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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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De Nederlandsche Bank NV
P.O. Box 98
1000 AB AMSTERDAM
The Netherlands

Optimal risk-sharing in pension funds when stock and labor markets are co-integrated*

Ilja Boelaars[a] and Roel Mehlkopf[b]

[a] *The University of Chicago*
[b] *Tilburg University and De Nederlandsche Bank*

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Abstract

A well established believe in the pension industry is that collective pension funds should take more stock market risk (compared to individual retirement accounts) since risk may be shared with future generations. We extend the OLG model of Gollier (2008) by adding labor income risk in the spirit of Benzoni, Collin-Dufresne, and Goldstein (2007) and show that this idea may be misguided. For the empirical range of parameter values reported by Benzoni et. al., we find that optimal risk-sharing actually implies that collective pension funds should take less stock market risk, not more. If labor income and dividend income are co-integrated, efficient risk-sharing policies should transfer risk from future generations to current generations instead of the other way around. Furthermore, we find that the potential welfare gains from intergenerational risk-sharing are significantly lowered.

Keywords: Dynamic portfolio choice; Labor income risk; Pension; Retirement; Intergenerational risk-sharing; Funded pension systems.
JEL classifications: H55, G11, G23, J26, J32.

* Corresponding author: Ilja Boelaars: ilja@uchicago.edu. The introduction of this paper contains some material from Roel Mehlkopfs PhD Thesis chapter *Risk Sharing and Long-Run Labor Income Risk*, which has not been independently published earlier (Mehlkopf, 2011).

1 Introduction

Risk sharing between non-overlapping generations can theoretically be welfare improving. The economic intuition behind this result is that it is optimal to spread risks over a broader base (i.e. a larger number of generations) than is possible in financial markets in which only overlapping generations are able to trade risk with each other. This point was made by Diamond (1977), Merton (1983) and Gordon and Varian (1988). More recent contributions include Smetters (2006), Teulings and De Vries (2006), Ball and Mankiw (2007), Bovenberg, Nijman, Teulings, and Koijen (2007), Gollier (2008) and Cui et. al. (2010). Several papers suggest that, by using its financial buffer efficiently, a pension fund may able to facilitate intergenerational risk sharing. Pension funds that are able to facilitate intergenerational risk sharing can be found in many countries. Examples include the the Japan Government Pension Investment Fund, the Canada Pension Plan, the Government Pension Fund in Norway, the ATP funds in Denmark and the occupational pension funds in Switzerland and the Netherlands. Novy-Marx and Rauh (2014) describe how risk-sharing may also be relevant for the US pension industry.

If designed properly, a risk-sharing contract may lead to a welfare improvement for all generations from an *ex-ante* perspective (i.e. before the economic shocks materialize that determine the size and direction of risk-sharing transfers between generations). Many studies on risk-sharing in pension funds abstract from labor-income risk, e.g. Teulings and De Vries (2006), Bovenberg et al. (2007), Gollier (2008) and Cui et. al. (2010). The models of these papers find that it is optimal for a pension fund to increase their stock market exposure (compared to a setting without intergenerational risk-sharing) and spread financial-market risk over as many generations as possible. By smoothing financial shocks to the collective fund over a large number of generations, the time-diversification of risk is improved and welfare is increased. These papers report large welfare gains from risk sharing. In addition, the models in these papers suggest that it is optimal for a pension fund to apply high levels of 'smoothing': retirement benefits hardly respond to shocks in the funds funding levels. A low level of smoothing would be sub-optimal, because it would imply that a relatively large share of the current shock is born by current generations, while it is optimal when the shock is spread out over all current and future generations.

In this paper we point out that the long-run dynamics of labor-income risk crucially determine optimal risk-sharing rules. We show that, if labor income and dividends follow cointegrated processes, it may actually be optimal for a collective pension plan to take *less* stock market risk than one would take in the absence of intergenerational risk-sharing, instead of more. For all levels of labor-stock market cointegration within the parameter range reported by Benzoni et al. (2007) it turns out not to be optimal for the collective pension fund to smooth current shocks. Instead, the fund does the opposite: the funding level of the pension fund is less volatile than the level of retirement benefits. The policy of smoothing, which was optimal in absence of labor income risk, is replaced with a policy of - so to say - amplification. By doing so, the collective fund optimally transfers risk from future generations to current generations, instead of the other way around.

We consider the OLG model of Gollier (2008) but extend this by adopting the labor income and dividend processes from Benzoni et al. (2007) (BCG hereafter). BCG assume that labor earnings are cointegrated with dividends on a stock portfolio. This modeling environment is characterized by the property that stock and labor markets move together at long horizons. The economic idea behind this assumption is that, in particular in the long-run, there are general economic factors that affect both labor income and capital income in the same direction[1] Consistent with empirical findings, the cointegration-framework of BCG allows for a low (or zero) contemporaneous correlation between labor income shocks and stock returns, whereas long-run correlations are high.[2]

BCG show that co-integration causes the human capital of young investors to become strongly correlated with stock returns, which reduces their appetite to invest in the stock market directly. In contrast to other studies that ignore long-run labor income risk, they find that it can even be optimal for young investors to take a short position in stocks, as this provides a hedge against future labor income shocks. This result could be a justification for

[1]As pointed out by Baxter and Jermann (1997), the form of most production functions used in macroeconomic theory imply that the long-run restriction that the factor shares of labor and capital are stationary. Indeed, BCG provide empirical evidence for this hypothesis, by showing that labor income and dividends on stock holdings are co-integrated. The estimates for the cointegration coefficient are significant, but fall in a wide range. BCG find an estimate for the cointegration coefficient of 0.205 when using US data going back to 1929, while the estimate is as low as 0.0475 when relying on the post-World War II sample period.

[2]Many studies feature low (or zero) correlations between aggregate labor income and stock returns, both contemporaneously as well as at long horizons. See e.g. Lucas and Zeldes (2006), Jagannathan and Kocherlakota (1996), Sundaresanz and Zapatero (1997), Carroll and Samwick (1997), Gourinchas and Parker (2002), Campbell, Cocco, Gomes, and Maenhout (2001), Cocco, Gomes, and Maenhout (2005), Davis and Willen (2000), Gomes and Michaelides (2005), Haliassos and Michaelides (2003), Viceira (2001). The assumption of low correlations at long horizons is controversial, given the empirical evidence provided in BCG.

the high-levels of non-participation in stock markets by young investors (the stock participation puzzle).

