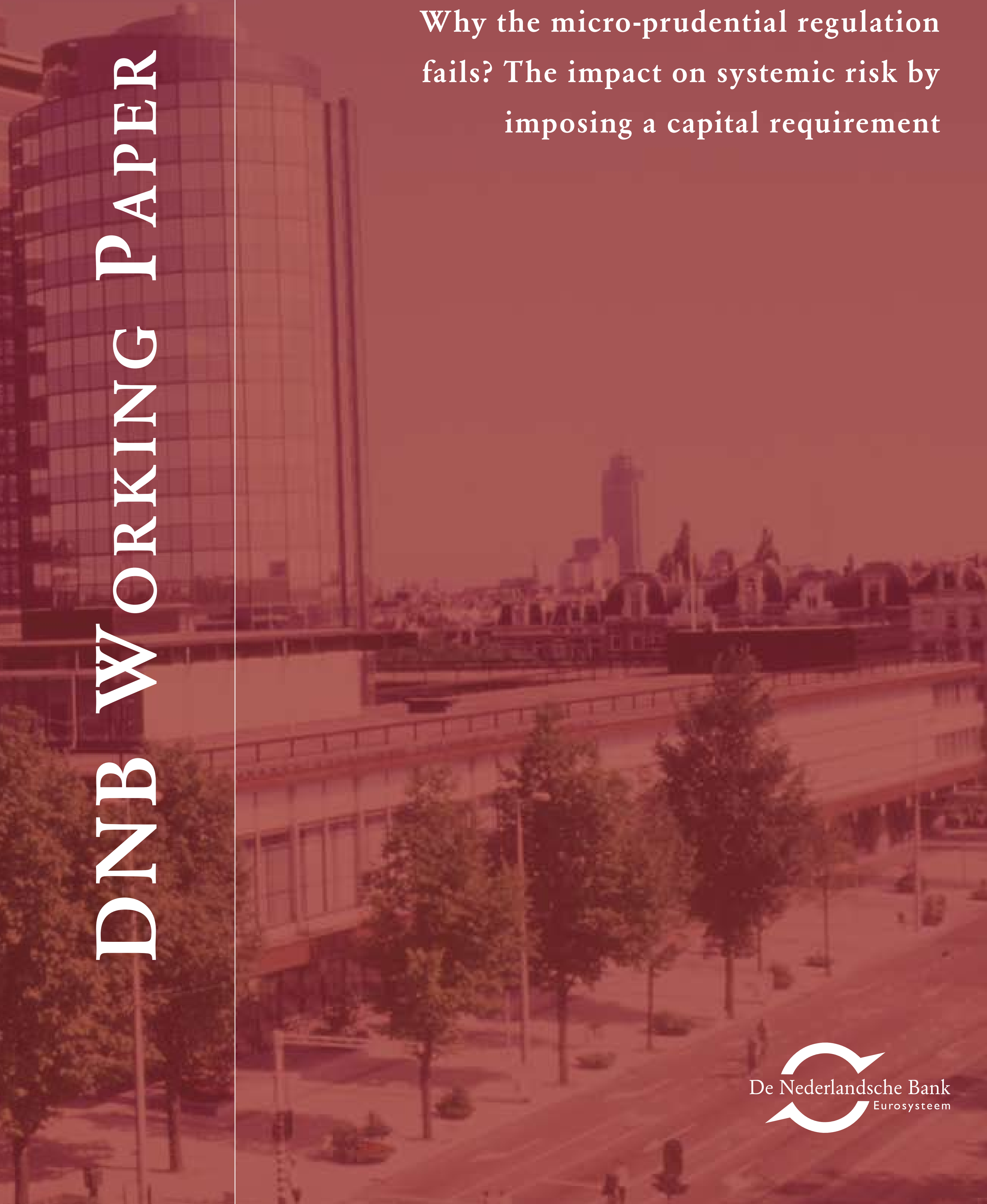
DNB Working Paper

No. 256 / July 2010

Chen Zhou

Why the micro-prudential regulation fails? The impact on systemic risk by imposing a capital requirement



De Nederlandsche Bank
Eurosysteem

Why the micro-prudential regulation fails? The impact on systemic risk by imposing a capital requirement

Chen Zhou*

* Views expressed are those of the author and do not necessarily reflect official positions of De Nederlandsche Bank.

Working Paper No. 256/2010

July 2010

De Nederlandsche Bank NV
P.O. Box 98
1000 AB AMSTERDAM
The Netherlands

Why the micro-prudential regulation fails?

The impact on systemic risk by imposing a capital requirement

Chen Zhou *

De Nederlandsche Bank and Erasmus University Rotterdam

July 15, 2010

Abstract This paper studies why the micro-prudential regulations fails to maintain a stable financial system by investigating the impact of micro-prudential regulation on the systemic risk in a cross-sectional dimension. We construct a static model for risk-taking behavior of financial institutions and compare the systemic risks in two cases with and without a capital requirement regulation. In a system with a capital requirement regulation, the individual risk-taking of the financial institutions are lower, whereas the systemic linkage within the system is higher. With a proper systemic risk measure combining both individual risks and systemic linkage, we find that, under certain circumstance, the systemic risk in a regulated system can be higher than that in a regulation-free system. We discuss a sufficient condition under which the systemic risk in a regulated system is always lower. Since the condition is based on comparing balance sheets of all institutions in the system, it can be verified only if information on risk-taking behaviros and capital structures of all institutions are available. This suggests that a macro-prudential framework is necessary for establishing banking regulations towards the stability of the financial system as a whole.

Keywords: Banking regulation; systemic risk; capital requirement; macro-prudential regulation.

JEL Classification Numbers: G01; G28; G32

*Postal address: Economics and Research Division, De Nederlandsche Bank, 1000AB Amsterdam, The Netherlands. E-mail: *C.Zhou@DNB.nl*.

1 Introduction

Regulations in the financial sector are designed to limit the risk-taking behavior of financial institutions and thus prevent potential financial crises. With the failure of the investment bank Lehman Brothers in 2008, the financial system in the US and the EU came close to a complete meltdown. This raises the questioning on the current financial regulations and supervision. Current policy debate focuses on imposing macro-prudential tools in reforming the incumbent regulations. The word "macro-prudential" is considered as the opposite of "micro-prudential" which refers to the Basel II type of regulation that focuses on the risk-taking behavior of individual financial institutions. In order to impose proper macro-prudential regulation, it is necessary to understand what went wrong with micro-prudential regulation.

The general critique on micro-prudential regulation is that it fails to achieve the goal of maintaining the stability of a financial system as a whole. In other words, it fails to limit the systemic risk within the system. There are two particular dimensions of systemic risk which micro-prudential regulations may not handle. One is on the time dimension: with micro-prudential regulations, the evolution of risk-taking behavior over time may result in a procyclicality problem. There is an extensive literature addressing the procyclicality problem caused by micro-prudential regulations[1]. In contrast, only in recent studies, the other dimension of systemic risk, the cross-sectional dimension, has caught attention. Because banks are interconnected, banking crises may occur simultaneously. This is regarded as a systemic risk on the cross-sectional dimension. The interconnectedness within the banking system are established from either a direct channel such as interbank lending[2] or an indirect channel that banks share common exposures due to diversification at individual level, see, e.g. Lagunoff and Schreft (2001), de Vries (2005) and Wagner (2010). For an overview on the causes of the systemic risk, we refer to de Bandt and Hartmann (2001) and Allen et al.

[1]See, for example, Borio et al. (2001), Borio and Zhu (2008), Brunnermeier et al. (2009), Shin (2009), Zhu (2008) and among others.

[2]See, for example, Allen and Gale (2000), Freixas et al. (2000), Dasgupta (2004) and among others.

(2009). Since micro-prudential regulations are designed for regulating individual financial institutions, they may not prevent the systemic risk on the cross-sectional dimension.

This study targets to pin down the impact of micro-prudential regulation on the cross-sectional dimension. Considering that financial institutions are interconnected because of common risk exposures, the interconnectedness, or in other words, systemic linkage, is then determined by the similarity between the investment strategies. We model the risk-taking behavior of financial institutions by optimizing their portfolio holdings and compare two cases: with and without a micro-prudential regulation rule–capital requirement. Firstly, we compare the difference on the individual risk-taking and the systemic linkage in the two cases. Secondly, we define a systemic risk measure that combines individual risk-taking with systemic linkage and compare the systemic risks in the two cases. We find that although in the regulated system, the individual risk of each institution is lower, the systemic linkage within the system is higher. Furthermore, under certain condition, the systemic risk can be higher in the regulated case. We discuss a sufficient condition under which the systemic risk in the regulated system is always lower. Since the sufficient condition is based on comparing the balance sheets of all institutions within the system, it can be only verified when having a helicopter view of the entire system. This suggests that a macro-prudential framework is necessary for establishing banking regulations towards the stability of the financial system as a whole.

Acharya (2009) also studied the impact of micro-prudential regulation on the cross-sectional dimension of systemic risk. Within a multi-period general equilibrium model, Acharya (2009) found that micro-prudential regulations based only the *own* risk of individual banks can in fact accentuate systemic risk. Differently, our study considers a static model, without imposing dynamics on the time dimension. Such a model is thus simpler. Nevertheless, it is sufficient to show similar conclusion as in Acharya (2009).

