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* Views expressed are those of the author and do not necessarily reflect official positions of De Nederlandsche Bank.

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US monetary regimes and optimal monetary policy in the Euro Area*

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Abstract

Monetary policy in the US has been documented to have switched from reacting weakly to inflation fluctuations during the '70s, to fighting inflation aggressively from the early '80s onwards. In this paper, I analyze the impact of the US monetary policy regime switches on the Eurozone. I construct a New Keynesian two-country model where foreign (US) monetary policy switches regimes over time. I estimate the model for the US and the Euro Area using quarterly data and find that the US has switched between those two regimes, in line with existing evidence. I show that foreign regime switches affect home (Eurozone) inflation and output volatility and their responses to shocks, substantially, as long as the home central bank commits to a time invariant interest rate rule reacting to domestic conditions only. Optimal policy in the home country instead requires that the home central bank reacts strongly to domestic producer price inflation and to international variables, like imported goods relative prices. In fact, I show that currency misalignments and relative prices play a crucial role in the transmission of foreign monetary policy regime switches internationally. Interestingly, I show that only marginal gains arise for the Euro Area when the ECB adjusts its policy according to the monetary regime in the US. Thus, a simple time-invariant monetary policy rule with a strong reaction to PPI inflation and relative prices is enough to counteract the effects of monetary policy switches in the US.

Keywords*:* Monetary Policy, Markov-switching DSGE and Bayesian estimation, optimal monetary policy, international spillovers.
JEL classifications*:* C3, E52, F3, F41, F42.

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1 Introduction

The analysis of international monetary policy and its effects has attracted much research since the seminal work of Canzoneri and Gray (1985) and Canzoneri and Henderson (1991). The analysis, so far, has focused on the international spillovers of optimal monetary policy and on the conditions necessary to guarantee large, marginal or even zero gains from international monetary policy cooperation (Benigno and Benigno (2006); Corsetti and Pesenti (2001, 2005); Coenen et al. (2008); Pappa (2004)). The consensus is that large economies are interdependent and as such, the effects of their monetary policies are international. However, the evidence about the international effects of regime changes in the monetary policy of one country is very poor. Is one country affected when another changes its monetary policy, and if so, how? Shall the former react to changes in the policy of the latter? In this paper, I address these questions.

In the empirical literature, it has been widely documented that the systematic behavior of the Fed has not been stable at all times (Bianchi (2013a); Bianchi (2012); Boivin (2006); Boivin and Giannoni (2006, 2002); Lubik and Schorfheide (2004)). In particular, its reaction to inflation fluctuations has been documented to have been weak during the 1970s, leading to high inflation. On the contrary, following Volcker's appointment, US inflation started to fall continuously from 1980 onwards, until it stabilized to a substantially lower level due to a switch in Fed's policy towards fighting inflation more aggressively. However, little evidence exists regarding the international effects of changes in US monetary policy. How would another big country be affected by those switches, and how should it react? To address these questions, I construct a two country DSGE model and estimate it for the US and the Euro Area, considering the latter as the home country.

I address the first question of the paper using a two-country model with monopolistic competition, nominal rigidities and local currency pricing, in the spirit of Benigno (2004); Benigno and Benigno (2006); Corsetti and Pesenti (2005, 2001); Devereux and Engel (2003). I extend the model by assuming that monetary policy in the foreign country (US) varies over time.[1] In particular, I allow the coefficients on inflation and output gap in the foreign interest rate rule and the volatilities of the structural shocks to change over time. I assume that the volatilities change independently of the changes in the interest rate rule parameters. Specifically, I model those changes as two independent Markov-switching (MS) processes, in the spirit of Hamilton (1989). Agents in both regions are aware of the possibility of regime shifts. Therefore, the law of motion of the variables depends on agents' beliefs around alternative regimes as well. Monetary policy in the home country (Euro Area) instead is assumed to remain unchanged over time.

[1] Adopting the language of the relevant literature on monetary policy regime switches, a hawkish regime is one in which the foreign central bank (US) reacts aggressively to inflation fluctuations (Taylor principle). A dovish regime instead is one in which the foreign central bank reacts passively to inflation fluctuations (i.e. the coefficient on inflation in its interest rate rule is less than one).

I perform a Bayesian estimation of the model using quarterly data, following the approach of Liu and Mumtaz (2011). In line with estimated closed economy Markov-switching (MS-DSGE) models, I find that the monetary policy of the Federal Reserve has indeed varied over time and is characterized by a weaker response of the federal funds rate (FFR) to inflation fluctuations during the '70s, mainly (dovish regime). I find that the probability of a switch to a regime with a stronger reaction to inflation increases substantially from the early '80s onwards and stays persistently high until the early 2000s. The estimated monetary policy of the ECB yields a strong-enough reaction to Euro Area inflation fluctuations, in line with the mandate of the bank.

I compute the impulse responses in each regime conditionally and find that regime changes in US monetary policy have non-negligible effects not only domestically, but also in the Euro Area. Specifically, in the dovish regime, I show that the responses of inflation and output in both countries are amplified, especially after shocks originating in the US. The new result in this part is that inflation in the Euro Area can become more volatile, even though the ECB commits to an interest rate rule with a consistently strong reaction to inflation fluctuations. Apart from the exchange rate channel, I show that the international effects of US monetary policy shifts are transmitted through relative prices, currency misalignment and deviations from the law of one price, all of which affect output and firm's marginal costs in the Euro Area, directly.[2]

In order to address the second question of the paper, I design the optimal monetary policy problem of the home country when foreign monetary policy switches regimes. I derive the objective function of the home central bank from a second order approximation to the welfare of the representative home household and focus on the optimal discretionary policy. I compute both the optimal time varying and the optimal time invariant policy for the Euro Area conditional on foreign monetary policy regime switches. Using an appropriate welfare cost measure (as in Ravenna and Walsh (2011)), I show that only marginal welfare gains can arise for the Euro Area when the ECB reacts to each foreign regime optimally by adjusting its policy according to the latter. Therefore, the ECB does not need to change its policy over time according to the monetary regime in the US. However, I show that the ECB needs to react not only to domestic conditions, but also to international ones. That is, it is important to take into account how those switches are transmitted internationally. More importantly, I show that under this policy, the ECB is able to control the effects of US monetary policy regime switches better, leaving inflation in the Euro Area almost unaffected while dampening the effects on output substantially.

To compute the optimal policy in the home country, I extend Soderlind (1999) algorithm to account for regime switches. Optimal discretionary policy suggests an interest rate rule

[2] Currency misalignment and deviations from the law of one price contribute to the transmission of foreign monetary policy regime switches since producers engage in local currency pricing (LCP). If instead they were engaging in producer currency pricing (PCP), then foreign regime switches would have been transmitted into the home country through their effect on the terms of trade only.

where the control variable (i.e. interest rate) reacts to all state variables in the maximization problem. This rule though is tough to implement. For this reason, I compute an implementable rule which is able to replicate the optimal allocation under the optimal time invariant discretionary monetary policy. I follow an approach similar to that in Schmitt-Grohe and Uribe (2007) and show that the rule that is able to replicate this allocation is one in which the ECB targets home producer-price inflation, the output gap and imported goods relative price. Therefore, the ECB should target both domestic and international variables.[3] This is how the ECB can counteract the effects of foreign regime shifts on home agents expectations. Finally, the importance of imported goods relative price in the rule is attributed to the fact that it is one of the main channels through which US monetary policy regime shifts are transmitted to the Euro Area.

To the best of my knowledge this is the first paper that estimates an MS-DSGE model for the Euro Area and the US, while providing an analysis of the international transmission of monetary policy regime switches in the latter region and then designing the optimal monetary policy response of the former.[4] With the above analysis, I show that the Eurozone can benefit a lot when the ECB takes into account the way regime switches in US monetary policy are transmitted internationally.

The paper is organized as follows. In section 2, I present the literature related to the study of this paper. I describe the theoretical model in section 3. I introduce Markov switching monetary policy in section 4 where I also describe how the model is solved and estimated. In section 5, I compute the impulse responses and explore how regime switches in the US are transmitted internationally. The optimal policy problem of the Euro Area is developed in section 6, while section 7 concludes.

2 Related Literature

In the literature, there is a large number of papers focusing on regime shifts in monetary policy. On the empirical side, the majority of those papers analyze the monetary policy of the Fed in the last fifty years in order to explore the causes of the Great Inflation of the 1970s (pre-Volcker era) and the low inflation volatility from 1980 onwards (post-Volcker era). In particular, many authors support the view that it was mainly Fed's weak reaction to inflation during the 1970s that led to high volatility, while it was again Fed's aggressive reaction to inflation fluctuations that lowered inflation and output volatility from 1980 onwards (Boivin

[3]In the literature on optimal monetary policy for open economies with local currency pricing, home optimal monetary policy should not be 'inward' looking if expressed through a targeting rule. That is, it faces trade offs concerning foreign and international variables as well (foreign inflation, foreign output, real exchange rates and/or currency misalignments). However, if optimal monetary policy is expressed through an instrument rule, the interest rate rule that is able to implement the optimal allocation is one in which the home central bank targets home CPI only. With the analysis in this paper, I show that this is no longer the case whenever foreign monetary policy switches regimes. For a more detailed analysis on optimal monetary policy policy with LCP see Corsetti et al. (2010) and Engel (2011) and the references therein.

[4]Similar works on the design of optimal monetary policy with regime switches are those of Blake and Zampolli (2011) and Zampolli (2006). However, in those papers regime switching is introduced in the data generating process for the nominal exchange rate. Moreover, they do not have a two-country setup and they use an ad-hoc objective function for the central bank.

and Giannoni (2002, 2006); Boivin (2006); Clarida et al. (2000); Lubik and Schorfheide (2004)).[5] On the other hand, a number of authors attribute the lower inflation and output volatility in the post-Volcker era to lower shock volatility instead of better policy (Cogley and Sargent (2005); Primiceri (2005); Sims and Zha (2006); Stock and Watson (2002)). The target, though, of this paper is to explore the consequences of changes in the monetary policy of one country for another. For this reason, I abstract from changes in volatilities and focus on regime switches in monetary policy, only.

Recently, Markov-switching techniques in DSGE models for monetary policy analysis have been introduced in a number of papers. Specifically, monetary policy regime switches are introduced within a single model in order to analyze the effects of those on inflation and output. The virtue of these models rests on the fact that agents have now a richer information set in which possible regime switches in monetary policy are incorporated. Such an analysis is superior to other fixed regime models that have been used in the literature and in which agents do not have this enlarged information set (Boivin and Giannoni (2006); Lubik and Schorfheide (2004)). When agents know that monetary policy is likely to switch to another regime in the near future, they update their inflation expectations accordingly. Consequently, regime switches affect inflation and output a number of periods before the actual regime change date, as shown in calibrated models (Davig and Leeper (2007); Liu et al. (2009)). Another virtue of MS-DSGE models is that policy regime switches and changes in variances of the structural disturbances can be modeled within the same model. Baele et al. (2011); Bianchi (2013a); Bianchi and Ilut (2017); Davig and Doh (2009) estimate a MS-DSGE model for the US and find evidence of both regime shifts in monetary policy as well as on shock volatilities.

Apart from the estimation of a two country MS-DSGE model for the Euro Area and the US, what also distinguishes this paper from the current literature on MS-DSGE models is the analysis of the international effects of monetary policy regime switches. As already mentioned, there is ample evidence in the literature about the effects of regime switches on agents expectations and, hence, on macro variables in a closed economy framework. However, there is very poor evidence as regards whether and how regime switches in one country are transmitted to another country. In this paper, I analyze those channels to show that monetary policy regime switches have non-negligible effects to other countries as well.

On the international dimension, there is ample theoretical evidence in the new open economy macroeconomic literature as regards the design of (and the gains from) optimal monetary policy with or without cooperation between two countries and the transmission of nominal and real shocks (Benigno and Benigno (2006); Benigno (2002); Coenen et al. (2008); Corsetti and Pesenti (2001, 2005); Enders and Muller (2009); Engel (2011); Lombardo and Sutherland (2006); Pappa (2004)). However, there is very little evidence as regards the design of optimal monetary policy in one country when another switches regimes. For this reason, I bring the

[5] Other authors find only modest changes in the policy of the Fed from 1960 onwards (Primiceri (2006); Sargent et al. (2006)).

analysis a step further by designing the optimal monetary policy of one country, conditional on regime switches in another country, to show that when the former reacts to the policy regime of the latter the effects of regime switches are dampened substantially.

3 The model

In this section, I specify a two country stochastic general equilibrium model with nominal rigidities in the lines of Benigno (2004). The economy consists of two regions, the euro area, denoted as home country and the United States, denoted as foreign country. Each country is populated by a continuum of infinitely lived and identical households in the interval $[0, 1]$. Households derive utility from the consumption of home and foreign goods and disutility from labor supply. In each country, a continuum of goods is produced, each by a monopolist. There is no migration, in the sense that households supply labor to firms located within their own country. However, they get utility from goods produced in both countries. There is home bias in consumption in each country. That is, home households have a preference towards home goods, while foreign households have a preference towards foreign goods. Foreign variables are denoted with an asterisk.

Goods are produced by monopolistically competitive firms. Each firm specializes in the production of a specific good using only labor, subject to technology shocks. Firms set their prices in a sticky manner *à ℓa* Calvo (1983) and engage in local currency pricing (LCP). That is, they set the price for their good according to the country it is sold. This implies that the law of one price (LOOP) does not hold.

Monetary policy in conducted by the central bank of each country through an interest rate feedback rule. I assume that the foreign central bank changes its reaction to inflation and output gap over time according to a Markov-Switching process. Agents in both countries are fully aware of those regime shifts and share the same transition probabilities. Before the optimal monetary policy design of the home country, I assume that its central bank commits to a standard Taylor rule with some interest rate smoothing. This allows me to focus on the effects of foreign monetary policy regime shifts when the home central bank is naive and ignores completely what the foreign central bank is doing. Next, I proceed to the design of the optimal policy in the home country conditional on the regime of foreign monetary policy.

