal expected shortfall; systemically important financial institutions; extreme value theory

JEL Classification Numbers: G21; C14; G32

*The research of Xiao Qin has received funding from the NSFC grant (No. 71001070) and the NSFC-NWO grant (No. 71211130310 and No. 2012/07448/BOO). The fundings are gratefully acknowledged. A large part of the work is done when Xiao Qin was visiting Erasmus University Rotterdam and she thanks Erasmus School of Economics for their hospitality. Views expressed are those of the authors and do not reflect the official positions of De Nederlandsche Bank.

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1 Introduction

Regulating financial institutions from a macro-prudential perspective aims to mitigate the risk of the financial system as a whole. A crucial issue in designing such regulatory policies is on how to allocate systemic risk of a financial system to individual banks. For that purpose, Acharya et al. (2010) propose an indicator of systemic risk, the marginal expected shortfall (MES). The indicator serves the purpose of systemic risk allocation due to its additive feature. Nevertheless, as we shall point out, the estimation strategy of the MES indicator requires an extra assumption on the dependence between financial institutions and the system. The assumption is valid only for a system consisting of a large number of banks. In this paper, we provide a new estimation method for the MES based on multivariate extreme value theory (EVT). The new method does not depend on particular dependence assumptions, and is applicable to both large and small systems. Our MES estimator inherits the additive property. We apply this new method to evaluate the systemic risk allocation among global systemically important financial institutions (G-SIFIs), released by the Financial Stability Board (FSB) in 2011.[1]

A simple, albeit naive, way for allocating systemic risk is proportional to bank size, in line with the "too big to fail" argument that large banks are more systemically important. An alternative is to evaluate individual risk of each financial institution first, and then allocate systemic risk according to individual risk indicators. However, these solutions are far from forming a fair allocation of systemic risk, see e.g., Brunnermeier et al. (2011) and Zhou (2010). A more sensible strategy is then to measure the systemic risk contribution of each financial institution directly. For that purpose, several indicators have been designed.[2] Among the existing systemic risk indicators, we follow Acharya et al. (2010) to use the MES measure, defined as the expected loss on a bank's equity conditional on

[1] G-SIFIs are financial institutions whose distress or disorderly failure, due to their size, complexity and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activities globally (FSB (2011)).

[2] To name a few, the marginal expected shortfall (MES) in Acharya et al. (2010), CoVaR in Adrian and Brunnermeier (2011), CoRisk measure in Chan-Lau (2010), Shapley value measure in Tarashev et al. (2009), distress insurance premium (DIP) in Huang et al. (2009, 2012), principal component analysis in Billio et al. (2012), probability of at least one extra failure (PAO) given a failure in the system in Segoviano and Goodhart (2009) and systemic impact index (SII) in Zhou (2010). A detailed survey on systemic risk analytics can be found in Bisias et al. (2012).

the occurrence of an extreme loss in the aggregated returns of the system.

Theoretically, the MES is a nice tool for allocating systemic risk due to its additive property.[3] The sum of MES from all banks is equal to a measure of the total systemic risk. As stated in Tarashev et al. (2010), such an additivity property allows for macro-prudential tools to be implemented at bank level. For example, prudential requirements can simply be a linear function of the systemic risk contribution. Empirically, Acharya et al. (2010) employ a single factor model to capture the dependence between the financial system and individual banks and derive an estimator of the MES from that. We show that the single factor model is not valid for systems with a small number of banks, because in such cases the idiosyncratic risks cannot be fully diversified away in the aggregated return of the system. The details on this discussion are given in Section 2.

The MES measure proposed in Acharya et al. (2010) has triggered a number of studies on establishing estimation strategies to it. For example, Engle and Brownlees (2010) apply the Dynamic Conditional Correlation (DCC) model and develop a dynamic estimator on MES. The major contribution therein is on the dynamic feature, whereas the estimation of MES on each time point is based on a nonparametric kernel estimator. Such a kernel estimator cannot handle extreme cases, which is relevant in measuring financial risks. Cai et al. (2012) propose a MES estimator based on multivariate EVT which can handle extreme cases. They do not assume any specific dependence structure between individual banks and the system. Although such an approach is flexible, it disregards the fact that the system is an aggregation of individual banks, and consequently ignores the additive feature of the MES.

In this paper, we estimate the MES based on multivariate EVT, which is able to handle extreme events.[4] We model the dependence structure across the financial institutions with a broad non-parametric assumption, which does not impose specific parametric models. Furthermore, our method can be applied to systems consisting of either a large or a small number of banks. Lastly, we take into account the fact that the system return is

[3]Among the existing systemic risk indicators, the MES, Shapley value and DIP are measures with the additive property.

[4]For more research on applying multivariate EVT to the financial context, one may refer to Hartmann et al. (2004), Poon et al. (2004), Straetmans et al. (2008) and De Jonghe (2010).

a weighted average of individual bank returns, as in the MES definition. Therefore, the consequent estimator of MES maintains the additive property.

We apply the new estimation method to estimate the MES of 29 G-SIFIs (FSB (2011)). We estimate their systemic risk contribution as measured by the MES ratios compared to the total systemic risk. We provide the ranking of their systemic risk contributions in a static study and the dynamics of systemic risk contributions in a moving window analysis. To explore the fairness of allocating systemic risk based on bank characteristics, we further construct the Gini coefficient based on the MES ratios vis-à-vis size and individual risk of the financial institutions, two potential determinants of systemic risk.

We observe that allocating systemic risk according to either size or individual risk is imperfect and can be unfair. Before 2008, the EU banks take excessive systemic risk with respect to the size, while the US banks take less systemic risk. This pattern is reversed after 2008. We find an opposite result for the allocation fairness with respect to individual risk. Between the allocation with respect to individual risk and that with respect to size, the former is less unfair. On the time dimension, both allocation fairness across all the G-SIFIs has decreased since 2008.

The rest of the paper is structured as follows. Section 2 clarifies the main difference between our model setup and that in Acharya et al. (2010). Section 3 provides our estimation strategy on the MES. Section 4 applies the method to a global system with a small number of banks. Section 5 discusses the fairness of allocating systemic risk according to two bank characteristics, size and individual risk. Section 6 concludes with a few remarks.

2 The Marginal Expected Shortfall

In this section, we first review the general framework in Acharya et al. (2010) on measuring systemic risk which leads to the definition of the MES measure. Then, we discuss the limitation of their empirical strategy when applying to systems with a small number of banks. Generally speaking, Acharya et al. (2010) formulate an optimal policy for managing systemic risk which aligns banks' incentive with the regulator. The optimal policy imposes a tax as the sum of banks' individual expected default loss and their expected contribution to a system crisis. The latter is further related to the marginal expected shortfall (MES).

The framework on measuring systemic risk in Acharya et al. (2010) is given as follows. Consider a system consisting of d banks. Each bank i has a total asset a^i, with a net value w_0^i at time 0. At time 1, the net value is stochastic and denoted as w_1^i. The return on equity over the period is thus $r^i = \frac{w_1^i}{w_0^i} - 1$. The occurrence of a system distress is given by $\bar{I} = 1_{\sum w_1^i < z \sum a^i}$, where z denotes a threshold level: once the total capital is below the z fraction of the total assets, a system distress occurs. Then the systemic risk of the system is measured by the loss function $e \cdot E[\bar{I} \cdot (z \sum a^i - \sum w_1^i)]$, where the parameter e measures the sensitivity of the externality imposed on the economy when the financial sector is in distress. Apparently, the systemic risk can be decomposed as

$$\begin{aligned} e \cdot E[\bar{I} \cdot (z \sum_i a^i - \sum_i w_1^i)] &= e \cdot \Pr(\bar{I}) \cdot \left(\sum_i E(za^i - w_1^i | \bar{I}) \right) \\ &= e \cdot \Pr(\bar{I}) \cdot \sum_i \left(za^i - w_0^i - w_0^i E(r^i | \bar{I}) \right). \end{aligned}$$

A main factor in the systemic risk contribution of bank i is thus

$$MES^i = -w_0^i E(r^i | \bar{I}).^5$$

Denote the weighted return of the system as $r^m = \sum_i w_0^i r^i$. We rewrite the indicator of system distress as

$$\bar{I} = 1_{\sum w_1^i < z \sum a_1^i} = 1_{\sum w_0^i r^i < \sum z a_1^i - w_0^i} = 1_{r^m < \bar{r}},$$

[5] Notice that this is an unscaled MES measure, whereas in Acharya et al. (2010), the MES is defined as scaled by w_0^i.

where $\bar{r}$ is an extremely low threshold. Consequently

$$\sum_i MES^i = -\sum_i w_0^i E(r^i|\bar{I}) = -E(r^m|r^m < \bar{r}).$$

Hence, the sum of the MES of all the banks is equal to the expected shortfall of the system return. If we consider the expected shortfall as the risk measure of the system, then the MES for individual banks provides a scheme which allocates the total systemic risk to each bank. This gives the additive property for the MES measure, see also equations (3) and (4) in Acharya et al. (2010).

