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Liquidity regulation, the central bank and the money market*

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Abstract

Money markets play a central role in monetary policy implementation. Money market functioning has changed since the financial crisis. This arguably reflects the interaction of two forces: Changes in monetary policy, and changes in regulation. This interaction is not yet well understood. We focus on the newly introduced Liquidity Coverage Ratio (LCR) and how it influences the behaviour of banks and the equilibrium on the money market. We develop a theoretical model to analyse how liquidity regulation may interfere with the central bank's implementation of monetary policy. We find that when the market equilibrium is suboptimal due to asymmetric information, both the central bank and the regulator can act to improve welfare. These actions can be complementary or conflicting, depending on the environment. The main insight from the central bank perspective is that the regulator can reach the welfare optimum, but at the expense of the central bank moving away from its optimum. The central bank will thus need to adjust its implementation of monetary policy accordingly, to address the effects of liquidity regulation.

Keywords: regulation; Basel III; central bank; interbank lending; money market; asymmetric information.
JEL classifications: E43; E58; G01; H12; L51.

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1 Introduction

Money markets play a central role in monetary policy implementation. Most modern central banks implement monetary policy by steering short-term interest rates, the rates on money markets. From these short-term interest rates, monetary policy impulses are then transmitted onwards via the monetary policy transmission mechanism, to ultimately influence price stability.

Money market functioning has changed since the financial crisis that started in 2007: Most importantly, activity has decreased in the unsecured segment of the money market. The secured segment has held up better. This arguably reflects the interaction of two forces: Changes in monetary policy, and changes in regulation. Most notably, huge asset purchase programmes have led to significant excess liquidity in the euro area, which is suppressing money market activity. At the same time, Basel III liquidity regulation has been developed as a reaction to the crisis, which also has an impact on the functioning of the money market.

In particular, the Liquidity Coverage Ratio (LCR) forces banks to hold a short-term liquidity buffer in the form of High-Quality Liquid Assets (HQLA). This makes such assets more attractive. It also alters the relative value of short-term versus longer-term funding and of secured versus unsecured interbank loans, because the LCR treats secured and unsecured short-term funding differently. Roughly, unsecured short-term funding below 30 days is treated as outflowing liquidity that needs to be covered by HQLA, while this is not the case for secured short-term funding below 30 days where collateral is received back in return. Therefore, unsecured short-term funding below 30 days becomes less attractive for borrowers, while it becomes more attractive for lenders. The lending rate is expected to decrease at the very short end, with the unsecured yield curve steepening accordingly. Volumes would probably decrease as well, as demand would shift to longer tenors and/or the secured segment.

The interaction between the effects of monetary policy and regulation is not yet well understood, but crucial for the central bank: What will be the effect of regulation on money market activity once the monetary policy environment has normalized? How should the central bank implement monetary policy in that context? This is very difficult to assess empirically in the current environment.

We provide insights by developing a theoretical model. As a starting step, we build a very stylized model. The model is designed to be as simple as possible while still capturing the key features that we want to understand, namely the behaviour of the central bank, the regulator, and banks on the money market.

In order to study the interaction of the regulatory and central bank actions on money markets, we develop a model of the interbank market with asymmetric information. Our approach to modelling asymmetric information is similar to that taken in Stiglitz and Weiss (1981). That the choice of risk in a portfolio is closely related to the degree of asymmetric information in a market has been shown by e.g. Flannery (1986) and Diamond (1991). Our model relates to this strand of literature in the sense that asymmetric information is the cause of a sub-optimal outcome. Our approach has similarities with the model of Heider et al. (2015), who also introduce private information about bank risk as the friction that can lead to a failure of interbank markets to distribute liquidity efficiently. Contrary to their work, we focus our attention on the central bank, the regulator and their interaction and a corresponding welfare analysis. On the empirical side, Ashcraft and Bleakley (2006) provide evidence for the role of asymmetric information impacting interbank market behaviour.

Furthermore, we explicitly separate the role of the secured and the unsecured interbank market. This is key for the understanding of liquidity regulation and monetary policy implementation as liquidity regulation alters the relative "cost" of acting on these two markets while the implementation of monetary policy by the central bank classically focuses on the unsecured market. The distinction between the two markets is modelled similarly to Heider and Hoerova (2009), where unsecured interbank lending is risky as banks may become insolvent due to the risk of their illiquid investments, and therefore unable to repay their interbank loan. To compensate lenders, borrowers have to pay a premium for funds obtained in the unsecured interbank market. Our treatment of the repo market also has similarities with the introduction of a repo market in Freixas and Holthausen (2004), who find that a repo market reduces interest rate spreads and improves upon the segmentation equilibrium, but may destroy the unsecured integrated equilibrium.

Finally, we introduce a central bank and a regulator in our model. The central bank acts as a mediator on the interbank market, via a corridor system implemented through a deposit facility and a lending facility.[1] The eligible collateral accepted by the central bank at the lending facility is a wider set of collateral than what is accepted in the interbank market. This treatment of the central bank is motivated by the role of the European Central Bank (ECB) and the broad set of assets accepted as collateral by the ECB, which was widened even more during the recent crisis.[2] We show that the intermediation of a central bank in the interbank market can improve social welfare compared to the market outcome.

The regulator can implement liquidity regulation in different ways, as motivated by Perotti and Suarez (2011), e.g. by taxing risky behaviour or by subsidizing investment in liquid assets.[3] We show that liquidity regulation can also improve welfare. However, the actions of the central bank and the regulator can be complementary in some situations and conflicting in others. For example, when the central bank is at its utility optimum regarding the implementation of monetary policy, the introduction of liquidity regulation can decrease the central

[1]The model holds both in the situation of balanced liquidity conditions and in the situation of excess liquidity conditions. In the case of balanced liquidity conditions, the central bank steers market rates via the middle of the corridor. It steers market rates with the deposit facility rate in case of excess liquidity conditions.

[2]In a similar vein, Hoerova and Monnet (2016) provide a theory for the joint existence of lending on decentralized money markets and lending by a central bank.

[3]The latter interpretation is motivated by the introduction of the concept of High Quality Liquid Assets (HQLA) in the LCR. The fact that certain assets count towards the LCR increases their value for the banks that hold those assets, compared to the assets that do not count towards the LCR.

bank's utility by changing conditions on money markets. Conversely, when the central bank decides to change its behaviour in the implementation of monetary policy, e.g. to widen its corridor, the situation may become suboptimal for the regulator. This calls for close cooperation between the central bank and the regulator.

The results of our stylized model are in line with intuition: The regulator will have an impact on the equilibrium in the money market, also changing the way that central bank actions affect this market. However, it should be possible for the central bank to adapt its operational framework to the new equilibrium that exists once liquidity regulation is in place. In future research, we plan to enrich the model by introducing a detailed balance sheet analysis of borrowers, including bank capital. We will also include an endogenous description of collateral in this context. Furthermore, we want to study the effect of haircut changes, e.g. in a crisis, on the market equilibrium and on the potential response of the central bank and the regulator. Finally, while we are basing our model on the assumption that the central bank's operational target is the unsecured rate, the model could be extended to study the question of the optimal operational target.

Our paper is structured as follows. Section 2 discusses the literature that is relevant in this context. Section 3 introduces the basic model set-up, characterised by asymmetric information. Section 4 gives the normative analysis for the social planner, compares the normative and the positive outcome and establishes the case for an intervention in the interbank market. We discuss the role of the central bank in section 5 and the role of the regulator in section 6. Section 7 discusses the interaction of the central bank and the regulator. Section 8 concludes.

2 Related literature

The interbank market and regulatory reaction to its frictions have received some attention since the recent financial crisis, while of course many of the theoretical approaches used in this analysis have their roots before the crisis. These are often based on the classical banking model developed by Diamond and Dybvig (1983). Allen and Gale (2017) present an overview of the literature on possible market failures that can make liquidity regulation necessary in the context of a model of financial institutions and markets based on Allen and Gale (2004*a*) and Bhattacharya et al. (1985). Allen and Gale (2004*b*) study the regulation of the financial system using a welfare analysis in the context an integrated theoretical model of banks and markets and find that there may be a role for regulating liquidity provision in an economy in which markets for aggregate risks are incomplete. An integrate model of demand deposits and anonymous markets with market frictions is also studied by Von Thadden (1999). Freixas and Holthausen (2004) study cross-country interbank market integration under asymmetric information. Heider and Hoerova (2009) also study the functioning of secured and unsecured interbank markets in the presence of credit risk and show that interest rates decouple across secured and unsecured markets following an adverse shock to credit risk. Heider et al. (2015) study a model of interbank lending and borrowing with counterparty risk and identify a market breakdown that arises from adverse selection in the interbank market.

The response of the central bank to frictions on the interbank market has also been studied, in particular since the start of the financial crisis. Allen et al. (2009) study how central banks should react to malfunctions on the interbank market and show that a central bank can implement the constrained efficient allocation by using open market operations to fix the short-term interest rate. In that context, market freezes can be a feature of the constrained efficient allocation. The role of the central bank in this context is explored further in Allen et al. (2014), who find that the combination of nominal contracts and a central bank policy of accommodating commercial banks' demand for money leads to first best efficiency in a wide range of circumstances. Freixas et al. (2011) examine the efficiency of the interbank lending market in allocating funds and the optimal policy of a central bank in response to liquidity shocks. Freixas and Jorge (2008) analyse the impact of asymmetric information in the interbank market and its relationship with the monetary policy transmission mechanism. Martin et al. (2014) develop a model of financial institutions with distinct liquidity and collateral constraints to study the behaviour of repo markets during the recent financial crisis.

