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* Views expressed are those of the author and do not necessarily reflect official positions of De Nederlandsche Bank.

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Fundamental uncertainty about the natural rate of interest: Info-gap as guide for monetary policy

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Abstract
In this paper we assume that the natural rate of interest is fundamentally uncertain. Based on a small scale macroeconomic model, info-gap theory is used to rank different monetary policy strategies in terms of their robustness against this uncertainty. Applied to the euro area, we find that a strategy that is responsive to deviations from the policy targets is more robust against natural rate uncertainty than the historical response of the ECB as reflected in an estimated Taylor rule. An inert or passive monetary strategy is least robust. Our analysis presents a methodology that is applicable in a wide range of policy analyses under deep uncertainty.

Keywords: Monetary Policy, Monetary Strategy, Knightian uncertainty, info-gaps, satisficing
JEL classifications: E42, E47, E52

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1. **Introduction**

The decline of interest rates to the lower bound has stirred the debate on the use of natural rate (r^*) in monetary policy. In the literature, r^* is the real interest rate that serves as a benchmark for the real policy rate in the steady state. It is part of the Taylor rule, which describes the standard reaction of the central bank to output and inflation. In equilibrium, the output gap is closed and there is price stability, implying that the policy rate equals the natural rate. The main complication in the use of r^* for monetary policy is that it is an unobservable variable. It is a theoretical concept and there are several definitions in existence. For instance, r^* can be defined as real long-term interest rate where there is equilibrium on the capital markets, or as the real short-term interest rate consistent with equilibrium in the economy.

Since r^* cannot be directly observed, the empirical literature uses several methods to estimate proxies of r^*. For instance, r^* is estimated by time series models; semi-structural models and general equilibrium models (see Bonam et al., 2018 for an overview). A widely-accepted model to estimate r^* is the semi-structural model developed by Holston et al. (2017). This model relates the natural rate of interest to the potential economic output, while filtering r^* out of the observable macroeconomic data series. In general, the outcomes for r^* are sensitive to the model assumptions. Research shows that estimations of the natural rate can be highly inaccurate and can vary widely depending on the model specification applied (Beyer and Wieland (2017).

The unobservable nature of r^* and the large modelling and estimation uncertainties underscore that r^* goes with fundamental uncertainty. According to Knight (1921) this type of uncertainty describes events with unknown or objectively unmeasurable probabilities. Keynes (1921) also argued against probabilising the unknown. Because an unknown probability is indeterminate, he considered that it cannot be modelled as random variables. This contrasts to measurable uncertainty or risk, which can be quantified based on known probability distributions of events. In case of Knightian uncertainty however, the data distribution might be unknowable, either intrinsically or because of practical limitations. This goes with an unknown event space and indeterminate outcomes, because there are no laws that govern the process. This makes fundamental uncertainty barely manageable. Assuming fundamental uncertainty with regard to r^* raises the question whether r^* can be used by the central bank as benchmark for the policy rate in equilibrium.

There are several risk management strategies to deal with uncertainty in monetary policy. A well-known strategy to deal with measurable uncertainty (risk) is Brainard's attenuation principle (1967). This assumes that with uncertainty the central bank should respond to shocks more cautiously and in smaller steps than in conditions without uncertainty. It would call for policy gradualism and less aggressive responses to economic shocks. Brainard referred to uncertainty about the monetary transmission mechanism, but also data uncertainty is associated with policy gradualism in the literature (e.g. by Aoki, 2003). There are also risk management strategies that call for a more aggressive response by the central bank, for instance if there is uncertainty about the persistence in the rate of inflation (Tetlow, 2018). More aggressive policy measures could then be needed to prevent an adverse shock from destabilizing inflation expectations. When the policy rate is at the effective lower bound, several studies conclude that it is optimal for the central bank to raise inflation expectations by committing to keep interest rates low for a time in the future (Eggertsson and Woodford, 2003).

