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Maurice Bun and Jasper de Winter

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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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De Nederlandsche Bank NV
P.O. Box 98
1000 AB AMSTERDAM
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Measuring trends and persistence in capital and labor misallocation

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Abstract

We analyze trends and persistence in the misallocation of labor and capital using firm-level panel data for the Netherlands in the period 2001–2017. We use the dispersion in marginal revenue products of labor and capital to measure the extent of misallocation. Compared to a counterfactual efficient allocation we find that misallocation has had a sizable negative impact on aggregate productivity. Especially capital misallocation has increased over time. The relatively high and rising capital misallocation is caused by a combination of small, highly productive firms facing relatively high capital wedges and large and unproductive firms facing relatively low capital wedges. Exploiting a panel data error components model we find that capital misallocation has a much more permanent character than labor misallocation. Moreover, it is the permanent component of capital misallocation that has increased over time. Finally, we show that in our sample the measurement of misallocation is largely insensitive to capital adjustment costs and alternative specifications of the production function. The contribution of heterogeneous markups to observed misallocation, however, is non-negligible.

Keywords: misallocation; panel data; persistence; productivity.
JEL-Codes: C23, D24, O47.

* Economics and Research Division, De Nederlandsche Bank and Amsterdam School of Economics, University of Amsterdam, m.j.g.bun@dnb.nl.

† Economics and Research Division, De Nederlandsche Bank, j.m.de.winter@dnb.nl.

1 Introduction

Many countries have experienced an aggregate productivity slowdown in the last decades. To shed light on the causal mechanisms behind the stagnation of macro-economic productivity growth, the recent literature has increasingly made use of micro-economic data sets. It is now widely accepted that the large heterogeneity in firm-level outcomes plays a key role in explaining macro-economic developments. An often analyzed micro-founded explanation for the aggregate productivity slowdown is misallocation of resources across firms. The empirical evidence suggests that the productivity loss caused by misallocation can be large, see e.g. Hsieh and Klenow (2009) or Gopinath et al. (2017) among others.

In this paper we estimate the negative impact of misallocation of labor and capital on aggregate productivity in the Netherlands. To the best of our knowledge the size of resource misallocation in the Dutch business sector has not been analyzed before. In our empirical analysis we use the model of Hsieh and Klenow (2009), further referred to as the Hsieh and Klenow model. This set up is attractive because it is able to estimate distortions in output and capital allocation for each year at the firm-level. Furthermore, the Hsieh and Klenow model is able to construct counterfactual productivity levels assuming an efficient allocation of production factors. We show that misallocation in the Netherlands has increased by 14 percentage points in the period 2001–2017 compared to this counterfactual.

Apart from providing firm-level evidence on misallocation for the Netherlands, our contribution to the empirical literature on allocative efficiency is twofold. First, we analyze the relation between estimated firm-level distortions and firm-characteristics in more detail. Throughout the analysis we distinguish industry, size and productivity as relevant firm-characteristics. We use ordered probit-regressions to estimate the probability of a high or low distortion conditional on these firm-characteristics. We find that smaller firms at the productivity frontier have the highest probability of facing relatively large distortions, while non-productive firms have the highest probability of facing relatively small distortions.

Second, we analyze the persistence of the variables underlying the misallocation measures, i.e. the Marginal Revenue Product of Capital (MRPK) and the Marginal Product of Labor (MRPL). We exploit error component models from the earnings dynamics literature (see e.g. Moffitt and Gottschalk, 2011 and Doris et al., 2013) to discriminate between the permanent component and the transitory component of the variances of the marginal revenue products. We find that the permanent component of capital misallocation has increased over time. Furthermore, the permanent component of capital misallocation is highest for small firms and for firms operating at the productivity frontier, corroborating the earlier empirical findings.

The recent literature on misallocation has questioned the underlying assumptions of the Hsieh and Klenow model. Crucial question in this literature is whether we should attribute measured distortions to misallocation, or alternative explanations for the observed distortions. In other words, are the estimated distortions due to misallocation or misspecification? We assess the importance of some often raised conjectures (see e.g. David and Venkateswaran, 2019) to the modeling framework of Hsieh and Klenow (2009). First, we analyze the importance of capital adjustment costs (see e.g. Asker et al., 2014) and we find that it explains only 5% of the observed misallocation. Second, we show that allowing for heterogeneous production technologies across firms does not have an impact on our empirical results. Also we analyze how our measure of misallocation changes if we allow for non-unitary substitution between the production factors labor and capital. Third, we analyze the impact of heterogeneous markups across firms. Using methodology of De Loecker and Warzynski (2012) we find that firm-level heterogeneity in markups is able to explain a sizable part of the observed variation in marginal revenue products, i.e. approximately 25%. Taken together we conclude that in our case the majority of the measured distortions (around 70%) is a consequence of either capital or labor misallocation.

The remainder of this paper is structured as follows. Section 2 describes our baseline model to measure misallocation and presents empirical results for the Netherlands. Section 3 analyzes the relation between the firm-level distortions and firm-characteristics in more detail. Section 4 analyzes the evolution of misallocation focusing on persistence and trends. Section 5 analyses the sensitivity of our outcomes to recent criticism in the literature. Section 6 concludes.

2 Measuring misallocation

To quantitatively assess the effect of within-industry resource misallocation on aggregate total factor productivity we build on the framework developed by Hsieh and Klenow (2009), recently applied by i.e. Gopinath et al. (2017), Gamberoni et al. (2016), Calligaris (2015), Calligaris et al. (2018) and others. The Hsieh and Klenow model formally shows that frictions distorting the marginal revenue products of capital and labor lower aggregate factor productivity. We estimate the model using a rich and tailored data set containing the annual balance sheet and the profit and loss statements of all Dutch firms that are legally obliged to declare corporate income tax. Our final data set contains 1,831,5757 firm-year observations for the period 2001–2017, and contains 342,245 unique firms. Appendix A contains a detailed description of the sources and cleaning procedure used to construct this data set.

2.1 Model

The Hsieh and Klenow model makes two crucial assumptions. First, firms are heterogeneous in their productivity level and in the extent of factor-market distortions they face. Second, every firm supplies a heterogeneous good which is priced individually in the market. Aggregate economy wide output is defined by a Cobb Douglas (CD) production technology:

$$Y_t = \prod_{S=1}^{S} Y_{s,t}^{\theta_s} \text{ where } \sum_{s=1}^{S} \theta_s = 1, \tag{1}$$

where $Y_{s,t}$ denotes industry-specific output in industry s in year t and Y_t is the product of all industry-specific outputs $Y_{s,t}$ raised to their individual industry-output share θ_s. At the industry level, s, the output is a Constant Elasticity of Substitution (CES) aggregate of M_s differentiated products:

$$Y_{s,t} = \left(\sum_{i=1}^{M_s} Y_{is,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where $Y_{is,t}$ denotes output of firm i –i.e. the heterogeneous good produced by firm i– in industry s at time t. The parameter σ is the time-invariant elasticity of substitution between firm value added or the substitutability of competing manufacturers. The higher σ the more substitutable the goods become and the less the firm can control the market price via its mark-up. For comparability with Hsieh and Klenow (2009) and Gopinath et al. (2017) we set σ at 3.[1]

Each individual firm produces its unique good according to a standard CD production function:

$$Y_{is,t} = A_{is,t} K_{is,t}^{\alpha_s} L_{is,t}^{1-\alpha_s}, \tag{3}$$

where $A_{is,t}$, $K_{is,t}$ and $L_{is,t}$ are total factor productivity, real capital and labor input of firm i, in industry s at time t, respectively. We measure the firm's nominal value added $P_{is,t}Y_{is,t}$ as the difference between gross turnover minus materials used in production. Real firm-level output $Y_{is,t}$ equals nominal value added deflated with a firm-specific output price

[1] Note that as σ goes to ∞ the economy leaves monopolistic competition and approaches perfect competition.

deflator $P_{is,t}$. Since we do not observe prices at the firm-level in our data set, we revert to using two-digit industry deflators, following e.g. Dias et al. (2016), Gorodnichenko et al. (2018) and Gopinath et al. (2017). This implies we calculate $Y_{is,t}$ actually as $P_{is,t}Y_{is,t}/P_{s,t}$, where $P_{s,t}$ is the industry deflator. We measure labor input $L_{is,t}$ as the firm's wage bill deflated by the same two-digit industry deflators used to deflate nominal firm-level output, i.e. $W_{is,t}L_{is,t}/P_{s,t}$. We use the nominal wage bill instead of employment to control for differences in the quality of the workforce across firms, as argued in e.g. Hsieh and Klenow (2009) and Gopinath et al. (2017). Finally, we measure the firm-specific real capital stock $K_{is,t}$ with the book value of fixed tangible assets deflated with the total deflator of non-financial corporations gross fixed capital formation, P_t^K. This implies we actually calculate $K_{is,t}$ as $P_{is,t}^K K_{is,t}/P_t^K$.

The idea of the Hsieh and Klenow model is that each firm acts a monopolist in its differentiated product and faces two types of distortion: a capital wedge $\tau_{is,t}^K$, that changes the relative marginal revenue product of capital with respect to labor, and an output wedge $\tau_{is,t}^Y$, that changes the marginal product of capital and labor by the same proportion. Market distortions appear in the firm's profit equation:

$$\pi_{is,t} = (1 - \tau_{is,t}^Y)P_{is,t}Y_{is,t} - w_t L_{is,t} - (1 + \tau_{is,t}^K)R_s K_{is,t}, \tag{4}$$

where w_t is the wage faced by all firms and R_s is the time-invariant industry specific real rental price of capital, i.e. the sum of the nominal interest rate per industry r_s and the industry-specific depreciation rate δ_s minus the average headline inflation, $\Delta HICP_t$, where $HICP_t$ is the natural logarithm of the Harmonized Index of Consumer Prices (HICP). We assume an average inflation rate of 2% in our sample, in line with the average HICP-inflation rate in the Netherlands during the period 2001–2017. We estimate r_s and δ_s from the firm-level data in two steps. First, we calculate the firm-specific time-varying implicit interest rate $r_{is,t}$ and depreciation rate $\delta_{is,t}$, by dividing the firm's interest payment by total debt and depreciation divided by total fixed tangible assets. Second, we aggregate the firm-specific time-varying implicit interest rate and depreciation rate to industrial averages by weighting with the average industrial real value added shares over the period 2001–2017.[2] Before aggregation we clean $\delta_{is,t}$ and $r_{is,t}$ by setting $\delta_{is,t}$ or $r_{is,t}$ equal to missing when it is in the top/bottom 5% of the respective distributions.

As a result of the wedges $\tau_{is,t}^Y$ and $\tau_{is,t}^K$, there will be differences in the marginal products of labor and capital across firms. The first order conditions for profit maximization with

[2] All deflators, the HICP-index and the real value added shares are publicly available data from Statistics Netherlands

respect to labor and capital, are given by:

$$P_{is,t}\frac{\partial Y}{\partial L} = MRPL_{is,t} = (1-\alpha_s)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{P_{is,t}Y_{is,t}}{L_{is,t}}\right) = \left(\frac{1}{1-\tau^Y_{is,t}}\right)w_t, \tag{5}$$

$$P_{is,t}\frac{\partial Y}{\partial K} = MRPK_{is,t} = \alpha_s\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{P_{is,t}Y_{is,t}}{K_{is,t}}\right) = \left(\frac{1+\tau^K_{is,t}}{1-\tau^Y_{is,t}}\right)R_{s,t}, \tag{6}$$

The estimated distortions to capital ($\tau^K_{is,t}$) and output ($\tau^Y_{is,t}$) are estimated *relative* to labor. An observationally equivalent, and in our case more useful, expression is in terms of distortions in *absolute* levels of capital and labor. Denote level distortions of capital and labor as $\tau^{K*}_{is,t}$ and $\tau^{L*}_{is,t}$, respectively. Hsieh and Klenow (2009) show that the firm's first order conditions are identical to those with $\tau^K_{is,t}$, $\tau^Y_{is,t}$ assuming $1-\tau^Y_{is,t} = 1/(1+\tau^{L*}_{is,t})$ and $1+\tau^K_{is,t} = (1+\tau^{K*}_{is,t})/(1+\tau^{L*}_{is,t})$.

Following Foster et al. (2008) we make a distinction between revenue productivity (TFPR) and physical productivity (TFPQ), i.e. the Solow-residual. Using this terminology we can define TFPQ by rewriting equation (3):

$$TFPQ_{is,t} = A_{is,t} = \kappa_{s,t}\frac{P_{is,t}Y_{is,t}^{\frac{\sigma}{\sigma-1}}}{K^{\alpha_s}_{is,t}L^{(1-\alpha_s)}_{is,t}}, \tag{7}$$

Where the scalar is $\kappa_{s,t} = (P_{s,t}Y_{s,t})^{\frac{1}{\sigma-1}}/P_{s,t}$. The calculation of $TFPQ_{is,t}$ warrants some explanation. We do not observe each firm's real output $Y_{is,t}$, but only its nominal output $P_{is,t}Y_{is,t}$, i.e. we lack data on firm's prices ($P_{is,t}$), and only have information on sectoral prices ($P_{s,t}$). Plants with high real output, however, must have a lower price to explain why buyers would demand the higher output. Following the assumption of an isoelastic inverse demand curve we raise $P_{is,t}Y_{is,t}$ to the power $\sigma/(\sigma-1)$ to arrive at $Y_{is,t}$. That is, we infer price versus quantity value added from revenue and our assumed (constant) elasticity of demand. Elegantly, equation (7) requires only the assumptions about technology and demand plus maximization. Then TFPR is defined as:

$$TFPR_{is,t} = P_{is,t}A_{is,t} = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{MRPK_{is,t}}{\alpha_s}\right)^{\alpha_s}\left(\frac{MRPL_{is,t}}{1-\alpha_s}\right)^{1-\alpha_s}. \tag{8}$$

In this model TFPR does not vary across firms within an industry unless firms face output and/or capital distortions. The idea is that in a friction-less economy more capital and labor should be allocated to firms with higher physical productivity, $A_{is,t}$, to the point where higher output results in lower price and the same TFPR compared to firms with lower physical productivity. When there are frictions, however, economy wide output can be (much) lower. Imagine an economy with two firms that have identical technology but in which one firm benefits from subsidized credit (say from a state-owned bank), and the other firm can only borrow at high rates from informal capital markets. Assuming that both firms equate their MRPK with the interest rate, the marginal revenue product of the firm with access to subsidized credit will be lower than the MRPK of the firm that only has access to informal financial markets. This is a clear case of capital misallocation: aggregate output would be higher if capital was reallocated from the firm with a low MRPK to the firm with a high MRPK. The misallocation of capital results in lower aggregate output per worker and TFPQ (see e.g. Gopinath et al., 2017).

We follow Hsieh and Klenow (2009), and estimate the impact of misallocation thus defined on the TFPQ level by defining the "efficient" level of TFPQ, as the TFPQ-level we would observe in the first-best allocation in absence of dispersion in MRPK, MRPL and TFPR such that $TFPR_{is,t} = \overline{TFPR}_{s,t}$, where:

$$\overline{TFPR}_{s,t} = \frac{P_{s,t}Y_{s,t}}{K_{s,t}^{\alpha_s} L_{s,t}^{(1-\alpha_s)}}, \tag{9}$$

It can then be shown that the difference in log(TFPQ) arising from misallocation – the misallocation-gain $\Lambda_{s,t}$– can be written as a combination of the variables introduced in equations (3)–(9). See equations (1)–(11) in Gopinath et al. (2017) for a formal derivation of this misallocation measure. In Section 2.2 we will express the misallocation-measure $\Lambda_{s,t}$ in percentage point differences with respect to the first-best allocation in absence of dispersion in MRPK, MRPK and TFPR in 2001.