This study is connected to another branch of studies on systemic risk: measuring systemic risk, and further extending that to evaluate the contribution of one financial institution to the systemic risk, see Adrian and Brunnermeier (2008), Segoviano and Goodhart (2009), Tarashev et al. (2009a), Tarashev et al. (2009b), Huang et al. (2009) and Zhou (2010a). In our study, we considers the systemic risk measure proposed by Segoviano and Goodhart (2009). Moreover, we employ a heavy-tailed framework to calculate the systemic risk measure. Besides the analysis under the heavy-tailed framework, we also provide the result in the conventional variance and co-variance framework. The conventional framework is sufficient to illustrate the impact of micro-prudential regulation on the individual risk-taking and the systemic linkage, but fails to assess the systemic risk. With such a comparison, we show that the heavy-tailed framework is necessary for systemic risk analysis. It on the one hand addresses the heavy-tailed feature existing in the downside risks, on the other hand provides an easy framework for systemic risk analysis.

Our finding on the limitation of micro-prudential regulations has direct policy implications. The model suggests that it is necessary to have a macro-prudential regulator holding a helicopter view on all financial institutions in the system. That includes monitoring banking activities as well as liability compositions. From our result, we conclude that when regulating a financial system consisting of institutions with similar banking activities, a micro-prudential regulation can be sufficient for reducing systemic risk. In contrast, the macro-prudential regulation is particularly important when regulating a diversified financial system which contains heterogeneous financial institutions focusing on different banking activities. For such a system, it is necessary to identify the systemically important institutions and impose proper prudential regulations on them. This is crucial for managing the systemic risk in the system.

The paper is organized as follows. Section 2 presents the setup of the general model. Section 3 discusses a simple framework assuming normally distributed asset returns. We also discuss the limitation of the normal framework. In Section 4, we consider a heavy-tailed framework and establish the main result. Section 5 concludes the paper and provides further discussion on potential extensions. Proofs of the results are gathered in Appendix.

2 The model

We set up a static model to study the impact of micro-prudential regulation on systemic risk. With the model, we can analyze the risk-taking behavior of individual financial institutions, and evaluate the consequent systemic linkage and systemic risk in the two cases: the regulation-free case and the regulated case. The comparison is between the two cases, thus, there is no issue on the time dimension.

For the micro-prudential regulation tool, we consider the capital requirement as in Basel II. In its elementary form, a capital requirement is calculated from the Value-at-Risk (VaR) of the portfolio holding and multiplied by a risk-weight appointed by the regulator. Financial institutions are required to hold sufficient equity capital to achieve the level of the requirement. In our model, instead of requiring a certain amount of capital holding, we regard the capital structure of a bank as a non-adjustable characteristic in short term, while allow banks to adjust their portfolios in order to obey the regulation rule. This setup is in line with the situation in financial crisis: raising new capital is extremely difficult or very expensive during a crisis; instead, financial institutions choose to fire sale their assets. Under such a framework, the capital requirement regulation turns to be a restriction on the VaR of the portfolio held by a bank.

Consider a financial system consisting of two banks. Each bank can invest in two risky projects and the risk-free rate. The expected returns of the two projects R_1 and R_2 are μ_1 and μ_2 respectively. Without loss of generality, we assume that the risk-free rate is zero and $\mu_2 > \mu_1 > 0$. Moreover, the two projects are independent.

From the bank side, suppose Bank j holds a portfolio $P_j = w_{j1}R_1 + w_{j2}R_2$, $j = 1, 2$. For simplicity, short selling is not allowed, i.e. $w_{ji} \geq 0$ and $w_{j1} + w_{j2} \leq 1$, for $j = 1, 2$.

We consider a mean-downside risk utility for the two banks with different levels of risk aversion λ_j, $j = 1, 2$. Without loss of generality, we assume that $\lambda_1 \leq \lambda_2$, i.e. Bank 1 is less risk averse. More precisely, the utility function of Bank j is given as

$$U_j = w_{j1}\mu_1 + w_{j2}\mu_2 - \lambda_j D(w_{j1}, w_{j2}), \tag{2.1}$$

where $D(w_{j1}, w_{j2})$ is a measure of the downside risk. An example of the downside risk measure D is the variance of the portfolio. Then the utility function is a usual mean-variance approach. In the regulation-free case, the portfolio holding of each bank is determined by maximizing the utility in (2.1).

In the regulated case, a capital requirement regulation is imposed to the two-bank system. The capital requirement is determined by the VaR of the portfolios hold by the banks and a multiplier (risk-weight) chosen by the regulator.

For a given probability level p, the VaR of P_j, $VaR_j(p)$, is defined by the relation $P(P_j < -VaR_j(p)) = p$. From the VaR calculation, the capital requirement for Bank j is $I_j VaR_j(p) d_j$, where I_j is the total investment on the portfolio, and d_j is a multiplier chosen by the regulator. The capital requirement should be covered by the total (equity) capital raised by the bank, denoted by E_j. Hence, we get the restriction as $I_j VaR_j(p) d_j \leq E_j$, for $j = 1, 2$. It can be rewritten as

$$VaR_j(p) \leq T_j := \frac{Q_j}{d_j}, \tag{2.2}$$

where $Q_j := E_j / I_j$ is the equity ratio of the bank.

As discussed above, we regard the equity ratios as fixed within a short period. Moreover, the regulator chooses the regulatory probability level p and the bank specific multiplier d_j ex ante. Hence the threshold T_j in the capital requirement rule (2.2) is regarded as a characteristic of the bank which is ex ante determined. By fixing the threshold T_j, the capital requirement rule (2.2) should be read as a restriction on the VaR of the portfolio held by each bank.

In the case a capital requirement is imposed, banks rebalance their portfolios to obey the rule. Therefore, they solve the constrained utility maximization problem, that is to maximize the utility in (2.1) with the constrain (2.2).

We make a few assumptions in order to simplify the analysis. Notice that the assumptions are not essential. It is possible to omit those assumptions while having a full discussion on all scenarios. The stylized results will be similar. Therefore, they are imposed only for simplicity.

Assumption 1 *In the regulation-free case, the optimal portfolios held by the banks are not corner solutions which assign all portfolio weights to one asset.*

Assumption 2 *In the regulation-free case, the optimal portfolios held by the banks are not partial investment solutions which assign positive weight to the risk-free asset.*

Assumption 3 *Any fully invested risky portfolio can not obey the regulation rule.*

We remark that Assumption 1 implies that the risk aversion levels are not too low, while Assumption 2 implies that the risk aversion levels are not too high. Assumption 3 implies that the thresholds T_j are sufficiently low such that the regulation rule is effective. Together with Assumption 2, the optimal portfolio in the regulation-free case can not satisfy the regulation requirement. Hence, banks must adjust their investment strategy in order to obey the regulation rule. Without Assumption 3, banks may simply keep the optimal portfolio in the regulation-free case while still obeying the capital requirement. In that case, there is nothing to compare between the regulated case and regulation-free case. To prevent this trivial situation, we impose Assumption 3. Moreover, because T_j is partially determined by the regulator due to the choice of d_j, the regulator can make sure that T_j is sufficiently low such that the regulation is effective. Hence Assumption 3 is also reasonable.

3 The impact on individual risk-taking and systemic linkage: a normal framework

We consider a simple framework to illustrate the impact of capital requirement on the individual risk-taking and the systemic linkage. It is called "simple", because we assume that the returns of the risky assets are normally distributed. This is not in accordance with the fact that the distributions of asset returns, particularly the downside risks, are heavy-tailed. The heavy-tailed feature for risk modeling is widely acknowledged in literature, see, e.g. Jansen and De Vries (1991), Embrechts et al. (1997) and among others. Nevertheless, we start with the simple normal framework, because it provides a natural individual risk measure–the variance and a natural systemic linkage measure–the correlation coefficient. We show that within the simple normal framework, in a regulated system, individual risks will be lower, while the systemic linkage becomes higher.

The disadvantage of the normal framework is that it is not convenient for analyzing the systemic risk due to the normal distribution setup. We discuss this at the end of this section. Due to such difficulty and other shortages, it is necessary to consider a heavy-tailed framework instead of the simple normal framework.

3.1 The regulation-free case

Suppose the returns of the two projects follow normal distributions, denoted as

$$R_i \sim N(\mu_i, \sigma_i^2), \quad \text{for} \quad i = 1, 2,$$

where $\mu_2 > \mu_1 > 0$ and $\sigma_2^2 > \sigma_1^2$, i.e. R_2 is more risky than R_1. Consider the downside risk measured by the variance, i.e.

$$D(w_{j1}, w_{j2}) = \frac{1}{2}(\sigma_1^2 w_{j1}^2 + \sigma_2^2 w_{j2}^2).^3$$

In the regulation-free case, the utility function becomes the usual mean-variance utility. The solution to the utility maximization problem is given as in the following proposition.

Proposition 3.1 *With Assumptions 1 and 2 on the risk aversion levels, the solution to the utility maximization problem in the regulation-free case is*

$$w_{j1}^* = \frac{\sigma_2^2 - \frac{\mu_2-\mu_1}{\lambda_j}}{\sigma_1^2 + \sigma_2^2}, \quad w_{j2}^* = \frac{\sigma_1^2 + \frac{\mu_2-\mu_1}{\lambda_j}}{\sigma_1^2 + \sigma_2^2}. \tag{3.1}$$

From the optimal portfolios held by the two banks in Proposition 3.1, we can calculate their individual risks measured by the variances, and their systemic linkage measured by the correlation between their portfolio returns. The variance of the portfolio return for Bank j is

$$IR_j^0 := (w_{j1}^*)^2\sigma_1^2 + (w_{j2}^*)^2\sigma_2^2 = \frac{\sigma_1^2\sigma_2^2 + \frac{(\mu_2-\mu_1)^2}{\lambda_j^2}}{\sigma_1^2 + \sigma_2^2}. \tag{3.2}$$

It is clear that a higher risk aversion level corresponds to a lower individual risk-taking.

