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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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The economics of sharing macro-longevity risk*

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Abstract:

Pension funds face macro-longevity risk or uncertainty about future mortality rates. We analyze macro-longevity risk sharing between cohorts in a pension fund as a risk management tool. We show that both the optimal risk-sharing rule and the welfare gains from risk sharing depend on the retirement age policy. Welfare gains from sharing macro-longevity risk measured on a 10-year horizon in case of a fixed retirement age are between 0.2 and 0.3 percent of certainty equivalent consumption after retirement. Cohorts experience a similar impact of macro-longevity risk on post retirement consumption and it is not optimal for young cohorts to absorb risk of older cohorts. However, in case the retirement age is fully linked to changes in life expectancy, the welfare gains are substantially higher. The risk bearing capacity of workers is larger when they use their labor supply as a hedge against macro-longevity risk. As a result, workers absorb risk from retirees in the optimal risk-sharing rule, thereby increasing the welfare gain up to 2.7 percent.

Keywords: Macro-longevity risk, risk sharing, welfare analysis, retirement age
JEL classifications: D61, G22, J26, J32

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1 Introduction

Macro-longevity risk is the uncertainty about future mortality rates. Mortality rates may, e.g., decrease as a consequence of medical improvements, or may increase because of new diseases. Macro-longevity risk is a systematic risk. It affects the entire population. Macro-longevity risk does not decrease by sharing it within a pool of participants of the same cohort. Nonetheless, sharing macro-longevity risk with other cohorts can be beneficial if cohorts are differently affected by macro-longevity risk. Macro-longevity risk differs from micro-longevity risk or the individual uncertainty about the time of death. Micro-longevity risk is an idiosyncratic risk that can be fully diversified by pooling enough participants in a pension fund.

Macro-longevity risk can have a significant impact on pension benefits. The impact depends on the configuration. In a defined benefit (DB) pension scheme macro-longevity risk increases the uncertainty in the funding ratio. The risk is, e.g., borne by the employer and employees that contribute to the pension scheme. In a defined contribution (DC) pension scheme with a fixed annuity pension benefits are guaranteed after retirement and macro-longevity risk is borne by the pension provider, for example the shareholders of an insurance company. In a DC pension scheme with a variable annuity pension benefits can be adjusted to changes in future mortality rates. As a consequence, the participants bear macro-longevity risk. Retirees are especially vulnerable to macro-longevity risk because they cannot compensate lower pension benefits by working longer or saving more. However, also future pension benefits of employees may be negatively affected if mortality decreases. Either their benefits are reduced or their contributions are increased to finance a decrease in mortality rates. Hence, macro-longevity risk affects both retirees and employees. However, it does not affect all cohorts in the same way. Medical progress or diseases may affect cohorts in a different way. Furthermore, workers have more risk-absorbing capacity compared to retirees. They can adjust their labor supply. These differences create a clear case for risk sharing. This is strengthened by the fact that the market for macro-longevity risk is close to absent.

The economic problem central to this paper is optimal risk sharing between cohorts in a pension fund. Collective risk sharing is a risk management method that allocates risks to cohorts. We maximize aggregate expected utility of all generations in the situation where a social planner is present. In this way we find the Pareto-efficient risk-sharing rule and calculate the welfare gain of the Pareto improvement. All participants experience the same welfare gain since we use a utility-based fairness criteria. We find that the design of the retirement age policy has a large impact on the optimal risk-sharing rule and size of the welfare gains. If the retirement age is fixed, the welfare gains from sharing macro-longevity risk measured on a 10-year horizon are between 0.2 and 0.3 percent of certainty equivalent consumption after retirement. In this case, the impact of macro-longevity risk on consumption after retirement is more or less equal for different cohorts. Young cohorts do not absorb macro-longevity risk of other cohorts. As a result, the welfare gains from sharing macro-longevity risk are limited. However, if the retirement age is linked to life expectancy, the welfare gains from sharing macro-longevity risk are substantially higher up to 2.7 percent. The risk bearing capacity of workers is larger, because they can use their labor supply as a hedge against macro-longevity risk. As a result, workers absorb risk from retirees. After all, the human capital of workers increases if they work longer.

This paper contributes to the literature on macro-longevity risk. We approach this actuarial topic from an economic perspective. It is to the best of our knowledge the first paper to optimize the risk-sharing rule of macro-longevity risk based on welfare analyses. Related works are De Waegenaere et al. (2017) and De Waegenaere et al. (2018). These papers investigate ad hoc risk-sharing rules for micro- and macro-longevity risk. Other related studies (for example Piggott et al. (2005), Qiao and Sherris (2013) and Boon et al. (2017)) consider group self-annuitisation schemes (GSAs). In these schemes both micro- and macro-longevity risk are shared uniformly among all participants in the pool. Moreover, we are the first to investigate the impact of different retirement age policies when sharing macro-longevity risk. Investigating different retirement age policies is relevant as several countries have linked the retirement age to life expectancy. Stevens (2017) investigates the impact of different retirement age policies on the individual retirement age, expected remaining lifetime at retirement and value of pension benefits but does not consider collective risk sharing.

Insurance is an alternative way to manage macro-longevity risk. Insurance is a risk management method in which a third party guarantees to compensate specified losses in return for a levy. For example, macro-longevity risk can be transferred to financial markets via financial products, bought by for example investors. This is called securitization (Cairns et al. (2006a), Blake et al. (2006a), Ngai and Sherris (2011), Hunt and Blake (2015)). Securitization can be welfare improving because it achieves a more efficient risk allocation by distributing the risk among market participants who can better bear the risk. Moreover, literature suggests that governments can establish solutions to manage macro-longevity risk by issuing longevity bonds (Brown and Orszag (2006), Blake et al. (2014)). In practice, the amount of financial products that transfer macro-longevity risk is small (Basel Committee on Banking Supervision (2013)) and insurance companies and governments are reluctant to underwrite macro-longevity risk. There are several reasons for the lack of a well-functioning market. Blake et al. (2006b) divide these reasons into design issues, pricing issues and institutional issues. Finally, buy-outs and buy-ins insure macro-longevity risk (Lin et al. (2015)). A disadvantage of pension buy-outs and buy-ins is that they are expensive. Natural hedging is a third way to manage macro-longevity risk (Cox and Lin (2007)). Macro-longevity risk in annuity policies can be hedged with mortality risk in life insurance policies.[1] Participants living longer than expected have a negative impact on annuity policies but a positive impact on life insurance products since less participants die at a young age. However, mortality risk only provides a partial hedge to longevity risk due to the different nature of both risks and the different age groups. Moreover, the mortality risk market is more than five times smaller than the longevity risk market (EIOPA (2011)).

The remainder of this paper is organized as follows. Section 2 describes the modeling of macro-longevity risk. Section 3 explains the concept of collective risk sharing. Section 4 describes the different retirement age policies. Section 5 presents the results. Section 6 concludes and gives a policy evaluation.

2 Macro-longevity risk

We consider three sources of macro-longevity risk. These are visualized in Figure 1. The first source is stochastic variation. This is the random variation in the aggregate realized number of deaths. A stochastic mortality model captures stochastic variation.[2] The second source is parameter risk. It is the uncertainty about the true value of the parameters of the stochastic model. The third source is model risk. This is the uncertainty about the appropriateness of the mortality model. For instance, model risk can occur due to structural breaks that are not captured by the model. Medical innovations or a rapid increase of obesity can cause these structural breaks. All three sources of uncertainty can lead to mis-estimation of mortality rates. A stochastic mortality model only takes into account stochastic variation while ignoring the other sources of risk. Ideally, one wants to model macro-longevity risk including all these sources of risk.

In this paper the main source of macro-longevity risk is stochastic variation. However, we also consider a type of parameter risk. This will be discussed in more detail in Subsection 2.2. In a sensitivity analysis in Subsection 5.1.3 we address model risk by considering an alternative model for macro-longevity risk.

We employ the widely used Lee and Carter (1992) model which is a stochastic mortality model that allows for stochastic variation in death rates. It is fitted to historical data to forecast death rates and to quantify macro-longevity risk. Cairns et al. (2011) discuss the suitability of six stochastic mortality models for forecasting mortality and conclude that the Lee-Carter model is both reasonably robust relative to historical data and produces plausible

[1] In this context mortality risk is the risk that people live shorter than expected.

[2] Stochastic variation in death rates of individuals within cohorts, i.e. individual uncertainty about the time of death, is excluded. We assume that cohorts are large enough so that micro-longevity risk is fully diversified.

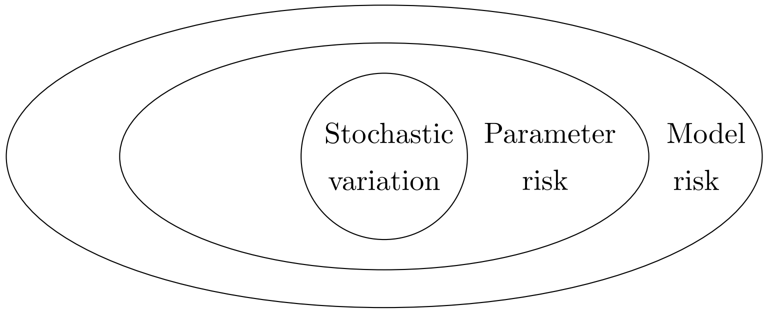

Figure 1: Sources of macro-longevity risk.

forecasts.[3] Several academics use the Lee-Carter model to model macro-longevity risk, for example Hari et al. (2008), Cocco and Gomes (2012), Stevens (2017) and De Waegenaere et al. (2017). Moreover, the model is the basis of several mortality table forecasts in practice.[4]

We discuss the Lee-Carter model in Subsection 2.1 and elaborate on macro-longevity risk in the Lee-Carter model in Subsection 2.2. In Subsection 4 we discuss different retirement age policies.

2.1 Lee-Carter model

The central death rate $\mu_{x,t}$ for a cohort of age x in year t equals

$$\mu_{x,t} = \frac{D_{x,t}}{E_{x,t}}, \tag{1}$$

where $D_{x,t}$ is the number of deaths in year t among the people in the cohort of age x and $E_{x,t}$ is the number of people in the cohort of age x in year t.

The Lee-Carter model estimates the log central death rates with the following expression[5]

$$\ln(\mu_{x,t}) = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t}, \tag{2}$$

where α_x is an age-specific constant, κ_t is a time trend and β_x represents the sensitivity of the log central death rates to the time trend. The time trend reflects the development of death rates over time. This trend is generally downward implying an increasing life expectancy over

[3] Alternative stochastic mortality models are for example the model of Renshaw and Haberman (2006) that is an extension of the Lee-Carter model including a cohort effect and the two-factor model of Cairns et al. (2006b).

[4] For example the U.S. Census Bureau and the U.S. Social Security Administration. The Actuarial Society in the Netherlands ('Koninklijk Actuarieel Genootschap') uses an alternative specification of this model.

[5] The logarithm of $\mu_{x,t}$ ensures that death rates cannot be negative. However, death rates can exceed unity but this is not a problem in practice. This can be avoided by modeling $\ln(\mu_{x,t}/(1-\mu_{x,t}))$, but in that case a linear trend in k does not imply a constant geometric rate of decline for each age-specific death rate (Lee (2000)).

time. The error term $\epsilon_{x,t}$ is normally distributed with mean zero and age-dependent variance $\sigma^2_{\epsilon,x}$.

The Lee Carter model assumes that the central death rates are constant during a year, i.e. $\mu_{x+s,t+s} = \mu_{x,t}(0 \le s \le 1)$. Therefore, we can approximate the one-year death probability $q_{x,t}$ in the following way

$$q_{x,t} \approx 1 - \exp(-\mu_{x,t}). \tag{3}$$

The one-year death probability is the probability that an individual of age x and alive at the beginning of year t dies before year $t+1$. The one-year survival probability $p_{x,t}$ equals

$$p_{x,t} = 1 - q_{x,t} \approx \exp(-\mu_{x,t}). \tag{4}$$

One-year survival probabilities can be used to calculate the probability that an individual of age x in year t is still alive after i years. This is called the cumulative survival probability $cp^t_{x,i}$

$$cp^t_{x,i} = \prod_{j=0}^{i-1} p_{x+j,t+j}. \tag{5}$$

The Lee-Carter model forecasts survival probabilities by estimating the time trend κ_t in (2) with a standard univariate time series model. Lee and Carter (1992) conclude after testing several $ARIMA$ specifications that the $ARIMA(0,1,0)$ model, a random walk with drift, is most appropriate to fit the data. This model equals

$$\kappa_t = c + \kappa_{t-1} + \eta_t, \tag{6}$$

where c is the drift and η_t is the error term that is normally distributed with mean zero and variance σ^2_η. The Lee-Carter model assumes that the error terms $\epsilon_{x,t}$ in (2) and η_t in (6) are independent. This independency implies that for each cohort mortality develops at an own age-specific exponential rate.

