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## ABSTRACT

# Central Bank Communication and Output Stabilization

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Some central banks have a reputation for being secretive. A justification for that behavior that we find in the literature is that being transparent about its operations and beliefs hinders the central bank in achieving the best outcome. In other words, a central bank needs flexibility and therefore cannot be fully transparent. Using a forward-looking New-Keynesian model, we find exactly the opposite. A central bank that is conservative improves output stabilization by being transparent about the procedures it uses to assess the economy and, especially, about the forecast errors it makes. Under certain conditions transparency by a conservative central bank also improves interest rate stabilization. We also find that higher transparency makes it optimal for the central bank to be more conservative as the benefits from higher transparency in terms of output stabilization are greater the more conservative the central bank is.

Key words: monetary policy, central bank, information processing, expectation formation

JEL codes: D83, E52, E58

## 1 INTRODUCTION

Monetary policy makers broadly agree that communication is a very important part of their business. Communication gives central bankers a tool to shape private sector expectations, which are crucial for effective monetary policy.

Blinder (1998) argues that openness and communication with the public improve the effectiveness of monetary policy as a macroeconomic stabilizer because: "Central banks generally control only the overnight interest rate, an interest rate that is relevant to virtually no economically interesting transactions. Monetary policy has important macroeconomic effects only to the extent that it moves financial market prices that really matter – like long-term interest rates, stock market values, and exchange rates."

In theoretical models of monetary policy one often assumes an informational asymmetry between the central bank and the private sector. Most of the times, the central bank has an informational advantage when it sets its policy. However, as Cukierman (2001) remarked at least theoretically, the issue of whether it is desirable to communicate central bank forecasts is far from being settled. A reading of the literature shows that the social desirability of communicating to the public any private information possessed by the central bank depends very much on the specific nature of the information. For example Faust and Svensson (1999) consider a case where the central bank has shifting objectives about its employment target and conclude that making this available to the public is socially desirable. Geraats (1999) also reaches at similar conclusion for the case where the central bank has private information about its inflation target. On the other hand in a model where the central bank has private information about upcoming shocks, Cukierman (2001) has shown that advance communication of central bank forecasts reduces social welfare. Taking a different direction, Eijffinger, Hoeberichts and Schaling (2000) demonstrate that transparency about central bank's inflation-output preferences depends on the degree of the credibility problem (leading to inflationary bias) relative to the stabilization problem (i.e. the need for flexibility to react to shocks).

The present paper confirms Cukierman's remark by looking at a rather different aspect of private information, namely, the central bank's own assessment of private sector expectations. We study a case where information is asymmetric in two ways. First, in our forward-looking model the private sector has private information about its own expectations of future inflation and output. Then, the central bank sets its policy based on an imperfect assessment of private sector expectations regarding next period's level of output and inflation. Likewise the private sector can not perfectly observe these assessments made by the central bank unless the central

bank publishes them. If it wishes the central bank can provide information about the way its assessment is produced and thereby make it easier for the public to forecast the assessment errors the central bank is making (see Tarkka and Mayes (1999)).

The aim is to investigate the effect of communication by the central bank of its assessment errors on private sector expectations and macroeconomic outcomes. However, the aim of communication is not to reduce the variance of forecast errors; that is fixed by assumption.

Finally, we look at the effect of communication on the macroeconomic variables that we are concerned with in this model: the rate of inflation and the output gap. We find that communication about assessment errors of private sector expectations increases the volatility of inflation but decreases the volatility of the output gap.

## 2 THE MODEL

In order to give a prominent role to expectations and communication, we base our analysis on a forward-looking IS-LM model, as described by King (2000).

We have a forward looking Phillips equation that determines inflation:

$$\mathbf{p}_t = \mathbf{b}E_t^p \mathbf{p}_{t+1} + \mathbf{I}x_t + u_t \quad (1)$$

where  $\mathbf{p}$  is the inflation rate,  $x$  is the output gap, and  $u$  is an inflation shock. The parameters  $\mathbf{b}$  and  $\mathbf{I}$  satisfy  $0 \leq \mathbf{b} \leq 1$  and  $\mathbf{I} > 0$ . The superscript  $p$  in  $E_t^p \mathbf{p}_{t+1}$  stands for private sector expectations. Thus inflation depends on private sector expectations of future inflation, the output gap and inflation shock.