Our misallocation measure represents an upper limit to intra-industry misallocation, given that *all* variation in the marginal revenue product is attributed to misallocation. This is assuming that firms are at their long-term static equilibrium at any given moment in time, irrespective of the scope, type and frequency of firm-specific shocks. Each diversion from the equilibrium is regarded as misallocation, which constitutes a substantial assumption. The model assumptions are relatively strict: all firms have a productivity level with the same mark-up and the same capital intensity, adjustment costs are absent, and the substitution elasticity with respect to capital and labor is equal to a CD-production function with constant revenues of scale. In our view the the model is still a suitable model to measure misallocation in the economy, despite these limitations. We elaborate on the sensitivity of our misallocation measures to the model assumptions in Section 5.

2.2 Estimation results misallocation total economy

Figure 1 shows the allocative efficiency for both factors of production, labor and capital, in the period 2001–2017 according to the Hsieh and Klenow model outlined above, reflecting the standard deviation of the log of the marginal revenue product of capital (black line) and the log of the marginal revenue product of labor (grey line). In the remainder of the text we will also refer to these measures as capital misallocation and labor misallocation.[3] The larger the standard deviation, the higher the degree of misallocation. Capital misallocation shows a clearly discernible upward trend over the period 2001–2013, interrupted briefly between 2005 and 2007. After 2013 misallocation of capital has leveled out. On balance, capital misallocation in the Netherlands increased by 18% in the period 2001–2017. Labor misallocation developed quite differently. Labor misallocation increased much less strongly than capital misallocation but fluctuated much more. In the period 2002–2006 labor misallocation even dropped below the level recorded in 2001. In 2017 the level of labor misallocation exceeded the 2001 level by a "mere" 6%.

[INSERT FIGURE 1 ABOUT HERE]

Figure 2 shows the possible gain from reducing capital and labor misallocation to zero using our misallocation-gain measure $\Lambda_{s,t}$ introduced in Section 2.1. According to this measure, removing all distortions would increase aggregate TFPQ in the Netherlands by 43% in 2001. This gain increases to 57% in 2017, indicating that since 2001 there has been a decline in allocative efficiency by 14 percentage points. Interestingly, the worsening of allocative efficiency started long before the Great Recession (2008) and the European Debt crisis (2011). During these crisis-years misallocation losses kept rising. After 2013, the misallocation loss dropped somewhat and has been fluctuating around 57% in the period 2014–2017. Comparing our results to recent findings in the literature seems to indicate that the measured increase in misallocation is more comparable to the Southern EU-member states than the Northern EU-member states. Similar to the Netherlands, Gopinath et al. (2017) observe significant increased in misallocation in Spain and Italy, but quite stable levels of misallocation in Germany, France and Norway. Calligaris (2015) and Calligaris et al. (2018) also find steadily increasing misallocation in Italy.

[INSERT FIGURE 2 ABOUT HERE]

[3] Economy-wide measures are calculated by the weighted mean, using real industrial value added weights, calculated as the average share in real value added over the period 2001–2017, according to the National Accounts.

2.3 Misallocation per firm-characteristic

In the period 2001–2017, the level and development in misallocation was not evenly spread across industries. The level of our misallocation measure in the services sector clearly exceeds that in the manufacturing sector.[4] This is clear from Figure 3, which plots the measured capital and labor misallocation in the total economy, the manufacturing sector (dashed line) and the services sector (dotted line). Panel A and B show the evolution of capital and labor misallocation, respectively. These results confirms the outcomes of earlier studies addressing misallocation in the services sector. For Portugal, Dias et al. (2016) find that the level of misallocation in the services sector exceeds that in the manufacturing sector by a large amount. Busso et al. (2013) find the same result for a number of Latin American countries. de Vries (2014) finds substantial misallocation in the Brazilian retail sector. Dias et al. (2016) suggest that the relatively high misallocation in the services sector vis-à-vis the manufacturing sector could be driven by lower competition, limited international trade-ability, relatively high regulatory barriers an the location-specific nature of services.[5]

[INSERT FIGURE 3 ABOUT HERE]

Moreover, the *evolution* of labor misallocation in manufacturing is markedly different from the services sector (Panel B of Figure 3). In the manufacturing sector labor misallocation has steadily increased in the period 2005–2017, whereas misallocation in the services sector more or less stabilized during this period. Unlike the diverging evolution of labor misallocation in the manufacturing and services sector, capital misallocation rose in roughly the same amount in both the manufacturing sector and services sector.

Figure 4 shows a more detailed picture of capital misallocation at the five-digit industry level in 2001 and 2017, respectively. Our data set contains 93 industries in the manufacturing sector and 240 industries in the services sector. In both 2001 and 2017 most firms were active in the services sector, i.e. 78% and 81% of all firms in our sample, respectively. Panel A and B show scatter diagrams of capital misallocation for each five-digit industry in manufacturing and services, respectively. The grey dots show cap-

[4] The manufacturing sector is comprised of (NACE-industry two-digit codes in brackets): manufacturing (10–33) and construction (41–43). The services sector is comprised of: wholesale and retail trade; repair of motor vehicles and motorcycles (45–47), transportation and storage (49–53), accommodation and food services (55–56), information and communication (58–63), professional, scientific and technical activities (69–75) and administrative and support service (77–82).

[5] As pointed out by Restuccia and Rogerson (2017) the difference in the level of misallocation between the manufacturing and services sector might also be partly driven by measurement problems specific to the services sector. Defining and measuring quality change is perhaps the single most severe problem contributing to poorer measurement of real output in services.

ital misallocation measures in 2001 and 2017, expressed against the same misallocation measure for the total economy in 2001. Dots lying above/below the 45°-line indicate that capital misallocation has increased/decreased. The main take-away from Figure 4 is that capital misallocation increased in the period 2001–2017 for the vast majority of industries in both the manufacturing and services sector, i.e. most grey dots are above the 45°-line. We observe increases of capital misallocation in 90% and 93% of all five-digit industries in the manufacturing and services sectors, respectively. Figure 5 shows the same measures as plotted in Figure 4, but for labor misallocation instead of capital misallocation. In contrast to capital misallocation, the number of industries showing an increase or decrease in labor misallocation is approximately the same in the services sector. This implies that the industry-level heterogeneity in labor misallocation is much larger than capital misallocation.

[INSERT FIGURES 4 AND 5 ABOUT HERE]

Misallocation is also related to firm-size for various reasons. First, larger firms tend to be older than smaller firms, and may therefore be able to self-finance more easily via build up reserves than the smaller ones and, hence, be less exposed to financial constraints (Gopinath et al., 2017). Consequently, larger firms might be less vulnerable to misallocation caused by changing conditions in capital markets and bank-credit-standards. Second, the size of a firm might be a signal for good management practices and more efficient allocation of resources (see e.g. Calligaris, 2015). Recently, Gopinath et al. (2017) provided evidence that credit restrictions in a low-interest environment prompt large enterprises to invest too much and small enterprises to invest too little, because the latter face credit-restrictions, also in a low inflation environment.

[INSERT FIGURE 6 ABOUT HERE]

Figure 6 reports the evolution of the capital misallocation, according to firm-size. We categorize all firms as either micro, small, medium, or large based on the number of employees and annual balance sheet following the firm-size division of the European Commission.[6] During the period 2001–2012, capital misallocation increased for all business sizes. Moreover, there is a (strong) negative relation between the level of capital misallocation and the size of the firm. Averaged over the period 2001–2017 capital misallocation of micro

[6] Micro (small) enterprises have less than 10 (50) employees and an annual balance sheet total of below EUR 2 (10) million. Medium-sized (large) enterprises have less (more) than 250 employees and an annual balance sheet total of no more (less) than EUR 43 million. A firm's number of employees includes the firm's owner(s). This means that sole proprietors are classified as firms with one employee.

firms is around 24% higher than capital misallocation of large firms. Remarkably, this difference was much larger in the years leading up to the financial crisis of 2008–2009. In the period 2001–2007 the difference was 36%, but in the years 2008–2011 capital misallocation of large firms increased much faster than for small firms decreasing the gap in MRPK dispersion between micro and large firms to 15% averaged over the period 2008–2011. In 2017 the gap was "just" 10%.

Figure 7 reports the evolution of the labor misallocation, according to firm-size. The figure broadly resembles the Figure 6, but the convergence of the relatively low level of labor misallocation of large firms to the average level is much faster. Remarkably, the level of labor misallocation of large firms was even higher than the average level in the period 2010–2017.

[INSERT FIGURE 7 ABOUT HERE]

3 Misallocation in terms of the capital and labor wedge

As discussed in the previous section the within-industry dispersion of revenue productivity of firms is quite large. Moreover, there are sizable differences according to industry and firm-size. What the misallocation measures do not indicate is whether the distortion lead to a firm to use "too much" or "too little" capital and labor. However, in the Hsieh and Klenow framework this can be analyzed by estimating the absolute capital and labor "wedges", described in Section 2.1. A relatively high capital/labor wedge indicates a firm has "too little" capital/labor, whereas a relatively low capital/labor wedge indicates a firm has "too much" capital/labor. In this section we correlate the estimated wedges with firm characteristics to determine which type of firm has relatively high/low wedges.

3.1 Estimation results wedges total economy

Figure 8 shows the distribution of the estimated capital and labor wedges in our sample of firms. The figure reports the distribution of the logarithm of the absolute wedges on capital and labor, defined as $\ln(\tau_{is,t}^{K*}+1)$ and $\ln(\tau_{is,t}^{L*}+1)$ respectively. Both measures were described in Section 2.1. Noticeably, the distribution of the capital wedge has a much higher standard deviation than the labor wedge, which is in line with the relatively low standard deviation of labor misallocation presented in Section 2.2.

[INSERT FIGURE 8 ABOUT HERE]

Figure 9 reports the evolution of the capital and labor wedges. Panel A shows the distribution of the capital wedge in 2001, 2008 and 2017. The distribution of the capital wedge has become much more compressed over time. Among other things, the right tail of the distribution has become thicker, i.e. the percentage of firms with a relative high capital wedge has increased in our sample period. Panel B of Figure 9 reports the labor wedge in 2001, 2008 and 2017. In contrast to the capital wedge, the shape of the labor wedge distribution has changed very little over time. Interestingly, the distribution of the labor wedge has shifted to the right, indicating that firms on average faced somewhat higher labor distortions in the later years of our sample.

[INSERT FIGURE 9 ABOUT HERE]

As emphasized in Restuccia and Rogerson (2017) and Hsieh and Klenow (2009), distortions would be particularly costly if they are positively correlated with a firm's physical productivity. To investigate this, we examine the relation between the capital and labor wedges on the one hand and the firm's factor inputs (K, L) and TFPQ on the other hand by calculating the correlations between these variables. We denote deviations of variables from their industry means as, $\widetilde{TFPQ}_{is,t}$, $\tilde{\tau}^{K*}_{is,t}$, $\tilde{\tau}^{L*}_{is,t}$, $\widetilde{L}_{is,t}$ and $\widetilde{K}_{is,t}$, respectively.

The correlation matrix, shown in Table I, provides a number of interesting insights. First, firms with a higher TFPQ-level face higher "taxes" on both capital and labor. The correlation between the capital wedge and total factor productivity is 0.49, while between the labor wedge and total factor productivity is only 0.21. Second, the negative effect of the labor distortions on the level of total factor productivity is lower than for the capital distortion, because the correlation between labor input and total factor productivity is relatively high. This is different for the firm's capital input, which seems to be much more distorted. Third, the correlation between capital input and total factor productivity is very modest, signaling that firms with a high level of total factor productivity are unable to attract enough capital. Finally, the labor and capital wedges are very weakly correlated in our sample. In line with the outcomes for for capital and labor misallocation presented in the Section 2.2, we can conclude from these simple correlations that the capital wedge seems to be the most important factor behind the total factor productivity loss reported in Figure 2.

[INSERT TABLE I ABOUT HERE]

3.2 Capital wedges per firm-characteristic

The high correlation between the capital wedge and total factor productivity warrants further analysis. To establish what type of firm is especially influenced by the high capital wedges, we estimated an ordered probit-regression of the capital wedge on a set of firm-characteristics. The dependent variable is the capital wedge split up in five categories.[7] Each category contains 20 percent of all capital wedges ordered from low to high, based on the distribution of the capital wedge in the period 2001–2017. We label firms that fall into the first two deciles of the capital wedge distribution as "very low", the third and fourth decile as "low", the fifth and sixth decile as "average", the seventh and eight decile as "high" and the ninth and tenth decile as "very high". As regressors we included firm-size, industry, year fixed-effects and the position in the firm's productivity distribution. The inclusion of the latter variable was induced by recent research of, among others, Andrews et al. (2016), Andrews et al. (2018) and van Heuvelen et al. (2018). Using cross-country firm-level data, Andrews et al. (2016) reveal an increasing productivity-gap between so-called "frontier" and "laggard" firms. For each year they define" frontier" firms as the top 5% of firms in terms of labor productivity levels within each industry. All other firms are defined as "laggards". We deviate somewhat from this definition and define three groups of firms, i.e firms at the national frontier ("frontier" firms), firms that have an average productivity level ("average" firms), and firms at the low-end of the productivity distribution ("laggard" firms). Frontier, average and laggard firms are defined as firms being in the 10th decile, the 2nd up to the 9th decile, and 1st decile of the TFPQ-distribution within each two-digit industry, respectively. We determine these groups for each year separately.

Figure 10 shows the estimated marginal effects of the ordered probit-regression, with the absolute capital wedge as dependent variable. All reported marginal effects are calculated by varying one of the following variables, i.e. firm-size, productivity-level (frontier, average, laggards) or sample year, while keeping all other covariates at their sample mean.

[INSERT FIGURE 10 ABOUT HERE]

Panel A reports the marginal effects for firm-size. The figure shows that the capital wedges are unevenly distributed across firm-size. Large firms have a high probability (53%) facing "very low" capital wedges, while the opposite holds for small and, especially, micro firms. The probability of facing a "very low" capital wedge is 23% for small firms

[7] All model-estimates, marginal effects and expected values of the models presented in this section are available in the online appendix, here.

and only 12% for micro firms. The probability of facing a high capital wedges is inversely related to firm-size too. According to our estimation results, large firms face a 2% probability of facing "very high" capital wedges, whilst micro firms face a 21% probability of "very high" capital wedges instead.

Panel B reports the marginal effect of the firms position in the TFPQ distribution. Interestingly, laggard firms have the highest probability of facing a relatively "very low" capital wedge (61%), whilst firms at the productivity frontier face a 53% probability of facing a relatively "very high" capital wedge. From a policy perspective this is a disturbing outcome, because it indicates that low productivity firms have a "subsidy" on capital inducing them to produce more than would be optimal. Moreover, highly productive firms face a "tax" on capital inducing them to produce less than optimal, thereby reducing economy-wide TFPQ. Panel C indicates that in the period 2001–2017 (very) high capital wedges have increased and (very) low capital wedges have decreased.

To get a better understanding of the interaction between TFPQ-level, firm-size and the evolution of the capital wedge, we estimated a separate ordered probit-model where the dummy variables on firm-size, position in the TFPQ-distribution and year are interacted. This specification enables enough flexibility for possible time-dependent relations between the capital wedge on the one hand and the firm's position in the TFPQ-distribution, firm-size and survey year on the other hand.[8] Figure 11 shows the expected capital wedge from this exercise. The expected values vary between -2 (very low) to +2 (very high).[9] The reported expected values in panels A, B and C are calculated by varying firm-size and sample year and setting the firm's TFPQ level in the laggards, average and frontier category respectively. For readability we used a color-code to indicate the size of the expected capital wedge. A "very low" capital wedge is color-coded white and a "very high" is color-coded black.

The expected values in Figure 11 show a number of interesting results. First, the (slight) increase in capital wedges, which we observed in Figure 10, seem to be largely concentrated among frontier firms, as can be seen from the darker colors in 2017 in this category at the end of our sample period. Second, micro firms face the highest capital wedges, which holds for laggards, average and frontier firms. According to our estimation

[8] We included a total of 201 dummy variables in this specification, i.e. we constructed a dummy for all possible interactions of firm-size, position in the TFPQ-distribution and year, i.e. 132 (triple and double) interaction dummy variables, 2 dummy variables for the position in the TFPQ-distribution, 2 size-class dummy variables, 16 year dummy variables and 49 sector dummy variables.