3.1 Households

The representative household l in the home country derives utility from consumption C_t and disutility from labor supply L_t. The utility function is specified as

$$U_t(l) = E_t \left\{ \sum_{t=t_0}^{\infty} \beta^t \left[\frac{(C_t(l))^{1-\sigma}}{1-\sigma} - \frac{(L_t(l))^{1+\gamma}}{1+\gamma} \right] \right\} \tag{1}$$

where σ is the degree of relative risk aversion and γ the inverse of the Frisch elasticity of labor supply. Home households consume home and foreign goods. The consumption aggregate $C_t(l)$ is a composite index described as

$$C_t(l) = \left[\delta^{\frac{1}{\rho}} C_{H,t}(l)^{\frac{\rho-1}{\rho}} + (1-\delta)^{\frac{1}{\rho}} C_{F,t}(l)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{2}$$

where ρ captures the intratemporal elasticity of substitution between home and foreign goods. $\delta > \frac{1}{2}$ is the parameter of home bias in preferences. $C_{H,t}$ and $C_{F,t}$ is the home and foreign goods consumption index, in the home country. In the foreign country $C^*_{H,t}$ and $C^*_{F,t}$ is the home and foreign goods consumption index, respectively. Consumption indices in the home country for household l are defined as

$$C_{H,t}(l) = \left[\int_0^1 c_t(l,h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \; C_{F,t}(l) = \left[\int_0^1 c_t(l,f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}} \tag{3}$$

where $c_t(l,h)$ and $c_t(l,f)$ are respectively consumption of home brand h and foreign brand f by the home household l at time t. θ is the elasticity of substitution of goods produced within the same country and is assumed to be the same across countries. Consumption indices in the foreign country, $C^*_{H,t}(l^*)$ and $C^*_{F,t}(l^*)$, are analogously defined. The aggregate consumption price index for the home country is specified as

$$P_t = \left[\delta (P_{H,t})^{1-\rho} + (1-\delta) P_{F,t}^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{4}$$

where P_H and P_F are price indices for home and foreign goods, expressed in the domestic currency and defined as

$$P_{H,t} = \left[\int_0^1 p_t(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}, \; P_{F,t} = \left[\int_0^1 p_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}} \tag{5}$$

Foreign households have analogous preferences. Capital markets are complete. The consumers of both countries purchase state uncontingent bonds denominated in the domestic currency, B_t for home households and B^*_t for foreign households at price Q_t. That is B_t denotes the home agent's holdings of a one period nominal bond paying one unit of the home currency. The home household maximizes its utility subject to the period budget constraint

$$P_t C_t(l) + Q_{t,t+1} B_{t+1}(l) = B_t(l) + W_t L_t(l) + \int_0^1 \Pi_t(h) dh - Tr_t(l) \tag{6}$$

where W_t is the nominal wage, Π_t are nominal profits the household receives and Tr_t are lump-sum taxes. Foreign households face an analogous budget constraint

3.2 First order conditions

Maximizing the utility function (1) subject to the budget constraint (6) and using the fact that home households are identical, in order to drop the index for the household, yields the following first order conditions

$$1 = \beta E_t \left[\frac{P_t}{Q_{t,t+1} P_{t+1}} \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \right] \tag{7}$$

$$L_t = C_t{}^{-\frac{\sigma}{\gamma}} w_t^{\frac{1}{\gamma}} \tag{8}$$

where the first equation is the usual Euler equation while the second determines the labor supply schedule. Home consumers demand functions for varieties h and f of goods produced in the home and foreign country, respectively, are expressed as

$$c_t(h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} \left(\frac{P_{H,t}}{P_t} \right)^{-\rho} \delta C_t, \; c_t(f) = \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\theta} \left(\frac{P_{F,t}}{P_t} \right)^{-\rho} (1-\delta) C_t \tag{9}$$

Since financial markets are complete internationally and bonds are denominated in the home currency the foreign household analogues of first order conditions (7) and (8) are:

$$1 = \beta E_t \left[\frac{P_t^* z_t}{Q_{t,t+1} P_{t+1}^* z_{t+1}} \left(\frac{C_t^*}{C_{t+1}^*} \right)^{\sigma} \right] \tag{10}$$

$$L_t^* = (C_t^*)^{-\frac{\sigma}{\gamma}} (w_t^*)^{\frac{1}{\gamma}} \tag{11}$$

where z_t is the nominal exchange rate defined as the domestic currency price of the foreign currency.

3.3 Price setting

Firms set one price for their good at home and one at the foreign country periodically. At each date, each firm changes its price with a probability $1-\omega$, regardless of the time since it last adjusted its price. The probability of not changing the price, thus, is ω. The probability of not changing the price in the subsequent ς periods is ω^{ς}. Consequently, the price decision at time t determines profits for the next ς periods. The price level for home goods at date t is defined as[6]

$$P_{H,t} = \left[\omega P_{H,t-1}^{1-\theta} + (1-\omega) \widetilde{p}_t(h)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{12}$$

Each brand h is produced by a single firm following the linear technology

$$Y_t(h) = A_t L_t(h) \tag{13}$$

where A_t is a country specific productivity shock at date t which is assumed to follow a log stationary AR(1) process, $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$. Firms set their prices by maximizing their expected discounted profits. Hence, their maximization problem is described as

[6] Here, I present the profit maximization problem of home firms. The respective problem for foreign firms is symmetric.

$$maxE_t \sum_{\varsigma=0}^{\infty} \omega^{\varsigma} \Lambda_{t,t+\varsigma} \left\{ p_{t+\varsigma}(h) c_{t+\varsigma}(h) + z_t p^*_{t+\varsigma}(h) c^*_{t+\varsigma}(h) - (1-\tau) W^h_{t+\varsigma} L^h_{t+\varsigma}(h) \right\} \tag{14}$$

where $\Lambda_{t,t+\varsigma} = \beta^{\varsigma} \left(C_{t+\varsigma}/C_t\right)^{-\sigma} \left(P_{t+\varsigma}/P_t\right)$ is the consumer intertemporal marginal rate of substitution, τ is a labor subsidy and $c_t(h)$ is the domestic demand for the home-produced good and $c^*_t(h)$ is the foreign demand for the home-produced good, respectively. The firm maximizes its objective function (14) subject to (13) and (9) in order to determine the optimal prices for the home good in the home ($\widetilde{p}_t(h)$) and the foreign country ($\widetilde{p}_t{}^*(h)$). The optimal prices for the home produced good in the home and foreign country are specified as

$$\tilde{p}_t(h) = \frac{\theta}{\theta-1} \frac{E_t \sum_{\varsigma=0}^{\infty} \omega^{\varsigma} \Lambda_{t,t+\varsigma} MC_{t+\varsigma} \tilde{c}_{t+\varsigma}(h)}{E_t \sum_{\varsigma=0}^{\infty} \omega^{s} Q_{t,t+\varsigma} \tilde{c}_{t+\varsigma}(h)} \tag{15}$$

$$\tilde{p}^*_t(h) = \frac{\theta}{\theta-1} \frac{E_t \sum_{\varsigma=0}^{\infty} \omega^{s} \Lambda_{t,t+\varsigma} MC_{t+\varsigma} \tilde{c}^*_{t+\varsigma}(h)}{E_t \sum_{\varsigma=0}^{\infty} \omega^{s} Q_{t,t+\varsigma} z_{t+\varsigma} \tilde{c}^*_{t+\varsigma}(h)} \tag{16}$$

where $\tilde{c}_{t+\varsigma}(h)$ and $\tilde{c}^*_{t+\varsigma}(h)$ are the demand from the home and the foreign country, respectively, under the condition that $\tilde{p}_t(h)$ and $\tilde{p}^*_t(h)$ still apply. Clearly, the optimal subsidy removing the monopolistic distortion satisfies $(1-\tau)(\frac{\theta}{\theta-1}) = 1$. This subsidy renders the steady state and the flexible price equilibrium efficient. Finally, let $T_t = P_{F,t}/P_{H,t}$ and $T^*_t = P^*_{H,t}/P^*_{F,t}$ denote the relative price of imported and exported goods, respectively.

3.4 Equilibrium

As in Gali (2002), I assume that the government in each country purchases domestically produced final goods. Home and foreign government expenditures G_t and G^*_t constitute a fraction χ_t and χ^*_t of the output of the home and foreign good, respectively. Therefore, total demands for the home good h and foreign good f read as follows

$$Y_t(h) = C_{H,t}(h) + C^*_{H,t}(h) + G_t(h) \quad and \quad Y^*_t(f) = C_{F,t}(f) + C^*_{F,t}(f) + G^*_t(f)$$

Goods market clearing in the home and foreign country are thus summarized by

$$Y_t = C_{H,t} + C^*_{H,t} + G_t \quad and \quad Y^*_t = C_{F,t} + C^*_{F,t} + G^*_t \tag{17}$$

In what follows I assume a stationary $AR(1)$ process for the demand shocks (g_t, g^*_t) in each country. That is, $g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$ and $g^*_t = \rho_{g^*} g^*_{t-1} + \varepsilon^*_{g,t}$, where $g_t = log(G_t) = -log(1-\chi_t)$ and $g^*_t = log(G^*_t) = -log(1-\chi^*_t)$.

Equations (7) and (10) yield the familiar condition describing perfect risk sharing:

$$\left(\frac{C^*_t}{C_t}\right)^{-\sigma} = \varpi q_t \tag{18}$$

where $\varpi \equiv \left(\frac{C_0^*}{C_0}\right)^{-\sigma} \frac{P_0}{z_0 P_0^*}$ depends on initial conditions and $q_t = \frac{z_t P_t^*}{P_t}$ is the real exchange rate.

4 Markov Switching

4.1 Solution and Estimation Technique

In this section, I describe how Markov switching is introduced into the model and how the resulting model is estimated. I follow the approach of Liu and Mumtaz (2011). I consider the Euro Area as the home country and the US as the foreign. I assume that the parameters in the Taylor rule of the Fed are subject to regime shifts while those in the respective rule of the ECB are time invariant. Moreover, I allow the variances of all the shocks in the model to be subject to regime shifts as well. I allow for independent regime switching in the volatility of the structural shocks that the model features. To specify the MS-DSGE model, I partition the parameter vector Φ into three blocks

$$\Phi = \left\{\Phi^S; \Sigma^s; \bar{\Phi}\right\}$$

where Φ^S is the set of parameters subject to regime shifts, Σ^s is the variance of the regime switching volatilities and $\bar{\Phi}$ denotes the remaining time-invariant parameters. The time invariant Taylor rule of the home country and the Markov-switching interest rate rule of the foreign country are specified as

$$R_t = R_{t-1}^{i_r} \left(\left(\frac{\pi_t}{\tilde{\pi}} \right)^{\phi_\pi} \tilde{Y}_t^{\phi_y} \right)^{1-i_r} e^{\varepsilon_{r,t}} \tag{19}$$

$$R_t^* = R_{t-1}^{*i^*_{r*,S_t}} \left(\xi^*_{S_t} \left(\frac{\pi_t^*}{\tilde{\pi}^*} \right)^{\phi^*_{\pi^*,S_t}} \tilde{Y}_t^{*\phi^*_{y^*,S_t}} \right)^{1-i^*_{r^*,S_t}} e^{\varepsilon^*_{r^*,t}} \tag{20}$$

where, in the home rule, $\tilde{\pi}$ is the home inflation target, $\tilde{Y}_t$ is the home output gap and $\varepsilon_{r,t}$ is a home monetary policy shock with time varying variance σ^2_{r,s_t}. In the foreign rule, $\xi^*_{S_t}$ is a scale parameter, $\tilde{\pi}^*$ is the foreign inflation target, $\tilde{Y}_t^*$ is the foreign output gap, defined as the deviation of output from its natural rate, which corresponds to the efficient flexible price equilibrium and $\varepsilon^*_{r^*,t}$ is a shock to foreign monetary policy with time varying variance $\sigma^2_{r^*,s_t}$. The superscript S denotes the unobserved regime associated with the foreign monetary policy parameters taking on values 1 or 2. Foreign monetary policy regime follows a Markov process with transition probabilities $p_{ji} = P\left[S_t = i | S_{t-1} = j\right]$, where $i, j = 1, 2$. The superscript $s = 1, 2$ in variances denotes the unobserved regime associated with the volatilities and which evolves independently of S. The two state variables S and s follow a first-order Markov chain with the following transition probability matrices, respectively:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad and \quad Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}$$

where $P_{ji} = p\left[S_t = i | S_{t-1} = j\right]$ and $Q_{ji} = p\left[s_t = i | s_{t-1} = j\right]$ where $i, j = 1, 2$. The model can be written in a matrix form as

$$A(S_t)X_{t+1} = B(S_t)X_t + \Psi(S_t)\varepsilon_t + \Pi(S_t)\eta_t, \quad where \quad \varepsilon_t \sim N(0, Q_s) \tag{21}$$

where $X_{t+1} = [y_{t+1}, y^*_{t+1}, \pi_{H,t+1}, \pi^*_{H,t+1}, \pi_{F,t+1}, \pi^*_{F,t+1}, z_{t+1}, T_{t+1}, T^*_{t+1}, y_t, y^*_t, \pi_{H,t}, \pi^*_{H,t}, \pi_{F,t}, \ldots$

$\ldots \pi^*_{F,t}, z_t, T_t, T^*_t, q_t, q_{t-1}, i_t, i^*_t, a_t, a^*_t, g_t, g^*_t, \epsilon_{cp}, \epsilon^*_{cp^*}]$, ε_t is a 8×1 vector of structural shocks of mean zero and whose variance is allowed to vary over time as specified above.[7] η_t is an 9×1 vector of endogenous random variables.

I solve the model using the approach of Farmer et al. (2011) a detailed description of which is described in Appendix B.3.[8] Farmer et al. (2011) show that if a unique solution exists then this can be cast as a Markov switching VAR of the following form:

$$X_t = g_{1,S_t}X_{t-1} + g_{2,S_t}\varepsilon_t \tag{22}$$

Equation (22) above can be combined with an observation equation giving the following state space model with Markov switching:[9]

$$\begin{aligned} X_t &= g_{1,S_t}X_{t-1} + g_{2,S_t}\varepsilon_t, \qquad where \quad \varepsilon_t \sim N(0, Q^s) \\ D_t &= HX_t \end{aligned} \tag{23}$$

where, as described above, the Markov states S and s evolve independently with transition probability matrices P and Q, respectively. D_t represents the observed data and matrix H is the loading matrix. As Liu and Mumtaz (2011) point out, the presence of the unobserved DSGE states X_t and the unobserved Markov states makes the standard Kalman filter not operational in order to provide inference on X_t and to calculate the value of the likelihood. Inference now has to be conditioned on both current and past values of S and s. Following their approach, I define a new state variable S^* indexing both S_t and s_t and which has a four state transition matrix, $P^* = P \otimes Q$. As in Kim and Nelson (1999) and Davig and Doh (2014), I track S^*_t, S^*_{t-1} and S^*_{t-2} which means that I account for $4^3 = 64$ possible paths

[7] As in Justiniano and Preston (2010) when loglinearizing the model I add an import cost-push shock, denoted by $\epsilon_{cp,t}$, to the home goods PPI inflation in the foreign country, $\pi^*_{H,t}$, and another to the respective PPI inflation of foreign goods imported in the home country, $\pi_{F,t}$, denoted by $\epsilon^*_{cp^*,t}$. The reason is mainly to have a number of structural shocks equal to the number of observables in the state space representation of the model.

[8] I log-linearize the model around the symmetric zero inflation steady state. The log-linearized model is summarized in Appendix B.2. The steady state must be unique, in order for cross regime comparisons in the log-linearized model to be valid. I provide the conditions guaranteeing steady state uniqueness in Appendix A.

[9] I assume no measurement errors.

for the state variables at each point in time.[10] As mentioned above, I follow a Bayesian approach to estimate the model, where I combine the approximate likelihood function with the assumed prior distributions and use a random walk Metropolis Hastings algorithm with 200,000 replications in order to approximate the posterior.

4.2 Priors

A summary of the prior distributions and the relevant bounds for the model parameters is provided in table 1. I specify those trying to remain close to the assumed prior distributions in the literature on either closed or open economy models (e.g. Justiniano and Preston 2010; Lubik and Schorfheide 2007; Liu and Mumtaz 2011; Smets and Wouters 2003, 2007). Given the size of the model and the number of parameters to be estimated, I assume that the risk aversion, σ, the inverse elasticity of the Frisch labor supply parameter, γ, and the home bias parameter, δ, are the same across the two regions. I calibrate the home bias parameter δ to equal 0.82, which is close to the average share of imports and exports in the two regions (captured by $1-\delta$ in the model). For simplicity, I assume that the degree of relative risk aversion σ is the same across the two regions and follows a Normal distribution with mean 1.5 and a standard deviation of 0.375. I borrow the assumed distribution from Smets and Wouters (2003, 2007). I follow the same approach for the inverse Frisch elasticity of labor supply γ which follows a Normal distribution with mean 2 and standard deviation of 0.75. As for the elasticity of substitution between home and foreign goods ρ, I also assume it to be same across the two regions following a Gamma distribution with a mean of 1.5 and a standard of 0.75.