The output gap is governed by a forward looking IS equation:

$$x_t = E_t^p x_{t+1} - \mathbf{j}r_t + v_t \quad (2)$$

where  $r$  is the real interest rate and  $v$  is a demand shock. The parameter  $\mathbf{j}$  satisfies  $\mathbf{j} > 0$ .

The current output gap depends on private sector expectations of next period's output gap, the real interest rate and a demand shock.

Finally, the real interest rate is determined by the Fisher equation, linking the nominal interest rate with the real interest rate.

$$r_t = i_t - E_t^p \mathbf{p}_{t+1} \quad (3)$$

where  $i$  is the nominal interest rate. Combining (2) and (3) we write the output gap as a function of private sector expectations and the central bank's policy instrument.

$$x_t = E_t^p x_{t+1} - \mathbf{j}i_t + \mathbf{j}E_t^p \mathbf{p}_{t+1} + v_t \quad (4)$$

The central bank sets the period  $t$  nominal interest rate that minimizes its period  $t$  loss function:

$$L_t^c = \mathbf{p}_t^2 + \mathbf{a}x_t^2 \quad (5)$$

where  $\mathbf{a}$  is the weight on output stabilization. In other words we are looking for optimal discretionary policy where the central bank optimizes period by period by taking as given its assessment of private sector expectations. However, since the central bank has an imperfect assessment of private sector expectations, we write the optimality condition with the actual values replaced by expectations from the central bank's perspective<sup>1</sup>

$$E_t^c x_t = -\frac{\mathbf{l}}{\mathbf{a}} E_t^c \mathbf{p}_t \quad (6)$$

where central bank's expectations of the Phillips equation is based on its assessment of private sector inflationary expectations

$$E_t^c \mathbf{p}_t = \mathbf{b}E_t^c \mathbf{p}_{t+1} + \mathbf{l}E_t^c x_t + u_t \quad (7)$$

using the central bank's assessment of the Phillips curve (7) in optimality condition (6) we get

$$E_t^c x_t = -\frac{\mathbf{bl}}{\mathbf{a} + \mathbf{l}^2} E_t^c \mathbf{p}_{t+1} - \frac{\mathbf{l}}{\mathbf{a} + \mathbf{l}^2} u_t \quad (8)$$

Taking central bank's expectations of the IS equation (4)

<sup>1</sup> We get the optimality condition (6) by minimizing the expected value of (5) subject to the central bank expectation of the Phillips curve, which is equation (7) below.

$$E_t^c x_t = E_t^c x_{t+1} - \mathbf{j} i_t + \mathbf{j} E_t^c \mathbf{p}_{t+1} + v_t \quad (9)$$

Combining the IS curve (9) with the optimality condition and the Phillips curve (7) implies the following expression for the nominal interest rate:

$$i_t = \frac{1}{\mathbf{j}} \left( \frac{\mathbf{j}(\mathbf{a} + \mathbf{l}^2) + \mathbf{b}\mathbf{l}}{\mathbf{a} + \mathbf{l}^2} E_t^c \mathbf{p}_{t+1} + E_t^c x_{t+1} + \frac{\mathbf{l}}{\mathbf{a} + \mathbf{l}^2} u_t + v_t \right) \quad (10)$$

Now, the idea, in the spirit of Tarkka and Mayes (1999), is that the central bank's assessment of private sector expectations about the future output gap and the future rate of inflation is imperfect. Evans and Honkapohja (2002) also discuss the issue of observability of current private expectations in the context of the adaptive learning literature. They point out that although survey data on private forecasts of future inflation and output are available to central banks, there are apparent concerns about the accuracy of this data. Although most experts would agree that it is very hard for the central bank to accurately measure the public's expected output gap, opinions differ about the extent to which the central bank is uncertain about the public inflationary expectations (see, however, Mankiw, Reis and Wolfers, 2003). We choose a general setup, where the central bank makes an assessment error in both private sector inflationary expectations and private sector output gap expectations. However, the variances of these errors may be different.

$$\begin{aligned} E_t^c x_{t+1} &= E_t^p x_{t+1} - w_t^x \\ E_t^c \mathbf{p}_{t+1} &= E_t^p \mathbf{p}_{t+1} - w_t^p \end{aligned} \quad (11)$$

where superscript  $c$  stands for central bank's expectations of private sector expectations and superscript  $p$  stands for private sector expectations. The assessment errors follow an AR(1) process  $w_t = \mathbf{r}w_{t-1} + \mathbf{h}_t$  where the innovations are independent and normally distributed  $\mathbf{h}_t \sim N(0, \mathbf{s}_h^2)$  and  $\mathbf{r}$  is a measure for the persistence of the assessment errors. The central bank's assessment errors can be persistent because the central bank only sluggishly adjusts its procedures.

