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Abstract

This paper empirically assesses the impact of liquidity and capital constraints on the allocation of defined benefit pension funds to illiquid assets. Liquidity constraints result from short-term pension payments and collateral requirements on derivatives. Capital constraints follow from the requirement to retain sufficient capital to absorb unexpected losses. Liability duration and hedging affect the allocation to illiquid assets through both these constraints. First, we find a hump-shaped impact of liability duration on the illiquid assets allocation. Up to 17.5 years, liability duration positively affects the illiquid asset allocation. However, beyond this point the effect is reversed as the capital constraint dominates the liquidity constraint. Second, we find no evidence that interest rate hedging affects the illiquid assets allocation. Third, we do find that currency risk hedging positively impacts the illiquid assets allocation.

Keywords: Illiquid assets, asset liability management, asset allocation, liquidity constraints, capital constraints, pension funds, regulation.

JEL classifications: G11, G23.

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1 Introduction

Over recent years pension funds increased their investments in illiquid assets, such as real estate, mortgages, private equity, hedge funds and infrastructure. Several studies provide evidence for this. A broad OECD study assessing the asset allocation of pension funds in 34 countries shows that allocations to illiquid assets on average increased from 14 percent in 2010 to 15 percent in 2014[^1]. The 2015 Towers Watson Global Pension Asset Study reveals that the 16 largest pension markets in the world increased the allocation to illiquid assets from about 5 percent in 1995 up to 20 percent in 2015[^2]. In this paper, we analyze the impact of liquidity and capital constraints and other pension funds’ characteristics on the allocation to illiquid assets.

The liquidity of an asset is defined by three dimensions: price, quantity and immediacy. An asset is considered less liquid if the investor cannot quickly sell a significant quantity of the asset at a price near fundamental value. Although this is a hurdle for short-term investors to invest in illiquid assets, for pension funds this might not be the case as they

require less short-term liquidity. More precisely, pension funds might choose to invest in illiquid assets for three reasons. First, the possibility to be compensated for the lack of liquidity through the so-called liquidity premium. Conceptually, the liquidity premium consists of two components, the liquidity level premium and the liquidity risk premium. The liquidity level premium compensates for the expected liquidity of an asset. However, over time the level of liquidity changes in an unpredictable way. The liquidity risk premium compensates for the exposure to time variation in the liquidity level. Several studies find the existence of a liquidity premium in traditional asset classes such as stocks and bonds (e.g., Acharya and Pedersen (2005), Driessen and De Jong (2012), and Pastor and Stambaugh (2003)). The literature on liquidity premiums in illiquid asset classes is however scarce, typically as a result of lack of data. Existing papers reveal a mixed view. Qian and Liu (2012) report a positive effect of illiquidity on expected returns in case of unlisted real estate, although the effect is relatively small. In case of private equity, there is little empirical evidence for a liquidity level premium (Driessen and De Jong (2015)). There is however some evidence for a liquidity risk premium. Franzoni et al. (2012) study the compensation for liquidity risk in the market for private equity funds. They estimate the annual compensation for liquidity risk in private equity returns to be 3 percent. Sadka (2010) estimates that in case of hedge
funds, the liquidity risk premium is about 6 percent. Despite a lack of convincing evidence for liquidity premiums in some illiquid asset classes, the prolonged low-return environment in traditional stock and bond markets makes pension funds engaged in a search for additional returns in alternative assets, typically less liquid markets. Using a large database of hedge funds returns and a sample of U.K. pension funds, Jackwerth and Slavutskaya (2016) do for instance show that the addition of hedge funds to pension funds portfolios improves the average pension fund performance.

Second, pension funds invest in illiquid assets to match the liabilities in terms of inflation and interest rate risk. For instance, real estate is considered as a potential hedge against inflation risk. The rents on residential and institutional real estate are often correlated with inflation. Third, illiquid asset classes may offer diversification benefits if their returns have low correlations with the returns on traditional asset class such as stocks and bonds. Jacobs et al. (2014) show that diversification gains are mainly driven by a balanced allocation over different asset classes. Hoevenaars et al. (2008) show that besides a diversification perspective, illiquid asset classes are also important from a hedging perspective. They show that illiquid assets are more valuable for a long-term investor with
liabilities than for an asset-only investor.

Next to these advantages, there are also reasons why pension funds are constrained to invest in illiquid assets. We analyse two in this paper: liquidity and capital constraints. We introduce them already briefly here. First, pension funds are liquidity constrained. Pension funds require sufficient liquid resources to fulfill immediate liquidity needs. These liquidity needs consist of two components: short run pension payments and collateral requirements on interest rate, currency and other derivatives.

The cash flows from the pension funds’ liabilities are well predictable. Specifically if the pension fund runs a defined benefit scheme. The cash flows from collateral requirements however, are much less predictable. If the market value of a derivative declines, the pension fund is required to transfer cash or highly liquid short-term bonds to a margin account in order to limit the risk the counter party faces. Cash flows arising from margining on interest rate and currency derivatives can become quite substantial, especially in financial crises. For instance, during the 2008 financial crisis, there was a high demand for cash and T-bills in the U.S., leading to an appreciation

\footnote{Investing in illiquid asset classes may require specific knowledge about the asset class. This could be considered a third constraint but is not analysed in our paper.}
of the dollar vis a vis the euro in the period from 2007 until 2009. This resulted in a significant decline of the market value of currency derivatives for Dutch pension funds. Consequently, they were required to post substantial collateral.

Second, defined benefit pension funds are capital constrained. They need to have sufficient capital to manage the risks they are exposed to, such as interest rate risk, market risk, currency risk and longevity risk. As a result, pension funds are constrained to increase their exposure to illiquid assets given a certain level of available capital. In choosing the optimal investment strategy, a pension fund optimizes the trade-offs between different risk factors for a given level of required capital. For instance, Dutch pension funds derive the marked-to-market value of their liabilities based on the prevailing term structure of interest rates. A longer liability duration therefore increases the pension fund’s exposure towards interest rate risk. Given a certain level of available capital, this decreases the opportunities to invest in illiquid assets. Sias (2004) and Andonov et al. (2016) indeed show that regulation can have a significant impact on pension fund’s investment decisions.

Liquidity and capital constraints interact. Take, for instance, liability
duration as a measure of a pension fund’s investment horizon. On the one hand, a pension fund with a higher liability duration is lower liquidity constrained as it will have to pay less pensions in the short run. This allows for higher allocations to illiquid assets. On the other hand, a higher liability duration implies that the pension fund is more exposed to interest rate risk through its liabilities. This might restrict the opportunity to invest in illiquid assets as more of the available capital serves to manage interest rate risk. Another example of the interaction between liquidity and capital constraints reveals if we look at derivatives. Hedging interest rate and currency risk increases the liquidity constraint as a result of collateral requirements. However, by hedging interest rate and currency risk, the pension fund is less exposed to these two risk factors and might take additional risk elsewhere before the capital constraint is binding. As pension funds are generally not required to hold capital for liquidity risk, hedging might create opportunities to invest more in illiquid assets. Pension funds assess both the capital and liquidity constraint when making investment decisions. This paper, to the best of our knowledge, is the first to empirically study the effect of liquidity and capital constraints on the illiquid asset allocation.
We analyze the investment decisions of Dutch occupational pension funds in our paper. The Dutch pension system is a useful setting for such an analysis for several reasons. First, the Dutch pension system is large in terms of size. The total assets under management (AUM) of Dutch pension funds’ equals approximately 1.3 trillion euro. Or roughly 1.5 times the GDP of the Netherlands. Second, Dutch pension funds do not face quantitative investment restrictions. Regulation allows them to invest in any asset class in any country, as long as the pension fund satisfies the capital constraint. As a result, Dutch pension funds invest in a broad range of asset classes, including many illiquid assets. Over two-third of Dutch pension funds invest in illiquid assets. Third, Dutch pension funds mainly have defined benefit pension liabilities. This means that they have a clear asset-liability perspective when deriving the optimal asset allocation. Fourth, the pension funds value their defined benefit liabilities marked-to-market using the current term structure of interest rates. As a result, we are able to analyze in which way interest rate risk impacts the allocation to illiquid assets. The unique data are therefore particularly well suited to study the effect of pension fund’s liquidity and capital constraints on the illiquid asset allocation.