[9] The expected capital wedge is calculated as -2*P(very low capital wedge)-1*P(low capital wedge)+1*P(high capital wedge)+2*P(very high capital wedge). We combined medium and large firms in one group, because we had problems with convergence of the model if we specified both medium and large firms separately, most probably because of the relatively small number of large firms in our sample.

results, in 2017, micro firms at the productivity frontier had a 48% probability of facing a very high capital wedge, versus 26% for medium and large firms. Micro firms that were classified as laggards faced a 2% probability of a very high capital wedge, versus 0.1% for medium and large firms.

[INSERT FIGURE 11 ABOUT HERE]

3.3 Labor wedges per firm-characteristic

We estimated similar probit models for the labor wedge.[10] The estimation results are reported in Figure 12 and Figure 13.

[INSERT FIGURE 12 AND 13 ABOUT HERE]

Overall, Figure 12 and Figure 13 show the same picture as the outcomes for the capital wedge, i.e. smaller firms face higher labor wedges, firms at the frontier face low wedges and laggard firms face low labor wedges. There are two main differences between the probit-outcomes for the capital and labor wedges, however. First, in contrast to the large differences in capital wedges according to firm-size and the firm's position in the TFPQ-distribution, the differences in labor wedges are (much) smaller. Second, the evolution of relatively high and low wedges is much more dynamic than the capital wedge, i.e. the decrease/increase in "very low"/"very high" labor wedges is much larger compared to capital wedges. This evolution is largely disguised in the evolution of the average labor wedge, because the shifts occurred in the top and bottom deciles of the distribution. The decrease/increase in "very low"/"very high" labor wedges seems to be relatively evenly spread over firm-size and the firm's position in the TFPQ-distribution as can be seen from the darker colors moving from 2001 to 2017 in Panel A through C in Figure 13.

These results imply that the loss in TFPQ compared to an efficient allocation of capital and labor can largely be attributed to a combination of Dutch highly productive (frontier) firms facing relatively high capital wedges and Dutch low productive (laggard) firms facing relatively low wedges. The effect of firm-size class is interesting as well, because the high wedges are concentrated among micro firms at the productivity frontier, while the low wedges are concentrated among large firms in the lowest decile of the TFPQ distribution (laggards). Furthermore, the clustering of high and low wedges by size class and level of TFPQ increased over time.

[10] All model-estimates, marginal effects and expected values of the models presented in this section are available in the online appendix, here.

4 Trends and persistence in misallocation

Section 2 indicates that capital misallocation is trending upward over time and that there can be substantial differences in capital and labor misallocation depending on firm-characteristics. A related question is for what period the observed firm-specific differences in capital and labor misallocation persist. To analyze the trends and persistence of capital and labor misallocation we use panel data specifications from the literature on individual earnings dynamics, see e.g. Ng (2008), Guvenen (2009), Moffitt and Gottschalk (2011), Browning and Ejrnæs (2013) and Doris et al. (2013). The basic idea is that misallocation can be modeled with an error component model, which allows identification of the permanent and transitory components of the observed capital and labor misallocation.

4.1 Permanent and transitory misallocation

To identify the separate contributions of the permanent and transitory component of capital and labor misallocation we use the following model:

$$y_{i,t} = y_{i,t}^{P} + y_{i,t}^{T}, \tag{10}$$

where $y_{i,t}$ is either log(MRPK) or log(MRPL) of firm i in year t. The first and second term in (10) are the permanent and transitory components, respectively. There is a large literature on the precise formulation of such a decomposition and many specifications are possible. For example, the basic random effects model assumes an i.i.d. time-invariant permanent component ($y_{i,t}^{P} = \eta_i$) independent from an i.i.d. transitory component. Extensions allow for (1) a random walk in the permanent component; (2) a stationary ARMA-process for the transitory component; (3) time series heteroscedasticity or factor-loadings for both components; (4) heterogeneity in model parameters across individuals. We note that, irrespective of the chosen specification, there is inevitably some degree of arbitrariness in distinguishing permanent and transitory components.

We use the modeling framework of Doris et al. (2013) and estimate:

$$y_{i,t} = p_t \eta_i + \lambda_t v_{i,t}, \tag{11}$$

where $p_t \eta_i$ and $\lambda_t v_{i,t}$ are defined as the permanent ($y_{i,t}^{P}$) and transitory error component ($y_{i,t}^{T}$), respectively. The factor-loadings p_t and λ_t allow the variances of the permanent and transitory misallocation to change over time. Furthermore, $v_{i,t}$ is modeled as a stationary

AR(1) process to account for persistence in transitory shocks:[11]

$$v_{i,t} = \rho v_{i,t-1} + \varepsilon_{i,t}. \qquad (12)$$

Finally, regarding the initial observation it is assumed that $Var(v_{i1}) = \sigma^2_{v1}$. It is easily seen (Doris et al., 2013) that the cross-sectional variance implied by the model (11)–(12) is equal to:

$$\begin{aligned} V_{t,\infty} &= p_t^2\sigma_\eta^2 + \lambda_t^2\sigma_{v1}^2, \quad t = 1, \\ V_{t,\infty} &= p_t^2\sigma_\eta^2 + \lambda_t^2(\rho^{2t-2}\sigma_{v1}^2 + \sigma_\varepsilon^2\sum_{w=0}^{t-2}\rho^{2w}), \quad t > 1. \end{aligned} \qquad (13)$$

The permanent component of the cross-sectional variance at time t is given by the first term $p_t^2\sigma_\eta^2$, while the second term is the transitory component. In case of covariance stationarity we have that $p_t = \lambda_t = 1$ and $\sigma^2_{v1} = \sigma^2_\varepsilon/(1-\rho^2)$. Any deviation from these values will cause the cross-sectional variance to change over time. Note that p_t and λ_t are unknown parameters, hence can be trending over time. Note also that when $-1 < \rho < 1$ the impact of a non-stationary initial observation will gradually vanish over time.

We estimate the model by using the Generalized Method of Moments (GMM) estimator of Doris et al. (2013). The parameters are ρ, σ^2_ε, σ^2_η, σ^2_{v1}, $p_1 - p_T$ and $\lambda_1 - \lambda_T$. The coefficient estimates are then used to predict the portion of variance due to the permanent and transitory components, e.g. the former is estimated by $\hat{p}_t^2\hat{\sigma}_\eta^2$. Note that the pattern of p_t fully determines the trend in the permanent component. Regarding the transitory component more parameters are involved, hence there is no one-to-one mapping from parameters to cross-sectional variance.[12]

Figure 14 reports the evolution of the permanent and transitory variances for capital and labor misallocation. The figure shows some interesting results. First, the permanent component of capital misallocation (Panel A) strongly increased over our sample period, whilst the transitory component was approximately the same size in 2017 as in 2001. From 2008 onwards, the permanent capital misallocation is larger than the transitory misallocation. Second, the relative size and evolution of the permanent and transitory component of capital an labor misallocation are quite different. In contrast to the evolution of capital misallocation, the transitory labor misallocation has been larger than the permanent component over our whole sample period. Third, the transitory nature of labor misallocation

[11] We experimented with adding a random walk element to the permanent component and higher-order autoregressive or moving-average processes in $v_{i,t}$, but these components were generally not significant.

[12] All model-estimates and predicted permanent and transitory components presented in this section are available in the online appendix, available here.

has been on a steady increase since 2009. The relative stability of labor misallocation over the whole sample period can therefore be largely ascribed to a simultaneous increase and decrease in the transitory and permanent component, respectively. Overall, these outcomes suggest that the Dutch firms have become more adaptive to changing labor demand and supply conditions, i.e. quite effective in removing distortions. The opposite is true for firm's acquisition of capital, where the permanent component of misallocation has increased by much more than the transitory component.[13]

[INSERT FIGURE 14 ABOUT HERE]

4.2 Variance decomposition per firm-characteristic

To determine the link between the permanent and transitory components of capital and labor misallocation on the one hand and firm-level characteristics on the other hand, we estimated the model (11)–(12) for various sub-samples, as defined in earlier sections. Figure 15 shows the evolution of the permanent and transitory components of capital misallocation for the manufacturing and services sectors. Interestingly, at the end of our sample capital misallocation has a more permanent character in both the manufacturing and services sectors. Since 2009 the transitory component and permanent components of capital misallocation are of comparable size in the manufacturing sector, whilst the permanent component of capital misallocation in the services sector has been larger than the transitory component since 2006, albeit the difference has shrunk since 2013. These results imply that the services sector not only has a higher level of capital misallocation (see Figure 3), but the misallocation has become also more permanent in nature.

[INSERT FIGURES 15 ABOUT HERE]

Figure 16 shows the evolution of the permanent and transitory components of labor misallocation for the manufacturing and services sector. In contrast to capital misallocation, the evolution of the permanent and temporary components in both sectors is roughly the same, i.e. an increase of the temporary component after 2009 and a stabilization (manufacturing sector) or small decline (services sector) in the permanent component. Another take-away is that the convergence of the level of labor misallocation in the manufacturing sector to the level of labor misallocation in the services sector, documented in Figure 3,

[13] The online appendix, available here, also includes a decomposition of the capital and labor wedges. Generally, the outcomes are quite comparable to the outcomes for capital and labor misallocation, so we do not discuss them here, in order to keep the main text contained.

can be attributed to the combination of a strong increase in the transitory component of labor misallocation in the manufacturing sector and a decline of the permanent component in the services sector. Overall, labor misallocation has gotten a less permanent character in both the manufacturing and services sector.

[INSERT FIGURES 16 ABOUT HERE]

Figures 17 and 18 show the evolution of the permanent and transitory components of misallocation for different firm-sizes. For all firm-sizes the permanent component of capital misallocation has increased and surpassed the size of the transitory component after 2001. Interestingly, the negative relation between the level of capital misallocation and firm-size documented earlier (see Figure 6) seems to be mainly caused by differences in the permanent component. This means that capital misallocation is most persistent for micro firms. The evolution of the permanent and transitory component for large firms is quite erratic, possibly caused by less precise GMM estimation due to the relatively small number of firms in this firm-size category.

[INSERT FIGURES 17 AND 18 ABOUT HERE]

Finally, Figures 19 and 20 document the difference in persistence of capital and labor misallocation depending on the firm's position in the productivity distribution. For all three productivity groups (frontier, average, laggards) the permanent component of capital misallocation has increased over the period 2001–2017. The increased permanent character of capital misallocation is most for frontier firms. This indicates that the relatively high capital wedges for frontier firms that we estimated in Section 3.2 have become more persistent over time. Turning to the persistence of labor misallocation according to the firm's position in the productivity distribution we note that the evolution of the permanent and transitory components are comparable across productivity level, but the bulk of labor misallocation is concentrated among the frontier firms.

[INSERT FIGURES 19 AND 20 ABOUT HERE]

4.3 Validation by Monte Carlo simulation

To determine if the error components model (11)–(12) is capable of reproducing the pattern of the estimation results in Figure 1 and also the other figures we run a Monte Carlo experiment. We use the estimated model to generate artificial MRPK and MRPL distributions and analyze if the resulting time series pattern of the simulated cross-sectional variances coincides with the empirical cross-sectional variances. The outcome of the Monte Carlo experiments are also used to evaluate the accuracy of the GMM estimator in finite samples.

We generate data according to equations (11)–(12). For the error components we assume η_i i.i.n.$(0, \sigma_\eta^2)$ independent from ε_{it} i.i.n.$(0, \sigma_\varepsilon^2)$. Regarding the initial observations we assume v_{i1} i.i.n.$(0, \sigma_{v1}^2)$ independent from the error components. For the parameters ρ, σ_ε^2, σ_η^2, σ_{v1}^2, $p_1 - p_T$ and $\lambda_1 - \lambda_T$ we choose the GMM estimates underlying Figure 14. Finally, we choose $T = 17$ and $N = 10,000$. The cross-sectional dimension is different from the sample size in the empirical analysis, but we get similar results for different N. Other differences with the empirical analysis are that in the simulations we use balanced panel data and calculate unweighted cross-sectional variances.

For both MRPK and MRPL we create 1,000 data sets and calculate the time series of cross-sectional variances for each replication. The mean of these 1,000 replications is then compared to the empirical cross-sectional variances. Figure 21 shows the empirical MRPK and MRPL variances (normalized at 1 in base year 2001), the average cross-sectional variances of the Monte Carlo replications and the 99%-confidence interval. Panel A shows that the simulated cross-sectional variances implied by the error component model are capable of reproducing the upward trend in the MRPK dispersion. The error components model seems flexible enough to describe the sometimes irregular development, e.g. in the crisis years 2007–2009, of the empirical variances. Panel B shows the simulation results for MRPL, which are also close to the observed empirical measure of labor misallocation. Given the differences between simulation design and empirical estimates, we conclude that the estimated error components model provides an accurate description of the evolution of MRPK and MRPL dispersion.

[INSERT FIGURE 21 ABOUT HERE]

5 Sensitivity analysis

Recently, several studies have raised concern about the Hsieh and Klenow model. A first source of concern is that capital might be subject to adjustment costs in investment ("time-to-build"), which can lead to a higher dispersion simply due to technology-driven adjustment processes, which in itself are not inefficient (see e.g. Cooper and Haltiwanger, 2006; Asker et al., 2014; David and Venkateswaran, 2019). The Hsieh and Klenow (2009) model neglects this distinction between technology-driven adjustment costs, such as the natural time needed to build a new plant, and wasteful frictions, such as the bureaucratic procedures of authorization that may delay the construction and activation of a new plant. We estimate the importance of these non-wasteful adjustment costs for our measurement in Section 5.1. Another concern is the functional form. Recently Haltiwanger et al. (2018) showed that the model is only valid under quite strict assumptions which generally do not hold in reality. In order to investigate the impact of the functional form we estimate the effect of allowing for firm-specific labor and capital shares, and a different functional form of the production function (i.e. CES instead of a CD functional form) in Section 5.2.[14] Finally, in Section 5.3 we analyze the impact of heterogeneous markups using methodology developed by De Loecker and Warzynski (2012).

5.1 Adjustment costs of capital

In order to explore whether adjustment costs are a significant driver of our measurement of misallocation we use the methodology of David and Venkateswaran (2019). The model is an extension of the Hsieh and Klenow (2009) framework to include dynamic considerations in firm's investment decisions. Furthermore, in the model a number of forces can contribute to the observed dispersion in MRPK and MRPL, i.e.: (i) capital adjustment costs, (ii) informational frictions, in the form of imperfect knowledge about firm-level fundamentals and (iii) firm-specific factors, meant to capture all other forces influencing investment decisions, such as unobserved heterogeneity in markups and/or production technologies, financial frictions or institutional distortions. Appendix C.1 presents the main model equations. David and Venkateswaran (2019) model capital adjustment costs as a quadratic function:

$$\Phi\left(K_{is,t+1}, K_{is,t}\right) = \frac{\hat{\xi}}{2}\left(\frac{K_{is,t+1}}{K_{is,t}} - (1-\delta)\right)^2 K_{is,t}, \tag{14}$$

where δ is the depreciation rate.[15] The coefficient $\hat{\xi}$ determines the slope of the marginal adjustment costs. For example, if $\delta = 0.10$ and capital is doubled form time t tot $t+1$ ($K_{is,t+1} = 2K_{is,t}$), $\hat{\xi} = 1$ implies that the adjustment costs are 60.5% of the investment ($\Delta K_{is,t+1}$). There is a rather large variation in estimates of $\hat{\xi}$ in the literature. Investment-regressions, derived from the Q-theory of investment usually find values for $\hat{\xi}$ (> 10), see

[14] Another concern with our quantification relates to measurement error in firms' revenues and inputs. As Bils et al. (2018) point out, mismeasurement distorts our analysis because a firm's TFPR is higher when revenues are overstated and/or inpts are understated, the dispersion of measured TFPR is unequivocally biased upward. In our sample this form of mismeasurement is likely not a main concern, since we use tax-data. Though self reported by the firm, these data are thoroughly checked by the tax authorities.