Many other papers have assumed that labor income and dividend flows are cointegrated, see e.g. Baxter and Jermann (1997), Menzly, Santos, and Veronesi (2004), Santos and Veronesi (2006) and Geanakoplos and Zeldes (2010). Earlier studies that investigate a link between aggregate labor income and asset prices include Mayers (1974), Fama and Schwert (1977), Black (1995), Jagannathan and Kocherlakota (1996), and Campbell (1996). In the study of Campbell (1996), a high correlation between human capital and market returns is obtained in a model in which there is no strong interrelation between stock and labor markets. Campbell (1996) uses the same highly time-varying discount factor to discount both labor income and dividends, which results in a high correlation between human capital and market returns.

This paper contributes to the existing literature in two ways. First, we rederive the model of Gollier (2008) in continuous time (instead of discrete time), which allows us to derive the results fully analytically. Secondly, we relax the assumption that labor income is risk-free and allow for long-run correlation between labor and capital income and show that this will have large implications for optimal risk-sharing.

There are two other papers that explore risk-sharing in a setting in which stock and labor markets are subject to a common risk factor: van Hemert (2005) and Bohn (2009). In the framework of van Hemert (2005), labor income and capital returns follow a joint Markovian process, thereby allowing for horizon-dependent correlations. However, the Markov process in van Hemert (2005) is imposed to be stationary, implying that labor income is not risky in the long run. Bohn (2009) uses a VAR model to estimate 30-year correlations between productivity and capital returns. He reports a positive correlation between 30% and 60%, depending on the specification of the VAR model. In line with our findings, Bohn (2009) finds that due to risky labor income, workers bear systematically more risk than retirees. Efficient risk-sharing policies should therefore shift risk away from workers to retirees. He concludes that safe pensions can be rationalized as efficient only if preferences display age-increasing risk aversion, such as habit formation.

The structure of the remainder is as follows. In section 2 we summarize the general setup of the model by Gollier. In 3 we will (re)derive its results in continuous time. We continue with a brief discussion of the model in 4. This then sets the stage for our extension with labor income risk which is introduced in section 5 and in section 6 we derive our results. Finally, section 7 concludes.

2 General setup

For comparability we will stay as close as possible to the model of Gollier (2008). The main difference will be that we will work in continuous time and we will represent stock market returns by a geometric Brownian motion with drift. Gollier works in discrete time and uses an empirical distribution of past S&P 500 returns. Working in continuous time has the advantage that we can derive all results analytically and it makes it easier to add labor income risk later on.

We consider a model of overlapping generations with a defined contribution pension system. Once a generation enters the labor market, it saves a fixed portion of income for retirement, denoted by $L(t)$. After n years, a generation retires and receives a one-time retirement benefit of $b(t)$[3]. There is a continuum of generations which we will index by their retirement date T. Each generation is of the same size, which is normalized to one. Hence, the total number of generations that contribute to their retirement savings at any point in time is n. Following Gollier, we will set $n = 40$ in our numerical illustrations.

We will be comparing two alternative institutional arrangements. First we consider the situation in which each generation solves its own optimization problem, which we will refer to as 'autarky'. Then, we will compare this to the case in which a hypothetical planner runs a collective pension fund, which invests the collective wealth of all current and future generations. We will refer to this as the solution with intergenerational risk-sharing.

Financial market

The financial market features two assets: a risk-less cash account and a stock index. The return processes are:

$$\frac{dB(t)}{B(t)} = rdt$$
$$\frac{dS(t)}{S(t)} = \mu_S dt + \sigma dz_S$$

[3] We could think of $b(t)$ as the value of an annuity that is being paid out during retirement.

Markets are assumed to be complete, so asset prices are captured by a single stochastic discount factor:

$$\frac{dM(t)}{M(t)} = -rdt + \phi_S dz_S \tag{2.1}$$

where it follows from the definition of the stochastic discount factor that $\mu_S = r - \sigma\phi_S$. For comparability, we will set the parameters values in our illustrations and numerical examples in accordance with the return distribution used by Gollier: $r = 0.02$, $\sigma = 0.136$ and the risk premium $-\sigma\phi_S = 0.039$.

Preferences

The preferences of each generation are represented by a CRRA utility function. Since the model considers a defined contribution setting, utility during the contribution phase is irrelevant for the optimal investment problem. Therefore, we only consider utility defined over the retirement benefit $b(T)$ here. So, if $U(t,T)$ denotes the expected utility at time t of the generation that retires at date T, then:

$$U(t,T) = \mathbb{E}_t\left[\frac{b(T)^{1-\gamma}}{1-\gamma}\right] \tag{2.2}$$

In autarky, it is assumed that individual generations cannot expose themselves to equity risk before they enter the labor force. Once they enter, we will not impose a borrowing constraint. So, individuals may borrow and take leverage. The advantage of this assumption is that it allows us to derive our results analytically, while the utility effect of not imposing a no-borrowing constraint during the lifetime of a generation has only a marginal effect on the level of utility (as also pointed out by Gollier). In absence of a borrowing constraint, the inefficiency in autarky comes solely from the fact that future generations are unable to get exposure to shocks. that occur before their labor market entry. For generations that are currently alive ($T < (t+n)$) the problem is to maximize 2.2 with respect to $b(T)$ subject to:

$$W(t,T) = \mathbb{E}_t\left[b(T)\frac{M(T)}{M(t)}\right] \tag{2.3}$$

where $W(t,T)$ denotes total wealth of generation T at time t, which includes the present value of future retirement savings. The optimal wealth process is subsequently found by determining the evolution of the present value of the optimal retirement benefit over time.

3 Optimal risk-sharing with deterministic labor income

Let us first consider the model with deterministic labor income, as studied by Gollier. So, we have that $L(t) = L$. Total wealth at time t of generation T is given by:

$$W(t,T) = \begin{cases} F(t,T) + \frac{1-e^{-rT}}{r}L & \text{for } T \leq t+n \\ \frac{e^{-r(T-n)}-e^{-rT}}{r}L & \text{for } T > t+n \end{cases} \tag{3.1}$$

where $F(t,T)$ is accumulated financial wealth, which is zero for generations that did not enter the labor force yet.