We use two pension fund characteristics that impact both liquidity
and capital constraints: the pension fund’s liability duration and collateral requirements on interest rate and currency derivatives. In short we offer the following contributions. First, we find a hump-shaped impact of liability duration on the illiquid asset allocation (Figure 2). Up to 17.5 years, liability duration positively affects this allocation. Up to this point of reversal the marginal benefit of a lower liquidity constraint outweighs the marginal costs of an increase in the capital constraint. A one year increase in the liability duration from 10 to 11 years implies an increase in the illiquid asset allocation of 0.64 percentage points (Table 2, Panel A). However, beyond this point, the effect is reversed. The allocation to illiquid assets decreases. The marginal costs of an increase in the capital constraint now dominates the marginal benefits of a decrease in the liquidity constraint. A one year increase in the liability duration from 25 to 26 years implies a decrease in the illiquid asset allocation of 0.75 percentage points (Table 2, Panel A). A pension fund’s liquidity constraint decreases exponentially with longer liability duration. On the other hand, a pension fund’s interest rate risk exposure increases quadratically with a longer liability duration due to the non-linear relation between the discount rate and the marked-to-market value of liabilities. Pension funds with a higher liability duration require extra capital to manage this risk.
Second, we do not find evidence that interest rate risk hedging and collateral requirements on interest rate derivatives impact the illiquid assets allocation. This indicates that neither the liquidity nor the capital constraint of hedging interest rate risk dominates. In case of currency risk hedging, however, collateral requirements on currency derivatives do impact the illiquid asset allocation positively. In line with the positive (less capital requirement) and negative (more liquidity needs) implications of hedging, currency risk hedging creates the opportunity for pension funds to take additional risks by investing in illiquid assets. Liquidity risk is not priced in Dutch pension regulation and as a result, given a certain capital, a pension fund is actually able to increase the exposure to illiquid assets. A reasonable increase in hedging currency risk leads to an increase in the illiquid asset allocation of approximately 0.6 percentage points (Table 2, Panel B). This implies a relative increase in the illiquid asset allocation of 6 percentage points.

Finally, we find that also other pension fund characteristics impact investment policy. Size positively affects the allocation to illiquid assets, which is in line with previous literature, [Dyck and Pomorski (2011)] and [Andonov (2014)]. A pension fund that is ten times larger in terms of assets
under management has a 5 percentage points higher allocation to illiquid asset (Table 2). Furthermore, corporate pension funds tend to invest 5 percentage points less in illiquid assets compared to industry-wide and professional group pension funds (Table 2). Corporate pension funds generally take less mismatch risk as this risk reflects on the corporate balance sheet (Jin et al. (2006)).

The remainder of this paper is organized as follows. Section 2 describes the capital and liquidity constraints of pension and in which way these constraints affect the illiquid asset allocation. The data description is given in Section 3. The model and results are discussed in Section 4. The robustness checks are in Section 5 and the last section concludes.

2 Liquidity and capital constraints

Pension funds have access to a large pool of asset classes to invest in. A key responsibility of a pension fund is to optimize the asset allocation given the structure of its liabilities. This is known as Asset Liability Management (ALM). Key input parameters for ALM are the expected returns, the variance-covariance matrix of returns and the risk appetite of
the pension fund’s stakeholders. Here, we focus on a specific part of the ALM process, i.e., the optimal allocation to illiquid assets. In finding their optimal allocation to illiquid assets pension funds assess the benefits and costs imposed by these investments. We specifically look at two key drivers of the optimal asset allocation: liquidity and capital constraints. More specifically, we look at two pension fund characteristics that impact both the liquidity and capital constraint: liability duration and collateral requirements on interest rate and currency derivatives.

2.1 Liquidity constraints

A pension fund must have sufficient liquidities to fulfill its immediate obligations. The liquidity constraints of a pension fund consist of two components: short run pension payments and collateral requirements on interest rate and currency derivatives. Salary payments to pension fund’s staff, administrative expenses and investment costs are also sources that require short-term liquidity, but are outside the scope of this paper. Liquidity risk arises when a pension fund lacks sufficient liquid resources to fulfill its immediate liquidity needs. Several theoretical studies investigate the optimal asset allocation with illiquid assets and short-term pension payments. For instance, Ang et al. (2014) solve the optimal asset allocation
for an investor with short-term liquidity needs. They show that an investor should significantly reduce the allocation to illiquid assets in order to avoid states of the world in which it would be short liquidity and could not cover, e.g., pension payments. This reduction is stronger for higher short-term liquidity needs.

A pension fund’s liability duration shows the weighted average time to maturity of the pension payments. A higher liability duration is associated with lower pension payouts in the short-term. In line with the findings in Ang et al. (2014), a higher liability duration therefore creates opportunities to invest in illiquid assets. The inability to frequently trade illiquid assets is less of a constraint for a pension fund with a long liability duration. As a result, we hypothesize that pension funds with a longer liability duration invest more in illiquid assets due to a lower liquidity constraint:

Hypothesis 1: A higher liability duration increases the allocation to illiquid assets due to lower liquidity constraints, all else equal.

Pension funds use interest rate and currency derivatives to lower the exposure towards interest rate and currency risk. Hedging of both interest
rate and currency risk comes at a cost of an increase in liquidity risk. Interest rate and currency derivatives involve collateral requirements to reduce counter-party risk. If the market value of a derivative decreases, the pension fund has to transfer eligible, liquid collateral to the counter-party of the derivative contract. These cash flows are hard to predict and can become quite substantial. We hypothesize that the allocation to illiquid assets is lower for pension funds that hedge more interest rate or currency risk as a result of the higher collateral requirements:

*Hypothesis 2: Hedging interest rate and currency risk lowers the allocation to illiquid assets due to higher liquidity constraints, all else equal.*

### 2.2 Capital constraints

Dutch pension funds mainly execute defined benefit pension contracts, which implies that the benefit payments are highly predictable. A key metric to assess the extent to which a defined benefit pension fund is able to meet its liabilities is the funding ratio. It measures the ratio of the market value of the assets over the marked-to-market value of the liabilities. A pension fund’s capital is the amount of assets that remain after
deducting the value of the liabilities from the value of the assets. Similar to banks and insurance companies, Dutch defined benefit pension funds are required to retain sufficient capital over the liabilities to be able to absorb losses in case of adverse events on the financial markets or in longevity (Broeders and Pröpper (2010)). The capital requirement is based on the well-known Value-at-Risk (VaR) risk measure. The required capital is calculated such that the probability that the funding ratio drops below 100 percent on a one year horizon equals 2.5 percent. If a pension fund does not meet its capital requirement, it files a recovery plan to the supervisor. In this recovery plan the pension fund specifies what actions it will take in order to comply again with the capital requirement.

Next to the funding ratio, liability duration is also a key metric for defined benefit pension funds. The liability duration not only measures liquidity needs. It also serves as a proxy of the interest rate sensitivity of the value of the liabilities. The higher the liability duration the more sensitivity the marked-to-market value of the liabilities is to interest rate changes. This implies a higher liability duration is associated with a higher capital requirement for interest rate risk. As a result, given a certain capital, a pension fund with higher liability duration might have less
opportunities to invest in illiquid assets:

\textit{Hypothesis 3: A higher liability duration lowers the allocation to illiquid assets due to a higher capital constraint, all else equal.}

Hedging interest rate and currency risk decreases the exposure towards these two underlying risk factors, but increases liquidity risk. However, liquidity risk is not penalized in Dutch pension regulation.\footnote{There is no regulatory penalty for the liquidity risk of using derivatives and investments in illiquid assets.} As a result, by using interest rate and currency derivatives the pension fund can increase risks elsewhere given a certain required capital ratio. The pension fund can for instance increase its allocation to illiquid assets by hedging additional interest rate and currency risk:

\textit{Hypothesis 4: Hedging interest rate and currency risk increases the allocation to illiquid assets due to a lower capital constraint, all else equal.}

\footnote{Obviously, pension funds will have internal or self imposed restrictions for liquidity and collateral management.}

\footnote{The capital charge is generally higher for illiquid assets compared to liquid ones. This is the result of higher underlying risk, but does not reflect that the pension fund is not able to trade the illiquid asset.}
The four hypotheses will be tested for a large sample of Dutch defined benefit pension funds. Before we go into the data and model, we first describe the trade-off between the liquidity and the capital constraint when it comes to liability duration.

### 2.3 Hump-shaped effect liability duration on the illiquid asset allocation

A higher liability duration affects both the liquidity and the capital constraint. A higher liability duration, all else equal, lowers the liquidity constraint (fewer short-term pension payments) but increases the capital constraint (higher interest rate risk). We expect that the effect of a lower liquidity constraint dominates the effect of a higher capital constraint for relative low values of the liability duration. However, this will reverse for relative high values of the liability duration. Up to a certain liability duration we therefore expect a longer liability duration positively affects the illiquid assets allocation. At some point however, a longer liability duration negatively affects the illiquid assets allocation. We can explain this by the shape of both the liquidity and the capital constraint as a function of the liability duration.
First we consider the impact of liability duration on the liquidity constraint. We define the liquidity constraint as the pension payments to be paid in the first year. In Appendix A we show that in case a pension fund has an equal age composition and fixed pension payments $x$ in each year, the liquidity constraint can be approximated as follows:

$$x \approx \frac{L}{2D_L}.$$ (1)

where $L$ represent the value of the liabilities and $D_L$ the duration of the liabilities.