Calibration of the Lee-Carter model

In this paper we use mortality data of Dutch females from 1985 until 2014 from the Human Mortality Database to calibrate the parameters of the Lee-Carter model.[6,7] The central death rates $\mu_{x,t}$ are calculated using the number of deaths $D_{x,t}$ and number of people $E_{x,t}$ as in (1).

For very high ages no death rates are available in the database. When excluding the death rates beyond the age of 90 the expected remaining lifetime will be underestimated. We apply

[6] Human Mortality Database (HMD). University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany): http://www.mortality.org/.

[7] A calibration period of 30 years is conventional. For statistical reliability, one would prefer a longer calibration period (HMD). However, a shorter calibration period leads to a better estimate of the current trend in mortality improvements.

the method of Kannisto (1994) to extrapolate the central death rates for ages $x \in \{91, ..., 110\}$ using the death rates of younger cohorts. This method uses a logistic regression based on $\mu_{x,t}$ for ages $x \in \{80, 81, ..., 90\}$. Death rates above age $x = 110$ are assumed to be equal to the death rates at age $x = 110$.

We estimate parameters α_x, β_x and κ_t in (2) using a singular value decomposition. However, this method does not produce uniquely identified parameters. Therefore, we impose restrictions to identify the model. We use the standard identification choice of Lee and Carter (1992) that imposes the following constraints

$$\sum_{x=0}^{110} \beta_x = 1$$
$$\sum_{t=1985}^{2014} \kappa_t = 0.$$

The age-specific constant α_x is the average log central death rate of cohort of age x over time, i.e. $\alpha_x = \frac{1}{30} \sum_{t=1985}^{2014} \ln(\mu_{x,t})$. Subsequently the drift c and variance σ_η^2 in (6) are estimated using the κ_t's.

Figure 2 displays the estimates of the three key parameters in the Lee-Carter model in (2) using mortality data of Dutch females from 1985 until 2014. The top graph shows that the age-specific constant increases with age x. This implies higher death rates at higher ages. This is intuitive as older people have a higher change of dying. The middle graph shows that the sensitivity of death rates to the time trend in general decreases with age but in a non-monotonic way. A decreasing sensitivity implies that death rates for high ages are less effected by the time trend compared to death rates for young ages. The bottom graph shows that the time trend k_t decreases over time. This implies that death rates decrease over time. It is result of for example medical innovations and better nutrition. The estimated drift equals $\hat{c} = -1.3$. Each year the time trend κ_t decreases with 1.3 in expectation. The graph also contains the expected future time trend including the 90% confidence interval that is a result of the stochastic variation in the time trend.

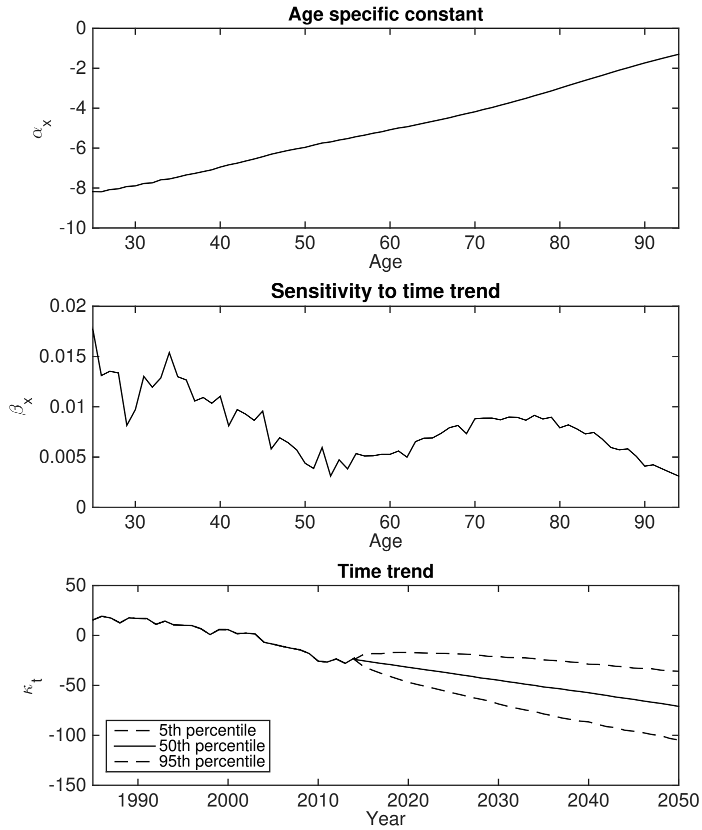

Figure 2: Parameter estimates of the Lee-Carter model using mortality data of Dutch females from 1985 until 2014. The top graph shows the age-specific constant α_x, the middle graph shows the sensitivity of death rates to the time trend β_x and the bottom graph shows the time trend κ_t. The bottom graph also contains the expected future time trend including a 90% confidence interval.

2.2 Macro-longevity risk in Lee-Carter model

As already mentioned at the start of Section 2 the main source of macro-longevity risk in this paper is stochastic variation. Macro-longevity risk in the Lee-Carter model arises from two random variables:

- *Uncertainty in time trend*: random shock η_t in the time trend κ_t in (6). It reflects the uncertainty in the time trend, i.e. development of death rates over time. The impact of this shock on future death rates depends on the size of σ_η and β_x.
- *Uncertainty in death rates*: random shock $\epsilon_{x,t}$ in the log central death rate $\mu_{x,t}$ in (2). It reflects particular age-specific historical influences not captured by the model. The impact of this shock on future death rates depends on the size of $\sigma_{\epsilon,x}$.

We model the first source of macro-longevity risk, stochastic variation, as the aggregate effect of those two random variables. We assume that η_t and $\epsilon_{x,t}$ are independent and normally distributed. The sum of two independent normal random variables is again normally distributed

$$\left.\begin{array}{lll}\eta_t & \sim & N(0,\sigma_\eta^2)\\ \epsilon_{x,t} & \sim & N(0,\sigma_{\epsilon,x}^2)\end{array}\right\} \quad \Rightarrow \quad \beta_x\eta_t+\epsilon_{x,t}\sim N(0,\beta_x^2\sigma_\eta^2+\sigma_{\epsilon,x}^2). \tag{7}$$

The trend risk η_t is multiplied with the sensitivity of to the time trend β_x because the sensitivity parameter β_x determines the impact of the time trend on death rates. Macro-longevity risk has zero mean because it is the risk that future mortality rates deviate from the best estimate mortality rates.

In this research we do not consider yearly macro-longevity shocks but consider macro-longevity risk on a 10-year horizon because a pension contract has a long horizon and we want to focus on structural changes in life expectancy only. We determine macro-longevity shocks on a 10-year horizon by summing up the independent normal random variables in (7) over 10 years

$$\sum_{i=0}^{9}(\beta_{x+i}\eta_{t+i}+\epsilon_{x+i,t+i})\sim N\left(0,\sigma_\eta^2\sum_{i=0}^{9}\beta_{x+i}^2+\sum_{i=0}^{9}\sigma_{\epsilon,x+i}^2\right). \tag{8}$$

The second source of risk is parameter risk. We calibrate the parameters in the mortality model using mortality data. When more recent mortality data are available we can recalibrate the parameters. Recalibration changes the parameter estimates (Cairns (2013)). In this paper we include recalibration risk. We use the realized death rates $\mu_{x,t}$ including the trend shocks η_t and estimation shocks $\epsilon_{x,t}$ to recalibrate the parameters in (2) and (6). Subsequently, we use these recalibrated parameters to forecast future death rates. By considering recalibration risk we include the influence of parameter risk.[8]

The third source of macro-longevity risk is model risk. We initially exclude model risk in our analysis and address this separately in Subsection 5.1.3.

Figure 3 visualizes the impact of macro-longevity risk measured on a 10-year horizon in the Lee-Carter model on the expected remaining lifetime (top graphs) and the value of a (deferred) variable annuity (bottom graphs) by displaying different percentiles of the distribution.[9] Besides the absolute impact on the expected remaining lifetime and the value of a (deferred) annuity (lefthand graphs), it is also interesting to look at the relative change of these variables (righthand graphs). We assume that the interest rate - used to determine the value of a (deferred) annuity - equals $r = 2\%$ and the retirement age equals $R = 67$. One can also make the retirement age contingent on life expectancy which is the case in several countries. This will be discussed in the next section.

[8] A more formal way to include parameter risk is to use standard Bayesian methods (Cairns et al. (2006b)).

[9] Negative (positive) macro-longevity shocks, i.e. negative (positive) random shocks in log central death rates, have a positive (negative) impact on life expectancy and annuity values. To avoid confusion we denote negative (positive) macro-longevity shocks by unexpected increases (decreases) in life expectancy.

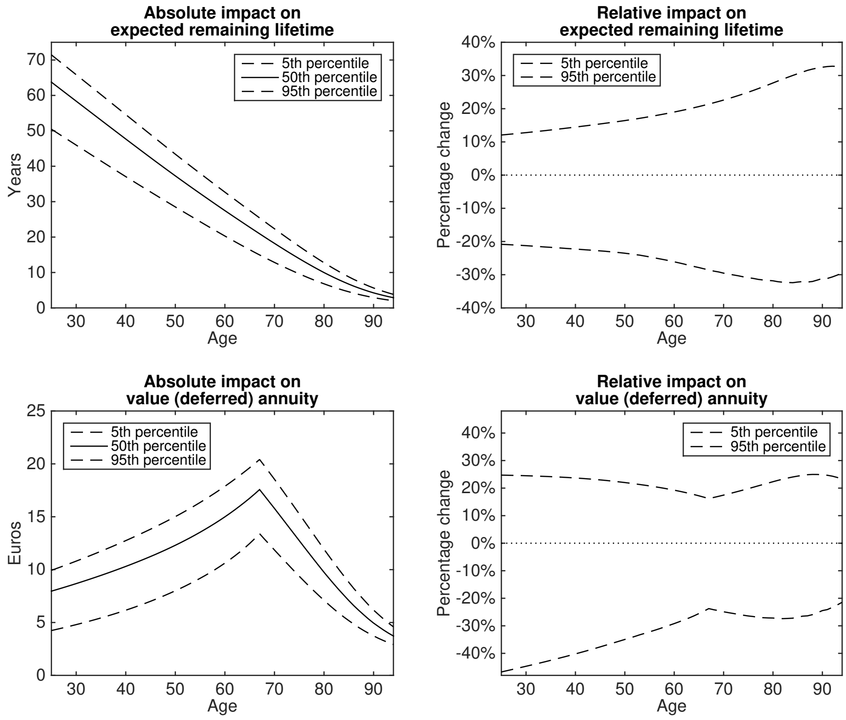

Figure 3: Impact of macro-longevity risk measured on a 10-year horizon in the Lee-Carter model on the expected remaining lifetime and the value of a (deferred) variable annuity for a Dutch female in 2014 in absolute terms (lefthand graphs) and relative change (righthand graphs) assuming a constant interest rate $r = 2\%$ and fixed retirement age $R = 67$.

The top lefthand graph shows that the expected remaining lifetime decreases with age. E.g. at age 25 it is 64 years and 11 years at age 80. This decrease is intuitive as older people have a higher chance of dying. Moreover, we notice that the impact of macro-longevity risk also decreases with age. E.g. the difference between the 5th and 95th percentile at age 25 is 21 years and 6 years at age 80. There are two reasons for this decreasing impact. *First*, a longevity shock has an impact on all future death probabilities. The expected remaining lifetime of young cohorts depends on more future death probabilities compared to the expected remaining lifetime of old cohorts. *Second*, the impact of both trend and estimation risk decreases with age. The sensitivity of the death rates β_x decreases with age implying a decreasing impact of the trend risk. The variance of the estimation risk $\sigma^2_{\epsilon,x}$ generally decreases with age as there is less uncertainty at higher death rates. This implies a decreasing impact of estimation risk.

The value of a deferred annuity (bottom lefthand graph) increases before retirement because of two reasons:

- The probability that a participant reaches the retirement age increases with age.
- The value of a deferred annuity is lower for young cohorts compared to cohorts just

before retirement because of a larger discounting effect.

The relative change of the value of a (deferred) annuity as a result of a macro-longevity shock is in the same order of magnitude for all age cohorts. Later in this paper we will see that this explains the small welfare gains in case of collective risk sharing when the retirement age is fixed.