### 3 INFORMATION TRANSMISSION THROUGH A LIMITED CAPACITY CHANNEL

In our model, the central bank is a rational agent that minimizes its loss-function, using all the information that is available. Therefore, the central bank does not know the realization of its assessment errors. However, it can communicate to the public the *procedure* that it uses to assess private sector expectations. When the private sector understands this procedure, it will be able to find out the forecast error, although it *cannot influence the size* of the forecast error. By applying information theory as developed by Shannon (1948), Sims (1998, 2003) has studied the effects of constrained information processing on the behavior of macroeconomic time series. Adam (2003) uses the information channel concept to look at optimal monetary policy when firms have private information about shocks hitting the economy. We apply the same concept to expectation formation in a monetary policy framework where the central bank communicates about its assessment of expectations to the public through an information channel with limited capacity.

The central bank communicates with the public about the model it uses to assess private sector expectations through a channel with limited capacity. In the model, the central bank sends a signal  $w$  over a channel with limited capacity  $C$  and the receiver (i.e. the public) observes the signal with noise  $\mathbf{e}$ . The public observes  $W$ :

$$W_t = w_t + \mathbf{e}_t \tag{12}$$

where  $w_t \sim N(0, \mathbf{s}_w^2)$ <sup>2</sup> and  $\mathbf{e}_t \sim N(0, \mathbf{s}_e^2)$  and  $w_t$  and  $\mathbf{e}_t$  are independent.

In order to model the information channel with limited capacity, we define a measure of uncertainty of a random variable, called *entropy*. This measure has several attractive properties compared to other measures of uncertainty (see Cover and Thomas, 1991 for a textbook treatment). The entropy for the input  $w_t$  is defined as (we have dropped time-subscripts):

<sup>2</sup> Where  $\mathbf{s}_w^2 = \frac{\mathbf{s}_h^2}{1 - \mathbf{r}^2}$ . Here we have made the simplifying assumption that the world starts at  $t$ . If this is not the

case, the public receives information about the innovation and the part of the noise that is carried over from last period. Then, the input signal will be  $w_t = \mathbf{h}_t - \mathbf{r}\mathbf{e}_{t-1}$  with  $w_t \sim N(0, \mathbf{s}_h^2 + \mathbf{r}^2\mathbf{s}_e^2)$ . This has no implications for qualitative results.



$$H(w) = - \int_{-\infty}^{\infty} p(w) \ln p(w) dw = \frac{1}{2} (\ln 2\pi e + \ln \mathbf{s}_w^2)$$

where  $p(w)$  is the probability density function of  $w$ , which we choose to be normal. The entropy of the stochastic variable  $w$  is an increasing function of its variance  $\mathbf{s}_w^2$ .

Based on this definition we compute the conditional and unconditional entropy of output signal  $W$ .

$$H(W|w) = H(\mathbf{e}) = \frac{1}{2} (\ln 2\pi e + \ln \mathbf{s}_e^2)$$

$$H(W) = \frac{1}{2} (\ln 2\pi e + \ln(\mathbf{s}_w^2 + \mathbf{s}_e^2))$$

Unless variables  $w$  and  $W$  are independent, conditioning reduces the entropy. The information about  $w$  obtained by observing  $W$ , denoted by  $I(w, W)$ , is called the *mutual information*. Using a basic theorem from information theory (see, for instance, Cover and Thomas, 1991) we can write the following expression:

$$I(w, W) = H(w) - H(w|W) = H(W) - H(W|w)$$

In words, the amount of uncertainty reduction for the two jointly distributed variables is the same whether we use observations on  $W$  to infer about  $w$  or vice versa. Of course, in our model we are interested in the first equality because the public uses observations on  $W$  to inform itself about the input signal  $w$ . But this theorem allows us to use the (equivalent but computationally more attractive) second expression<sup>3</sup>

$$\begin{aligned} I(w, W) &= H(W) - H(W|w) \\ &= \frac{1}{2} (\ln 2\pi e + \ln(\mathbf{s}_w^2 + \mathbf{s}_e^2)) - \frac{1}{2} (\ln 2\pi e + \ln \mathbf{s}_e^2) \\ &= \frac{1}{2} \ln \left( 1 + \frac{\mathbf{s}_w^2}{\mathbf{s}_e^2} \right) \end{aligned} \tag{13}$$

<sup>3</sup> From equation (12) it is easier to compute the conditional probability distribution for  $W|w$  than for  $w|W$ .