This shows that the liquidity constraint is an exponential decreasing function of the liability duration. As a result, the drop in the liquidity constraint if the liability duration increases with one year is stronger for low initial values of the liability duration.

Second, we determine the impact of liability duration on the capital constraint. The capital constraint is the exposure of the funding ratio to interest rate risk. This involves both the assets and the liabilities. By investing in bonds and by using interest rate derivatives, a pension fund can lower the exposure of the funding ratio to interest rate changes. We
assume the pension fund hedges $\phi$ of its interest rate risk embedded in the liability structure. Then the capital constraint of the pension fund equals by approximation:

$$y = -(1 - \phi) D_L \Delta r L + 0.5(1 - \phi) C_L (\Delta r)^2 L$$ (2)

where $D_L$ is the liability duration, $C_L$ the liability convexity, and $\Delta r$ is the applicable change in the continuously compounded interest rate.

It is important to take convexity into account as this effect is more profound for cash flows that have to be paid further out in the future. However, in our data we have no information on liability convexity. And since we do not have access to the expected cash flows of each pension fund either, we need to approximate the convexity. In Appendix B we show that the liability convexity $C_L$ can be approximated by the liability duration squared $D_L^2$. This allows us to approximate the capital constraint $y$ as a function of the liability duration only:

$$y \approx -(1 - \phi) D_L \Delta r L + 0.5(1 - \phi) D_L^2 (\Delta r)^2 L$$ (3)

Equation (3) shows that the capital constraint is a quadratically
increasing function of the liability duration. This implies the increase in the
capital constraint is more profound for high values of the liability duration.

Now we can analyse the impact of a one year higher liability duration.
The marginal benefit (a lower liquidity constraint) of a one year increase in
the liability duration equals

\[
\frac{\partial x}{\partial D_L} = \frac{L}{2D_L^2}
\]

(4)

The marginal cost (a higher capital constraint) of a one year increase in
the liability duration follows from:

\[
\frac{\partial y}{\partial D_L} = -(1 - \phi)L\Delta r + (1 - \phi)D_LL(\Delta r)^2
\]

(5)

To see if there is a cut-off point we equate the marginal benefits to the
marginal costs to find

\[
\frac{1}{2D_L^2} = -(1 - \phi)\Delta r + (1 - \phi)D_LL(\Delta r)^2
\]

(6)

There is no straightforward analytical solution for the turning point of

\footnote{The liquidity constraint is modeled as a cost. Taking the first derivative implies that we are calculating the marginal costs. In order to transfer this into marginal benefits, we multiply the marginal benefit by minus one.}
the liability duration. To find a numerical solution we set the interest rate change \( \Delta r = -0.005 \) and assume the pension fund hedges 60 percent of the interest rate risk. The former is in accordance with the Dutch capital requirement.\(^7\) The latter is equal to the average hedging policy of Dutch pension funds.\(^8\) The duration for which the marginal benefits equal the marginal costs is \( D_L = 15.2 \) years. Up to this point, the marginal benefit (lower liquidity constraint) outweighs the marginal cost (higher capital constraint). After this point, the marginal costs outweigh the marginal benefit. The marginal benefits and costs are graphically shown in Figure 1.

Equation (6) is highly sensitive to the parameter specification. The liability duration corresponding to the cut-off point of the hump-shaped effect, \( D_L = 15.2 \), is therefore only indicative. Key is that we expect to find a hump-shaped impact of liability duration on the allocation to illiquid assets in our sample. The higher the absolute change in the interest rate \( \Delta r \), the lower the turning point of the liability duration \( D_L \). This is intuitive as a larger change in the interest rate increases the capital constraint. If pension

\(^7\)In order to calculate the interest rate sensitivity the pension fund has to determine the change in the pension funds surplus for a lower and a higher term-structure of interest rates. These new term-structures are derived by multiplying the current term structure with fixed factors determined for each maturity. Suppose the 15 year interest rate equals 2%. The corresponding fixed factor equals 0.76 in case of an interest rate decrease. This implies the new interest rate becomes \( 0.76 \times 2\% = 1.52\% \). This is equal to a decrease of 0.48% in the 15 year interest rate.

\(^8\)Renteafdekking van Pensioenfondsen' (2015), Onderzoek op verzoek van het Ministerie van Sociale Zaken en Werkgelegenheid, De Nederlandsche Bank, the average reported is an equally weighted average.
funds hedge more of their interest rate risk, i.e. a higher $\phi$, the turning point of the liability duration $D_L$ increases. This pension fund is less exposed towards interest rate risk, leading to a lower capital constraint. However, in case interest rate derivatives are used as hedging device it might experience a higher liquidity constraint due to collateral requirements on the interest rate derivatives.

Figure 1 Marginal benefits $\{4\}$ versus marginal costs $\{5\}$ as a function of the liability duration. The discounted value of the liabilities is normalized to one, i.e. $L = 1$. 
3 Data

We use data of 220 Dutch pension funds including their asset allocations, interest rate and currency derivatives, and other characteristics such as size and pension fund type. We only consider defined benefit pension funds as those are subject to capital constraints. Defined benefit pension funds guarantee retirement benefits that depend on an employees earnings history and years of service. The data is obtained from the quarterly statements Dutch pension funds mandatory report to De Nederlandsche Bank, the prudential supervisor. The sample runs from the beginning of 2012 to the end of 2015, or 16 quarters. As the reporting requirements changed as of 2015, we carefully merged data before 2015 with the data from the start of 2015 to ensure consistency in the reported variables. We exclude pension funds that are liquidated during the sample period. Pension funds in such a run-off scenario gradually sell their assets, resulting in non-representative asset allocations. The data are free from reporting biases as pension funds report mandatory.

\[\text{In 2015 the reporting requirements distinguish a larger scale of different asset categories. Besides that, the total amount invested in each asset category was divided into larger subsets in order to improve knowledge on pension fund investment behavior.}\]
A - Illiquid assets

Our data distinguishes between the following 12 asset classes: government bonds, stocks in mature markets, credits, stocks in emerging markets, inflation index-linked bonds, listed indirect real estate, commodities, indirect non-listed real estate, direct real estate, mortgages, private equity, and hedge funds. The sum of all asset classes except government bonds and inflation-index bonds is defined as the allocation to risky assets ($w_{\text{risky}}^i$) of pension fund $i$ at time $t$. Privately issued inflation-index bonds constitute only a small portion of the market. As a result, we assume that all inflation-index bonds are issued by governments.

To distinguish between illiquid and liquid assets we need to define the concept of liquidity. As already touched upon in the introduction an asset is considered less liquid if the investor cannot quickly sell a significant quantity of the asset at a price near fundamental value. In illiquid markets it is generally more difficult to quickly find counter-parties to trade with at a price close to the asset’s fundamental value. Asset classes such as private equity require the counter-parties to have significant capital and particular knowledge about the asset class, which are often limited in supply. Therefore, the costs associated with transactions in illiquid assets can
become substantial. For some illiquid assets, legal impediments make it impossible to trade for a particular time period at all, such as lock-up periods some hedge funds and private equity funds require. Certainly, each asset class has some (time-varying) degree of liquidity, e.g., trading in a corporate bond may become illiquid if the corporate comes close to bankruptcy. As a result, no clear line can be drawn to distinguish a liquid asset class from an illiquid one. However, some asset classes are substantially more illiquid than others in terms of three dimensions mentioned above. As pension funds are long term investors, we use immediacy as the key criterion to distinguish between liquid and illiquid asset classes. We classify the sum of non-listed and direct real estate, mortgages, private equity, and hedge funds allocations as total allocation to illiquid assets \( w_{it}^{ILLIQ} \). The sum of allocations to stocks in mature markets, credits, stocks in emerging markets, listed indirect real estate and commodities is defined as total allocation to liquid risky assets \( w_{it}^{LIQ} \). So, the allocation to risky assets is equal to the sum of the allocation to risky illiquid assets and to risky liquid assets

\[
w_{it}^{risky} = w_{it}^{ILLIQ} + w_{it}^{LIQ}.
\]

Pension funds report both strategic and actual asset allocations. We
focus on the strategic asset allocations as those are not sensitive to market fluctuations. Strategic asset allocations therefore better reflect actual decisions made by the pension funds. Furthermore, the actual asset allocation is less useful as it may deviate from the strategic one due to imperfect re-balancing. It is important to note that the private equity allocation includes both listed and non-listed private equity, infrastructure investments and micro finance investments. The allocation to private equity contains only the commitments already made, so future commitments are not included. The asset category mortgages contains mortgage-backed securities and direct mortgage lending.