Another important observation in Figure 3 is that the impact of macro-longevity risk on the expected remaining lifetime and (deferred) annuity value is asymmetric. Unexpected increases in life expectancy have a smaller impact than unexpected decreases in life expectancy. This can be explained by the exponential distribution of death rates. A consequence of this asymmetry is that the expectations of future survival probabilities and therefore also the expected remaining lifetime and expected (deferred) annuity value are smaller than its forecasted values. We present a derivation in Appendix A.1.

3 Sharing macro-longevity risk

The previous section discusses the modelling of macro-longevity risk. In this section we consider the concept of collective risk sharing. Pension providers can create an internal market for macro-longevity risk. We refer to this as collective risk sharing. Collective risk sharing can be welfare enhancing because the risk is not traded on a liquid market and cohorts are affected differently by the risk.[10]

We discuss the concept of collective risk sharing in Subsection 3.1. We use a stylized two-agent model in Subsection 3.2, related to Gollier (2008), in which risk-sharing solutions can be derived analytically and which gives economic intuition. Subsequently, we present a full model that consists of many generations representing the population of a pension fund. In this model risk-sharing solutions cannot be derived analytically anymore.

3.1 Pareto-efficient risk sharing

We investigate collective risk sharing under the notion of Pareto efficiency. Pareto efficiency means that the utility of no agent can be improved without hurting the utility of any other agent. The literature describes two ways to evaluate Pareto-efficient risk sharing: using a social planer or looking for an equilibrium. In this paper we make use of a social planner who maximizes total welfare of all agents and thereto reallocates risk across agents. This method is used by, e.g., Gordon and Varian (1988), Gollier (2008), Cui et al. (2011) and Bovenberg and Mehlkopf (2014). We determine the Pareto-efficient risk-sharing rule and calculate the welfare gain of the Pareto improvement by maximizing the aggregate expected

[10] Collective risk sharing can also be welfare enhancing if the risk is traded with future cohorts. In this paper we abstract from this dimension.

utility of all generations. We use a utility-based fairness criteria that yields a unique risk-sharing solution within the set of Pareto-efficient solutions. It requires that all participants experience the same welfare gain from risk sharing. An alternative way to achieve Pareto-efficient risk-sharing is by looking for an equilibrium. In this approach the agents can trade in a fictitious market. This method is used by, e.g., Krueger and Kubler (2006), Ball and Mankiw (2007) and Gottardi and Kubler (2011). Collective risk sharing can be Pareto improving if the risk is not traded on a liquid market. Pareto-efficient risk-sharing is often applied in the context of pension scheme design and intergenerational risk sharing. Intergenerational risk sharing is welfare improving since there is no market for risk sharing with future generations. We focus on macro-longevity risk which is not traded on a liquid market.

3.2 Benefits of collective risk sharing: stylized two-agent model

We determine the welfare gains from ex-ante Pareto-efficient risk-sharing solutions. To understand how collective risk sharing can lead to welfare gains we first consider an overlapping-generations model (OLG) consisting of two agents and two periods that we solve analytically. This stylized modeling framework is an adjusted version of the two-agent model of Gollier (2008). The difference is the nature of the risk. The total exposure to macro-longevity risk in our model is exogenous for a pension fund while investment risk in the model of Gollier (2008) is endogenous because an investor can choose the equity exposure.

Figure 4 visualizes our model featuring two agents. Agent 1 consumes in period 1 and

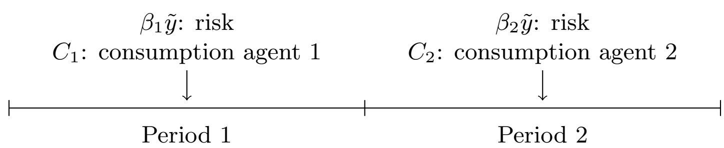

Figure 4: Stylized model in autarky.

agent 2 in period 2. Both agents are exposed to the same exogenous risk factor $\tilde{y}$ with mean zero and variance σ^2. Risk $\tilde{y}$ represents the unexpected component of a macro-longevity shock that affects both agents but in different ways. The variables β_1 and β_2 determine the exposure of each agent to $\tilde{y}$. We first consider consumption in autarky, i.e. without risk sharing. In autarky consumption consists of initial wealth and the exposure to the exogenous risk

$$\begin{aligned} C_1^a &= W_1 + \beta_1\tilde{y} \\ C_2^a &= W_2 + \beta_2\tilde{y}. \end{aligned} \tag{9}$$

The agents have identical preferences given by the following power utility function with risk aversion γ[11]

$$U(C_i) = \begin{cases} \dfrac{C_i^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \ln(C_i) & \text{if } \gamma = 1. \end{cases} \tag{10}$$

Risk sharing implies a (partial) transfer of the risk $t(\tilde{y})$ from agent 1 to agent 2 or vice versa. We restrict ourselves to the following linear risk-sharing rules

$$t(\tilde{y}) = t_0 + \eta\tilde{y}, \tag{11}$$

where the risk transfer η is the amount of the risk that agent 1 transfers to agent 2 and t_0 is a constant risk compensation that agent 1 pays up in front to agent 2 or vice versa.[12] In the rest of this paper we use the term risk-sharing rule to refer to $t(\tilde{y})$ and risk transfer rule to refer to η. The model does not allow non-linear functions such as put options. The model imposes that the risk transfer fraction η is the same for negative and positive macro-longevity shocks. In case agent 2 transfers risk to agent 1 η is negative. The risk transfer should satisfy $-\beta_2 \leq \eta \leq \beta_1$ as agents cannot transfer more than the entire risk. Consumption in case of risk sharing equals

$$\begin{aligned} C_1^s &= W_1 + \beta_1\tilde{y} - t(\tilde{y}) \\ C_2^s &= W_2 + \beta_2\tilde{y} + t(\tilde{y}). \end{aligned} \tag{12}$$

How much risk η should agent 1 optimally transfer to agent 2 and at what price? We solve this using a welfare analysis. A common measure of welfare is certainty equivalent consumption.[13] Risk sharing is Pareto improving if the welfare of at least one agent improves and all other agents do not become worse off. We derive the optimal risk-sharing solution by maximizing the expected utility of agent 1 under the condition that the expected utility of agent 2 does not decrease, or formally

$$\max_{\eta,t_0} \mathbb{E}[U(C_1^s)] \quad \text{such that} \quad \mathbb{E}[U(C_2^s)] \geq \mathbb{E}[U(C_2^a)]. \tag{13}$$

This yields the following optimal risk transfer η^* as shown in Appendix A.2

$$\eta^* = \frac{\beta_1 W_2 - \beta_2 W_1}{W_1 + W_2}. \tag{14}$$

[11] We justify the assumption that agents have the same risk aversion γ because collective risk sharing within a pension fund often occurs within a group of participants with similar characteristics such as education, salary, etc.

[12] Similar to Gollier (2008). In a linear risk-sharing rule the risk compensation t_0 can be interpreted as a risk premium for absorbing risk. One could also consider non-linear risk-sharing rules. However, these are more difficult to interpret.

[13] The certainty equivalent consumption is equal to the constant certain consumption level that yields the same ex-ante utility at retirement as the stochastic consumption.

The risk transfer η^* increases linear in β_1 and decreases linear in β_2.[14] In case $\beta_1 W_2 = \beta_2 W_1$, risk sharing is not welfare improving. We apply Arrow-Pratt approximations to find η^*. This solution can slightly deviate from the true optimal solution because of the remainder term in the Taylor series. We do not matter too much about this inaccuracy because we use the two-agent model to derive an analytical framework and get intuition but not for numerical precision.

The optimal risk transfer η^* can also be derived via alternative maximizations. One can also maximize aggregate equivalent variation or aggregate expected utility. These derivations are also presented in Appendix A.2.

The maximization does not deliver a unique solution for the risk compensation t_0. The risk compensation determines how the welfare gain from risk sharing is distributed among the agents. There is a range of values for t_0 that yield a Pareto improvement. However, not every value for t_0 yields a Pareto improvement. Requiring the risk-sharing solution to be Pareto improving in comparison to autarky ensures that the welfare gain can be fully attributed to gains from risk sharing and is not a result from ex-ante redistribution between agents. Under the condition that the risk-sharing solution is Pareto improving the risk compensation should lie between the following lower and upper bound

$$\frac{1}{2}\frac{\gamma}{W_1}\sigma^2\eta^*(2\beta_1 - \eta^*) \leq t_0 \leq \frac{1}{2}\frac{\gamma}{W_2}\sigma^2\eta^*(2\beta_2 + \eta^*), \tag{15}$$

that is shown in Appendix A.3. If $t_0 = \frac{1}{2}\frac{\gamma}{W_1}\sigma^2\eta^*(2\beta_1 - \eta^*)$, the full welfare gain from risk sharing goes to agent 1 and if $t_0 = \frac{1}{2}\frac{\gamma}{W_2}\sigma^2\eta^*(2\beta_2 + \eta^*)$, the full welfare gain from risk sharing goes to agent 2. There exist different fairness criteria that yield a unique risk-sharing solution within the set of Pareto-efficient solutions. We use a utility-based fairness criterion (comparable to Gollier (2008) and Bovenberg and Mehlkopf (2014))) which requires that all agents experience the same increase in certainty equivalent consumption as a result of risk sharing relative to autarky. Under the utility-based fairness criterion t_0 equals

$$t_0 = \frac{1}{4}\gamma\sigma^2\eta^*\left(\frac{1}{W_1}(2\beta_1 - \eta^*) + \frac{1}{W_2}(2\beta_2 + \eta^*)\right). \tag{16}$$

The derivation is in Appendix A.3.

An alternative fairness criterion is financial fairness which sets ex-ante market values of risk transfers between agents to zero (e.g. Teulings and De Vries (2006), Bovenberg and Mehlkopf (2014) and Bao et al. (2017)). This criterion implies t_0.

[14] This linearity results from the Arrow-Pratt approximations and because we assume a linear risk-sharing rules (11).

Numerical example

Suppose agent 1 is a retiree and agent 2 is a worker. Initial wealth of agent 1 equals $W_1 = 2$, the exposure of agent 1 to the risk is $\beta_1 = 1$, initial wealth of agent 2 equals $W_2 = 4$ and the exposure of agent 2 to the risk is $\beta_2 = 0.5$. The exposure to risk of agent 1 is thus higher than the exposure of agent 2. This exogenous risk $\tilde{y}$ has mean $\mu = 0$ and variance $\sigma = 0.3$. We assume that all agents have identical preferences given by the power utility function with risk aversion $\gamma = 5$. The welfare gain from risk sharing depends on the risk transfer η. Figure 5 (lefthand figure) visualizes this.

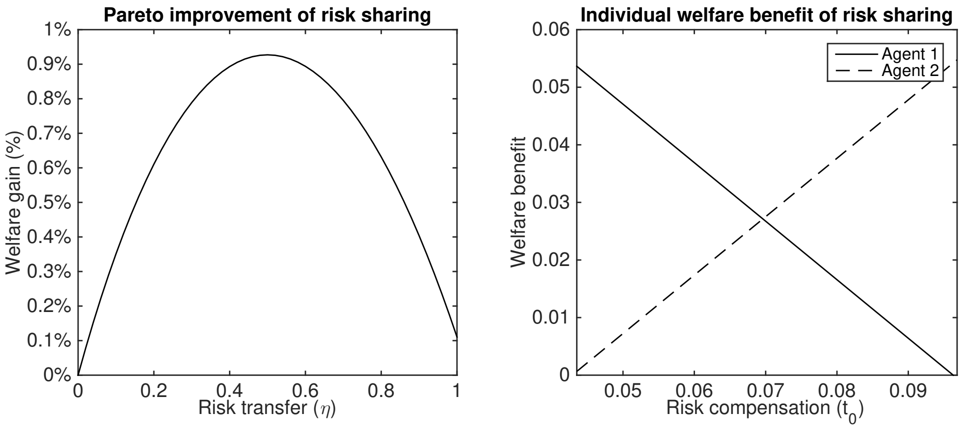

Figure 5: Welfare gain in terms of relative increase in aggregate certainty equivalent consumption (lefthand figure) and welfare gain in terms of absolute increase individual certainty equivalent consumption (righthand figure).

Maximizing (13) yields the optimal risk transfer $\eta^* = 0.5$. This risk transfer is only Pareto improving if at least one agent benefits from risk sharing and the other agent does not become worse off. This requires that the risk compensation, that is paid by agent 1 to agent 2, should satisfy $0.04 \leq t_0 \leq 0.1$. Figure 5 (righthand graph) visualizes the individual welfare gain for both agents, in terms of increase in individual certainty equivalent consumption. In case $t_0 = 0.07$ both agents benefit equally from risk sharing in terms of the increase in certainty equivalent consumption as in (16).