As is clear from (13), the mutual information  $I(w, W)$  is an increasing function of the signal-to-noise ratio  $\frac{\mathbf{s}_w^2}{\mathbf{s}_e^2}$ . The larger the variance of the noise is, the lower the mutual information.

The *capacity* of the channel is defined as its maximum mutual information. Since communication goes through a channel with limited capacity  $C$ , the maximum reduction in entropy that can be achieved by communicating is  $C$ :

$$I(w, W) = \frac{1}{2} \ln \left( 1 + \frac{\mathbf{s}_w^2}{\mathbf{s}_e^2} \right) \leq C$$

When we assume that capacity is used to the maximum, the capacity constraint is binding and the previous equation holds with equality. It follows that

$$\mathbf{s}_e^2 = \frac{\mathbf{s}_w^2}{e^{2C} - 1} \tag{14}$$

This equation (14) shows us that the variance of the communication error is a negative function of the capacity of the communication channel.

The intuition behind this result is that given the variability of the actual assessment error  $w$ , larger information transmission capacity reduces the magnitude of the noisy part in the observed assessment error  $W$ . In the extreme, with infinite capacity of the information channel ( $C \rightarrow \infty$ ) the variance of the noise goes to zero and the receiver observes the central bank's signal about the assessment errors without noise. In economic terms, the public perfectly understands the central bank's assessment of private sector expectations. If, on the other hand, capacity tends to zero the variance of the noise tends to infinity. In that case, the uncertainty about  $w$  after observing  $W$  equals the uncertainty of  $w$  before observing  $W$ , so that the observation of  $W$  adds no information at all. With a low capacity, the noise dominates the signal.

#### 4 EXPECTATION FORMATION

As described above in equation (12) the public receives an output-signal  $W$  that indicates this period's assessment error. With this signal, the agent solves a standard signal-extraction problem to form expectations about the assessment error given all available information<sup>4</sup>

$$E_t[w_t|W_t] = \frac{\mathbf{s}_w^2}{\mathbf{s}_w^2 + \mathbf{s}_e^2} W_t = (1 - e^{-2C}) W_t \equiv K W_t \quad (15)$$

where  $K \equiv 1 - e^{-2C}$  so that  $C \rightarrow 0 \Leftrightarrow K \rightarrow 0$  and  $C \rightarrow \infty \Leftrightarrow K \rightarrow 1$

So, communication by the central bank that is received by the private sector contains a noise term  $\mathbf{e}$  and is weighted with a factor  $0 \leq K \leq 1$  that depends on the capacity of the communication channel.

#### 5 INSERTING THE COMMUNICATION CHANNEL INTO THE MODEL

We solve the model by applying the method of undetermined coefficients (see e.g. McCallum (1983)).

First for the output gap and inflation rate we conjecture that they depend on the assessment errors, the inflation shock, the demand shocks and the noise that is introduced by the limited capacity channel:

$$x_t = B_{11} w_t^p + B_{12} w_t^x + B_{13} u_t + B_{14} v_t + B_{15} \mathbf{e}_t^p + B_{16} \mathbf{e}_t^x \quad (16)$$

$$p_t = B_{21} w_t^p + B_{22} w_t^x + B_{23} u_t + B_{24} v_t + B_{25} \mathbf{e}_t^p + B_{26} \mathbf{e}_t^x \quad (17)$$

Then from these follow private sector expectations. The only information that the private sector has is the signal about the assessment errors  $W$  and the AR(1)-structure of the assessment errors. This signal is used to form expectations about the future output gap and inflation rate:

$$E_t^p x_{t+1} = B_{11} r K W_t^p + B_{12} r K W_t^x = B_{11} r K w_t^p + B_{12} r K w_t^x + B_{11} r K \mathbf{e}_t^p + B_{12} r K \mathbf{e}_t^x \quad (18)$$

<sup>4</sup> For an early application of signal-extraction to economics see Lucas (1973).

$$E_t^p \mathbf{p}_{t+1} = B_{21} \mathbf{r} K W_t^p + B_{22} \mathbf{r} K W_t^x = B_{21} \mathbf{r} K w_t^p + B_{22} \mathbf{r} K w_t^x + B_{21} \mathbf{r} K e_t^p + B_{22} \mathbf{r} K e_t^x \quad (19)$$

Essential here is that the public, using the signal of today's error and its persistence, is able to forecast the error that the central bank is going to make in the next period.