Table 1 presents the summary statistics. Panel A highlights the strategic asset allocations. The averages are equally weighted over pension funds. Government bonds, stocks mature markets and credits are the most important asset classes, with average allocations of 30.31, 24.25 and 16.43 percent respectively. Of the illiquid asset classes, non-listed indirect real estate is the largest asset class with an average allocation of 2.51 percent. The 90th percentile shows that 10 percent of the pension funds invest more than 7.44 percent in this asset class. The other four illiquid asset classes are around 1 percent allocations on average.
Turning to Panel B, we see that the average allocation to the illiquid assets is relatively small and equal to 7.30 percent. Approximately 1/3 of Dutch pension funds do not invest in illiquid assets at all. The 90th percentile shows that 10 percent of the pension funds allocate over 18.18 percent of their total AUM to illiquid assets.

B - Liability duration

Pension funds report the modified duration of their liabilities ($D_{L,t}$). In Section 2 we hypothesize that the capital constraint becomes more prominent for higher liability durations as a result of the quadratic increase in the capital constraint. We include a quadratic term in our model to test for the hump-shaped impact of liability duration on the illiquid asset allocation. We call this quadratic term the liability convexity ($C_{L,t}$), as in the previous section we found that liability duration squared is an approximation of the liability convexity.

The liability duration and liability convexity are summarized in Panel B of Table 1. The average liability duration equals 18.5 years. However,
10 percent of the pension funds have a liability duration below 14.2 years and 10 percent have a liability duration in excess of 23.5 years. The liability convexity based on our approximation is on average 358.70.

C - Collateral requirements

Pension funds report the market value of the interest rate and currency derivatives. In addition, they also report the simulated values of the derivatives after four predetermined shocks in the underlying risk factors. The shocks are performed on the underlying risk factor and not on the market value of the derivative itself. In case of interest rate derivatives, the four shocks imply a decrease (increase) in the term structure of interest rates with 0.5 percentage points and 1 percentage point respectively. In case of currency derivatives, the four shocks imply an appreciation (depreciation) of the foreign currency with respect to the euro by 12.5 percentage points and 25 percentage points respectively.

Pension funds mainly use interest rate derivatives to reduce the risk of a decline in the interest rate. The most often used interest rate derivatives are receiver swaps and swaptions. The market value of the receiver swaps and
swaptions decline in case of an interest rate increase. In such an event, the pension fund is required to transfer eligible, liquid collateral to the counterparty. The change in the market value of the derivatives after an interest rate risk increase is therefore used to assess the liquidity needs of a pension fund as a result of collateral requirements on interest rate derivatives. We define the collateral requirements on interest rate derivatives (CRr_{it}) as the absolute difference between the market value of the portfolio of derivatives after a predetermined shock, \( MV_{s,it} \), minus its current market value, \( MV_{c,it} \). We express the absolute value of the change relative to the pension fund’s total assets under management (AUM):

\[
CRr_{it} = \frac{|(MV_{s,it} - MV_{c,it})|}{AUM_{it}}
\]

In addition to hedging interest rate risk, pension funds also want to limit the risk of a depreciation of foreign currencies to maintain the market value of the investment in its own currency. Pension funds mainly use forward contracts to hedge currency risk, where the pension fund agrees to sell an amount in the foreign currency in the future for a predetermined exchange rate. If however the exchange rate of the foreign currency relative to the euro increases, the market value of the forward contracts decreases. Similar to interest rate derivatives, the pension fund is required to transfer eligible,
liquid collateral to the counter-party. The change in the market value of the currency derivatives after an increase in the foreign currency relative to the euro is therefore used to assess the liquidity needs of a pension fund as a result of collateral requirements on currency derivatives. We define the collateral requirements on currency derivatives \( \text{CRcurr}_{it} \) as the absolute difference between the market value of the portfolio of derivatives after a predetermined shock, \( MV_{curr}^{s, it} \), minus its current market value, \( MV_{curr}^{c, it} \). We express the absolute value of the change relative to the pension fund’s total AUM:

\[
\text{CRcurr}_{it} = \frac{|(MV_{curr}^{s, it} - MV_{curr}^{c, it})|}{AUM_{it}}
\]

In the model specification the collateral requirements are determined by using an increase in the interest rate of 1 percentage points and an increase of the foreign currency relative to the euro of 25 percentage points. In Section 5, we show that our results are robust against the size of the predetermined shock. As we only observe the aggregate market value of interest rate and currency derivatives, we cannot observe each interest rate and currency derivative individually. This implies we assume that all derivative contracts do have collateral requirements.
The collateral requirements on interest rate and currency derivatives are summarized in Panel B of Table 1. The collateral requirements arising from interest rate derivatives and currency derivatives are approximately of the same order of magnitude. The average collateral requirement on interest rate derivatives equals 4.95 percent of total AUM. The average collateral requirements on currency derivatives equals 4.75 percent. The 10th percentiles are in both cases equal to zero, revealing that part of the pension funds do not hedge interest rate and currency risk at all.

### D - Control variables

As control variables we include pension fund type \( \text{Type}_i \), size \( \text{Size}_{it} \) and the required funding ratio \( \text{Rfr}_{it} \). We distinguish three different pension fund types. The dataset covers 56 industry-wide pension funds, 10 professional group pension funds and 153 corporate pension funds. Industry-wide pension funds are generally mandatory pension funds and organize pensions for a specific industry or sector, e.g., civil servants and hospital staff. Professional group pension funds provide pensions for a single profession such as hairdressers and doctors. Corporate pension funds arrange pensions for a particular company. We measure size as the log of total assets under management.
The required funding ratio is also added as control variable. The required funding ratio measures the funding ratio that is required to comply with regulation. It is determined risk-based, depending on the specific risk profile of an individual pension funds. In order to ensure that the capital constraint argument applies in case a positive effect of the collateral requirements on the allocation to illiquid assets is found, we have to control for the required funding ratio. If two pension funds differ in their required funding ratios, the positive effect of interest rate and/or currency hedging on the allocation to illiquid assets might be the result of additional risk taking instead of risk-shifting. This could be the case if pension funds that hedge more interest rate and/or currency risks are also the pension funds that have higher required funding ratios.

The pension fund’s size and required funding ratio are summarized in Panel B, Table 1. The 10th and 90th percentile of the log of total AUM reveals that the distribution is right skewed. This shows that the largest pension funds in our sample are considerably larger compared to the mean pension fund. The average required funding ratio equals 115.67 percent and does only slightly vary across pension funds. The actual average funding ratio
equals 110.45, which is slightly below the required funding ratio of pension funds.

4 The model and empirical results

In order to assess the relationship between a pension fund’s illiquid asset allocation and liquidity and capital constraints, we estimate a static random effects Tobit model\textsuperscript{[10]} The Tobit model controls for left-censoring of the allocation to illiquid assets at zero, as a substantial number of pension funds does not invest in illiquid assets at all. In the model the dependent variable is the allocation to illiquid assets \((w_{illiq}^{it})\). In alternative model specifications we also use the allocation to liquid risky assets \((w_{liq}^{it})\) and total risky assets \((w_{risky}^{it})\) as dependent variables. The general model specification is as follows:

\[
w_{illiq}^{it} = \beta_0 + \beta_1 D_L^{it} + \beta_2 C_L^{it} + \beta_3 CRR^{it} + \beta_4 CRcurr^{it} + \beta_5 Size^{it} + \beta_6 Type_i + \beta_7 Rfr^{it} + \lambda_t + \epsilon^{it}
\]

The main explanatory variables of interest are the pension fund’s \footnote{Random effects structure corrects for time-invariant characteristics of the pension fund that we do not observe and potentially play a role.}
liability duration ($D_{L,it}$), the liability convexity ($C_{L,it}$), the collateral requirements on interest rate derivatives (CRr$_{it}$) and the collateral requirements on currency derivatives (CRcurr$_{it}$). The control variables are the log of total AUM (Size$_{it}$), the pension fund type (Type$_i$), and the required funding ratio (Rfr$_{it}$). We include dummies for the pension fund type, where the reference group is the industry-wide pension fund. The professional group pension fund is denoted by Prof$_i$ and the corporate pension fund is denoted by Corp$_i$. Moreover, we control for time-fixed effects ($\lambda_t$) in all our model specifications. Notice that in the model as specified in Equation (7) we implicitly assume that pension funds first decide the optimal hedging strategies and based on those strategies the allocation to illiquid assets is determined.

There are not sufficient long-term (government) bonds to cover all pension liabilities. This issue is more severe for pension funds with long liability duration. Therefore, it seems intuitive that pension funds with a higher liability duration are more likely to hedge interest rate risk through derivatives. This implies that $D_{L,it}$ and CRr$_{it}$ are correlated. Therefore, in the regression analysis we first include separately $D_{L,it}$ and $C_{L,it}$ and CRr$_{it}$ and CRcurr$_{it}$, then secondly we include all four variables of interest. We
will show that the coefficients do only slightly change relative to the
coefficient estimates of our main model. This indicates that the correlation
between the collateral requirements as a result of interest rate risk and the
liability duration of a pension fund do not lead to biased coefficient
estimates.