As regards the Calvo pricing parameters ω and ω^*, I follow Smets and Wouters (2003, 2007) once again and assume that they follow a Beta distribution with a mean of 0.75 and standard deviation of 0.05 in the Euro Area and a mean of 0.5 and a standard deviation of 0.1 for the US, respectively. In the interest rate rule for the Euro Area, I assume that the smoothing parameter i_r follows a Beta distribution with a mean of 0.8 and standard deviation equal to 0.1, the inflation coefficient ϕ_π follows a Gamma distribution with a mean of 1.5 and a standard deviation of 0.25, while the coefficient on output gap follows also a Gamma distribution with a mean of 0.125 and a standard deviation of 0.05. The time varying foreign interest rate rule coefficients are specified as follows. The smoothing coefficient i^*_{r*} has a Beta prior distribution with a mean of 0.5 and a standard deviation of 0.25 in both regimes. For the reaction coefficients $\phi^*_{\pi^*}$ and $\phi^*_{y^*}$, they are assumed to follow a Gamma distribution with means 1.5 and 0.12 with standard deviations of 0.45 and 0.05, respectively, in the hawkish regime. In the dovish regime, I assume the same distributions but impose a mean on $\phi^*_{\pi^*}$ equal to 0.95 and standard deviation equal to 1.1 while I assume the same mean and standard

[10]For a more detailed description of Kim and Nelson (1999)'s algorithm see the technical appendix in Liu and Mumtaz (2011).

deviation for $\phi^*_{y^*}$ as in the hawkish regime.[11]

The autoregressive parameters for government spending and productivity shock in the Euro Area are assumed to follow a Beta distribution with a mean of 0.85 and a standard deviation of 0.1, as in Smets and Wouters (2003). Those parameters for the respective shocks in the US are assumed to follow a Beta distribution as well with a mean of 0.5 and a standard deviation of 0.2, as in Smets and Wouters (2007). Since I have borrowed the assumed cost push shocks in the imported goods inflation from Justiniano and Preston (2010), I adopt their assumption imposing a Beta prior with a mean for the autoregressive parameter of 0.5 and a standard deviation of 0.15, in each region. Finally, regarding the transition probabilities, as in Liu and Mumtaz (2011), I follow Sims and Zha (2006) in assuming a Dirichet prior, with the scale matrix chosen to reflect the belief that regimes are persistent. The relevant parameters for the Dirichet distribution imposed are $\alpha_1 = 18$ and $\alpha_2 = 1$. This yields a probability of staying in the same regime equal to 0.95. I also assume the priors on the model's structural parameters are symmetric across the different regimes.

4.3 Data

For the euro area, all quarterly data are from the AWM database of the ECB (see Fagan et al. (2001)) spanning from 1970:q1 to 2006:q4. The series for the real GDP per capita ($\hat{Y}_t$) are constructed by the ratio of real GDP to the labor force (people between 15 to 64 years of age). The series for CPI inflation (π_t) are constructed by first difference of the overall HICP seasonally adjusted (codename: HICPSA). The series for the relative price of imports ($\hat{T}_t$) are constructed by the ratio of the import price deflator (codename: MTD) over the export price deflator (codename: XTD). Short-term nominal interest rate data are used for the central bank policy rate (R_t). Quarterly U.S. data span from 1970q1 to 2006:q4 as well and are obtained from the relevant NIPA tables. In particular, the series for the real GDP ($\hat{Y}^*_t$) are constructed using data on nominal GDP (NIPA Table 1.1.5 line 1) over GDP deflator (NIPA Table 1.1.4 line 1) and civilian non-institutional population, over 16 (given by LNU00000000Q, at Bureau of Labor Statistics). The series for CPI inflation (π^*_t) are constructed by the first difference of GDP deflator. GDP deflator data obtained as described before. Monthly FFR series from St. Louis FRED website averaged to receive quarterly series are used for the foreign central bank policy rate (R_t). The data for the real exchange rate ($\hat{q}_t$) are constructed by using the Euro per USD exchange series from the AWM dataset (codename: EXR) times the US GDP deflator series over the Euro Area overall HICP series. All the series are in logs apart from the nominal interest rates. I detrend the Euro Area and U.S. real GDP series separately using a quadratic trend, in order to obtain the home and foreign output deviations, respectively, from the model steady state. The constructed

[11] I assume a high standard deviation for the reaction coefficient on inflation, $\phi^*_{\pi^*}$, in the dovish regime because the sample is not big enough to allow for a smaller variance. Thus a higher standard deviation is necessary in order for the estimator to identify potential regime shifts.

series of the relative prices and the real exchange rate are also detrended separately using a quadratic trend, in order to remove nonstationarities. The constructed series comprise the elements of the observed data matrix, D_t, in the state space representation used in the estimation of the model, (23).

5 Parameter Estimates

Table 1 reports the posterior parameter estimates. Since the focus of the paper is on regime switches in US monetary policy, I will focus on the estimates of the parameters of the two Taylor rules in the two regions. In figure 1, I display the estimated posterior distribution of the inflation reaction coefficient, $\phi^*_{\pi^*,S_t}$, in the Taylor rule for the US. As shown in the figure, the way the Fed has been reacting in each regime differs. In line with existing evidence, the Fed has spent periods during which its reaction to inflation has been weak. The model estimation clearly identifies that the reaction of the Fed has changed over time with the mass of the estimated dovish regime distribution of $\phi^*_{\pi^*,2}$ lying to the left of the hawkish regime distribution of $\phi^*_{\pi^*,1}$. The dovish regime distribution of the inflation coefficient receives values that lie below 1 and slightly above 1, and it does not overlap at all with the hawkish regime distribution that receives values above 1.5. The mean of the posterior estimate in the dovish regime is 1.017, lower than the 1.5784 posterior mean in the hawkish regime. Such change in reaction to inflation reflects the higher determination by the Fed to curb inflation and also to anchor inflation expectations.

The dovish regime is also associated with higher interest rate smoothing. This is in line again with the literature on estimated MS-DSGE models. In the literature, there are various explanations regarding the different degrees of interest rate smoothing . One view is that higher smoothing may be due to the fact that the estimated rule includes less variables than those that the monetary authority might be targeting. Alternatively, according to Rudebusch (2002), more persistent shocks could trigger higher interest rate smoothing. Finally, another view supports the idea that higher smoothing is associated with a more gradual adjustment of the policy rate instead of quicker changes followed from the early ’80s onwards.

The coefficient on output in the Taylor rule for the US seems to also be subject to switches over time. Specifically, the estimated posterior mean in the hawkish regime is higher, $\phi^*_{y^*,1} = 0.0368$, than that in the dovish regime, $\phi^*_{y^*,2} = 0.0292$. Hence, the Fed seems to have been reacting weakly not only to inflation, but also to output fluctuations in the latter regime. Lubik and Schorfheide (2004) have similar findings with higher means in both regimes though.[12]

The estimated probability of the hawkish regime, P_{11}, suggests that this is the most

[12] Lubik and Schorfheide (2004) estimate a closed economy DSGE model for the US. They estimate the model for the pre- and post-Volcker period separately and do not impose a Markov-Switching structure in the Taylor rule.

recurrent and persistent regime. This regime is associated with higher reaction coefficient on inflation in the Taylor rule, especially from the '80s onwards. This finding is in line with other papers in the literature that have estimated the monetary policy stance in the US for the pre- and post-Volcker era (see Bianchi (2013a); Boivin (2006); Boivin and Giannoni (2006); Lubik and Schorfheide (2004) among others). The top panel of figure 2 shows the estimated filter probability of the dovish regime. In line with other estimated Markov-Switching models, the probability of this regime increases considerably from the early to mid '70s, with $Pr(S_t = 2) > 0.5$, and rises again slightly before the late '70s until the early '80s. In fact, during that period, the US economy was subject to hyperinflation which many authors have attributed to the Fed's weaker reaction to inflation fluctuations.

From the early '80s onwards instead the Fed has been documented to have been more committed towards battling inflation and anchoring inflation expectations. The model estimation is in line with this result. The hawkish regime appears to be the most recurrent one from the early '80s onwards. From that point in time onwards the probability of the dovish regime stays persistently low until the mid '90s where it rises shortly. Therefore, the hawkish regime dominated during most of the '80s and the '90s.[13] This probability rises shortly again in the early 2000s. One explanation for the increase in the likelihood of the dovish regime in the late '90s could be that inflation was fluctuating around the Fed's target and inflation expectations were well anchored so that policy did not have to do much. Hence, the increasing likelihood of the dovish regime during that period may not necessarily reflect a weaker reaction, but instead the fact that the Fed was getting the rewards of good policies since the early '80s. The same explanation mayhold for the short increase in the probability in the 2000s.

The estimated coefficients in the Taylor rule for the Euro Area yield a posterior mean of 2.7154 for inflation coefficient, ϕ_π, and 0.3909 for the output coefficient, ϕ_y, respectively. The posterior mean of the interest rate smoothing parameter 0.6390.

The estimation of the model also shows that there have been changes in the volatilies of the shocks hitting the two economies. Specifically, there is a high- and a low-variance regime, with the latter being the most recurrent. In particular, as shown in the bottom panel in figure 2, the probability of the high volatility regime increases substantially in the early until the mid '70s and for a short period towards the end of that decade while it stays high from the early until slightly after the mid '80s. Then it rises again for a short period in the late '90s until the early 2000s.[14] Note that here the volatilities in the two regions are assumed to change jointly. This might be the reason why small differences arise with the timing of the high volatility regime estimated in closed economy models for the US in the

[13]Bianchi (2013a) also finds that the probability of the hawkish regime falls slightly in the late '90s and reverts back to high levels until the onset of the recent financial crisis where US monetary policy switched to the dovish regime. The key difference with his model is that he estimates a closed economy model. The model in the current paper though incorporates richer dynamics which possibly reflect on the estimated US monetary policy stance over time.

[14]Bianchi and Ilut (2017) estimate a closed economy MS-DSGE model for the US and also find evidence for a high volatility regime in the early '80s along with an increase in the probability of this regime almost during the same period in the early 2000s.

literature. Bianchi (2013b) for instance finds that the high volatility regime is the dominant regime during the '70s until the mid '80s, with a small break in the late '70s. However, he estimates a closed economy model for the US. The main picture though is similar. That is, the high volatility regime seems to happen mostly during the '70s and the early '80s.

5.1 Impulse responses

In this section, I look at the impulse response functions of the estimated model. The model features eight shocks. In order to save space I display the responses to four of them, namely monetary and productivity in each region, respectively. First, I look at the effect of foreign (US) shocks on home (Eurozone) and foreign variables in each regime conditionally.[15] In figure 3, I display the responses of inflation, output, interest rate in each region along with the real exchange rate to a foreign monetary policy shock. The left-hand side shows the home block, while the right-hand side the foreign. In each case, the first column displays the median impulse responses for the hawkish and the dovish regime, while the second column displays the median and the 68% error bands for the difference between the two responses.

As in closed economy MS-DSGE models, the responses of foreign inflation and output (right hand-side in figure 3) in the country where the shock originates (i.e. US) are amplified in the dovish regime. In the hawkish regime (solid blue lines), both foreign inflation and output revert back to the steady state faster which also explains why the foreign interest rate stays above the steady state for a shorter period compared to the dovish regime (dashed red lines). Looking at the responses in the Euro Area (left-hand side), both inflation and output responses are sensitive to the regime of US monetary policy. Inflation and output jump on impact, in both regimes, but they increase more when US monetary policy is dovish. The first channel that explains this is the way the real exchange rate responds. The latter depreciates more on impact in the dovish regime which boosts the demand for home goods keeping output in the Eurozone higher than in the hawkish regime. The real exchange rate though is not the only channel through which US monetary regime shifts are transmitted to the Eurozone. Relative prices and deviations from the law of one price also contribute to firms' pricing decisions and, more importantly, can affect home agents' expectations. I analyze those channels in the following section.

In figure 4, I display the responses to a foreign productivity shock. The responses of inflation and output in the Euro Area are again sensitive to the monetary regime in the US. Interestingly, the response of inflation and, hence, of the interest rate are of the opposite sign in the two regimes. In the hawkish regime, inflation in the Euro Area drops on impact, while it jumps in the dovish regime. Given the estimated monetary policy stance of the ECB, a

[15] Following Sims and Zha (2006), I compute the impulse responses in each regime separately assuming that there is no switch during the period over which the responses are computed. Note, however, that because of the Markov-Switching structure agents account for possible regime shifts in the future when forming their expectations.

similar pattern is observed in the interest rate. The drop in inflation in the hawkish regime is due to the way the Fed responds to the shock and to the subsequent effect on the real exchange rate. In fact, in this regime the Fed lowers the FFR more than in the dovish regime which triggers a stronger real appreciation of the Euro compared to the dovish regime. This causes an expenditure switch towards US goods lowering the demand for goods produced in the Euro Area driving thereby their prices down. The expenditure switching effect is further enhanced by the drop in the relative price of goods imported from the US. The switch towards US produced goods allows for a faster increase in their prices after the initial drop due to the productivity shock.

The responses of the variables in the Euro Area to a monetary policy shock in the Eurozone are not sensitive to the monetary policy regime in the US, as shown in the left-hand side of figure 5. However, the responses of inflation and output in the US show higher persistence in the dovish regime. Following a productivity shock in the Euro Area (figure 6), the responses of inflation and output in this region are not very sensitive either to the monetary regime in the US. On the contrary, US variables are more sensitive. In fact, US output in the dovish regime shrinks more after an increase in productivity in the Euro Area, and the response of US inflation is more persistent and of the opposite sign on impact. The fall of US output in the dovish regime is determined by the way imported goods prices react to a productivity shock, given the weak reaction of the Fed to inflation in this regime. In particular, the productivity shock in the Euro Area implies lower prices for goods that are exported to the US. Given the weak monetary policy response, the prices of imported goods in the US, $P^*_{H,t}$, stay low for a longer period, which also explains the amplified US inflation response in this regime. The persistent expenditure switching of US households towards imported goods also explains the persistently lower output in the dovish regime.

Therefore, the impulse response analysis reveals that the responses of variables in the Euro Area are sensitive to the monetary regime in the US only following shocks originating from the latter. However, when shocks originate from the Euro Area, the monetary regime in the US seems irrelevant to the way variables in the former respond. Variables in the US instead are sensitive to the domestic monetary regime regardless of the origin of a shock.

5.1.1 How are foreign monetary policy regime switches transmitted internationally?

Foreign monetary policy regime switches are transmitted in the home country through relative prices ($\hat{T}_t$, $\hat{T}^*_t$), currency misalignments and the deviations from the law of one price for foreign goods. Those three variables are among the key determinants of home firm's marginal cost and, thereby, home inflation.[16] Following Engel (2011), I define "currency misalignment" as

$$\hat{m}_t \equiv \frac{1}{2}\left(\hat{z}_t + \hat{P}^*_{F,t} - \hat{P}_{F,t} + \hat{z}_t + \hat{P}^*_{H,t} - \hat{P}_{H,t}\right) \tag{24}$$

Using (8) and the total demand for the home good h, the marginal cost of the home firm is specified as

$$MC_t = \left(\frac{\widehat{(1-\tau)\,W_t}}{A_t P_{H,t}}\right) = \gamma\left(\hat{Y}_t(h) - a_t\right) + \sigma \hat{C}_t - a_t + (1-\delta)\,\hat{T}_t \tag{25}$$

where

$$\hat{Y}_t(h) = -\rho\delta\hat{\tilde{p}}_t(h) + (1-\delta)\left(\rho\delta + \frac{(1-\delta^*)}{\sigma}\right)\hat{T}_t + \hat{C}_t - \rho\,(1-\delta^*)\,\hat{\tilde{p}}^*_t(h)\ldots$$

$$-\rho\delta^*\left((1-\delta^*) - \frac{(1-\delta^*)}{\sigma}\right)\hat{T}^*_t - \frac{(1-\delta^*)}{\sigma}\left(2\hat{m}_t - \widehat{zf_t}\right) + g_t$$

From equation (25), the marginal cost depends on the relative price of imported goods $\hat{T}_t$ and, through the demand for home good h, on the relative price of exported goods $\hat{T}^*_t$, currency misalignment and the deviations from LOOP $\widehat{zf_t}$.[17] Regime switches in foreign monetary policy affect both relative prices ($\hat{T}_t$, $\hat{T}^*_t$) through imported goods inflation ($\pi_{F,t}$) and foreign producer-price inflation ($\pi^*_{F,t}$).[18] This has a direct impact on home firm's marginal cost and, hence, on home inflation. Moreover, LCP, and the resulting currency misalignment along with the deviations from the law of one price, create an additional channel through which foreign policy shifts are transmitted into the home economy.[19] In fact, both depend on foreign goods prices which are highly sensitive to the foreign monetary policy regime. This is something the home central needs to take into account. That is, it needs to account for those channels when setting its policy rate. However, one should not interpret this as an indication towards adjusting the reaction coefficients in the home interest rate rule. These transmission channels point towards which variables the home central bank should look at when setting its policy rate accounting for the international spillovers following foreign monetary regime switches.