Two main strategies for managing unmeasurable, or Knightian uncertainty have emerged in the literature: robust control and info-gap. Robust control insures against the maximally worst outcome (min-max) as defined by the policymaker (see Hansen et al., 2006; Hansen and Sargent, 2008 and Williams, 2007). Olalla and Gómez (2011) apply the robust control tool to a Neo-Keynesian model to study the effect of model uncertainty in monetary policy. Typically, policies derived through min-max are more aggressive by comparison to those derived under no uncertainty. Intuitively, when mechanisms at work are poorly understood, aggressive policies allow decisions makers to learn about them. The literature has raised two objections to this: (i) policymakers do not like experimenting for the purposes of learning; (ii) worst events are rare and hence poorly known. It is odd therefore, to design policies for events about which one knows the least (Sims, 2001).

However, the most important drawback in our view is that robust control does not account for the fundamental choice between robustness against uncertainty on the one hand, and aspiration for good performance outcomes, on the other. This is where the alternative approach, info-gap, makes an important contribution by mapping explicitly this trade-off (Ben-Haim, 2006, 2010). If the central bank adopts an ambitious inflation target, it needs to compromise on the degree of confidence in achieving that target (Ben-Haim and Demertzis, 2008, 2016). Conversely, if the central bank requires high confidence in achieving specified goals, it needs to moderate how ambitious these goals are. Info-gap theory quantifies this intuitive trade off. Ben-Haim et al. (2018) apply info-gap to evaluate different monetary policy reaction functions. They show that a traditional Taylor rule, in which the inflation and output gap are targeted, is preferred over more complex reaction functions that include financial variables. Our paper builds on that approach.

2. Contribution to the literature

This paper contributes to the literature by assessing various monetary policy strategies to cope with fundamental uncertainty with regard to r^*. The assessment is based on info-gap theory. Several monetary strategies to deal with uncertainty about r^* can be distinguished. A monetary policy strategy is defined in this paper as a particular response of the central bank to deviations from the inflation and output objectives in a Taylor rule. Most approaches that call for expansionary monetary policy measures are based on the secular stagnation hypothesis. It assumes that r^* has fallen to negative levels due to deficient aggregate demand and a savings surplus, caused by structural developments like ageing and fallen labor productivity (see e.g. Summers, 2014). In that situation the lower bound of interest rates impedes the ability of the central bank to stimulate the economy by lowering the policy rate (adjusted for inflation) to below r^*. Alternatively the central bank could raise the inflation target or pursue a price-level targeting strategy (Bernanke, 2017; Williams, 2017), thereby reducing the real interest rate. Inflation expectations could also be raised by keeping interest rates low for longer (Yellen, 2018; Bernanke et al., 2019). In contrast, the financial cycle hypothesis argues that persistently low policy rates have contributed to a reduction in r^*. This view assumes that accommodative monetary conditions encouraged a misallocation of production factors and thus depressed production growth and thereby r^* (Borio, 2017). Following from this, a less expansionary monetary policy would be needed. Others argue that the fundamental uncertainty regarding the level of r^* constrains its practical usefulness as a benchmark for monetary policy (Bonam et al., 2018). Not only are model estimations of r^* beset with great uncertainty, there is no uniform definition of the natural rate of interest either. This is in line with Tarullo (2017), who argues that policymakers ought to be cautious in basing their policy on non-observable variables. The various hypotheses illustrate that the view on r^* is a crucial – and highly uncertain – determinant of the preferred monetary policy strategy.

Some studies take the uncertainty about r^* into account in monetary policy reaction functions. According to Williams and Orphanides (2007), taking into account natural rate uncertainty implies that in the optimal policy strategy the interest rate is more inert (higher degree of interest rate smoothing), reacts more aggressively to inflation and less strongly to the unemployment gap than would be optimal if agents had perfect knowledge of the economy. Their conclusions follow from Bayesian and min-max or robust control approaches. Kiley and Roberts (2017) propose a risk adjustment factor to r^* in the Taylor rule when shocks to the economy and a subsequent low level of r^* constrain the ability for monetary accommodation. The risk adjustment assumes that monetary policy is more accommodative, on average, than a Taylor rule would imply. Ferrero et al. (2019) assume uncertainty about the value of the natural interest rate in a basic (two-equation) forward looking new-Keynesian framework, in which the policy rate is set to maximize an objective function for the inflation and output gap. They conclude that optimal monetary policy should not react either more aggressively or more cautiously to exogenous shocks relative to the full information case. The rationale for this result is that the shocks affecting r^* do not interact with any endogenous variable in the model, so that the uncertainty can be integrated out without affecting the rest of the problem. Ferrero et al. seem to assume that their best estimate of the model is in fact the most robust to uncertainty. That is a basic and common confusion between a best putative model and a best response to uncertainty, and does not acknowledge deep uncertainty. We show that the central bank may err if it adopts a monetary policy strategy based on its best putative model or best current understanding and that a strategy that robustly satisfies critical requirements is preferable.