A - Liability duration

Table 2 shows the results of the main model specification in (7). The
first column in Panel A shows that liability duration has a positive effect on
the illiquid asset allocation, whereas liability convexity has a negative impact.
Both coefficients are statistically significant at the 1 percent significance level.
Up to a liability duration of 17.5 years \(\frac{\partial w_{it}}{\partial D_{L, it}} = \beta_1 + 2\beta_2 D_{L, it} = 0 \Rightarrow D_{L, it} = -\frac{\beta_1}{2\beta_2}\), the effect of liability duration is positive. After this point however,
the effect is reversed. A liability duration increase from 10 to 11 years is
followed by an increase in the allocation to illiquid assets of 0.64 percentage
points, all else equal. A liability duration increase of 25 to 26 leads to a
decrease in the allocation to illiquid assets of 0.75 percentage points, all else
equal\(^{11}\). In the figure below, the hump-shaped impact of liability duration

\(^{11}\) Notice that the coefficients are estimates based on the uncensored latent variable, not
the observed outcome. The coefficients are the right interpretation for all observations
on the dependent variable, \(w_{it}^{ILLIQ}\), above zero. In order to get the effect on the
on the illiquid asset allocation is shown.

Figure 2 The effect of the liability duration on the illiquid asset allocation. The calculations are based on assuming an industry-wide or corporate pension fund of average size (other variables are excluded as they are not statistically significant).

Up to a liability duration of 17.5 years, the results support our first hypothesis. A pension fund with a long liability duration has less short-term liabilities relative to a pension fund with a short liability duration. A longer liability duration implies it is less of a constraint for a pension fund not actual observed dependent variable, the coefficient estimates have to be multiplied by the probability of the dependent variable being above zero.
being able to trade frequently in illiquid assets. Beyond a liability duration of 17.5 years however, the second hypothesis is supported. The quadratic increase in the interest rate sensitivity that is associated with higher liability duration negatively affects the allocation towards illiquid assets. As a larger part of the available capital is absorbed by interest rate risk. This lowers the opportunity to invest in illiquid assets.

As an alternative specification we test the allocation to liquid risky assets $w_{it}^{LIQ}$. The results are shown in Panel A, Column (2). The liability duration does not have the same hump-shaped impact on the liquid risky asset allocation. This is in line with Merton (1969), namely that the allocation to liquid risky assets is independent of the investment horizon. The capital constraint argument should however also apply in case of liquid risky assets. The quadratic increase in the interest rate risk exposure associated with a higher liability duration also limits the opportunities for the pension fund to invest in liquid risky assets. In an additional test we estimate the allocation of liquid risky assets on the liability duration only and find a negative coefficient, significant at the 1% significance level. In the figure below, this linear decreasing effect of the liability duration on the liquid risky asset allocation is shown.
Figure 3 The effect of the liability duration on the liquid risky assets allocation. The calculations are based on assuming a pension fund with an average funding ratio (other variables are excluded as they are not statistically significant).

In Panel A, Column (3), we test for the total allocation to risky assets. As the largest part of the total allocation to risky assets consists of liquid risky assets, the liability duration does also not show a hump-shaped impact on the allocation to total risky assets.
B - Collateral requirements

Panel B of Table 2 focuses on the liquidity constraints of interest rate risk and currency risk hedging. Column (1) shows the impact on our key dependent variable, the allocation to illiquid assets. The collateral requirements of interest rate risk hedging ($CR_{rit}$) do not have an effect on the allocation to illiquid assets. This implies that neither the capital constraint nor the liquidity constraint argument dominates. However, we do find that the collateral requirements on hedging currency risk has a significant and positive impact on illiquid asset allocation. According to hypothesis 4 this implies the capital constraint dominates. The increased liquidity risk as a result of hedging additional currency risk is offset by a smaller capital constraint in the allocation to illiquid assets. A one standard deviation increase in the collateral requirements (an increase of 0.0462 $CR_{curr_{it}}$) implies an increase increase in the illiquid assets allocation of approximately 0.61 percentage points. The average allocation to illiquid assets of pension funds that do have a positive allocation to illiquid assets equals 10 percent. This implies an relative increase in the total illiquid asset allocation of approximately 6 percentage points.

There are several explanations for the different impacts of interest rate
and currency hedging on the illiquid asset allocation. First, currency derivatives do not always have collateral requirements, whereas interest rate derivatives do. This implies that if the market value of the currency forwards decreases, the pension fund is not always required to transfer cash or other highly liquid, risk-free assets to a margin account. Second, a margin call in case of interest rate derivatives due to an increase in the interest rate is associated with a decreases in other fixed income securities as well. If there is not enough cash at hand, pension funds have to use a relative larger part of their short-term liquid bonds as collateral. In case of a margin call on currency derivatives, the value of the assets however increases. This increase is the result of the increased value of the non-Euro investments. In this case, if there is not enough cash at hand, pension funds have to use a relative smaller part of their eligible, liquid assets as collateral.

The impact of collateral requirements on the allocation to liquid risky assets is shown in Panel B, column (2). Here the capital constraint dominates. Both interest rate risk hedging and currency risk hedging have a positive impact on the allocation to liquid risky assets. As the allocation to illiquid assets does not increase if the pension fund hedges more interest rate risk, it
seems that pension funds take into account liquidity risk in case of interest rate hedging. With respect to currency risk hedging, the capital constraint argument applies to both liquid risky and illiquid asset allocations. The magnitude of the coefficients for collateral requirements on currency hedging is approximately the same for the allocation to risky liquid as to illiquid assets.

C - Control variables

Furthermore, Table 2 shows that size has a positive and significant impact on the allocation to illiquid assets. A pension fund that is ten times larger in terms of total assets under management invests an additional 5 percentage points in illiquid assets. Illiquid assets are generally complex products and therefore the pension fund needs to have sufficient knowledge to be able to manage the risk of those asset classes. The larger a pension fund, the better the pension fund can afford to pay high costs to make investments decisions regarding different projects or hire external managers to make those decisions. These findings are consistent with Dyck and Pomorski (2011) and Andonov (2014), who show that the increase in the allocation to illiquid assets is relatively more pronounced for large institutional investors. Moreover,
Stoughton and Zechner (2011) argue that economies of scale in alternative assets exist because only large investors can afford to pay high fixed search costs to identify profitable projects or skilled external managers. On top of that, larger institutional investors are more qualified to access good quality projects at lower fees because they have more negotiation power.

We also find that corporate pension plans invest less in illiquid assets compared to industry wide pension plans. Relative to compulsory pension funds, corporate pension funds invest about 5 percentage points less in illiquid assets, all else equal. We believe that this difference is due to the fact that a corporate needs to report on its pension fund in the annual accounts. The riskiness of the pension plan impact the risk profile of their corporate (Jin et al. (2006)). An additional explanation is that corporate pension funds are to a higher extent exposed to sponsor default risk compared to compulsory and professional group pension funds. Therefore, they can take less risk (Broeders (2010)).
5 Robustness checks

5.1 Alternative measure of maturity

The liability duration of the pension fund is an implicit measure of the maturity of a pension fund. The higher the liability duration, the less short-term pension payments and thus the higher the relative number of young participants. We do not have information on the demographics of the pension fund participants. We do however know the total pension payments per year and the value of the liabilities per quarter. The ratio of pension payments to liabilities (Benefits$_{it}$) is an alternative measure for the maturity of the pension fund. If this ratio is high, a larger part of the pension fund’s participants consists of retirees.

The correlation between Benefits$_{it}$ and $D_{L,it}$ equals $-0.78$. This indeed reflects the expected strong negative correlation between the liability duration and the ratio of pension payments relative to total liabilities. Table 3 shows that replacing duration and duration squared by the ratio of pension payments to liabilities (Benefits$_{it}$) and the square root of the ratio (Benefits$_{it}^2$) also results in a hump-shaped impact on the illiquid assets allocation. Up to a ratio of pension payments to
pension liabilities of 5.5 percent, the allocation to illiquid assets is positively affected. After this point, the effect is reversed. Column (2) shows that this hump-shaped effect is absent when considering the allocation to liquid risky assets only.

In Section 2 we elaborate a theoretical motivation for the hump-shaped impact of the liability duration on the illiquid asset allocation. We define the liquidity constraint as the pension payments to be paid in the first year. The liquidity constraint as a fraction of the total liabilities exactly equals \( \text{Benefits}_{it} \). In the theoretical model however, we approximate \( \text{Benefits}_{it} \) as \( \frac{1}{2D_{L_{it}}} \). Figure 4 shows that \( \text{Benefits}_{it} \) is indeed an exponentially decreasing function of liability duration. The shape of the line provides empirical evidence for the assumed shape of the liquidity constraint in the theoretical framework.
Figure 4  This figure shows the observed ratios of pension payments to pension liabilities ($\text{Benefits}_{it}$) and the fitted curve for the observed ratios of pension payments to pension liabilities. This figure uses observations from the second quarter of 2015.