3.3 Collective risk sharing of macro-longevity risk: full model

We extend the stylized two-agent model of Subsection 3.3 to a full model with many cohorts representing the population of a pension fund. In the full model macro-longevity risk impacts survival probabilities and therefore also retirement consumption in a non-linear way based on the Lee-Carter model.

The full model is an OLG model consisting of $N = 70$ cohorts. Cohort 1 is aged 25 and cohort 70 is aged 94.[15] We base the number of participants n_i in cohort i on the cumulative probability that a participant is still alive at age $i + 24$. So old cohorts consist of less participants compared to young cohorts. The lefthand graph in Figure 6 visualizes this population composition. Participants have identical preferences given by a power utility function with risk aversion $\gamma = 5$. The total wealth W_i of a participant in cohort i depends on his or her age. The righthand graph in Figure 6 visualizes the development of wealth over the life-cycle of a participant. Wealth increases during the working period as the participant contributes to the pension fund. Wealth at the start of the working period is positive because wealth consists of financial wealth and human wealth (i.e. future pension contributions).[16] Furthermore, we initially assume that the participants retire at age $R = 67$, the interest rate - used to determine the value a (deferred) annuity - equals $r = 2\%$.

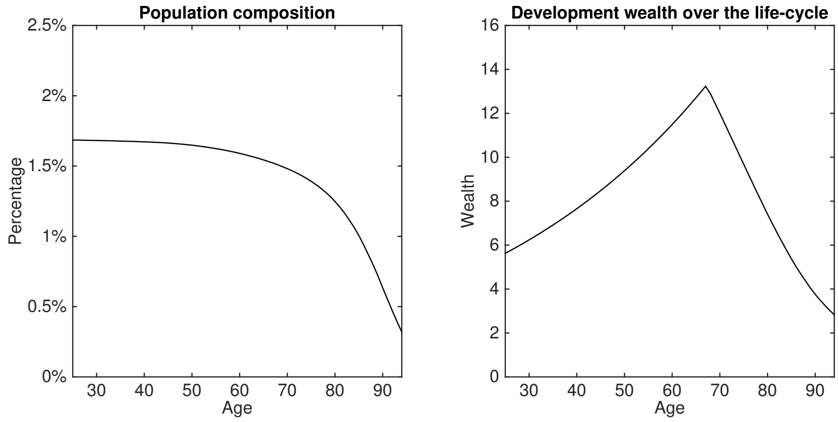

Figure 6: Population composition (lefthand graph) and development of wealth over the life-cycle of a participant (righthand graph).

We consider a DC pension scheme. Consumption after retirement depends on the value of an annuity. We assume that the participant buys a variable annuity which value varies with future survival probabilities.[17] If for example life expectancy increases the annuity value increases. This has a negative effect on consumption after retirement and implies that macro-longevity risk is borne by the participant. The value of a (deferred) variable annuity a_x^t, that pays 1 dollar annually during retirement, for an individual of age x in year t is calculated as follows

$$a_x^t = \sum_{j=\max(x,R)}^{M} \frac{1}{(1+r)^{j-x}} cp_{x,j-x}^t. \tag{17}$$

[15] We exclude cohorts older than age 94 because the number of participants in these cohorts is very small and therefore do not influence the results significantly.

[16] Human wealth is equal to the present value of future pension contributions and not the present value of future labor income because the pension contributions are fixed in our model.

[17] Variable annuities can also vary with realized investment returns. Because we exclude investment risk, this is not the case in this paper.

In this formula R equals the retirement age, M is the maximum age an individual can reach and $cp_{x,i}^t$ is the probability of still being alive after i years as in (5). We assume a constant interest rate r.

For ease of reference we denote the value of a (deferred) annuity in (17) by $a_x^t = a_i$. A macro-longevity shock impacts future survival probabilities which influence the value of a (deferred) annuity as stated in (17). The value of a (deferred) annuity changes for cohort i from a_i to $\tilde{a}_i$ due to a macro-longevity shock in the Lee-Carter model. The expected annual consumption after retirement in autarky C_i^a for cohort i after a shock is given by

$$C_i^a = \frac{W_i}{\tilde{a}_i}. \tag{18}$$

To determine the impact of macro-longevity risk on consumption we calculate for each cohort how much money is needed (or is left) to fully compensate the impact of a macro-longevity shock.[18] We denote this by $\tilde{y}_i$

$$\begin{aligned}\frac{W_i}{a_i} &= \frac{W_i + \tilde{y}_i}{\tilde{a}_i} \\ \tilde{y}_i &= W_i\left(\frac{\tilde{a}_i}{a_i} - 1\right).\end{aligned} \tag{19}$$

$\tilde{y}_i$ represents the amount of money to offset the effect of a macro-longevity shock on consumption in autarky. If the annuity value increases (decreases) due to an unexpected increase (decrease) in life expectancy, $\tilde{y}_i$ is positive (negative) and money is needed (left). $\tilde{y}_i$ is not the same for each cohort i because the impact of a macro-longevity shock on future death rates depends on age. We can calculate the total money needed (or left) to fully compensate the impact of a macro-longevity shock for all N cohorts. We denote this by $\widetilde{y}_T$

$$\widetilde{y}_T = \sum_{i=1}^{N} n_i \tilde{y}_i, \tag{20}$$

where n_i is the number of participants in cohort i. In this paper macro-longevity risk is shared by distributing the total macro-longevity shock $\tilde{y}_T$ among cohorts. Similar to the stylized two-agent model we restrict ourselves to linear risk sharing rules. Each participant absorbs part of the total macro-longevity shock η_i and receives (or pays) a risk compensation $t_{0,i}$. Consumption after risk sharing equals

$$C_i^s = \frac{W_i + \tilde{y}_i - \eta_i \widetilde{y}_T - t_{0,i}}{\tilde{a}_i}. \tag{21}$$

We determine the optimal risk-sharing solution, similar to proof 2 in Appendix A.2, by maximizing aggregate equivalent variation

$$\max_{\substack{\eta_1,\eta_2,\dots\eta_N \\ t_{0,1},t_{0,2},\dots t_{0,N}}} \sum_{i=1}^{N} n_i EQV_i = \sum_{i=1}^{N} n_i \left(\mathbb{E}[U(C_i^s)] - \mathbb{E}[U(C_i^a)]\right), \tag{22}$$

[18] In this paper we assume that consumption before retirement is fixed, i.e. a macro-longevity shock can only be absorbed by changing consumption after retirement. In case a participant can also change consumption before retirement, the impact of a macro-longevity shock on the consumption level after retirement will be smaller for workers.

where the following restrictions should be satisfied

$$\sum_{i=1}^{N} n_i \eta_i = 1 \tag{23a}$$

$$\sum_{i=1}^{N} n_i t_{0,i} = 0. \tag{23b}$$

We do not make use of any Taylor expansions in the full model because we determine the optimal solution numerically. The first restriction makes sure that the macro-longevity shock is fully distributed among all participants. The second restriction guarantees that the total risk compensation that participants receive is paid by the other participants. The direction and size of the wealth transfer $\tilde{y}_i - \eta_i \widetilde{y}_T - t_{0,i}$ for cohort i depends on the direction and size of the macro-longevity shock, the wealth of the participants and the population composition. Similar to the stylized model we use the utility-based fairness criterion to determine the risk compensations $t_{0,i}$.

4 Retirement age policies

The significant increase in life expectancy during the last decades had a major impact on the sustainability of pension systems. As a response several countries are linking the state pension age to life expectancy developments. In the United Kingdom for example the government plans to link the state pension age at future dates to the projected longevity of the population in such a way that people receive state pension during a fixed proportion of adult life (Hammond et al. (2016)). Under this policy both the working and retirement period increase if life expectancy increases. In the Netherlands the retirement age is linked to life expectancy in a different way. The Dutch government implemented a law that links the retirement age to the remaining life expectancy of the population at age 65. Under this policy the absolute length of the retirement period is fixed and independent of life expectancy while the working period increases if life expectancy increases.

In this paper we focus on occupational pension schemes. The retirement age in occupational pension schemes is often equal to the state pension age. As a consequence, the retirement age policy of the government also impacts the retirement age in occupational pension schemes and thus the ability to share macro-longevity risk in occupational pension schemes. We consider three policies:

1. **Fixed retirement age** (FRA): the retirement age is fixed, i.e. the retirement age does not change after macro-longevity shocks. In this policy the length of the working period is constant. This policy supports the belief that if people live longer, they extent their retirement period. In most countries, for example in the United States and Australia, the retirement age is not linked to life expectancy.

2. **Partial adjustment of the retirement age** (PARA): the retirement age automatically adjusts to life expectancy developments in a such a way that retirement consumption remains the same.[19] This means, e.g., that if life expectancy increases (decreases) with 12 months, the retirement age should increase (decrease) with roughly 9 months.[20] In this policy consumption after retirement is constant. The adjustment only holds for working participants, since retirees cannot adjust their retirement age anymore. This policy is close to the retirement age policy in the United Kingdom.[21]

3. **Full adjustment of the retirement age** (FARA): the retirement age automatically keeps up fully with life expectancy changes. This means, e.g., that if the remaining life expectancy at retirement increases (decreases) with 12 months, the retirement age also increases (decreases) with 12 months. In this policy the length of the retirement period is constant. The adjustment holds for working participants only, since retirees cannot adjust their retirement age anymore. This policy supports the belief that if people live longer, they increase their labor supply by extending their working period. This policy is similar to the retirement age policy in the Netherlands.[22]

Stevens (2017) investigates the effect of different retirement age policies on the distribution of the (forecasted) retirement age. He concludes that if the retirement age is linked to life expectancy macro-longevity risk is effectively hedged. However, such a policy also leads to substantial uncertainty in the retirement age and length of the retirement period.

	Working period	Retirement period	Retirement consumption	Value annuity	Wealth at retirement
FRA	constant	++	-	++	constant
PARA	+	+	constant	+	+
FARA	++	constant	+	-	++

Table 1: Impact of an unexpected increase in life expectancy on several variables for working participants in case of different retirement age policies.

[19] There are also countries in which the retirement age is not automatically linked to life expectancy but the government decides to increase the retirement age based on life expectancy improvements incidentally. We do not investigate such a policy.

[20] The exact increase (decrease) does not only depend on the size of the longevity shock but also on the impact of the longevity shock on survival probabilities at different ages and the life expectancy before the longevity shock.

[21] The retirement age adjustment in the UK proposal depends on the proportion of adult life that people receive state pension.

[22] The Dutch law states that the retirement age R is only adjusted in case the remaining life expectancy at age 65 increases but it remains the same if it decreases. In this paper we assume a symmetric rule, i.e. the retirement age is adjusted in case of both positive and negative shocks.

Table 1 presents the impact of an unexpected increase in life expectancy on several variables for the three retirement age policies. Consumption after retirement is determined by the value of a (deferred) annuity and accumulated wealth at retirement (see (18)). The righthand graph in Figure 6 shows the development of wealth over the life-cycle in case of a fixed retirement age. If the retirement age is linked to life expectancy the development of wealth over the life-cycle is different because the participant accrues more (less) wealth by paying pension premia for a longer (shorter) period.[23] The table presents the impact for working participants only because retirees cannot adjust their retirement age as response to longevity shocks. In case of an unexpected decrease in life expectancy, the signs in Table 1 revert.

In case of a fixed retirement age the length of the working period is constant. As a result the (expected) length of the retirement period increases in case of an unexpected increase in life expectancy. The annuity value increases as a result of higher survival probabilities. Wealth at retirement remains the same. As a result retirement consumption will decrease.

In case of a partial adjustment of the retirement age both the working and retirement period are extended. The annuity value increases as a result of higher survival probabilities. The wealth at retirement also increases because the participant will work longer. The annuity value and wealth at retirement increase such that consumption after retirement remains the same.

If the retirement age is fully adjusted the length of the retirement period is constant. The (expected) length of the working period increases in case of an unexpected increase in life expectancy. The annuity value is lower than before the longevity shock. Higher survival probabilities have a positive impact on the annuity value, but later retirement has a negative impact on the annuity value. It turns out that the latter effect outweighs. The wealth at retirement increases because the participant will work longer. As a result, retirement consumption will increase.