In this paper we also recognize that the level and dynamics of variable r^* are fundamentally uncertain. Based on this we evaluate various possible monetary strategies with the info-gap method for managing this uncertainty. This evaluates the trades-off of the potential loss for the central bank against the robustness of each strategy to fundamental uncertainty about r^*. We use a small scale macro model with a standard Taylor rule and a loss function that measures the deviation of the inflation and output gap from target (model 1). We also consider a model variant model with an augmented Taylor rule and a loss function which both include a financial variable (model 2). Based on both economic models we use the info-gap method to assess four monetary policy strategies, testing their robustness against uncertainty in r^*. The four strategies we evaluate are: 1) the historical response by the ECB, based on the estimated parameters of the Taylor rule; 2) an inert or passive monetary strategy in which the policy rate is kept constant at a low level (equivalent to a low-for-long if the interest rate is low from the start); 3) a strategy that is moderately responsive to the inflation gap, output gap and financial stability; 4) a responsive monetary policy strategy, which implies a stronger reaction to deviations from the policy targets than the historical response. In all strategies, except strategy 2, the central bank also responds to r^*, which is the benchmark of the policy rate in the long run.

The outcomes show that in both economic models the responsive monetary policy strategy is most robust to fundamental uncertainty about r^*, while achieving specified policy goals. This strategy performs best with regard to the trade-off between reaching the objectives (price stability and financial stability) and the robustness against uncertainty. This outcome is found in both economic models. The strategy that follows the historical ECB response ranks next and the moderately responsivestrategy ranks third in the trade-off between performance (objectives) and robustness. The inert or passive strategy performs worst according to this trade-off criterion[2].

The rest of this paper is organized as follows. Section 3 presents the two economic models and section 4 describes the policy strategies which the central bank can apply in the presence of fundamental uncertainty about r^*. Section 5 describes the model outcomes in case of no uncertainty. Section 6 introduces the info-gap approach, which is used in section 7 to evaluate the policy strategies. Section 8 concludes.

[2] This ranking of policy strategies is ordinal.

3. Model

This section presents a small scale macroeconomic model in which r^* plays a role. The model is similar to the basic three equations (backward looking) new-Keynesian model. The model has two variants; one with a standard Taylor rule (model 1) and one with an augmented Taylor rule and loss function which both include a financial variable (model 2). The model is used in the subsequent sections to simulate the effects of four different monetary policy strategies on key macroeconomic variables.

The benchmark model (model 1) consists of a Phillips curve (eq. 1), an aggregate demand curve (eq. 2) and a traditional Taylor rule (eq. 3a):

$$\hat{\pi}_t = \alpha_1 + \beta_1 \hat{\pi}_{t-1} + \beta_2 \hat{y}_t \tag{1}$$

$$\hat{y}_t = \alpha_2 + \beta_3 \hat{y}_{t-1} + \beta_4 \hat{r}_{t-2} \tag{2}$$

$$i_t = \alpha_3 r_t^* + \beta_5 \hat{\pi}_{t-1} + \beta_6 \hat{y}_{t-1} \tag{3a}$$