[15] More general specification that also allow non-convex elements can be found in e.g. Cooper and Haltiwanger (2006), Bloom (2009) and Asker et al. (2014).

for instance Hayashi and Inoue (1991). Estimates based on the method of moments are usually much lower, e.g. between 0.8 and 1.6 in Eberly et al. (2008) or close to zero in Cooper and Haltiwanger (2006) and Bloom (2009). This could be partly driven by different country samples. Asker et al. (2014) show that there is quite a large difference in the estimated $\hat{\xi}$ for the US (4.4) on the one hand and France (0.1) on the other.

The relevant model parameters to determine $\hat{\xi}$ can be estimated from the following dynamic model of the capital stock:

$$k_{is,t} = \pi_1 k_{is,t-1} + \pi_2 a_{is,t-1} + \eta_i + u_{is,t}, \tag{15}$$

where $k_{is,t}$ and $a_{is,t}$ are the log of capital $k_{is,t}$ and $TFPQ_{is,t}$, respectively. Moreover, π_1 measures capital adjustment costs, η_i measures firm-specific factors influencing investment decisions and the disturbance term $u_{is,t}$ captures all other possible frictions. In the hypothetical case of no adjustment costs of capital $\pi_1 = 0$ and when there are adjustment costs $\pi_1 > 0$. In the limit, when $\hat{\xi} \to \infty$ it holds that $\pi_1 \to 1$. The relevance of adjustment costs is straightforward to test with the null-hypothesis $H_0 : \pi_1 = 0$ versus the alternative hypothesis $H_1 : \pi_1 > 0$.

We estimate the dynamic panel data model in equation (15) directly using the fixed effects OLS estimator.[16] The estimate for π_1 is 0.62 and the standard error is 0.002, which implies a value of $\hat{\xi} \approx 0.4$. The estimate indicates adjustment costs are a relevant source of misallocation, although the coefficient of 0.4 is on the low-end of the estimations found in the literature. Based on the estimated coefficients David and Venkateswaran (2019) derive a measurement of the empirical relevance of adjustment costs. First, they assume that adjustment costs are the sole source of between-firm variation in MRPK. In that special case, equation (15) reduces to:

$$k_{is,t} = \pi_1 k_{is,t-1} + \pi_2 a_{is,t-1}, \tag{16}$$

where $a_{is,t}$ is assumed to follow a stationary AR(1) process is. We can use equation (16) to determine the variance of MRPK (σ^2_{MRPK}) when adjustment costs are the only source of misallocation. Appendix C.1 presents the formal derivation of σ^2_{MRPK} in this special case. Using our estimate of π_1 we find $\sigma^2_{MRPK} = 0.16$. This indicated that approximately 5% of the measured total MRPK variance –2.97 in our sample– can be attributed to adjustment costs. We therefore conclude that capital adjustment costs are not a sizable source of MRPK dispersion in our sample. This outcome is in line with our previous outcome on the persistent nature of capital misallocation as documented in Section 4.

[16] Our estimation method is a simplification of the method of moments estimator of David and Venkateswaran (2019), who use a set of non-linear moment conditions to estimate the parameters. We are interested only in the adjustment costs parameter, hence the linear model in equation (15).

5.2 Alternative functional form

Another concern of the Hsieh and Klenow approach is the strict assumptions on the functional form of the production function, i.e. a CD-production function with firm-invariant capital elasticities and a substitution elasticity of 1 between labor and capital. We relax these assumptions one-by-one in Section 5.2.1 and Section 5.2.2. Our main finding is that only a small fraction of the observed misallocation can be attributed to one of these factors.

5.2.1 Firm-level heterogeneity in the production function

In the standard Hsieh and Klenow model the specified CD-production function has industry varying labor and capital shares, but these shares are constant within an industry, as can be seen from equation (3). In other words, the capital and labor elasticities are assumed to be equal for all firms within a five-digit industry. It follows from the model equations (1)–(8) that *all* firm-specific variation in labor or capital-elasticities automatically lead to an increase in misallocation. We are able to relax this strong assumption, because we are able to calculate firm-specific capital shares based on the data. In Appendix C.2 we derive an expression for the dispersion in (the log of) MRPK where misallocation is solely caused by differences in firm-specific capital elasticities. By comparing this measure with the observed dispersion in (the log of) MRPK in the data we can get a sense of the impact on our results of the assumption of equal capital shares within an industry. We find that the overall impact is rather small. The total variance of MRPK in the data is 2.97, whilst our alternative firm-specific variation is only 0.03. This implies that only approximately 1% of the observed variance of the (log of) MRPK can be attributed to differences in firm-specific capital elasticities.

5.2.2 Constant Elasticity of Substitution production function

Another possible explanation for the observed misallocation misallocation could be that the assumption of unity substitution in the CD-production function in equation (3) is invalid. An alternative –often used– functional form is the CES-production function, where the substitution-elasticity between capital and labor is freely estimated and not a priori equalized to unity, i.e.:

$$Y_{is,t} = A_{is,t} \left(\alpha_s K_{is,t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha_s) \left(L_{is,t}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{17}$$

where σ is the substitution elasticity between capital and labor. The CES-production function reduces to the Hsieh and Klenow (2009) imposed CD-production function (3) when $\sigma = 1$. MRPK is defined as $P_{is,t}\frac{\partial Y}{\partial K}$, the product of marginal revenue ($P_{is,t}$) and marginal product ($\frac{\partial Y}{\partial K}$). In Appendix C.3 we derive that, in the absence of wedges $\tau^Y_{is,t}$ and $\tau^K_{is,t}$, in this case the MRPK can be written as:

$$MRPK_{is,t} = \alpha_s^{1-\sigma} R_t^{\sigma} \left(\alpha_s^{\sigma} R_t^{1-\sigma} + (1-\alpha_s)^{\sigma} w_t^{1-\sigma} \right), \tag{18}$$

Notice that when $\sigma = 1$ (CD-production function), the expression reduces to $MRPK = R$. In the standard Hsieh and Klenow model the only source of variation in MRPK are the firm-specific wedges $\tau^Y_{is,t}$ and $\tau^K_{is,t}$. In the case of a CES-production-function ($\sigma \neq 1$) this is not any different since the parameters α and σ are not firm-specific, but only vary between industries.

We also considered a CES-production function with labor-augmenting technological process. In Appendix C.3 we derive that labor-augmenting technology can only have impact on our measure of misallocation if it varies between firms within the same five-digit industry. In policy discussions a concern is the negative impact of industrial robots on employment growth (Acemoglu and Restrepo, 2017), which is an example of a labor saving technology. Only if the use of robots has unequal effects on employment growth across firms within an industry, the misallocation estimation based on a CES-production function will lead to different results compared to the base model of Hsieh and Klenow (2009). We leave it to future research to determine how relevant the variation in the adaption of such technologies varies between firms in the same industry. We conclude that, relaxing the (strong) assumption of a unity substitution elasticity between capital and labor does not alter our estimate of misallocation.

5.3 Heterogeneous markups

In the standard Hsieh and Klenow (2009) model all firms within an industry have identical technologies and demand structure. Therefore, markups are identical as well. All dispersion in marginal revenue products is therefore attributed to misallocation, while heterogeneous markups could explain part of the observed MRPK and MRPL variance. To measure dispersion in markups we follow the approach of David and Venkateswaran (2019), which is based on the methodology of De Loecker and Warzynski (2012). We generalize the CD-production function (3) with intermediate inputs $M_{is,t}$, which have an industry specific elasticity ζ_s constant over time. Cost minimization then implies the following optimality condition (David and Venkateswaran, 2019):

$$\frac{P^M_{is,t} M_{is,t}}{P_{is,t} Y_{is,t}} = (1-\zeta_s)\frac{MC_{is,t}}{P_{is,t}}, \tag{19}$$

where $MC_{is,t}$ are (unobserved) marginal costs of the firm. From this equation it is seen that the dispersion in the log share of intermediate inputs can be used as an estimator of the heterogeneity in log markups. The materials share, i.e. the left hand side of (19), is directly observed in the data. Using this estimator we find that heterogeneous markups explain around 25% of the variation in MRPK.

5.4 Summary

Overall, our sensitivity analysis indicates that the factors raised in recent critique on the misallocation measure of Hsieh and Klenow (2009) seem to have a limited impact on the size of the misallocation loss we measured for the Netherlands. Capital adjustment costs lower our misallocation loss with 5 percentage points, firm-level heterogeneity in the production function with 1 percentage point, whilst the functional form has a negligible effect. The most sizable effect on our measure stems from heterogeneity in markups (25%).

Taken together these factors –not taking into account inter-dependencies of the factors– would lower our measured level of misallocation by roughly 31%, which would lower the estimated efficiency loss from 43% to 30% percent in 2001, and from 56% to 39% in 2017.[17] We have been mindful of the effect of these factors on the *level* of misallocation in the previous sections by focusing on the *differences* in misallocation over time, i.e. we normalized on the level of misallocation in the first year of our sample. The idea is that, following Hsieh and Klenow (2009), some "base level" misallocation can be understood as the result of misallocation originating from misspecification, and that a reasonable, although admittedly debatable, starting point is to assume that this level is constant over time (Restuccia and Rogerson, 2017).

6 Conclusion

Misallocation of capital has been on the rise since the turn of the millennium in several European countries (see e.g. Gopinath et al., 2017, Gamberoni et al., 2016 and Calligaris et al., 2018). Our analysis focuses on the evolution of resource misallocation in the Netherlands. We use a very rich data set containing annual balance sheets as well as profit and loss statements for all Dutch firms that had to declare corporate income tax during the period 2001–2017. Our results shed new light on the evolution of capital and

[17] We assumed adjustment costs, firm-level heterogeneity in the production function and the dispersion of the markups remained constant in the period 2001–2017. Unreported results show that the dispersion of markups, which is by far the largest contributing factor, has indeed remained constant in the period 2001–2017. Detailed results are available upon request with the authors.

labor misallocation and underlines the importance of looking at the level and persistence of misallocation at the firm-level.

Using conventional measures of misallocation, i.e. the dispersion in firm-level marginal revenue products, we find a combination of steeply rising capital misallocation and relatively stable labor misallocation in the period 2001–2017, indicating that capital frictions account for most of the increase in measured misallocation in the Netherlands. Compared to a counterfactual efficient allocation we find that total misallocation has increased 14 percentage points in the period 2001–2017. The level of capital and labor misallocation is much larger in the services sector than the manufacturing sector. This result might be driven by less competitive pressures in the services sector compared to the manufacturing sector. The misallocation of capital and labor are inversely related to firm-size, i.e. misallocation is smaller among large firms than under small firms.

The standard misallocation measures are silent about the magnitude of distortions across firms. We therefore use the analytical framework to estimate capital and labor wedges at the level of the firm. We find that small, highly productive firms face the highest capital and labor wedges, whilst large, unproductive firms face relatively low distortions. From a policy perspective this is a disturbing outcome, because it implies that in the Netherlands, highly productive firms tend to be too small compared to the optimal size and non-productive firms are too large compared to their optimal size.

We have exploited panel data specifications to analyze the persistence of misallocation over time. Using an error components model we distinguish between permanent and transitory components of misallocation. We find that the observed increase in capital misallocation is permanent rather than transitory. The majority of the observed labor misallocation is transitory in nature. In other words, the persistence of labor misallocation is lower than capital misallocation and, on average, shocks in the allocation of labor die out in a couple of years.

Finally, our findings are quite insensitive to recently raised concerns with the model of Hsieh and Klenow (2009). We show that in our sample the measurement of misallocation is largely insensitive to observed heterogeneity in the production function and to the presence of capital adjustment costs. Although the contribution of heterogeneous markups is non-negligible, we conclude that the majority of the observed dispersion in marginal revenue products can be attributed to misallocation.

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A Data set

Our data set contains the complete population of Dutch firms that legally had to declare corporate income tax during the period 2001–2017 (i.e., all firms with a legal person). These confidential micro data are provided by Statistics Netherlands (CBS: Centraal bureau voor de statistiek) and are based on a merger of the Dutch general business register (ABR: Algemeen bedrijven register) and corporate tax declarations (NFO: Financiën van niet-financiële ondernemingen). The matched data set includes annual balance sheets as well as profit and loss statements. We restrict our sample to non-agricultural private firms in the non-financial sectors. For multi-establishment firms we take the five-digit industry code (NACE Rev. 2) of the establishment with the largest number of employees. The raw data set contains 2,752,359 firm-year observations, and 491,313 unique firms for the period we analyzed (2001–2017).

The original data come as a repeated cross-section, with unique identifiers for the firm, and can be viewed as highly unbalanced panel data. We clean our data set for outliers and minimum number of observation per industry following previous research of e.g. Gamberoni et al. (2016) and Gopinath et al. (2017) by taking the following steps. First, we drop firm-year observations for which no fixed tangible assets are available. These steps reduce our sample to 2,305,977 firm-year observations and 396,235 unique firms. Second, we drop observation when the ratio of tangible assets to the balance sheet total is greater than one. This step reduces the sample by 915 observations. Next, we drop firm-year observations where the ratio of the wage bill to value added is in the top/bottom 1% of the distribution. This step reduces the sample by 42,798 observations. In addition, following Gopinath et al. (2017), we drop firm-year observations if the wage bill to value added ratio is larger than 1.1 or smaller than 0.1. This step reduces the sample by 265,133 and 55,226 observations, respectively. Finally, we drop industries where the minimum number of yearly observations or the average number of observations over the whole sample is less than 30. This step reduces our sample by 110,330 observations. After cleaning, our final data set contains 1,831,575 firm-year observations and 342,245 unique firms.

B Figures and tables main text

Figure 1: Evolution of MRPK dispersion and MRPL dispersion, 2001–2017, the Netherlands.

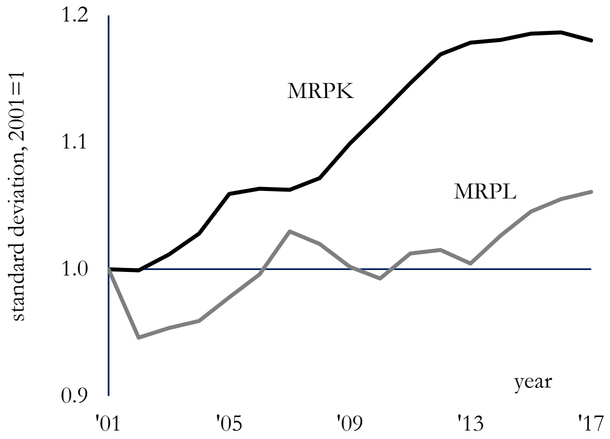


Figure 2: Evolution of misallocation in the Netherlands, 2001–2017.

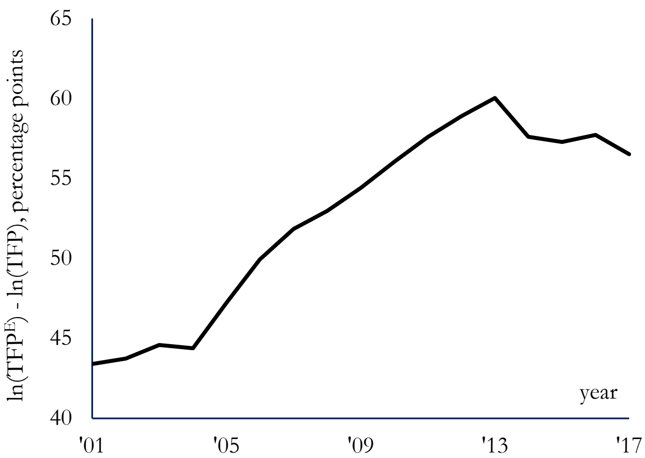


Figure 3: Evolution of MRPK and MRPL dispersion in the manufacturing sector and the services sector, 2001–2017, the Netherlands.