The interest rate rule (10) will now be

$$i_t = \frac{1}{\mathbf{j}(\mathbf{a} + \mathbf{I}^2)} \left( [\mathbf{j}(\mathbf{a} + \mathbf{I}^2) + \mathbf{b}\mathbf{I}] (E_t^p \mathbf{p}_{t+1} - w_t^p) + (\mathbf{a} + \mathbf{I}^2) (E_t^p x_{t+1} - w_t^x) + \mathbf{I} u_t + (\mathbf{a} + \mathbf{I}^2) v_t \right) \quad (20)$$

Using (18) and (19) in (20) we can express the interest rate as a function of structural parameters and shocks.

Then, output and inflation will be

$$x_t = E_t^p x_{t+1} - \mathbf{j} i_t + \mathbf{j} E_t^p \mathbf{p}_{t+1} + v_t \quad (21)$$

$$\mathbf{p}_t = \mathbf{b} E_t^p \mathbf{p}_{t+1} + \mathbf{I} x_t + u_t \quad (22)$$

Using (18), (19) and (20) in (21) and (22) and then solving for undetermined coefficients to make (21) and (22) consistent with (16) and (17) gives us the expressions for B11-B36. (see appendix)

Coefficients B11, B12, B21, B22 are all positive, indicating that the existence of assessment errors makes inflation and the output gap more volatile. For small values of  $K$ , however, the coefficients B31 and B32 are negative. This means that an underestimation of, for instance, inflationary expectations (positive assessment error) makes policy too lax, which is also what one would expect.

To analyze the effect of communication, we look at the first derivatives of the coefficients with respect to  $K$ . We find that the coefficients B11, B12, B15 and B16 in the output equation **decrease** monotonically with communication, while B21, B22, B25, B26 in the inflation equation as well as those for the interest rate equation - B31, B32, B35, and B36 **increase** monotonically with communication. That means that the output gap and the interest rate (for

$K$  small) become less volatile with communication, whereas the inflation rate becomes more volatile.

## 6 SOCIAL WELFARE AND COMMUNICATION

In order to analyze the effect of communication by the central bank about the assessment errors, we use a loss function that punishes deviations of the inflation rate, output gap and interest rate from its target value zero. We allow for the possibility that society weights the objectives differently from the central bank.

$$L_t^S = \mathbf{p}_t^2 + \bar{a}x_t^2 + \bar{q}i_t^2$$

where  $\bar{a}$  and  $\bar{q}$  are society's weights on output and interest rate stabilization respectively.

For the analysis of the welfare effects of communication, we look separately at the assessment errors for the expected output gap and the expected rate of inflation and we disregard the inflation and demand shocks  $u$  and  $v$ . The reason why we do that is that the equilibrium solution for output and inflation under communication or not have the same coefficients attached to  $u$  and  $v$ . Thus, in evaluating the *change* in the welfare loss, these shocks vanish. That is why for clarity we dropped them from the expected losses themselves.

Only focusing on assessment errors on inflationary expectations, society's expected loss is:

$$EL_t^{S,p} = E\mathbf{p}_t^2 + \bar{a}Ex_t^2 + \bar{q}Ei_t^2 = \left( B_{21}^2 + \bar{a}B_{11}^2 + \bar{q}B_{31}^2 + \frac{(B_{25}^2 + \bar{a}B_{15}^2 + \bar{q}B_{35}^2)(1-K)}{K} \right) \mathbf{s}_w^{p^2}$$

Only focusing on assessment errors on output gap expectations, society's expected loss is:

$$EL_t^{S,x} = E\mathbf{p}_t^2 + \bar{a}Ex_t^2 + \bar{q}Ei_t^2 = \left( B_{22}^2 + \bar{a}B_{12}^2 + \bar{q}B_{32}^2 + \frac{(B_{26}^2 + \bar{a}B_{16}^2 + \bar{q}B_{36}^2)(1-K)}{K} \right) \mathbf{s}_w^{x^2}$$

In this case, given the central bank's preferences, if the society cares much about output stabilization communication turns out to be welfare enhancing.<sup>5</sup>

<sup>5</sup> This is a situation where the central bank decides on interest rate policy based on its own weight on output stabilization  $\mathbf{a}$  while the society assigns a higher weight on output stabilization  $\bar{a} > \mathbf{a}$ . Thus given  $\mathbf{a}$  we can assign a value for  $\bar{a}$  such that communication is worthwhile for the society's welfare.