There has been little (theoretical) literature focusing on liquidity regulation, particularly before the crisis. If mentioned in this context, the central bank appears mostly in its function as a lender of last resort. Following Holmstrom and Tirole (1997) and Holmström and Tirole (1998), Rochet (2004) and Rochet et al. (2008) study possible institutional (regulatory) arrangements that solve market failures in the provision of liquidity. More explicitly, Rochet (2004) discusses prudential regulation and the lender of last resort function of the central bank in the presence of moral hazard and suggests a differential regulatory treatment of banks according to their exposure to macroeconomic shocks. Rochet et al. (2008) argues that a simple liquidity ratio seems appropriate to attain a micro-prudential objective, i.e. to limit the externality associated with individual bank failures, while the macro-prudential objective of liquidity regulation seems harder to attain. In an earlier contribution, Rochet and Tirole (1996) provide a stylized theoretical framework to analyse systemic risk and study how one might protect central banks while preserving the flexibility of the interbank market. Other studies of the role of the central bank as lender of last resort include Repullo (2005), who finds that the existence of a lender of last resort does not increase the incentives to take risk, while penalty rates do, and Cao and Illing (2009), who find that imposing minimum liquidity standards for banks ex ante is a crucial requirement for a sensible lender of last resort policy. In a recent contribution, Diamond and Kashyap (2016) find that regulation similar to the liquidity coverage ratio and the net stable funding ratio can make bank runs less likely. On the empirical side, Banerjee and Mio (2017) study the impact of liquidity regulation on banks and find that, in response to tougher liquidity regulation, banks replaced claims on other financial institutions with cash, central bank reserves and government bonds.

Another strand of literature studies liquidity regulation from the perspective of aggregate welfare. Perotti and Suarez (2011) discuss liquidity regulation when short-term funding enables credit growth but generates negative systemic risk externalities, focusing on the relative merit of price versus quantity rules. They present a baseline model where a price regulation (via linear taxes) is optimal and another version of the model where a quantity regulation is optimal. Furthermore, both Tirole (2012) and Philippon and Skreta (2012) study optimal intervention in markets with adverse selection.

The failure of the interbank market during the recent financial crisis has been analysed both empirically and theoretically in a number of studies. Taylor and Williams (2009) find that increased counterparty risk contributed to these failures. Eisenschmidt and Tapking (2009) relate it to the funding liquidity risk of lenders in unsecured term money markets. Brunnermeier (2009) offers a comprehensive analysis of the liquidity and credit crunch 2007-2008, exploring four economic mechanisms through which the mortgage crisis amplified into a severe financial crisis, namely borrowers' balance sheet effects, the drying-up of the lending channel, runs on financial institutions, and network effects. Brunnermeier and Oehmke (2013) show that extreme reliance on short-term financing may be the outcome a maturity rat race. Brunnermeier and Pedersen (2008) explain the sudden dry-up of markets with a model that links an asset's market liquidity and traders' funding liquidity. Huang and Ratnovski (2011) show that inefficient liquidations can be the result of asymmetric information.

Our paper adds to the existing literature by analysing the interaction of the central bank and the liquidity regulator from a theoretical perspective. While some of the above-mentioned work looks at one or the other, the behaviour of both of these policy-makers is rarely studied together. A notable exception is Bech and Keister (2017), who find that the introduction of the liquidity coverage ratio may impact the efficacy of the central bank's current operational framework. Taking a more practical perspective, Committee et al. (2015) explicitly studies regulatory change and monetary policy. Furthermore, Carlson et al. (2015) look at liquidity regulation and the central bank, focussing on in its lender-of-last-resort function.

3 The model

The basic set-up consists of banks that want to finance an investment on the money market. A bank can invest either into a safe, liquid asset that is classified as HQLA (e.g. a government bond) or into a risky, illiquid asset (e.g. a loan). When a fixed amount I is invested, the safe, liquid investment returns A with certainty, while the risky, illiquid investment returns θ with probability p_i and 0 with probability $1 - p_i$, where $I, \theta, A > 0$.[4] Note that we do not impose any

[4] Of course, it can be considered an extreme assumption that the value of the loan can only take these two extreme values. A more complex payout structure could also be modelled, but would make the exposition much more complex without adding insight, so we decided to use

restrictions on the amount I such that it can also be interpreted as a refinancing requirement. The borrower always needs to invest the full amount I, i.e. he cannot partition his resources to invest in both types of investment.[5]

The bank finances this investment on the money market, which has both an unsecured and a secured segment. On the secured segment, a loan is collateralised by a fixed collateral amount that covers the outstanding debt (plus interest) and that can be seized by the lender if the borrower defaults on the loan. A haircut could also be applied to the value of the collateral. We do not assume additional possibilities for litigation. However, modelling unsecured versus collateralised borrowing can also be interpreted as different forms of limited liability. We assume that all agents (borrowers and lenders on the money market) are risk-neutral, implying that they maximise their expected profit.

We introduce asymmetric information by assuming the probability $p_i \in [0, 1]$ to be borrower-specific. The probability p_i can thus be interpreted as the borrower's type. The borrowers' type is distributed along the interval $[0, 1]$ according to the probability distribution function f. A borrower i will know about his type p_i, but the lenders cannot observe p_i. Both borrowers and lenders know the distribution f of types in the population.

Borrowers finance their investment on the money market. We allow for the option to combine borrowing on the secured and on the unsecured market, i.e. the borrower can borrow a share ρ of the funding on the secured market and a share $1 - \rho$ on the unsecured market. Borrowers have thus two options to access funding: they can use collateral for collateralised borrowing on the secured market, but they can also access the unsecured market directly without using collateral. We assume that borrowers default if their investments are unsuccessful.[6]

We assume that the investment itself can be used as collateral, but with a haircut $1 - \lambda$.[7] Thus, if $\lambda < 1$, collateral is scarce, and collateral constraints are the same for all borrowers. Borrowers can only borrow the share $\lambda < 1$ of the total loan I on the secured market and have to borrow the rest on the unsecured market. In this context, we can study the effect of changes in the haircut (and thus changes in the parameter λ) on the market equilibrium.

the simplest stochastic payout structure possible.

[5]This assumption can easily be relaxed, but keeps the model more parsimonious.

[6]Again, this assumption is extreme and could be softened. However, we make this assumption in order to keep the model as simple as possible while reflecting the credit risk that is inherent in unsecured market transactions.

[7]In order to remain in our framework where the lender cannot find out the type of the borrower, the lender would need to be able to take this as collateral without knowing whether the investment was done in A or θ. For example, one can assume a third party (central counterparty) arranging the repo contract, without disclosing the exact choice of collateral. Furthermore, one needs to assume that taking the investment (even in the risky, illiquid asset) as collateral does not create a risk for the lender, i.e. the investment being "unsuccessful" would then need to be interpreted in a way that makes the borrower default but still creates enough recovery value for the lender to recover his loan. For example, one can consider that this recovery value takes time to materialise and the lender has more patience than the borrower. If these assumptions seem to restrictive, one can simply assume that both collateral and the parameter λ are exogenously given.

Let R^s be the interest rate on the secured market and R^u be the interest rate on the unsecured market. The interest rates are determined in the interplay between borrowers and lenders. Given the fact that borrowers and lenders have the choice between the secured and the unsecured market, and that the lender will need to be compensated for the additional risk borne when lending is unsecured, $R^u \geq R^s$.

Corollary 1 *If the equilibrium market interest rate on the secured market is R^s, then $A \geq R^s I$ or $\theta \geq R^s I$ is a necessary condition for market activity.*

If we assume that there is a safe store of assets (that does not bear interest), there is always the risk-free alternative of not conducting any investment or lending activity. In this case, it is clear that $R^s \geq 1$, since the lender always has the alternative to keep his funds.[8]

Corollary 2 *If there is a safe store of assets, then the equilibrium market interest rate on the secured market is $R^s \geq 1$.*

To keep the model simple, we assume that the lender can always claim the collateral in case the investment does not pay off and that the lender does not bear any risk when lending secured, because of the haircut applied to the collateral.

In order to simplify the model, we could assume for simplicity that $R^s = 1$. This can, for example, be rationalised by assuming perfect competition between lenders on the secured market and the existence of a safe store of assets. In order to keep both markets comparable in a risk-neutral setting, we then assume perfect competition also on the unsecured market, i.e. that expected profits of lenders are 0 on both markets. [9]

For a borrower to have an incentive to invest in the liquid safe project, $A \geq R^s I$ must hold, and in order to have an incentive to invest in the risky, illiquid project, $\theta \geq R^s I$ must hold. In order to have an incentive to invest in the illiquid risky project instead of the liquid safe project, $\theta > A$ must hold.

Corollary 3 *A necessary condition for investment to take place in the illiquid, risky project instead of the liquid safe project is $\theta > A$.*

[8]Including collateral liquidation costs in the model would lead to a secured rate which is slightly higher than 1, because the lender would have to take these collateral liquidation costs into account when setting the appropriate secured interest rate. As this does not change the basic structure of the model, we ignore these costs.

[9]This means that all profit from the investments arise with the borrowers, none with the lenders. However, it is noted that we could also make different assumptions about how the profit from investing is split between borrowers and lenders - considerations on the respective market power of the two parties could determine where these interest rates lie precisely, in the spirit of Stiglitz and Weiss (1981). This would then give a range of possible equilibria and corresponding constellations of interest rates. We do not follow that route at present, but the model can easily be generalised in this way. For example, one can assume that lenders make a certain positive profit from lending on both markets and that this profit is equal on both markets. Alternatively, one can also argue that the profit from lending on the unsecured market should be higher than that from lending on the secured market because of the additional risk borne, thereby dropping the assumption of risk-neutrality.

In the following, we assume that $\theta > A$ and that $A \geq R^s I$, as otherwise the situation becomes trivial.

3.1 Strategies for borrowers

The borrower aims to maximise his expected profit. He can choose whether to borrow secured or unsecured and whether to invest in the liquid safe or in the illiquid risky asset. The expected payoff under the four possible "corner solutions" (where $s\lambda$ stands for borrowing as much as possible on the secured market and u stands for borrowing all on the unsecured market) is given by the equations below:

$$
\begin{aligned}
\Pi_B^{s\lambda}(\text{liquid-safe}) &= A - (R^s\lambda + R^u(1-\lambda))I \\
\Pi_B^{u}(\text{liquid-safe}) &= A - R^u I \\
\Pi_B^{s\lambda}(\text{illiquid-risky}) &= (\theta - R^u(1-\lambda)I)p_i - R^s\lambda I \\
\Pi_B^{u}(\text{illiquid-risky}) &= (\theta - R^u I)p_i + (0)(1-p_i) = (\theta - R^u I)p_i.
\end{aligned}
$$

Given that all relevant equations are linear, the optimisation behaviour of borrowers will lead them to choose a corner solution, namely the one that maximises their expected payoff.[10]

The optimal strategy for the borrower depends on the individual value of p_i. For very low p_i, borrowers borrow on the secured market (as much as possible) and invest in the safe asset. For very high p_i, borrowers borrow on the secured market (as much as possible) and invest in the risky, illiquid asset. In between, there can be a region where it is optimal for the borrowers to borrow on the unsecured market and invest in the risky, illiquid asset.