We use exogenous rules in the retirement age policies. An alternative is an endogenous retirement age. The participant optimizes his retirement age based on realized life expectancy improvements. In that case it is necessary to include leisure time besides consumption in the utility function to take the labor-leisure trade-off into account. Otherwise a high retirement age would always be optimal because a shorter retirement period implies a higher consumption after retirement. Cocco and Gomes (2012) investigate the impact of macro-longevity risk on the optimal saving and retirement decision in an individual life-cycle model. They conclude that individuals decide to retire later even if this entails a utility cost in terms of

[23] We assume that the labor market functions perfectly so participants do not experience any difficulties with staying employed.

foregone utility of (additional) leisure. Although we do not explicitly model the labor-leisure trade-off in this paper, the retirement age policies represent different preferences regarding consumption and leisure. In case of a fixed retirement age, a life expectancy increase implies a lengthening of the retirement period (leisure) at the expense of the consumption level. In case of a partial adjusted of the retirement age both consumption and leisure (relative to labor) remain approximately equal. A full adjustment of the retirement age implies a higher consumption level at the expense of leisure.

5 Results

In this research we quantify the welfare gains from collective risk sharing in terms of aggregate certainty equivalent consumption after retirement. Table 2 presents the aggregate welfare gains for the three retirement age policies in Section 4.

Fixed retirement age (FRA)	0.3%
Partial adjustment retirement age (PARA)	0.5%
Full adjustment retirement age (FARA)	2.7%

Table 2: Welfare gains in terms of aggregate certainty equivalent consumption after retirement from sharing macro-longevity risk measured on a 10-year horizon.

We observe that for each retirement age policy collective risk sharing of macro-longevity risk is welfare improving compared to autarky. The design of the retirement age policy impacts the welfare gains from sharing macro-longevity risk. In case of a fixed retirement age, the welfare gain equals 0.3 percent. This relatively small welfare gain is a result of the fact that in this policy the impact of macro-longevity risk on retirement consumption for different cohorts is more or less equal (Figure 3). As a result, the welfare gain from risk sharing is limited. In case the retirement age is partially adjusted the welfare gain from risk sharing is higher. This is a result of the fact that the expected retirement consumption of workers is not affected by macro-longevity shocks. In case of a full adjustment of the retirement age the aggregate welfare gain increases significantly. This is a result of the large risk bearing capacity of workers. They adjust their labor supply as a hedge against macro-longevity shocks. This increases the risk appetite of the workers to provide insurance to retirees.

In this research we measure the welfare gains of sharing macro-longevity risk and not the welfare gains of different retirement age policies since the retirement age policy is given for both autarky and risk sharing. We do not focus on the suitability of retirement age policies. This is a different research question and requires the inclusion of leisure time besides consumption in the utility function to take the labor-leisure trade-off into account.

Figure 7 (lefthand graph) visualizes the optimal risk transfer relative to autarky for a participant in cohort i as a percentage of total risk. A positive risk transfer for cohort i means that participants in cohort i absorb risk of other cohorts. A negative risk transfer means that the own exposure to macro-longevity risk is (partly) transferred to other cohorts.[24] In case of a fixed retirement age the risk transfer increases with age for the workers until retirement and decreases with age for retirees. Macro-longevity risk of the young workers and old retirees is (partly) absorbed by the other cohorts. The development of wealth over the life-cycle (righthand graph in Figure 6) primarily explains this shape. Cohorts who have relatively more wealth can absorb more risk. The risk transfer rule in case of a fixed retirement age significantly differs from the risk transfer rule in case the retirement age is adjusted to macro-longevity shocks. The risk transfer rule in case the retirement age is partially adjusted is very similar to the risk transfer rule in case the retirement age is fully adjusted. The workers absorb risk and the retirees transfer risk. This makes sense because the workers adjust their labor supply to macro-longevity shocks. As a result, they are able to absorb risk of the retirees.

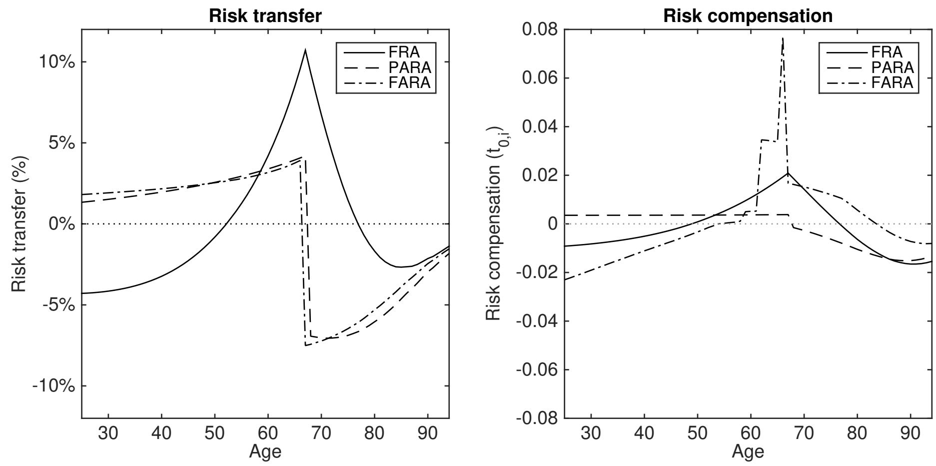

Figure 7: Optimal risk transfer relative to autarky for all cohorts as percentage of total risk (lefthand graph) and corresponding risk compensation (righthand graph) in case of sharing macro-longevity risk measured on a 10-year horizon. A positive (negative) risk transfer for cohort i means that participants in cohort i absorb risk of (transfer risk to) other cohorts. A positive (negative) risk compensation for cohort i means that participants receive (pay) a risk compensation.

The righthand graph in Figure 7 displays the risk compensation $t_{0,i}$ corresponding to the optimal risk transfer for a participant in cohort i under the utility-based fairness criterion

[24] The sum of risk transfers in the graph is not exactly equal to zero because each cohort does not consist of an equal number of participants.

(lefthand graph).[25] A positive risk compensation for cohort i means that participants receive a risk compensation. A negative risk compensation for cohort i means that participants pay a risk compensation. In general, cohorts who absorb risk from other cohorts receive a risk compensation and cohorts who transfer risk have to pay a risk compensation. However, this does not hold if the retirement age is fully adjusted. Young cohorts absorb risk from other cohorts but do not receive a risk premium; the risk premium is even negative. Under this policy workers adjust their labor supply as a hedge against macro-longevity shocks. This implies a reverse effect of macro-longevity shocks for workers and retirees (Table 1). As a result, a positive risk compensation is not required for young cohorts to absorb risk of retirees. A final observation is the peak in the risk compensation around age 66 in case of a fully adjusted retirement age. This peak is due to the fact that cohorts just before retirement cannot fully adjust their retirement age in case of an unexpected decrease in life expectancy, i.e. the retirement age cannot be lower than their current age. As a result, the certainty equivalent consumption of these cohorts is relatively high in autarky so risk sharing is less welfare improving for these cohorts. Therefore, these cohorts require a higher risk compensation.

We consider macro-longevity risk on a 10-year horizon. The welfare gains from sharing macro-longevity risk over the whole life-cycle are most likely higher. Another sidenote is that this paper applies a first-best risk-sharing solution as its benchmark for evaluating welfare effects. In practice, however, the first-best risk-sharing solution may not always be feasible. Policy makers might want to limit the maximum risk a participant can absorb to prevent very large wealth transfers in case of extreme macro-longevity shocks.

5.1 Sensitivity analyses

In this section we verify whether the welfare gains and risk-sharing rules are sensitive to mortality data and model assumptions by performing three types of sensitivity analyses:

1. **Alternative mortality data**: macro-longevity risk in the Lee-Carter model depends on the parameters in (2) and (6) that are calibrated using historical mortality data. We investigate the impact of alternative mortality data on the welfare gains from risk sharing and corresponding risk-sharing rule.
2. **Alternative population compositions**: the welfare gains from sharing macro-longevity risk also depend on the population composition. We will investigate the impact of alternative population compositions on the welfare gains from risk sharing and corresponding risk-sharing rule.
3. **Alternative model macro-longevity risk**: instead of macro-longevity risk in the Lee-Carter model we assess the impact of alternative shocks in death rates on the welfare

[25] The sum of risk compensations in the graph is not exactly equal to zero because each cohort does not consist of an equal number of participants.

gains from risk sharing and corresponding risk-sharing rule.

5.1.1 Alternative mortality data

Macro-longevity risk in the Lee-Carter model depends on the parameters in (2) and (6). In our main analysis we calibrate the parameters using historical mortality data of Dutch females. Using alternative mortality data changes the parameters and therefore also the size and distribution of macro-longevity shocks.

Mortality data	**Dutch females**	**Dutch males**	**US females**	**US males**
Fixed retirement age (FRA)	0.3%	0.2%	0.2%	0.3%
Partial adjustment retirement age (PARA)	0.5%	0.5%	0.2%	0.3%
Full adjustment retirement age (FARA)	2.7%	1.9%	0.7%	1.5%

Table 3: Welfare gains in terms of aggregate certainty equivalent consumption after retirement from sharing macro-longevity risk measured on a 10-year horizon for alternative mortality data.

The welfare gains from risk sharing using alternative mortality data are presented in Table 3. We look at Dutch males, US females and US males. In case of a fixed retirement age or partial adjustment of the retirement age, the welfare gains do not change significantly. However, in case the retirement age is fully adjusted the welfare gains from risk sharing are lower compared to the mortality data of Dutch females. This especially holds for mortality data of US females. This lower welfare gain is caused primarily by lower volatility parameters in (7). A lower volatility implies smaller risk and therefore lower welfare gains from risk sharing.

Figure 8 visualizes the optimal risk transfer relative to autarky as percentage of total risk. The black lines represent the optimal risk transfer rule using the mortality data of Dutch females and the grey lines for the alternative mortality data. We can conclude that for each retirement age policy the optimal risk transfer rule is robust to the alternative mortality data we consider.

5.1.2 Alternative population compositions

We determined the welfare gains in Table 2 and risk transfers and risk compensations in Figure 7 for a population composition of an entire country (lefthand graph in Figure 6). In practice the population composition of a pension fund is generally not equal to this standard population composition. Therefore, it is interesting to also consider alternative population compositions: a population composition of a green and grey pension fund. We assume that

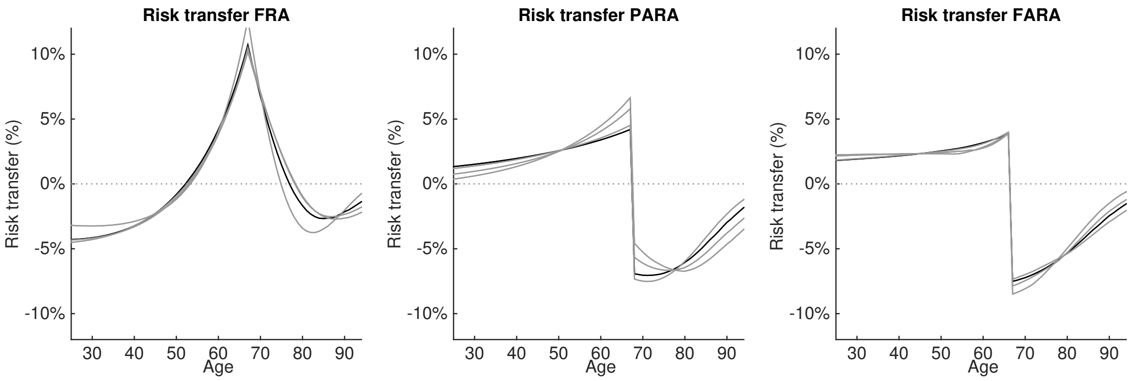

Figure 8: Optimal risk transfer relative to autarky as percentage of total risk in case of sharing macro-longevity risk measured on a 10-year horizon. The black lines represent the risk transfer rules based on Dutch females and the grey lines represent the risk transfer rules using alternative mortality data.

the green pension fund has a relatively young population. We approximate this by assuming that the number of participants in a cohort decreases with 1 percent per age year compared to the standard population composition. In the grey pension fund the number of participants in a cohort increases with 1 percent per age year compared to the standard population composition. The standard and alternative population compositions are displayed in Figure 9.

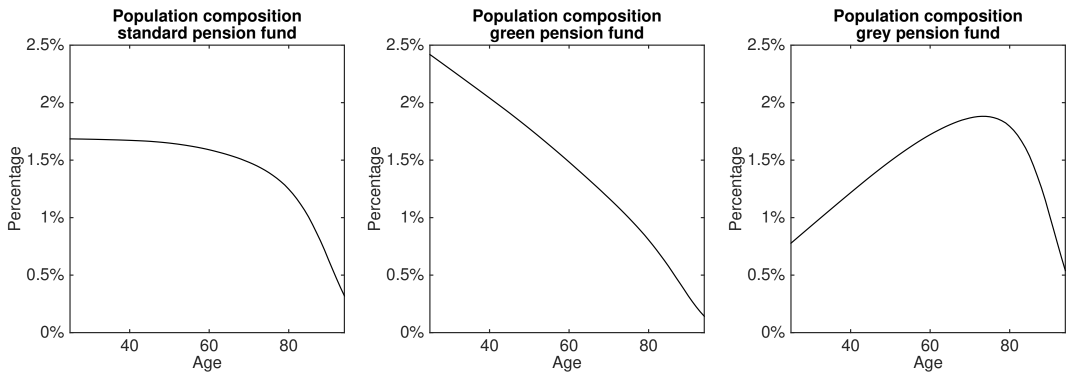

Figure 9: Different population compositions: a standard, green and grey pension fund.