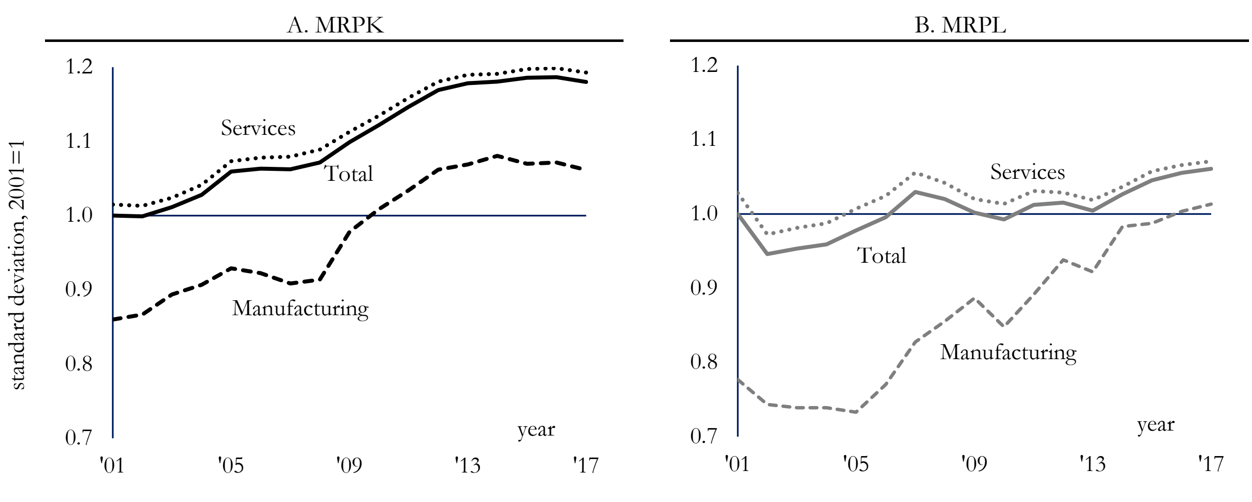
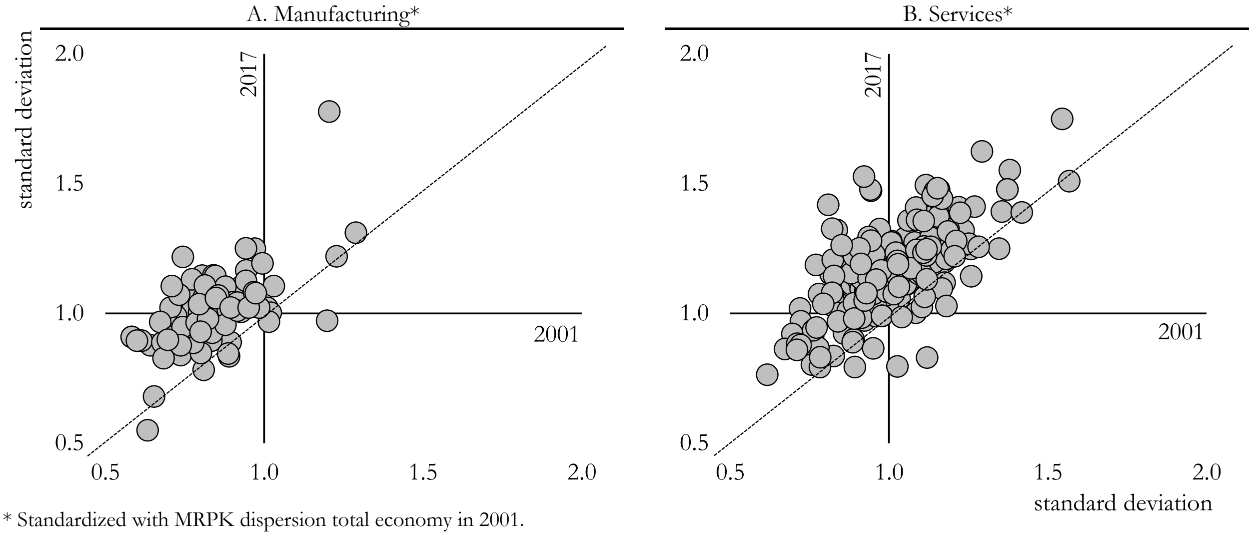
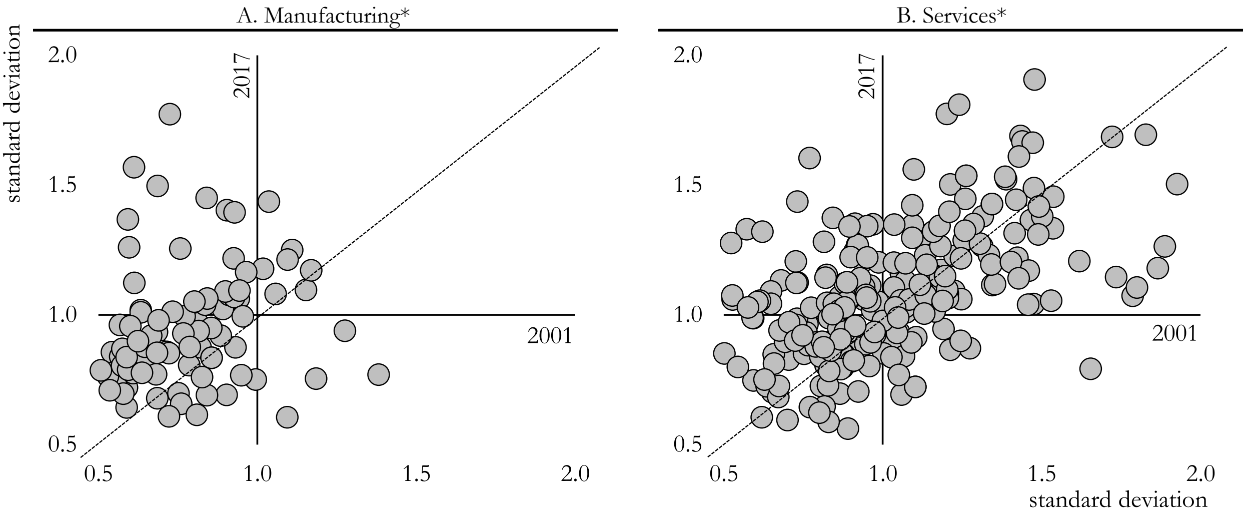


Figure 4: Evolution of MRPK dispersion in the manufacturing sector and services sector, 2001 vs. 2017, NACE five-digit level, the Netherlands.



* Standardized with MRPK dispersion total economy in 2001.

Figure 5: Evolution of MRPL dispersion in the manufacturing sector and services sector, 2001 vs. 2017, NACE five-digit level, the Netherlands.



* Standardized with MRPK dispersion total economy in 2001.

Figure 6: Firm-size and the evolution of MRPK dispersion, 2001–2017, the Netherlands.

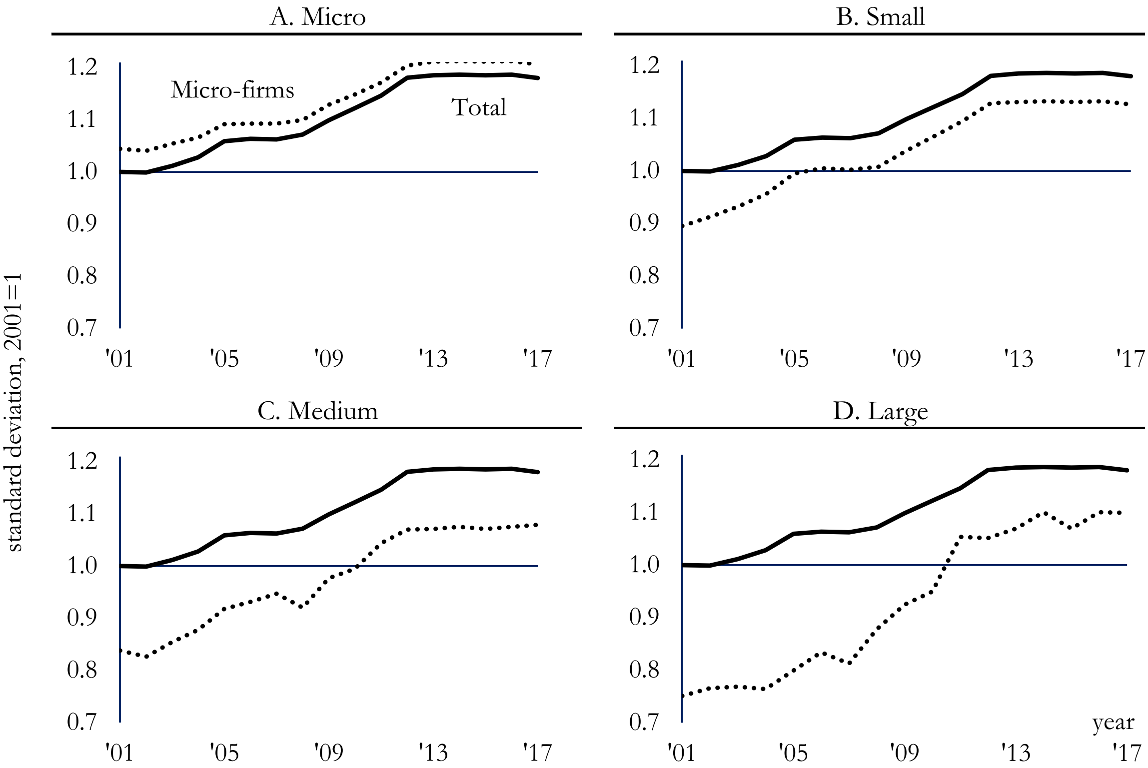


Figure 7: Firm-size and the evolution of MRPL dispersion, 2001–2017, the Netherlands.

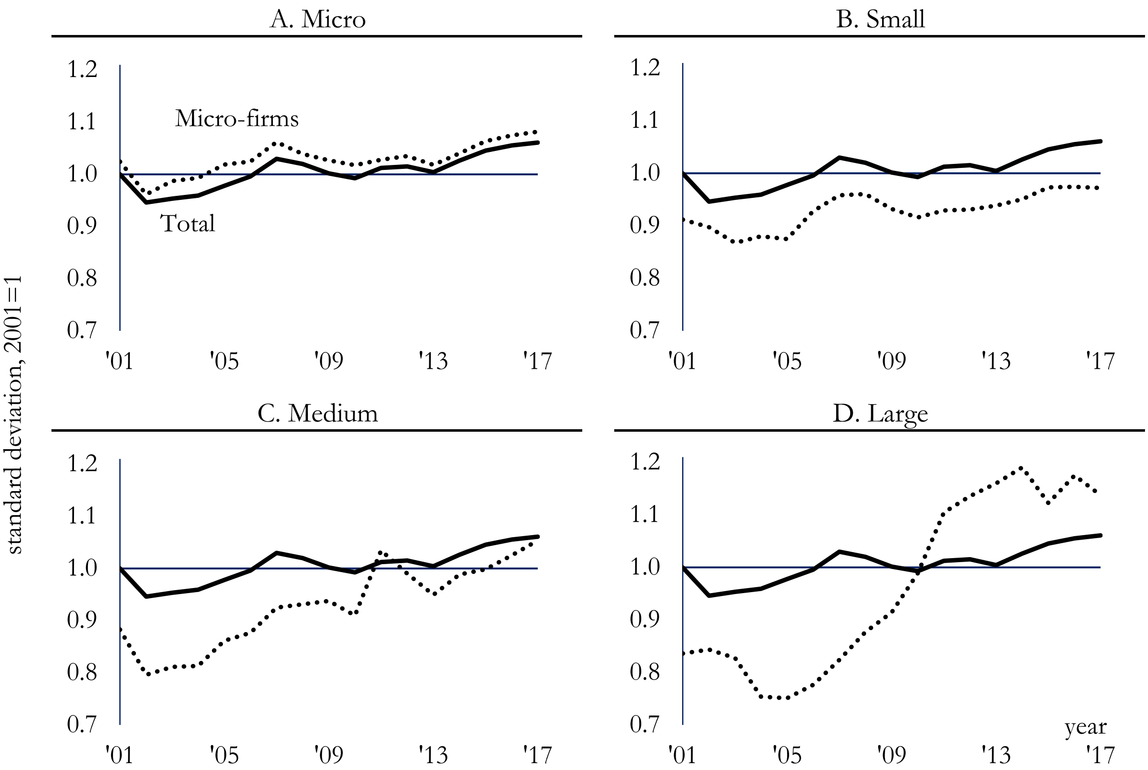


Figure 8: Distribution of capital and labor wedge, 2001–2017, the Netherlands.

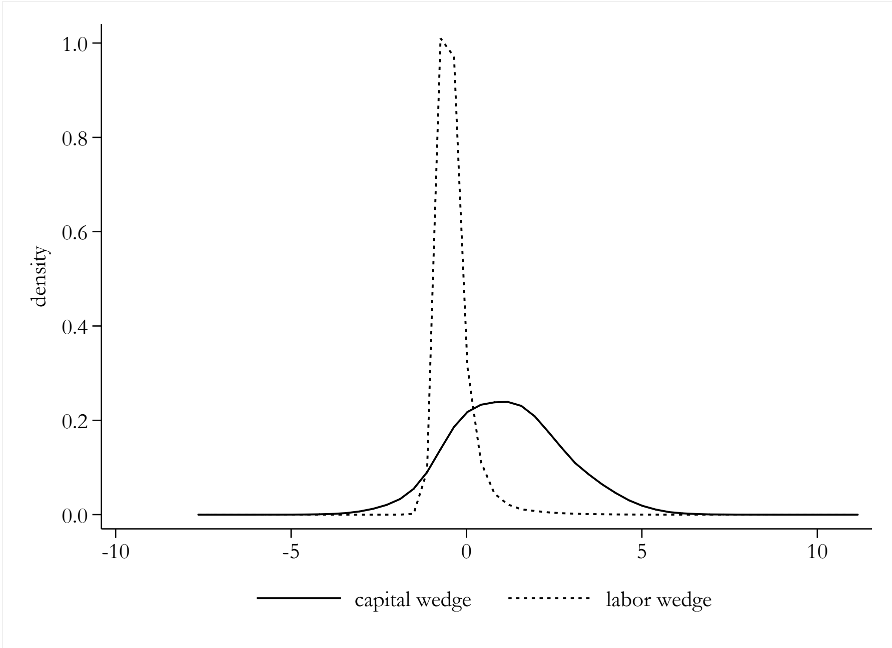


Figure 9: Distribution of capital and labor wedge, 2001, 2008 and 2017, the Netherlands.

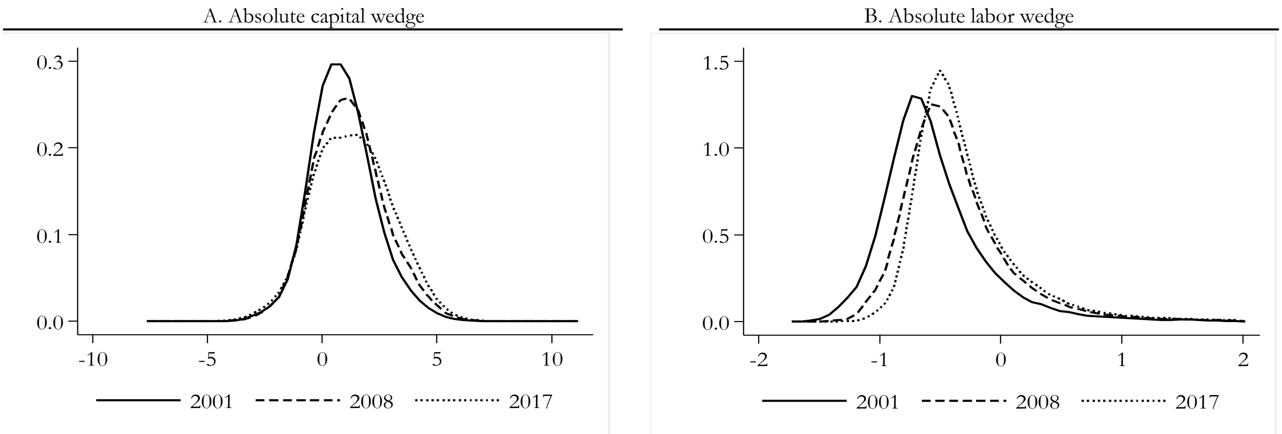


Table I: Correlation matrix of TFPQ, factor-inputs and capital and labor wedges

	$\widetilde{TFPQ}_{is,t}$	$\widetilde{K}_{is,t}$	$\widetilde{L}_{is,t}$	$\widetilde{\tau}^{K*}_{is,t}$
$\widetilde{K}_{is,t}$	0.06	1	0.54	-0.70
$\widetilde{L}_{is,t}$	0.51	0.54	1	0.10
$\widetilde{\tau}^{K*}_{is,t}$	0.49	-0.70	0.10	1
$\widetilde{\tau}^{L*}_{is,t}$	0.21	0.08	-0.33	-0.08

Note: variables are in deviation from sectoral averages.