The key elements for the subsequent analysis are the three intersection points between the lines given by the payoff functions.

A borrower who borrows (as much as possible) on the secured market is indifferent between the two investment strategies when $p_i = p^T$, where

$$p^T := \frac{A - R^u(1-\lambda)I}{\theta - R^u(1-\lambda)I}.$$

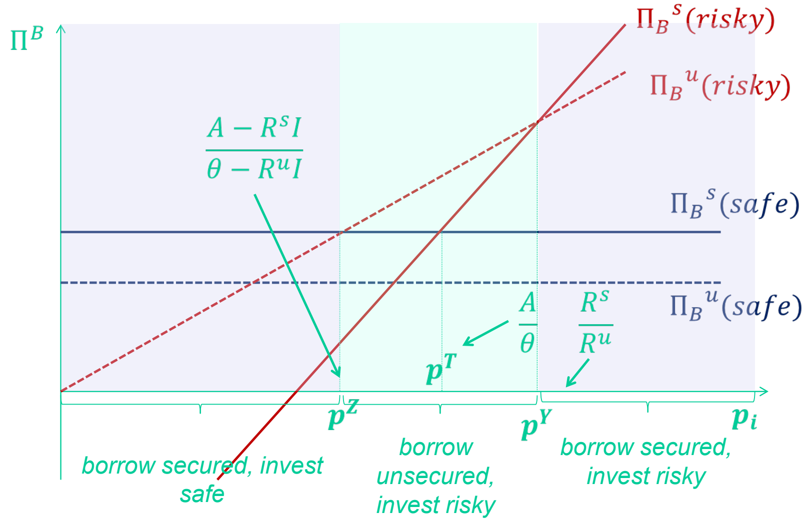
Lemma 1 *A borrower that borrows a share λ on the secured market and the rest on the unsecured market will choose the safe asset whenever $p_i \leq p^T$ and the risky, illiquid asset otherwise.*

A borrower who invests in the risky, illiquid asset is indifferent between the two borrowing strategies if $p_i = p^Y$, where

$$p^Y := \frac{R^s}{R^u}.$$

[10] We assume that parameter values are such that borrowers have an incentive to undertake one of the two investments. Otherwise, no market transactions will take place.

Figure 1: Borrower payoff structure



Finally, it will never be preferable for a borrower who invests in the safe asset to borrow only on the unsecured market, as $R^u \geq R^s$.

A borrower is indifferent between (i) borrowing (as much as possible) on the secured market and investing safe and (ii) borrowing only on the unsecured market and investing risky, illiquid if $p_i = p^Z$, where

$$p^Z := \frac{A - I(R^s\lambda + R^u(1-\lambda))}{\theta - IR^u}.$$

This is illustrated by Figure 1 (where we set $\lambda = 1$ for simplicity of notation in the equations that illustrate the figure). We note that a similar picture is also derived in Stiglitz and Weiss (1981).

Depending on the parameter constellation, two cases are possible. CASE 1 (boring case) arises if it is never advantageous to borrow fully on the unsecured market. (On the graph above, this corresponds to the red dotted line crossing the two solid lines below their intersection point.) CASE 2 (interesting case) arises if there is a range of types p_i for whom it is advantageous to borrow fully on the unsecured market. (In Figure 1, this corresponds to the red dotted line crossing the two solid lines above their intersection point, as shown.)

Proposition 1 *In the borrower's optimization problem, we can distinguish two cases:*

CASE 1 (boring case): CASE 1 arises if and only if $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} \geq \frac{R^s}{R^u}$, *i.e.* $p^Y \leq p^T \leq p^Z$. *In CASE 1, the borrower will always borrow on the secured market as much as possible. He will invest in the safe asset whenever* $p_i \leq p^T$ *and in the risky, illiquid asset whenever* $p_i > p^T$.

CASE 2 (interesting case): CASE 2 arises if and only if $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} < \frac{R^s}{R^u}$, *i.e.* $p^Z < p^T < p^Y$. *In CASE 2, there is an area of borrowers that do not use the secured market at all: Namely, if* $p_i \in [p^Z, p^Y]$, *the borrower will fully borrow on the unsecured market and invest in the risky, illiquid asset. When* $p_i < p^Z$ *or* $p_i > p^Y$, *the borrower will borrow as much as possible on the secured market. The borrower then invests in the safe asset if* $p_i \in [0, p^Z]$ *and in the risky, illiquid asset if* $p_i \in [p^Y, 1]$.

3.2 Strategies for lenders

In our model, lenders are modelled as simply as possible: They can lend funds to borrowers either on the secured or on the unsecured market. They will only lend to borrowers if the expected profit is non-negative.

For simplicity, as mentioned above, we can assume that the lending market is fully competitive, i.e. that profits for lenders are zero. This then means that $R^s = 1$ (assuming that we have a safe store of assets), because $\Pi_L^s = R^s I - I = 0$.[11] Furthermore, it means that $\Pi_L^u = R^u I \text{Prob(loan is paid back)} - I = 0$.

When lending on the unsecured market, the situation is complex, as it depends on the probability that the loan is paid back. The lenders do not know the individual borrower's success probability p_i, only the distribution f of these success probabilities. The expected payoff from each individual loan depends on p_i and on whether the borrower will invest in safe, liquid or risky, illiquid assets, which the lender does not know. Thus, the lender will need to form an expectation of the aggregate behaviour of borrowers on the unsecured market. As the borrower's behaviour is not only influenced by his type but also by the interest rate constellation which results from the interplay between borrowers and lenders, we obtain a recursive equation that cannot be solved analytically. Rather, numerical simulation can be used to yield the equilibrium value of the interest rate R^u for a given parameter constellation.

In the analysis, we need to distinguish the two cases outlined above.

CASE 1 (boring case): In CASE 1, all borrowers finance themselves as much as possible on the secured market. The expected profit of the lender from lending I on the unsecured market is then the following (see Annex 1 for details):

$$\Pi_L^u = R^u I \left(1 - \int_{p^T}^{1} (1-p) f dp\right) - I$$

Lending on the unsecured market takes place if for the given parameter constellation an interest rate R^u on the unsecured market can be found so that $\Pi_L^u = 0$.[12]

CASE 2 (interesting case): This is the (more interesting) case where there is a range of borrowers that have an incentive to finance themselves fully on the

[11] We do not drop R^s from the notation as this will make it easier to generalise the approach, as outlined earlier.

[12] As discussed above, this can easily be generalised to positive profit of the lender.

unsecured market. The key equation for the expected profit for the lender from lending I on the unsecured market is derived by finding the equation for the probability of repayment of the unsecured loan, which is given by the fraction of different integrals below (easily seen by inspection of borrowing and repayment behaviour on the three intervals discussed above, see Annex 1 for details):

$$\Pi_L^u = R^u I \frac{\int_0^{p^Z}(1-\lambda)f dp + \int_{p^Z}^{p^Y} 1 p f dp + \int_{p^Y}^1 (1-\lambda) p f dp}{\int_0^{p^Z}(1-\lambda)f dp + \int_{p^Z}^{p^Y} 1 f dp + \int_{p^Y}^1 (1-\lambda) f dp} - I$$

We now study the special case of $\lambda = 1$, i.e. the case without collateral constraints.

CASE 1: In case 1, all borrowers finance themselves as much as possible on the secured market. As, with $\lambda = 1$, 100% financing on the secured market is possible, there is no unsecured market activity.

CASE 2:

In case 2, with $\lambda = 1$ and noting that $p^Z < p^Y = \frac{R^s}{R^u}$, the equation becomes

$$\begin{aligned}\Pi_L^u &= R^u I \frac{\int_{p^Z}^{p^Y} 1 p f dp}{\int_{p^Z}^{p^Y} 1 f dp} - I \\ &< R^u I p^Y - I = R^s I - I = 0\end{aligned}$$

Given that lending activity will not take place if profits are negative, no unsecured market activity can take place.[13]

We summarise our findings in the following proposition.

Proposition 2 *Without collateral constraints, market activity on the unsecured market does not take place. With collateral constraints, we have a* pooling equilibrium *in one case (CASE 1, where* $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} > \frac{R^s}{R^u}$*, i.e.* $p^T > p^Y$*) and a* partial pooling equilibrium *in the other case (CASE 2).*

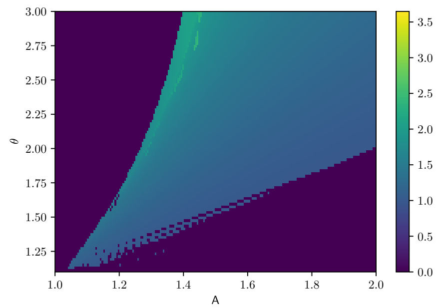
In CASE 1, all borrowers borrow on the secured market as far as possible and are not distinguishable. In CASE 2, borrowers adjust their market behaviour according to type (borrowing either on the secured market as far as possible, or fully on the unsecured market), but not sufficiently for lenders to clearly distinguish the borrower's type.

3.3 Simulation: Market equilibrium determination

With the recursive definition of the market equilibrium derived above, the crucial question is whether an equilibrium exists at all. This question cannot be

[13] In the more general case, where lenders' profits can be positive, still no activity would take place because profits on the unsecured market would lie strictly below those on the secured market.

Figure 2: Simulation parameters for which an equilibrium exists (coloured area; graph shows CASE 2 equilibrium values for R^u given θ and A)



answered easily analytically. In order to solve the equations, simulations are necessary. For the sake of simplicity, we therefore make a few assumptions in the following: We assume that the probability distribution f is the uniform distribution on the interval $[0, 1]$. Furthermore, we assume that $R^s = 1$, $I = 1$, and that lenders' profits are zero.