Table 4 presents the welfare gains from risk sharing using alternative population compositions. The welfare gains are not significantly different from the welfare gains for the standard population composition, even if the retirement age is fully adjusted. Figure 10 visualizes the optimal risk transfer relative to autarky as percentage of total risk. The black lines represent the optimal risk transfer rules using the original population composition and the grey lines represent the risk transfer rules using alternative population compositions. The shape of the risk transfer rule is reasonably robust to the population composition but the percentage of total risk an individual participant absorbs or transfers can be different in case of alternative population compositions. A different population composition leads to a different ratio

Population composition	Standard	Green	Grey
Fixed retirement age (FRA)	0.3%	0.3%	0.3%
Partial adjustment retirement age (PARA)	0.5%	0.4%	0.7%
Full adjustment retirement age (FARA)	2.7%	2.2%	2.7%

Table 4: Welfare gains in terms of aggregate equivalent consumption after retirement from sharing macro-longevity risk measured on a 10-year horizon for alternative population compositions.

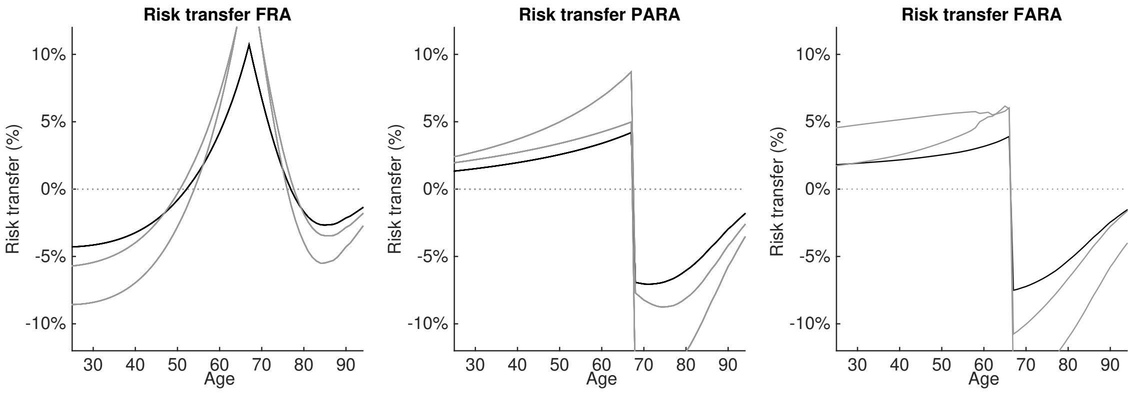

Figure 10: Optimal risk transfer relative to autarky for each cohort as percentage of total risk in case of sharing macro-longevity risk measured on a 10-year horizon. The black lines represent the original risk transfers and the grey lines represent the optimal risk transfers using alternative population compositions.

between the individual macro-longevity shock and total macro-longevity shock. This impacts the optimal risk transfer as percentage of total risk.

5.1.3 Alternative model macro-longevity risk

Several academics use the Lee-Carter model to model macro-longevity risk. Moreover, it is the basis of several mortality table forecasts in practice. However, the model is not a perfect representation of reality because there is uncertainty about structural breaks. For example, medical innovations can cause structural breaks that are not captured by the Lee-Carter model. Therefore it is interesting to also look at the impact of alternative shocks in the death rates.

There is no scientific consensus on the development of future survival probability at old ages. Buettner (2002) suggests that there are two alternative views about the future survival probability at old ages: compression versus expansion. In case of mortality compression mortality continues to decline over a widening range of adult ages, but meets natural limits for very advanced ages. This development implies that the survival probability approaches a rectangle

(Figure 11). Einmahl et al. (2017) and Dong et al. (2016) find evidence for the existence of a maximum age. In case of mortality expansion mortality continues to decline for all ages, i.e. there is no maximum age. Wilmoth (2000) and Oeppen and Vaupel (2002) argue that there is indeed no maximum age. Wilmoth (2000) states that, based on available demographic evidence, the human life span shows no sign of approaching a certain limit imposed by biology or other factors. There are even scientists who believe in the possible realization of longevity escape velocity. In this scenario death rates fall so fast that people's remaining life expectancy increases with time because therapies restore health faster than the rate of body deterioration due to biological ageing (De Grey (2004)).

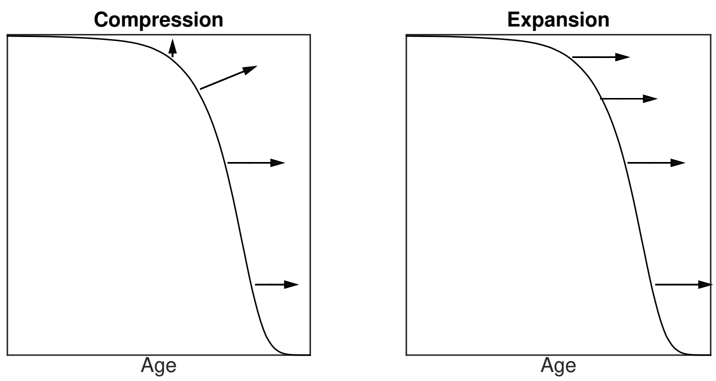

Figure 11: Different views of future survival probability: compression (lefthand graph) and expansion (righthand graph).

The development of future mortality in the Lee-Carter model is in line with the mortality compression view. The sensitivity of the death rates to the time trend decreases in age x to almost zero at very high ages. An alternative shock in death rates is the macro-longevity shock in the Solvency II framework for insurers. The Solvency II capital requirements for longevity risk are determined by applying a uniform shock, i.e. a 20 percent decrease, to all future death probabilities $q_{x,t}$.[26] For mortality risk the capital requirements are determined by applying an increase of 15 percent to all future death probabilities. The longevity shock in the Solvency II framework is in line with the expansion view because all death probabilities decrease at the same rate. Figure 12 visualizes both types of shocks, i.e. macro-longevity shocks in the Lee-Carter model and in the Solvency II framework. The graphs show that the development of future mortality in the Lee-Carter model is in line with the compression view and the Solvency II framework is in line with the expansion view.

The shocks for longevity and mortality risk in the Solvency II framework are deterministic, i.e. no stochastic mortality model is used to determine the distribution of future death rates. Because we have to make an assumption about the distribution of future death rates when

[26] These capital requirements are based on the 99.5% VaR of the available capital over a one-year horizon.

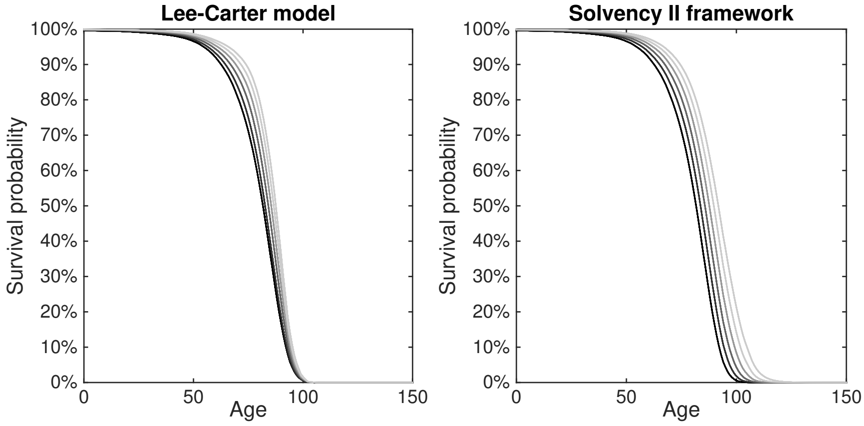

Figure 12: Impact of several consecutive macro-longevity shocks in the Lee-Carter model (lefthand graph) and in the Solvency II framework (righthand graph) on the survival probability.

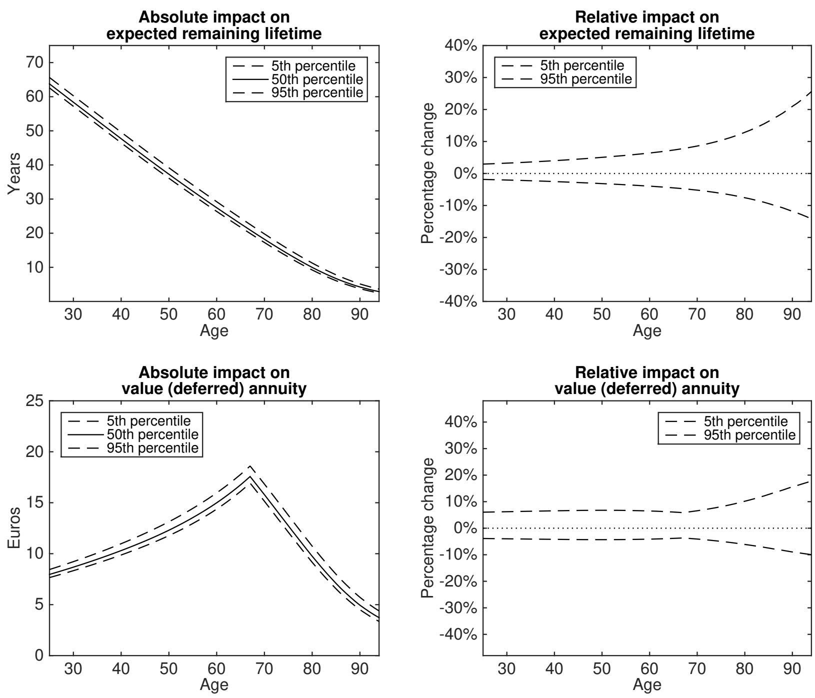

Figure 13: Impact of macro-longevity risk in the Solvency II framework on the expected remaining lifetime and the value of a (deferred) variable annuity for a Dutch female in 2014 in absolute terms (lefthand graphs) and relative change (righthand graphs) assuming a constant interest rate of 2% and fixed retirement age $R = 67$.

sharing macro-longevity risk, we assume that the shocks for longevity and mortality risk both occur with probability 50%. Figure 13 visualizes the impact of those shocks on the expected remaining lifetime and the value of a (deferred) variable annuity.

We cannot compare the size of the impact of macro-longevity risk in the Lee-Carter model (Figure 3) and Solvency II framework (Figure 13) directly, because the shocks in the Lee-Carter model are on a 10-year horizon while the shocks in the Solvency II framework are one-off shocks. However, we can still compare the distribution of macro-longevity risk over different cohorts in both models. We notice that the relative change of the expected remaining lifetime and (deferred) annuity value per cohort (righthand figures) differ significantly. While the relative change in the Lee-Carter model decreases with age, it increases with age in the Solvency II framework. This is due to the fact that the impact of a uniform improvement of death probabilities on survival probabilities is much higher at high ages compared to low ages because death probabilities are higher at high ages. As a result, the relative change increases with age in the Solvency II framework. In the Lee-Carter model the impact of macro-longevity risk on death probabilities decreases with age.

Model	**LC**	**SII**
Fixed retirement age (FRA)	0.3%	0.3%
Partial adjustment retirement age (PARA)	0.5%	0.3%
Full adjustment retirement age (FARA)	2.7%	0.4%

Table 5: Welfare gains from sharing macro-longevity risk in terms of aggregate certainty equivalent consumption after retirement in the Lee-Carter model and in the Solvency II framework.

Table 5 shows the welfare gains from risk sharing in the Solvency II framework for the three retirement age policies. We cannot compare the size of the welfare gains in the Lee-Carter model and Solvency II framework directly because both shocks have a different interpretation as mentioned above. In the Solvency II framework the welfare gain does not increase significantly in case of a full adjustment of the retirement age. Recall that the high welfare gain in case the retirement age is fully adjusted in the Lee-Carter model is a result of the hedge effect of the adjusted labor supply to macro-longevity shocks for workers. In the Solvency II framework the impact of macro-longevity risk on the expected remaining lifetime (Figure 13) is small for workers. As a result, the hedge effect is much smaller in the Solvency II framework compared to the Lee-Carter model.