The intuition behind this results is as follows. With a positive assessment error, the central bank underestimates the expected future output gap and rate of inflation. The policy it has planned is therefore too loose and the interest rate it plans to set too low. If the public is aware of the fact that the procedure used by the central bank leads to an underestimation of the expected output gap and inflation (i.e. this error is communicated) the public will expect a positive assessment error next period (because of persistence in the error). It will therefore have a higher inflationary expectation. This is picked-up by the policymaker (still with an assessment error, though) and it makes policy tighter than without communication. The opposite reasoning holds for a negative assessment error.

Note that the parameters for the interest rate equation are negative. The increase in the parameters for this equation mean less interest volatility.

Since the coefficients either monotonically decrease or monotonically increase, it is sufficient to consider only the extreme cases of no communication  $K = 0$  and full communication  $K = 1$ .

**Proposition 1:** *If the public has no preference for interest rate stabilization ( $\bar{q} = 0$ ), and the policymaker is sufficiently conservative communication about the central bank's assessment errors improves welfare. It is welfare improving to communicate the assessment error of inflation expectations and/or the assessment error of output gap expectations if:*

$$\frac{\bar{a}}{\mathbf{a}} > \frac{2\mathbf{I}^2 + \mathbf{a}(2 - \mathbf{b}\mathbf{r})}{\mathbf{I}^2(2 - \mathbf{b}\mathbf{r}) + \mathbf{a}(2 - 2\mathbf{b}\mathbf{r})} \geq 1$$

**Proof:**  $\frac{\partial EL_t^{s,p}}{\partial K} < 0$  iff  $\frac{\bar{a}}{\mathbf{a}} > \frac{2\mathbf{I}^2 + \mathbf{a}(2 - \mathbf{b}\mathbf{r})}{\mathbf{I}^2(2 - \mathbf{b}\mathbf{r}) + \mathbf{a}(2 - 2\mathbf{b}\mathbf{r})}$

$$\frac{\partial EL_t^{s,x}}{\partial K} < 0 \text{ iff } \frac{\bar{a}}{\mathbf{a}} > \frac{2\mathbf{I}^2 + \mathbf{a}(2 - \mathbf{b}\mathbf{r})}{\mathbf{I}^2(2 - \mathbf{b}\mathbf{r}) + \mathbf{a}(2 - 2\mathbf{b}\mathbf{r})}$$

The intuition behind this result is as follows. The positive effect of communication on stabilization of the output gap is stronger when the central bank is conservative ( $\mathbf{a}$  low). On the other hand stabilization of the output gap contributes more to social welfare if society puts more weight on output gap stabilization ( $\bar{a}$  large).

As an extension of the above analysis, we ask under what conditions communication turns out to be welfare improving when society's welfare depends on the variability of inflation, output

and the nominal interest rate. The new element is that now the society has an additional goal, namely, the nominal interest rate.<sup>6</sup> For this purpose let us fix  $\bar{a}$  such that

$$\bar{a} = \left( \frac{2I^2 + \mathbf{a}(2 - \mathbf{br})}{I^2(2 - \mathbf{br}) + \mathbf{a}(2 - 2\mathbf{br})} \right) \mathbf{a} .^7$$

That means under the case without additional interest rate goal for the society (i.e.  $\bar{q}=0$ ), communication would not affect social welfare. Proposition 2 gives the condition under which communication improves social welfare when we allow the society to care about interest rate stabilization.

**Proposition 2:** *Suppose the public has a preference for interest rate stabilization (i.e.  $\bar{q} > 0$ ).*

*Suppose also  $\bar{a} = \left( \frac{2I^2 + \mathbf{a}(2 - \mathbf{br})}{I^2(2 - \mathbf{br}) + \mathbf{a}(2 - 2\mathbf{br})} \right) \mathbf{a}$ , so that the central bank is sufficiently conservative (but to a lesser extent than when  $\bar{q} = 0$ ). Then communication about the central bank's assessment errors improves social welfare if:*

$$\mathbf{j} < \frac{(2 - \mathbf{r})(1 - \mathbf{br})\mathbf{a} + (2 - \mathbf{r}(1 + \mathbf{b} - \mathbf{br}))I^2}{\mathbf{r}I(\mathbf{a} + I^2)}$$

**Proof:** Since the condition in proposition 1 holds with equality, social welfare increases if and only if the nominal interest rate is less volatile with better communication. It is then easy to show that the equilibrium nominal interest rate is less volatile with better communication if and only if the above condition is satisfied.