Autarky

The solution to the (unconstrained) portfolio choice problem is well known since Merton (1969) and Samuelson (1969). The law of motion of the optimal wealth process for those in the labor force, is:

$$\frac{dW(t,T)}{W(t,T)} = \left(r + \frac{\phi^2}{\gamma}\right)dt - \frac{1}{\gamma}\phi_S dz_S, \tag{3.2}$$

which implies that the optimal fraction of total wealth invested in the stock index is $-\frac{1}{\gamma}\frac{\phi_S}{\sigma}$. The optimal retirement benefit is equal to terminal wealth, which, given time 0 information, is

$$b(T) = W(0,T)\exp\left\{rT + \left(\frac{\phi^2}{\gamma} - \frac{1}{2}\frac{\phi^2}{\gamma^2}\right)\min[n,T] - \frac{\phi}{\gamma}\int_{\max[0,T-n]}^{T} dz_s(u)\right\} \tag{3.3}$$

where the minimum and maximum operator capture the fact that a generation may only take stock market exposure while in the labor force.

When considering welfare, it will be useful to translate utility units into certainty equivalent units. Define the certainty equivalent at time t, $\mathrm{CE}(t,T)$, as the certain retirement benefit at time T that would yield the same expected utility at time t as the stochastic benefit $b(T)$:

$$CE(t,T) \equiv \mathbb{E}_t\left[b(T)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{3.4}$$

Then, in autarky, the certainty equivalent for each generation is given by:

$$\text{CE}(t, T) = W(t, T) \exp \left\{ r(T - t) + \left(\frac{1}{2} \frac{\phi_S^2}{\gamma} \right) min[T - t, n] \right\} \tag{3.5}$$

The term $\frac{1}{2} \frac{\phi_S^2}{\gamma}$ captures the benefit per unit of time of being able to invest in the risky asset. This is only possible during a maximum of n years for each generation.

First best solution

In autarky the benefit of taking stock market risk can only be reaped during a maximum of n years. This is what makes the autarky solution inefficient in this model. There is potential welfare to be gained by allowing future generations to take stock market exposure before their own lifetime. If a social planner could somehow invest on behalf of future generations and commit these future generations to actually bear the risk, the certainty equivalent would become:

$$\text{CE}_{FB}(t, T) = W(0, T) \exp \left\{ rT + \left(\frac{1}{2} \frac{\phi_S^2}{\gamma} \right) T \right\} \tag{3.6}$$

Note that current generations would not gain anything, since they were not constrained to begin with. Future generations however would gain proportionally to the number of extra years the planner can take risk on their behalf.

First best risk-sharing in a collective pension fund

Gollier suggests that this potential welfare gain may be realized by forming a collective pension fund. If a benevolent social planner can force all generations to participate, the planner can expose future generations to contemporaneous stock market risk, by making the benefits of future generations contingent on today's returns. The collective fund could act as a vehicle to transfer today's gains or losses to future generations

Let the utility function of the planner be a simple weighted sum of all individual generations utilities:

$$U(t) = \mathbb{E}_t \left[\int_t^{\infty} \beta(T)^{\gamma} \frac{b(T)^{1-\gamma}}{1-\gamma} dT \right] \tag{3.7}$$

where $\beta(T)^\gamma$ is the welfare weight the planner puts on generation T. We will be more specific about these weights in a moment. Let us furthermore assume that the planner is not borrowing constrained. Again, not imposing a constraint has the benefit that the optimal solution can be found analytically. The planner then maximizes 3.7 subject to

$$\tilde{W}(0) = \mathbb{E}_0\left[\int_t^\infty \tilde{b}(T)\frac{M(T)}{M(0)}dT\right] \tag{3.8}$$

where $\tilde{W}(0)$ is the planners initial total wealth, which is equal to aggregate financial wealth of current generations, plus the present value of all future contributions by all generations. Let us refer to the present value of future contributions as human capital[4].

The solution to the planners problem is again the well known solution to Mertons' problem, but now in a context with intermediate consumption and an infinite horizon. So, the process for optimal total wealth is given by:

$$\frac{d\tilde{W}(t)}{\tilde{W}(t)} = \left(r + \frac{\phi_S^2}{\gamma}\right)dt - \frac{\tilde{b(t)}}{\tilde{W}(t)}dt - \frac{\phi_S}{\gamma}dz_S \tag{3.9}$$

And the process for the collective pension funds wealth, $\tilde{F}(t)$ is:

$$\frac{d\tilde{F}(t)}{\tilde{F}(t)} = \left(r + \frac{W(t)}{F(t)}\frac{\phi_S^2}{\gamma}\right)dt - \frac{nL - \tilde{b(t)}}{\tilde{F}(t)}dt - \frac{\tilde{W}(t)}{\tilde{F}(t)}\frac{\phi_S}{\gamma}dz_S \tag{3.10}$$

The optimal fraction of total wealth invested in the stock index is the same as in the autarky problem. The difference, however, is that the planner applies this weight to aggregate total wealth, which includes the human capital of future generations, resulting in a higher overall investment in the stock index. Total investment in the stock index in autarky is equal to $-\frac{1}{\gamma}\frac{\phi_S}{\sigma}\int_0^n W(t,T)dT$, while the collective planner invests $-\frac{1}{\gamma}\frac{\phi_S}{\sigma}\int_0^\infty W(t,T)dT$ in the stock market. So, the planner will always take more stock market risk on aggregate compared to autarky. For our parameter choices so far, the present value of aggregate human capital is $\tilde{H}_0 = \frac{yn}{r} = 2000L$. If we assume that in the initial distribution of financial wealth corresponds to the median scenario in autarky aggregate initial financial wealth is $\tilde{F}_0 = 1732L$. The median investment in stocks by the collective fund is then $1540L$, while in autarky median aggregate

[4]Note that in our defined contribution setting, this definition of human capital only captures the part of human capital which is saved for retirement.

investment in the stock market is only $972L$.