Figure 14 visualizes the optimal risk transfer relative to autarky for a participant in cohort i as percentage of total risk. We can conclude that for each retirement age policy the optimal risk transfer rule is reasonably robust to the alternative mortality model.

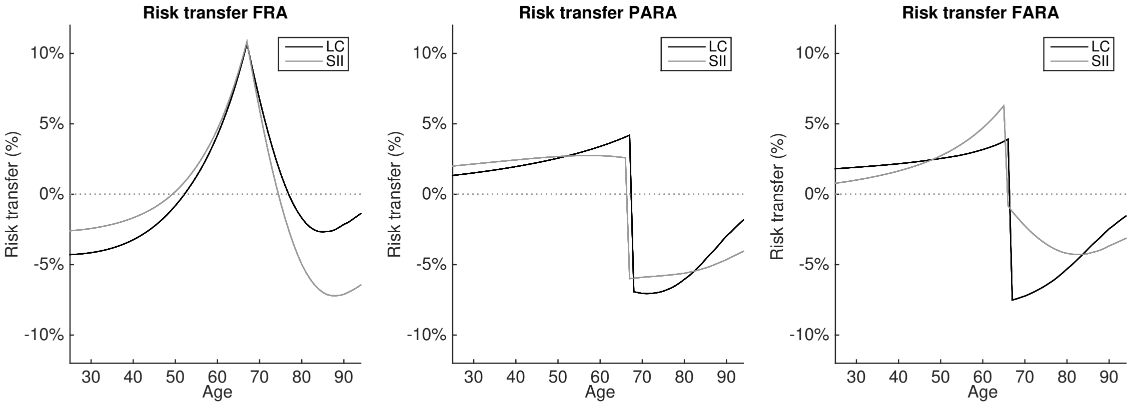

Figure 14: Optimal risk transfer relative to autarky as percentage of total risk for different retirement age policies. The black lines represent the original risk transfers in the Lee-Carter model and the grey lines represent the optimal risk transfers in the Solvency II framework.

6 Conclusion and policy evaluation

Pension funds face macro-longevity risk or uncertainty about future mortality rates. We analyze macro-longevity risk sharing between cohorts in a pension fund as a risk management tool. We explore this economic problem as macro-longevity risk is not traded on a liquid market and cohorts are affected differently by macro-longevity risk. We derive the optimal risk-sharing rule and the welfare gains from ex-ante Pareto efficient risk-sharing solutions for different retirement age policies.

The retirement age policy impacts both the optimal risk-sharing rule and the welfare gains from sharing macro-longevity risk. In case the retirement age is fixed, the welfare gains from sharing macro-longevity risk are between 0.2 percent and 0.3 percent of certainty equivalent consumption after retirement. Under this policy, the impact of macro-longevity risk on retirement consumption for different cohorts is more or less equal. Young cohorts do not absorb macro-longevity risk of other cohorts in the optimal risk transfer rule. As a result, the welfare gains from risk sharing are limited. The risk transfer rules and corresponding welfare gains are reasonably robust to the alternative mortality data and model assumptions we consider in the sensitivity analyses.

Some countries link the retirement age to life expectancy developments. In case of a full adjustment of the retirement age the welfare gains from sharing macro-longevity risk measured on a 10-year horizon are substantially higher up to 2.7 percent. The risk bearing capacity of workers is larger, because they can use their labor supply as a hedge against macro-longevity shocks. As a result, workers absorb risk from retirees in the optimal risk transfer rule, thereby increasing the welfare gain from risk sharing. The size of the welfare gains from risk sharing is sensitive to the mortality data and model assumptions. For example, the welfare gains are lower in case of US mortality data. However, the optimal risk transfer rules are reasonably robust to the alternative mortality data and model assumptions.

The findings in this paper are relevant for pension policy, especially because of the general trend of transferring risks to pension participants. *First*, we determine the optimal risk-sharing rule. In practice macro-longevity risk is shared in different ways. In the first pillar macro-longevity risk is shared between retirees and active participants. In DC schemes macro-longevity risk is usually not shared before retirement and in pooled annuity schemes retirees share macro-longevity risk uniformly. In DB schemes macro-longevity risk impacts the funding ratio. This implies that all cohorts share macro-longevity risk uniformly. The results in this paper show that uniform risk sharing is suboptimal. Moreover, it is sometimes argued that workers can provide insurance to macro-longevity risk of retirees. The results in this paper show that such a risk distribution is optimal only in case the retirement age is linked to life expectancy. If the retirement age is fixed it is not optimal for young cohorts to absorb risk of retirees. *Second*, we determine a fair risk compensation for cohorts who absorb macro-longevity risk of other cohorts using a utility-based fairness criterion. In practice, there is usually no risk compensation for absorbing macro-longevity risk.

Sharing macro-longevity risk results in high welfare gains in case of a full adjustment of the retirement age. However, we do not want to make a statement about the suitability of retirement age policies in this paper. This is a different research question and requires the inclusion of leisure time besides consumption in the utility function to take the labor-leisure trade-off into account. Moreover, it is up to policy makers to decide whether it is appropriate to link the retirement age to life expectancy. The suitability of a retirement age policy involves a broader perspective. For example, healthy life expectancy and practical implementation are relevant but outside the scope of this paper.

Sensitivity analyses show that the size of the welfare gains depends on the population composition and the mortality data. For example, welfare gains from sharing macro-longevity risk are smaller for US compared to Dutch mortality data. An interesting area for future research is to investigate sharing macro-longevity risk between pension funds or between countries. For example, Van Binsbergen et al. (2014) propose to share risks between heterogeneous pension funds by trading pension guarantees. Bodie and Merton (2002) propose swaps to achieve risk-sharing benefits of broad international diversification.

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A Appendix

A.1 Expected survival probability

The random shocks in (7) in the log central death rates are normally distributed with mean zero, i.e. $\mathbb{E}[\beta_x \eta_t + \epsilon_{x,t}] = 0$. The following holds for the expected survival probability

$$\begin{aligned}
\mathbb{E}[p_{x,t}] &\approx \mathbb{E}[\exp(-\mu_{x,t})] = \mathbb{E}[\exp(-\exp(\alpha_x + \beta_x \kappa_t + \epsilon_{x,t}))] \\
&\leq \mathbb{E}[\exp(-\exp(\alpha_x + \beta_x c + \beta_x \kappa_{t-1} + \beta_x \eta_t + \epsilon_{x,t}))] \\
&= \exp(-\exp(\alpha_x + \beta_x c + \beta_x \kappa_{t-1} + \mathbb{E}[\beta_x \eta_t + \epsilon_{x,t}])) \\
&\leq \exp(-\exp(\alpha_x + \beta_x c + \beta_x \kappa_{t-1}) = \hat{p}_{x,t},
\end{aligned} \tag{24}$$

using Jensen's inequality $\mathbb{E}[f(x)] \leq f(\mathbb{E}[x])$ with $f(x) = \exp(-\exp(x))$ being a concave function for $x \leq 0$.

A.2 Proof Pareto-efficient risk-sharing rule

The Pareto-efficient risk-sharing rule can be derived in three different ways. *First*, the optimal risk-sharing solution can be found by maximizing individual expected utility. *Second*, the solution can be found by maximizing aggregate equivalent variation. *Third*, it can be found by maximizing aggregate expected utility. All three maximizations lead to the same optimal risk-sharing rule. We present the derivations below. In our full model we maximize aggregate equivalent variation because it is most easy to implement.

Proof 1: maximizing individual expected utility

Risk sharing is Pareto improving in comparison to autarky if the welfare of at least one agent improves and all other agents do not become worse off. Therefore, we maximize the expected utility of agent 1 under the condition that the expected utility of agent 2 does not decrease (similar to Gottardi and Kubler (2011))

$$\max_{\eta, t_0} \mathbb{E}[U(C_1^s)] \quad \text{such that} \quad \mathbb{E}[U(C_2^s)] \geq \mathbb{E}[U(C_2^a)]. \tag{25}$$

By using the power utility function and applying the Arrow-Pratt approximation we get the following expression for the expected utility of agent 1

$$\begin{aligned}
\mathbb{E}[U(C_1^s)] &= \mathbb{E}\left[\frac{(W_1 + \beta_1 \tilde{y} - t(\tilde{y}))^{1-\gamma}}{1-\gamma}\right] \\
&= \mathbb{E}\left[\frac{(W_1 + (\beta_1 - \eta)\tilde{y} - t_0)^{1-\gamma}}{1-\gamma}\right] \\
&\approx \frac{(W_1 - \frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta)^2 - t_0)^{1-\gamma}}{1-\gamma}.
\end{aligned} \tag{26}$$

In the Arrow-Pratt approximation[27] we determine z in $\mathbb{E}[U(W+\tilde{y})] \approx U(W+z)$ by applying a Taylor expansion to both sides of the equation.[28] We apply a second-order Taylor expansion to $U(W+\tilde{y})$ around W and take the expectation

$$\mathbb{E}[U(W+\tilde{y})] \approx \mathbb{E}[U(W)+U'(W)\tilde{y}+\frac{1}{2}\tilde{y}^2U''(W)] = U(W)+\frac{1}{2}\sigma^2U''(W). \tag{27}$$

We apply a first-order Taylor expansion to $U(W+y)$ around W

$$U(W+z) \approx U(W)+U'(W)z, \tag{28}$$

which implies

$$z = \frac{1}{2}\sigma^2\frac{U''(W)}{U'(W)} = \frac{1}{2}\sigma^2\frac{-\gamma W^{-\gamma-1}}{W^{-\gamma}} = \frac{1}{2}\sigma^2\frac{\gamma}{W}. \tag{29}$$

In the same way we derive an expression for the expected utility of agent 2

$$\begin{aligned}\mathbb{E}[U(C_2^s)] &= \mathbb{E}\left[\frac{(W_2+\beta_2\tilde{y}+t(\tilde{y}))^{1-\gamma}}{1-\gamma}\right] \\ &= \mathbb{E}\left[\frac{(W_2+(\beta_2+\eta)\tilde{y}+t_0)^{1-\gamma}}{1-\gamma}\right] \\ &\approx \frac{(W_2-\frac{1}{2}\frac{\gamma}{W_2}\sigma^2(\beta_2+\eta)^2+t_0)^{1-\gamma}}{1-\gamma}.\end{aligned} \tag{30}$$

Maximizing individual expected utility in (25) is equivalent to maximizing certainty equivalent consumption

$$\max_{\eta,t_0}\left(W_1-\frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1-\eta)^2-t_0\right) \quad \text{such that} \quad W_2-\frac{1}{2}\frac{\gamma}{W_2}\sigma^2(\beta_2+\eta)^2+t_0 \geq W_2-\frac{1}{2}\frac{\gamma}{W_2}\sigma^2\beta_2^2. \tag{31}$$

The Lagrange function of this maximization problem equals

$$\mathcal{L} = W_1-\frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1-\eta)^2-t_0-\lambda\left(-\frac{1}{2}\frac{\gamma}{W_2}\sigma^2((\beta_2+\eta)^2-\beta_2^2)+t_0\right), \tag{32}$$

with first order conditions

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\eta} &= \frac{\gamma}{W_1}\sigma^2(\beta_1-\eta)+\lambda\frac{\gamma}{W_2}\sigma^2\beta_2+\lambda\frac{\gamma}{W_2}\sigma^2\eta = 0 \\ \frac{\partial\mathcal{L}}{\partial t_0} &= -1-\lambda = 0 \quad \Longleftrightarrow \lambda = -1.\end{aligned} \tag{33}$$

Solving the first order conditions leads to the optimal risk transfer η^*

$$\begin{aligned}\frac{\gamma}{W_1}\sigma^2\beta_1-\frac{\gamma}{W_2}\sigma^2\beta_2 &= \left(\frac{\gamma}{W_1}\sigma^2+\frac{\gamma}{W_2}\sigma^2\right)\eta \\ \frac{\beta_1}{W_1}-\frac{\beta_2}{W_2} &= \eta\left(\frac{1}{W_1}+\frac{1}{W_2}\right) \\ \eta^* &= \frac{\beta_1W_2-\beta_2W_1}{W_1+W_2}.\end{aligned} \tag{34}$$

[27] An Arrow-Pratt approximation can be used under the condition that the risk is small.

[28] For the sake of simplicity we exclude t_0 from the Taylor expansion. This is allowed because t_0 is small relative to W.

The risk transfer η^* increases linear in β_1 and decreases linear in β_2.[29] In case $\beta_1 W_2 = \beta_2 W_1$, risk sharing is not welfare improving. We apply Arrow-Pratt approximations to find η^*. This solution can slightly deviate from the true optimal solution because of the remainder term in the Taylor series. We do not matter too much about this inaccuracy because we use the two-agent model to derive an analytical framework and get intuition but not for numerical precision.