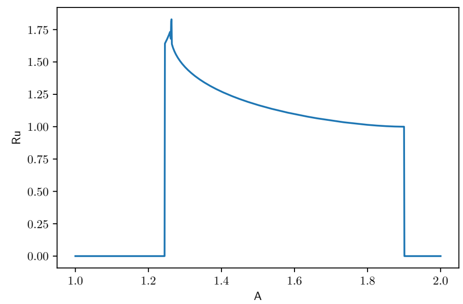
CASE 1 (boring case): In CASE 1, all borrowers finance themselves as much as possible on the secured market. Lending on the unsecured market takes place if for the given parameter constellation an interest rate R^u on the unsecured market can be found so that $\Pi^u_L = 0$.

Note that CASE 1 never arises when the probability distribution is continuous around the intersection points of the lines given by the borrower profit. Namely, with $R^u > 1$, the lenders makes a profit from borrowers investing in the safe, liquid asset. In order to make an expected profit of zero, there also have to be borrowers where the lenders make a loss, the borrowers who invest in θ but who do not have very high p_i. These borrowers have an incentive to borrow fully on the unsecured market - for those whose p_i is not very high, the higher loan payments are more than compensated by the fact that they can pass on all losses in case their investment is unsuccessful. Thus, CASE 1 does not arise when f is the uniform distribution on $[0, 1]$.

Simulations show that the interesting CASE 2 indeed exists, i.e. that values R^u can be found that satisfy the recursive equations. For reasonable values of A and θ, continuous f and $\lambda < 1$ (e.g. f uniform distribution on $[0, 1]$ and $\lambda = 0.7$), we find solutions of these recursive equations. The interest rate R^u depends on the parameters θ and A, as can be seen in Figure 2, showing R^u for the range of permissible combinations of θ and A that yield a CASE 2 equilibrium. (Figure 2 thus also shows that under our simulation assumptions all reasonable parameter combinations yield CASE 2, in line with the argumentation above.)

For illustration, Figure 3 shows that, given a fixed value of $\theta = 1.9$, the interest rate R^u declines with rising A. This is in line with intuition: The more investors in the safe asset exist, which are borrowing on the unsecured money

Figure 3: Equilibrium interest rate R^u declines with increasing A



market and thus cross-finance the losses lenders may make on loans to risky borrowers on the unsecured money market, the lower the equilibrium rate on the unsecured market (that yields zero profit for the lenders) can be.

4 Normative analysis

We now derive a benchmark allocation to which we can compare the market outcome. We define total welfare as the sum of lenders' and borrowers' expected payoffs. The social planner would choose the borrowers that should invest in the risky, illiquid asset to maximise total welfare. The social planner is risk-neutral.

We assume that investments are worth undertaking, i.e. that both θ and A are greater than I.[14] In this case, it is the interest of the social planner to ensure that investments are always undertaken. The only question is whether a borrower should invest in the safe, liquid or the risky, illiquid asset.

From the perspective of the social planner, the distribution of losses from an unsuccessful investment does not play a role, and neither do interest payments between borrowers and lenders. Moreover, the distribution of collateral between market participants is not relevant for total welfare, so the choice of market (secured or unsecured) does not play a role either.

With a cutoff value p^c being the threshold between borrowers that invest in the safe, liquid asset and borrowers that invest in the risky, illiquid asset, total welfare is

$$\begin{aligned} W(p^c, A, \theta) &= \int_0^{p^c} (A - I) f(p) dp + \int_{p^c}^1 (\theta p - I) f(p) dp \\ &= \left[F(p^c) A + \int_{p^c}^1 \theta p f(p) dp \right] - I. \end{aligned}$$

[14] This assumption can be relaxed easily - if A is less than 1, then it is not in the interest of the social planner that investments are always undertaken, but only if θp_i is greater than 1. Replacing 'safe, liquid investment' by 'no investment', the discussion below can easily be generalised to this case.

Here, F is the cumulative distribution function associated with f. The social optimum for a parameter combination A, θ is given by a cutoff value p^c to maximise $W(p^c, A, \theta)$.

For a borrower of type i, the sum of the lenders' and the borrowers' payoff is $A - I$ for the safe, liquid asset and $\theta p_i - I$ for the risky, illiquid asset. The social planner will wish this borrower to invest in the risky, illiquid asset whenever $A - I < \theta p_i - I$. Thus, he will wish all borrowers with $p_i \leq A/\theta$ to invest in the safe, liquid asset.

We obtain the following proposition:

Proposition 3 *With $p_{SP}^T := A/\theta$, it is in the interest of the social planner to ensure that borrowers with $p_i \leq p_{SP}^T$ invest in the safe, liquid asset and that borrowers with $p_i > p_{SP}^T$ invest in the risky, illiquid asset.*[15]

We see that $F(p_{SP}^T)$ borrowers invest in the safe, liquid asset and $1 - F(p_{SP}^T)$ borrowers invest in the risky, illiquid asset.

The total optimal welfare, according to the social planner, is then

$$\begin{aligned} W_{SP}(A, \theta) &= W(p_{SP}^T, A, \theta) \\ &= \int_0^{p_{SP}^T} (A - I) f(p) dp + \int_{p_{SP}^T}^1 (\theta p - I) f(p) dp \\ &= \left[F(p_{SP}^T) A + \int_{p_{SP}^T}^1 \theta p f(p) dp \right] - I. \end{aligned}$$

We now analyse welfare in the market equilibrium. In CASE 1, the cutoff value was $p^c = p^T < p^Z$, in CASE 2 (the more interesting case) the cutoff value was $p^c = p^Z < p^T$. Thus, overall, $p^c = \min(p^T, p^Z)$.

We define market welfare as

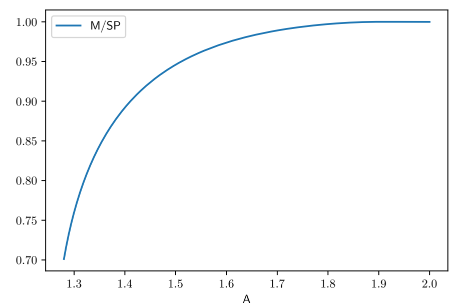
$$W_M(A, \theta) := W(\min(p^T, p^Z), A, \theta)$$

We note that p_{SP}^T is greater than p^Z or p^T (for $\theta > A > R^u I(1 - \lambda)$ and $\lambda < 1$). Thus, in case of a market equilibrium, the resulting welfare $W_M(A, \theta)$ is suboptimal. This reflects "moral hazard" behaviour of borrowers, who invest overly risky as they can shift risks to the lenders, investing in the risky, illiquid asset when they would have invested in the safe, liquid asset if they would have to take the losses themselves.[16]

[15] Of course, for borrowers with $p_i = p_{SP}^T$, the social planner is indifferent, as the expected payout from the safe, liquid and the risky, illiquid asset is the same. For simplicity of notation, we always favour the safe, liquid asset in that case.

[16] The lender compensates for his expected losses by charging higher interest rates on the unsecured market on average. But he cannot distinguish between borrowers that will invest in the safe, liquid asset and those that will invest in the risky, illiquid asset. Thus, borrowers that invest in the safe, liquid asset (or which have a very high probability of success) cross-subsidise borrowers which have a medium-high probability of success for the risky, illiquid asset and invest in this anyway.

Figure 4: Market welfare as share of optimal welfare



In both cases, the market solution differs from the socially optimal one that would be chosen by the social planner (where the borrower would invest in the safe, liquid asset if and only if $p_i \leq p^T_{SP}$). Thus, collateral shortage and asymmetric information will always lead to a sub-optimal market outcome. The suboptimal market outcome shows the need for intervention by a public authority.

4.1 Simulation: Welfare analysis

We include the welfare analysis in our simulation. The simulation indicates that market welfare and socially optimal welfare can indeed differ substantially, calling for an intervention of the regulator. For parameter values as described previously, Figure 4 shows market welfare as a share of socially optimal welfare for a given A, leaving $\theta = 1.9$ fixed. We see that market welfare can lie considerably below the optimal welfare, approaching optimal welfare as A increases.

5 The central bank

We focus on the central bank's role of implementing monetary policy on money markets. In our model, the central bank implements monetary policy via a corridor system. It sets two interest rates, the interest rate R^{DF} of the deposit facility and the interest rate R^{LF} of the lending facility.[17]

The central bank provides a deposit facility with an interest rate R^{DF}, to which the lender has access. For the lender, the option to hold deposits with

[17]Of course, the central bank has the principal aim of influencing economic conditions via its policy interest rates such that price stability is maintained. We do not study the effect of interest rates on the economy here, but focus on the implementation of monetary policy. With two interest rates, the central bank has two degrees of freedom, and one could see one aimed at influencing economic conditions and the other affecting market functioning. For example, with balanced liquidity conditions, one could interpret the middle of the corridor as imfluencing economic conditions and the width of the corridor as influencing market functioning.

the central bank is an alternative to lending on the secured market, as both actions are risk-free. By setting the interest rate on the deposit facility, the central bank can thus give a lower bound for the interest rate on the secured market. Assuming perfect competition of lenders, we have $R^s = R^{DF}$. (This holds as long as there is no safe store of assets or $R^{DF} \geq 1$. If there is a safe store of assets, R^s cannot fall below 1. Otherwise, R^s could become negative if R^{DF} is negative.)

The central bank also provides a lending facility with an interest rate R^{LF}, where it lends (unlimited) funds against central bank eligible collateral. Obviously, the central bank will set its interest rates such that $R^{DF} < R^{LF}$. Then, the width of the corridor is $R^{LF} - R^{DF}$.

To model the central bank as a lender of last resort for banks, we assume that the collateral range accepted by the central bank is wider than that assumed by markets. We assume that market participants have enough central bank eligible collateral available, even if they have used up all collateral eligible on the secured market.[18] Thus, even in the collateral-constrained case, market participants can satisfy all their funding needs by borrowing from the central bank. The central bank interest rates provide a corridor for market interest rates.

This assumption is motivated by the concrete situation in the case of the ECB, by the fact that the central bank in general plays the role of a lender of last resort, and by the fact that the central bank is not liquidity constrained and can thus take illiquid, but otherwise valuable, collateral.

Corollary 4 *If there is a central bank that offers a deposit facility (to which the lenders have access) and if the interest rate at the deposit facility is R^{DF}, then $R^s \geq R^{DF}$ holds for the equilibrium market interest rate on the secured market.*

For simplicity, we could again assume that $R^{DF} = R^s = 1$. This does not change the line of argumentation.