To estimate the MES for an individual bank, Acharya et al. (2010) employ the single factor model as follows. Suppose each bank holds a portfolio of S assets. Each asset s has a stochastic return following the single-factor Capital Asset Pricing Model (CAPM). Then as a portfolio, the equity return of the bank must follow the CAPM as well, i.e. $r^i = \beta^i r^m + \epsilon^i$.[6] Such a linear model helps to relate the MES to the expected shortfall of the system return. By assuming heavy-tailedness on the system return, Proposition 2 in Acharya et al. (2010) provides an extrapolation method on estimating the MES measure. The MES under extreme cases cam be extrapolated from a MES estimator corresponding to the 5% worst scenario of the system.

We now discuss why the single factor model may contradict with the definition of MES when the system consists of a small number of banks. In the definition of the MES measure, the system distress is defined via the weighted return of the system r^m, i.e. $r^m = \sum_i w_0^i r^i$. At the same moment, the system return is the common factor in the single factor model. To accommodate the two facts, it is necessary to have that $\sum w_0^i \beta^i = 1$ and $\sum w_0^i \epsilon^i = 0$. These are usual assumptions in the classical CAPM: the validity of $\sum w_0^i \epsilon^i = 0$ is ensured by the law of large number. When considering sufficiently many idiosyncratic factors, their weighted average converges to the mean, zero. However, when adopting the single factor in the context of measuring systemic risk, the assumption is valid only if the number of banks within the system is sufficiently large. It may fail in a

[6]Acharya et al. (2010) do not state the fact that the bank equity return follows the CAPM. However, it can be derived from their setup.

system consisting of a small number of banks.

To demonstrate the problem of applying the single factor model in a small system, we consider an extreme case: $d = 2$, i.e., there are only two banks in the system. Then it is impossible to have the following statements hold simultaneously:

1) $r^i = \beta^i r^m + \epsilon^i$, for $i = 1, 2$, where ϵ^1, ϵ^2 and r^m are independent;

2) $r^m = w_0^1 r^1 + w_0^2 r^2$.

This can be proved by contradiction. Suppose both statements hold. We get that $w_0^1 \epsilon^1 + w_0^2 \epsilon^2 = 0$. It contradicts with the fact that ϵ^1 and ϵ^2 are independent. Nevertheless, the definitoin of MES does not necessarily depend on the single factor model. When abolishing the single factor model in the statement 1), it is still possible to maintain the statement 2) as in the definition of the MES. Theoretically, the additive property remains.

To summarize, the MES measure stemming from the general framework in Acharya et al. (2010) is a suitable measure for systemic risk allocation due to its additive property. This is valid regardless of whether the system consists of a large or small number of banks. However, the estimation procedure proposed therein can only be applied to systems with a large number of banks. This is what we intend to overcome in this paper.

3 Methodology

3.1 Preliminaries on EVT

We adopt EVT techniques in constructing an estimator of the MES. We first introduce some preliminary results in EVT. Since our focus is on the downside tail risks, we denote $X = (X_1, \cdots, X_d)$ as the negative returns of d individual banks. Denote $VaR_i(\delta)$ as the Value at Risk (VaR) of X_i at tail probability level δ, i.e., $P(X_i > VaR_i(\delta)) = \delta$. In addition, denote $ES_i(\delta)$ as the expected shortfall (ES) of X_i at tail probability level δ, i.e., $ES_i(\delta) = E\left(X_i | X_i > VaR_i(\delta)\right)$. The VaR and ES can be regarded as measures of individual risk.

Univariate EVT considers models on tail properties of a distribution function, for example, heavy-tailedness. Suppose X_i follows a heavy-tailed distribution, i.e., $P(X_i > t) \sim A_i t^{-\alpha_i}$, where $\alpha_i > 0$ is called the *tail index* and A_i is the *scale*. Then, according to univariate EVT, we have that as $\delta \to 0$,

$$VaR_i(\delta) \sim \left(\frac{A_i}{\delta}\right)^{1/\alpha_i}, \tag{3.1}$$

$$ES_i(\delta) \sim VaR_i(\delta)\frac{\alpha_i}{\alpha_i - 1}. \tag{3.2}$$

Multivariate EVT provides broad models on tail dependence, i.e. the extreme co-movements among the negative returns of financial institutions. More specifically, for any $x_1, x_2, \cdots, x_d > 0$, we assume that as $\delta \to 0$

$$\frac{P\left(X_1 > VaR_1(\delta x_1) \text{ or } \cdots \text{ or } X_d > VaR_d(\delta x_d)\right)}{\delta} \to L(x_1, x_2, \cdots, x_d),$$

where L is a finite positive function. From the definition, the L function characterizes the co-movement of extreme events. It is independent from marginal information and does not contain information on the dependence at a moderate level.[7]

There are several ways to further express L function into an spectral representation. Here we adopt one that is commonly used, see de Haan and Ferreira (2006). Let H be any probability measure on

$$W = \left\{w = (w_1, \cdots, w_d) : w_1 + \cdots + w_d = 1, w_i \geq 0, i = 1, 2, \cdots, d\right\},$$

such that

$$\int_W w_i H(dw) = \frac{1}{d}, \quad \text{for } 1 \leq i \leq d.$$

Any qualified H leads to the following L function

$$L(x_1, \cdots, x_d) = d \int_W \max_{1 \leq i \leq d} x_i w_i H(dw).$$

Conversely, any L function has the above representation with a suitable H. H is called

[7]We refer the readers to de Haan and Ferreira (2006) for some basic properties on the L function.

the *spectral measure on* W, which characterizes the tail dependence.

3.2 The calculation of MES under EVT

In this section, we apply the EVT model to derive a calculation formula on the MES. Suppose that the weighted negative return of the system is given by $Y = \sum_{i=1}^d s_i X_i$ for some $s_1, \cdots, s_d > 0$. Under the framework in Acharya et al. (2010), the weights are the net values of the banks at time 0, w_0^i. In that case, $Y = -r^m$. Consider that the systemic risk is measured by the expected shortfall of the system, i.e. $E(Y|Y > t)$ for some high threshold t. Then it can be further decomposed as $E(Y|Y > t) = \sum_{i=1}^d s_i E(X_i|Y > t)$. Consequently, the relative systemic risk contribution of bank i to the system is given as

$$MES_i = \lim_{t\to\infty} s_i \frac{E(X_i|Y > t)}{E(Y|Y > t)}.$$

We call it the *MES ratio*. Theoretically $\sum_{i=1}^d MES_i = 1$, which gives the additive property.

The following theorem shows the calculation formula on the MES ratios. The proof of this theorem is postponed to the Appendix.

Theorem 3.1 *Suppose that* $X = (X_1, \cdots, X_d)$ *follow a* d*-dimensional EVT setup with all marginal tail indices* α*, marginal scale parameter* A_i *for* $1 \leq i \leq d$ *and spectral measure* H *on* W*. Then the weighted return of the financial system* $Y = \sum_{i=1}^d s_i X_i$ *must follows a heavy-tailed distribution with the same tail index* α*, i.e.,* $P(Y > t) \sim At^{-\alpha}$*, where* A *is the scale parameter for* Y*. Furthermore, we have that*

$$\lim_{t\to\infty} \frac{E(X_i|Y > t)}{E(Y|Y > t)} = \frac{\int_W (A_i w_i)^{1/\alpha} \left(\sum_{i=1}^d s_i (A_i w_i)^{1/\alpha}\right)^{\alpha-1} H(dw)}{\int_W \left(\sum_{i=1}^d s_i (A_i w_i)^{1/\alpha}\right)^{\alpha} H(dw)} \tag{3.3}$$

and consequently

$$MES_i = s_i \frac{\int_W (A_i w_i)^{1/\alpha} \left(\sum_{i=1}^d s_i (A_i w_i)^{1/\alpha}\right)^{\alpha-1} H(dw)}{\int_W \left(\sum_{i=1}^d s_i (A_i w_i)^{1/\alpha}\right)^{\alpha} H(dw)} \tag{3.4}$$

We give a few examples under specific tail dependence structures.

Example 3.2 *The returns of individual banks are tail independent.*

In this case, the H measure assigns only positive measures to the corner points of W: it is a discrete measure which concentrates its measure on d points $(1, 0, \cdots, 0), (0, 1, \cdots, 0), \cdots, (0, 0, \cdots, 1)$ with probability $1/d$ each. Thus, from Theorem 3.1 and Eq.(3.1), we get that $MES_i = \frac{s_i^\alpha A_i}{\sum_{i=1}^d s_i^\alpha A_i}$.

Example 3.3 *The returns of individual banks are completely tail dependent.*

In this case, the H measure concentrates all its measure on a single inner point $(1/d, 1/d, \cdots, 1/d)$. From Theorem 3.1 and Eq.(3.1), we get that $MES_i = \frac{s_i A_i^{1/\alpha}}{\sum_{i=1}^d s_i A_i^{1/\alpha}}$.

3.3 The Estimation of MES

To estimate the MES, we first estimate all the elements in the MES calculation formula: the marginal tail indices α, the scales A_i and the spectral measure H on W.