Proof 2: maximizing aggregate equivalent variation

The optimal risk transfer can also be found by maximizing aggregate equivalent variation (similar to Bovenberg and Mehlkopf (2014))

$$\max_{\eta, t_0} EQV_1 + EQV_2. \tag{35}$$

The equivalent variation EQV_i is defined as the amount of wealth which agent i should be given ex-ante in autarky to obtain the same ex-ante welfare in case of risk sharing. This is equal to the certainty equivalent consumption in case of risk sharing minus the certainty equivalent consumption in autarky. Risk sharing is potentially Pareto improving only if $EQV_1 + EQV_2 > 0$. Equivalent variation is an attractive welfare measure in the context of risk sharing, because it is unaffected by redistribution between agents. The derivation below shows that the maximization is unaffected by the deterministic risk compensation t_0 between agents.

Using (26) and (30) the sum of EQV_1 and EQV_2 approximately equals

$$\begin{aligned} EQV_1 + EQV_2 \approx\ & W_1 - \frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta)^2 - t_0 - W_1 + \frac{1}{2}\frac{\gamma}{W_1}\sigma^2\beta_1^2 \\ & + W_2 - \frac{1}{2}\frac{\gamma}{W_2}\sigma^2(\beta_2 + \eta)^2 + t_0 - W_2 + \frac{1}{2}\frac{\gamma}{W_2}\sigma^2\beta_2^2 \\ =& -\frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta)^2 + \frac{1}{2}\frac{\gamma}{W_1}\sigma^2\beta_1^2 - \frac{1}{2}\frac{\gamma}{W_2}\sigma^2(\beta_2 + \eta)^2 + \frac{1}{2}\frac{\gamma}{W_2}\sigma^2\beta_2^2 \\ =& \frac{1}{2}\frac{\gamma}{W_1}\sigma^2(2\beta_1\eta - \eta^2) - \frac{1}{2}\frac{\gamma}{W_2}\sigma^2(\beta_2 + \eta)^2. \end{aligned} \tag{36}$$

The first order condition with respect to η equals

$$\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta) - \frac{\gamma}{W_2}\sigma^2(\beta_2 + \eta) = 0, \tag{37}$$

which leads to the optimal risk transfer η^*

$$\begin{aligned} \frac{\beta_1}{W_1} - \frac{\beta_2}{W_2} &= \eta\left(\frac{1}{W_1} + \frac{1}{W_2}\right) \\ \frac{\beta_1 W_2 - \beta_2 W_1}{W_1 W_2} &= \eta\frac{W_1 + W_2}{W_1 W_2} \\ \eta^* &= \frac{\beta_1 W_2 - \beta_2 W_1}{W_1 + W_2}. \end{aligned} \tag{38}$$

[29] This linearity results from the Arrow-Pratt approximations and because we assume a linear risk-sharing rules (11).

Proof 3: maximizing aggregate expected utility

Finally, the optimal risk transfer can also be obtained by taking a social planner's view and maximizing a weighted sum of the expected utility of agents

$$\max_{\eta,t_0} \mathbb{E}[U(C_1^s)] + \delta\mathbb{E}[U(C_2^s)]. \tag{39}$$

The social planner chooses parameter δ that weights the relative importance of the agents. A low δ means that the utility of future generations is less important.

$$\begin{aligned}
\mathbb{E}[U(C_1^s)] + \delta\mathbb{E}[U(C_2^s)] =& \mathbb{E}\left[\frac{(W_1 + (\beta_1 - \eta)\tilde{y} - t_0)^{1-\gamma}}{1-\gamma}\right] + \delta\mathbb{E}\left[\frac{(W_2 + (\beta_2 + \eta)\tilde{y} + t_0)^{1-\gamma}}{1-\gamma}\right] \\
\approx& \frac{(W_1 - \frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta)^2 - t_0)^{1-\gamma}}{1-\gamma} + \frac{\delta(W_2 - \frac{1}{2}\frac{\gamma}{W_2}\sigma^2(\beta_2 + \eta)^2 + t_0)^{1-\gamma}}{1-\gamma}.
\end{aligned}$$

The first order condition with respect to t_0 equals

$$\begin{aligned}
-\left(W_1 - \frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta)^2 - t_0\right)^{-\gamma} + \delta\left(W_2 - \frac{1}{2}\frac{\gamma}{W_2}\sigma^2(\beta_2 + \eta)^2 + t_0\right)^{-\gamma} &= 0 \\
\left(W_1 - \frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta)^2 - t_0\right)^{-\gamma} &= \delta\left(W_2 - \frac{1}{2}\frac{\gamma}{W_2}\sigma^2(\beta_2 + \eta)^2 + t_0\right)^{-\gamma}.
\end{aligned} \tag{40}$$

The first order condition with respect to η equals

$$\begin{aligned}
\left(W_1 - \frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta)^2 - t_0\right)^{-\gamma}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta) + \delta\left(W_2 - \frac{1}{2}\frac{\gamma}{W_2}\sigma^2(\beta_2 + \eta)^2 + t_0\right)^{-\gamma}\frac{-\gamma}{W_2}\sigma^2(\beta_2 + \eta) &= 0 \\
\left(W_1 - \frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta)^2 - t_0\right)^{-\gamma}\frac{\beta_1 - \eta}{W_1} &= \delta\left(W_2 - \frac{1}{2}\frac{1}{W_2}(\beta_2 + \eta)^2 + t_0\right)^{-\gamma}\frac{\beta_2 + \eta}{W_2},
\end{aligned}$$

where plugging in (41) yields

$$\begin{aligned}
&\left(W_1 - \frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta)^2 - t_0\right)^{-\gamma}\frac{\beta_1 - \eta}{W_1} = \left(W_1 - \frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta)^2 - t_0\right)^{-\gamma}\frac{\beta_2 + \eta}{W_2} \\
&\frac{\beta_1 - \eta}{W_1} = \frac{\beta_2 + \eta}{W_2} \\
&\beta_1 W_2 - \eta W_2 = W_1\beta_2 + \eta W_1 \\
&\eta^* = \frac{\beta_1 W_2 - \beta_2 W_1}{W_1 + W_2}.
\end{aligned} \tag{41}$$

A.3 Proof of optimal risk compensation

Maximizing individual expected utility (proof 1) or aggregate equivalent variation (proof 2) does not deliver a unique solution for the risk compensation t_0. The risk compensation determines how the welfare gain from risk sharing is distributed among the agents. Requiring the risk-sharing solution to be Pareto improving in comparison to autarky ensures that the welfare gain can be fully attributed to gains from risk sharing and is not a result from ex-ante redistribution between agents. Under the condition that the risk-sharing solution is Pareto improving the risk compensation should lie between the following upper bound

$$\begin{aligned} & CEQ_1^s \geq CEQ_1^a \\ & W_1 - \frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta^*)^2 - t_0 \geq W_1 - \frac{1}{2}\frac{\gamma}{W_1}\sigma^2\beta_1^2 \\ & -\frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1 - \eta^*)^2 + \frac{1}{2}\frac{\gamma}{W_1}\sigma^2\beta_1^2 \geq t_0 \\ & \frac{1}{2}\frac{\gamma}{W_1}\sigma^2\eta^*(2\beta_1 - \eta^*) \geq t_0, \end{aligned} \tag{42}$$

and lower bound

$$\begin{aligned} & CEQ_2^s \geq CEQ_2^a \\ & W_2 - \frac{1}{2}\frac{\gamma}{W_2}\sigma^2(\beta_2 + \eta^*)^2 + t_0 \geq W_2 - \frac{1}{2}\frac{\gamma}{W_2}\sigma^2\beta_2^2 \\ & t_0 \geq \frac{1}{2}\frac{\gamma}{W_2}\sigma^2(\beta_2 + \eta^*)^2 - \frac{1}{2}\frac{\gamma}{W_2}\sigma^2\beta_2^2 \\ & t_0 \geq \frac{1}{2}\frac{\gamma}{W_2}\sigma^2\eta^*(2\beta_2 + \eta^*). \end{aligned} \tag{43}$$

So risk sharing is Pareto improving if t_0 satisfies the following condition

$$\frac{1}{2}\frac{\gamma}{W_1}\sigma^2\eta^*(2\beta_1 - \eta^*) \leq t_0 \leq \frac{1}{2}\frac{\gamma}{W_2}\sigma^2\eta^*(2\beta_2 + \eta^*). \tag{44}$$

We use a utility-based fairness criterion which yields a unique risk-sharing solution within the set of Pareto-efficient solutions. This requires that all agents experience the same increase in certainty equivalent consumption as a result of risk sharing relative to autarky

$$\begin{aligned} CEQ_1^s - CEQ_1^a &= CEQ_2^s - CEQ_2^a \\ \frac{1}{2}\frac{\gamma}{W_1}\sigma^2(\beta_1^2 - (\beta_1 - \eta^*)^2) - t_0 &= -\frac{1}{2}\frac{\gamma}{W_2}\sigma^2((\beta_2 + \eta^*)^2 - \beta_2^2) + t_0 \\ t_0 &= \frac{1}{4}\frac{\gamma}{W_1}\sigma^2\eta^*(2\beta_1 - \eta^*) + \frac{1}{4}\frac{\gamma}{W_2}\sigma^2\eta^*(2\beta_2 + \eta^*) \\ t_0 &= \frac{1}{4}\gamma\sigma^2\eta^*\left(\frac{1}{W_1}(2\beta_1 - \eta^*) + \frac{1}{W_2}(2\beta_2 + \eta^*)\right). \end{aligned} \tag{45}$$

Maximizing aggregate expected utility (proof 3) does deliver a unique solution t_0 which depends on δ. In this case the parameter δ can be chosen such that the risk-sharing solution is Pareto improving compared to autarky. Because we maximize aggregate equivalent variation in our full model we do not elaborate further on δ here.

A.4 Definitions

Parameter	Definition
α_x	Age-specific constant in log central death rates
Annuity value (a_x^t)	Value of an annuity that pays 1 dollar annually during retirement for an individual of age x in year t
Autarky	Situation without risk sharing
C_i^a	Consumption after retirement in autarky for a participant in cohort i
C_i^s	Consumption after retirement after risk sharing for a participant in cohort i
Certainty equivalent consumption	Guaranteed consumption level that someone would accept rather than a higher uncertain consumption
Central death rate ($\mu_{x,t}$)	Average yearly death rate of an individual of age x in year t
Cumulative survival probability ($cp_{x,i}^t$)	Probability that an individual of age x in year t is still alive after i years
c	Drift in time trend
Equivalent variation (EQV_i)	Amount of wealth which agent i should be given ex-ante in autarky to obtain the same ex-ante welfare in case of risk sharing
Uncertainty in death rates ($\epsilon_{x,t}$)	Random variation in log central death rates
Fixed retirement age (FRA)	Constant retirement age
Full adjustment retirement age (FARA)	Retirement age keeps up fully with life expectancy
Longevity risk	Risk that people live longer than expected
Macro-longevity risk	Uncertainty about future mortality rates
Micro-longevity risk	Uncertainty about individual time of death
Mortality risk	Risk that people live shorter than expected
One-year death probability ($q_{x,t}$)	Probability that an individual of age x and alive in year t dies before year $t+1$
One-year survival probability ($p_{x,t}$)	Probability that an individual of age x and alive in year t is still alive in year $t+1$
Parameter risk	Uncertainty in the true value of the parameters
Partial adjustment retirement age (PARA)	Retirement age adjusts to life expectancy such that the value of an annuity remains the same
β_x	Sensitivity of log central death rates to time trend
Risk compensation ($t_{0,i}$)	Financial compensation for absorbing risk for a participant in cohort i
Risk sharing	Allocate risks to cohorts via a predetermined rule
Risk-sharing rule ($t(\tilde{y})$)	Risk transfer plus risk compensation
Risk transfer (η_i)	Part of total macro-longevity shock a participant in cohort i absorbs
Stochastic variation	Random variation in the aggregate realized number of deaths
Time trend (κ_t)	Development of death rates over time
Uncertainty in trend (η_t)	Random variation in the time trend
σ_ϵ^2	Variance death rates
σ_η^2	Variance trend
W_i	Wealth of a participant in cohort i
Welfare gain	Relative increase certainty equivalent consumption after retirement
$\tilde{y}_i$	Amount of money needed to offset effect of macro-longevity shock for a participant in cohort i
$\tilde{y}_T$	Amount of money needed to offset effect of macro-longevity shock for all cohorts

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