For the systemic linkage measured by the correlation coefficient, we get that

$$\begin{aligned}
\rho^0 :=& \frac{Cov(w_{11}^*R_1 + w_{12}^*R_2, w_{21}^*R_1 + w_{22}^*R_2)}{\sqrt{Var(w_{11}^*R_1 + w_{12}^*R_2)Var(w_{21}^*R_1 + w_{22}^*R_2)}} \\
=& \frac{w_{11}^*w_{21}^*\sigma_1^2 + w_{12}^*w_{22}^*\sigma_2^2}{\sqrt{((w_{11}^*)^2\sigma_1^2 + (w_{12}^*)^2\sigma_2^2)((w_{21}^*)^2\sigma_1^2 + (w_{22}^*)^2\sigma_2^2)}} \\
=& \frac{\lambda_1\lambda_2 + K}{\sqrt{(\lambda_1^2 + K)(\lambda_2^2 + K)}},
\end{aligned} \tag{3.3}$$

where $K := \left(\frac{\mu_2-\mu_1}{\sigma_1\sigma_2}\right)^2$ is a constant. From Cauchy inequality, we get that $0 < \rho^0 \leq 1$.

[3] We impose a multiplier 1/2 to make the utility function idential to the usual mean-variance approach. It has no impact on the stylized outcome of the model.

Moreover, $\rho^0 = 1$ holds if and only if $\lambda_1 = \lambda_2$. Notice that, a high ρ^0 corresponds to a high systemic linkage between the two banks, while $\rho^0 = 1$ corresponds to a fully linked system.

It can be verified that

$$\frac{\partial \rho^0}{\partial \lambda_1} > 0 \quad \text{and} \quad \frac{\partial \rho^0}{\partial \lambda_2} < 0.$$

Thus, an increase on λ_1 or an decrease on λ_2 corresponds to an increase on ρ^0. Because increasing λ_1 or decreasing λ_2 will increase the similarity between the risk aversions of the two banks, we conclude that increasing the homogeneity between the risk aversions of the two banks corresponds to an increase in their systemic linkage. We summarize the results on the systemic linkage into the following proposition.

Proposition 3.2 *If the asset returns are normally distributed and there is no regulation, the two banks are systemically connected at a level $\rho^0 \leq 1$. If and only if the risk aversion levels of the two banks are identical, we get the fully connected case: $\rho^0 = 1$. Moreover, increasing the similarity between the risk aversion levels of the two banks will increase their systemic linkage.*

3.2 The regulated case

We consider the case a capital requirement regulation is imposed to the two-bank system. Under the normal framework, we have that

$$P_j = w_{j1}R_1 + w_{j2}R_2 \sim N(w_{j1}\mu_1 + w_{j2}\mu_2, w_{j1}^2\sigma_1^2 + w_{j2}^2\sigma_2^2).$$

Hence, the VaR of P_j is calculated as

$$VaR_j(w_{j1}, w_{j2}; p) = -\left((w_{j1}\mu_1 + w_{j2}\mu_2) - z_p\sqrt{w_{j1}^2\sigma_1^2 + w_{j2}^2\sigma_2^2}\right),$$

where z_p solves the equation $\Phi(z_p) = 1-p$ with Φ the standard normal distribution function.

With Assumptions 1-3, both banks adjust their portfolio to solve the constrained utility maximization problem. The following proposition gives the solution.

Proposition 3.3 *Denote $e_i = \mu_i/\sigma_i^2$ for $i = 1, 2$, and*

$$S = \frac{\mu_1^2}{\sigma_1^2} + \frac{\mu_2^2}{\sigma_2^2}.$$

With Assumptions 1-3, the constrained utility maximization is solved by $(\tilde{w}_{j1}, \tilde{w}_{j2})$ given as

$$\tilde{w}_{j1} = \frac{T_j e_1}{z_p\sqrt{S} - S}, \quad \tilde{w}_{j2} = \frac{T_j e_2}{z_p\sqrt{S} - S}. \tag{3.4}$$

From Assumption 3, we get that the total investment of the optimal portfolio $\tilde{w}_{j1} + \tilde{w}_{j2}$ is less than 1. Moreover, the relative proportion between the two risky assets $\tilde{w}_{j1}/\tilde{w}_{j2}$ equals to e_1/e_2 which is irrelevant to the risk aversion level.

Similar to the regulation-free case, we evaluate the individual risk-taking behavior. For Bank j, the individual risk under regulation, measured by the variance of its portfolio return is given as

$$IR_j^1 := (\tilde{w}_{j1})^2\sigma_1^2 + (\tilde{w}_{j2})^2\sigma_2^2 = \frac{T_j^2 S}{\left(z_p\sqrt{S} - S\right)^2}. \tag{3.5}$$

It is compared to that in the regulation-free case as in the following proposition.

Proposition 3.4 *In the regulated case, individual bank takes less risk compared to the regulation-free case, i.e.*

$$IR_j^0 > IR_j^1, \qquad \text{for } j = 1, 2.$$

Next, we evaluate the systemic linkage. Notice that the relative proportion between the two risky assets is a constant e_1/e_2 across the two banks. Therefore, the two banks are in fact holding portfolios with the same construction in terms of relative proportion. The only difference is that they have different total investment due to the different constrains on the regulation thresholds T_j. Considering the systemic linkage measured by the correlation of their portfolio returns, ρ^1, clearly, we have that $\rho^1 = 1$, which corresponds to the fully connected case. Intuitively, when imposing a capital requirement, the regulation plays a dominate role in guiding the optimal strategy. It overrides the impact of the individual risk aversion levels λ_j, $j = 1, 2$. This results in extremely similar portfolio holdings across banks and thus creates high systemic linkage.[4] We summarize this as the following proposition.

Proposition 3.5 *If the asset returns are normally distributed and the banks are regulated by a capital requirement, the systemic linkage between the two banks measured by the correlation coefficient is* $\rho^1 = 1$. *This corresponds to the fully connected case.*

We summarize the comparison on individual risk-taking and systemic linkage between the regulation-free and the regulated cases in the following theorem.

Theorem 3.6 *With in the normal framework, when imposing a capital requirement as micro-prudential regulation rule, the risk-taking of individual bank is lower, while the systemic linkage within the banking system is higher.*

3.3 Discussion on the limitation of the normal framework

The normal framework in this section follows the conventional variance and co-variance analysis on risk modeling. We show that, under such a simple framework, it is sufficient to demonstrate the two-folded impact of micro-prudential regulation: although imposing a micro-prudential regulation may reduce individual risk-taking as it intends to, because it overrides the diversified individual risk aversions,financial institutions tend to hold more similar portfolios and thus generates higher systemic linkage.

A natural question following the two-folded result is that: how to evaluate the tradeoff? To answer such a question it is necessary to consider a systemic risk measure that combines individual risk with systemic linkage. For instance, a fully connected system with no individual risk should be regarded as having no systemic risk. Only with a proper systemic risk measure, it is possible to evaluate the tradeoff between reducing individual risk and

[4] In our models, the regulation leads to exactly the same portfolio constructions in terms of relative proportion. This results in a fully connected system.

increasing systemic linkage and further assess whether a regulated system corresponds to a lower systemic risk.

An example of such a systemic risk measure within a two-bank system is the probability that both of the two banks are insolvent. With the notation in Section 2, the measure is given as

$$SR := P(P_1 < -Q_1, P_2 < -Q_2), \tag{3.6}$$

where Q_j is the capital ratio for Bank j. This measure is a special case of the banking stability index discussed in Segoviano and Goodhart (2009).

Notice that $Q_j = d_j T_j$ is higher than the threshold T_j in (2.2) because the regulators usually set a multiplier $d_j > 1$. The corresponding probability in (3.6) must be at an extremely low level, much lower than the probability p used in the regulation rule. Hence it is a probability of a tail event. Apparently, this probability is associated to both individual risk-taking and the dependence between them.

We try to calculate the systemic risk within the normal framework. In the regulation-free case, (P_1, P_2) follows a bivariate normal distribution. The calculation of the joint probability in (3.6) is complicated: it has no explicit expression. Therefore, the normal framework is not convenient for comparing the systemic risks in the two cases with and without a capital requirement. It is thus necessary to introduce a model that is accessible for calculating the systemic risk measure as in (3.6).

We remark that the normal framework bears some other shortages. Firstly, the normality assumption is not in line with empirical observations. Empirical literature on asset returns widely acknowledged the so-called "heavy-tail" feature, particularly for the downside of the distribution. That is, the downside tail of the distribution function decays at a power speed rather than an exponential speed. The tail of the normal distribution follows an exponentially decaying property. Therefore, the normal framework may not reflect the reality.

Secondly, in the normal framework, we consider the variance as a measure of individual risk-taking. Variance is a measure of how the asset return varies around its mean value.

It is thus a measure of risk at a moderate level, rather than a measure of the downside risk. For instance, if the probability of earning a high positive return is high, the variance is correspondingly high; however, in this case, the high variance does not correspond to a high risk. Therefore, when evaluating the downside risk, VaR or other measures that focus on the downside tail of the distribution of the asset returns only should be considered.

Thirdly, parallel to the variance, in the normal framework, the correlation coefficient is considered as a measure of systemic linkage. It is in fact a measure of the dependence at a moderate level, rather than the tail dependence. It is known that the tail dependence and the dependence at a moderate level are irrelevant, see e.g. Zhou (2010b). Therefore, when measuring the linkage between banking crises, we need to consider a measure on the dependence of the extreme losses. The correlation coefficient does not qualify for that purpose.