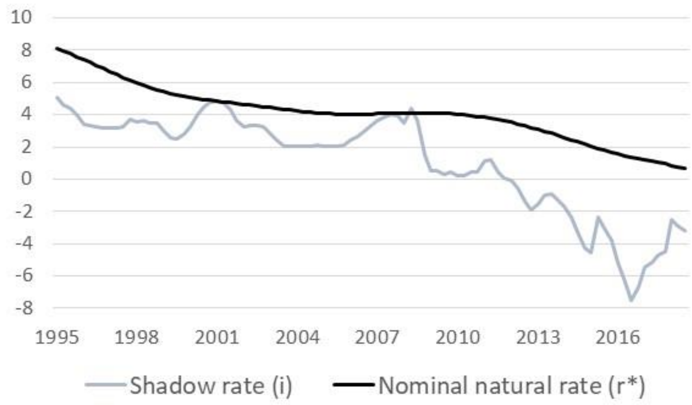
Eq. (1) is the Phillips curve, with $\hat{\pi}_t$ the inflation gap and $\hat{y}_t$ the output gap. The former is the inflation rate π_t as deviation from its target π^*(assumed to be 2% in the euro area). Eq. (2) links aggregate demand to the real interest rate gap $\hat{r}$, which equals the nominal policy or shadow rate (i_t) minus the natural rate in nominal terms (r^*) and the inflation gap.[3] In eq. (2) we take the 2-quarters lagged interest rate, to allow for lags in the monetary transmission mechanism (higher order lags are used as instruments in the estimation of the model, see Section 5). A reduction of policy rate i_t has an expansionary effect on output and vice versa. We assume that the policy rate (i_t) is not bound by zero; when it does go below zero it reflects a shadow rate that is a proxy for unconventional monetary policy measures. The shadow rate is an indicator for the effect of unconventional monetary policy on the monetary stance (e.g. Krippner, 2013; Lombardi and Zhu, 2018). It measures the effect of quantitative easing and forward guidance on the expectations component and the term premium component of bond yields. In our model we use the shadow rate as developed by Krippner (2013). It measures the expected lift-off of the policy rate from the zero lower bound. Since the shadow rate is based on forward interest rates it introduces a forward looking element in the model, reflecting financial market expectations.

Eq. (3) is a backward-looking standard Taylor rule, which assumes that the ECB reacts to the inflation gap and the output gap, next to changes in r^*. The backward-looking nature of the rule implies that the estimated coefficients reflect the extent to which the ECB reacted to those variables in the sample period. So it is a description of the historical policy (which may guide future policy strategies) and not

[3] The correction for the inflation gap makes the difference between the nominal policy rate and the nominal natural rate a real rate gap which is not influenced by differences in (implied) inflation rates included in the policy rate and natural rate.

a policy rule which optimizes a loss function, such as the one presented below. The natural rate (r_t^*) features in eq. (3) and in eq. (2) through $\hat{r}$. In the model, r^* is (implicitly) made up of the real interest rate plus inflation due in the steady state.[4] We include a time-varying natural rate, proxied by the trend in the long-term euro average government bond yield (Figure 1[5]). Approximating r^* as the trend in the long-term interest rate is in line with other time-series approaches for r^* (e.g. Del Negro et al., 2017; Johanssen and Mertens, 2016). Including the long-term interest rate in the Taylor rule assumes that this rate would eventually stabilize the economy, and thus it focuses on long-run stabilization (see for instance Roberts, 2018). While the shadow rate and r^* are different interest rate concepts, they both reflect the expected future policy rate (the shadow rate of Krippner is based on short and long-term forward interest rates).

Figure 1. Interest rate variables (nominal values)



In the augmented model 2, the Taylor rule is augmented with a financial variable (f in eq.(3b)) to which the central bank is assumed to react with its policy rate or with unconventional measures (the money market spread variable f is modelled in eq. (4) as a function of its own lag and the real interest rate gap $\hat{r}$). It is likely that monetary policy is relaxed when financial stress is high and vice versa. In augmenting the Taylor rule with a risk spread variable we follow for instance Woodford (2010) and Taylor and Zilberman (2016). The coefficient of the f reflects to what extent the ECB has reacted to this financial factor in the sample period.

$$i_t = \alpha_3 r_t^* + \beta_5 \hat{\pi}_{t-1} + \beta_6 \hat{y}_{t-1} + \beta_7 f_t \quad (3b)$$

[4] By taking the nominal natural rate we abstain from assumptions about the inflation rate that can be used to calculate the real natural rate and so implicitly take the uncertainty about this deflator into account in the info-gap analysis.