Using the log-linearized Euler equation and the goods market clearing condition it becomes clear that the effect on home output comes from both the current and expected future relative

[16] Remember that home inflation is a linear combination of home producer-price inflation ($\pi_{H,t}$) and the change in imported goods relative price.

[17] In the demand function for home goods I have substituted out the real exchange rate $\hat{q}_t$ using the fact that it can be expressed as $\hat{q}_t = 2m_t - \widehat{zf_t} - \delta^*\hat{T}^*_t - (1-\delta)\,\hat{T}_t$, where $\widehat{zf_t} = \hat{z}_t + \hat{P}^*_{F,t} - \hat{P}_{F,t}$ denotes the deviations from the law of one price for foreign goods.

[18] Imported goods inflation and foreign producer-price inflation ($\pi_{F,t}$, $\pi^*_{F,t}$) depend on foreign firm's marginal cost which is specified by the foreign counterpart of equation (22) above. Foreign firm's marginal cost is thus a function of foreign output through which the effects of foreign monetary policy regime shifts are transmitted in the foreign country.

[19] With producer currency pricing, the transmission channel of foreign monetary policy shifts would work through the terms of trade, only.

prices, currency misalignment and deviations from the law of one price.[20] The possibility of a change in foreign monetary policy affects home agents expectations about the future path of those three variables, having a direct impact on home output.

As Liu et al. (2009) argue, dovish monetary policy increases volatility in macroeconomic variables not only in the dovish regime per se, but also in the hawkish. Increased volatility in the latter regime builds up due to the knowledge from households and firms of the existence of the dovish regime and of the possibility that the monetary authority might switch to that regime in the future. Assigning a probability of a switch to the dovish regime increases the volatility of inflation and output in the hawkish regime as well. In the current model, inflation volatility increases not only in the foreign country (US), but also in the Euro Area through the transmission channel described above. This suggests that the home central bank (ECB) might need to curb inflation volatility, when US monetary policy is dovish, and also to anchor inflation expectations while US monetary policy is hawkish but is expected to become dovish with a positive probability. Consequently, the ECB might need to react stronger to CPI or to its determinants (i.e. $\pi_{H,t}$ and $\hat{T}_t$) in both regimes, but not necessarily differently across regimes. Whether or not the Euro Area can benefit when the ECB not only reacts strongly to those variables, but also differently according to the foreign monetary regime requires welfare analysis. I elaborate on this matter specifically in the following section.

6 Optimal policy with regime switches

I now turn to the design of optimal monetary policy in the home country. I focus on the optimal discretionary policy 1) conditional on the regime of foreign monetary policy (time varying policy) and 2) independently of the foreign regime (time invariant policy). In order to compute the optimal policy, I extend the algorithm by Soderlind (1999) to account for regime shifts.

6.1 Quadratic approximation to welfare

Before the formulation of the optimal policy problem of the home central bank I derive its quadratic objective function from a second order approximation to the welfare of the average home household as in Rotemberg and Woodford (1998). The second order approximation to welfare under the efficient steady state[21] receives the form

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U_t \approx \frac{1}{1-\beta} \bar{U} - \frac{1}{2} \Theta U_C \bar{C} \sum_{t=t_0}^{\infty} \beta^{t-t_0} X_t' R X_t + t.i.p. \tag{26}$$

[20] The log-linearized IS equation is derived in Appendix B.2.

[21] The full derivation of the second order approximation is presented in the Appendix B.4.

where Θ is a constant that is a nonlinear function of the structural parameters of the model, $\bar{C}$ is steady state state consumption, X_t is a 28×1 vector of state variables, R is a 28×28 symmetric matrix and constant function of structural model parameters. Finally, $t.i.p.$ denotes terms independent of policy.

6.2 Formulation

In this section I adopt a formulation similar to that in Blake and Zampolli (2011). The policy maker chooses the control r_t (i.e. the interest rate) which maximizes the expected value of the intertemporal loss function

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \widetilde{L}(X_t, r_t) \tag{27}$$

subject to X_0, s_{-1} and the reduced form model describing the world economy

$$X_{t+1} = \Omega(s_t)X_t + \Lambda(s_t)r_t + C(s_t)\varepsilon_t \qquad t \geq 0 \tag{28}$$

where $\widetilde{L}(X_t, r_t) = -\frac{1}{2}\Theta U_C \bar{C} \left\{ X_t' R X_t + r_t Q r_t \right\}$ is the period loss function where I have added an ad hoc interest rate stabilization objective. Q is a 28×28 symmetric matrix. X_t and ε_t are 28×1 and 8×1 vectors of endogenous variables and i.i.d. shocks, respectively. C is a 28×8 matrix.[22] Matrices Ω and Λ are stochastic and take on different values depending on the regime $s_t \in \{1, 2\}$.[23]

The Bellman equation. The policy-maker in a Markov-switching environment needs to find the interest rate rule that is state-contingent. This rule describes the way that the control variable, the interest rate, should be set as a function of both the state variables X_t and the regime s_{t-1} occurring at date $t-1$. Therefore, a Bellman equation is associated with each regime. The regime j dependent Bellman equation is specified as follows

$$V(X_t, j) = \max_{r_t} \left\{ \widetilde{L}(X_t, r_t) + \beta \sum_{i=1}^{2} p_{ji} E_t \left[V(X_{t+1}, i) \right] \right\} \tag{29}$$

where $V(X_t, j)$ is a function of the state variables X_t, the regime prevailing at date $t-1$, $s_{t-1} = j$, and represents the continuation value of the optimal dynamic programming problem at t. The home central bank has to find the sequence $\{r\}_{t=0}^{\infty}$ that maximizes both the current $\widetilde{L}(X_t, r_t)$ and the discounted sum of future losses. The value function for this problem is

$$V(X_t, j) = X_t' P_j X_t + d_j, \quad j = 1, 2 \tag{30}$$

[22]Note that the results presented in the following section are robust to different values for the weight on interest rate stabilization. The set of different values tried lies in the interval $[0, 3]$.
[23]Matrices Ω and Λ are specified in detail in Appendix B.5.

where P_j is a 28×28 symmetric positive semi-definite matrix, while d_i is a scalar. The optimal policy is described by

$$r(h_t, j) = -F_j X_t, \quad j = 1, 2 \tag{31}$$

where F_j is a 28×1 matrix, depending on P_j.[24] That is, matrix F_j specifies the coefficients in the policy rule of the central bank. Those coefficients are regime specific. Maximizing the Bellman subject to the constraints, the matrix F_j is specified as

$$F_j = \left(Q + \beta \sum_{i=1}^{2} \Lambda_i' \left(p_{ji} P_i \right) \Lambda_i \right)^{-1} \beta \left(\sum_{i=1}^{2} \Lambda_i' \left(p_{ji} P_i \right) \Omega_i \right) \tag{32}$$

where matrix P_i has is determined by a set of interrelated Riccati equations.

6.3 How should the home central bank react?

The optimal monetary policy for the Euro Area is computed using the median of the estimated posterior for each parameter. Having specified the formulation of the policy problem of the home central bank, in this section, I present the optimal rules conditional and unconditional on regime shifts in foreign monetary policy, respectively. The probabilities used are the medians of the posterior estimates, with $P_{11} = 0.9678$ and $P_{22} = 0.9549$. The results are summarized at table 2. For convenience, only the coefficients different to zero are reported.[25] In the table, I display the reaction coefficient both from the optimal time varying and the optimal time invariant policy.

Let me focus on home producer-price inflation and imported goods relative price ($\pi_{H,t}$, $\hat{T}_t$). The reaction of the home central bank to those two variables, both of which determine home CPI inflation, is strong regardless of the foreign regime, with both coefficients being greater than one in both the optimal time varying and the optimal time invariant rule. Moreover, in the varying rule both coefficients are higher in the dovish regime. Those findings - especially the strong reaction to those two variables in both regimes - are in line with the argument in section 5.1.1 regarding the way foreign monetary policy switches are transmitted internationally. The high reaction coefficients on those two variables are explained by the fact that the home central bank tries to counteract the effects of a possible foreign policy switch to the dovish regime on home agents inflation expectations today.[26] That is, a future

[24] Note that due to certainty equivalence, the resulting optimal policy rule is the same regardless of the properties of the shocks. That is, the changes in the volatilities of shocks over time do not affect the optimal choice for the control variable, nor do they enter the optimal policy problem.

[25] If one introduces habits and/or indexation in price setting in order to allow for endogenous persistence, then coefficients on lagged endogenous variables in the optimal rule (31) would be different from zero.

[26] Note that, even though I focus on optimal discretionary policy, the central bank and home agents know that today's home central bank's actions affect the path of the variables the day after. This is so because the model contains endogenous state variables like T_{t-1}, T^*_{t-1} and q_{t-1}.

US monetary policy dovish regime must be accompanied by a more aggressive response from the ECB to the determinants of home CPI inflation.

The question that is raised though is whether the ECB should change its policy whenever the Fed switches regimes or is it enough to react strongly to domestic and international conditions in order to counteract the effects of foreign regime switches. That is, whether the ECB should adjust its reaction coefficients depending on the US monetary policy regime. I answer this in the following section by computing the welfare costs associated with each policy.

6.4 The welfare costs of regime invariant monetary policy

In order to gauge the importance of reacting optimally conditional on the regime of foreign monetary policy, I turn now to the computation of the welfare costs associated with a regime invariant policy. Welfare costs are computed as in Ravenna and Walsh (2011). In particular, I compute the percent increase in steady state consumption that would make the home representative agent indifferent between the time invariant policy and the optimal regime specific home monetary policy.

Given that the economy in the MS-DSGE model can lie in either of the two regimes, I compute the costs conditional on the regime. Such a comparison is valid given that the steady state is unique and independent of monetary policy regime changes, each regime is determinate and the minimum state variable solution is stationary.

Let tv be the policy regime where the home central bank follows the time varying (regime specific) optimal policy as described in (31). Let ti be the policy regime where the home central bank follows the optimal time invariant policy.[27] Let $\varsigma^{s,ti}$ capture the welfare cost of not implementing a regime specific optimal time varying policy and be measured as the percent increase in steady state consumption that would make the individual indifferent between the regime invariant policy ti and the regime specific optimal policy tv. More specifically, $\varsigma^{h,ti}$ denotes the welfare cost in the hawkish regime and $\varsigma^{d,ti}$ denotes the welfare cost in the dovish regime. Formally, $\varsigma^{h,ti}$ and $\varsigma^{d,ti}$ solve the following

$$\frac{1}{1-\beta}\bar{U}((1+\varsigma^{s,ti})\bar{C},\bar{L}) + W^{s,ti} + t.i.p. = \frac{1}{1-\beta}\bar{U}(\bar{C},\bar{L}) + W^{s,tv} + t.i.p$$

with

$$W^{s,ti} = -\frac{1}{2}\Theta U_C \bar{C} E \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ X'_{t,s|ti} R X_{t,s|ti} + r_{t,s|ti} Q r_{t,s|ti} \right\}$$

$$W^{s,tv} = -\frac{1}{2}\Theta U_C \bar{C} E \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ X'_{t,s|tv} R X_{t,s|tv} + +r_{t,s|tv} Q r_{t,s|tv} \right\}$$

[27] The optimal time invariant policy is of the same form as (31) but with coefficients being regime invariant and coincides with the regime specific optimal policy when each regime has an equal probability to occur (i.e. $p11 = p22 = 0.5$).

for $s = h,\ d$.

At table 3, I compute the welfare costs of time invariant optimal policy. In the hawkish regime, the home household should receive a 0.1189% increase in its steady-state consumption, in order to be as well off as it would have been under the optimal time varying policy. The respective increase in foreign steady state consumption is 0.1860%. Welfare costs are higher in the dovish regime for the home country, but lower for the foreign country. Home steady state consumption must rise by 0.8427%, while foreign must rise by 0.0196%. Although the necessary increase in steady state home consumption in the dovish regime is not negligible, in general only modest gains can be obtained when the ECB follows a optimal time varying rule instead of a time invariant one. Therefore, the welfare analysis does not provide strong evidence in favor of changes in the reaction coefficients in the monetary policy rule of the ECB when the Fed switches regimes. Given the structure of the optimization problem of the ECB, a weakness of the resulting optimal rule is that it is tough to be implemented in reality. This is because in the optimal linear regulator problem the control variable (i.e. the policy rate) needs to react to all the endogenous variables in the system. For this reason, in the next section I compute a simple rule that is able to replicate the allocation under an optimal time invariant policy rule.

6.4.1 An implementable rule

In this section, I compute a simple interest rate rule that can either replicate or generate losses that are negligible relative to the allocation under the time invariant optimal policy computed in the last section.

The task of specifying an implementable rule is challenging given the size of the model. This is because of three reasons. First, the rule either replicate the optimal allocation or generate negligible welfare costs in both regimes. Second, the coefficients in the rule must be such that determinacy is achieved in each regime, conditionally. Third, the rule must be such that the MS-DSGE model has a stationary minimum state variable solution.

I experiment with a number of different simple interest rate rules. The family of rules considered includes the standard Taylor rule (either lagged or forward looking), a strict inflation targeting rule (either PPI or CPI), nominal income rules in the spirit of Schmitt-Grohe and Uribe (2007), and Taylor rules expanded by international variables (i.e. real exchange rate and/or relative prices). For each of these rules I do a grid search. Specifically, I limit my attention to policy coefficients in the interval [0, 5].[28] In each step of the search, I compute the associated welfare costs. The only rule that is able to generate negligible welfare costs is one in which the home nominal interest rate reacts to home PPI, output gap and the relative price of foreign goods. It is specified as follows

[28]The size of the interval is arbitrary. I have also tried wider bands but the results are robust qualitatively. Here, I exclude negative values as they can never produce an allocation close to the optimal.

$$r_t = \phi_{\pi,s_t}\pi_{H,t} + \phi_{y,s_t}\hat{\hat{Y}}_t + \phi_{T,s_t}\hat{T}_t \tag{33}$$

The values of the coefficients and the resulting welfare costs relative to the optimal time invariant discretionary policy are summarized at table 4. Determinacy is achieved in each regime, conditionally, and the corresponding minimum state variable solution is stationary in each case.

The results from the grid search are in line with the optimal discretionary rule in table 2, as far as the reaction to inflation is concerned. In particular, the home central bank must be always aggressive to home producer-price inflation fluctuations. Moreover, the coefficient on relative prices, $\hat{T}_t$, is also high implying that the ECB needs to react strongly to variables also affecting the international competitiveness of goods produced in the Euro Area. This is also in line with the analysis in section 5.1.1 regarding the way foreign regime switches are transmitted into the home country. Again, a strong - and not a time varying - reaction to fluctuations in relative prices is enough in order to offset the effects of foreign regime switches on the volatility of domestic prices.

The welfare costs associated with the optimal simple rule are negligible. In particular, the simple rule is associated with a welfare cost of 0.0009% and 0.0003% in the hawkish and the dovish regime, respectively. I consider such increases in steady state consumption necessary to make home households as well off as under an optimal time invariant rule as negligible.

As I showed in section 5.1.1, the relative price of foreign goods $\hat{T}_t$ is one of the key variables determining the international transmission of foreign monetary policy regime switches. It is not surprising thus that the inclusion of this variable in the simple interest rate rule is crucial in order for the latter to be able to generate an allocation that is closer to that under the optimal time invariant discretionary policy. Finally, given local currency pricing, one should not expect a strictly inward looking interest rate rule to be able to replicate or even generate negligible welfare costs.[29]

Finally, at table 5 I compute the relative volatilities of inflation and output in each region under the simple optimal time-invariant rule (33). In each case, the relative volatility is computed as the ratio of the standard deviation of the variable in the dovish regime over its standard deviation in the hawkish regime. For convenience, I also display the relative volatilities of the variables under the baseline interest rate rule as estimated in section 5. As shown in the table, inflation in the Euro Area is almost unaffected by foreign regime switches while output volatility increases only marginally in the hawkish regime under the simple optimal rule. Under the baseline rule though inflation volatility is 1.22 times higher when US monetary policy is dovish compared to the case when it is hawkish, while output is approximately 1.15 times more volatile in the dovish regime. Hence, the simple optimal rule allows the ECB to counteract the effects of regime switches in US monetary policy on

[29] An inward interest rate rule is one in which the home central bank reacts to domestic variables only. LCP implies deviations from the law of one price which are distortionary and are a separate source of loss. This renders inward looking monetary policy suboptimal. For a more detailed analysis see Engel (2011) and the references therein.

domestic inflation and output almost completely.