Figure 10: Marginal effects ordered probit, capital wedge, 2001–2017, the Netherlands.

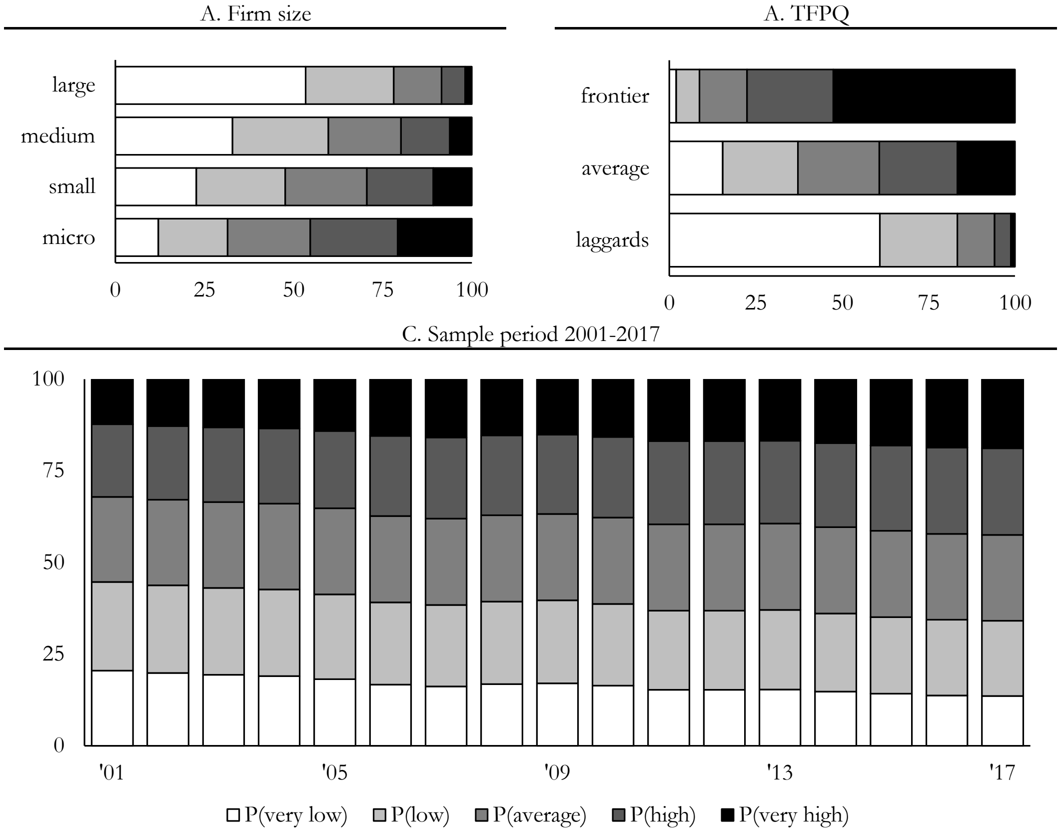


Figure 11: Expected value of capital wedge, ordered probit, full set of interactions, 2001–2017, the Netherlands

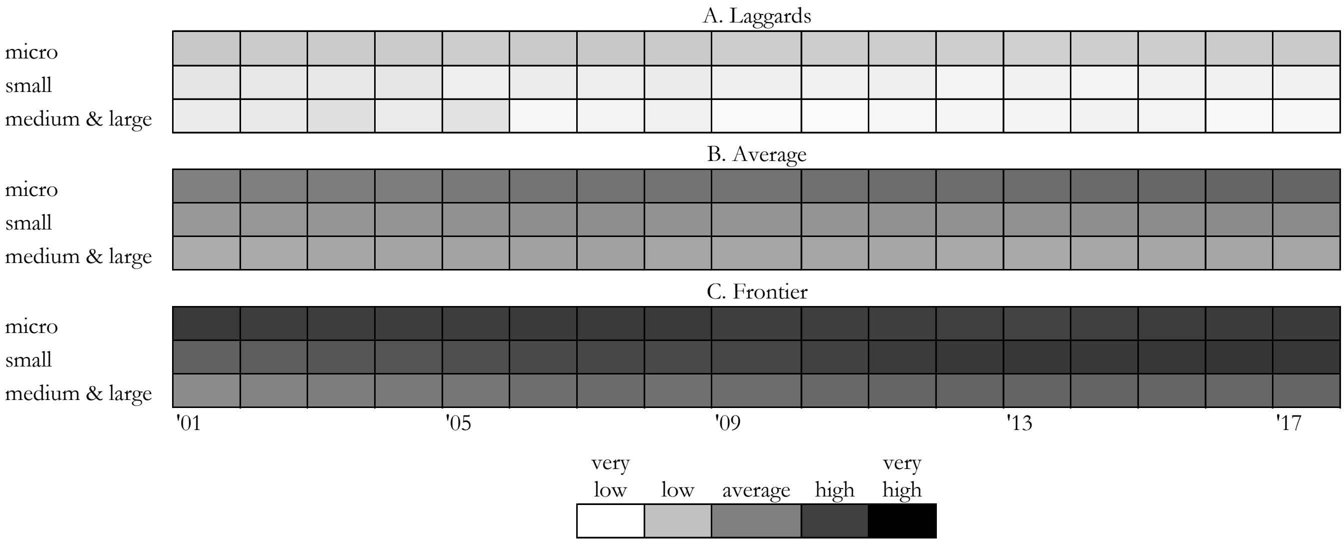


Figure 12: Marginal effects ordered probit, labor wedge, 2001–2017, the Netherlands.

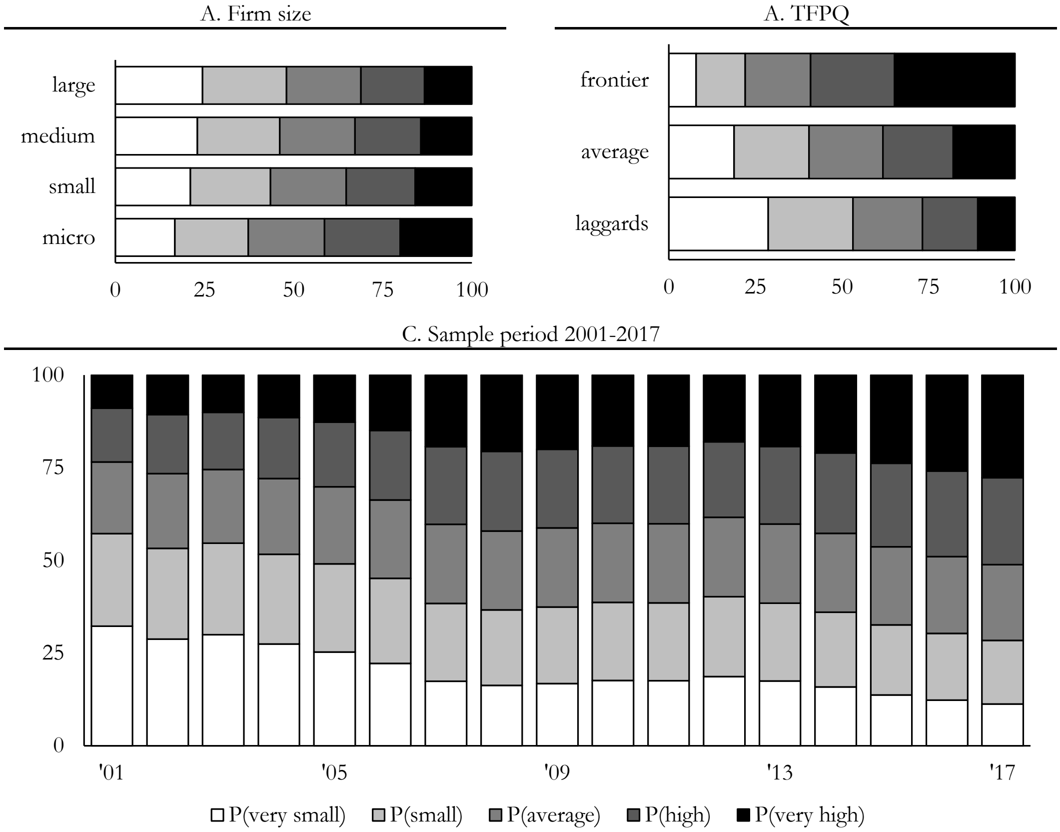


Figure 13: Expected value of labor wedge, ordered probit, full set of interactions, 2001–2017, the Netherlands

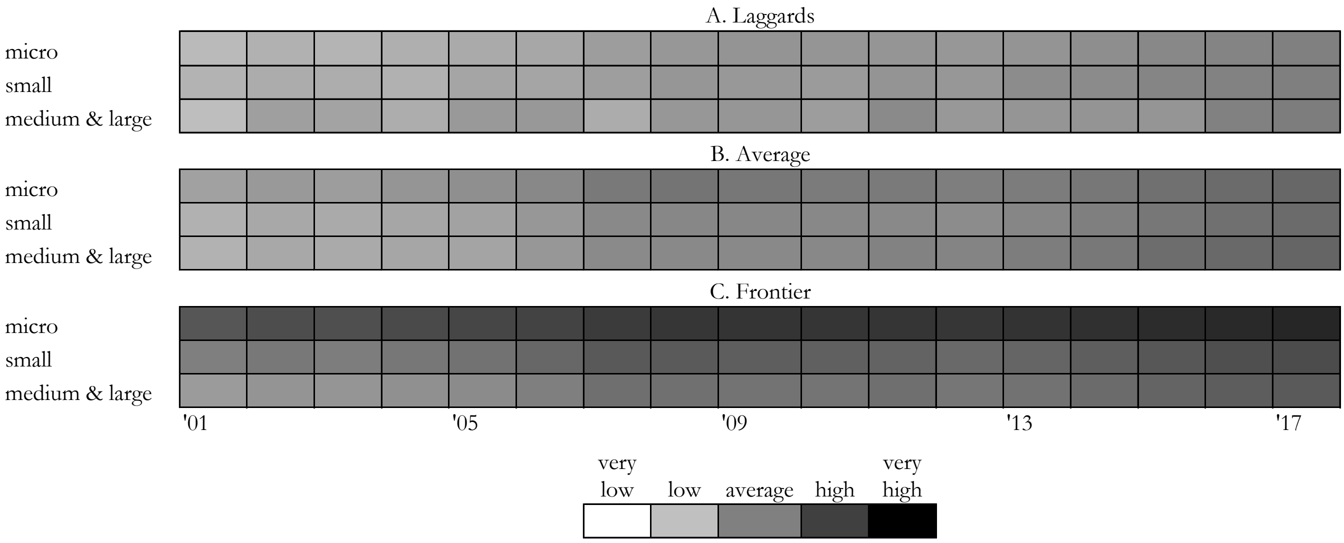


Figure 14: Evolution of permanent and transitory components of MRPK and MRPL dispersion, 2001–2017, the Netherlands.

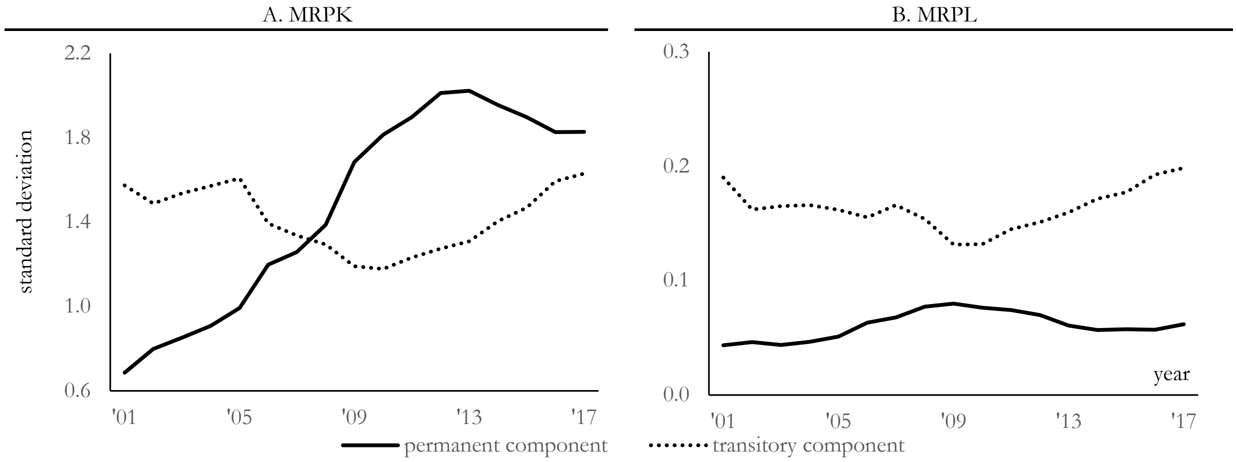


Figure 15: Evolution of permanent and transitory components of MRPK dispersion, manufacturing and services, 2001–2017, the Netherlands.

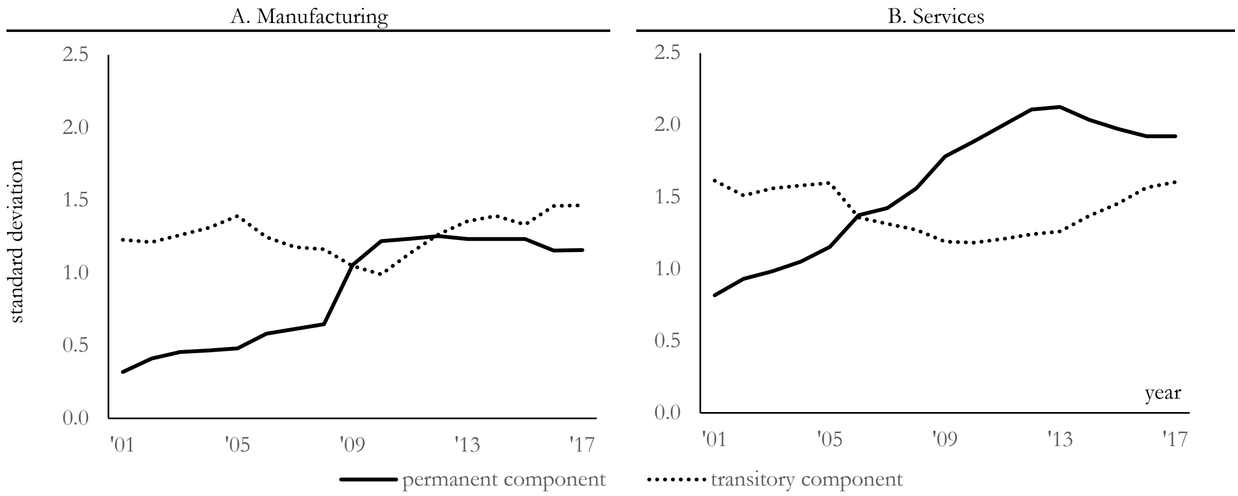


Figure 16: Evolution of permanent and transitory components of MRPL dispersion, manufacturing and services, 2001–2017, the Netherlands.