To summarize, the normal assumption on the distributions of the asset returns is not ideal. It is necessary to introduce a model such that, on the one hand, it accommodates the observed heavy-tail feature for the asset returns and addresses tail risk and tail dependence, on the other hand, it is convenient for evaluating systemic risk measures as in (3.6). In the next session, we consider a heavy-tailed framework which overcomes all discussed shortages.

4 The impact on systemic risk: a heavy-tailed framework

We inherit the general framework as in Section 2, but consider the heavy-tail feature on the downside distribution of the asset returns. The left tails of the distributions of the asset returns are given as

$$P(R_i < -t) = A_i t^{-\alpha}(1 + o(1)),$$

where $A_2 > A_1 > 0$. The parameter α is called the *tail index*, while A_i is called the *scale*. The right tails of the two asset returns are assumed to be thinner than the left tails, i.e.

$$P(R_i > t) = o(t^{-\alpha}).$$

This ensures that when constructing a portfolio based on R_1 and R_2, the downside risk of the portfolio is dominated by the downside risks of the two asset returns, and the right tails do not intervene.[5] Moreover, we assume equal tail indices α for the two assets. Theoretically, this is the only case in which the aggregation of risk factors is non-trivial, see Zhou (2010b). Empirical evidence also supports the equal tail indices assumption; see, e.g., Jansen and De Vries (1991). Finally, the existence of a finite mean implies that $\alpha > 1$.

With such a setup, the left tail of the portfolio return held by Bank j, $P_j = w_{j1}R_1 + w_{j2}R_2$ is also heavy-tailed, i.e.

$$P(P_j < -t) = A_{P_j} t^{-\alpha}(1 + o(1)),$$

where the scale of the left tail is $A_{P_j} = w_{j1}^{\alpha}A_1 + w_{j2}^{\alpha}A_2$. This comes from the properties of aggregating independent heavy-tailed risks; see Feller (1971). Moreover, the left tail of (P_1, P_2) follows the bivariate Extreme Value Theory (EVT) setup, and exhibits tail dependence. For details on multivariate (or bivariate) EVT, see de Haan and Ferreira (2006).

With the equal tail indices among all risky assets, the scale is then a downside risk measure, which is similar to the variance in the normal framework. The difference is that the scale only measures the risk in the downside tail. Therefore, we use the scale as the measure of the downside risk in the utility function, i.e.

$$D(w_{j1}, w_{j,2}) = \frac{1}{\alpha}(w_{j1}^{\alpha}A_1 + w_{j2}^{\alpha}A_2).^6$$

[5] Empirical literature supports this assumption, see Jansen and De Vries (1991).

[6] The denominator α is imposed for simplifying the calculation just as the 1/2-multiplier in the normal case. Again, it has no impact on the stylized outcome.

The solution of the unconstrained utility maximization problem is given in the following proposition. Since it is parallel to the normal case, we omit the proof.

Proposition 4.1 *With Assumptions 1 and 2 on the risk aversion levels, the solution of the unconstrained utility maximization problem in the regulation-free case, (w^*_{j1}, w^*_{j2}), is given by firstly solving the equation*

$$(w^*_{j2})^{\alpha-1}A_2 - (1-w^*_{j2})^{\alpha-1}A_1 = \frac{\mu_2-\mu_1}{\lambda_j}, \tag{4.1}$$

and then taking $w^*_{j1} = 1 - w^*_{j2}$.

Assumption 1 and 2 ensure that there exists a unique solution of equation (4.1).

Combining the facts that $\frac{\mu_2-\mu_1}{\lambda_2} \leq \frac{\mu_2-\mu_1}{\lambda_1}$ and the left hand side of (4.1) is an increasing function of w^*_{j2}, we get that $w^*_{12} \geq w^*_{22}$. Intuitively, since Bank 1 is less risk averse, it assigns more weight on the risky asset R_2. The equality holds if and only if $\lambda_1 = \lambda_2$.

Next, we consider a capital requirement in the heavy-tailed model and the VaR-constrain as in the inequality (2.2).

Under the heavy-tailed framework, the calculation of VaR is convenient thanks to the explicit expansion of the tails. Since the left tail distribution of the portfolio return P_j is a heavy-tailed distribution with tail index α and scale $A_{P_j} = A_1 w^\alpha_{j1} + A_2 w^\alpha_{j2}$, we get that

$$VaR_j(w_{j1}, w_{j2}; p) \approx \left(\frac{A_1 w^\alpha_{j1} + A_2 w^\alpha_{j2}}{p}\right)^{1/\alpha}. \tag{4.2}$$

Here the approximation is for low level of p.

With a capital requirement, the optimal portfolio construction for each bank is then determined by the constrained utility maximization problem. The following proposition gives the solution to that.

Proposition 4.2 *Denote $e_i' = (\mu_i/A_i)^{\frac{1}{\alpha-1}}$ for $i = 1, 2$, and*

$$c_j' = \frac{T_j p^{1/\alpha}}{((e_1')^\alpha A_1 + (e_2')^\alpha A_2)^{1/\alpha}}.$$

With Assumption 1-3, the constrained utility maximization problem is solved by $(\tilde{w}_{j1}, \tilde{w}_{j2})$ as

$$\tilde{w}_{j1} = e_1' c_j', \quad \tilde{w}_{j2} = e_2' c_j'. \tag{4.3}$$

Consider the individual risk measured by the scale A_{P_j}. Similar to the simple model, it is not difficult to verify that in the regulated case, the individual risk is lower than that in the regulation-free case. Therefore, we conclude that with the regulation rule, the individual tail risks are lower. This result is parallel to that stated in Theorem 3.6 under the normal framework.

Since the left tail of (P_1, P_2) follows the bivariate EVT, we consider a measure on the systemic linkage stemming from a measure on tail dependence in EVT. Define a distress of a bank with a low tail probability p as the loss exceeding the corresponding VaR. In other words, a distress occurs with a frequency $1/p$. The systemic linkage of distresses between the two banks can be measured by

$$R(1,1) := \lim_{p \to 0} \frac{P\left(P_1 < -VaR_1(p) \text{ and } P_2 < -VaR_2(p)\right)}{p}. \tag{4.4}$$

Here $VaR_j(p)$ is the VaR of the portfolio return held by Bank j.

The limit in and (4.4) exist because the left tail of (P_1, P_2) follows a bivariate EVT setup. Notice that the measure $R(1,1)$ indicates the quotient ratio between the probability of a joint distress and that of an individual distress. Therefore, $0 \leq R(1,1) \leq 1$. Moreover, a higher $R(1,1)$ corresponds to a higher systemic linkage. $R(1,1) = 1$ corresponds to the full tail dependence case. We remark that $R(1,1)$ measure contains only information on the tail dependence: it is irrelevant to the individual risk-taking of the banks as well as the dependence at a moderate level. Hence the $R(1,1)$ measure plays a similar role as the correlation coefficient in the normal framework, but focuses on the tail dependence only. We calculate this measure for the regulation-free case as well as the regulated case. To compare them, we have the following proposition.

Proposition 4.3 *Consider the systemic linkage measured by the $R(1,1)$ measure and denote them as SL^0 and SL^1 for the regulation-free case and the regulated case respectively. Then, we have that $SL^0 < 1 = SL^1$.*

To summarize, under the heavy-tailed framework, we confirm similar statements as in Theorem 3.6 in the normal framework: when imposing a capital requirement as the micro-prudential regulation rule, the individual risk of each bank is lower, while the systemic linkage within the banking system is higher.

Different from the normal framework, under the heavy-tailed model, it is now possible to assess the systemic risk measure as in (3.6). The following lemma shows how to calculate SR given the portfolio structure of the two banks.

Lemma 4.4 *Suppose Bank j holds a portfolio (w_{j1}, w_{j2}) for $j = 1, 2$. Then the systemic risk measure in (3.6) is calculated as*

$$SR \approx A_1 \left(\frac{w_{11}^\alpha}{Q_1^\alpha} \wedge \frac{w_{21}^\alpha}{Q_2^\alpha} \right) + A_2 \left(\frac{w_{12}^\alpha}{Q_1^\alpha} \wedge \frac{w_{22}^\alpha}{Q_2^\alpha} \right). \tag{4.5}$$

From (4.5), when increasing the capital ratio of a bank, the systemic risk may decrease or remain at the same level due to the minimum feature in the formula.

A modification of the formula on SR is that

$$SR \approx A_1 \frac{w_{21}^\alpha}{Q_1^\alpha} \left(\frac{w_{11}}{w_{21}} \wedge \frac{Q_1}{Q_2} \right)^\alpha + A_2 \frac{w_{22}^\alpha}{Q_1^\alpha} \left(\frac{w_{12}}{w_{22}} \wedge \frac{Q_1}{Q_2} \right)^\alpha. \tag{4.6}$$

From this representation, we observe that for calculating SR, it is necessary to compare $\frac{w_{11}}{w_{21}}$ and $\frac{w_{12}}{w_{22}}$ with $\frac{Q_1}{Q_2}$.

Finally, we compare the systemic risk measures in the regulation-free and the regulated cases. The result is presented in the following proposition.