6.5 Implications for the reaction of the Home central bank and comparison to existing rules.

In the optimal monetary policy analysis above, I showed that the rule that is able to replicate the optimal allocation is one in which the home central bank adjusts its interest rate to home producer-price inflation, the output gap and the relative price of imported goods. This implies that the home central bank should focus both on domestic variables as well as international, since relative prices can be explicitly written as a function of the real exchange rate and currency misalignments. This rule is an extension of that suggested by Clarida et al. (2002)(CGG), where there interest rate rule able to implement the optimal allocation is a function of producer price inflation. What is interesting, though, is the fact that CGG assume PCP, whereas firms in this paper set a different price for each country. More importantly, the optimal simple rule described by (33) above is at odds with that in Engel (2011), where the interest rate rule replicating the optimal allocation under discretion includes a target for CPI inflation, only.

In the literature, it has been widely documented that whenever firms follow LCP, monetary policy should outward looking (i.e. targeting foreign variables as well), or that the central bank should target CPI when an optimal instrument rule is used instead (Engel (2011); Corsetti and Pesenti (2005); Devereux and Engel (2003)). That is, the central bank in each country faces trade offs that concern not only domestic variables, but also international (e.g. currency misalignments, real exchange rates). On the contrary, whenever firms engage in PCP, monetary policy should be inward looking, when optimal targeting criteria are derived, or it should target domestic producer-price inflation only, when an optimal instrument rule is used instead.

The reason why the optimal simple interest rate rule in this paper differs substantially from the ones suggested in models where LCP holds is due to foreign monetary policy regime switches and the way they are transmitted into the home country. As already argued, foreign monetary policy switches are transmitted through relative prices, currency misalignments and the deviation from the law of one price. By targeting the relative price of imported goods, the home central bank counteracts the effects of one of the main transmission channels of foreign regime shifts. At the same time, by targeting producer price inflation and output, the home central bank is able to counteract the adverse effects that deviations from the law of one price and currency misalignments have on output and inflation, directly. Consequently, when foreign monetary policy switches regimes, optimal monetary policy in the home country, expressed through an optimal interest rate rule, is both inward and outward looking.

7 Concluding remarks

A large part of the empirical literature on US monetary policy has argued in favor of changes in the behavior of the Fed throughout the years. In particular, this strand of the literature attributes the hyperinflation of the '70s to the Fed's weak reaction to inflation fluctuations and the sharp fall in inflation (and its lower volatility) from the '80s onwards to a strong reaction to inflation fluctuations. MS-DSGE models have managed to put these regime switches in US monetary policy into a single model in which agents are fully aware of those. However, little evidence, particularly theoretical, exists regarding the transmission of monetary policy switches in one country to other countries.

Taking into account the empirical findings in the literature, I have examined the international effects of changes in monetary policy. I have constructed and estimated a two country DSGE model for the Eurozone and the US in which foreign monetary policy (US) switches regimes over time. Home monetary policy (Eurozone) is assumed to be time invariant and follow a Taylor rule with some interest rate smoothing. In the estimation of the model parameters I have found that monetary policy in the US has switched between a hawkish and a dovish regime with the latter being the dominant regime in the '70s and the early '80s, in line with the existing literature. I have shown that the way inflation and output in the Euro Area respond to shocks coming from the US depends on the monetary regime of the latter. Specifically, I have shown that when US monetary policy is dovish inflation and output responses in the Euro Area are more volatile. This suggests that the monetary policy of the ECB, even though strongly committed to keeping medium-run inflation close but below 2%, needs to account for those facts. However, when shocks originated from the Euro Area the responses of those variables are not sensitive to the monetary regime in the US. I have argued that relative prices and currency misalignments play a crucial role in the transmission of volatility to the Euro Area when US monetary policy switches regimes. Therefore, I have argued in favor of the importance of international variables in the design of the monetary policy of the ECB.

In fact, through the solution of the optimal monetary policy problem for the Euro Area, conditional on foreign monetary policy switching regimes over time, I have shown that the ECB needs to react strongly not only to domestic conditions (i.e. PPI inflation and output), but also to international ones, like the relative price of imported goods. More importantly, I have shown that the ECB need not adjust its policy over time whenever the Fed switches regimes, since little welfare gains can arise from such a policy. However, I have argued in favor of an outward looking policy rule.

Given the complexity of the optimal interest rate rule suggested by optimal discretionary policy, I have moved a step further to compute an optimal and implementable interest rate rule for the Euro Area. The only rule that replicates the optimal allocation under discretionary monetary policy is one in which the ECB targets home producer-price inflation, output gap and foreign imported goods relative price. The existence of the latter in the rule is explained by the fact that it is one of the main channels through which foreign monetary policy regime switches are transmitted to the home country. The key message thus of this paper is that the Eurozone can benefit a lot if the ECB takes into account the effects of regime changes in US monetary policy, first, and, second, if it adjusts its policy appropriately by looking at international variables as well so that to isolate the economy from the effects of foreign regime shifts.

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Tables

Table 1
Prior Distributions and Posterior Estimates

	Posterior Estimates				Prior Specification		
Parameters	Mode	Mean	5%	95%	Distribution	Mean	St.dev
σ	1.2751	1.2024	1.1155	1.2867	Normal	1,.50	0.375
γ	1.1308	1.0956	1.0257	1.1562	Normal	2.00	0.75
ω	0.3917	0.4508	0.3946	0.5334	Beta	0.75	0.05
$\omega*$	0.5920	0.5963	0.5652	0.6479	Beta	0.50	0.1
ρ	1.4879	1.5217	1.4863	1.5625	Gamma	1.50	0.75
ϕ_π	2.6662	2.7154	2.6462	2.7840	Gamma	1.70	0.1
ϕ_y	0.4057	0.3909	0.2955	0.4538	Gamma	0.125	0.05
i_r	0.7333	0.6390	0.5223	0.7456	Beta	0.80	0.1
$\phi^*_{\pi^*,1}$	1.5530	1.5784	1.5242	1.6325	Gamma	1.50	0.45
$\phi^*_{\pi^*,2}$	1.0088	1.0168	0.9615	1.0768	Gamma	0.95	1.1
$\phi^*_{y^*,1}$	0.0807	0.0367	0.0078	0.0863	Gamma	0.12	0.05
$\phi^*_{y^*,2}$	0.0089	0.0292	0.0067	0.0576	Gamma	0.12	0.05
$i^*_{r^*,1}$	0.7908	0.8208	0.7012	0.8935	Beta	0.50	0.25
$i^*_{r^*,2}$	0.9814	0.9069	0.8258	0.9813	Beta	0.50	0.25
ρ_a	0.8751	0.9018	0.8677	0.9327	Beta	0.85	0.1
ρ_g	0.8059	0.7818	0.7537	0.8180	Beta	0.85	0.1
ρ_{cp}	0.5762	0.5747	0.5129	0.6300	Beta	0.50	0.15
$\rho^*_{a^*}$	0.7630	0.6733	0.5894	0.7709	Beta	0.50	0.2
$\rho^*_{g^*}$	0.2665	0.3519	0.2494	0.6035	Beta	0.50	0.2
$\rho^*_{cp^*}$	0.4024	0.5131	0.3739	0.7524	Beta	0.50	0.15
$\sigma_{a,1}$	0.1018	0.1036	0.0837	0.1363	Inverse Gamma	0.40	2
$\sigma_{a,2}$	0.3298	0.3168	0.2652	0.3532	Inverse Gamma	0.40	2
$\sigma_{g,1}$	0.6713	0.6714	0.6142	0.7276	Inverse Gamma	0.40	2
$\sigma_{g,2}$	0.8288	0.7900	0.7132	0.8368	Inverse Gamma	0.40	2
$\sigma_{cp,1}$	0.1854	0.2278	0.1730	0.3547	Inverse Gamma	0.50	10
$\sigma_{cp,2}$	0.7289	0.8128	0.7227	0.9031	Inverse Gamma	0.50	10
$\sigma_{r,1}$	0.0167	0.0183	0.0156	0.0215	Inverse Gamma	0.10	2
$\sigma_{r,2}$	0.1221	0.1906	0.0921	0.3194	Inverse Gamma	0.10	2
$\sigma^*_{a^*,1}$	0.0148	0.0167	0.0146	0.0191	Inverse Gamma	0.10	2
$\sigma^*_{a^*,2}$	0.3512	0.3301	0.2982	0.3548	Inverse Gamma	0.10	2
$\sigma^*_{g^*,1}$	0.0304	0.0385	0.0256	0.0659	Inverse Gamma	0.10	2
$\sigma^*_{g^*,2}$	0.9704	0.9815	0.9324	1.0341	Inverse Gamma	0.10	2
$\sigma^*_{cp^*,1}$	0.1793	0.1761	0.1570	0.1987	Inverse Gamma	0.50	10
$\sigma^*_{cp^*,2}$	0.4627	0.3937	0.2865	0.4842	Inverse Gamma	0.50	10
$\sigma^*_{r^*,1}$	0.2876	0.2719	0.2169	0.3112	Inverse Gamma	0.10	2
$\sigma^*_{r^*,2}$	0.1377	0.1678	0.0808	0.2722	Inverse Gamma	0.10	2
P_{11}	0.9471	0.9559	0.9362	0.9786	Dirichet	18.0	1
P_{22}	0.8836	0.9423	0.8546	0.9961	Dirichet	18.0	1
Q_{11}	0.9962	0.9922	0.9821	0.9985	Dirichet	18.0	1
Q_{22}	0.9073	0.9382	0.8832	0.9945	Dirichet	18.0	1

This table displays the modes, mean, 95% probability intervals, and prior distributions of the parameters of the Markov-switching DSGE model.

Table 2

Reaction Coefficients

	$\hat{Y}_t$	$\hat{Y}_t^*$	$\pi_{H,t}$	$\pi_{H,t}^*$	$\pi_{F,t}$	$\pi_{F,t}^*$	$\hat{q}_{t-1}$	$\hat{T}_t$	$\hat{z}_t$	$\hat{z}_{t-1}$	r_{t-1}^*
Optimal Time Varying Monetary Policy											
Hawkish regime	1.0315	0.4427	4.7639	0.1454	-0.0224	-2.9816	-0.6354	1.6300	0.9033	0.6354	3.4396
Dovish regime	1.1280	0.4869	5.4009	-0.0459	-0.0117	-4.1453	-0.6303	1.7500	0.9401	0.6303	4.1349
Optimal Time Invariant Monetary Policy											
Both regimes	1.0809	0.4492	5.1469	0.0446	0.0051	-3.6223	-0.6805	1.6846	0.8711	0.6805	3.7862

This table displays the reaction coefficients in the optimal interest rate rule resulting from the optimal time varying and time invariant monetary policy problem, respectively, of the home central bank.

Table 3

Welfare Costs

Optimal Time Invariant Rule vs Optimal Time Varying Rule		
	Home	*Foreign*
Hawkish	0.1189	0.1860
Dovish	0.8427	0.0196

This table displays the welfare costs related to the optimal time invariant policy relative to the optimal time varying policy. Optimal time invariant policy is the policy corresponding to the case where the home country in the two country Markov-switching model does not react to the regime shifts in foreign monetary policy. Optimal time varying policy is the the policy according to which the home central bank adjusts its monetary policy according to the foreign monetary policy regime. The value of the welfare cost corresponds to the percentage increase in steady state consumption that is necessary in order to make the home and foreign household, respectively, as well off as in the allocation resulting from the optimal time varying discretionary policy.

Table 4

Implementable Rule Coefficients

ϕ_π	ϕ_y	ϕ_T	*Welfare Costs*	
			Hawkish Regime	*Dovish Regime*
4.73	1.1015	2.3812	0.0009	0.0003

This table displays the coefficients of a simple optimal time varying interest rate rule for the home country of the form $r_t = \phi_{\pi,s_t}\pi_{H,t} + \phi_{y,s_t}\hat{\hat{Y}}_t + \phi_{T,s_t}\hat{T}_t$ for different values of the elasticity of substitution between home and foreign goods. r_t is the home nominal interest rate, $\pi_{H,t}$ is home producer price inflation, $\hat{\hat{Y}}_t$ home output gap and $\hat{T}_t$ the relative price of foreign imported goods over the price of home goods consumed domestically. The values of the coefficients result from a grid search within an interval (0,5) for each coefficient. The values displayed are the those leading to the smallest welfare loss. The value of the welfare cost corresponds to the percentage increase in steady state consumption that is necessary in order to make the home household as well off as in the allocation resulting from the optimal time invariant discretionary policy.

Table 5

Inflation and Output relative volatilities

	Inflation		*Output*	
	EA	*US*	*EA*	*US*
Baseline Rule	1.2260	1.8271	1.1465	1.6931
Simple Optimal Rule	1.0073	2.0174	1.0568	1.4719

This table displays the relative volatilities of inflation and output in the Euro Area and the US, respectively under the baseline rule from the model estimation and under the simple optimal and implementable rule computed in section 6.4.1. Relative volatilities are computed as the ration of the standard deviation of the variable in the dovish regime over its standard deviation in the hawkish regime.

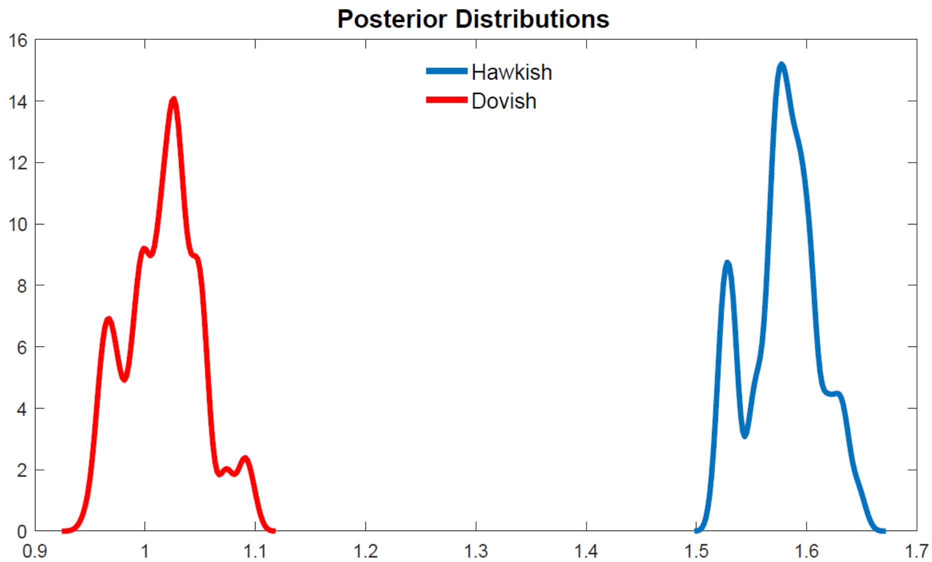
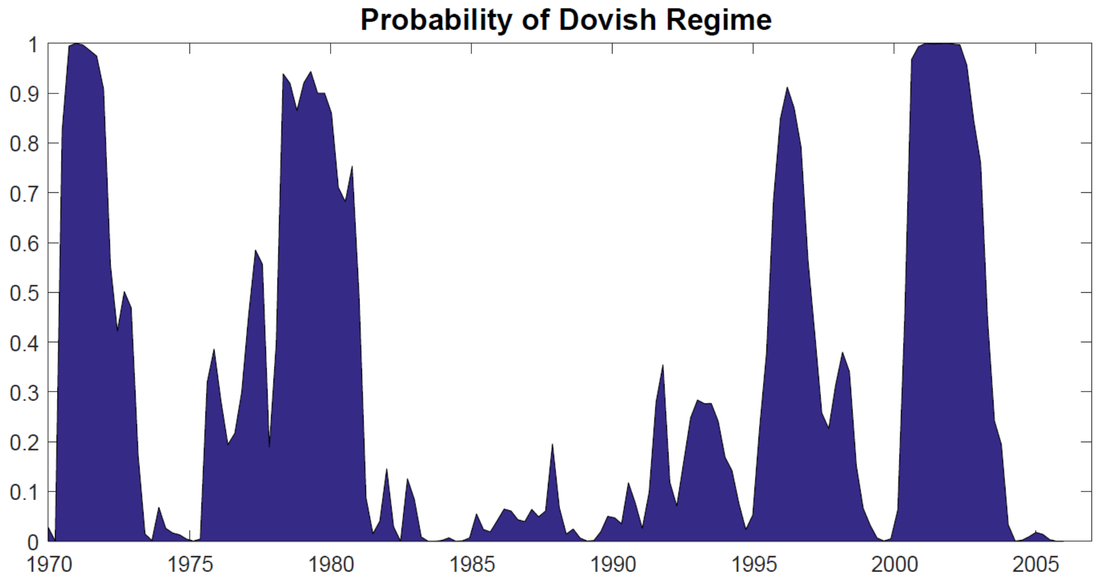


Figure 1 - Estimated Posterior distribution of inflation reaction coefficient $\phi^*_{\pi^*}$ in both regimes. The distribution in the dovish regime is in red while the distribution in the hawkish regime in blue.