We start with the estimation of the H measure by following the procedure in de Haan and Ferreira (2006)(Section 7.3). Denote X_i^j as the observed negative returns for bank i at time j, where $i = 1, \cdots, d$ and $j = 1, \cdots, n$. Here n is the sample size and d is the number of banks. By ranking $\{X_i^j\}_{1 \le j \le n}$, we obtain d rank sequences $Z_i^j = rank(X_i^j)$, for $i = 1, \cdots, d$, and derive $S^j = \sum_{i=1}^d \frac{n+1}{n+1-Z_i^j}$. By choosing a proper m, we select those observations corresponding to the m highest S^j values:

$$S := \left\{ j : rank \left(\sum_{i=1}^d \frac{n+1}{n+1-Z_i^j} \right) \ge n+1-m \right\}.$$

Then, for each $s \in S$, we connect the point

$$(n+1-Z_1^s, \cdots, n+1-Z_d^s) \in \mathbb{R}_+^d$$

and the origin by a straight line, and find the intersection with the plane

$$W := \left\{ (x_1, \cdots, x_d) \in \mathbb{R}_+^d : \sum_{i=1}^d x_i = 1 \right\}$$

at the points

$$\frac{1}{\sum_{i=1}^{d} \frac{n+1}{n+1-Z_i^s}} \left(n+1-Z_1^s, \cdots, n+1-Z_d^s\right).$$

By assigning equal weights to those points on W, we get the estimation of the spectral measure H on W. Due to the fact that the dataset is finite, the estimated H measure is always a discrete measure on m points. The theoretical requirement on the choice of m to guarantee the consistency is that as $n \to \infty$, $m \to \infty$ and $m/n \to 0$.

As to the estimation on marginal information, i.e., tail index α and scales A_i, they follow the usual univariate EVT methods. By ranking the negative returns of each individual bank i, we get the order statistics $X_i^{(1)} \leq X_i^{(2)} \leq \cdots \leq X_i^{(n)}$. Hill (1975) proposes the so-called *Hill estimator* in estimating the tail index as

$$\hat{\alpha}_i = \left(\frac{1}{k} \sum_{j=1}^{k} \log X_i^{(n-j+1)} - \log X_i^{(n-k)} \right)^{-1},$$

where $k = k(n)$ is a suitable intermediate sequence such that $k(n) \to \infty$ and $k(n)/n \to 0$ as $n \to \infty$. Then we use the average of all $\hat{\alpha}_i$ to be the estimator as the common tail index α, i.e. $\hat{\alpha} = \frac{1}{d} \sum_{i=1}^{d} \hat{\alpha}_i$. The estimator of marginal scales A_i can be consequently obtained as

$$\hat{A}_i = \frac{k}{n} \left(X_i^{(n-k)} \right)^{\hat{\alpha}}.$$

With estimating α_i and A_i, one can estimate the VaR of each bank i, by replacing α_i and A_i in Eq. (3.1) with their estimators. The VaR estimate gives a measure of individual risk for each bank.

The intermediate sequences k in univariate EVT procedure and m in multivariate EVT procedure play a similar role. The selection of such intermediate sequences is conducted following a standard procedure in EVT.[8] We estimate the function $L(1,1)$ for various pairs of banks i and j with varying choices of m values. Then, we plot the estimates against ascending m values. The value m is chosen by picking up the first stable part of the plot starting from a low m. Such a choice can balance the tradeoff between a larger

[8] Here we do not further distinguish the two sequences m and k, while always using $k = m$.

variance when using a low value of m and a larger bias when using a high value of m. Because m is chosen from a stable part of the plot, a small variation of m value should not change the empirical results to a great extent.

With the estimation of the marginal tail index α, scales A_i and spectral measure H, we can use the Eq. (3.4) to calculate MES_i. We remark that according to Eq. (3.4), $\sum_{i=1}^{d} MES_i = 1$ holds for any H measure and parameters s_i, A_i and α. Therefore, with plugging in our estimates, the additive property $\sum_{i=1}^{d} \widehat{MES}_i = 1$ still holds. In other words, our estimation procedure is consistent with the additive property. This guarantees that the estimated MES ratios are suitable for systemic risk allocation. Furthermore, our estimation procedure is valid regardless of whether the number of banks, d, is high or low. Hence, it is applicable to financial systems with any size.

4 Allocating systemic risk according to MES

4.1 Data

We apply our MES estimator to analyze 29 global systemically important financial institutions (G-SIFIs). The choices of the G-SIFIs follows the list released by the Financial Stability Board (FSB) in November 2011.[9] The list includes 8 US banks, 17 EU banks (10 EZ and 7 non-EZ banks) and 4 Asian banks (1 Chinese and 3 Japanese banks).[10]

We collect market data for equity prices and balance sheet data for market capitalization (denoted as MV throughout the rest of the paper) for these 29 banks from Datastream. Equity prices are in local currencies at a daily frequency. MV is in billions of USD at a monthly frequency.[11]

We construct two panels: (1) A broader panel with all 29 banks in the period from

[9]These G-SIFIs are assessed by scoring selected indicators from five categories with equal weights, i.e., the size, interconnectedness, substitutability, global activity and complexity based on data till the end of 2009 (see BIS (2011)).

[10]The list is updated by the FSB in November 2012 with some changes, i.e., adding two banks (Banco Bilbao Vizcaya Argentaria and Standard Chartered) and deleting three banks (Dexia, Commerzbank and Lloyds); see FSB (2012).

[11]One exceptional case is Groupe BPCE. Since the banking group is unlisted, we use data for its investment management and financial services arm, the publicly traded bank Natixis, instead. Similar adjustment has been adopted in other studies; see e.g., Acharya and Steffen (2012).

August 2006 to December 2012; (2) A longer panel with 24 banks from December 1996 to December 2012 (with 7 US, 16 EU and 1 Asian bank). Therefore, Panel 1 and 2 both encompass the recent financial crisis, and Panel 2 also covers the periods on the Asian crisis, the bust of internet bubble and the introduction of the Euro currency. We conduct a static analysis based on data in Panel 1, while a dynamic analysis based on data in Panel 2.

4.2 Static MES estimates

For Panel 1, we carry out a static analysis. As to the size used in the MES measure, we employ the end-of-period MV of individual banks and normalize them by the total market capitalization in the system. This reflects the size ratio at the end of the period for these banks. Following the selection procedure on k, we choose $k = 60$ for 1675 observations in total, which corresponds to a ratio k/n around 3.6%. We calculate the MES ratio and rank them across the 29 banks. The ranking demonstrates their relative systemic risk contribution. On top of that, we estimate the level of MES in terms of its original definition. To achieve this, we first derive the total systemic risk, estimated as the expected shortfall of the system return multiplied by the total MV of the system. The system return is constructed as the weighted average return of all the banks in the system. We again use the size ratios as the weights. The expected shortfall of the system at a probability level 0.1% is estimated based on univariate EVT, as in Eq. (3.2). The MES level (in billion USD) of each bank is then calculated as the MES ratio multiplying with the expected shortfall of the system. These results are shown in Table 1.

From the results, the banks from the US generally have a higher rank compared to the others, with Citigroup, Wells Fargo, Bank of America, and JP Morgan Chase as the top four in the list. Within the EU, in general, the non-EZ banks have higher ranks than the EZ banks. The banks from the EZ take up the last four seats. Among them, Dexia and Commerzbank are removed in the updated list by FSB (2012). Four Asian banks all have low ranks in the ranking list.

We also list in Table 1 the 29 banks' size measured by the MV and individual risk measured by the VaR at the probability level 0.1% following Eq. (3.1), as well as the relevant ranks. It is clear that the MES ranking is different from (albeit similar to) the other two rankings, indicating that the systemic risk contribution is closely related with but not determined alone, by either size or individual risk. The largest difference in the rankings between the MES and size for individual banks is for Bank of China. It is ranked at the 6th by the size but only the 22nd by the MES. Similarly large rank difference arises for the Japanese banks. This implies that the Asian banks contribute less systemic risk to the global financial system, compared to their size share. In contrast, all the US banks have MES ranks higher than or similar to their corresponding MV ranks, which demonstrate the great systemic importance of the US banking system. For example, Morgan Stanley has a MES rank at the 8th, but a MV rank at the 18th. Comparing the MES and the VaR, we find that the rank difference is relatively smaller than that between the MES and the MV. Bank of China has again the largest difference. It is ranked at the 11th in the VaR ranking and the 22nd in the MES ranking, indicating that its individual risk is larger compared to its systemic risk. An opposite example is Deutsche Bank, which is ranked at the 15th in the MES ranking but 21st in the VaR ranking.

Lastly, we investigate whether the static MES estimates are robust when considering a small banking system. Theoretically, our estimation method on the MES can be applied to system with either large or a small number of banks. Nevertheless, this does not guarantee that the MES ratios calculated in a subsystem are equivalent to those calculated in a full system. We conduct additional analyses within regional systems: we estimate the MES ratios of the EU/US/Asian banks when considering the EU/US/Asia itself as the system. These results are then compared with the MES estimates within a global system, i.e., the MES ratios in Table 1 normalized within each region. The results are listed in Table 2. There is no large difference between the MES ratios calculated within the regional system and those calculated within the global system. Qualitatively, all banks have the same rank in systemic risk contribution globally and regionally, except for four adjacent pairs. Hence, the static results are robust to different setups of system.

4.3 Dynamic MES estimates

To investigate on the dynamics of the MES over time, we conduct a moving window analysis on Panel 2 with 24 banks. The MES ratios are estimated in estimation windows with a 4-year horizon (1,043-1,045 daily observations in each window), and the estimation windows are shifted month by month. In total, we get the monthly MES ratios in 145 windows from December 2000 to December 2012.[12] Within each window, the end-of-period MV is used as the size weight for constructing the system return. When estimating the MES measure, we follow the selection procedure introduced in Section 4.2, and choose $k = 50$ for each window. This corresponds to a ratio k/n around 4.8%. The estimation of the MES ratios and the MES levels follows the same statistical procedure as in Section 4.3.