In addition to choosing the optimal portfolio allocation, the planner also decides what the optimal rule is for the distribution of retirement wealth. The optimal retirement benefit to generation T is given by:

$$\tilde{b}(T) = \tilde{b}(0)\beta(T)\exp\left\{\frac{1}{\gamma}\left(r + \frac{1}{2}\phi_S^2\right)T - \frac{1}{\gamma}\phi_S\int_0^T dz_S(u)\right\} \tag{3.11}$$

where we normalized $\beta(0)$ to one. This implies that the retirement benefit follows the following process:

$$\frac{d\tilde{b}(t)}{\tilde{b}(t)} = (...)dt - \frac{\phi_S}{\gamma}dz_S \tag{3.12}$$

where we suppress the drift term for simplicity (note that it will depend on our choice of $\beta(T)$). The retirement benefit fluctuates one-to-one with aggregate total wealth and all generations share proportionally in todays shock. Notice that collective pension fund wealth is always more volatile than the retirement benefit, due to the $\frac{\tilde{W}(t)}{\tilde{F}(t)}$ term in 3.10. In the pension industry this is often described as 'smoothing'. All generations that still have some human capital prefer to take more stock market risk (as a share of financial wealth), than the currently retiring generation. The fund takes this extra risk on their behalf, but, to not over expose the currently retiring generation, shocks to the funding level are only gradually transmitted into the retirement benefit. This could be seen as a justification for the fact that collective pension funds in practice often apply various kinds of smoothing mechanisms, either explicitly or through their valuation assumptions of assets and liabilities.

Welfare effects

Let us now turn to the welfare effect of the risk-sharing arrangement. The certainty equivalent for each generation can be found by pluging the optimal retirement benefit in 3.11 into the definition of the certainty equivalent (3.4). This gives us that

$$\tilde{CE}(0,T) = \tilde{b}(0)\beta(T)\exp\left\{\frac{1}{\gamma}\left[\left(r + \frac{1}{2}\frac{\phi_S^2}{\gamma}\right)T\right]\right\} \tag{3.13}$$

Now, we still need to specify $\beta(T)$. Our choice of $\beta(T)$ will determine how the funds wealth is distributed across generations. One choice of $\beta(T)$ could be to set it such that the market consistent present value of the reitrement benefit for each generation is unchanged compared to the autarky solution. We could think of this particular choice of $\beta(T)$ as a non-re-distributive choice. If we do so, the welfare gain for all generations would be exactly as given in equation 3.6. This would imply though, that the expected utility of generations far into the future goes to infinity. Hence, the welfare weights would have to be exploding too. If not, the planner would rather bring some consumption forward. Gollier suggests to choose welfare weights such that all generations obtain the same certainty equivalent[5]. A downside of this approach is that this does not ensure that the introduction of risk-sharing is a Pareto improvement. As a matter of fact, in our calculations, it is not. Older generations have a higher certainty equivalent in autarky simply because they do not face much uncertainty anymore. If the introduction of risk-sharing implies that their certainty equivalent is equalized with future generations, this lowers their utility. We therefore decide to take a slightly different approach. We will choose the welfare weights such that all generations see their certainty equivalent level of consumption at the initial date increase by the same factor α. So, we choose $\beta(T)$ such that:

$$\tilde{CE}(0,T) = \alpha CE(0,T) \tag{3.14}$$

Combining 3.14 with 3.13 then implies that:

$$\beta(T) = \frac{CE(0,T)}{CE(0,0)} \exp\left\{-\frac{1}{\gamma}\left[\left(r + \frac{1}{2}\frac{\phi_S^2}{\gamma}\right)T\right]\right\} \tag{3.15}$$

If we, like Gollier, set $\gamma = 5$ we find that $\alpha = 1.113$, so the certainty equivalent for all generations increases by 11.3 percent. Gollier, using his choice of $\beta(T)$ reports a 19 percent welfare gain for future generations. If we use the welfare same welfare weights as Gollier, we find that future generations gain 23 percent[6]. Current generations however lose in this case. For example, the currently retiring generation would face a welfare loss of 3.7 percent.

[5]The certainty equivalent for all generations is particularly simple in this case: $\tilde{CE}(0,T) = \tilde{CE}_0 = \tilde{W}(0)\left(r + \frac{1}{2}\frac{\phi^2}{\gamma}\right)$

[6]Since Gollier uses an empirical S&P 500 return distribution and works in discrete time, we may expect to see some differences here.

4 Modeling discussion

The Gollier model, like other models that assume risk-free labor income, leads to three main conclusions with respect to optimal risk-sharing in a collective pension fund. Firstly, optimal risk-sharing significantly improves welfare. Secondly, if this welfare improvement is implemented by the introduction of a collective pension fund, the collective fund would optimally take more stock market risk than one would take in a collection of individual (or generational) retirement accounts. This is often used as a justification in practice for large collective pension funds to choose relatively risky asset protfolios. Thirdly, the collective fund optimally smooths retirement benefits. Retirement benefits fluctuate less then one-for-one with the pension funds return realization. This is a logical consequence of the fact that future generations want to take relatively more risk with their financial wealth due to the presence of risk-free human capital.

Of course, we should be careful when applying this model to real-world pension plan design. One important problem is that social planners do not exist outside our modeling environment. The question is if the optimal solution is politically feasible. Gollier also points this out. He mentions that future participants may not be willing to bear the risky outcomes that were realized before their life-time. We would like to add to this that this may not be the only concern, since typically older generations have more political power. A bigger concern may actually be that the currently older generations use their political power to take a larger share from the collective fund than the social planner would find optimal.

In practice, this is a particularly difficult political problem since the optimal distribution of benefits depends on what one beliefs about the 'true' model. Even if all generations could agree to implement the planners solution, one still has to find agreement about the appropriate choice of parameter values. For example, the future equity premium is not objectively observable. Yet, in our model setting, the current level of the retirement benefit is very sensitive to this parameter. If one were to argue that the equity risk-premium is one percentage point higher, 4.9% instead of 3.9% in our illustration, the optimal retirement benefit of the current generation would increase by 20%. This highlights that there are significant incentives for the older generations to be overly optimistic about the future risk-return trade-off.