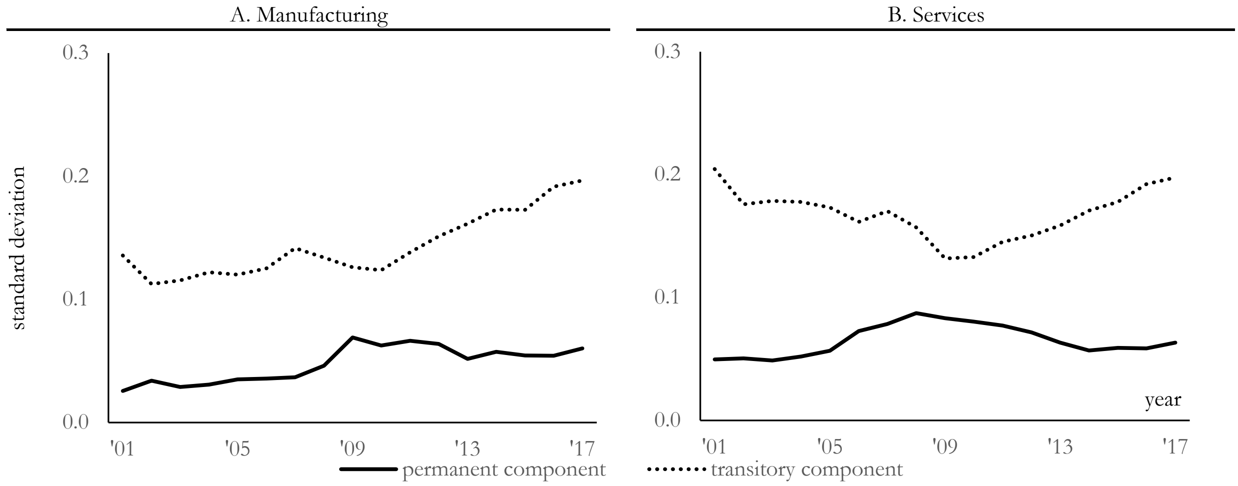


Figure 17: Evolution of permanent and transitory components of MRPK dispersion, firm-size, 2001–2017, the Netherlands.

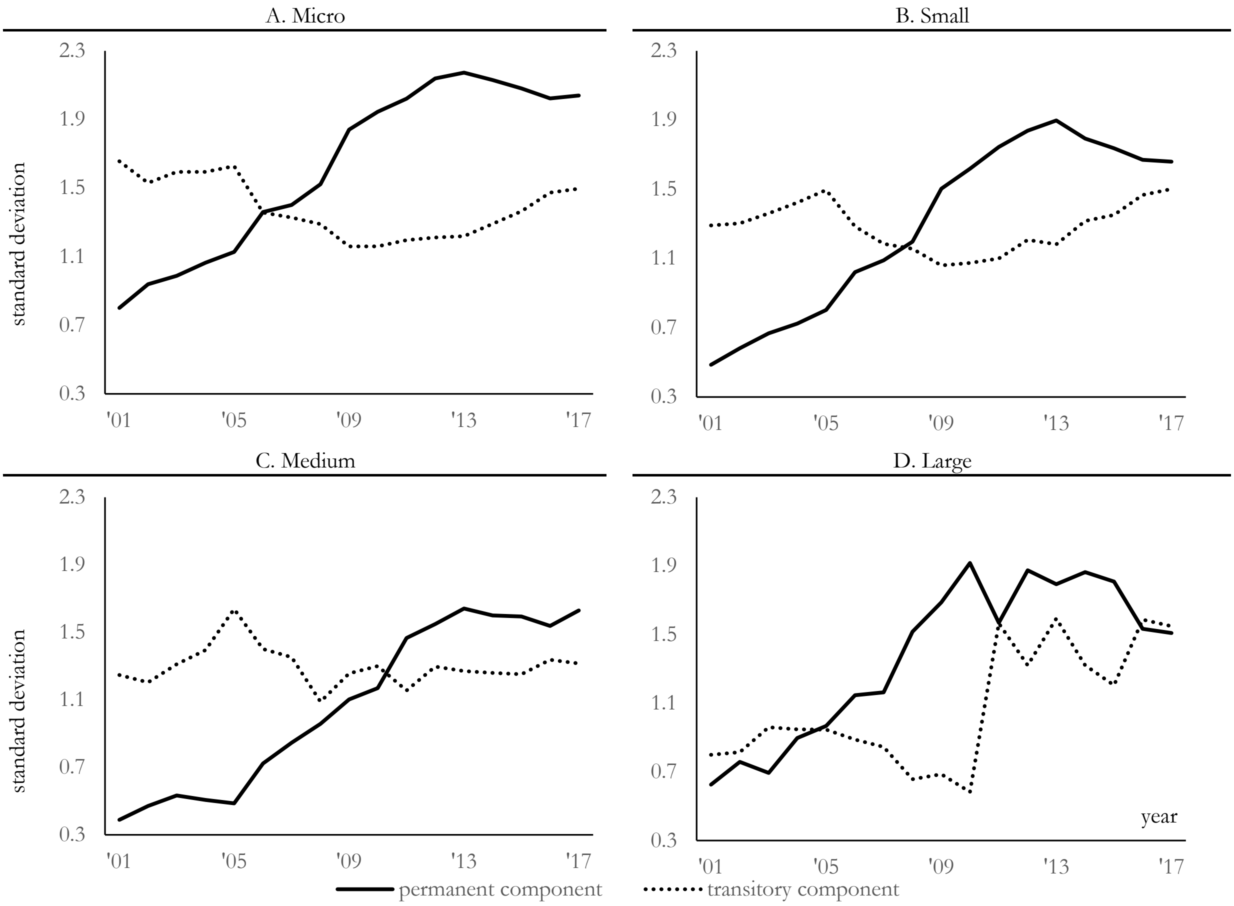


Figure 18: Evolution of permanent and transitory components of MRPL dispersion, firm-size, 2001–2017, the Netherlands.

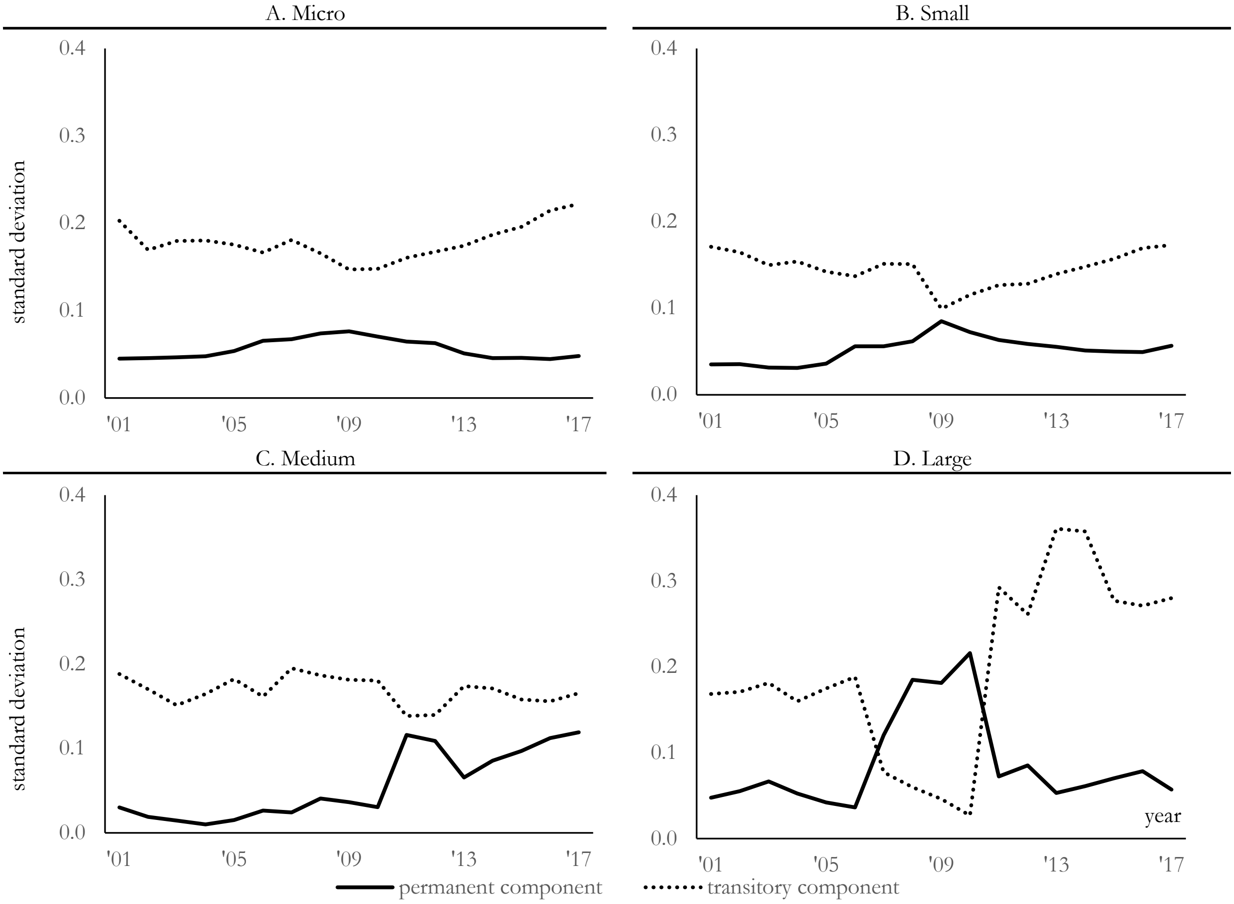


Figure 19: Evolution of permanent and transitory components of MRPK dispersion, productivity levels, 2001–2017, the Netherlands.

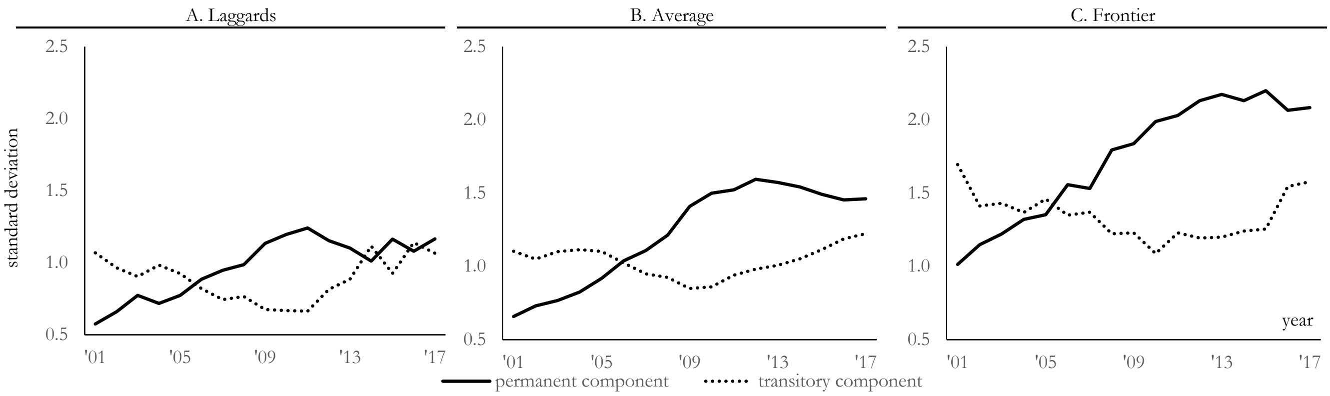


Figure 20: Evolution of permanent and transitory components of MRPL dispersion, productivity levels, 2001–2017, the Netherlands.

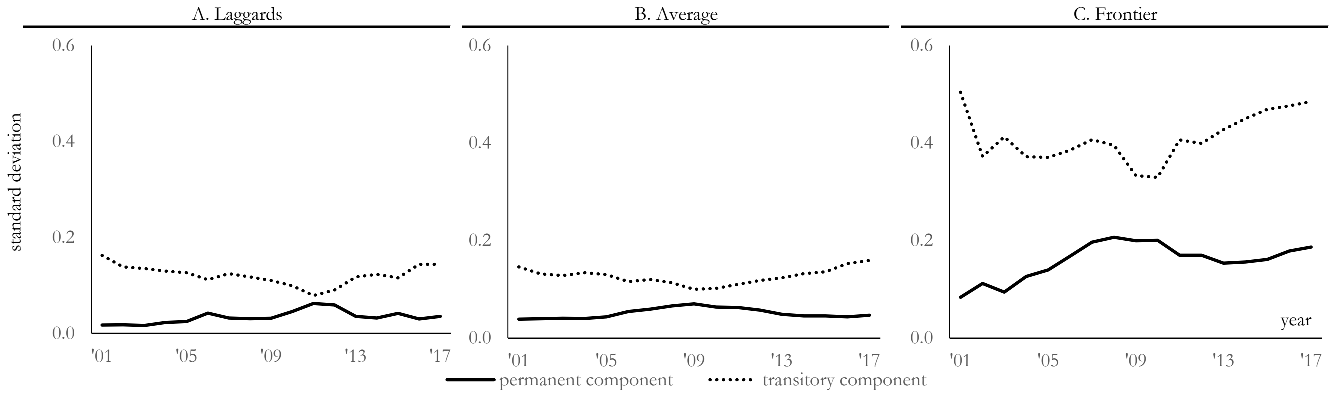
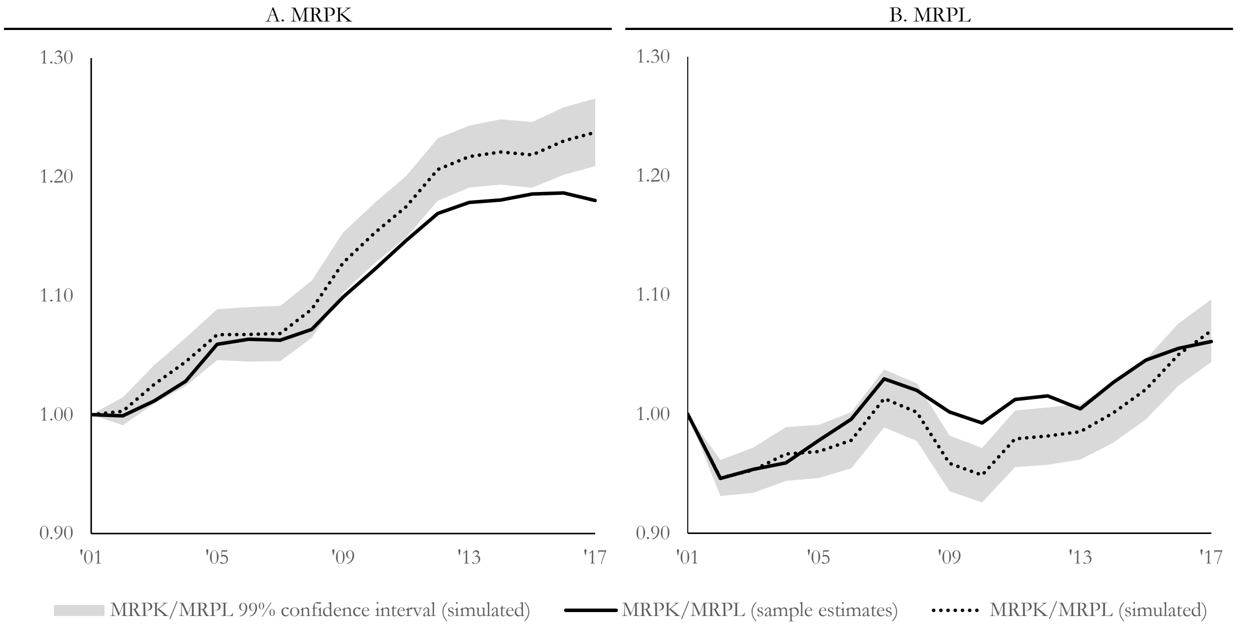


Figure 21: Empirical and simulated evolution of MRPK and MRPL dispersion, 2001–2017, the Netherlands.



C Technical Appendix

C.1 Misallocation with capital adjustment costs

David and Venkateswaran (2019) develop a model to distinguish the various sources of measured capital misallocation, i.e. dispersion in MRPK. They distinguish capital adjustment costs, informational frictions and firm-specific factors. We first describe the main model equations in David and Venkateswaran (2019). Next, we derive the variance of MRPK when adjustment costs are the only source of misallocation.