Note that the right hand side of this inequality condition is positive. Given our assumption that  $\mathbf{j} > 0$ , what the condition requires is that  $\mathbf{j}$  should not be too large. This makes sense since the effect of more communication on the variability of interest rate depends on the degree to which private sector expectations of next period's inflation and output respond to the current assessment errors (see the central bank's reaction function (20)). It turns out that as  $\mathbf{j}$  gets smaller, private sector expectations of output and inflation (see equations (18) and (19)) respond less strongly to the (current) assessment error on inflation expectations.

<sup>6</sup> For discussions interest rate stabilization as related to instability in financial markets and financial crises, see for example Cukierman (2001, p. 61) and the references there in.

<sup>7</sup> Note that with this assumption the central bank is at least as conservative as the public since  $\bar{a} \geq \mathbf{a}$ .

**Proposition 3:** *Communication about the assessment error will increase optimal conservatism of the central bank if the persistence of the assessment error on the expected output gap is not too large, i.e. if  $\mathbf{r} < \frac{\mathbf{j}^2(\bar{a} + \mathbf{I}^2) + \bar{q}}{\bar{q}(1 + \mathbf{j} \mathbf{I})}$ .*

**Proof:**

$$\frac{\partial \mathbf{a}^*}{\partial K} \Big|_{\mathbf{s}_w^{p2} = 0, K = 0, \mathbf{a} = \mathbf{a}^*} = - \frac{b\mathbf{r}(\mathbf{j}^2(\bar{a} + \mathbf{I}^2) + \bar{q})(\bar{q}(1 - \mathbf{r}(1 + \mathbf{j} \mathbf{I})) + \mathbf{j}^2(\bar{a} + \mathbf{I}^2))\mathbf{s}_w^{x2}}{\mathbf{j}^4 \mathbf{s}_u^2}$$

$$\text{with } \mathbf{a}^* \Big|_{\mathbf{s}_w^{p2} = 0, K = 0} = \bar{a} + \frac{\bar{q}}{\mathbf{j}^2}$$

$$\frac{\partial \mathbf{a}^*}{\partial K} \Big|_{\mathbf{s}_w^{p2} = 0, K = 0, \mathbf{a} = \mathbf{a}^*} < 0 \text{ iff } \mathbf{r} < \frac{\mathbf{j}^2(\bar{a} + \mathbf{I}^2) + \bar{q}}{\bar{q}(1 + \mathbf{j} \mathbf{I})}$$

The intuition behind Proposition 3 is that increased communication (from the zero level) improves stabilization of the output gap. Therefore, better communication (larger  $K$ ) makes it optimal for the central bank to become more conservative (smaller  $a$ ). Note that under the benchmark case  $\bar{q} = 0$ , Proposition 3 requires no relevant restrictions on the persistence parameter  $\mathbf{r}$  since in that case we would have that  $\frac{\partial \mathbf{a}^*}{\partial K} \Big|_{\mathbf{s}_w^{p2} = 0, K = 0, \mathbf{a} = \mathbf{a}^*} < 0$  if  $\mathbf{r} < \infty$ .

## 7 CONCLUDING REMARKS

It is sometimes argued that central banks need to be secretive in order to maintain flexibility. This flexibility enables central banks to stabilize the economy. In a standard New-Keynesian model we arrive at an opposite result. By communicating and being transparent about procedures that lead to assessment errors of private sector expectations on inflation and the output gap, the central bank is better able to stabilize the output gap than when its assessment errors come as a surprise to the public. The inflation rate, however, will become more volatile. The reason is that the public's reaction to the errors will cause the bank to adjust its interest rate in the direction that helps to stabilize the impact of the error on the output gap.

A crucial element in our analysis is that, with communication by the central bank, the public is able to forecast the error that the central bank will make in assessing private sector expectations. In practice, a central bank can help the public by providing information about the procedures it uses to assess expectations or by openly evaluating past performances of central bank assessments.