The discussion should not be limited to parameter choices, only. The question is if we are considering the 'right' model more generally. Alternative specifications may lead to different conclusion. The rest of this paper could be considered an illustration of this. A particularly strong assumption so far has been that future labor income is risk-free. Consequently in the model future generations are not exposed to risk at all. We will consider what happens when we relax this assumption and introduce labor income risk. In particular, we will focus on the case where labor income and stock market performance are related in the long-run. In the setup we considered so far the stock index may diverge indefinitely from labor income, which means that either dividends diverge or that the discount rate diverges. This seems to be an unwanted feature of the model. For short horizons this may not be such an issue, but when we consider risk-sharing over multiple generations, it should be a concern.

Over long-horizons, it seems reasonable to believe that capital and labor income are cointegrated. This point is, for example, made in Benzoni et al. (2007) (BCG hereafter). BCG subsequently argue that, if labor income is cointegrated with the dividend process, investors may want to invest significantly less in stocks. BCG show that the optimal investment in equity may even be negative for the very young. Due to the long-run correlation, shorting stocks could provide a hedge against negative developments in future labor income.

We will next investigate how the conclusions regarding intergenerational risk-sharing and optimal pension fund investment change if we introduce labor income risk as modeled by BCG.

5 Adding labor income risk

BCG suggest to model the long-run relation between labor income risk and stock market risk by letting the labor-to-dividend income ratio follow a mean-reverting process. We will follow their example here. We will first introduce the dividend process and subsequently the labor income process. After that we will determine the sensitivity of human capital to stock market risk.

Dividends and the stock price process

The dividend process follows a geometric Brownian motion with drift:

$$\frac{dD(t)}{D(t)} = g_d dt + \sigma dz_S, \tag{5.1}$$

where g_d is the average growth rate of dividends. The pricing kernel is still defined as before (2.1). Hence, the stock price, the price of a claim to the stream of dividends between now and infinity, can be found to be:

$$P(t) = \frac{D(t)}{r + \phi_3 \sigma - g_d} \tag{5.2}$$

Let $S(t)$ denote the value of a stock index that reinvests any dividends received. It then follows from the pricing kernel that the instantaneous return on the stock index will be:

$$\frac{dS(t)}{S(t)} = (r - \phi_S \sigma)dt + \sigma dz_S \tag{5.3}$$

This is exactly the same stock return we saw before. The fact that we have have explicitly modeled the stock price as a function of the underlying uncertain dividend flow, now allows us to specifically model labor income such that its correlation with the stock process is low over short horizons but higher over long horizons.

Labor income

BCG suggest to model the cointegration of labor income and the stock market by letting the dividend-labor income ratio follow a mean-reverting process. Define:

$$y(t) \equiv \ell(t) - d(t) - \bar{\ell d} \tag{5.4}$$

where $\ell(t)$ is log labor income, $d(t)$ is log dividend income and $\bar{\ell d}$ is the long-run mean log labor income to dividend ratio. It is then assumed that $y(t)$ follows a standard mean reverting Ornstein-Uhlenbeck process:

$$dy(t) = -\kappa y(t)dt + \nu_L dz_L(t) - \nu_S dz_S(t) \tag{5.5}$$

where $dz_L(t)$ is another standard Brownian motion that captures the part of labor income risk that is uncorrelated to stock market risk. We will here set this source of risk to zero, since our focus is on the sharing of financial market risk in collective pension schemes[7]. Combining 5.1, 5.4 and 5.5, and setting $\nu_l = 0$ we find that the process for log labor income is:

$$d\ell(t) = \left(-\kappa y(t) + g_d + \bar{\ell d} - \frac{1}{2}\sigma^2\right) dt + (\sigma - \nu_S)dz_S(t) \tag{5.6}$$

We will assume that labor income is contemporaneously uncorrelated with stock market risk by setting $\nu_S = \sigma$[8]. Notice that in the long-run labor income will still be correlated through the mean-reversion in $y(t)$. We can see this more clearly when we solve for labor income at time t conditional on time $s < t$ information:

$$\begin{aligned} L(t) = L(s) \exp \Bigg\{ &-\kappa B(t-s)y(s) + (g_d + \bar{\ell d} - \frac{1}{2}\nu_S^2)(t-s) \\ &+\kappa \nu_S \int_s^t B(t-v)dz_S(v) \Bigg\} \end{aligned} \tag{5.7}$$

where $B(x) = \frac{1}{\kappa}\left(1 - e^{-\kappa(x)}\right)$. In our numerical illustrations, we will set $(g_d + \bar{\ell d} - \frac{1}{2}\nu_S^2) = 0$, such that there is no expected real income growth in case $\kappa = 0$. By doing so, $\kappa = 0$ corresponds exactly to the Gollier setting (which does not feature income growth).

[7] Misschien nog een verdere rechtvaardiging of toelichting hier?
[8] Justify?

Human capital process

Now we have specified the labor income process, let us consider what the present value of labor income looks like. In what follows it will be particularly useful to know what the correlation of human capital with stock market risk looks like.

In the appendix we show that the present value at time t of the future labor income cash-flow at time τ , denoted by $PV_L(y,t,\tau)$ can be written as:

$$PV_L(y,t,\tau) = L(t)\exp\left\{A(\tau - t) - \kappa B(\tau - t)y(t)\right\} \tag{5.8}$$

where $A(x)$ is a function of the horizon only (which can be found in the appendix (A6)) and $B(x)$ is as specified above. Hence, the present value of human capital for generation T is:

$$H(y,t,T) = L(t)\int_{\max[t,T-n]}^{T} \exp\left\{A(\tau) - \kappa B(\tau)y(t)\right\} d\tau \tag{5.9}$$

From 5.9 we can find the exposure of human capital to dz_S:

$$\frac{dH(y,t,T)}{H(t,T)} = (...)dt + \sigma_h(y,t,T)dz_S \tag{5.10}$$

with

$$\sigma_h(y,t,T) \equiv \bar{B}(y,t,T) = \frac{\int_{\max[t,T-n]}^{T} PV_L(y,t,\tau)B(\tau - t)d\tau}{\int_{\max[t,T-n]}^{T} PV_L(y,t,\tau)d\tau}\,\kappa\,\nu_S \tag{5.11}$$

6 Optimal risk-sharing when labor income and dividend growth are co-integrated

Now, let us turn to the optimization problem in autarky. We did not add new sources of risk, so the optimization problem did not significantly change. The stochastic discount factor is as before and each individual generation still maximizes 2.2 subject to 2.3. Hence, the law of motion of optimal total wealth is as before (see 3.2). The crucial difference however, is that the present value of labor income is now stochastic and correlated with long-term stock-returns.