Note that we do not assume any further liquidity-providing operations as this does not unduly restrict our model. Some central banks operate with a corridor system only, so that the model would perfectly describe their behaviour. For other central banks, such as the ECB, the model describes the key features of the framework that is currently in place. In the current situation of a liquidity surplus, the market rate is effectively steered with the rate at the deposit facility.[19] With the fixed rate full allotment procedure, the interest rate at the main refinancing operations takes the role of R^{LF} in our model.

[18] Alternatively, in case we interpret λ not only in terms of collateral constraints but rather as a haircut $1 - \lambda$ on collateral, the central bank can be introduced by arguing that the central bank charges no haircut. While this is not realistic, a situation where central bank haircuts are lower than market haircuts can well arise, notably in case of a financial crisis where market haircuts rise unduly. (Of course, this would then mean that borrowers cannot combine market and central bank funding - if they choose central bank funding, it will have to be for the full amount. We do not elaborate on these technical details further here, as they are driven essentially by the desire to keep all model-parameters endogenous but do not seem crucial.)

[19] The liquidity surplus has been created by central bank action, e.g. by generous liquidity

Given our model setup there is no need for a central bank when there is enough collateral available, but the existence of the central bank can be welfare-improving when collateral is scarce. In this case, the unsecured rate can be higher than the central bank rate as a result of the combination of insufficient collateral and asymmetric information. In such a case, the existence of the central bank can move the market outcome closer to the first best outcome.

We recall that there were two cases: CASE 1, the *pooling equilibrium*, arising if $p^T > p^Y$, and CASE 2, the *partial pooling equilibrium*, arising if $p^T < p^Y$. The analysis is similar in the two cases.[20]

If $R^{LF} > R^u$, then the existence of the central bank has no effect. If $R^{LF} < R^u$,[21] then some borrowers will move from the unsecured market to the central bank. In particular, borrowers which are planning to invest safe, liquid anyway or borrowers with very high success probabilities will move towards central bank funding. This will induce lenders to increase the unsecured rate (as an increased share of "moral hazard" borrowers would participate in the unsecured market), again pushing more borrowers to borrow at the central bank. An equilibrium arises when central bank lending has completely crowded out the unsecured market. In this case, borrowers invest in the safe, liquid asset exactly if $p_i < p^T_{SP}$, and social welfare is optimal.

Corollary 5 *If R^u rises above R^{LF}, central bank intermediation replaces the unsecured market.*

We note that welfare is optimal when the central bank replaces the unsecured market. Central bank lending is collateralised, so no moral hazard arises. However, the central bank has interest in not always intermediating. As we are modelling monetary policy implementation, we assume that the central bank has the aim to steer the unsecured rate close to the middle of the corridor, which forms the first step in the monetary policy transmission mechanism.[22] As the central bank should act in line with a market economy, it aims to preserve market activity, but not at any price.

Thus, the central bank utility function is given as follows:

$$U_{CB} = \max\left(-\left(R^u - \frac{R^{LF}+R^{DF}}{2}\right)^2, -\left(\frac{R^{LF}-R^{DF}}{2}\right)^2\right)$$

provision in liquidity-providing operations to attenuate stress in the interbank market after the onset of the financial crisis as well as by outright purchases.

[20] In case there are no collateral constraints, no borrower will borrow at the central bank, as borrowing on the secured market is always cheaper. Namely, $R^{LF} > R^{DF}$ and $R^{DF} = R^s$, so we have $R^{LF} > R^s$. The central bank cannot exert an influence on market conditions in this case, which is also not necessary, as they are socially optimal.

[21] In case we interpret collateral constraints as haircuts on collateral and the central bank as taking no haircuts, this condition could become $R^{LF} < (1-\lambda)R^u + \lambda R^s$, because borrowers would have to fully move over to the CB.

[22] This assumes balanced liquidity conditions. Of course, with unbalanced liquidity conditions, e.g. excess liquidity, other implementation setups are possible.

Central bank utility is maximal when R^u is at the middle of the corridor. It decreases as R^u moves to the edge of the corridor. Central bank utility does not decrease further once central bank intermediation takes over (as R^u disappears).

Thus, the central bank would most likely set the interest rate R^{LF} somewhere above a normal market rate R^u, in order to only step in when the deviation of R^u from R^s is too large.

Finally, we also see that it is important for the central bank to take a wide range of collateral (as otherwise investment opportunities could not be realised), but that it is likewise important that this collateral is valued appropriately (as otherwise a moral hazard region could arise, similar to the case of the unsecured market that we have analysed above, when the borrower expects that he could pass some share of the costs of a failure to the central bank).

6 The model with a regulator

We now introduce a regulator into the model to see whether regulation can improve welfare.

The regulator seeks to maximise social welfare, so the regulator's utility function is given by the aggregate social welfare (which depends on the cutoff value p^c and the parameters A and θ):

$$U_{reg} = W(p^c, A, \theta)$$

If the share of borrowers investing in the risky, illiquid asset is above the social optimum, there is a case for the regulator to intervene.[23] Contrary to the central bank, the regulator cannot provide liquidity directly. As the secured market cannot supply all liquidity needed for an adequate overall level of investment, the regulator must not remove the unsecured market completely.

The regulator introduces liquidity regulation to improve welfare. Regulatory intervention can be modelled in a variety of ways. In general, a regulator that wants to achieve the optimal outcome from the perspective of the social planner could act via regulating either quantities or prices.[24] After a general discussion of Basel III liquidity regulation in the context of our model, we focus on each of these in turn.

6.1 The Liquidity Coverage Ratio (LCR)

In this section, we discuss the effect of the Liquidity Coverage Ratio (LCR) introduced via Basel III liquidity regulation (see Basel III (2013)) in the context

[23] Without collateral constraints, there is no need for the regulator to intervene, as the market outcome is socially optimal.

[24] As regards price action, the regulator can intervene via a tax on θ or a subsidy of A; the regulator could also intervene via a tax on R^u or a subsidy of R^s. As regards quantity action, the regulator could limit investment in θ or activity on the unsecured market, or set a minimum level for investment in A. In line with the current design of liquidity regulation, we do not model direct regulatory action on market prices R^u and R^s.

of our model.

Regulation prescribes that the LCR has to be above 100% at all times, where

$$LCR = \frac{HQLA}{NetOut}.$$

Here, $NetOut$ stands for net outflows over a 30-day horizon in a stress scenario that is prescribed by regulators. In our model, the LCR can be seen as a form of regulation, via a price effect (by making certain investments more attractive than others, as they will improve the LCR) and/or a quantity effect (by limiting the activity of certain economic agents on certain markets, given the restriction that they have to comply with the LCR requirements). How exactly these effects play out depends on the original situation of the market player, but we take a stylized approach in the model.

Regarding money market functioning, we model short-term interbank markets (maturity below 30 days), thus borrowing/lending on the interbank market affects the net outflows over a 30-day horizon in a stress scenario (NetOut). LCR-constrained banks have an incentive to reduce these net outflows, to the extent possible.

We note that the regulatory treatment favours borrowing from the central bank. Namely, the secured funding run-off rate for secured funding obtained from the central bank is 0%, independent of the collateral used.[25] If an amount I is borrowed from the central bank, then a bank which had $LCR = HQLA/NetOut$ before will have LCR' with

$$LCR' = \frac{HQLA + I}{NetOut}.$$

In principle, a bank could increase the LCR to any desired value by borrowing at the central bank. Thus, LCR-constrained banks have a strong incentive to borrow at the central bank (against non-HQLA collateral) instead of borrowing on the unsecured market or borrowing on the secured market against non-HQLA collateral.

We furthermore note that borrowing/lending on the secured market against high-quality liquid assets (HQLA) has no effect on the LCR. The numerator remains unchanged, as both the liquidity obtained and the collateral used form part of HQLA. Furthermore, the outflow of funds is matched with an inflow of collateral of the same magnitude.

By contrast, borrowing/lending on the unsecured market will have an effect. Namely, if an amount I is borrowed on the unsecured market, then a bank which had $LCR = HQLA/NetOut$ before will have LCR' with

$$LCR' = \frac{HQLA + I}{NetOut + I}.$$

[25]This is thus similar to secured funding against HQLA collateral, but better than the treatment of secured funding against non-HQLA collateral or unsecured funding, which both have run-off rates of 100%.

This moves the LCR closer to 1.[26] Thus, if a bank is not LCR-constrained, borrowing unsecured worsens the LCR. At the same time, if a bank is LCR-constrained (i.e. has an $LCR < 1$), then it has an incentive to borrow unsecured. However, such behaviour will never allow the bank to raise its LCR above 1, thus it is not sufficient to become LCR-compliant.

Regarding the investment side, we observe the following: As the safe, liquid asset is included in HQLA, while the risky, illiquid asset is not included therein, LCR-constrained banks have an incentive to invest in the safe, liquid asset.

To summarise, we can identify two main effects of the LCR in our model:

- LCR-constrained banks have an incentive to borrow from the central bank.
- LCR-constrained banks have an incentive to invest in the safe, liquid asset.

The impact of the LCR depends on the number of banks that are LCR-constrained. In case this number of LCR-constrained banks is not too high and the LCR can be satisfied by the banking system as a whole, the unsecured market can be used to shift LCR-leeway from one bank to the other. This shift can be seen as an additional reason for the existence of an unsecured market.

If this path cannot be pursued to satisfy the LCR-requirements of all banks, or if banks choose not to fully exploit this "socially neutral" way of fulfilling the LCR, LCR-constrained banks remain.

In this case, the LCR requirements may lead to a suboptimal outcome in our model. First, the fact that LCR-constrained banks have a strong incentive to borrow from the central bank will lead to the use of central bank funding instead of market funding. This may have a negative impact on market functioning. However, there is no effect on the allocation of investment, as seen by the social planner. Second, depending on the relationship between p_i and LCR_i at the individual bank level, the fact that LCR-constrained banks have an incentive to invest in the safe, liquid asset may lead banks with good risky, illiquid investment opportunities (i.e. where $\theta p_i > A$) to nevertheless invest in the safe, liquid asset. This would happen if costs of not fulfilling the LCR, e.g. fines charged by the regulator or the financial effect of credibility losses (that could for example lead to higher bank funding costs in general), would be greater than the additional expected gain from investing in the risky, illiquid opportunity.