5.2 Alternative specification of collateral requirements

In our main model specification, we use a 1 percentage points increase in the interest rate as predetermined shock in order to proxy the potential margin calls due to interest rate hedging. We have used a 25 percentage
points increase in the foreign currency relative to the euro as the predetermined shock in case of currency derivatives. In Table 3 we use a predetermined shock in the interest rate of 0.5 percentage points and an appreciation of the foreign currency of 12.5 percentage points. Table 3 shows that our results are robust against the choice for the predetermined shocks in order to proxy the potential margin calls as a result of interest rate and currency hedging.

6 Conclusion

In this paper we study the impact of liquidity and capital constraints on pension fund’s illiquid assets allocation. We use unique and proprietary data of all Dutch defined benefit pension funds covering the period 2012-2015. Liquidity constraints result from short-term pension payments and collateral requirements on interest rate and currency derivatives being used for hedging purposes. In addition, defined benefit pension funds are capital constrained. They need to have sufficient capital to manage risks such as interest rate risk, market risk, currency risk and longevity risk. Capital is the surplus of the value of assets minus the value of liabilities.
Liquidity and capital constraints interact. Fewer short-term liabilities, i.e. a longer liability duration, creates opportunities to invest in illiquid assets. However, a longer liability duration is associated with a quadratic increase in interest rate risk, which limits the opportunity to invest in illiquid assets due to a higher capital requirement. Furthermore, a pension fund can hedge certain risk exposures to reduce the capital requirement. This way hedging policies create opportunities to invest in illiquid assets. For instance, by hedging interest rate and currency risk, the pension fund is less exposed to these two risk factors and can take additional risk elsewhere before the capital constraint is binding. However, hedging strategies using derivatives involves collateral requirements. This hampers pension funds to invest in illiquid assets as they impose a liquidity constraint.

The key conclusions of our empirical analysis are as follows. First, we find a hump-shaped impact of liability duration on the illiquid asset allocation. Up to 17.5 years, the liability duration positively affects this allocation. A higher liability duration means that a pension is less constrained by short-term pension payments. This creates opportunities to invest in illiquid assets. However, beyond this point, the effect is reversed. The allocation to illiquid assets decreases. The reason is the non-linear
nature of interest rate risk. A higher liability duration is associated with a penalty in the form of stronger liability convexity. Pension funds with a higher liability duration require extra capital to manage this risk.

Second, we do not find evidence that interest rate hedging and collateral requirements on interest rate derivatives impact the illiquid assets allocation. This indicates that neither the capital or the liquidity constraint of hedging interest rate risk dominates. In case of currency risk hedging, however, the collateral requirements on currency derivatives impact the illiquid asset allocation positively. In line with the positive (less capital requirements) and negative (more liquidity needs) implications of hedging, currency risk hedging creates the opportunity to invest in illiquid assets. Liquidity risk is not priced in Dutch regulation. There is no regulatory penalty for the liquidity risk of using derivatives and investments in illiquid asset. As a result, given a certain capital, a pension fund is actually able to increase the exposure to illiquid assets by hedging more currency risk.

Finally, we find that also other pension fund characteristics impact the investment policy. Size positively affects the allocation to illiquid assets, which is in line with Dyck and Pomorski (2011) and Andonov (2014).

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Furthermore, corporate pension funds tend to invest less in illiquid assets compared to industry-wide and professional group pension funds. Corporate pension funds generally take less mismatch risk as this risk reflects on the corporate balance sheet (Jin et al. (2006)).

These findings offer important policy implications. It does not appear obvious for long term constrained institutional investors to automatically invest more in illiquid assets. The capital constraint for defined benefit pension funds becomes binding if the duration is substantially high. Although relaxing capital requirements for interest rate risk might seem a reasonable solution to mitigate the constraint, this is not what we recommend. The interest rate risk is inherent to the nature of the pension liabilities in a defined benefit pension contract. If a pension fund offers guaranteed benefits the interest rate risk becomes one of the key risk factors to manage. The interest rate risk by definition increases for guarantees that stretch over a longer horizon. A better approach would be to redefine the nature of the liabilities such that they no longer embed long term interest rate guarantees. This however means that the risk increases for the pension fund’s beneficiaries. Those beneficiaries should be able to understand and bear the increased risk.
Another policy implication concerns the importance of liquidity and collateral management for pension funds. Pension funds increasingly use derivatives to hedge risks. Although the exposure to interest rate risk and currency risk decrease through the application of derivatives, it also involves greater liquidity needs. Those liquidity needs can increase exponentially in times of market turbulence. Collateral management is key and becomes even greater once pension funds are integrated in central clearing of derivatives. Pension funds should prepare adequate contingency planning to be able to manage collateral through periods of market turbulence. More emphasis could be placed on liquidity and collateral management in pension regulation, as this is largely missing today.

Appendix A

In this appendix we derive the liquidity constraint defined in Section 2.3. Suppose a pension fund with an equal age composition such that pension payments are fixed and equal to $x$ each year over a finite horizon $T$. The value of the liabilities $L$ is the discounted value of the fixed pension payments $x$ from time $t = 0$ to time $t = T$. 
\[ L = x \int_0^T \exp(-rt) dt = x \left( \frac{1 - \exp(-rT)}{r} \right) \]  

where \( r \) is the continuously compounded interest rate.

The pension payment in the first year relative to the value of the liabilities equals

\[ \frac{x}{L} = \frac{r}{1 - \exp(-rT)} \]  

For \( r \) close enough to zero we obtain

\[ \lim_{r \to 0} \frac{r}{1 - \exp(-rT)} = \frac{1}{T} \]

As a result, an approximation for the pension payment in the first year is given by

\[ x \approx \frac{L}{T} \]  

The duration of the liabilities is defined as the first derivative with respect to the interest rate

\[ D_L = -\frac{dL}{dr}/L = \frac{1}{r} + \frac{T}{1 - \exp(rT)} \]
Again, for \( r \) close enough to zero we obtain

\[
\lim_{r \to 0} \frac{1}{r} + \frac{T}{1 - \exp(rT)} = \frac{T}{2}
\]  

(13)

This implies we can rewrite the pension payment in the first year as a function of duration as

\[
x \approx \frac{L}{2D_L}
\]

(14)

This is equation (1) in Section 2.3.

**Appendix B**

In this appendix we show that the liability convexity \( C_L \) can be approximated by the liability duration \( D_L \) squared, used in Section 2.3.

Therefore, suppose that all future cash flows of the pension fund can be represented by a single cash flow of \( X \) at time \( T \). Using continuous compounding the value of the liabilities equals

\[
L = X \exp(-rT)
\]

(15)
It follows easily that the liability duration of this cash flow equals

\[ D_L = - \frac{dX \exp(-rT)}{dr} / L = T \] (16)

The liability convexity in this case satisfies

\[ C_L = \frac{d^2X \exp(-rT)}{dr^2} / L = T^2 \frac{X \exp(-rT)}{L} = T^2 = D_L^2 \] (17)

This shows that \( C_L \) can be approximated by \( D_L^2 \).

References


Table 1: Descriptive statistics
This table provides summary statistics of pension funds’ strategic asset allocation (Panel A) and the variables specified in Section 6.1 (Panel B): allocation to illiquid assets ($w_{it}^{ILLIQ}$), allocation to liquid risky assets ($w_{it}^{LIQ}$), allocation to risky assets ($w_{it}^{risky}$), log of total AUM ($Size_{it}$), liability duration $D_{L, it}$, convexity ($C_{L, it}$), collateral requirements on interest rate derivatives ($CR_{r, it}$), collateral requirements on currency derivatives ($CR_{curr, it}$) and, required funding ratio ($Rfr_{it}$). The summary statistics are computed as the equally weighted average over all pension funds and all quarters in the 2012-2015 period.