Let dG denote actual total wealth as a function of the chosen portfolio weight in stocks, then we have now that:

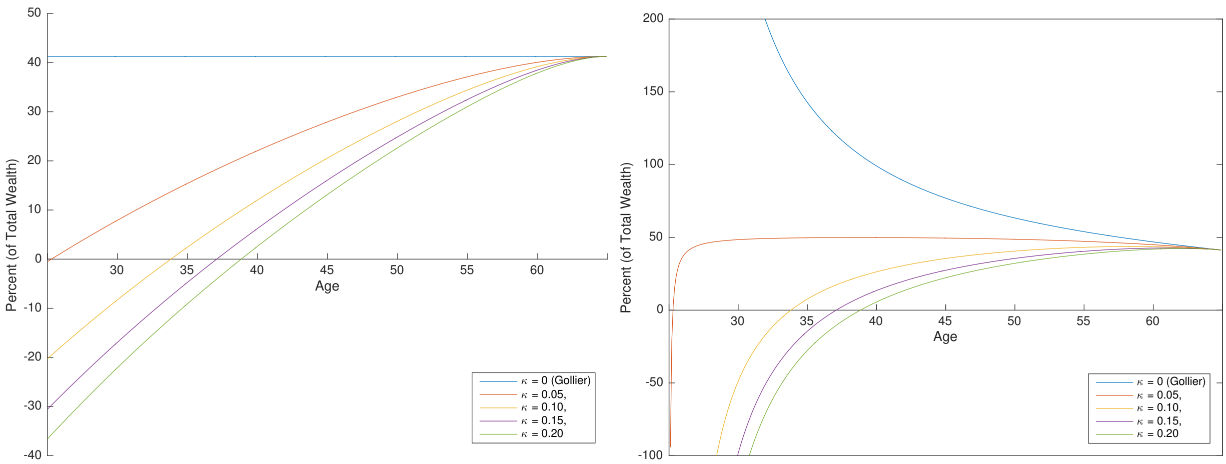
$$\frac{dG}{G} = (...)dt + w_s(t,T)\sigma dz_S + h(t,T)\sigma_h(t,T)dz_S \tag{6.1}$$

where $w_s(t,T)$ is the share of total wealth invested in the stock index, $h(t,T)$ is human capital as a share of total wealth and $\sigma_h(t,T)$ is the exposure of human capital to the equity shock, dz_S. We suppressed the drift term for simplicity. The optimal level of $w_s(t,T)$ can be found by setting the volatility term of the actual wealth process 6.1 equal to the volatility term of the optimal wealth process 3.2. This gives us:

$$w_S^*(t,T) = -\frac{1}{\gamma}\frac{\phi_S}{\sigma} - h(t,T)\,\frac{\sigma_h(t,T)}{\sigma} \tag{6.2}$$

In addition to the standard speculative demand we had before, we now have a hedging demand that compensates for the fact that labor income now also provides exposure to dz_S. Since $\sigma_h(t,T)$ is a weighted average of strictly positive terms, the hedging term in 6.2 is strictly negative. So, as BCG pointed out, a consequence of co-integration between labor income and dividend income, is that the optimal investment in the stock market is lower than in a model without co-integration.

How big the impact of co-integration is, is driven by our choice of κ, the strength of mean-reversion in the dividend-labor-income ratio. BCG argue that empirical estimates of κ are rather imprecise, due to the limited availability of long-horizon data. They use $\kappa = 0.15$ as their baseline parameter choice. Depending on the different sample periods they consider though, they find levels of κ ranging from 0.05 to 0.2. At the end of the day, the value of κ is a subjective belief and we do not intend to make a claim about its "true" value here. Instead, we merely illustrate how different beliefs about κ will change the results. The optimal equity exposure for different values of κ are given in figure 1. Notice that the limiting case where $\kappa = 0$ coincides with the setup we considered in the previous section without labor income risk.



(a) Equity weight (% of total wealth)

(b) Equity weight (% of financial wealth)

Figure 1. Optimal stock market exposure Panel (a) illustrates the optimal exposure to the stock index for different levels of mean-reversion in the dividend-labor income ratio (κ). Panel (b) shows the same exposures, but now as a percentage of financial wealth. Both graphs are based on the median scenario with intitial condition $y(0) = 0$.

Certainty equivalent

For completeness, let us finish by considering the certainty equivalent levels of consumption in autarky. Since the optimal total wealth process is unchanged, the certainty equivalent level of consumption also looks very similar, with one important difference: unlike in the Gollier setup, total wealth of unborn generations is now also stochastic. The certainty equivalent for future generations is therefore slightly different.

$$\mathrm{CE}(0,T) = \begin{cases} W(0,T)\exp\left\{\left(r + \frac{1}{2}\frac{\phi_S^2}{\gamma}\right)T\right\} & T \leq n \\ \mathbb{E}_0\left[H(T-n,T)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}\exp\left\{\left(r + \frac{1}{2}\frac{\phi_S^2}{\gamma}\right)n\right\} & T > n \end{cases} \tag{6.3}$$

where $H(t,T)$ is the value of human capital of generation T at time t. The term in expectations is the present value of human capital on the moment a generation enters the labor market. As a matter of fact it is not straight-forward to derive an analytic expression for this expectation, so we will determine it later numerically when comparing the certainty equivalent in autarky to the setting with intergenerational risk-sharing.