We first present our results on an aggregation level. The results are aggregated within three regions: the US, the EU and Asia. The dynamics of MES ratios and levels are illustrated in Figs. 1a-1b respectively. We observe an increase in the MES ratio of the US banks since October 2008, whereas a corresponding decrease in that of the EU banks. In addition, there is a large increase in the MES levels after August 2008 for banks in all three regions, with a peak in December 2009.

We further analyze the dynamics of MES ratio and level for individual banks from the US and EU. In Fig. 2a the MES ratios for the top four US banks in our list (Citigroup, Wells Fargo, Bank of America and JP Morgan Chase), which are also the top four in the Supervisory Capital Assessment Program (SCAP) stress testing results (FED (2009)). The four banks have different dynamics of MES ratios in the past twelve years. For Citigroup, its MES ratio has been continuously decreasing in the 2000s, with a notably large drop after December 2008. Citigroup was rescued by the U.S. government in November 2008 with a massive rescue package. This action might be the cause of the large reduction in the systemic risk contribution of Citigroup. After March 2009, its MES ratio starts to increase. This implies that the rescue effect of the government is short-term if there is any. For Wells Fargo, though its systemic risk contribution measured by the MES ratio is

[12] When shifting the windows day by day, we can obtain the daily MES as what the V-lab of NYU Stern School of Business provides; see http://vlab.stern.nyu.edu

the lowest in the top four before 2008, it has been increasing since then. The increasing systemic risk contribution of Wells Fargo might be a consequence of acquiring Wachovia at the height of the 2008 financial crisis. In contrast, the systemic risk contribution of Bank of America does not increase after acquiring Merrill Lynch in September 2008. To summarize, increasing size by merge and acquisition does not necessarily corresponds to an increase in systemic risk contribution.

To illustrate to what extent the change in systemic risk contribution is due to that in size, we plot the dynamic of an adjustment factor F_i^s as the MES ratios divided by the MV ratios in Fig. 2b. Except for JP Morgan Chase, the adjustment factors of the other three US banks have increased since 2008. JP Morgan Chase shows a very stable adjustment factor around 1, which implies that its relative systemic risk is fairly reflected by its relative size. For Citigroup, this adjustment factor is around 1 before 2008, has a jump in 2008, and doubles in September 2012. This suggests that the increase in its systemic risk contribution is not solely due to the effect of its size increment.

Parallel to the US analysis, we also display in Figs. 3a and 3b respectively the MES ratios and the adjustment factors with respect to size (F_i^s) for several EU banks. We choose Dexia and ING Bank from the EZ and HSBC from the non-EZ. HSBC has a relatively large systemic risk contribution during the entire twelve years. This is largely due to its size, because its adjustment factor is always below 1. The fact that Dexia has a low systemic risk contribution is mainly attributed to its small size, as the adjustment factor is close to 1. In particular, its systemic risk contribution decreases after 2008. ING Bank has the largest adjustment factor with respect to size among all 24 banks during the period from 2002 to 2006. However, on the time dimension, its adjustment factor decreases largely during the 2007-2008 period. All the obtained dynamics are robust when considering smaller banking systems as in Section 4.1.

5 Allocation fairness with respect to bank characteristics

In this section, we explore quantitatively the extent to which the MES is related to two potential determinants: the size measured by the MV and the individual risk measured by the VaR and the ES. Theoretically, the systemic risk of a bank may be associated to its size ("too big to fail"), individual risk, interconnectedness ("too interconnected to fail"), and other potential characteristics, such as non-traditional activities (Brunnermeier et al. (2011); Moore and Zhou (2012); López-Espinosa et al. (2012)). We focus on the first two characteristics because they are also input factors for estimating the MES measure.

As a preliminary evidence, we demonstrate in Fig. 4 the dynamics of the cross-sectional Pearson's correlation coefficient between the MES and two characteristics when shifting the estimation window. The correlation coefficient between the MES and size falls in the range from 0.77 to 0.98. It is close to 1 at the beginning of the 2000s, drops from 0.96 to 0.77 during the period 2004-2007 and remains around 0.85 after 2010. This implies a strong albeit varying relation between systemic risk and size. The correlation coefficient between the MES and VaR ranges from 0.87 to 0.98, and that between the MES and ES is from 0.75 to 0.97. This suggests a strong relation between systemic risk and individual risk.

Although we observe a strong correlation between the MES ratio and the two bank characteristics, the variations in the correlation coefficients hint that it may not be always fair to allocate the systemic risk according to bank characteristics. Thus, we conduct measures to quantify the fairness and examine their dynamics on the time dimension.

5.1 Regional level analyses

We first focus on the MES-size discrepancy at regional level. In Fig. 5a we show the differences between the MES and size ratios aggregated by region. A positive bar in the plot indicates a larger MES share than size, and thus excessive systemic risk with respect to size. A negative bar implies less systemic risk compared to size. A zero difference shows a fair allocation of systemic risk with respect to size. From Fig. 5a, the allocation of systemic risk with respect to size is fair at the beginning of the 2000s. The unfairness arises since 2002. Before 2008, the EU banks have a larger MES share than size, while their US counterparties are at the opposite. We can observe a turning point at the beginning of 2008. After that, the US banks have a much larger MES share than size, while the EU banks take the opposite position.

We check statistically whether there is any structural break in the regional gap of the MES-size discrepancies between the US and EU banks during the past twelve years. We construct a measure on the difference between the logarithms of the MES and MV ratios for each bank i in each window s as $Diff_i^s = \log(MES_i^s) - \log(MV_i^s)$.[13] Then, we aggregate the difference by region, and calculate the difference in difference between the US and EU to obtain

$$D^s = \sum_{i \in US} Diff_i^s - \sum_{i \in EU} Diff_i^s.$$

We test whether there is a structural break on D^s by the Andrews (1993) test. We find a structural break in June 2008 as indicated in Fig. 5a. Before this date, there is excessive systemic risk with respect to size for the EU banks but less systemic risk with respect to size for the US banks. This pattern is reversed after the break.

Similarly, we define a difference in difference measure D^{IR} on the MES-VaR discrepancy at a regional level. We find a structural break in October 2007 as indicated in Fig. 5b. From the two identified structural breaks, we conclude that the US banks as a whole increase their systemic risk contribution since the recent financial crisis.

5.2 Bank level analyses

The structural break analysis in Section 5.1 on the allocation fairness is based on the aggregation of the MES-MV or MES-VaR differences by region. We further conduct an analysis at bank level. The idea is similar to the Gini coefficient for quantifying the inequality of wealth distribution.

We start with the MES-size discrepancy. In each month, we sort the 24 banks in an

[13] This is equivalent to the logarithm of the adjustment factor to size.

ascending order by $Diff_i^s$, the difference in the logarithms of the MES ratios and the MV ratios. Then we plot the cumulative MES ratios against the cumulative MV ratios following the sorted order. By connecting the cumulative ratios, we get a convex curve starting from (0,0) and ending at (1,1), which lies below the diagonal line. In case the two ratios of the MES and MV perfectly match each other, the curve coincides with the diagonal line. Therefore, this curve is similar to the Lorenz curve, which represents the inequality in the income distribution. More specifically, the curve shows the relative systemic risk distribution with respect to size: for each point (x, y) on the curve, the bottom $100x\%$ banks of the total MV, contribute to $100y\%$ of the total MES. Accordingly, one can calculate the Gini coefficient, as $1-2*B$, where B is the area below the curve. This coefficient is in a scale from 0 to 1, where 0 represents a perfect fairness when allocating systemic risk according to size, 1 represents that such an allocation is completely unfair. Within the range [0,1], the higher the coefficient the more the unfairness.

For each month, we calculate such a Gini coefficient to measure the unfairness of allocating systemic risk with respect to size. The Gini coefficients are plotted over time in Fig. 6a and tested for the existence of structural break. From Fig. 6a, the Gini coefficients range from 0.1 to 0.22 with a large increase during the period from 2005 to 2007. The overall low level suggests a moderate unfairness when allocating systemic risk with respect to size. The Andrews (1993) test on the Gini coefficient identifies a structural break in February 2005. The average unfairness is low before the break date and high afterwards. The break date of allocation fairness at bank level does not coincide with what we obtain from the regional analysis.

Similarly, we carry out an analysis on the MES-VaR discrepancy. The Gini coefficient based on the MES and VaR ratios are shown in Fig. 6b. The Andrews (1993) test identifies a break point in October, 2007. The average unfairness of systemic risk allocation with respect to individual risk is low before the break date and high after that. The break date of allocation fairness with respect to individual risk at bank level coincides with that at regional level.

Comparing the two analyses, the Gini coefficients based on the MES and VaR ratios range from 0.07 to 0.19, which is lower than that based on the MES and MV ratios. Hence, allocating systemic risk with respect to the individual risk is relatively less unfair than that to the size.

6 Concluding remarks

The MES measure proposed by Acharya et al. (2010) serves the purpose of allocating systemic risk to individual financial institutions thanks to its additive feature. Nevertheless, as we point out, the specific dependence model for estimating MES employed therein is only valid for systems with a large number of banks. In this paper, we develop a new estimation method for the MES based on multivariate EVT. We do not impose specific parametric models, which leads to a method that can be applied to systems consisting of either a large or a small number of banks. Our consequent estimator of the MES maintains the additive property.