In crisis times, when interbank markets do not work properly, the central bank may assume an intermediation role. LCR-constraints cannot be shifted around via the unsecured market, so this exacerbates the central bank's role and leads to more central bank funding for LCR-constrained banks.

Furthermore, there is a higher incentive to invest in safe, liquid assets if banks are LCR-constrained (independent of the value of p_i), which means that less of the risky, illiquid projects are realised, even if they would be profitable. This in turn leads to an inferior social outcome. For example, this could be seen as exacerbating a credit crunch that may be one of the big risk factors in crisis times anyway.

[26] Of course, the effect is marginal if I is small compared to $HQLA$ and $NetOut$.

6.2 Regulating via quantity restrictions

The interpretation of the LCR as a quantity restriction (e.g. Perotti and Suarez (2011)) results from the fact that the LCR requires a minimum level of safe, liquid assets or, equivalently, restricts the amount of what would correspond to investments in risky, illiquid assets in our model. Such a quantity restriction can be analysed within our model, while the insight gained remains limited. Implemented at the individual level, it would restrict investment in the risky, illiquid asset for all borrowers, also for those with a high success rate. Thus, this would not be effective to reach social welfare in our model, as no investment in the risky, illiquid asset would take place at all.

The regulator would thus have to impose this restriction on an aggregate level. The effect of this regulatory activity depends on how the remaining investment possibilities would be distributed among borrowers.

If an aggregate quantity restriction is imposed and we assume a market/price-driven mechanism, it is possible that borrowers sort according to their type, whereby the borrowers with the highest success probabilities invest in the risky, illiquid asset and the borrowers with lower success probabilities invest in the safe, liquid asset. In this case, if the regulator sets the quantity thresholds at the socially optimal amounts corresponding to a share of $F(p^T)$ (to be set as a minimum share) for the safe, liquid asset or a share of $1 - F(p^T)$ (to be set as a maximum share) for the risky, illiquid asset, a socially optimal outcome can be achieved.

If an aggregate quantity restriction is imposed and we assume a mechanism whereby the restriction is allocated to individual borrowers without taking their type into account, i.e. by chance or via a process driven by another characteristic of the borrower, independent of their success probability, the outcome will be suboptimal.

Proposition 4 *Liquidity regulation via a quantity restriction (limiting the volume of risky, illiquid investments or setting a minimum volume for safe, liquid investments) can lead to a socially optimal outcome if adequate market mechanisms for the allocation of the restriction are assumed.*

6.3 Regulating via subsidising prices

We now focus on modelling liquidity regulation as an action on prices. In our model, it is most intuitive to model liquidity regulation as a (non-financial) subsidy of the safe, liquid asset A. This is motivated by the important role that HQLA plays in Basel III liquidity regulation, notably for the LCR. We see the introduction of the LCR as giving an additional value to investment in an asset classified as HQLA. The safe, liquid asset has additional (non-financial) value from the fact that it can be used to satisfy the LCR restriction. Thus, the value that the borrower has from investing in this asset increases, instead of A (its monetary value) it becomes $A + a$ (where a can be seen as a non-financial subsidy).

This can be motivated as follows: The borrowing bank can be seen as maximising a utility function U_B, where without regulation the utility is equal to the expected profit Π_B. With regulation, there is a reputational value V attached to the LCR of the bank, thus $U_B = \Pi_B + V(LCR)$, with V a strictly increasing function of the LCR. In turn, the LCR can be seen as a function of the investment choice (liquid or illiquid), where the function is determined by the definition of the LCR and the borrower's initial balance sheet. We know that the LCR is higher when investing in a safe, liquid asset than when investing in a risky, illiquid asset, as the former is classified as HQLA. Thus, $LCR(liquid) > LCR(illiquid)$. Therefore, we have:

$$\begin{aligned} U_B(\text{liquid-safe}) &= \Pi_B(\text{liquid-safe}) + V(LCR(\text{liquid-safe})) \\ U_B(\text{illiquid-risky}) &= \Pi_B(\text{illiquid-risky}) + V(LCR(\text{illiquid-risky})) \end{aligned}$$

Setting

$$a := V(LCR(\text{liquid-safe})) - V(LCR(\text{illiquid-risky})),$$

we see that

$$U_B(\text{liquid-safe}) - U_B(\text{illiquid-risky}) = (\Pi_B(\text{liquid-safe}) + a) - \Pi_B(\text{illiquid-risky}).$$

Expressing the utility to the borrower with profit alone, this equation can be interpreted as the safe asset being worth $A + a$ to borrowers, not only A.

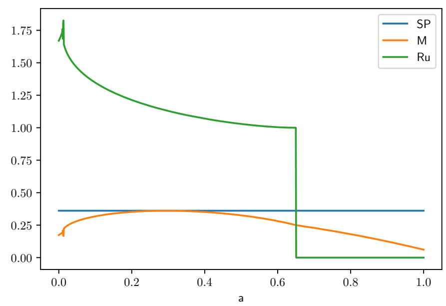
Alternatively, one can consider that the borrower investing into the illiquid asset enters a collateral swap to swap the illiquid asset into a liquid asset, in order to reach the same LCR as he would have reached with a direct investment into a liquid asset. If the costs of the collateral swap are a, then the value of the risky asset for the borrower is reduced by a (independent of the success probability), or equivalently, the safe asset is worth $A+a$ to borrowers, not only A. [27] In either way, the optimisation problem is the same.

With p^c the new cutoff value when the safe, liquid asset is worth $A + a$ to borrowers, we obtain the following utility function of the regulator (dependent on the size of the "subsidy" a).

$$\begin{aligned} U_{Reg}(a) &= W_M(A + a, \theta) - aF(p^c) \\ &= \left[F(p^c)(A + a) + \int_{p^c}^{1} \theta p f(p) dp\right] - I - aF(p^c) \\ &= \left[F(p^c)A + \int_{p^c}^{1} \theta p f(p) dp\right] - I \end{aligned}$$

[27] There is anecdotal evidence that banks are actively using such collateral swaps to improve their LCR, in particular around LCR reporting dates. Banks that want to receive HQLA pay for these collateral swaps. Thus, the interpretation of liquidity regulation as a tax on illiquid investment, or a financial subsidy on liquid investment, is not as theoretical as it may sound at first.

Figure 5: Increasing welfare by (non-financial) subsidy a



We note that this looks like the previous welfare function $W_M(A, \theta)$, but now p^c is based on the value $A + a$ of the safe, liquid asset instead of the value A.

If U_{Reg} is continuous in a and has a unique maximum, the regulator can increase a precisely to the point where the cutoff value is $p^c = A/\theta$, and thus welfare is optimal. With higher a, welfare decreases again. We note that p^c is monotonously increasing in a, as more and more borrowers will wish to invest in the safe, liquid assets as its value (to them) increases. (The increase is strictly monotone if f is continuous.) Thus, $F(p^c)$ also increases in a, while $\int_{p^c}^{1} \theta p f(p) dp$ decreases in a. (Again, both strictly monotone if f is continuous.) Furthermore, U_{Reg} becomes negative for a going to $\pm\infty$. Thus, U_{Reg} assumes a maximum. If f is continuous, U_{Reg} is continuous and the maximum is unique.

Proposition 5 *Liquidity regulation via (an implicit) subsidy of the price of safe, liquid assets can improve social welfare. With a continuous distribution function f of borrower types, a socially optimal outcome can be reached.*

6.4 Simulation: Regulating via subsidising prices

We illustrate the effect of a (non-financial) subsidy of the price of safe, liquid assets with a numerical simulation. Under the same simulation assumptions as before, starting with $A = 1.25$ and $\theta = 1.9$, the regulator can increase welfare by a (non-financial) subsidy a. The interest rate R^u decreases (see Figure 5).[28]

7 The model with a central bank and a regulator

As seen in previous sections, both the central bank and the regulator influence money market behaviour. However, their tools and their aims (utility functions)

[28] This is in line with the expectations formulated by practitioners (see Committee et al. (2015)).

differ.

This leads to a conflict of interest between the central bank and the regulator: The central bank has two aims: (1) to steer funding conditions for the economy;[29] and (2) allowing for activity on the unsecured market while addressing tail risks, i.e. a situation where the unsecured rate would rise too much above the secured rate.[30] In case of central bank intermediation, social welfare becomes optimal. Thus, the central bank addresses inefficiencies that come from asymmetric information to some extent, but not fully.

The regulator has the aim of reducing inefficiencies in the market, as they reduce aggregate welfare. Given that inefficiencies exist as soon as there is an unsecured market, the regulator would in principle design regulation such that as much activity as possible is pushed onto the secured market - just leaving enough activity on the unsecured market so that the collateral constraints do not keep investors from investing at all. However, as this is not possible directly, the regulator introduces incentives for socially optimal behaviour, e.g. via liquidity regulation.

In this section, we analyse the interaction between the central bank's and the regulator's action. Do they influence each other? If so, is the interaction conflicting (such that the action of one policy-maker undermines the utility of the other) or complementary (such that the action of one policy-maker supports the utility of the other)? We see that the interaction can be conflicting or complementary, depending on the constellation.

7.1 Interaction between central bank and regulator can be conflicting

7.1.1 Regulator negatively impacting central bank

Regulatory action can decrease central bank utility. We illustrate this by going through the effects of regulatory action, as modelled in our context.

Focusing on regulation via an implicit subsidy of the safe, liquid asset A, we see that this regulatory action negatively impacts a central bank that had optimally calibrated its implementation parameters to maximise its utility. Namely, a regulatory subsidy a of A has the side effect of decreasing the unsecured interest rate R^u. We call the new unsecured interest rate $\tilde{R}^u$, with $\tilde{R}^u < R^u$.

For a central bank that was at its utility optimum before, i.e. where the equation $R^u = \frac{R^{LF}+R^{DF}}{2}$ held, so that $U_{CB} = 0$, the central bank utility decreases.[31] As $\tilde{R}^u < R^u$, we have $\tilde{R}^u < \frac{R^{LF}+R^{DF}}{2}$. Thus,

[29] This is done by setting the middle of the corridor $\frac{R^{DF}+R^{LF}}{2}$ (in balanced liquidity conditions) or the lower bound for the corridor R^{DF} (in excess liquidity conditions), which becomes the anchor for the secured rate R^s.