[5] Taking the euro average long-term bond yield as proxy for r* assumes country-specific natural rates in the euro area, reflecting for instance country-specific differences in potential growth amongst other factors..

The augmented model equations 1, 2 and 3b are supplemented with an equation for the risk spread (*f*),

$$f_t = \alpha_4 + \beta_8 f_{t-1} + \beta_9 \hat{r}_{t-1} \tag{4}$$

Besides its own lag, the risk spread is driven by the real interest rate gap $\hat{r}$. It is likely that the spread rises if the rate gap widens (if monetary policy becomes tighter due to an increase of the policy rate relative to the natural rate) and falls if the rate gap declines (monetary policy becoming looser), as found by Cenesizoglu and Essid (2012).

The outcome of each model is evaluated by a loss function, which weights the objectives of monetary policy. For model 1 we specify a standard quadratic loss function (Woodford, 2003; Rotemberg and Woodford, 1997),

$$L = \lambda_\pi \hat{\pi}_t^2 + \lambda_y \hat{y}_t^2 \tag{5a}$$

It assumes that monetary policy attempts to manage losses that are caused by inflation and output being away from their target (see Table 1.A in Annex 1 for a detailed definition). The central bank chooses an interest rate (and/or unconventional monetary policy measures as reflected in the shadow rate) to obtain adequately low values for the quadratic loss function. Coefficients λ are the relative weights with which the central bank pursues its objectives. Preference parameters in eq. (5a) take values that proxy the ECB preferences (and the estimated coefficients of the Taylor rule, see section 5), where λ_π=1.5 and λ_y=0.5. Since the Taylor rules in eq. 3a and 3b include the estimated coefficients, they reflect the historical response of the central bank over the sample period. This is not necessarily the most optimal loss minimizing reaction function. In section 4 we define some alternative response functions and test their performance in terms of loss as well as robustness against uncertainty in sections 5-7.

The loss function to evaluate economic outcomes of model 2 is extended with the modified financial variable, $\hat{f}$, to reflect that financial stability risks are also part of the loss function:

$$L = \lambda_\pi \hat{\pi}_t^2 + \lambda_y \hat{y}_t^2 + \lambda_f \hat{f}_t^2 \tag{5b}$$

Variable $\hat{f}$ is the deviation of variable f_t from its long-term mean. The mean value is assumed to reflect financial stability.[6] So in quadratic terms the loss function emphasizes the very high and low values of *f*, which reflect stressful and very quiet market conditions respectively. Both conditions may come with financial stability costs. In very quiet market conditions debts and risks usually build up, stimulated by low costs of financing (i.e. low risk spreads), while periods of very high risk spreads can reflect a

[6] In the loss function $\hat{f}_t^2$ is included in terms of the principal fifth root to scale the weight of *f* in the loss function, so $\hat{f}_t^2 = \sqrt[5]{\hat{f}_t^2}$.

financial crisis. We assume preference parameters in eq. (5b) taking values λ_π=1.5, λ_y=0.5 and λ_f=0.5. The three variables in the loss function are each included in percentage points.

4. Monetary policy strategies

In this section we distinguish four possible monetary policy strategies for the central bank to deal with fundamental uncertainty about the estimated natural rate (r^*). Since r^* captures all kinds of economic trends, such as in productivity growth and saving and investment patterns, the uncertainty about r^* captures uncertainty on trends in the economy in general. The policy strategy is assumed to last for two years beyond 2018Q3. Based on that, the model generates out-of-sample predictions of the economic variables and the loss function for the period 2018Q4 – 2020Q3. Taking 2018Q3 as the starting point of the simulations implies that the policy response has to deal with economic conditions that do not necessarily reflect an equilibrium situation. So the analysis mimics a real-time policy decision, for which the central bank takes into account all available information and the conditions at the starting point. As a decision theory, info-gap can guide the central bank in choosing a robust policy strategy based on known information and on uncertain parameters such as r^*.

1. Historical reaction function. This assumes that the central bank follows a similar strategy in the out-of-sample period as the average strategy in the in-sample period. Hence the estimated Taylor rules (eq. (3a) and (3b)) also determine the future policy strategy. This reflects a central bank that opts for a steady hand in times of great uncertainty. One reason for this strategy can be to bolster the credibility of the central bank. Market participants know the historical reaction function and by following this function the central bank may confirm market expectations, enhancing its credibility.