We apply the new method to estimate the MES of 29 G-SIFIs from the US, EU and Asia, and test whether it is fair to allocate systemic risk based on simple bank characteristics, such as size and individual risk. We conclude that allocating systemic risk according to either size or individual risk is imperfect and can be unfair. Both allocation fairness across all the G-SIFIs has decreased since 2008. Between the allocation with respect to individual risk and that with respect to size, the former is less unfair.

At a first glance, our results seem to be counter intuitive: on the one hand, the MES ratio is highly correlated to size, while on the other hand, allocating systemic risk according size is imperfect. In fact, this can be explained by the calculation formula of the MES measure. The calculation of the MES is related to size, but in a non-linear way: from Eq.(3.4), the MES ratio is the product of size and an adjustment factor, while the adjustment factor is further related to size, individual risk and the dependence across the banks within the system. This argument can be further applied to explain the observed structural breaks in the allocation fairness: it is a potential consequence of the change in the dependence structure across banks. A similar explanation applies to the allocation fairness with respect to individual risk and its structural breaks on the time dimension.

Two recent developments in the global financial system may cause such changes on the dependence across banks. Firstly, financial innovation, in the form of credit derivatives, plays an important role in enhancing the interconnectedness within the global financial system. As pointed out by Allen and Carletti (2006), credit risk transfer can be damaging because of increasing the possibility of contagion. The development of such financial innovations may thus explain the observed structural break in the allocation unfairness with respect to size among the three regions, which is observed in February 2005. Secondly, the global financial crisis starting from 2007 may also steer the financial system towards a different level of interconnectedness. This may explain the structural break in the allocation fairness across all the G-SIFIs, which is observed in June 2008.

Our results can contribute to the debate on macro-prudential regulations as follows. Firstly, regulators may identify SIFIs from simple bank characteristics such as size and individual risks, albeit with caution. Systemic risk contribution of a bank is in general related to these characteristics, but the relation may vary across time. Secondly, information on the dependence structure within the financial system is crucial for regulators to identify SIFIs and allocate systemic risk. Thus a macro-prudential regulatory entity that has an overview of the dependence is necessary for implementing regulation rules towards mitigating the systemic risk.

Our analysis is limited to two simple characteristics of banks, size and individual risk. Because both characteristics are input factors for estimating the MES measure, they should be regarded as endogenous drivers. An extension of the current study would be to consider exogenous drivers of systemic risk, such as measures on leverage, interbank activities, etc. That is beyond the scope of the current paper and is thus left for future research.

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Appendix

Proof of Theorem 3.1 To prove the theorem, we first introduce an equivalent representation for the spectral measure H: the exponent measure ν. Denote $A_{x_1,\cdots,x_d} = \{(s_1,\cdots,s_d) : \exists 1 \le i \le d \text{ s.t. } s_i > x_i\}$. Define $Z_i = \frac{X_i^\alpha}{A_i}$ and $Z = (Z_1, ..., Z_d)$. Then,

$$
\begin{aligned}
\lim_{t\to\infty} tP(Z \in tA_{x_1,\cdots,x_d}) &= \lim_{t\to\infty} tP(X_1 > (A_1 t x_1)^{1/\alpha} \text{ or } \cdots \text{ or } X_d > (A_d t x_d)^{1/\alpha}) \\
&= \lim_{t\to\infty} tP\left(X_1 > VaR_1\left(\frac{1}{tx_1}\right) \text{ or } \cdots \text{ or } X_d > VaR_d\left(\frac{1}{tx_d}\right)\right) \\
&= L\left(\frac{1}{x_1}, \cdots, \frac{1}{x_d}\right).
\end{aligned}
$$

Hence, we can define $L\left(\frac{1}{x_1}, \cdots, \frac{1}{x_d}\right)$ as a measure on $A_{x_1,\cdots,x_d}$, i.e., $\lim_{t\to\infty} tP(Z \in tA_{x_1,\cdots,x_d}) = \nu(A_{x_1,\cdots,x_d})$. Since this holds for all $A_{x_1,\cdots,x_d}$, we get that for all Borel set $A \in \mathbb{R}_+^d$ satisfying

$$
\inf_{(x_1,\cdots,x_d)\in A} \max(x_1, \cdots, x_d) > 0, \tag{A.1}
$$

the relation $\lim_{t\to\infty} tP(Z \in tA) = \nu(A)$ holds.

The two measures H and ν can be transformed from one to the other. Define a one to one mapping π that maps any point $x = (x_1, \cdots, x_d) \in \mathbb{R}_+^d / \{(0, \cdots, 0)\}$ to $(r, w) \in (0, \infty) \times W$ by $r = \sum_{1\le i\le d} x_i$ and $w = x/(\sum_{i=1}^d x_i)$. For any Borel set A satisfying (A.1), $\pi(A)$ is a Borel set in $(0, \infty) \times W$. Thus, for any $(x_1, \cdots, x_d)^T \in \mathbb{R}_+^d / \left\{(0, \cdots, 0)^T\right\}$,

$$
\begin{aligned}
\pi(A_{x_1,\cdots,x_d}) &= \{(r, w) : \exists 1 \le i \le d \text{ s.t. } rw_i > x_i\} \\
&= \left\{(r, w) : r > \frac{x_1}{w_1} \wedge \cdots \wedge \frac{x_d}{w_d}\right\}.
\end{aligned}
$$

Hence, we have that

$$
\begin{aligned}
&\nu(A_{x_1,\cdots,x_d}) = L\left(\frac{1}{x_1}, \cdots, \frac{1}{x_d}\right) \\
=&d\int_W \frac{w_1}{x_1} \vee \cdots \vee \frac{w_d}{x_d} H(dw) = d\int_W \left(\int_{\frac{x_1}{w_1}\wedge\cdots\wedge\frac{x_d}{w_d}}^\infty \frac{1}{r^2} dr\right) H(dw) \\
=&d\int_{(r,w)\in\pi(A_{x_1,\cdots,x_d})} \frac{1}{r^2} dr H(dw).
\end{aligned}
$$

Since this relation holds for any set $A_{x_1,\cdots,x_d}$, it must hold for any Borel set A satisfying (A.1), i.e.

$$\nu(A) = d \int_{(r,w)\in\pi(A)} \frac{1}{r^2} dr H(dw). \tag{A.2}$$

Now, we prove Theorem 3.1 using the exponent measure ν. Because $X = (X_1, \cdots, X_d)$ follows the $d-$dimensional EVT setup with all marginal tail indices α, the linear combination $Y = \sum_{i=1}^d s_i X_i$ follows a heavy-tailed distribution with the same tail index α, i.e., $P(Y > t) \sim At^{-\alpha}$, as $t \to \infty$. Thus, as $t \to \infty$,

$$\frac{E(Y|Y > t)}{t} \to \frac{\alpha}{\alpha - 1}. \tag{A.3}$$

Next, we calculate $E(X_i|Y > t)$ as follows:

$$\begin{aligned} E(X_i|Y > t) &= E\left((A_i Z_i)^{1/\alpha} \middle| \sum_{i=1}^d s_i (A_i Z_i)^{1/\alpha} > t\right) \\ &= \int_0^\infty P\left((A_i Z_i)^{1/\alpha} > x \middle| \sum_{i=1}^d s_i (A_i Z_i)^{1/\alpha} > t\right) dx \\ &= t \int_0^\infty \frac{P\left((A_i Z_i)^{1/\alpha} > tu, \sum_{i=1}^d s_i (A_i Z_i)^{1/\alpha} > t\right)}{P\left(\sum_{i=1}^d s_i (A_i Z_i)^{1/\alpha} > t\right)} du. \end{aligned}$$

Denote the numerator and denominator in the integral as I_1 and I_2. As $t \to \infty$,

$$\begin{aligned} t^\alpha I_1 &= t^\alpha P\left(Z \in t^\alpha \left\{(A_i x_i)^{1/\alpha} > u, \sum_{i=1}^d s_i (A_i x_i)^{1/\alpha} > 1\right\}\right) \\ &\to \nu\left\{(A_i x_i)^{1/\alpha} > u, \sum_{i=1}^d s_i (A_i x_i)^{1/\alpha} > 1\right\}. \end{aligned}$$

Similarly, as $t \to \infty$,

$$t^\alpha I_2 \to \nu\left\{\sum_{i=1}^d s_i (A_i Z_i)^{1/\alpha} > 1\right\}.$$

Combining the limits of the two, we get that $\lim_{t\to\infty} \frac{E(X_i|Y>t)}{t} = \int_0^\infty \frac{\nu\left\{(A_i x_i)^{1/\alpha}>u, \sum_{i=1}^d s_i (A_i x_i)^{1/\alpha}>1\right\}}{\nu\left\{\sum_{i=1}^d s_i (A_i Z_i)^{1/\alpha}>1\right\}} du.$ To further derive the limit in terms of the spectral measure H, we apply the transformation relation (A.2). For any