[30] This is done by setting the width of the corridor $R^{LF} - R^{DF}$ (in balanced liquidity conditions) or the upper bound of the corridor R^{LF} (in excess liquidity conditions).

[31] We focus on the discussion for balanced liquidity conditions. Similar arguments hold for the situation of excess liquidity conditions.

$$U_{CB} = \max\left(-\left(\tilde{R}^u - \frac{R^{LF} + R^{DF}}{2}\right)^2, -\left(\frac{R^{LF} - R^{DF}}{2}\right)^2 \right) < 0.$$

The central bank needs to react: It needs to implement an appropriate corridor, i.e. appropriate parameter values $\tilde{R}^{DF}$ and $\tilde{R}^{LF}$, so that $\tilde{R}^u$ again lies in the middle of the corridor in the new (subsidized) market equilibrium.

Note that this effect, which we see in our theoretical model setup, is indeed what can be expected to arise as liquidity regulation is implemented in practice: As the regulatory environment shifts, the central bank has to change its monetary policy implementation framework to adopt to the new setup. After adaptation, it can again function normally.

7.1.2 Central bank negatively impacting regulator

Central bank action can also decrease the regulator's utility. We again illustrate this with our theoretical model, starting with a situation where central bank intermediation has fully taken over market functioning.

When the central bank intermediation role has taken over and no money market transactions are taking place, social welfare is optimal, and the regulator does not need to take any regulatory action. We now assume that the central bank decides to widen the corridor from $[R^{DF}, R^{LF}]$ to $[\tilde{R}^{DF}, \tilde{R}^{LF}]$ to re-initiate market functioning. This means a move away from the social optimum W_{SP} to market welfare $W_M < W_{SP}$.

For the regulator, who was at his utility optimum $U_{reg} = W_{SP}$ before (because of central bank intermediation), utility decreases.

Thus, the regulator needs to react: He needs to implement an appropriate subsidy a to achieve a social optimum.

7.2 Interaction between central bank and regulator can be complementary

The interaction between the central bank and the regulator can also be positive, in the sense that the action of one increases the welfare of the other. This can go in both directions.

7.2.1 Regulator positively impacting central bank

In order to see that regulatory action can increase central bank utility, consider the situation where the central bank is not at its utility optimum because R^u is too high ($R^u > \frac{R^{LF}+R^{DF}}{2}$). Furthermore assume that the regulator is also not at his utility optimum and decides to take action by implementing a (non-financial) subsidy a of the safe, liquid asset A.

As a regulatory subsidy a of A has the side effect of decreasing R^u, the central bank utility can increase (because R^u decreases). In an appropriate constellation, the central bank utility optimum can be reached. Even if not, the central bank may have to take less action to reach its optimum than in the case where the regulator did not intervene.

7.2.2 Central bank positively impacting regulator

Central bank action can also increase the regulator's utility. Consider the case when a central bank takes over the intermediation on the money market (either actively through narrowing the corridor, or passively, as exogenous factors change).

In this case, social welfare becomes maximal. Thus, for a regulator who was not at his utility optimum before (because the regulatory subsidy was not optimal, e.g. because exogenous factors had changed), utility increases to its optimum. The regulator does not have to take any additional action.

8 Conclusion

The introduction of liquidity regulation in the aftermath of the financial crisis has an impact on money market functioning. For the central bank, it is important to learn what the new equilibrium in the money market looks like and how liquidity regulation impacts the central bank's implementation of monetary policy. In line with other theoretical models of the money market, our model contains asymmetric information as a key driver of market activity and other constrains that justify the intervention of a central bank in the money market. We use our model to analyse the interaction of a central bank and a regulator when both have a reason to influence activity in the money market.

The model offers several important insights. First, even in the presence of asymmetric information, the market can lead to an efficient outcome if there are no collateral constraints, or if these are never binding. In this case, all trades are conducted on the secured market, and the social inefficiency stemming from the use of the unsecured market under asymmetric information does not arise.

Second, the activity of a central bank can be welfare-improving if the initial market outcome is not efficient, due to asymmetric information and collateral constraints. Then central bank lending can supplement the market by offering contracts that the market will not provide.[32]

Third, the regulator can achieve an efficient outcome. However, not all ways of implementing regulatory action are equally effective. It depends on the design of the regulatory response. This has direct implications for the implementation of global liquidity regulation. In our model, regulatory action on prices (e.g. subsidizing investment in liquid assets) works better than regulatory action on quantities (e.g. limiting risky behaviour). The latter is suboptimal: Implemented on the individual level, no investment may take place. Implemented

[32]This does not only hold during crisis times, when the central bank may widen its collateral set even further, but also during normal times. For example, the ECB's standard set of accepted collateral in monetary policy operations contains inter alia credit claims. It is much wider than the set of collateral that markets would accept.

on the aggregate level, the outcome is suboptimal if there is no mechanism to reach the right borrowers with the regulatory action, and such a mechanism is not part of our model. Price action, which can be interpreted as a subsidy of liquid assets either indirectly (because of the reputational value of a higher LCR) or directly (because of the costs of swapping illiquid assets into liquid assets via a collateral swap), works better. It can always be chosen such that the social optimum is reached (under the condition that the borrower distribution is continuous).

Finally, the interaction between the central bank and the regulator can be complementary or conflicting. It is complementary if the action of one policy-maker brings the other one closer to his optimum. It is conflicting if the action of one policy-maker forces the other one to take "more" action than he otherwise would have. If the regulator introduced liquidity regulation when the central bank was at its optimum, this means that the central bank has to adjust its operational framework to the new circumstances. Conversely, if the central bank decides to reduce its intermediary role which had led to a socially optimal outcome, the regulator may need to step in with a subsidy. As such actions may be associated with operational, reputational or other costs, this implies a need for coordination.

In particular, we have developed a theoretical model to analyse the impact of liquidity regulation on central bank implementation of monetary policy. The main insight from a central bank perspective is that the regulator can reach the welfare optimum, but at the expense of the central bank moving away from its optimum. The central bank needs to adapt by adjusting its monetary policy implementation framework accordingly. This insight is in line with practitioners' expectations.

References

Allen, Franklin and Douglas Gale (2004*a*), 'Financial fragility, liquidity, and asset prices', *Journal of the European Economic Association* **2**(6), 1015–1048.

Allen, Franklin and Douglas Gale (2004*b*), 'Financial intermediaries and markets', *Econometrica* **72**(4), 1023–1061.

Allen, Franklin and Douglas M Gale (2017), 'How should bank liquidity be regulated?'.

Allen, Franklin, Elena Carletti and Douglas Gale (2009), 'Interbank market liquidity and central bank intervention', *Journal of Monetary Economics* **56**(5), 639–652.

Allen, Franklin, Elena Carletti and Douglas Gale (2014), 'Money, financial stability and efficiency', *Journal of Economic Theory* **149**, 100–127.

Ashcraft, Adam B and C Hoyt Bleakley (2006), 'On the market discipline of informationally-opaque firms: Evidence from bank borrowers in the federal funds market'.

Banerjee, Ryan N and Hitoshi Mio (2017), 'The impact of liquidity regulation on banks', *Journal of Financial Intermediation* .

Basel III, BCBS (2013), 'The liquidity coverage ratio and liquidity risk monitoring tools', *Bank for International Settlements* .

Bech, Morten and Todd Keister (2017), 'Liquidity regulation and the implementation of monetary policy', *Journal of Monetary Economics* **92**, 64–77.

Bhattacharya, Sudipto, Douglas Gale, WA Barnett and K Singleton (1985), 'Preference shocks, liquidity, and central bank policy', *Liquidity and crises* **35**.

Brunnermeier, Markus K (2009), 'Deciphering the liquidity and credit crunch 2007–2008', *The Journal of economic perspectives* **23**(1), 77–100.

Brunnermeier, Markus K and Lasse Heje Pedersen (2008), 'Market liquidity and funding liquidity', *The review of financial studies* **22**(6), 2201–2238.

Brunnermeier, Markus K and Martin Oehmke (2013), 'The maturity rat race', *The Journal of Finance* **68**(2), 483–521.

Cao, Jin and Gerhard Illing (2009), 'Endogenous systemic liquidity risk'.

Carlson, Mark A, Burcu Duygan-Bump and William R Nelson (2015), 'Why do we need both liquidity regulations and a lender of last resort? a perspective from federal reserve lending during the 2007-09 us financial crisis'.

Committee, CGFS Markets et al. (2015), 'Regulatory change and monetary policy', *CGFS Papers* (54).

Diamond, Douglas W (1991), ‘Debt maturity structure and liquidity risk’, *The Quarterly Journal of Economics* **106**(3), 709–737.

Diamond, Douglas W and Anil K Kashyap (2016), ‘Liquidity requirements, liquidity choice, and financial stability’, *Handbook of Macroeconomics* **2**, 2263–2303.

Diamond, Douglas W and Philip H Dybvig (1983), ‘Bank runs, deposit insurance, and liquidity’, *Journal of political economy* **91**(3), 401–419.

Eisenschmidt, Jens and Jens Tapking (2009), ‘Liquidity risk premia in unsecured interbank money markets’.

Flannery, Mark J (1986), ‘Asymmetric information and risky debt maturity choice’, *The Journal of Finance* **41**(1), 19–37.

Freixas, Xavier, Antoine Martin and David Skeie (2011), ‘Bank liquidity, interbank markets, and monetary policy’, *The Review of Financial Studies* **24**(8), 2656–2692.

Freixas, Xavier and Cornelia Holthausen (2004), ‘Interbank market integration under asymmetric information’, *The Review of Financial Studies* **18**(2), 459–490.

Freixas, Xavier and José Jorge (2008), ‘The role of interbank markets in monetary policy: A model with rationing’, *Journal of Money, Credit and Banking* **40**(6), 1151–1176.

Heider, Florian and Marie Hoerova (2009), ‘Interbank lending, credit risk premia and collateral’.

Heider, Florian, Marie Hoerova and Cornelia Holthausen (2015), ‘Liquidity hoarding and interbank market rates: The role of counterparty risk’, *Journal of Financial Economics* **118**(2), 336–354.

Hoerova, Marie and Cyril Monnet (2016), ‘Money market discipline and central bank lending’.