2. Inertial or passive strategy. This strategy assumes that the policy rate, i_t, is kept constant at the level of 2018Q3 for two more years (since at this starting point the (shadow) interest rate is at a low level the strategy is equivalent to keeping the policy rate "low-for-long" in our simulations). The reason for such a strategy could be that the policymaker thinks r^* is probably at a lower level than estimated and uncertain anyway. The strategy can be viewed as a version of the precautionary principle or an extreme variant of the attenuation principle, which leads to an unchanged policy until the policymaker knows the change is beneficial. So an inert strategy can be viewed as optimal to prevent a worse outcome in terms of the robust control concept. Keeping i_t constant would seem to ignore insight and understanding that suggest the need for change. However, in times of great uncertainty, this approach may be justified, assuming that model relations (and the underlying insight and understanding) based on the past do not hold for the future.

A similar argument was made by Coeuré (2018) to justify the ECB's forward guidance to keep the key ECB interest rates at their present low levels. He motivated this "risk management" approach by saying that parameter and model uncertainties imply that the costs of committing a type II error – of failing to anticipate a much slower than usual return of inflation to pre-crisis levels – may be uncomfortably high. Moreover, Coeuré also stated that unwinding policy accommodation in a multi-dimensional policy space is terra incognita for both financial markets and policymakers. De Groot and Haas (2018) show that an increase in policy inertia strengthens the signalling channel of negative interest rates and so boosts economic activity and inflation.

This policy strategy differs from the other strategies as it implies that the uncertainty about r^* only enters via $\hat{r}$ in the demand curve, eq. (2). In the other policy strategies, the uncertainty about r^* enters also via the Taylor rule (through i_t).

3. Modestly responsive. This strategy follows Brainard (1967), meaning that the policy reaction is less vigorous relative to the optimal policy where r^* were known. It is based on the assumption that, when facing uncertainty, the central bank should respond to shocks more cautiously and in smaller steps than in conditions without uncertainty. It would call for policy gradualism and less aggressive responses to economic shocks (see Section 1). In the model the modest response strategy is applied by reducing the estimated parameters β_5, β_6 and β_7 in the Taylor rules by a multiplicative factor of 0.5. In our model simulations, the more cautious response is applied symmetrically over the business cycle, meaning that policy rates will be lowered less vigorously in a business cycle downturn (when output and inflation gaps are negative) and raised less aggressively in an upturn (when output and inflation gaps are positive).

4. Responsive. This strategy prescribes a reaction function that is sensitive to the output gap, inflation gap and the financial variable (in model 2). In our model simulations, such a response is applied symmetrically over the business cycle, meaning that policy rates are lowered in a business cycle downturn and raised in an upturn. With this strategy the central bank may want to prevent financial imbalances from building up by raising the interest rate more strongly if the credit spread (variable *f*) is below its mean and lowering the interest rate more aggressively if the spread is above its mean. This strategy is applied in model 2, which includes a financial stability objective. The responsive reaction is applied in the model by raising the estimated parameters β_5, β_6 and β_7 in the Taylor rules by a multiplicative factor of 2.

5. Putative outcomes

We estimate the benchmark and augmented economic models assuming no fundamental parameter uncertainty ("putative model", which is an expression used in other disciplines to describe the generally assumed model). Both models are estimated as a system of equations by generalized method of moments (GMM, see Hamilton, 1994). The system estimator uses more information than a single equation estimator (i.e. the contemporaneous correlation among the error terms across equations) and therefore produces more precise estimates. We do not impose cross-equation restrictions. GMM takes into account the interdependencies among the equations in the model, while controlling for the endogeneity of regressors and for the correlation between the lagged dependent variables and the error terms.[7] The model is estimated with quarterly data for the euro area over the period 1995-2018Q3 (see Table 1.A in Annex 1 for a detailed description of the data). We assume that the path of r^* beyond the sample period is fundamentally uncertain, which makes 2018Q3 a logical starting point for analyzing possible monetary policy strategies.