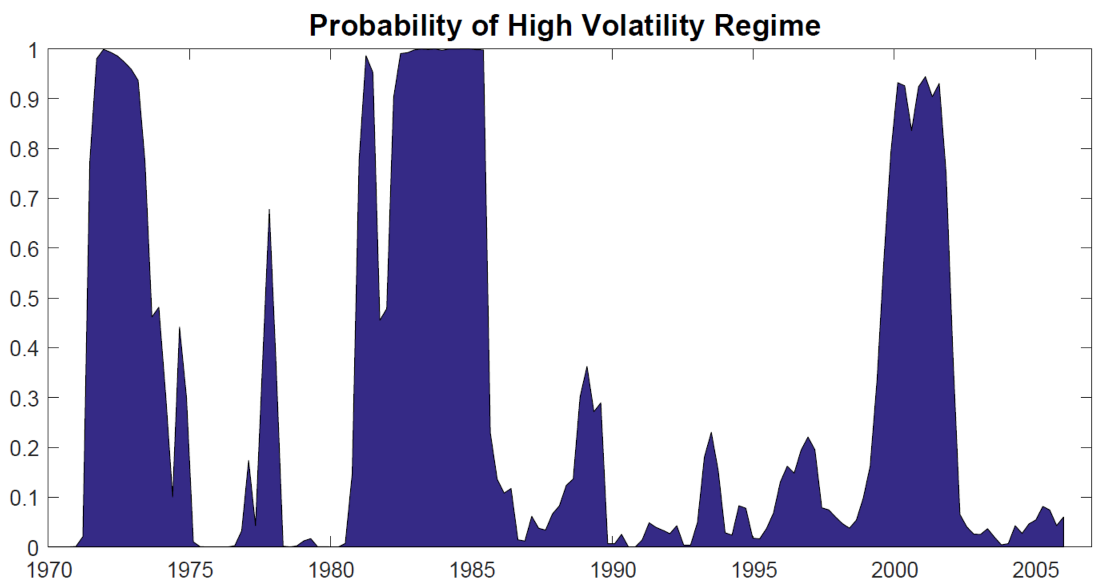


Figure 2 - MS-DSGE model, posterior median estimates. Top panel, probability of Dovish regime (Regime 2) for the structural parameters. Bottom panel, probability of high volatility regime (Regime 2) for the stochastic volatilities.

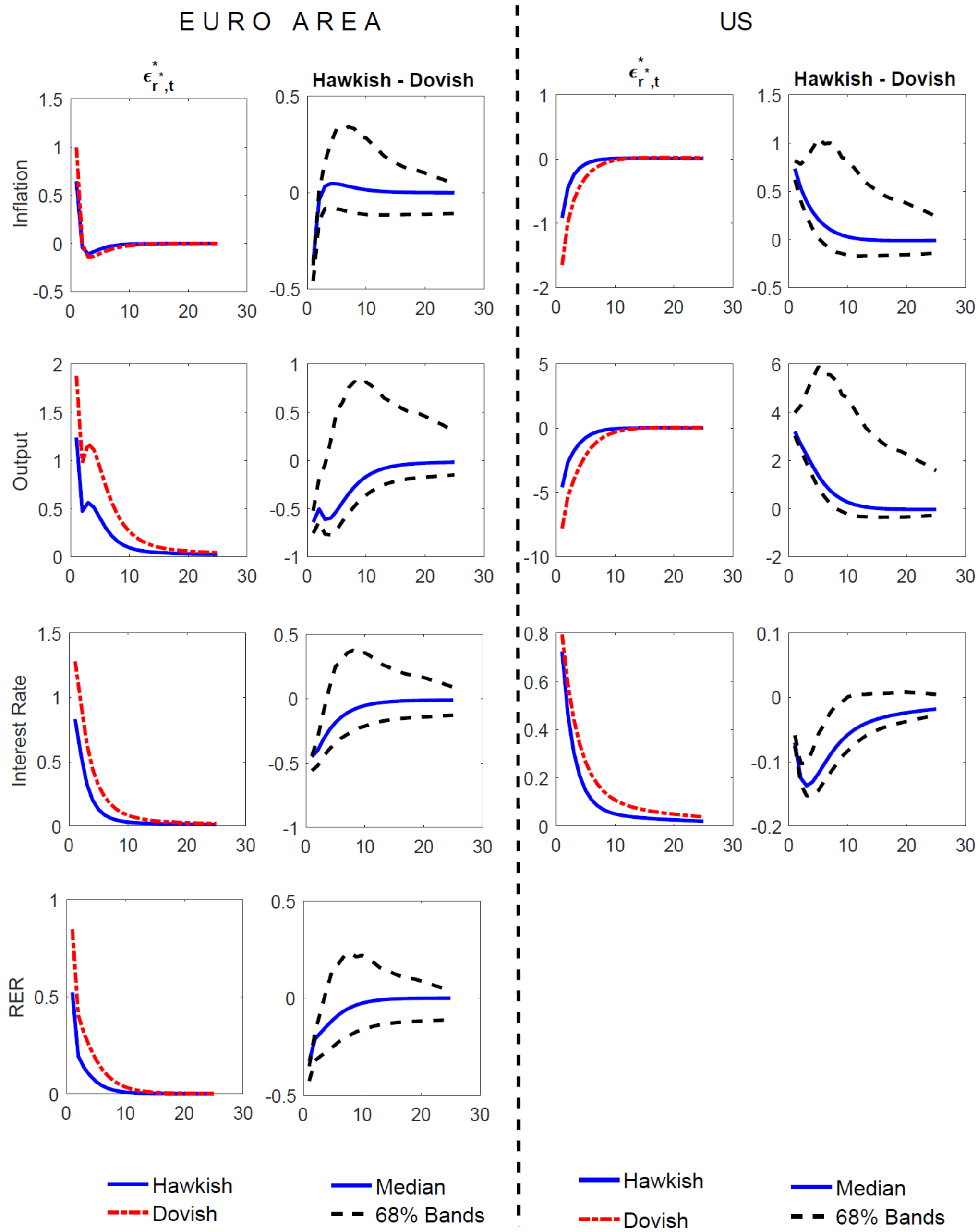


Figure 3 - The figure displays the impulse responses of inflation, output, interest rate in the Euro Area (left-hand side) along with the real exchange rate and the respective responses in the US (right-hand side) to a monetary policy shock in the US. In each case, the first column reports the median in the Hawkish and the Dovish regime, and the second column includes the median and the 68% error bands for the difference between the two regimes.

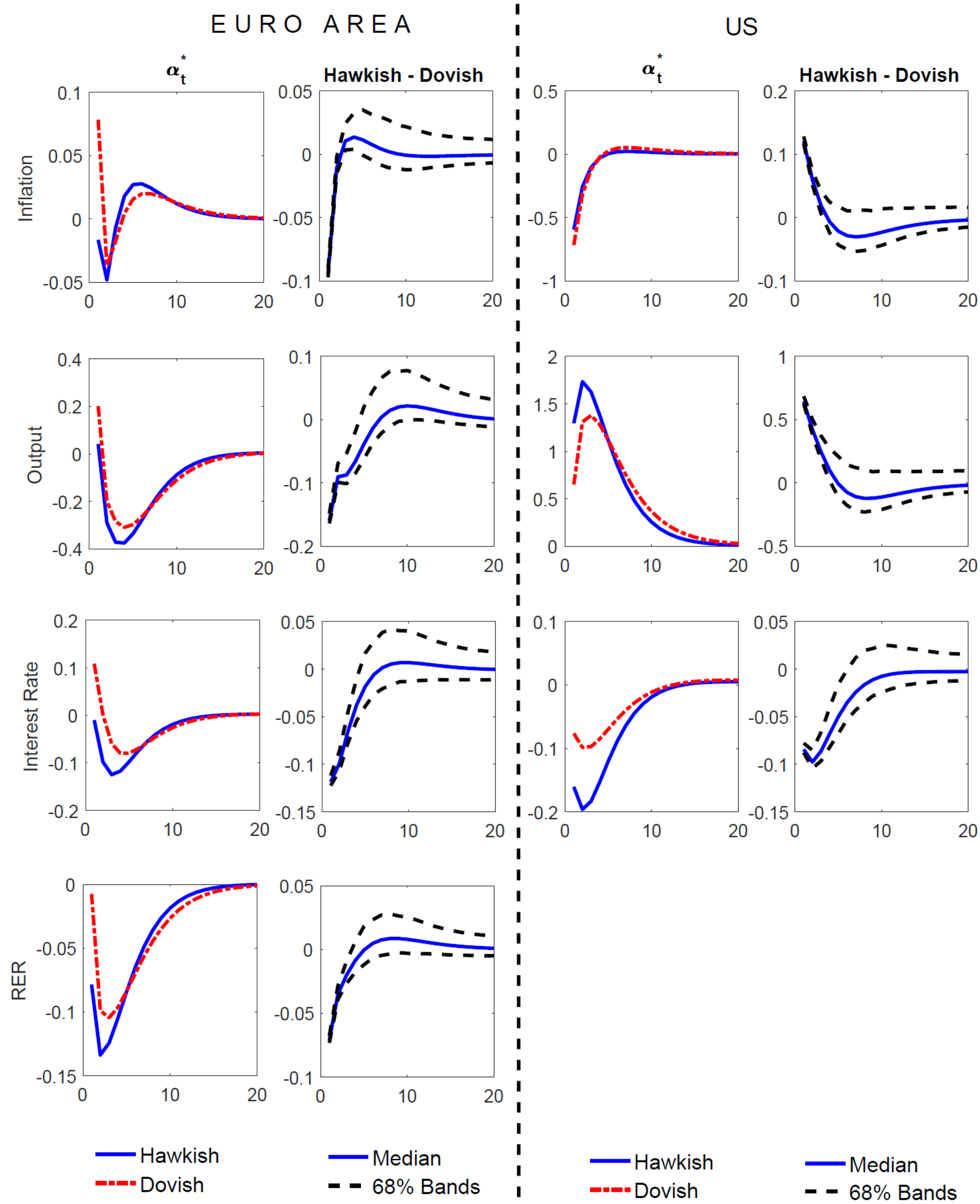


Figure 4 - The figure displays the impulse responses of inflation, output, interest rate in the Euro Area (left-hand side) along with the real exchange rate and the respective responses in the US (right-hand side) to a productivity shock in the US. In each case, the first column reports the median in the Hawkish and the Dovish regime, and the second column includes the median and the 68% error bands for the difference between the two regimes.

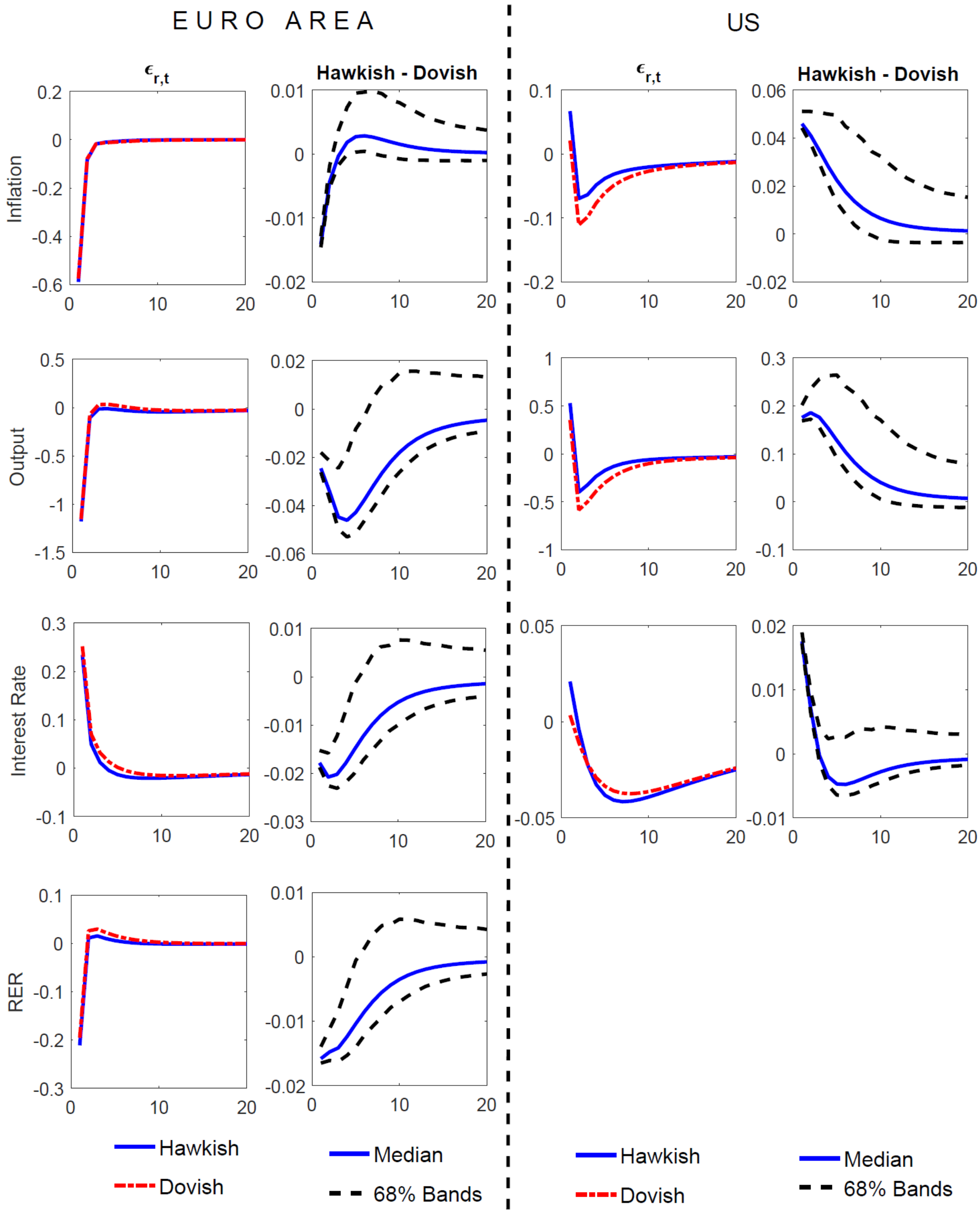


Figure 5 - The figure displays the impulse responses of inflation, output, interest rate in the Euro Area (left-hand side) along with the real exchange rate and the respective responses in the US (right-hand side) to a monetary policy shock in the Euro Area. In each case, the first column reports the median in the Hawkish and the Dovish regime, and the second column includes the median and the 68% error bands for the difference between the two regimes.