Proposition 4.5 *Consider the systemic risks measured by the SR measure in* (3.6)*. Denote the systemic risk measures in the regulation-free and regulated cases as SR^0 and SR^1 respectively.*

From the solution of the optimal portfolio in the regulation-free case, w^*_{ji}, $i = 1, 2$ and $j = 1, 2$, we define two thresholds as

$$\begin{cases} l(\lambda_1, \lambda_2; \mu_1, \mu_2, A_1, A_2) := \frac{w^*_{11}}{w^*_{21}} \\ r(\lambda_1, \lambda_2; \mu_1, \mu_2, A_1, A_2) := \frac{w^*_{12}}{w^*_{22}} \end{cases} \tag{4.7}$$

It is clear that $l < 1 < r$, provided by $\lambda_1 < \lambda_2$.

If $\frac{Q_1}{Q_2} \leq l$ or $\frac{Q_1}{Q_2} \geq r$, we have that $SR^0 > SR^1$, i.e. in the regulated system,the total systemic risk is lower.

If $l < \frac{Q_1}{Q_2} < r$, with suitable choices of the parameters λ_j, d_j, $j = 1, 2$ and μ_i, A_i, $i = 1, 2$, it is possible to have $SR^0 < SR^1$, i.e. the systemic risk in the regulated case can be higher than that in the regulation-free case.

We summarize the impacts of capital requirement on individual risk-taking, systemic linkage and systemic risk in the following theorem.

Theorem 4.6 *Within the heavy-tailed framework, when imposing a capital requirement, compared to the regulation-free case, we have that 1) the individual risk of each bank is lower; 2) the systemic linkage within the banking system is higher; 3) the systemic risk within the banking system is lower if the capital ratios of the two banks are sufficiently different, i.e. $\frac{Q_1}{Q_2}$ is out of the range (l, r), where l and r are determined by the risk aversion levels of the two banks as in* (4.7)*. If $l < \frac{Q_1}{Q_2} < r$, it is possible that the systemic risk in a regulated system is higher.*

From Theorem 4.6, whether the systemic risk in a system regulated by a micro-prudential regulation is lower than that in a regulation-free system depends on whether $\frac{Q_1}{Q_2}$ is out of the range (l, r). We further discuss this sufficient condition.

Firstly, by definition, both l and r are determined by the portfolio holding strategies of the banks, i.e. the asset side of the balance sheet. Meanwhile, $\frac{Q_1}{Q_2}$ is a comparison between the capital ratios of the two banks, i.e. the liability side of the balance sheet. The condition on whether $\frac{Q_1}{Q_2}$ is in between l and r is then a comparison between the asset and liability sides of the balance sheets of the two banks. It can not be verified by having information on only one of the two banks or only one side of the balance sheets. Therefore, Theorem 4.6 demonstrates the potential limitation of micro-prudential regulation and indicates that to overcome such a limitation it is necessary to have a helicopter view on the strategies and the liability compositions of all banks in the system. In other words, it is necessary to have a macro-prudential approach.

Secondly, it is not difficult to verify that

$$\frac{\partial l}{\partial \lambda_1} > 0, \frac{\partial r}{\partial \lambda_1} < 0, \frac{\partial l}{\partial \lambda_2} < 0, \frac{\partial r}{\partial \lambda_2} > 0.$$

Thus, fixing λ_2, an increase in λ_1 would increase l but decrease r. Notice that $\lambda_1 < \lambda_2$, increasing λ_1 is in fact reducing the heterogeneity between the two banks. Similar result can be observed when fixing λ_1 and varying λ_2. We thus conclude that when reducing the heterogeneity between λ_1 and λ_2, the range of (l, r) will be reduced. With a narrower range of (l, r), it is more likely that the ratio $\frac{Q_1}{Q_2}$ falls out of the range. Hence, when the two banks are more homogeneous in terms of risk aversion, the capital requirement regulation may be more effective in reducing systemic risk. This can be interpreted as follows. When the two banks are more similar in risk-taking, their systemic linkage in the regulation-free case would be at a high level. Imposing the capital requirement increases the systemic linkage further. However, that is a relatively minor effect compared to the reduction on individual bank risk-taking caused by the regulation. Therefore, the tradeoff is eventually on the beneficial side: the systemic risk in the regulated case will be lower. Policy wise, when regulating a banking system with banks having similar banking activities, a micro-prudential regulation such as capital requirement might be effective in reducing systemic risk.

Conversely, when the two banks are more heterogeneous in terms of risk-taking. Their systemic linkage in the regulation-free case would be at a low level. In our model, the range (l, r) will be wider. Then imposing a capital requirement regulation might increases the systemic risk because the ratio $\frac{Q_1}{Q_2}$ is more likely to fall into the range (l, r). To avoid this, it is necessary to have heterogeneity between the two banks liability side in order to achieve the sufficient condition that $\frac{Q_1}{Q_2}$ falls out of the (l, r) range. If $\frac{Q_1}{Q_2} < l$, from (4.6), we get that

$$SR \approx A_1 \frac{w_{21}^\alpha}{Q_1^\alpha} \left(\frac{Q_1}{Q_2}\right)^\alpha + A_2 \frac{w_{22}^\alpha}{Q_1^\alpha} \left(\frac{Q_1}{Q_2}\right)^\alpha = \frac{A_1 w_{21}^\alpha + A_2 w_{22}^\alpha}{Q_2^\alpha}.$$

Symmetrically, when $\frac{Q_1}{Q_2} > r$,

$$SR \approx \frac{A_1 w_{11}^\alpha + A_2 w_{12}^\alpha}{Q_1^\alpha}.$$

Therefore, when $\frac{Q_1}{Q_2}$ falls out of the range (l, r), the systemic risk is mainly from the risk of one of the two banks. In other words, one of the two banks is more *"systemically important"* than the other. In such a case, imposing a capital requirement that reduces risk-taking of the systemically important bank may effectively reduce the systemic risk. Policy wise, when regulating a financial system with different types of financial institutions having different banking activities, it is necessary to identify the systemically important institutions and imposing proper regulation rules to limit the risk-taking of the systemically important institutions. The identification of the systemically important institutions requires monitoring both the asset side and the liability side of the balance sheets of all financial institutions in the system.

To summarize, with the heavy-tailed model, we observe a potentially higher systemic risk in a system regulated by micro-prudential regulation than that in a regulation-free system. The condition on comparing the capital ratios and the portfolio compositions help avoid such a possibility. To verify the condition, it is necessary to have full information on the balance sheets of all financial institutions in the system. Therefore, introducing a macro-prudential framework is the only solution for the problem we raised.

5 Conclusion and discussions

This paper studies why a micro-prudential regulation may not reduce systemic risk and maintain the stability of a banking system as it intends to. As an example of a micro-prudential regulation tool, we consider the capital requirement rule as in Basel II. We start with a simple normal framework to show that, in a financial system regulated by a micro-prudential regulation rule, the individual risk-taking of all institutions can be lower, but the systemic linkage is higher simultaneously. Under the heavy-tailed framework, we further explore the systemic risk which is a combination of individual risks and the systemic linkage. We conclude that, the impact of a micro-prudential regulation can be two-folded.

If the liability sides of the balance sheets of the two banks are more heterogeneous than their asset sides, the systemic risk in the regulated system is lower than that in the regulation-free system. Otherwise, it is possible that the systemic risk in the regulated system is higher due to the enhanced systemic linkage.

Throughout the paper, we consider capital requirement as the micro-prudential regulation rule. A system with such a regulation may have a higher systemic risk, because the regulation rule can override the risk appetite of individual financial institutions in guiding the formation of portfolio holdings, and thus generates higher systemic linkage. This intuition is not limited to capital requirement regulation. It applies to all micro-prudential approaches based on a unified rule that applies to all financial institutions in a system. Therefore, we stress that the potential drawback raised in this study is a drawback of all micro-prudential regulations, rather than that of a particular micro-prudential tool. Hence, our result shows the limitation of micro-prudential regulation as a whole and the necessity of having a macro-prudential regulation framework. Particularly, the model suggests that it is necessary to have a general regulator holding a helicopter view on all financial institutions in the system. That includes monitoring the banking activities as well as the liability compositions. Such a macro-prudential framework helps justify whether a (micro-prudential) regulation too indeed help reduce systemic risk. It is worth mentioning that although a macro-prudential framework is necessary, we may not have to construct new "macro-prudential tools". With carefully monitoring the financial system from a macro-prudential view, the micro-prudential tools such as capital requirement may act as the practical tool for implementing regulations. In the end, a proper regulation scheme may consist of macro-prudential framework and micro-prudential tools: "macro-prudential" should be regarded as a general overview, while the practical regulation tools can still be "micro-prudential".

We also provide policy advice for regulating different types of financial systems. When regulating a system consisting of similar institutions, or in other words, the system is highly interconnected, considering a micro-prudential regulation can be sufficient for reducing the overall systemic risk. In contrast, the macro-prudential regulation is particularly important when regulating a diversified financial system consisting of heterogeneous financial institutions focusing on different banking activities. Regulations on the more systemically important institutions are necessary in such a case.

Our model bears two potential limitations.

Firstly, our model is a static model, i.e. it only considers systemic risk on the cross-sectional dimension, without addressing the potential impact of micro-prudential regulation on the time dimension. Although the model in this study intends to focus on the cross-sectional dimension, by varying the risk-weights d_j set by the regulator, we may partially address some impact on the time dimension within the current model. The static model in our study keeps d_j constant across time, which corresponds to a flat regulation rule. To consider a time-varying regulation rule, for example, a countercyclical regulation, one may consider our model with varying d_j according to macroeconomic environment. Notice that when d_j is in a very low level, the regulation rule may not be effective, i.e. Assumption 3 may not be valid. Increasing d_j to a high level makes the regulation rule effective. Therefore, increasing d_j may mimic the procedure of imposing the regulation rule to a regulation-free system. According to our result, this may actually impose higher systemic risk. From the calculation of the systemic risk, after d_j is sufficiently high for which the regulation is effective, increasing d_j further will reduce the systemic risk. Nevertheless, even in the latter case, the systemic risk is still possible to be higher than that in the regulation-free case. Therefore, if one intends to impose a time-varying regulation to deal with the procyclicality problem, it is not always better off increasing d_j. It is thus important to analyze the overall impact of such a regulation rule on the time dimension and the cross-sectional dimension by evaluating the time variation of the systemic risk measure.