Holmstrom, Bengt and Jean Tirole (1997), ‘Financial intermediation, loanable funds, and the real sector’, *the Quarterly Journal of economics* **112**(3), 663–691.

Holmström, Bengt and Jean Tirole (1998), ‘Private and public supply of liquidity’, *Journal of political Economy* **106**(1), 1–40.

Huang, Rocco and Lev Ratnovski (2011), ‘The dark side of bank wholesale funding’, *Journal of Financial Intermediation* **20**(2), 248–263.

Martin, Antoine, David Skeie and Ernst-Ludwig von Thadden (2014), ‘Repo runs’, *The Review of Financial Studies* **27**(4), 957–989.

Perotti, Enrico C and Javier Suarez (2011), 'A pigovian approach to liquidity regulation'.

Philippon, Thomas and Vasiliki Skreta (2012), 'Optimal interventions in markets with adverse selection', *The American Economic Review* **102**(1), 1–28.

Repullo, Rafael (2005), 'Liquidity, risk-taking and the lender of last resort'.

Rochet, Jean-Charles (2004), 'Macroeconomic shocks and banking supervision', *Journal of Financial Stability* **1**(1), 93–110.

Rochet, Jean-Charles and Jean Tirole (1996), 'Interbank lending and systemic risk', *Journal of Money, credit and Banking* **28**(4), 733–762.

Rochet, Jean-Charles et al. (2008), 'Liquidity regulation and the lender of last resort', *Financial Stability Review* **11**, 45–52.

Stiglitz, Joseph E and Andrew Weiss (1981), 'Credit rationing in markets with imperfect information', *The American economic review* **71**(3), 393–410.

Taylor, John B and John C Williams (2009), 'A black swan in the money market', *American Economic Journal: Macroeconomics* **1**(1), 58–83.

Tirole, Jean (2012), 'Overcoming adverse selection: How public intervention can restore market functioning', *The American Economic Review* **102**(1), 29–59.

Von Thadden, Ernst-Ludwig (1999), 'Liquidity creation through banks and markets: Multiple insurance and limited market access', *European Economic Review* **43**(4), 991–1006.

Annex 1 - Detailed derivation of lender strategy

This annex contains a detailed derivation of the strategy of the lender. The net profit for the lender from lending is the difference between the payoff when lending (which is R^u or R^s times the investment I) and the opportunity cost of lending (which is I, because we assumed that resources do not lose their value when they are not lent). When the borrower invests in the safe, liquid asset, the lender receives the profit from lending with certainty, while he only receives the profit with probability p_i if the borrower invests in the risky, illiquid asset. The lender's (certain) profit when lending on the secured market is $\Pi_L^s = (R^s - 1)I$. The lender's expected profit when lending on the unsecured market is $\Pi_L^u(risky) = (R^u I)p_i + (-I)(1 - p_i)$ if the borrower takes a risky, illiquid investment, and $\Pi_L^u(safe) = (R^u - 1)I$ otherwise.

As the lender neither knows p_i nor whether the borrower will invest risky and illiquid or safe and liquid, the lender will have to consider the expected return from lending on the unsecured market. Taking into account the known distribution function f, the lender has to form a belief about the borrower's type p_i conditional on whether the borrower participates in a certain (i.e. secured or unsecured) market. Let $q \in \{0; 1\}$ be an indicator that is 1 whenever a transaction takes place on the unsecured market.

The lender's expected profit when lending on the unsecured market is given by a conditional expectation, namely the expected return conditional on the borrower borrowing on the unsecured market. This is the agreed payout $(R^u - 1)I$ if the borrower invests in the safe, liquid asset, or if he invests in the risky, illiquid asset and is successful, and it is $-I$ if the borrower invests in the risky, illiquid asset and is unsuccessful. This means that we have to write the lender's expected return on the unsecured market as a function of conditional expectations on the borrower borrowing on the unsecured market (i.e. given that the transaction takes place on the unsecured market, or $q = 1$):

$$\Pi_L^u = (R^u - 1)I\phi((\mathcal{A} \cup \mathcal{B})|q = 1) - I\phi(\mathcal{C}|q = 1)$$

where the terms $\phi(\mathcal{A})$ denotes the probability of the case that a borrower invests safe, liquid, $\phi(\mathcal{B})$ denotes the probability that a borrower invests risky, illiquid and is successful and $\phi(\mathcal{C})$ denotes the probability that a borrower invests risky, illiquid and is unsuccessful. As a consequence, $\phi(\mathcal{A} \cup \mathcal{B} \cup \mathcal{C}) = 1$ or $\mathbf{1}_{\mathcal{A}} + \mathbf{1}_{\mathcal{B}} + \mathbf{1}_{\mathcal{C}} = 1$. $\phi((\mathcal{A} \cup \mathcal{B})|q = 1)$ and $\phi(\mathcal{C}|q = 1)$ denote the respective conditional probabilities. The complexity arises from the fact that the share which the borrower borrows on the unsecured market is determined by his type.

To simplify the notation for this conditional expectation, we define several functions on the interval $[0, 1]$: The probability to invest in the safe, liquid asset $\mathbf{1}_{\mathcal{A}}$ is 1 if the borrower of type p_i invests in the safe, liquid asset and 0 otherwise. The probability of investing in the risky, illiquid asset $\mathbf{1}_{\mathcal{B}\cup\mathcal{C}}$ is 1 if the borrower of type p_i invests in the risky, illiquid asset and 0 otherwise. The probability of an investor investing in the risky, illiquid asset and being successful $\mathbf{1}_{\mathcal{B}}$ is 1 if the borrower of type p_i invests in the risky, illiquid asset and is successful and

0 otherwise. Analogously, $\mathbf{1}_{\mathcal{C}}$ is 1 if the borrower of type p_i invests in the risky, illiquid asset and is unsuccessful and 0 otherwise.

Note that $\mathbf{1}_{\mathcal{B}}$ and $\mathbf{1}_{\mathcal{C}}$ can only be observed after the investments are realised. All other variables are deterministic functions of p_i, i.e. they are known if p_i is known, given the rational, profit-maximising behaviour of the borrower.

Using this notation, the expected return of the lender on the unsecured market, which is conditional on the borrower borrowing on the unsecured market, can be written as

$$\begin{aligned}\Pi_L^u &= E((R^u-1)I(\mathbf{1}_{\mathcal{A}}+\mathbf{1}_{\mathcal{B}})-I\mathbf{1}_{\mathcal{C}}|q=1)\\ &= (R^u-1)I\phi(\mathcal{A}\cup\mathcal{B}|q=1)-I\phi(\mathcal{C}|q=1).\end{aligned}$$

We can reformulate the above conditional expectation into an unconditional expectation by introducing some more notation: We define ψ as the function giving the share of funding that a borrower of type p_i will borrow on the unsecured market. We define id as the identity function. Using some basic mathematical identities, the lender's expected profit when lending on the unsecured market is then given by:

$$\Pi_L^u = \frac{E(R^u I\psi(\mathbf{1}_{\mathcal{A}}+(\mathbf{1}-\mathbf{1}_{\mathcal{A}})id)}{E(\psi)}-I \tag{A.1}$$

where the expected value is calculated with respect to the measure induced on $[0,1]$ by the density f.

The numerator is the sum of two components: The expected value in case of a safe borrower (where the payout probability is 1; it is multiplied by the share that is borrowed on the unsecured market) and the expected value in case of a risky, illiquid borrower (where the payout probability is p_i, which is taken up by including the function id in the formula; it is again multiplied by the share that is borrowed on the unsecured market). The denominator is the total share of funding obtained on the unsecured market. Finally, I has to be subtracted to calculate the lender's net profit.

Given the *collateral constraints*, all borrowers will borrow (at least a certain share of their funding) on the unsecured market.

We recall that we have to distinguish two cases: CASE 1 (where $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} > \frac{R^s}{R^u}$, i.e. $p^T > p^Y$) and CASE 2 (where $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} < \frac{R^s}{R^u}$, i.e. $p^T < p^Y$).

First, we consider CASE 1. In this case, all borrowers borrow as much as possible on the secured market, and they invest in the risky, illiquid asset if and only if $p_i > p^T$. The lender thus sets the unsecured rate based on the belief that every borrower borrows a share $(1-\lambda)$ on the unsecured market. The unsecured rate is somewhat higher than the secured rate, since losses are passed on to the lender in case of a non-successful risky, illiquid investment. We see this formally in the following calculation:

$$\begin{aligned}
\Pi_L^u &= \frac{E(R^u I\psi(\mathbf{1}_{\mathcal{A}} + (\mathbf{1} - \mathbf{1}_{\mathcal{A}})id)}{E(\psi)} - I \\
&= \frac{E(R^u I(1-\lambda)(\mathbf{1}_{\mathcal{A}} + (\mathbf{1} - \mathbf{1}_{\mathcal{A}})id)}{(1-\lambda)} - I \\
&= R^u I E(1_{safe} + (\mathbf{1} - \mathbf{1}_{\mathcal{A}})id) - I \\
&= R^u I \left[\int_0^{p^T} 1 f dp + \int_{p^T}^1 p f dp \right] - I \\
&= R^u I - R^u I \int_{p^T}^1 (1-p) f dp - I \\
&= R^u I \left(1 - \int_{p^T}^1 (1-p) f dp - I \right).
\end{aligned}$$

The last equation holds because $\int_0^1 f dp = 1$ (as f is a probability density).

Next, we consider CASE 2. ψ is equal to 1 for the risky, illiquid borrowers that borrow fully on the unsecured market (which are the ones with $p \in [p^Z, p^Y]$). It is equal to $1 - \lambda$ for the borrowers that borrow as far as possible on the secured market. We recall that borrowers with $p_i < p^Z$ invest in the safe, liquid asset, while the others invest in the risky, illiquid asset. Thus, the formula becomes

$$\Pi_L^u = \frac{R^u I(\int_0^{p^Z} (1-\lambda) f dp + \int_{p^Z}^{p^Y} 1 p f dp + \int_{p^Y}^1 (1-\lambda) p f dp)}{\int_0^{p^Z} (1-\lambda) f dp + \int_{p^Z}^{p^Y} 1 f dp + \int_{p^Y}^1 (1-\lambda) f dp} - I.$$

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