$(x_1, \cdots, x_d)^T \in \mathbb{R}_+^d / \left\{(0, \cdots, 0)^T\right\}$,

$$
\begin{aligned}
&\pi\left\{(A_i x_i)^{1/\alpha} > u, \sum_{i=1}^{d} s_i (A_i x_i)^{1/\alpha} > 1\right\} \\
=&\left\{(r, w) \,\middle|\, r^{1/\alpha} (A_i w_i)^{1/\alpha} > u, r^{1/\alpha} \sum_{i=1}^{d} s_i (A_i w_i)^{1/\alpha} > 1\right\} \\
=&\left\{(r, w) \,\middle|\, r > \frac{u^\alpha}{A_i w_i} \bigvee \left(\frac{1}{\sum_{i=1}^{d} s_i (A_i w_i)^{1/\alpha}}\right)^\alpha\right\}.
\end{aligned}
$$

Hence,

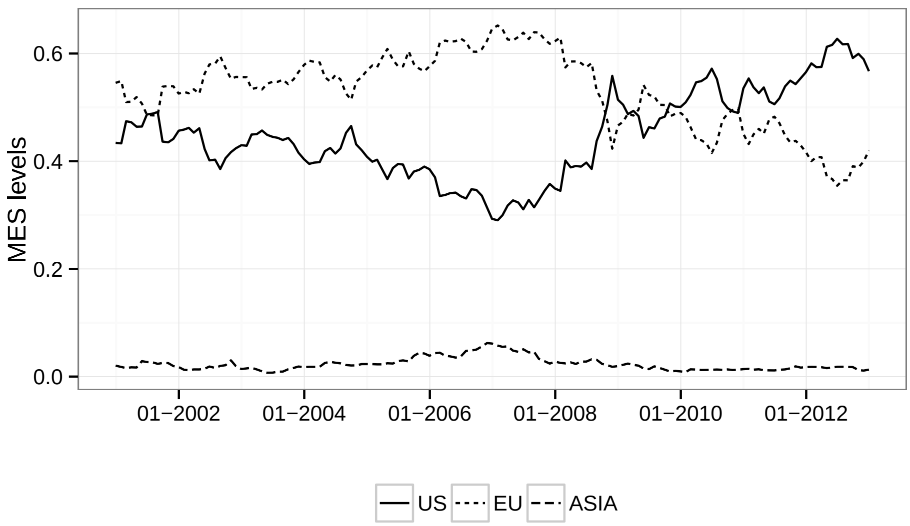
$$
\begin{aligned}
&\int_0^\infty \nu\left\{(A_i x_i)^{1/\alpha} > u, \sum_{i=1}^{d} s_i (A_i x_i)^{1/\alpha} > 1\right\} du \\
=&d \int_W H(dw) \int_0^\infty \frac{A_i w_i}{u^\alpha} \bigwedge \left(\sum_{i=1}^{d} s_i (A_i w_i)^{1/\alpha}\right)^\alpha du \\
=&d \int_W H(dw) \left(\int_0^{u_0} \left(\sum_{i=1}^{d} s_i (A_i w_i)^{1/\alpha}\right)^\alpha du + \int_{u_0}^\infty \frac{A_i w_i}{u^\alpha} du\right) \\
=&\frac{\alpha}{\alpha - 1} d \int_W (A_i w_i)^{1/\alpha} \left(\sum_{i=1}^{d} s_i (A_i w_i)^{1/\alpha}\right)^{\alpha - 1} H(dw). \qquad \text{(A.4)}
\end{aligned}
$$

where $u_0 = \frac{(A_i w_i)^{1/\alpha}}{\sum_{i=1}^{d} s_i (A_i w_i)^{1/\alpha}}$ is the break even point such that $\frac{A_i w_i}{u^\alpha} = \left(\sum_{i=1}^{d} s_i (A_i w_i)^{1/\alpha}\right)^\alpha$.

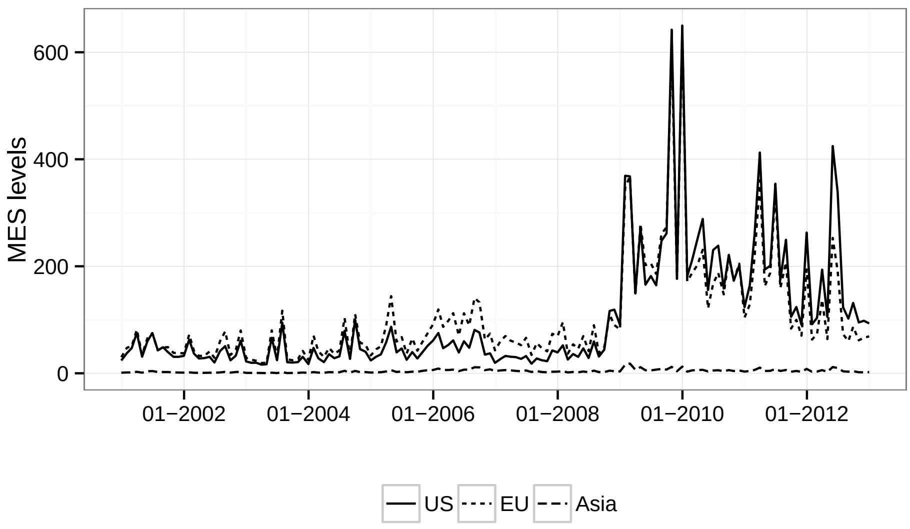
Similarly, we have that,

$$
\int_0^\infty \nu\left\{\sum_{i=1}^{d} s_i (A_i x_i)^{1/\alpha} > 1\right\} du = d \int_W \left(\sum_{i=1}^{d} s_i (A_i w_i)^{1/\alpha}\right)^\alpha H(dw). \qquad \text{(A.5)}
$$

By combining (A.3), (A.4) and (A.5), we prove Eq. (3.3). □



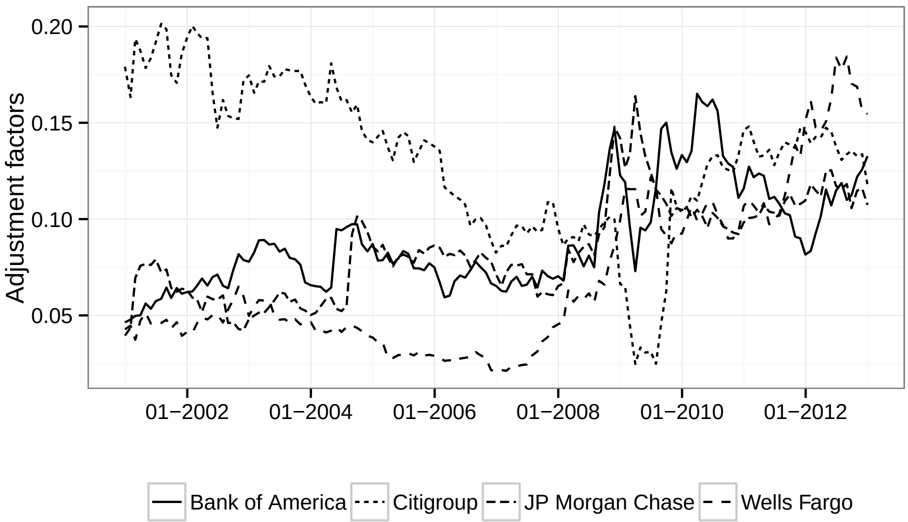
(a)



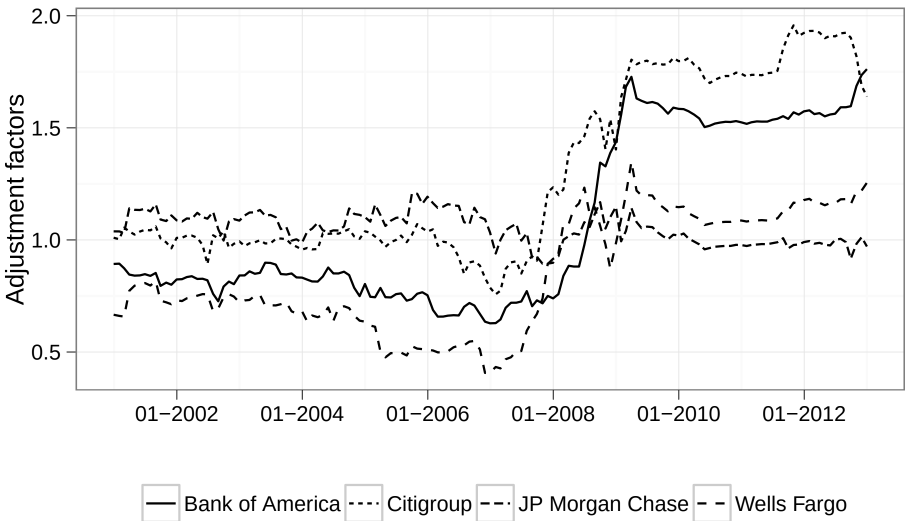
(b)

Figure 1: The MES ratios and MES levels

Note: At the end of each month, we estimate the MES ratios (in proportion) and MES levels (in billion USD) for 24 G-SIFIs by applying the estimation procedure in Section 3.3 using daily equity returns in the past 4 years. Then we aggregate the estimated MES ratios and MES levels by three regions, the US, the EU and Asia. The upper (lower) panel reports the aggregated MES ratios (levels).