Secondly, our model assumes that the capital ratios are fixed, at least in short term. This assumption implies that in order to obey the regulation, a bank must adjust its portfolio holding. In reality, financial institutions may raise more capital to achieve the same goal. To relax this assumption, the corresponding discussion within our framework is then to allow changes of Q_1 and Q_2. As we discussed, increasing capital ratio will decrease or maintain the level of systemic risk. Thus if possible, it is indeed better off increasing the capital ratios. On the other hand, changing Q_1 and Q_2 will correspondingly change the ratio $\frac{Q_1}{Q_2}$. A potential outcome is that the value of $\frac{Q_1}{Q_2}$ can move from out of the range (l, r) to be in this range, or vice versa. This will change the stylized property on whether the systemic risk is lower for the regulated case than the regulation-free case. If financial institutions raise capital such that their liabilities compositions are similar, then although the systemic risk is reduced in absolute level, the systemic risk under regulation is still possible to be higher than that in the regulation-free case. Particularly, if both banks follow a minimal capital requirement as in Basel I, i.e. $Q_1 = Q_2$, since $l < 1 < r$, the sufficient condition for a lower systemic risk in regulated system is always violated. With such a regulation rule, there is a possibility that the systemic risk is higher in the regulated case.

References

Acharya, V., 2009. A theory of systemic risk and design of prudential bank regulation. Journal of Financial Stability 5, 224–255.

Adrian, T., Brunnermeier, M., 2008. CoVaR. Federal Reserve Bank of New York Staff Report 348.

Allen, F., Carletti, E., Babus, A., 2009. Financial Crises: Theory and Evidence. Annual Review of Financial Economics 1 (1), 97–116.

Allen, F., Gale, D., 2000. Financial contagion. Journal of Political Economy 108, 1–33.

Borio, C., Furfine, C., Lowe, P., 2001. Procyclicality of the financial system and financial stability: issues and policy options. BIS Working Papers.

Borio, C., Zhu, H., 2008. Capital regulation, risk-taking and monetary policy: a missing link in the transmission mechanism? BIS Working Papers 268.

Brunnermeier, M., Crockett, A., Goodhart, C., Persaud, A., Shin, H., 2009. The Fundamental Principles of Financial Regulation. 11th Geneva Report on the World Economy.

Dasgupta, A., 2004. Financial contagion through capital connections: a model of the origin and spread of bank panics. Journal of the European Economic Association 2 (6), 1049–1084.

de Bandt, O., Hartmann, P., 2001. Systemic risk in banking: a survey. In: Goodhart, C., Illing, G. (Eds.), Financial Crisis, Contagion, and the Lender of Last Resort: A Reader. Oxford University Press, pp. 249–298.

de Haan, L., Ferreira, A., 2006. Extreme Value Theory: An Introduction. Springer.

de Vries, C., 2005. The simple economics of bank fragility. Journal of Banking and Finance 29 (4), 803–825.

Embrechts, P., Klüppelberg, C., Mikosch, T., 1997. Modelling extremal events: for insurance and finance. Springer.

Feller, W., 1971. An Introduction to Probability Theory and Its Applications. Vol. 2. Wiley & Sons, New York.

Freixas, X., Parigi, B., Rochet, J., 2000. Systemic risk, interbank relations, and liquidity provision by the central bank. Journal of Money, Credit and Banking 32, 611–638.

Huang, X., Zhou, H., Zhu, H., 2009. A framework for assessing the systemic risk of major financial institutions. Journal of Banking and Finance 33 (11), 2036–2049.

Jansen, D., De Vries, C., 1991. On the frequency of large stock returns: Putting booms and busts into perspective. Review of Economics and Statistics 73 (1), 18–24.

Lagunoff, R., Schreft, S., 2001. A model of financial fragility. Journal of Economic Theory 99 (1), 220–264.

Segoviano, M., Goodhart, C., 2009. Banking Stability Measures. Tech. rep., IMF Working Paper 09/04 (Washington: International Monetary Fund).

Shin, H., 2009. Financial Intermediation and the Post-Crisis Financial System. Mimeo, Princeton University.

Tarashev, N., Borio, C., Tsatsaronis, K., 2009a. Allocating systemic risk to individual institutions: methodology and policy applications. BIS Working Papers forthcoming.

Tarashev, N., Borio, C., Tsatsaronis, K., 2009b. The systemic importance of financial institutions. BIS Quarterly Review September.

Wagner, W., 2010. Diversification at financial institutions and systemic crises. Journal of Financial Intermediation 19 (3), 373–386.

Zhou, C., 2010a. Are banks too big to fail? International Journal of Central Banking forthcoming.

Zhou, C., 2010b. Dependence structure of risk factors and diversification effects. Insurance: Mathematics and Economics 46 (3), 531–540.

Zhu, H., 2008. Capital regulation and banks financial decisions. International Journal of Central Banking 4 (1), 165–211.

Appendix: Proofs

Proof of Proposition 3.1

To solve the unconstrained utility maximization problem, we first find the explicit boundaries for the risk aversion levels under Assumption 1 and 2. It is presented as in the following lemma.

Lemma A.1 *Assumption 1 and 2 are equivalent to the following inequality*

$$\frac{\mu_2 - \mu_1}{\sigma_2^2} < \lambda_j < \frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}, \tag{A.1}$$

for $j = 1, 2$.

Proof of Lemma A.1

For Bank j with a portfolio (w_{j1}, w_{j2}), the marginal utility on asset i is calculated as

$$MU_{ji}(w_{j1}, w_{j2}) := \frac{\partial U_j}{\partial w_{ji}} = \mu_i - \lambda_j w_{ji} \sigma_i^2.$$

It is clear that, in case $w_{j1} = 1$, $w_{j2} = 0$, $MU_{j1}(1,0) < MU_{j2}(1,0)$. Thus, if the optimal portfolio weights correspond to a corner solution, it must be a corner solution with $w_{j1} = 0$ and $w_{j2} = 1$, i.e. the optimal portfolio assigns all weights to the more risky asset R_2. That implies $MU_{j1} \leq MU_{j2}$ for all $w_{j1} + w_{j2} = 1$. Due to the monotonicity of the two marginal utilities, we only need to check $MU_{j1} \leq MU_{j2}$ at the point $w_{j1} = 0$ and $w_{j2} = 1$, which leads to

$$\mu_1 \leq \mu_2 - \lambda_j \sigma_2^2,$$

i.e. $\lambda_j \leq \frac{\mu_2 - \mu_1}{\sigma_2^2}$. Therefore, the assumption that there is no corner solution is equivalent to $\lambda_j > \frac{\mu_2 - \mu_1}{\sigma^2}$, which verifies the lower bound in (A.1).

For the upper bound, we consider the solution for utility maximization without restrictions on w_{j1}, w_{j2}, i.e. the solution of $MU_{j1} = MU_{j2} = 0$. That is

$$w'_{ji} = \frac{\mu_i}{\lambda_j \sigma_i^2}, \quad \text{for} \;\; i = 1, 2.$$

Then, Assumption 2 implies that $w'_{j1} + w'_{j2} > 1$, which gives exactly the upper bound of λ_j as in (A.1). $\square$

From the proof of Lemma A.1, we get that with Assumption 1 and 2, or equivalently under condition (A.1), it is not possible to achieve $MU_{j1} = MU_{j2} = 0$ within the area $w_{j1}+w_{j2} < 1$. Thus, we consider the constrained utility maximization problem with $w_{j1} + w_{j2} = 1$. By the Lagrange multiplier method, we maximize

$$U_j^1 = U_j - K(w_{j1} + w_{j2} - 1).$$

Denote

$$MU_{ji}^1 := \frac{\partial U_j^1}{\partial w_{ji}} = \mu_i - \lambda_j w_{ji} \sigma_i^2 - K.$$

By taking $MU_{j1}^1 = MU_{j2}^1 = 0$, we get that

$$\mu_1 - \lambda_j w_{j1}^* \sigma_1^2 = \mu_2 - \lambda_j w_{j2}^* \sigma_2^2.$$

Together with $w_{j1}^* + w_{j2}^* = 1$, we solve the equations system to obtain the solution as in (3.1). Notice that the condition on λ_j ensures that $w_{j1} > 0$. $\square$

Proof of Proposition 3.3

Denote the optimal solution of the VaR-constrained utility maximization problem as $(\tilde{w}_{j1}, \tilde{w}_{j2})$. We first show that the optimal solution matches VaR constrain, i.e. $VaR_j(\tilde{w}_{j1}, \tilde{w}_{j2}) = T_j$.