5.1 Benchmark model: Model 1

The estimation outcomes of model 1 (benchmark model) in column 1 of Table 1.B in Annex 1 show that most coefficients are significant and have the expected sign. The low J-statistic indicates that the model is well specified. The inflation gap has a significant relationship with respect to its own lag and the output gap in the Phillips curve (eq. (1)). The significant negative constant term $\alpha_1$in the Philips curve relation indicates a downward drift in the inflation gap (and in inflation itself given the constant inflation objective) over the sample period. The coefficient of the real interest rate gap in the demand curve of eq. (2) (the interest rate channel of monetary policy) is negative and significant, although it has a low value which implies that monetary policy has a limited effect on the output gap and thereby on inflation. The Taylor rule estimates of eq. (3a) show that monetary policy reacts more strongly to the inflation gap than to the output gap (given larger and significant coefficients) and in line with the ordering of objectives in the ECB. The estimated parameter for the inflation gap is somewhat lower and the estimated parameter for the output gap is somewhat higher than is generally assumed in the literature (1.5 and 0.5 respectively, these values we also include in the loss function as λ_π and λ_y respectively). The significant positive value of α_3 implies that the policy rate has followed our proxy of the natural rate over the sample period, although partly ($\alpha_3 \approx 0.5$), implying that monetary policy has been accommodative on average in the sample period.

Figure 2 shows the dynamic outcomes of the output and inflation gaps, $\hat{y}_t$ and $\hat{\pi}_t$, the nominal

[7] The model is estimated by heteroskedasticity and autocorrelation consistent GMM (HAC), applying prewhitening to soak up the correlation in the moment conditions. Lags of the variables (nearest lags) are used as instruments.

shadow rate (i_t) and the real interest rate gap ($\hat{r}$), for model 1, strategy 1 (historical response). These are the conditional forecasts for two quarters in-sample ($t = 1$ and $t = 2$) and 8 quarters out-of-sample *(t=3* to *t=10*). In all out-of-sample simulations, the value of r^* is kept constant at its 2018Q3 level, based on the assumption that the latest estimate of r^* is the best proxy for future observations, given the fundamental uncertainty about the level and dynamics of r^*. If the central bank would follow the historical reaction function (strategy 1) the shadow rate i_t jumps from around -3% in $t = 1$ and $t = 2$ ((the actual in-sample shadow rate) to positive numbers in the out-of-sample period. It illustrates that the actual shadow rate in 2018Q3 – the last in-sample quarter - deviates substantially from what the (estimated) Taylor rule would suggest to be the shadow rate. Hence, at the end of the in-sample period, monetary policy was much looser than it ought to be if the historical reaction rule would have been followed. According to that rule, the shadow rate responses to sthe inflation and output gap, which are both positive at the start of the simulation period. The positive value of i_t from *t=3* onward also relates to the positive value of r^*, which is partly tracked by the shadow rate in the Taylor rule. As a result, of that jump the real rate gap $\hat{r}$ quickly closes and stabilizes around 0% in the out-of-sample period. The negative interest rates in $t = 1$ and $t = 2$ support output so that the output gap becomes increasingly positive over the out-of-sample period, touching around 1.5% after 10 quarters. As a result, the inflation gap is closed in period 2 and becomes increasingly positive. The continuous rise of the output and inflation gap over the forecast horizon shows that the historical reaction function does not stabilize the economy, since that would require a higher policy rate. It indicates that the estimated model coefficients are not steady state values, but values that reflect the monetary policy as conducted in the (limited) sample period.

Figure 3 shows the conditional forecasts for model 1, strategy 2. This inert or passive strategy implies that i_t remains constant at -3.18% and that $\hat{r}$ becomes even more negative during the out-of-sample period. As a result, both $\hat{y}_t$ and $\hat{\pi}_t$ become increasingly positive over the out-of-sample period, also showing that this policy strategy does not stabilize the economy.