Main model equations David and Venkateswaran (2019)

For ease of exposition, we suppress the sector subscript, hence consider a single sector only. The main equations of the model, which are in logarithms, are as follows. An idiosyncratic firm-specific fundamental, which can be interpreted as demand shifter and/or level of efficiency, is generated by:

$$a_{i,t} = \rho a_{i,t-1} + \mu_{i,t}, \quad \mu_{i,t} \sim i.i.d.(0, \sigma_\mu^2), \tag{A.1}$$

where $0 < \rho < 1$ measures the persistence of firm-fundamentals. The capital distortion is modeled as:

$$\tau_{i,t} = \gamma a_{i,t} + \varepsilon_{i,t} + \chi_i, \quad \varepsilon_{i,t} \sim i.i.d.(0, \sigma_\varepsilon^2), \quad \chi_i \sim i.i.d.(0, \sigma_\chi^2), \tag{A.2}$$

where γ models the correlation of the distortion with the firm-fundamental. Furthermore, ε_{it} and χ_i are the uncorrelated time-varying and permanent components respectively. The equation for capital is:

$$k_{i,t+1} = \psi_1 k_{i,t} + \psi_2 (1+\gamma)\, \mathbb{E}_{i,t}\left[a_{i,t+1}\right] + \psi_3 \varepsilon_{i,t+1} + \psi_4 \chi_i, \tag{A.3}$$

where

$$\begin{aligned}
\xi\left(\beta\psi_1^2 + 1\right) &= \psi_1\left((1+\beta)\,\xi + 1 - \alpha\right), \\
\psi_2 &= \frac{\psi_1}{\xi\left(1 - \beta\rho\psi_1\right)}, \\
\psi_3 &= \frac{\psi_1}{\xi}, \\
\psi_4 &= \frac{1-\psi_1}{1-\alpha}.
\end{aligned}$$

The parameter β is the discount rate, which is an element of the optimized dynamic profit function of the firm. The parameter $\alpha = \frac{\alpha_1}{1-\alpha_2}$ where α_1 and α_2 are proportional to the capital and labor elasticities of the firm-level CD-production function. Finally, the marginal revenue product of capital (MRPK) is equal to:

$$MRPK_{i,t} = p_{i,t} + y_{i,t} - k_{i,t}, \tag{A.4}$$

where $p_{i,t}$ is the price of good i and $y_{i,t}$ is firm-level output. The assumed market structure furthermore implies:

$$p_{i,t} = -\frac{1}{\theta}(y_{i,t} - y_t) + \hat{a}_{i,t}, \tag{A.5}$$

where $a_{i,t} = \frac{1}{1-\alpha_2}\hat{a}_{i,t}$ and $\alpha_j = (1 - \frac{1}{\theta})\hat{\alpha}_j$, $j = 1, 2$. Furthermore, it can be shown that profit maximization yields the following for labor:

$$l_{i,t} \approx \frac{1}{1-\alpha_2}(\hat{a}_{i,t} + \alpha_1 k_{i,t}), \tag{A.6}$$

where $\approx$ means that we left out terms without cross-sectional dimension.

Derivation of σ^2_{MRPK} when adjustment costs are the only source of misallocation

Using the model's equations we can rewrite equation (A.4) as:

$$\begin{aligned} MRPK_{i,t} &= -\frac{1}{\theta}y_{i,t} + \frac{1}{\theta}y_t + \hat{a}_{i,t} + y_{i,t} - k_{i,t} \\ &= \left(1 - \frac{1}{\theta}\right)(\hat{\alpha}_1 k_{i,t} + \hat{\alpha}_2 l_{i,t}) + (1-\alpha_2)\,a_{i,t} - k_{i,t} + \frac{1}{\theta}y_t \\ &= \alpha_1 k_{i,t} + \alpha_2 l_{i,t} + (1-\alpha_2)\,a_{it} - k_{it} + \frac{1}{\theta}y_t \\ &\approx \alpha_1 k_{i,t} + \alpha_2\left(a_{it} + \frac{\alpha_1}{1-\alpha_2}k_{i,t}\right) + (1-\alpha_2)\,a_{i,t} - k_{i,t} \\ &= \frac{\alpha_1 + \alpha_2 - 1}{1-\alpha_2}k_{i,t} + a_{i,t}, \end{aligned} \tag{A.7}$$

where $\approx$ means that we left out terms without cross-sectional dimension. The reason is that these terms will not contribute to dispersion in MRPK, which is defined as:

$$\sigma^2_{MRPK} = \left(\frac{\alpha_1 + \alpha_2 - 1}{1-\alpha_2}\right)^2 \sigma^2_k + \sigma^2_a + 2\left(\frac{\alpha_1 + \alpha_2 - 1}{1-\alpha_2}\right)\sigma_{ka}, \tag{A.8}$$

where

$$\sigma^2_a = \frac{\sigma^2_\mu}{1-\rho^2},$$

due to the assumption that the firm-fundamental follows a stationary AR(1) process.

The model distinguishes three main sources of dispersion in MRPK: (1) capital adjustment costs; (2) imperfect information; (3) distortions. In absence of adjustment costs we have:

$$\psi_1 = 0, \quad \psi_2 = \psi_3 = \psi_4 = \frac{1}{1-\alpha},$$

while in case of perfect foresight we have:

$$\mathbb{E}_{it}\left[a_{i,t+1}\right] = a_{i,t+1},$$

and when there are no distortions we have:

$$\gamma = 0, \quad \sigma_\varepsilon^2 = \sigma_\chi^2 = 0.$$

Each of these three origins will show up in MRPK dispersion mainly through dispersion in capital (σ_k^2).

David and Venkateswaran (2019) assume the following:

$$\mathbb{E}_{i,t}\left[a_{i,t+1}\right] = \rho a_{i,t} + s_{i,t+1}^*, \tag{A.9}$$

where the error $s_{i,t+1}^*$ depends on the information a firm has on the next innovation $\mu_{i,t+1}$. We then have:

$$k_{i,t+1} = \psi_1 k_{i,t} + \psi_2(1+\gamma)\rho a_{i,t} + u_{i,t+1} + \psi_4 \chi_i, \tag{A.10}$$

where $u_{i,t+1} = s_{i,t+1}^* + \psi_3 \varepsilon_{i,t+1} \sim i.i.d.(0, \sigma_u^2)$. Using the lag operator, the model can be written as

$$k_{i,t+1} = \psi_2(1+\gamma)\rho \frac{1}{(1-\psi_1 L)(1-\rho L)} \mu_{it} + \frac{1}{(1-\psi_1 L)} u_{i,t+1} + \frac{\psi_4}{1-\psi_1} \chi_i. \tag{A.11}$$

The development in a firms' capital is the sum of three orthogonal components (μ_{it}, $u_{i,t+1}$ and χ_i are uncorrelated), which are an AR(2), AR(1) and i.i.d. process respectively. The AR(2) process:

$$\phi_{i,t+1} = \frac{1}{(1-\psi_1 L)(1-\rho L)} \mu_{i,t+1}$$

has variance:

$$\sigma_\phi^2 = \left(1 - (\psi_1 + \rho)\operatorname{corr}\left(\phi_{i,t+1}, \phi_{it}\right) + \psi_1 \rho \operatorname{corr}\left(\phi_{i,t+1}, \phi_{i,t-1}\right)\right)^{-1},$$

with

$$\begin{aligned} \operatorname{corr}\left(\phi_{i,t+1}, \phi_{i,t}\right) &= \frac{\psi_1 + \rho}{1 + \psi_1 \rho}, \\ \operatorname{corr}\left(\phi_{i,t+1}, \phi_{i,t-1}\right) &= \frac{(\psi_1 + \rho)^2}{1 + \psi_1 \rho} - \psi_1 \rho. \end{aligned}$$

Hence, we find that dispersion in capital is:

$$\sigma_k^2 = \left(\psi_2(1+\gamma)\rho\right)^2 \sigma_\phi^2 + \frac{1}{1-\psi_1^2}\sigma_u^2 + \left(\frac{\psi_4}{1-\psi_1}\right)^2 \sigma_\chi^2.$$

Consider now the case $\sigma_u^2 = \sigma_\chi^2 = 0$, i.e. only adjustment costs matter for dispersion in MRPK. The model reduces to

$$k_{i,t+1} = \psi_1 k_{i,t} + \psi_2(1+\gamma)\rho a_{i,t}, \tag{A.12}$$

which is basically equation (16). By repeated substitution we write:

$$k_{i,t+1} = \psi_2(1+\gamma)\rho \sum_{s=0}^{\infty} \psi_1^s a_{i,t-s}, \tag{A.13}$$

hence the covariance between capital $k_{i,t+1}$ and fundamental a_{it} becomes:

$$\begin{aligned}
\sigma_{ka} &= \mathbb{C}\mathrm{ov}\left(\psi_2(1+\gamma)\rho \sum_{s=0}^{\infty} \psi_1^s a_{i,t-s}, a_{i,t+1}\right) \\
&= \psi_2(1+\gamma)\rho \sum_{s=0}^{\infty} \psi_1^s \, \mathbb{C}\mathrm{ov}\left(a_{i,t-s}, a_{i,t+1}\right) \\
&= \psi_2(1+\gamma)\rho^2 \sum_{s=0}^{\infty} \psi_1^s \rho^s \sigma_a^2 \\
&= \frac{\psi_2(1+\gamma)\rho^2}{1-\psi_1\rho}\sigma_a^2. \qquad \text{(A.14)}
\end{aligned}$$

Substituting σ_a^2, σ_k^2 and σ_{ka} we find the analytical expression for σ_{MRPK}^2 for the specific case that only adjustment costs matter for MRPK dispersion.

C.2 Derivation of σ_{MRPK}^2 with firm-level heterogeneity in the production function

Consider the following generalized production function:

$$Y_{is,t} = A_{is,t} K_{is,t}^{\alpha_{is,t}} L_{is,t}^{1-\alpha_{is,t}}, \tag{A.15}$$

in which capital intensities are idiosyncratic and time varying. Suppose no distortions exist. The first-order conditions for profit maximization are:

$$P_{is,t}\frac{\partial Y}{\partial L} = w_t, \tag{A.16}$$

$$P_{is,t}\frac{\partial Y}{\partial K} = R_t, \tag{A.17}$$

with

$$\frac{\partial Y}{\partial K} = A_{is,t}\alpha_{is,t}K_{is,t}^{\alpha_{is,t}-1}L_{is,t}^{1-\alpha_{is,t}} = \alpha_{is,t}\frac{Y_{is,t}}{K_{is,t}}, \tag{A.18}$$

$$\frac{\partial Y}{\partial L} = ... = (1-\alpha_{is,t})\frac{Y_{is,t}}{L_{is,t}}. \tag{A.19}$$

Combining equation (A.17) and equation (A.18) and omitting constants we have:

$$MRPK_{is,t} = \log \frac{P_{is,t} Y_{is,t}}{K_{is,t}} \approx -\log(\alpha_{is,t}), \tag{A.20}$$

hence,

$$\mathbb{V}\text{ar}\left(MRPK_{is,t}\right) = \mathbb{V}\text{ar}\left(\log(\alpha_{is,t})\right). \tag{A.21}$$

Summarizing, in a model without distortions, but including heterogeneous technologies, all dispersion in MRPK is caused by dispersion in production technologies. Following Hsieh and Klenow (2009) we use 1 minus the labor share to estimate the elasticity of output with respect to capital. For the purpose of estimating dispersion at the level of the firm, we use firm-level labor shares.

C.3 Derivation of σ^2_{MRPK} in case of a CES-production function

For ease of exposition we suppress time subscripts. Consider the same market structure as in Hsieh and Klenow (2009), hence the demand curve and price setting for an individual firm's product follow from the first-order condition:

$$P_s Y_s^{\frac{1}{\sigma}} Y_{is}^{-\frac{1}{\sigma}} = P_{is}. \tag{A.22}$$

However, the individual firm has the CES-production function (17) with both a neutral productivity component A_{is} and labor augmenting productivity B_{is} (see e.g. Raval, 2019). Define $\rho = \frac{\sigma-1}{\sigma}$ with σ the substitution elasticity between capital and labor. Then we can express the CES-production function (17) as:

$$Y_{is} = A_{is}\left(\alpha_s K_{is}^{\rho} + (1-\alpha_s)\left(B_{is} L_{is}\right)^{\rho}\right)^{\frac{1}{\rho}}. \tag{A.23}$$

Note that the CD-production function is the special case $\sigma = 1$ or $\rho = 0$. The marginal products are:

$$\frac{\partial Y}{\partial L} = A_{is}\left(\alpha_s K_{is}^{\rho} + (1-\alpha_s)\left(B_{is} L_{is}\right)^{\rho}\right)^{\frac{1-\rho}{\rho}} (1-\alpha_s)\,\rho B_{is}^{\rho} L_{is}^{\rho-1}, \tag{A.24}$$

$$\frac{\partial Y}{\partial K} = A_{is}\left(\alpha_s K_{is}^{\rho} + (1-\alpha_s)\left(B_{is} L_{is}\right)^{\rho}\right)^{\frac{1-\rho}{\rho}} \alpha_s \rho K_{is}^{\rho-1}, \tag{A.25}$$

hence,

$$\frac{\frac{\partial Y}{\partial L}}{\frac{\partial Y}{\partial K}} = \frac{1-\alpha_s}{\alpha_s} B_{is}^{\rho} \left(\frac{K_{is}}{L_{is}}\right)^{1-\rho}. \tag{A.26}$$

In the absence of distortions profits of each producer are given by:

$$\pi_{is} = P_s Y_s^{\frac{1}{\sigma}} Y_{is}^{\frac{\sigma-1}{\sigma}} - wL_{is} - RK_{is}, \tag{A.27}$$

Each producer chooses L_{is} and K_{is} to maximize profits:

$$\max_{L_{is},K_{is}} \pi_{is}. \tag{A.28}$$

The first order conditions for profit maximization are:

$$P_s Y_s^{\frac{1}{\sigma}} \left(\frac{\sigma-1}{\sigma}\right) Y_{is}^{-\frac{1}{\sigma}} \frac{\partial Y}{\partial L} = w, \tag{A.29}$$

$$P_s Y_s^{\frac{1}{\sigma}} \left(\frac{\sigma-1}{\sigma}\right) Y_{is}^{-\frac{1}{\sigma}} \frac{\partial Y}{\partial K} = R. \tag{A.30}$$

Dividing both equations and rearranging we get for the capital-labor ratio:

$$\frac{K_{is}}{L_{is}} = \left(\frac{\alpha_s}{1-\alpha_s}\right)^{\frac{1}{1-\rho}} B_{is}^{\frac{-\rho}{1-\rho}} \left(\frac{w}{R}\right)^{\frac{1}{1-\rho}}. \tag{A.31}$$

Note that when $\rho = 0$, i.e. CD-production function, we get:

$$\frac{K_{is}}{L_{is}} = \frac{\alpha}{1-\alpha}\frac{w}{R}, \tag{A.32}$$

which does not depend on B_{is}.

The firm's output price P_{is} will be set as a fixed mark up $\frac{\sigma}{\sigma-1}$ over the firm's marginal costs. In case of the CES-production function (17), marginal costs are:

$$MC_{is} = \frac{\alpha_s^{\sigma} R^{1-\sigma} + (1-\alpha_s)^{\sigma} \left(\frac{w}{B_{is}}\right)^{1-\sigma}}{A_{is}\left(\alpha_s \left(\frac{R}{\alpha_s}\right)^{1-\sigma} + (1-\alpha_s)\left(\frac{w}{(1-\alpha_s)B_{is}}\right)^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}}, \tag{A.33}$$

and

$$P_{is} = \left(\frac{\sigma}{\sigma-1}\right) MC_{is}. \tag{A.34}$$

Combining these equations after some algebra we find that:

$$\begin{aligned} MRPK_{is} &= P_{is}\frac{\partial Y}{\partial K} \\ &= \alpha_s^{1-\sigma} R^{\sigma} \left(\alpha_s^{\sigma} R^{1-\sigma} + (1-\alpha_s)^{\sigma}\left(\frac{w}{B_{is}}\right)^{1-\sigma}\right). \end{aligned} \tag{A.35}$$

Without component B_{is}, i.e. the standard CES-production function, the result in equation (18) follows. The last equation shows that, for labor augmenting or labor saving productivity to cause variation in MRPK, it should vary across firms within the same industry.

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De Nederlandsche Bank N.V.
Postbus 98, 1000 AB Amsterdam
020 524 91 11
dnb.nl