From Assumption 3, it is clear that $\tilde{w}_{j1} + \tilde{w}_{j2} < 1$. Suppose $VaR_j(\tilde{w}_{j1}, \tilde{w}_{j2}) < T_j$, then a small increment on risky asset 1 is still possible and with such an increment, the new portfolio will still obey the regulation rule. Since $(\tilde{w}_{j1}, \tilde{w}_{j2})$ is the optimal solution, we must have that, $MU^0_{j1} = 0$ at $(\tilde{w}_{j1}, \tilde{w}_{j2})$. Similarly, we have $MU^0_{j2} = 0$. According to the proof of Lemma (A.1), this can not be achieved in the area $w_{j1} + w_{j2} < 1$. Thus, by contradiction, we proved that

$$VaR_j(\tilde{w}_{j1}, \tilde{w}_{j2}) = T_j. \tag{A.2}$$

With Assumption 3, (A.2) automatically implies that $\tilde{w}_{j1} + \tilde{w}_{j2} < 1$. Thus thus VaR-constrained utility maximization problem turns to be a maximization problem on U_j with the restriction (A.2). By the Lagrange multiplier method, we maximize

$$U^2_j = U_j - K'(VaR_j - T_j).$$

Denote

$$MU^2_{ji} := \frac{\partial U^2_j}{\partial w_{ji}} = \mu_i - \lambda_j w_{ji}\sigma^2_i - K'\left(z_p \frac{w_{ji}\sigma^2_i}{\sqrt{w^2_{j1}\sigma^2_1 + w^2_{j2}\sigma^2_2}} - \mu_i\right).$$

By taking $MU^2_{j1} = MU^2_{j2} = 0$, we get that

$$\mu_i(1 + K') = \tilde{w}_{ji}\sigma^2_i \left(\lambda_j + \frac{K' z_p}{\sqrt{\tilde{w}^2_{j1}\sigma^2_1 + \tilde{w}^2_{j2}\sigma^2_2}}\right),$$

for $i = 1, 2$.

The equation has two solutions.

Case 1) $K' \neq -1$

In this case, by taking the quotient of the two relations for $i = 1, 2$, we get that.

$$\frac{\mu_1}{\mu_2} = \frac{\tilde{w}_{j1}\sigma^2_1}{\tilde{w}_{j2}\sigma^2_2}.$$

Hence $\frac{\tilde{w}_{j1}}{\tilde{w}_{j2}} = \frac{e_1}{e_2}$, where $e_i := \mu_i/\sigma_i^2$, for $i = 1, 2$.

Denote the total amount of investment as $c := \tilde{w}_{j1} + \tilde{w}_{j2} < 1$. Then $\tilde{w}_{ji} = ce_i/(e_1 + e_2)$. We solve c from the relation $VaR_j(\tilde{w}_{j1}, \tilde{w}_{j2}) = T_j$ as

$$c = \frac{T_j(e_1 + e_2)}{z_p\sqrt{S} - S},$$

where $S = \frac{\mu_1^2}{\sigma_1^2} + \frac{\mu_2^2}{\sigma_2^2}$. Thus, we get the solution as in (3.4).

To verify that the solution maximizes the utility U_j^2, it is necessary to check the conditions on the second order derivatives. This is confirmed by further calculation. We omit the details.

We remark that from $c < 1$, we get the boundary of the threshold T_j for making the regulation effective as

$$\frac{z_p\sqrt{S} - S}{e_1 + e_2} > T_j, \tag{A.3}$$

for $j = 1, 2$.

Case 2) $K' = -1$

Then we must have

$$\lambda_j = \frac{z_p}{\sqrt{w_{j1}^2\sigma_1^2 + w_{j2}^2\sigma_2^2}}.$$

It can be verified that, for solutions satisfying the above equation, the second order derivatives does not satisfy the condition for being local maxima. Thus, such a solution does not maximize the utility U_j^2.

All in all, we have a unique solution as given in (3.4).□

Proof of Proposition 3.4

From inequality (A.3), we get

$$IR_j^1 < \frac{S}{(e_1 + e_2)^2}.$$

On the other hand, from (3.2) and (A.1), we get that

$$
\begin{aligned}
IR_j^0 &= \frac{\sigma_1^2\sigma_2^2 + \frac{(\mu_2-\mu_1)^2}{\lambda_j^2}}{\sigma_1^2+\sigma_2^2} \\
&> \frac{\sigma_1^2\sigma_2^2 + \frac{(\mu_2-\mu_1)^2}{(e_1+e_2)^2}}{\sigma_1^2+\sigma_2^2} \\
&= \frac{1}{(e_1+e_2)^2}\frac{\sigma_1^2\sigma_2^2(e_1+e_2)^2 + (\mu_2-\mu_1)^2}{\sigma_1^2+\sigma_2^2} \\
&= \frac{1}{(e_1+e_2)^2}\frac{\frac{\mu_1^2\sigma_2^2}{\sigma_1^2} + \frac{\mu_2^2\sigma_1^2}{\sigma_2^2} + \mu_2^2 + \mu_1^2}{\sigma_1^2+\sigma_2^2} \\
&= \frac{1}{(e_1+e_2)^2}\left(\frac{\mu_1^2}{\sigma_1^2} + \frac{\mu_2^2}{\sigma_2^2}\right) \\
&= \frac{S}{(e_1+e_2)^2}.
\end{aligned}
$$

Hence, we conclude that $IR_j^0 > IR_j^1$. $\square$

Proof of Proposition 4.2

Similar to the simple model case, the optimal portfolio must verifies the VaR restriction. By the Lagrange multiplier method, we maximize

$$U_j' := U_j - K\left(VaR_j - T_j\right),$$

where VaR_j is given in (4.2). The first order conditions for Bank j are that

$$\mu_i = A_i \tilde{w}_{ji}^{\alpha-1}\left(\lambda_j + \frac{K}{\left(A_1\tilde{w}_{j1}^{\alpha} + A_2\tilde{w}_{j2}^{\alpha}\right)^{1-\frac{1}{\alpha}}}\right),$$

for $i = 1, 2$.

Following a parallel discussion as in the simple model, the only solution of the equation systems which maximizes U_j' must satisfy

$$\frac{\mu_1}{\mu_2} = \frac{A_1 \tilde{w}_{j1}^{\alpha-1}}{A_2 \tilde{w}_{j2}^{\alpha-1}}.$$

Therefore, with the notation $e_i' = (\mu_i/A_i)^{\frac{1}{\alpha-1}}$ for $i = 1, 2$, we get that

$$\frac{\tilde{w}_{11}}{\tilde{w}_{21}} = \frac{e_1'}{e_2'},$$

which gives the relative proportion between the two risky assets. The restriction on VaR determines the total investment which results in the final solution as in (4.3). □

Proof of Proposition 4.3

The following lemma shows how to calculate $R(1, 1)$ in our model, given the portfolio holding of the two banks.

Lemma A.2 *Suppose Bank j holds a portfolio (w_{j1}, w_{j2}) for $j = 1, 2$. Then the systemic linkage between the two banks is given as*

$$R(1, 1) = s_1 \wedge s_2 + (1 - s_1) \wedge (1 - s_2), \tag{A.4}$$

where

$$s_j = \frac{A_1 w_{j1}^{\alpha}}{A_1 w_{j1}^{\alpha} + A_2 w_{j2}^{\alpha}}, \quad \textit{for} \;\; j = 1, 2.$$

Proof of Lemma A.2

The calculation comes from a generalized version of the properties on the heavy-tailed distributions, i.e. the Feller theorem, as follows. For any $p \to 0$,

$$
\begin{aligned}
&P(w_{11}R_1 + w_{12}R_2 < -VaR_1(p), w_{21}R_1 + w_{22}R_2 < -VaR_2(p)) \\
\sim &P(w_{11}R_1 \wedge w_{12}R_2 < -VaR_1(p), w_{21}R_1 \wedge w_{22}R_2 < -VaR_2(p)) \\
= &P\left(R_1 < -\left(\frac{VaR_1(p)}{w_{11}} \bigvee \frac{VaR_2(p)}{w_{21}}\right) \quad \text{or} \quad R_2 < -\left(\frac{VaR_1(p)}{w_{12}} \bigvee \frac{VaR_2(p)}{w_{22}}\right)\right) \\
\sim &A_1\left(\frac{w_{11}}{VaR_1(p)} \bigvee \frac{w_{21}}{VaR_2(p)}\right)^{\alpha} + A_2\left(\frac{w_{12}}{VaR_1(p)} \bigvee \frac{w_{22}}{VaR_2(p)}\right)^{\alpha}.
\end{aligned}
$$

Together with the marginal VaRs calculated in (4.2), we get that

$$
\begin{aligned}
R(1,1) :=& \lim_{p\to 0} \frac{1}{p} P(w_{11}R_1 + w_{12}R_2 < -VaR_1(p), w_{21}R_1 + w_{22}R_2 < -VaR_2(p)) \\
=& A_1\left(\frac{w_{11}^{\alpha}}{A_1 w_{11}^{\alpha} + A_2 w_{12}^{\alpha}} \bigvee \frac{w_{21}^{\alpha}}{A_1 w_{21}^{\alpha} + A_2 w_{22}^{\alpha}}\right) + A_2\left(\frac{w_{12}^{\alpha}}{A_1 w_{11}^{\alpha} + A_2 w_{12}^{\alpha}} \bigvee \frac{w_{22}^{\alpha}}{A_1 w_{21}^{\alpha} + A_2 w_{22}^{\alpha}}\right) \\
=& s_1 \wedge s_2 + (1 - s_1) \wedge (1 - s_2),
\end{aligned}
$$

with the notation on s_1 and s_2 as in the lemma. □

Notice that $s_j = \frac{A_1}{A_1 + A_2(w_{j2}/w_{j1})^{\alpha}}$ is only connected to the relative proportion of the weights on the two assets. We have shown that in the regulation-free case, because $\lambda_1 < \lambda_2$, $w_{12}^* > w_{22}^*$. Thus, $w_{12}^*/w_{11}^* > w_{22}^*/w_{21}^*$. Therefore, for the optimal solution of the regulation free case, we get that $s_1^* < s_2^*$. It implies that the systemic linkage of the crises between the two banks in the regulation-free case is

$$
SL^0 = s_1^* + 1 - s_2^* < 1. \tag{A.5}
$$

On the other hand, with the capital requirement regulation, as we shown in Proposition 4.3, the relative proportion of the two assets holding in the optimal portfolios is fixed to e_1'/e_2' regardless the risk aversions of the banks. Thus, $\tilde{s}_1 = \tilde{s}_2$. According to Lemma (A.2), we must have that the systemic linkage measure in the regulated case as

$$SL^1 = \tilde{s}_1 + 1 - \tilde{s}_1 = 1.$$

Compared to (A.5), we get that $SL^0 < SL^1$, i.e. the systemic linkage is increased. $\square$