In our welfare analysis we showed that a sufficiently conservative central bank improves society's welfare by communicating its assessment of private sector expectations. This holds in the benchmark case where society cares only about inflation and output stabilization and in a case where we allow the society to have interest rate stabilization goal on top of inflation and output.

Furthermore, we analyzed the relationship between communication and central bank conservativeness. It turns out that when the assessment errors on output gap expectations are not too persistent, a central bank deciding to be more transparent can afford to be more conservative since the benefits from higher transparency in terms of output stabilization are greater the more conservative the central bank is.

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## APPENDIX

### Undetermined coefficients

We conjecture the following structure for the output gap, the inflation rate, the nominal interest rate, the output gap expected by the public and the inflation rate expected by the public.

$$x_t = B_{11}w_t^p + B_{12}w_t^x + B_{13}u_t + B_{14}v_t + B_{15}e_t^p + B_{16}e_t^x \quad (16)$$

$$p_t = B_{21}w_t^p + B_{22}w_t^x + B_{23}u_t + B_{24}v_t + B_{25}e_t^p + B_{26}e_t^x \quad (17)$$

$$i_t = B_{31}w_t^p + B_{32}w_t^x + B_{33}u_t + B_{34}v_t + B_{35}e_t^p + B_{36}e_t^x$$

$$E_t^p x_{t+1} = B_{11}rKW_t^p + B_{12}rKW_t^x = B_{11}rKw_t^p + B_{12}rKw_t^x + B_{11}rKe_t^p + B_{12}rKe_t^x \quad (18)$$

$$E_t^p p_{t+1} = B_{21}rKW_t^p + B_{22}rKW_t^x = B_{21}rKw_t^p + B_{22}rKw_t^x + B_{21}rKe_t^p + B_{22}rKe_t^x \quad (19)$$

Then, the real interest rate will have the following structure:

$$r_t = (B_{31} - rKB_{21})w_t^p + (B_{32} - rKB_{22})w_t^x + B_{33}u_t + B_{34}v_t + (B_{35} - rKB_{21})e_t^p + (B_{36} - rKB_{21})e_t^x$$

From our model, it follows that the nominal interest rate will look like this:

$$i_t = \frac{1}{j(a + I^2)} \left( [j(a + I^2) + bI](E_t^p p_{t+1} - w_t^p) + (a + I^2)(E_t^p x_{t+1} - w_t^x) + Iu_t + (a + I^2)v_t \right)$$

(20)

The output gap and inflation rate will be

$$x_t = E_t^p x_{t+1} - j i_t + j E_t^p p_{t+1} + v_t \quad (21)$$

$$p_t = b E_t^p p_{t+1} + I x_t + u_t \quad (22)$$

Solving for undetermined coefficients we get the following results:

$$B_{21} = \frac{I(j(a + I^2) + Ib)}{(I^2 + a(1 - rKb))}$$

$$B_{11} = B_{21} \frac{1 - rKb}{l}$$

$$B_{12} = B_{11} \frac{a + l^2}{j(a + l^2) + lb}$$

$$B_{13} = -\frac{lj^2}{j^2(a + l^2)}$$

$$B_{14} = \frac{q}{j^2(a + l^2)}$$

$$B_{15} = B_{11} \frac{j(a + l^2) + lb}{a + l^2}$$

$$B_{16} = B_{12} - 1$$

$$B_{22} = B_{21} \frac{a + l^2}{j(a + l^2) + lb}$$

$$B_{23} = \frac{a}{(a + l^2)}$$

$$B_{24} = lB_{14}$$

$$B_{25} = B_{21} \frac{l(j(a + l^2) + lb)}{a + l^2}$$

$$B_{26} = B_{22} - l$$

$$B_{31} = B_{21} \frac{Kr(1 + b(1 - Kr) + jl) - 1}{jl}$$

$$B_{32} = B_{22} \frac{Kr(1 + b(1 - Kr) + jl) - 1}{jl}$$

$$B_{33} = -\frac{1}{j} B_{13}$$

$$B_{34} = \frac{1}{j}$$

$$B_{35} = B_{21} \frac{Kr((a + l^2)(1 + jl - Krb) + bl^2)}{lj(a + l^2)}$$

$$B_{36} = B_{22} \frac{Kr((a + l^2)(1 + jl - Krb) + bl^2)}{lj(a + l^2)}$$