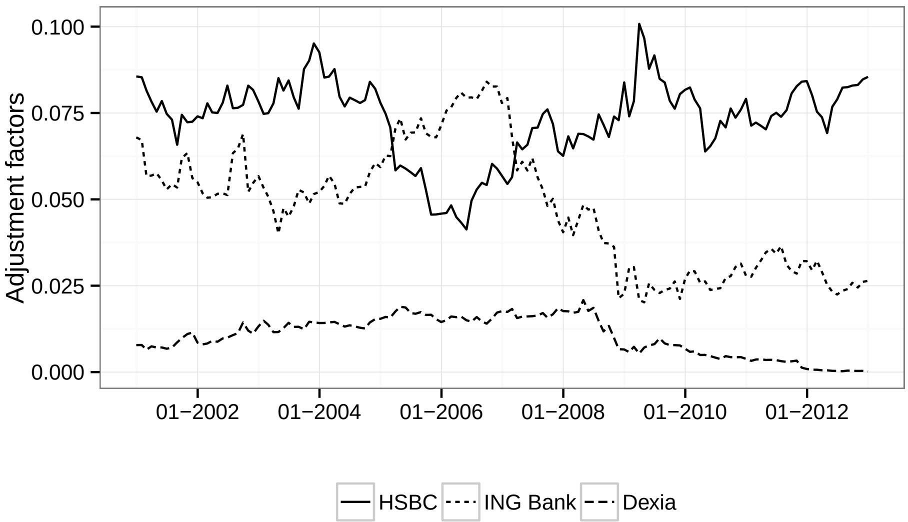
(a)



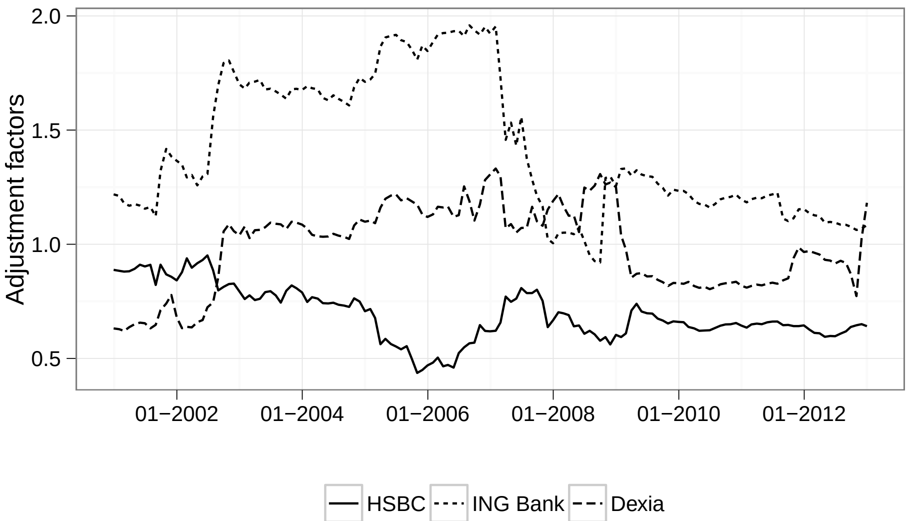
(b)

Figure 2: The MES ratios and the size adjustment factors for selected US banks

Note: At the end of each month, we estimate the MES ratios (in proportion) for 24 G-SIFIs by applying the estimation procedure in Section 3.3 using daily equity returns in the past 4 years. The size ratio of each bank is calculated as the share of the market value relative to the entire system at the end of the month. The size adjustment factor is the quotient between the MES ratio and the size ratio. The upper panel reports the MES ratios, while the lower panel reports the size adjustment factors. Both factors are reported for four selected US banks only.



(a)



(b)

Figure 3: The MES ratios and the size adjustment factors for selected EU banks

Note: At the end of each month, we estimate the MES ratios (in proportion) for 24 G-SIFIs by applying the estimation procedure in Section 3.3 using daily equity returns in the past 4 years. The size ratio of each bank is calculated as the share of the market value relative to the entire system at the end of the month. The adjustment factor is the quotient between the MES ratio and the size ratio. The upper panel reports the MES ratios, while the lower panel reports the size adjustment factors. Both factors are reported for three selected EU banks only.

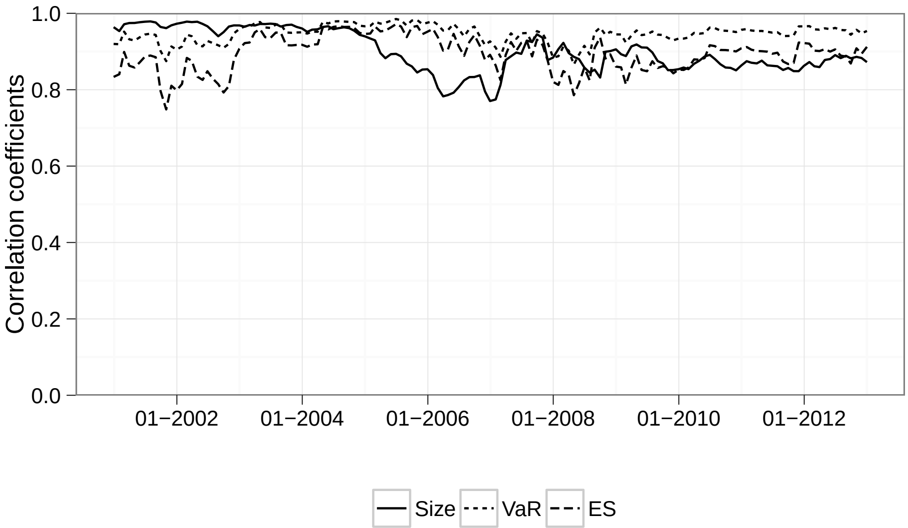
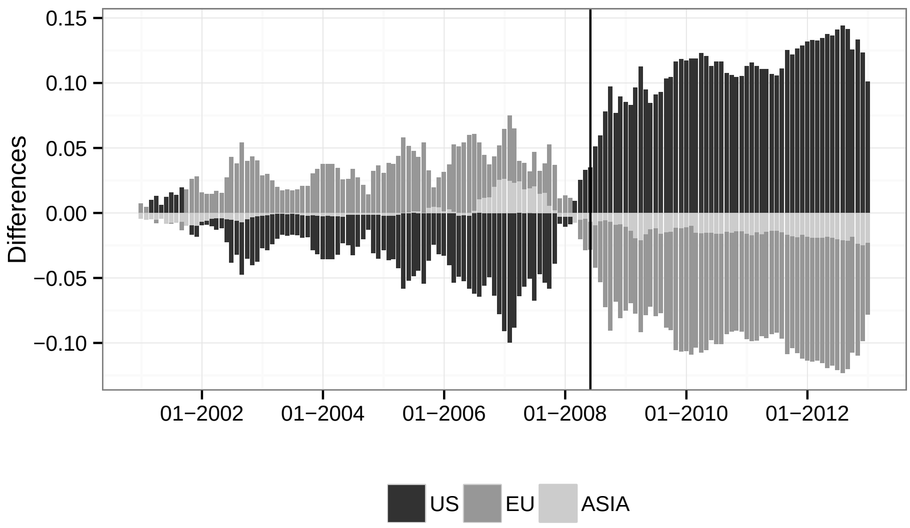
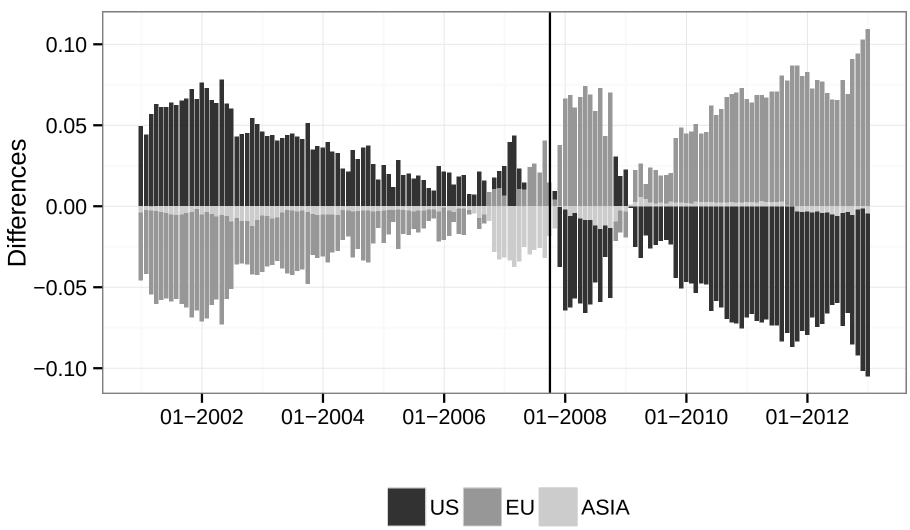


Figure 4: Pearson correlation between the MES and bank characteristics

Note: At the end of each month, we estimate the MES ratios (in proportion) for 24 G-SIFIs by applying the estimation procedure in Section 3.3 using daily equity returns in the past 4 years. The size of each bank is measured by its market value at the end of the month. The individual risk is measured by the VaR and the ES at a probability level 99.9%. They are calculated based on the same daily returns following Eqs. (3.1) and (3.2). The figure reports the cross-sectional Pearson correlations across the 24 banks.