Collective pension fund with risk-sharing

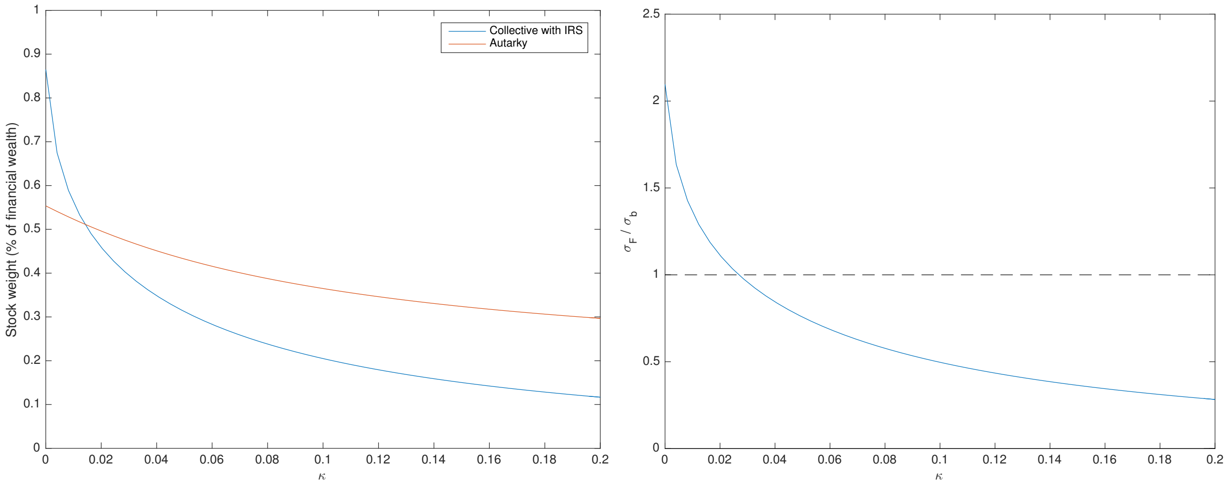
Let us now consider the collective pension fund planners problem. The planner still maximizes 3.7 subject to 3.8. Hence, like the individual generations problem, the optimal exposure of total wealth in the planners problem is unchanged. Also here, the main difference is that human capital already provides exposure to equity risk. The optimal exposure to the stock index for the planners problem therefore is:

$$\tilde{w}_S(t,T) = -\frac{1}{\gamma}\frac{\phi_S}{\sigma} - \tilde{h}(t)\,\frac{\tilde{\sigma}_h}{\sigma} \tag{6.4}$$

This looks very similar to to the autarky solution of an individual generation, except that $\tilde{h}(t)$ is aggregate human capital as a share of aggregate total wealth and $\tilde{\sigma}_h = \tilde{B}(t) = \frac{\int_t^\infty PV_L(\tau-t)B(\tau-t)d\tau}{\int_t^\infty PV_L(t,\tau)d\tau}\kappa\nu_S$ is the exposure of the aggregate human capital to stock market risk.

The optimal exposure to the stock index chosen by the planner is lower than in the setting without co-integration of labor income and dividend income. Since $\tilde{B}(t)$ is increasing in κ, the stronger the mean-reversion in the labor income to dividend ratio, the less the collective fund will invest in stocks. This is illustrated in figure 2. Panel (a) shows the allocation of aggregate financial wealth to stocks in the initial situation, both in autarky and with optimal risk-sharing. The difference between the two lines tells us how much risk the collective fund takes on behalf of future generations. Note that, given our parameter choice, only if κ is approximately below 1.8, the collective fund should take additional risk on behalf of future generations. When κ is higher, the planner finds it optimal to actually take less risk on aggregate and hence optimally allocate some risk from future generations to current generations.

Panel (b) compares the volatility of collective fund wealth and the retirement benefit. In the absence of long-run correlation ($\kappa = 0$) this ratio was bigger than one. Fund wealth was more volatile than the retirement benefit which we referred to as 'smoothing'. In Panel (b) we now see that only for levels of approximately $\kappa < 0.04$ it is optimal for the pension fund to



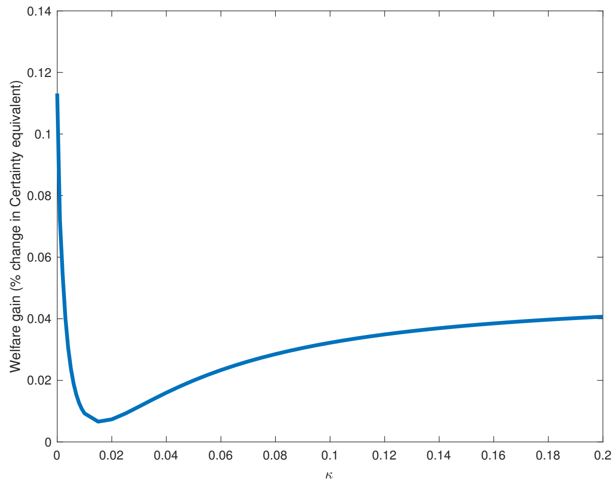
(a) Stock weight (% of aggregate financial wealth) (b) Level of benefit smoothing ($\tilde{\sigma}_F/\tilde{\sigma}_b$)

Figure 2. Optimal aggregate stock market exposure Panel (a) illustrates the optimal exposure to the stock index for different levels of mean-reversion in the dividend-labor income ratio as a percentage of aggregate total wealth both in Autarky and in the setup with a collective pension fund with intergenerational risk-sharing (κ). Panel (b) shows the instantaneous volatility of collective pension fund wealth relative to the volatility of the retirement benefit. Both panels assume wealth is at its initial level, where aggregate human capital is approximately 50% of total wealth and $y(0) = 0$

apply some level of smoothing. If $\kappa > 0.04$ the mechanism reverses. The pension fund takes less risk than is optimal for the retiring generation and hence the fund does not 'smooth' its shocks. Instead, shocks are amplified into the retirement benefit, so to say. At low levels of κ, the pension planner wants to take equity risk on behalf of future generations. So, the fund is rather volatile, but this risk is carried over to future generations. If κ is low instead, the planner optimally takes less risk and actually uses the fund to transfer some of the human capital risk of future generations to current generations by amplifying the funds risk into current retirement benefits.

Welfare effects

Let us again assume that the social planners sets the welfare weights such that the certainty equivalent of all generations increases by the same factor α. So, $\beta(T)$ is still defined as in 3.15. Unlike the setting with risk-free human capital, the certainty equivalent in autarky is now horizon dependent for future generations and can only be obtained numerically. We do so for the range of κ from 0 (risk-free human capital) to 0.2 (the upper bound reported by BCG). Figure 3 shows the welfare gain.