### Panel A: asset allocations

<table>
<thead>
<tr>
<th>Liquid assets</th>
<th>mean</th>
<th>std. dev.</th>
<th>p10</th>
<th>p90</th>
<th>obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>government bonds</td>
<td>0.3031</td>
<td>0.1990</td>
<td>0.0216</td>
<td>0.5750</td>
<td>3,693</td>
</tr>
<tr>
<td>stocks mature markets</td>
<td>0.2425</td>
<td>0.1036</td>
<td>0.1197</td>
<td>0.3541</td>
<td>3,693</td>
</tr>
<tr>
<td>credits</td>
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<td>0.1114</td>
<td>0.0000</td>
<td>0.2912</td>
<td>3,693</td>
</tr>
<tr>
<td>stocks emerging markets</td>
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<td>0.0315</td>
<td>0.0000</td>
<td>0.0810</td>
<td>3,693</td>
</tr>
<tr>
<td>inflation index-linked bonds</td>
<td>0.0196</td>
<td>0.0491</td>
<td>0.0000</td>
<td>0.0722</td>
<td>3,693</td>
</tr>
<tr>
<td>listed indirect real estate</td>
<td>0.0146</td>
<td>0.0239</td>
<td>0.0000</td>
<td>0.0500</td>
<td>3,693</td>
</tr>
<tr>
<td>commodities</td>
<td>0.0113</td>
<td>0.0192</td>
<td>0.0000</td>
<td>0.0460</td>
<td>3,693</td>
</tr>
<tr>
<td>Illiquid assets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>non-listed indirect real estate</td>
<td>0.0251</td>
<td>0.0369</td>
<td>0.0000</td>
<td>0.0744</td>
<td>3,693</td>
</tr>
<tr>
<td>direct real estate</td>
<td>0.0148</td>
<td>0.0415</td>
<td>0.0000</td>
<td>0.0539</td>
<td>3,693</td>
</tr>
<tr>
<td>mortgages</td>
<td>0.0141</td>
<td>0.0282</td>
<td>0.0000</td>
<td>0.0519</td>
<td>3,693</td>
</tr>
<tr>
<td>private equity</td>
<td>0.0101</td>
<td>0.0199</td>
<td>0.0000</td>
<td>0.0399</td>
<td>3,693</td>
</tr>
<tr>
<td>hedge funds</td>
<td>0.0081</td>
<td>0.0208</td>
<td>0.0000</td>
<td>0.0400</td>
<td>3,693</td>
</tr>
</tbody>
</table>

### Panel B: variables

| Allocation to illiquid assets | 0.0730 | 0.0761 | 0.0000 | 0.1818 | 3,652 |
| Allocation to liquid risky assets | 0.4710 | 0.1477 | 0.2755 | 0.6329 | 3,652 |
| Allocation to risky assets | 0.5440 | 0.1711 | 0.3079 | 0.7443 | 3,652 |
| Log of total AUM | 5.7788 | 0.8205 | 4.9561 | 6.8270 | 3,693 |
| Liability duration | 18.52 | 4.00 | 14.20 | 23.70 | 3,664 |
| Convexity | 358.70 | 157.83 | 201.64 | 561.69 | 3,664 |
| CR on interest rate derivatives | 0.0495 | 0.0452 | 0.0000 | 0.1032 | 3,690 |
| CR on currency derivatives | 0.0475 | 0.0462 | 0.0000 | 0.0948 | 3,690 |
| Required funding ratio | 1.1567 | 0.0709 | 1.0920 | 1.2225 | 3,690 |
| Actual funding ratio | 1.1045 | 0.1884 | 0.5993 | 2.9630 | 3,643 |
Table 2: The impact of capital and liquidity constraints on the illiquid assets allocation

In this table, we show the coefficient estimates based on the random-effects Tobit regression (7). The dependent variable in the first column is the allocation to illiquid assets ($w_{it}^{ILLIQ}$), the dependent variable in the second column is the allocation to liquid risky assets ($w_{it}^{LIQ}$) and the third column uses the allocation to risky assets as dependent variable ($w_{it}^{risky}$). The independent variables include the liability duration ($D_{L,it}$), the liability convexity ($C_{L,it}$), the collateral requirements on interest rate derivatives ($CR_{r,it}$), the collateral requirements on currency derivatives ($CR_{curr,it}$) and the log of total AUM ($Size_{it}$), dummy Corp$_i$, indicates whether a pension fund is a corporate pension fund, dummy Prof$_i$, indicates whether a pension fund is a professional pension fund, and Rfr$_it$ is the required funding ratio. Panel A shows the results including $D_{L,it}$ and $C_{L,it}$ together with control variables. Panel B shows the results including $CR_{r,it}$ and $CR_{curr,it}$ together with control variables. Panel C includes all four variables of interest: $D_{L,it}$, $C_{L,it}$, $CR_{r,it}$ and $CR_{curr,it}$. In each of the regressions we include time fixed effects. Standard errors are between parentheses; *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$.

<table>
<thead>
<tr>
<th>Panel A: liability duration</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable</td>
<td>$w_{it}^{ILLIQ}$</td>
<td>$w_{it}^{LIQ}$</td>
<td>$w_{it}^{risky}$</td>
</tr>
<tr>
<td>$D_{L,it}$</td>
<td>0.0161***</td>
<td>0.0037</td>
<td>0.0090</td>
</tr>
<tr>
<td></td>
<td>(0.0036)</td>
<td>(0.0061)</td>
<td>(0.0066)</td>
</tr>
<tr>
<td>$C_{L,it}$</td>
<td>−0.0005***</td>
<td>−0.0002</td>
<td>−0.0004**</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0002)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>$Size_{it}$</td>
<td>0.0514***</td>
<td>−0.0128</td>
<td>0.0209*</td>
</tr>
<tr>
<td></td>
<td>(0.0071)</td>
<td>(0.0107)</td>
<td>(0.0122)</td>
</tr>
<tr>
<td>Corp$_i$</td>
<td>−0.0509***</td>
<td>0.0117</td>
<td>−0.0268</td>
</tr>
<tr>
<td></td>
<td>(0.0136)</td>
<td>(0.0207)</td>
<td>(0.0241)</td>
</tr>
<tr>
<td>Prof$_i$</td>
<td>−0.0072</td>
<td>−0.0244</td>
<td>−0.0319</td>
</tr>
<tr>
<td></td>
<td>(0.0277)</td>
<td>(0.0421)</td>
<td>(0.0490)</td>
</tr>
<tr>
<td>Rfr$_it$</td>
<td>0.0074</td>
<td>0.1065***</td>
<td>0.1101***</td>
</tr>
<tr>
<td></td>
<td>(0.0140)</td>
<td>(0.0326)</td>
<td>(0.0348)</td>
</tr>
<tr>
<td>constant</td>
<td>−0.3503***</td>
<td>0.3974***</td>
<td>0.2626**</td>
</tr>
<tr>
<td></td>
<td>(0.0606)</td>
<td>(0.0943)</td>
<td>(0.1060)</td>
</tr>
<tr>
<td>time fixed effects</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>number of obs</td>
<td>3,558</td>
<td>3,558</td>
<td>3,558</td>
</tr>
<tr>
<td>left-censored observations</td>
<td>957</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>uncensored observations</td>
<td>2,601</td>
<td>3,558</td>
<td>3,558</td>
</tr>
</tbody>
</table>
### Panel B: hedging

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>( w_{it}^{III IQ} )</th>
<th>( w_{it}^{LIQ} )</th>
<th>( w_{it}^{risk} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CR_{it} )</td>
<td>-0.0355 ( (0.0284) )</td>
<td>0.2297*** ( (0.0481) )</td>
<td>0.2235*** ( (0.0512) )</td>
</tr>
<tr>
<td>( CR_{curr, it} )</td>
<td>0.1331*** ( (0.0200) )</td>
<td>0.1340*** ( (0.0441) )</td>
<td>0.2239*** ( (0.0469) )</td>
</tr>
<tr>
<td>( Size_{it} )</td>
<td>0.0530*** ( (0.0070) )</td>
<td>-0.0086 ( (0.0107) )</td>
<td>0.0264** ( (0.0122) )</td>
</tr>
<tr>
<td>( Corp_{i} )</td>
<td>-0.0474*** ( (0.0136) )</td>
<td>0.0280 ( (0.0208) )</td>
<td>-0.0073 ( (0.0241) )</td>
</tr>
<tr>
<td>( Prof_{i} )</td>
<td>-0.0045 ( (0.0280) )</td>
<td>-0.0127 ( (0.0428) )</td>
<td>-0.0176 ( (0.0497) )</td>
</tr>
<tr>
<td>( Rfr_{it} )</td>
<td>0.0086 ( (0.0138) )</td>
<td>0.1149*** ( (0.0323) )</td>
<td>0.1188*** ( (0.0345) )</td>
</tr>
<tr>
<td>constant</td>
<td>-0.2341*** ( (0.0479) )</td>
<td>0.3303*** ( (0.0779) )</td>
<td>0.2195** ( (0.0883) )</td>
</tr>
</tbody>
</table>

- Time fixed effects: \( Y \)
- Number of obs: 3,583, 3,583, 3,583
- Left-censored observations: 971, 2, 2
- Uncensored observations: 2,612, 3,581, 3,581
### Panel C: liability duration & hedging