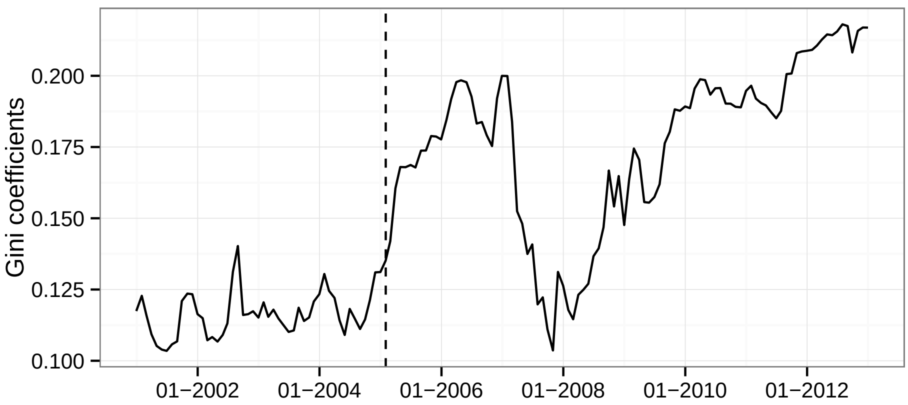
(a)



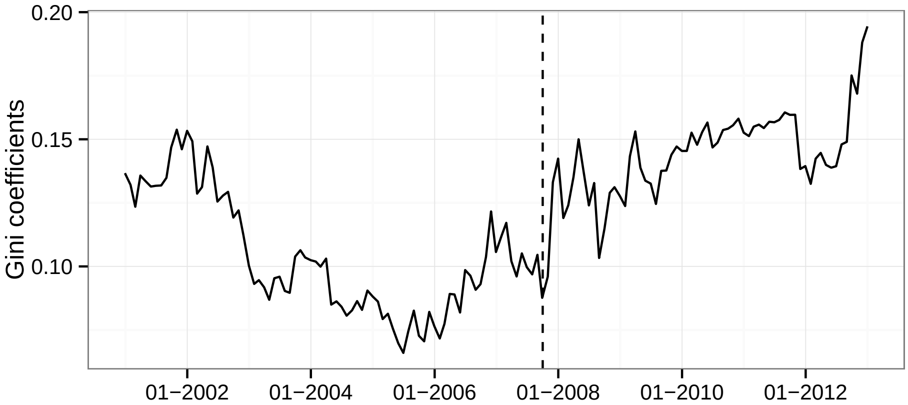
(b)

Figure 5: Differences between the MES ratios and the size (VaR) ratios by region

Note: At the end of each month, we estimate the MES ratios (in proportion) for 24 G-SIFIs by applying the estimation procedure in Section 3.3 using daily equity returns in the past 4 years. The size ratio of each bank is calculated as the share of the market value relative to the entire system at the end of the month. The VaR at a probability level 99.9% is calculated based on the same daily returns following Eq. (3.1). The VaR ratio of each bank is the share of its VaR relative to the entire system. Then we calculate for each bank the differences between the MES ratio and the size (VaR) ratio. The upper (lower) panel reports the aggregated differences between the MES ratio and the size (VaR) ratio by region. We further calculate the difference in the (log) differences between US and EU. The vertical lines indicate the break dates of the difference in differences identified from the Andrews (1993) test.



(a)



(b)

Figure 6: Gini coefficients between MES and MV (VaR) ratios

Note: At the end of each month, we estimate the MES ratios (in proportion) for 24 G-SIFIs by applying the estimation procedure in Section 3.3 using daily equity returns in the past 4 years. The size ratio of each bank is calculated as the share of the market value relative to the entire system at the end of the month. The VaR at a probability level 99.9% is calculated based on the same daily returns following Eq. (3.1). The VaR ratio of each bank is the share of its VaR relative to the entire system. Then we sort the 24 banks in an ascending order by the quotient of the MES ratios to the size (VaR) ratios, and plot the cumulative MES ratios against the cumulative size (VaR) ratios following the sorted order, which gives a convex curve starting from (0,0) and ending at (1,1) and lying below the diagonal line. We then calculate the Gini coefficient, as $1 - 2 * B$, where B is the area below the curve. The upper (lower) panel reports the Gini coefficients based on the size (VaR) ratios. The vertical lines indicate the break dates of the Gini coefficient identified from the Andrews (1993) test.

Table 1: The MES ratios and the ranking of G-SIFIs

Rank by MES	Bank	Region	MES ratio (%)	MES level (in bn USD)	MV ratio (%)	MV level (in bn USD)	Rank by MV	VaR (in bn USD)	Rank by VaR
1	Citigroup	US	12.54	29.03	6.48	116.01	5	35.60	1
2	Wells Fargo	US	12.19	28.20	10.04	179.93	2	22.87	3
3	Bank of America	US	11.67	27.02	6.99	125.13	4	24.65	2
4	JP Morgan Chase	US	9.99	23.12	9.33	167.14	3	19.14	4
5	HSBC	non-EZ	7.79	18.04	10.85	194.28	1	14.45	5
6	Lloyds Banking Group	non-EZ	4.04	9.35	3.06	54.79	12	8.45	6
7	Barclays	non-EZ	3.51	8.12	2.92	52.22	13	6.59	8
8	Morgan Stanley	US	3.13	7.25	2.11	37.75	18	8.17	7
9	Santander	EZ	3.12	7.22	4.63	83.01	7	3.71	13
10	UBS	non-EZ	2.97	6.87	3.34	59.74	11	4.41	12
11	Goldman Sachs	US	2.95	6.82	3.35	59.95	10	5.60	10
12	BNP Paribas	EZ	2.93	6.79	3.89	69.72	9	3.67	14
13	ING Bank	EZ	2.42	5.60	1.99	35.66	19	3.44	15
14	Royal Bank of Scotland	non-EZ	2.32	5.38	1.79	32.02	21	6.36	9
15	Deutsche Bank	EZ	2.04	4.71	2.26	40.46	16	2.69	21
16	Mitsubishi UFJ FG	Asia	1.69	3.91	4.21	75.49	8	3.23	17
17	Bank of New York Mellon	US	1.64	3.79	1.68	30.03	22	3.29	16
18	Credit Suisse	non-EZ	1.58	3.67	1.79	32.10	20	2.91	20
19	Société Générale	EZ	1.51	3.50	1.63	29.15	23	2.11	23
20	Nordea	non-EZ	1.41	3.26	2.16	38.66	17	1.89	25
21	Sumitomo Mitsui FG	Asia	1.40	3.23	2.84	50.94	14	3.18	19
22	Bank of China	Asia	1.32	3.05	5.12	91.64	6	4.59	11
23	State Street	US	1.32	3.05	1.22	21.85	25	3.23	18
24	Unicredit Group	EZ	1.27	2.95	1.58	28.28	24	1.95	24
25	Mizuho FG	Asia	1.10	2.54	2.44	43.75	15	2.38	22
26	Group Crédit Agricole	EZ	0.94	2.17	1.12	20.04	26	1.21	26
27	Commerzbank	EZ	0.60	1.39	0.61	11.01	27	0.83	28
28	Banque Populaire CdE	EZ	0.60	1.38	0.58	10.38	28	1.12	27
29	Dexia	EZ	0.01	0.02	0.01	0.18	29	0.03	29

Notes: This table presents the ratios and levels for the MES, the size (measured by market value at the end of Dec 2012), and the VaR at a probability level 99.9% for the 29 G-SIFIs. The MES and VaR are estimated using daily equity returns from August 2006 to December 2012 following the estimation procedure in Section 3.3 and Eq. (3.1) respectively. The rankings are based on the MES, MV and VaR of each bank.

Table 2: The MES ratios and rankings of G-SIFIs in global and regional analysis

Banks	MES ratio: regional(%)	MES ratio: global(%)	MES rank: regional	MES rank: global
US				
Wells Fargo	24.89	25.33	1	1
Citigroup	21.28	21.13	2	2
JP Morgan Chase	19.31	18.83	3	3
Bank of America	18.58	18.26	4	4
Goldman Sachs	5.34	5.44	5	6
Morgan Stanley	5.19	5.56	6	5
Bank of New York Mellon	2.86	2.98	7	7
State Street	2.54	2.48	8	8
EU				
HSBC	19.30	19.95	1	1
Lloyds Banking Group	10.23	10.34	2	2
Barclays	9.16	8.98	3	3
Santander	8.23	7.99	4	4
BNP Paribas	7.80	7.51	5	6
UBS	7.74	7.60	6	5
Royal Bank of Scotland	6.22	5.95	7	8
ING Bank	6.09	6.20	8	7
Deutsche Bank	5.01	5.21	9	9
Credit Suisse	4.10	4.06	10	10
Société Générale	3.93	3.87	11	11
Unicredit Group	3.41	3.26	12	13
Nordea	3.39	3.60	13	12
Group Crédit Agricole	2.40	2.39	14	14
Commerzbank	1.49	1.53	15	15
Banque Populaire CdE	1.48	1.53	16	16
Dexia	0.03	0.03	17	17
Asia				
Mitsubishi UFJ FG	30.93	30.68	1	1
Sumitomo Mitsui FG	24.70	25.38	2	2
Bank of China	24.16	23.98	3	3
Mizuho FG	20.22	19.96	4	4

Notes: This table presents the MES ratios calculated in the global system and each regional system. The MES ratios are estimated using daily equity returns from August 2006 to December 2012 following the estimation procedure in Section 3.3. The estimation is first conducted in the global system, then in subsystems consisting of banks from each region only. The rankings are across banks within the same region under both global and regional analysis.

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