(a) Welfare gain from risk-sharing (% of CE)

Figure 3. Welfare gain from optimal risk-sharing This figure shows the percentage increase in certainty equivalent retirement benefit between autarky and optimal risk-sharing. The welfare weights are chosen such that the percentage gain of all generations is the same. The figure assumes wealth is at its initial level, where aggregate human capital is approximately 50% of total wealth and $y(0) = 0$

We see that the boundary case, where kappa is 0, corresponds to the welfare gain of 11.3 percent we saw before. For values of kappa in the range reported by BCG (0.4 - 0.2), the welfare gain ranges from 1 to 4 percent. In this case the welfare gain is not only much smaller, as we saw before, it also comes from a different source. This welfare gain is no longer a benefit from extra overall risk-taking. It is a benefit from the fact that current generations accept some risk from future generations.

The welfare gain is minimized at approximately $\kappa = 0.015$. This roughly coincides with the point where the optimal portfolio allocation to the stock market is the same in autarky and under the optimal risk-sharing scheme. In this case, the optimal investment in the stock market on behalf of future generations is exactly 0. So, the social planner does not allocate any risk from current to future generations or vice-versa. The planner still achieves some welfare gain though by re-allocating some risk among future generations.

7 Conclusion

We show that the potential presence of co-integration between labor income risk and stock market risk has a significant impact on collective portfolio choice and optimal risk-sharing. Our findings imply that the commonly held idea that collective pension plans with mandatory participation can and should take more risk, should be considered a boundary case. For the empirical parameter range reported by Benzoni et al. (2007), our model actually suggests that a collective fund should optimally take less stock market risk. This observation is in line with the findings of Bohn (2009), who also concludes that an efficient policy should probably shift risk from workers to retirees, instead of the other way around. We furthermore find that the welfare gains from risk-sharing turn out to be much lower than in the boundary case where labor income risk is completely uncorrelated with stock market risk.

Our results highlight a challenge for policy makers running collective pension plans. Policy makers will have to decide what the 'right' model is and what the 'right' parameter values within that model are. As we showed, these beliefs will have a significant impact on the optimal portfolio allocation, the optimal distribution of collective risk and wealth and the potential welfare gains from risk-sharing. Not only does our analysis suggest that the potential welfare gains from collective risk-sharing are smaller. It also highlights that it is not easy for policy makers to reap these potential benefits. Picking the wrong model has significant implications for the optimal policy. Especially if we bear in mind that our analysis merely focused on one modeling dimension (albeit an important one). Other factors we could consider in future work include: interest rate risk, time-varying risk-premia, heterogeneity in risk aversion, etc.

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Appendix A: Derivation of the human capital process

Integrating the log-labor income process 5.6 gives that labor income at time t as a function of information at time s is :

$$L(t) = L(s)\exp\left\{-\kappa\int_s^t y(u)du + (g_d + \bar{\ell}d - \frac{1}{2}\sigma^2))(t-s) + (\sigma - \nu_S)\int_s^t dz_S(u)\right\} \quad \text{(A1)}$$

First, let us also express $y(t)$ as a function of time s information. Since $y(t)$ follows an Ornstein-Uhlenbeck process, this solution is well known (remember that we have set $\nu_L = 0$):

$$y(u) = y(s)e^{-\kappa(u-s)} - \nu_S\int_s^u e^{-\kappa(u-v)}dz_S(v) \quad \text{(A2)}$$

Substituting this into A1 gives:

$$\begin{aligned} L(t) = L(s)\exp\Bigg\{ & y(s)(e^{-\kappa(t-s)} - 1) + (g_d + \bar{\ell}d - \frac{1}{2}\sigma^2))(t-s) \\ & +\kappa\nu_S\int_s^t\int_s^u e^{-\kappa(u-v)}dz_S(v)du\Bigg\} \end{aligned} \quad \text{(A3)}$$

Which can be simplified into:

$$\begin{aligned} L(t) = L(s)\exp\Bigg\{ & -\kappa B(t-s)y(s) + (g_d + \bar{\ell}d - \frac{1}{2}\sigma^2)(t-s) \\ & +\kappa\nu_S\int_s^t B(t-v)dz_S(v)\Bigg\} \end{aligned} \quad \text{(A4)}$$

where $B(x) = \frac{1}{\kappa}(1 - e^{-\kappa(x)})$

Present value of human capital

The present value of labor income is simply the sum of present values of all future labor income cash-flows. Remember that T denotes the retirement date and n denotes the time the individual is in the labor force:

$$H(t,T) = \int_{\max[t,T-n]}^T PV_L(y,t,\tau)d\tau$$

where

$$PV_L(y,t,\tau) = \mathbb{E}_t \left[\frac{M(\tau)}{M(t)} L(\tau) \right]$$

Substituting the expression for $L(\tau)$ and $\frac{M(\tau)}{M(t)}$ given time t in formation in gives:

$$\begin{aligned} PV_L(y,t,\tau) = L(t)\mathbb{E}_t \Big[\exp \Big\{ & -\kappa B(\tau - t) y(u) + \bar{h} \, (\tau - t) \\ & \int_t^\tau \left(\phi_S + \nu_S \kappa B(\tau - v) \right) dz_S(v) \Big\} \Big] \end{aligned}$$

where $\bar{h} = (g_d + \bar{\ell} d - \frac{1}{2}\sigma^2) - \left(r + \frac{1}{2}\phi_S^2 \right)$.

Note that the expectation and variance of the term inside the exponential are given by

$$\begin{aligned} \mathbb{E}_t \left[... \right] &= -\kappa B(\tau - t) y(u) + \bar{h} \, (\tau - t) \\ Var_t \left[... \right] &= \mathbb{E}_s \left[\left(\int_t^\tau \left(\phi_S + \nu_S \kappa B(\tau - v) \right) dz_S(v) \right)^2 \right] \\ &= (\phi_S + \nu_S)^2 (\tau - t) - 2(\phi_S + \nu_S)\nu_S B(\tau - t) + \nu_S^2 \frac{B(2(\tau - t))}{2} \end{aligned}$$

So, we find that:

$$PV_L(y,t,\tau) = L(t) \exp \left\{ A(\tau - t) - \kappa B(\tau - t) y(t) \right\} \tag{A5}$$

where

$$A(\tau - t) = \left(g_d + \bar{\ell} d + \nu_S \phi_S - r \right) (\tau - t) - (\phi_S + \nu_S)\nu_S B(\tau - t) + \nu_S^2 \frac{B(2(\tau - t))}{4} \tag{A6}$$

which brings us to 5.8 in the main text.

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