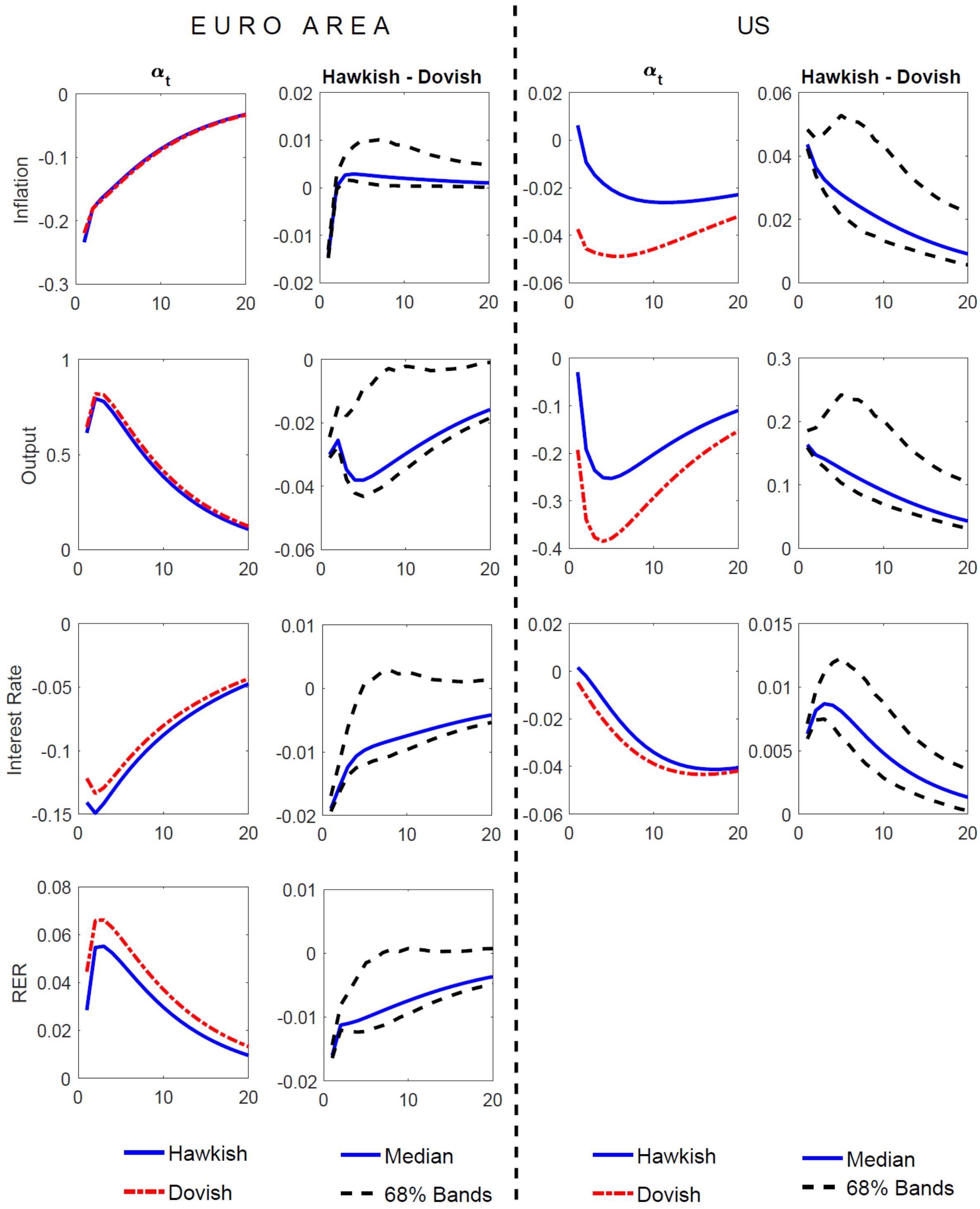


Figure 6 - The figure displays the impulse responses of inflation, output, interest rate in the Euro Area (left-hand side) along with the real exchange rate and the respective responses in the US (right-hand side) to a productivity shock in the Euro Area. In each case, the first column reports the median in the Hawkish and the Dovish regime, and the second column includes the median and the 68% error bands for the difference between the two regimes.

Appendix A (Supplemental Material)

Steady State Uniqueness

Following Liu et al. (2009) the necessary and sufficient condition that guarantees steady state uniqueness is the following:

> **Proposition:** *The steady state equilibrium values of aggregate output, consumption and the real wage are independent of monetary policy and are thus invariant to monetary policy regime shifts. Moreover, as long as domestic monetary policy does not change regimes, it is enough that*
>
> $$\xi_s^* = \frac{1}{\beta}\tilde{\pi}^*\bar{\tilde{Y}}^{*-\phi^*_{y^*,s}},$$
>
> *where $\bar{\tilde{Y}}$ is the steady state output gap, so that the steady state nominal variables are given by $\pi = \bar{\pi}$, $\quad \pi^* = \bar{\pi}^*$, $R = \frac{1}{\beta}\bar{\pi}$ and $R^* = \frac{1}{\beta}\bar{\pi}^*$, and which are independent of regime changes as well.*

Proof

The foreign households intertemporal decision (10) implies that in the steady state the following will be true for the nominal interest rate

$$R^* = \frac{\bar{\pi}^*}{\beta}$$

Additionally, after setting the interest rate smoothing coefficient to zero for simplicity, the assumed interest rate rule of the foreign country (20) receives the following form in the steady state

$$R^* = \xi_s \left(\frac{\pi^*}{\bar{\pi}^*}\right)^{\phi^*_{\pi^*}} \bar{\tilde{Y}}^{*\phi^*_{y^*,s}}$$

Combining the above two equations for the foreign interest rate, solving for ξ_s and recalling that the interest rate in the steady state is such that foreign inflation π^* hits its target $\tilde{\pi}^*$, I receive the following

$$\xi_s = \frac{1}{\beta}\bar{\pi}^*\bar{\tilde{Y}}^{*-\phi^*_{y^*,s}}$$

Therefore the steady state interest rate is

$$R^* = \frac{\bar{\pi}^*}{\beta}$$

and, as already mentioned, inflation at the steady state is $\pi^* = \bar{\pi}^*$. Nominal variables, thus, are independent of policy regime in the steady state. Moreover, as already shown above, the real variables (i.e. consumption, output, labor) are independent of policy regime, as well, in the steady state.

Appendix B (Supplemental Material): NOT FOR PUBLICATION

B.1: The steady State

In this section I compute the steady state of the the real variables, first and then through the proof of proposition 1, the steady state of the nominal variables. Given that in the steady state each firm will change the same price in both countries, the law of one price holds and, hence, PPP holds as well. Therefore the real exchange rate is pegged to one.

$$\bar{Q} = 1$$

Given an international risk sharing condition, PPP implies that at the steady state consumption levels will be equalized across the two countries. Hence

$$\bar{C} = \bar{C}^*$$

From the representative household's labor supply decision, I have for each country that

$$\bar{L}^{\gamma} = \bar{C}^{-\sigma} \frac{\bar{W}}{\bar{P}}$$

$$\bar{L}^{*\gamma} = \bar{C}^{*-\sigma} \frac{\bar{W}^*}{\bar{P}^*}$$

while from the firms production function in each country, I have that

$$\bar{Y} = \bar{A}\bar{L} \quad and \quad \bar{Y}^* = \bar{A}^* \bar{L}^*$$

As already mentioned, firms will set the same price in each country. From their maximization problem, and under the optimal subsidy, it follows that prices at the steady state will be specified as follows

$$\bar{p}_H = \bar{S}\bar{p}_H^* = \bar{P}_H = \frac{\bar{W}}{\bar{A}}$$

$$\frac{\bar{p}_F^*}{\bar{S}} = \bar{p}_F = \bar{P}_F^* = \frac{\bar{W}^*}{\bar{A}^*}$$

and since the law of one price holds, the demand for the home and foreign produced good respectively will be specified as

$$\bar{Y}_H = \left(\frac{\bar{P}_H}{\bar{P}}\right)^{-\rho} \bar{C}$$

$$\bar{Y}_F = \left(\frac{\bar{P}_F^*}{\bar{P}^*}\right)^{-\rho} \bar{C}$$

Combining, thus, the above equations, along with the household's optimal labor decision I end up to the following expressions for the consumption levels in the steady state

$$\bar{C} = \left[\frac{\theta - 1}{\theta}\left(\frac{\bar{P}_H}{\bar{P}}\right)^{1+\rho\gamma}\bar{A}\right]^{\frac{1}{\gamma+\sigma}}$$

$$\bar{C}^{*} = \left[\frac{\theta - 1}{\theta}\left(\frac{\bar{P}_F^{*}}{\bar{P}^{*}}\right)^{1+\rho\gamma}\bar{A}^{*}\right]^{\frac{1}{\gamma+\sigma}}$$

As in Benigno (2004), note that $\frac{\bar{P}_H}{\bar{P}}$ and$\frac{\bar{P}_F}{\bar{P}}$ are both functions of $\bar{T} \equiv \frac{\bar{P}_F}{\bar{P}_H}$, so that the two equations above uniquely determine $\bar{C}$ and $\bar{T}$. Having specified the steady state values of consumption output and relative prices, I can proceed to the proof of proposition in section 5.

B.2: Aggregate Supply and Aggregate Demand

Here, I derive the PPI inflation rates and the aggregate demand equation. Given the uniqueness of the steady state proved in Appendix A, the equilibrium equations are also log-linearized around the symmetric zero inflation steady state. To save space, I present the home economy block only given the symmetry across countries.

Aggregate Supply

Forward looking producers in the home country maximize their profits in the home market by choosing the optimal price specified as

$$\tilde{p}_t(h) = \frac{\theta}{\theta - 1}\frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} \tilde{c}_{t+s}(h)}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} \tilde{c}_{t+s}(h)}$$

The optimal price above rearranged can be written in the following form

$$E_t \sum_{s=0}^{\infty} (\omega\beta)^s \frac{C_{t+s}^{-\sigma} P_{H,t+s}}{P_{t+s}} \left[\left\{\frac{\tilde{p}_t(h)}{P_{H,t+s}} - \left(\frac{\theta}{\theta - 1}\right)\frac{(1-\tau)W_{t+s}}{A_{t+s}P_{H,t+s}}\right\}\tilde{c}_{t+s}(h)\right] = 0$$

and its log-linear approximation is summarized as follows

$$E_t \sum_{s=0}^{\infty} (\omega\beta)^s \left[\hat{p}_{t,t+s}(h) - \left(\frac{\widehat{(1-\tau)W}_{t+s}}{A_{t+s}P_{H,t+s}}\right)\right] = 0 \tag{34}$$

where $\hat{p}_t(h) = ln\left(\frac{\tilde{p}_t(h)}{P_{H,t+s}}\right)$. Using the household's optimality condition (8) I can expand the marginal cost term in the above relationship as follows

$$\frac{\widehat{(1-\tau)W}_{t+s}}{A_{t+s}P_{H,t+s}} = \gamma\left(\hat{y}_{t+s}(h) - a_t\right) + \sigma\hat{C}_{t+s} - a_{t+s} + (1-\delta)\hat{T}_{t+s}$$

Furthermore, by suing the total demand for the home good $\hat{y}_{t+s}(h)$ can be expanded as follows

$$\hat{y}_{t+s}(h) = -\rho\delta\hat{p}_{t,t+s}(h) + \rho\delta\left(1-\delta\right)\hat{T}_{t+s} + \hat{C}_{t+s} - \rho\left(1-\delta^{*}\right)\hat{p}^{*}_{t,t+s}(h)\ldots$$

$$-\rho\delta^{*}\left(1-\delta^{*}\right)\hat{T}^{*}_{t+s} - \frac{\left(1-\delta^{*}\right)}{\sigma}\hat{q}_{t+s} + g_{t}$$

for the home good in the home and the foreign market respectively. From (12) $\hat{p}_t(h)$ and $\hat{p}^{*}_t(h)$ can be expressed as follows

$$\hat{p}_{t,t+s}(h) = \hat{p}_{t,t}(h) - \sum_{i=1}^{s}\pi_{H,t+i}$$

$$\hat{p}^{*}_{t,t+s}(h) = \hat{p}^{*}_{t,t}(h) - \sum_{i=1}^{s}\pi^{*}_{H,t+i}$$

Combining the above relationships for the prices set at date t , I can express the price set by home firms as follows

$$\hat{p}_{t,t}(h) = \frac{1}{1-\omega}\pi_{H,t}$$

Solving for $\hat{p}_{t,t+s}(h)$ in (34) and combining all the above relationships I end to the following relationship for PPI inflation

$$\pi_{H,t} = \beta E_{t}\pi_{H,t+1} + \frac{\left(\omega-\omega^{*}\right)\left(\gamma\theta\left(1-\delta^{*}\right)\left(1-\omega\right)\right)}{\omega\left(1-\omega^{*}\right)\left(1+\theta\gamma\delta\right)}\pi^{*}_{H,t} + \ldots$$

$$\frac{\left(1-\omega\beta\right)\left(1-\omega\right)}{\omega\left(1+\theta\gamma\delta\right)}\hat{\Theta}_{t} + \frac{\gamma\theta\left(1-\delta^{*}\right)\left(1-\omega\right)}{\left(1-\omega^{*}\right)\left(1+\theta\gamma\delta\right)}\left(\beta E_{t}\pi^{*}_{H,t+1} - \pi^{*}_{H,t}\right)$$

where $\hat{\Theta}_t$ is specified as

$$\hat{\Theta}_{t} = \left(1+\gamma\rho\delta\right)\left(1-\delta\right)\hat{T}_{t} + \left(\gamma+\sigma\right)\hat{C}_{t} - \gamma\rho\delta^{*}\left(1-\delta^{*}\right)\hat{T}^{*}_{t} - \frac{\gamma\left(1-\delta^{*}\right)}{\sigma}\hat{q}_{t} - \left(\gamma+1\right)a_{t} + \gamma g_{t}$$

The supply of home produced goods in the foreign country is derived by following similar steps. Home producers set their price in foreign country according to the following maximization rule

$$E_{t}\sum_{s=0}^{\infty}(\omega^{*}\beta)^{s}\frac{C^{-\sigma}_{t+s}P_{H,t+s}}{P_{t+s}}\left[\left\{\frac{\tilde{p}^{*}_{t}(h)}{P^{*}_{H,t+s}}\frac{Z_{t+s}P^{*}_{H,t+s}}{P_{H,t+s}} - \left(\frac{\theta}{\theta-1}\right)\frac{(1-\tau)W_{t+s}}{A_{t+s}P_{H,t+s}}\right\}c^{*}_{t+s}(h)\right] = 0$$

and its log-linear approximation is summarized as follows

$$E_{t}\sum_{s=0}^{\infty}(\omega\beta)^{s}\left[\hat{p}^{*}_{t,t+s}(h) + \widehat{zh}_{t} - \left(\widehat{\frac{(1-\tau)W_{t+s}}{A_{t+s}P_{H,t+s}}}\right)\right] = 0 \qquad (35)$$

where $zh_t = \frac{Z_t P^*_{H,t}}{P_{H,t}}$. And after following similar steps as in the derivation of the supply in the home country I conclude to the following for the supply of home goods in the foreign country

$$\pi^*_{H,t} = \beta E_t \pi^*_{H,t+1} + \frac{(\omega^* - \omega)(\gamma\theta\delta(1-\omega^*))}{\omega^*(1-\omega)(1+\theta\gamma\delta)}\pi_{H,t} + \dots$$

$$\frac{(1-\omega^*\beta)(1-\omega^*)}{\omega^*(1+\theta\gamma\delta)}\hat{\Theta}^*_t + \frac{\gamma\theta\delta(1-\omega^*)}{(1-\omega)(1+\theta\gamma\delta)}(\beta E_t \pi_{H,t+1} - \pi_{H,t})$$

Having used $\widehat{zh_t} = \hat{q}_t - \delta^* \hat{T}^*_t + (1-\delta)\hat{T}_t$, $\hat{\Theta}^*_t$ is specified as

$$\hat{\Theta}^*_t = (\gamma\rho\delta - 1)(1-\delta)\hat{T}_t + (\gamma+\sigma)\hat{C}_t - \delta^*(\gamma\rho(1-\delta^*)-1)\hat{T}^*_t - (\frac{\gamma(1-\delta^*)}{\sigma}+1)\hat{q}_t - (\gamma+1)a_{t+s} + \gamma g_t$$