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>$w_{it}^{ILLIQ}$</th>
<th>$w_{it}^{LIQ}$</th>
<th>$w_{it}^{risky}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{L,it}$</td>
<td>0.0147***</td>
<td>0.0028</td>
<td>0.0076</td>
</tr>
<tr>
<td></td>
<td>(0.0036)</td>
<td>(0.0061)</td>
<td>(0.0066)</td>
</tr>
<tr>
<td>$C_{L,it}$</td>
<td>-0.0004***</td>
<td>-0.0002</td>
<td>-0.0003***</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0002)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>$CRR_{it}$</td>
<td>-0.0297</td>
<td>0.2359***</td>
<td>0.2334***</td>
</tr>
<tr>
<td></td>
<td>(0.0284)</td>
<td>(0.0484)</td>
<td>(0.0514)</td>
</tr>
<tr>
<td>$CR_{curr_{it}}$</td>
<td>0.1233***</td>
<td>0.1350***</td>
<td>0.2187***</td>
</tr>
<tr>
<td></td>
<td>(0.0203)</td>
<td>(0.0445)</td>
<td>(0.0472)</td>
</tr>
<tr>
<td>$Size_{it}$</td>
<td>0.0519***</td>
<td>-0.0134</td>
<td>0.0201*</td>
</tr>
<tr>
<td></td>
<td>(0.0070)</td>
<td>(0.0107)</td>
<td>(0.0122)</td>
</tr>
<tr>
<td>Corp$_i$</td>
<td>-0.0512***</td>
<td>0.0169</td>
<td>-0.0216</td>
</tr>
<tr>
<td></td>
<td>(0.0135)</td>
<td>(0.0208)</td>
<td>(0.0240)</td>
</tr>
<tr>
<td>Prof$_i$</td>
<td>-0.0081</td>
<td>-0.0246</td>
<td>-0.0326</td>
</tr>
<tr>
<td></td>
<td>(0.0275)</td>
<td>(0.0421)</td>
<td>(0.0488)</td>
</tr>
<tr>
<td>$Rfr_{it}$</td>
<td>0.0100</td>
<td>0.1183***</td>
<td>0.1241***</td>
</tr>
<tr>
<td></td>
<td>(0.0140)</td>
<td>(0.0325)</td>
<td>(0.0347)</td>
</tr>
<tr>
<td>constant</td>
<td>-0.3461***</td>
<td>0.3760</td>
<td>0.2418**</td>
</tr>
<tr>
<td></td>
<td>(0.0601)</td>
<td>(0.0943)</td>
<td>(0.1056)</td>
</tr>
</tbody>
</table>

| time fixed effects | Y               | Y               | Y               |
| number of obs      | 3,558           | 3,558           | 3,558           |
| left-censored observations | 957              | 0              | 0               |
| uncensored observations | 2,601         | 3,558           | 3,558           |
Table 3: Robustness check - alternative measure liability duration

In this table, we show the coefficient estimates based on the random-effects Tobit regression (7), using an alternative measure for the liability duration. The dependent variable in the first column is the allocation to illiquid assets ($w_{it}^{ILLIQ}$). The dependent variable in the second column is the allocation to liquid risky assets ($w_{it}^{LIQ}$). The third column uses the allocation to total risky assets as dependent variables ($w_{it}^{risky}$). As independent variables we include Benefits$_{it}$, the ratio of pension payments to pension liabilities, Benefits$_{it}^2$, the ratio of pension payments to pension liabilities squared, Size$_{it}$, the log of total AUM, Corp$_i$, indicating whether pension fund is a corporate pension fund, Prof$_i$, indicating whether the pension fund is a professional pension fund, and, Rfr$_{it}$, the required funding ratio. In each of the regressions we include time fixed effects. Standard errors are between parentheses; *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>(1) $w_{it}^{ILLIQ}$</th>
<th>(2) $w_{it}^{LIQ}$</th>
<th>(3) $w_{it}^{risky}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benefits$_{it}$</td>
<td>2.8396*** (0.3654)</td>
<td>2.8212*** (0.7277)</td>
<td>4.6143*** (0.7753)</td>
</tr>
<tr>
<td>Benefits$_{it}^2$</td>
<td>-25.8820*** (3.7975)</td>
<td>-12.4716 (7.5549)</td>
<td>-27.9385*** (8.0163)</td>
</tr>
<tr>
<td>Size$_{it}$</td>
<td>0.0471*** (0.0066)</td>
<td>-0.0151 (0.0097)</td>
<td>0.0185* (0.0107)</td>
</tr>
<tr>
<td>Corp$_i$</td>
<td>-0.0580*** (0.0130)</td>
<td>-0.0022 (0.0182)</td>
<td>-0.0451** (0.0201)</td>
</tr>
<tr>
<td>Prof$_i$</td>
<td>-0.0091 (0.0263)</td>
<td>-0.0403 (0.0367)</td>
<td>-0.0497 (0.0405)</td>
</tr>
<tr>
<td>Rfr$_{it}$</td>
<td>0.3167*** (0.0379)</td>
<td>0.9250*** (0.0758)</td>
<td>1.1544*** (0.0809)</td>
</tr>
<tr>
<td>constant</td>
<td>-0.5974*** (0.06053)</td>
<td>-0.5927*** (0.1062)</td>
<td>-0.9817*** (0.1149)</td>
</tr>
<tr>
<td>time fixed effects</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>number of obs</td>
<td>3,290</td>
<td>3,290</td>
<td>3,290</td>
</tr>
<tr>
<td>left-censored obs</td>
<td>887</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>uncensored obs</td>
<td>2,403</td>
<td>3,288</td>
<td>3,288</td>
</tr>
</tbody>
</table>
Table 4: Robustness check - definition collateral requirement

In this table, we show the coefficient estimates based on the random-effects Tobit regression (7). The dependent variable in Column (1) is the allocation to illiquid assets ($w_{it}^{ILIQ}$). The dependent variable in Column (2) is the allocation to liquid risky assets ($w_{it}^{LIQ}$) and in Column (3) the allocation to total risky assets ($w_{it}^{risky}$). The independent variables are $D_{L,it}$, the liability duration, $C_{L,it}$, the liability convexity, $CR_{r,it}$, a proxy for the potential margin call as a result of interest rate derivatives, $CR_{curr,it}$, a proxy for the potential margin call as a result of currency derivatives, $Size_{it}$, the log of total AUM, $Corp_{i}$, indicating whether pension fund is a corporate pension fund, $Prof_{i}$, indicating whether the pension fund is a professional pension fund, and, $Rfr_{it}$, the required funding ratio. Standard errors are between parentheses; *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>(1) $w_{it}^{ILIQ}$</th>
<th>(2) $w_{it}^{LIQ}$</th>
<th>(3) $w_{it}^{risky}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{L,it}$</td>
<td>0.0150***</td>
<td>0.0036</td>
<td>0.0085</td>
</tr>
<tr>
<td></td>
<td>(0.0036)</td>
<td>(0.0061)</td>
<td>(0.0066)</td>
</tr>
<tr>
<td>$C_{L,it}$</td>
<td>−0.0004***</td>
<td>−0.0002</td>
<td>−0.0004**</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0002)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>$CR_{r,it}$</td>
<td>0.0086</td>
<td>0.3078***</td>
<td>0.3188***</td>
</tr>
<tr>
<td></td>
<td>(0.0420)</td>
<td>(0.0597)</td>
<td>(0.0634)</td>
</tr>
<tr>
<td>$CR_{curr,it}$</td>
<td>0.1353***</td>
<td>0.0658</td>
<td>0.1563***</td>
</tr>
<tr>
<td></td>
<td>(0.0248)</td>
<td>(0.0548)</td>
<td>(0.0583)</td>
</tr>
<tr>
<td>$Size_{it}$</td>
<td>0.0522***</td>
<td>−0.0126</td>
<td>0.0212*</td>
</tr>
<tr>
<td></td>
<td>(0.0070)</td>
<td>(0.0107)</td>
<td>(0.0123)</td>
</tr>
<tr>
<td>$Corp_{i}$</td>
<td>−0.0507***</td>
<td>0.0153</td>
<td>−0.0229</td>
</tr>
<tr>
<td></td>
<td>(0.0135)</td>
<td>(0.0208)</td>
<td>(0.0241)</td>
</tr>
<tr>
<td>$Prof_{i}$</td>
<td>−0.0075</td>
<td>−0.0240</td>
<td>−0.0317</td>
</tr>
<tr>
<td></td>
<td>(0.0276)</td>
<td>(0.0423)</td>
<td>(0.0491)</td>
</tr>
<tr>
<td>$Rfr_{it}$</td>
<td>0.0094</td>
<td>0.1126***</td>
<td>0.1177***</td>
</tr>
<tr>
<td></td>
<td>(0.0140)</td>
<td>(0.0325)</td>
<td>(0.0347)</td>
</tr>
<tr>
<td>constant</td>
<td>−0.3500***</td>
<td>0.3767***</td>
<td>0.2404**</td>
</tr>
<tr>
<td></td>
<td>(0.0602)</td>
<td>(0.0945)</td>
<td>(0.1060)</td>
</tr>
</tbody>
</table>

| time fixed effects | Y                   | Y                   | Y                   |
| number of obs      | 3,556               | 3,556               | 3,556               |
| left-censored observations | 957     | 0               | 0                   |
| uncensored observations | 2,599 | 3,556           | 3,556               |
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