Proof of Lemma 4.4

The calculation follows exactly the same lines as in the proof of Lemma A.2. The only difference is that the thresholds are given by the capital ratios where in the proof of Lemma A.2, they are marginal VaRs. We omit the details. $\square$

Proof of Proposition 4.5

Firstly, in the regulated case, from Proposition 4.2, we get that

$$\frac{\tilde{w}_{11}}{\tilde{w}_{21}} = \frac{\tilde{w}_{12}}{\tilde{w}_{22}} = \frac{c_1'}{c_2'} = \frac{T_1}{T_2}.$$

In case $d_1 \geq d_2$, we have that $\frac{Q_1}{Q_2} \geq \frac{T_1}{T_2}$. From Lemma 4.4, the systemic risk measure in the regulated case is

$$SR^1 \approx A_1 \frac{\tilde{w}_{11}^{\alpha}}{Q_1^{\alpha}} + A_2 \frac{\tilde{w}_{12}^{\alpha}}{Q_1^{\alpha}} = d_1^{-\alpha} p.$$

Similarly, for the case $d_1 \leq d_2$ we have that $SR^1 \approx d_2^{-\alpha} p$. In all, we get that, for the regulated case,

$$SR^1 \approx (d_1 \vee d_2)^{-\alpha} p. \tag{A.6}$$

It means that with the capital requirement, the systemic risk measure is linked to the tail probability level used in regulation, p, and the maximum of the multipliers applied to the two banks.

Secondly, we calculate the systemic risk measure for the regulation-free case, SR^0. This is more complicated due to the lack of an explicit expression on w_{ji}^* for $i, j = 1, 2$. However, because the solutions are in the regulation-free case, $\frac{w^*_{11}}{w^*_{21}}$ and $\frac{w^*_{12}}{w^*_{22}}$ are independent from $\frac{Q_1}{Q_2}$. Since $\frac{w^*_{11}}{w^*_{21}} < 1 < \frac{w^*_{12}}{w^*_{22}}$, we consider the three different cases.

Case 1) $\frac{Q_1}{Q_2} \leq \frac{w^*_{11}}{w^*_{21}} =: l$

In this case, we get that

$$SR^0 \approx A_1 \frac{(w^*_{11})^\alpha}{Q_1^\alpha} + A_2 \frac{(w^*_{12})^\alpha}{Q_1^\alpha}.$$

Notice that the portfolio (w^*_{11}, w^*_{12}) does not satisfy the regulation rule. It implies that

$$\frac{A_1(w^*_{11}) + A_2(w^*_{12})^\alpha}{T_1^\alpha} > p.$$

Thus, $SR^0 > d_1^{-\alpha} p$. Comparing with SR^1 in (A.6), we get that $SR^0 > SR^1$. Hence the systemic risk is lower in the regulated case.

Case 2) $\frac{Q_1}{Q_2} \geq \frac{w^*_{12}}{w^*_{22}} =: r$

Similar to Case 1), we have in this case $SR^0 > d_2^{-\alpha} p \geq SR^1$. The systemic risk is lower in the regulated case.

Case 3) $l < \frac{Q_1}{Q_2} < r$

In this case, we get that

$$SR^0 \approx A_1 \frac{(w^*_{11})^\alpha}{Q_1^\alpha} + A_2 \frac{(w^*_{22})^\alpha}{Q_2^\alpha}.$$

We show that it is possible to have $SR^0 < SR^1$ by choosing particular values of the parameters.

Consider the case Bank 1 is extremely risk seeking and Bank 2 is extremely risk averse, i.e. λ_1 and λ_2 reach the lower bound and upper bound for λ respectively. Then (w^*_{11}, w^*_{12}) is the riskiest corner solution $(0, 1)$ and (w^*_{21}, w^*_{22}) is the unrestricted solution of maximizing the utility as

$$w^*_{22} = \frac{\left(\frac{\mu_2}{A_2}\right)^{\frac{1}{\alpha-1}}}{\left(\frac{\mu_1}{A_1}\right)^{\frac{1}{\alpha-1}} + \left(\frac{\mu_2}{A_2}\right)^{\frac{1}{\alpha-1}}} = \frac{1}{1 + \left(\frac{\mu_1}{\mu_2}\frac{A_2}{A_1}\right)^{\frac{1}{\alpha-1}}}.$$

For simplicity, we consider $d_1 = d_2 = d$. Then, the systemic risk measure is given as

$$SR^0 \approx d^{-\alpha} \frac{A_2}{T_2^\alpha} \left(\frac{1}{1 + \left(\frac{\mu_1}{\mu_2} \frac{A_2}{A_1} \right)^{\frac{1}{\alpha - 1}}} \right)^\alpha .$$

Similar to inequality (A.3), we can get the boundary of the threshold T_j in the heavy-tailed model as

$$T_j < \frac{(A_1 A_2)^{1/\alpha}}{p^{1/\alpha} \left(A_1^{\frac{1}{\alpha-1}} + A_2^{\frac{1}{\alpha-1}} \right)^{\frac{\alpha-1}{\alpha}}},$$

for $j = 1, 2$. We make further assumption on T_2 that it is very close to the upper bound. Thus,

$$\frac{1}{T_2^\alpha} = p \frac{\left(A_1^{\frac{1}{\alpha-1}} + A_2^{\frac{1}{\alpha-1}} \right)^{\alpha-1}}{A_1 A_2} = p \frac{\left(1 + \left(\frac{A_2}{A_1} \right)^{\frac{1}{\alpha-1}} \right)^{\alpha-1}}{A_2}.$$

Lastly, we make assumption on the parameters μ_1, μ_2, A_1, A_2 as $\frac{\mu_1}{\mu_2} = \left(\frac{A_1}{A_2} \right)^{\frac{1}{\alpha}}$. We get that

$$SR^0 \approx d^{-\alpha} p \frac{\left(1 + \left(\frac{A_2}{A_1} \right)^{\frac{1}{\alpha-1}} \right)^{\alpha-1}}{\left(1 + \left(\frac{A_2}{A_1} \right)^{1/\alpha} \right)^{\alpha}}.$$

Notice that $\frac{A_2}{A_1} > 1$. Thus for $\alpha > 1$, we have

$$\left(1 + \left(\frac{A_2}{A_1} \right)^{\frac{1}{\alpha-1}} \right)^{\alpha-1} < \left(1 + \left(\frac{A_2}{A_1} \right)^{\frac{1}{\alpha}} \right)^{\alpha}.$$

Together with (A.6), we get that $SR^0 < d^{-\alpha} p = SR^1$. Therefore, in the case $\frac{Q_1}{Q_2}$ is in (l, r), it is possible that the systemic risk in the regulated system is higher than that in the regulation-free case. $\square$

Previous DNB Working Papers in 2010

No. 242 **Leo de Haan and Jan Kakes**, Momentum or Contrarian Investment Strategies: Evidence from Dutch institutional investors
No. 243 **Ron Berndsen**, Toward a Uniform Functional Model of Payment and Securities Settlement Systems
No. 244 **Koen van der Veer and Eelke de Jong**, IMF-Supported Programs: Stimulating Capital to Solvent Countries
No. 245 **Anneke Kosse**, The safety of cash and debit cards: a study on the perception and behaviour of Dutch consumers
No. 246 **Kerstin Bernoth, Juergen von Hagen and Casper de Vries**, The Forward Premium Puzzle and Latent Factors Day by Day
No. 247 **Laura Spierdijk, Jacob Bikker and Pieter van den Hoek**, Mean Reversion in International Stock Markets: An Empirical Analysis of the 20th Century
No. 248 **F.R. Liedorp, L. Medema, M. Koetter, R.H. Koning and I. van Lelyveld**, Peer monitoring or cantagion? Interbank market exposure and bank risk
No. 249 **Jan Willem van den End**, Trading off monetary and financial stability: a balance of risk framework
No. 250 **M. Hashem Pesaran, Andreas Pick and Allan Timmermann**, Variable Selection, Estimation and Inference for Multi-period Forecasting Problems
No. 251 **Wilko Bolt, Leo de Haan, Marco Hoeberichts, Maarten van Oordt and Job Swank**, Bank Profitability during Recessions
No. 252 **Carin van der Cruijsen, David-Jan Jansen and Jakob de Haan**, How much does the public know about the ECB's monetary policy? Evidence from a survey of Dutch households
No. 253 **John Lewis**, How has the financial crisis affected the Eurozone Accession Outlook in Central and Eastern Europe?
No. 254 **Stefan Gerlach and John Lewis**, The Zero Lower Bound, ECB Interest Rate Policy and the Financial Crisis
No. 255 **Ralph de Haas and Neeltje van Horen**, The crisis as a wake-up call. Do banks tighten screening and monitoring during a financial crisis?