CPI

$$\pi_t = \pi_{H,t} + (1-\delta)\Delta\hat{T}_t$$

Aggregate Demand

The market clearing condition for home goods market satisfies the following

$$Y_t = C_{H,t} + C^*_{H,t} + G_t$$

or

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\rho}\delta C_{H,t} + \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\rho}(1-\delta^*)C^*_{H,t} + G_t$$

and after log-linearizing and solving for $\hat{C}_t$, I receive the following

$$\hat{C}_t = \hat{Y}_t - \rho\delta(1-\delta)\hat{T}_t + \rho(1-\delta^*)\delta^*\hat{T}^*_t + \left(\frac{1-\delta^*}{\sigma}\right)\hat{q}_t - g_t$$

Using the log-linearized Euler equation, I end up to the aggregate demand equation for the home country

$$\hat{Y}_t = -\frac{1}{\sigma}(i_t - E_t\pi_{t+1}) + E_t\hat{Y}_{t+1} - \rho\delta(1-\delta)E_t\hat{T}_{t+1} + \rho\delta^*(1-\delta^*)E_t\hat{T}^*_{t+1} + \dots$$

$$\frac{(1-\delta^*)}{\sigma}E_t\hat{q}_{t+1} + \rho\delta(1-\delta)\hat{T}_t - \rho\delta^*(1-\delta^*)\hat{T}^*_t - \frac{1-\delta^*}{\sigma}\hat{q}_t + (1-\rho_g)g_t$$

and similarly for the foreign country

$$\hat{Y}^*_t = -\frac{1}{\sigma}\left(i^*_t - E_t\pi^*_{t+1}\right) + E_t\hat{Y}^*_{t+1} - \rho\delta^*(1-\delta^*)E_t\hat{T}^*_{t+1} + \rho\delta(1-\delta)E_t\hat{T}_{t+1} - \dots$$

$$-\frac{(1-\delta)}{\sigma}E_t\hat{q}_{t+1} + \rho\delta^*(1-\delta^*)\hat{T}^*_t - \rho\delta(1-\delta)\hat{T}_t + \frac{1-\delta}{\sigma}\hat{q}_t + (1-\rho_{g^*})g^*_t$$

Real exchange rate and relative prices

$$\Delta\hat{q}_t = \Delta\hat{z}_t + \pi_t^* - \pi_t \tag{36}$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} \quad \hat{T}_t^* = \hat{T}_{t-1}^* + \pi_{H,t}^* - \pi_{F,t}^*$$

B.3: Model Solution

Given the Markov-Switching structure of the model, standard solution techniques cannot be applied in order to find a solution. In the recent literature on Markov-switching DSGE (MS-DSGE) models, various alternative solution algorithms have been suggested (Davig and Leeper (2007); Farmer et al. (2011); Liu et al. (2009); Svensson and Williams (2005)). The one I use is that of Farmer et al. (2011). The virtue of that technique is that it is able to find all possible minimal state variable (MSV) solutions. Moreover, the algorithm is able to find whether an MSV solution is stationary (mean square stable).[30] The model can be written in the following form

$$A(s_t)X_{t+1} = B(s_t)X_t + \Psi(s_t)\varepsilon_t + \Pi(s_t)\eta_t \tag{37}$$

where $X_{t+1} = [y_{t+1}, y_{t+1}^*, \pi_{H,t+1}, \pi_{H,t+1}^*, \pi_{F,t+1}, \pi_{F,t+1}^*, z_{t+1}, T_{t+1}, T_{t+1}^*, y_t, y_t^*, \pi_{H,t}, \pi_{H,t}^*, \pi_{F,t}, \ldots$
$\ldots \pi_{F,t}^*, z_t, T_t, T_t^*, q_t, q_{t-1}, i_t, i_t^*, a_t, a_t^*, g_t, g_t^*, \epsilon_{cp}, \epsilon_{cp^*}^*]$, ε_t is a 8×1 vector of i.i.d. stationary exogenous shocks and η_t is an 9×1 vector of endogenous random variables. The solution of the model can be characterized as a regime-switching vector autoregression, as the one studied by Sims and Zha (2006); Hamilton (1989):

$$X_t = g_{1,s_t}X_{t-1} + g_{2,s_t}\varepsilon_t \tag{38}$$

In order for the above minimal state variable solution to be stationary it must be that the the eigenvalues of

$$(P \otimes I_{24^2})diag\left[\Gamma_1 \otimes \Gamma_1, \Gamma_2 \otimes \Gamma_2\right] \tag{39}$$

are all inside the unit circle. $\Gamma_j = A(j)V_j$ for $j = 1, 2$, where V_j is a 28×17 matrix resulting from the Schur decomposition of $A(j)^{-1}B(j)$. Using the median posterior estimate of each parameter, the largest eigenvalue is equal to 0.9076.

B.4: The welfare criterion

In this section I derive the second order approximation (26) to the representative household's utility function (1) in the home country. The steps for the derivation of the welfare measure for the foreign country are exactly the same. I assume that there is a subsidy to labor. This implies that the steady state is efficient, given that the distortions form monopolistic competition are exhausted. Therefore, I derive the welfare criterion for each country

[30] For an extensive argument regarding the merits of the solution technique used in this paper over the alternative ones see Farmer et al. (2011) and the references therein.

using a second-order Taylor series expansion of (1) around the efficient steady state. Moreover, the welfare measure is expressed as deviations from the flexible price equilibrium, which is efficient as well, given the labor subsidy.

The second order approximation of the welfare of the representative optimizing household receives the following form

$$W_t = U + U_C(\hat{C}_t + \frac{1}{2}(1 + \frac{U_{CC}C}{U_C})\hat{C_t}^2) - U_L(\hat{L}_t + \frac{1}{2}(1 + \frac{U_{LL}L}{U_L})\hat{L}_t^2 \quad (40)$$

where $U_C = C^{-\sigma}$, $U_{CC} = C^{-\sigma-1}$, $U_L = L^{\gamma}$ and $U_{LL} = L^{\gamma-1}$. Using the fact that $\hat{y}_t(h) = a_t + \hat{L}_t$ and approximating it up to a second order I receive the following expression for labor

$$\hat{L}_t = 1 + \frac{y(h)}{L}E_t(\hat{y}_t(h)) + a_t + \frac{y(h)}{2L}var(\hat{y}_t(h)) + a_t^2 - \frac{1}{2}\hat{L}_t^2 \quad (41)$$

Moreover by Woodford (Ch. 6) I have that

$$var(\hat{y}(h)) = \delta\theta^2 var(\tilde{p}_t(h)) + (1-\delta)\theta^2 var(\tilde{p}_t^*(h)) \quad (42)$$

But $\tilde{p}_t(h)$ and $\tilde{p}_t^*(h)$ are determined according to (15) and (16) in the main text. Let $\bar{P}_{H,t} \equiv E_t\left[log(\tilde{p}_t(h))\right]$ and $\Delta_t \equiv var(log(\tilde{p}_t(h))$. Then,

$$\Delta_t \equiv var(log(\tilde{p}_t(h) - P_{H,t-1})$$

$$= E_t\left[(log(\tilde{p}_t(h) - P_{H,t-1})^2 - (E_t\left[log(\tilde{p}_t(h) - P_{H,t-1}\right])^2\right]$$

$$= \omega\Delta_{t-1} + (1-\omega)(log(\tilde{p}_t(h) - \bar{P}_{H,t-1})^2 - (\bar{P}_{H,t} - \bar{P}_{H,t-1}) \quad (43)$$

The same expression holds for $\tilde{p}_t^*(h)$. Before substituting the above expression in (39) and then in (38), note that $\bar{P}_{H,t} = log(\bar{P}_{H,t}) + O(||\xi||^2)$, so that $\bar{P}_{H,t} - \bar{P}_{H,t-1} = \pi_{H,t} + O(||\xi||^2)$. Additionally, the following relationships hold

$$\tilde{p}_t(h) = \frac{\omega}{1-\omega}\pi_{H,t} + P_{H,t}$$

$$\tilde{p}_t^*(h) = \frac{\omega}{1-\omega}\pi_{H,t}^* + P_{H,t}^*$$

Substituting the above expression into (40), I receive the following for Δ_t

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{1}{(1-\omega\beta)}\sum_{t=0}^{\infty}\beta^t\left[\frac{\omega}{1-\omega}\pi_{H,t}^2\right] + t.i.p. + O(||\xi||^3) \quad (44)$$

Similarly for the price set in the foreign country for the home good I receive the following

$$\sum_{t=0}^{\infty} \beta^t \Delta_t^* = \frac{1}{(1-\omega^*\beta)} \sum_{t=0}^{\infty} \beta^t \left[\frac{\omega^*}{1-\omega^*} \pi_{H,t}^{*2} \right] + t.i.p. + O(||\xi||^3) \tag{45}$$

where $t.i.p.$ represents terms independent of policy and $O(||\xi||^3)$ stands for terms of order higher than two.

Additionally, note that for the home output the following relationship holds (and similarly for foreign output)

$$\hat{Y}_t = E_t(\hat{y}_t(h)) + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) var(\hat{y}_t(h)) + O(||\xi||^3)$$

Using the above expression to substitute for $E_t(\hat{y}_t(i))$ in equation (41), I receive the following expression for $\hat{L}_t$

$$\hat{L}_t \approx 1 + \frac{Y}{L}\hat{Y}_t - \frac{1}{2\theta}\frac{Y}{L} var(\hat{y}_t(h)) - \frac{1}{2}\hat{L}_t^2 + t.i.p. \tag{46}$$

Finally, a second order approximation of the total demand function for home goods yields the following:

$$\begin{aligned}\hat{Y}_t \approx\ & \delta\rho(1-\delta)\hat{T}_t + \hat{C}_t - \delta^*\rho(1-\delta^*)\hat{T}_t^* - \frac{(1-\delta)}{\sigma}\hat{q}_t + \frac{1}{2}\hat{C}_t^2 + \\ & \frac{\delta\rho^2(1-\delta)}{2}\hat{T}_t^2 + \delta\rho(1-\delta)\hat{C}_t\hat{T}_t + \frac{\delta^*\rho^2(1-\delta^*)}{2}\hat{T}_t^* - \delta^*\rho(1-\delta^*)\hat{C}_t\hat{T}_t^* + \\ & \frac{\delta((1-\delta))^2}{2\sigma}\hat{q}_t^2 - \frac{(1-\delta)^2\delta\rho}{\sigma}\hat{q}_t\hat{T}_t - \frac{(1-\delta^*)}{\sigma}\hat{q}_t\hat{C}_t + \\ & \frac{(1-\delta^*)(1-\delta)\delta^*\rho}{\sigma}\hat{q}_t\hat{T}_t^* + t.i.p. + O(||\xi||^3)\end{aligned} \tag{47}$$

Substituting (44), (45), (46) and (47) into (41) and then in (40), I receive the following form for the welfare measure

$$W_t = -\tfrac{1}{2}u_c\bar{C}E_0\{\textstyle\sum_{t=0}^{\infty}\beta^t[\lambda_1\tilde{Y}_t^2 + \lambda_2\pi_{H,t}^2 + \lambda_3{\pi_{H,t}^*}^2 + \lambda_4\tilde{q}_t^2 + \lambda_5\tilde{T}_t^2 + \lambda_6\tilde{T}_t^{*2} + \lambda_7\tilde{C}_t\tilde{T}_t + \lambda_8\tilde{C}_t\tilde{T}_t^* + \ldots$$

$$+\lambda_9\tilde{C}_t\tilde{q}_t + \lambda_{10}\tilde{q}_t\tilde{T}_t + \lambda_{11}\tilde{q}_t\tilde{T}_t^* + \lambda_{12}\tilde{Y}_t\tilde{T}_t + \lambda_{13}\tilde{Y}_t\tilde{T}_t^* + \lambda_{14}\tilde{q}_t\tilde{Y}_t + \lambda_{15}\tilde{T}_t\tilde{T}_t^*]\} + t.i.p. + O(||\xi||^3)$$

where

$$\lambda_1 = \sigma + \overline{C}^{\frac{1-\sigma}{\gamma}}\left(\frac{W}{\overline{C}}\right)^{\frac{\gamma+1}{\gamma}}$$

$$\lambda_2 = \left(\overline{C}^{\frac{1-\sigma}{\gamma}}\left(\frac{W}{\overline{C}}\right)^{\frac{\gamma+1}{\gamma}}\right)\frac{\omega\theta}{1-\omega}$$

$\lambda_3 = \left(\overline{C}^{\frac{1-\sigma}{\gamma}} \left(\frac{\overline{W}}{\overline{C}}\right)^{\frac{\gamma+1}{\gamma}}\right) \frac{\omega^*\theta}{1-\omega^*}$

$\lambda_4 = \frac{(1-\delta)^2(\delta+1)}{\sigma}$

$\lambda_5 = \rho^2\delta\,(1-\delta)\,(1+\sigma\delta\,(1-\delta))$

$\lambda_6 = \rho^2\delta^*\,(1-\delta^*)\,(1+\sigma\delta^*\,(1-\delta^*))$

$\lambda_7 = \rho\delta\,(1-\delta)$

$\lambda_8 = \rho\delta^*\,(1-\delta^*)$

$\lambda_9 = \frac{(1-\delta^*)}{\sigma}$

$\lambda_{10} = \frac{\rho\delta(1-\delta)^2(\sigma+1)}{\sigma}$

$\lambda_{11} = \frac{\rho\delta^*(1-\delta^*)^2(\sigma+1)}{\sigma}$

$\lambda_{12} = 2\sigma\rho\delta\,(1-\delta)$

$\lambda_{13} = 2\sigma\rho\delta^*\,(1-\delta^*)$

$\lambda_{14} = 2\,(1-\delta)$

$\lambda_{15} = (\sigma+1)\,[\rho\delta\,(1-\delta)]^2$

B.5: Definition of Matrices in the Dynamic Programming Problem of section 6.2

As described in section 6.2 in the text, the policy maker chooses the control r_t (i.e. the interest rate) which maximizes the expected value of the intertemporal loss function

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \widetilde{L}(X_t, r_t) \tag{48}$$

subject to X_0, s_{-1} and the reduced form model describing the world economy

$$X_{t+1} = \Omega(s_t)X_t + \Lambda(s_t)r_t + C(s_t)\varepsilon_t \qquad t \geq 0 \tag{49}$$

The control variable is the nominal interest rate r_t. Therefore, the matrices of the model as specified in equation (28) in the main text have to be adjusted appropriately before being able to execute the optimal discretionary monetary policy problem. From equation (49), above it becomes clear that matrices $\Omega(s_t)$ and $\Lambda(s_t)$ are respectively given by

$$\Omega(s_t) = \tilde{A}(s_t)^{-1}B(s_t) \; and \; \Lambda(s_t) = \tilde{A}(s_t)^{-1}\Upsilon(s_t) \tag{50}$$

Matrix $B(s_t)$ carries the same structure as the one specified by the log-linearized model described in (34). Matrix $\tilde{A}(s_t)$ is specified as follows

$$\tilde{A}(s_t) = \begin{bmatrix} A_{11\times 20}(s_t) & 0_{11\times 1} & A_{11\times 7}(s_t) \\ 0_{1\times 20} & 1 & 0_{1\times 7} \\ A_{16\times 20}(s_t) & 0_{16\times 1} & A_{16\times 7}(s_t) \end{bmatrix}$$

where the A sub-matrices above are matrices containing the elements of matrix $A(s_t)$ in (34) corresponding to the specified dimensions. The home interest rate rule corresponds to the 12th line of the system, while the home nominal interest rate is the 21st element as specified in the X_t vector of equation (49). The 0 matrices are matrices of zeros. Finally, matrix $\Upsilon(s_t)$ in (50) above receives the following form

$$\Upsilon(s_t) = [0\ 1/\sigma\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]'$$

where the term $1/\sigma$ corresponds to the presence of the home nominal interest rate in the home aggregate demand equation in Appendix B.2.

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