Figures 4-5 show the conditional forecasts for model 1, strategies 3 and 4. They look similar to the outcomes of strategy 1 (historical response). If the central bank would follow an modest response strategy the output and inflation gaps increase somewhat more than in a historical strategy. If the central bank would apply the responsive strategy, both i_t and $\hat{r}$ keep on rising during the out-of-sample period (Figure 5). As a result of the latter strategy, both $\hat{y}_t$ and $\hat{\pi}_t$ stabilize at levels closer to zero than in the other policy strategies, although the output and inflation gaps do not close completely during the sample period, due to the low value of the interest rate gap coefficient in the demand equation, which limits the effect of monetary policy This is specific to our small scale economic model, which does not include the full set of monetary transmission channels through which the interest rate can influence aggregate demand and inflation.

Figure 2. Outcomes model 1, strategy 1

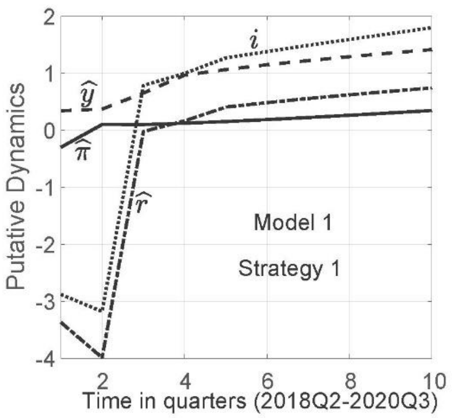
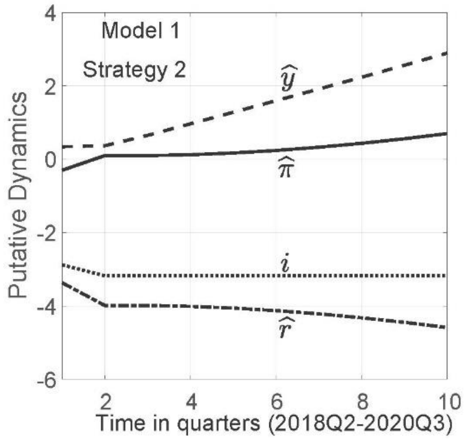


Figure 3. Outcomes model 1, strategy 2



Note: $\hat{y}_t$ and $\hat{\pi}_t$ are the output and inflation gaps, *i* is the nominal shadow rate and $\hat{r}$ is the real rate gap (the difference between *i* and $\hat{r}$ equals $r_t^* + \hat{\pi}_t$ (see Table 1.A in Annex 1).

Figure 4. Outcomes model 1, strategy 3

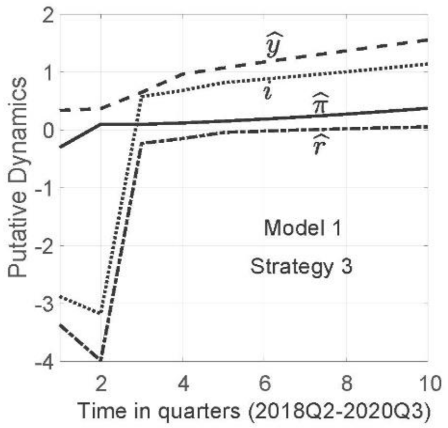
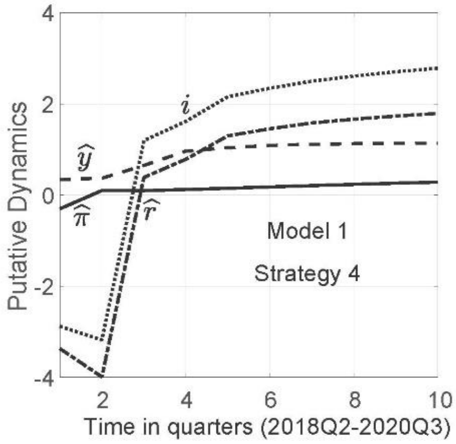


Figure 5. Outcomes model 1, strategy 4



Note: $\hat{y}_t$ and $\hat{\pi}_t$ are the output and inflation gaps, *i* is the nominal shadow rate and $\hat{r}$ is the real rate gap (the difference between *i* and $\hat{r}$ equals $r_t^* + \hat{\pi}_t$ (see Table 1.A in Annex 1).

Figure 6 compares the loss functions of the four policy strategies based on model 1; these are the outcomes of eq. (5a). Not surprising, the loss of strategy 2 is the largest, since following an inert or passive strategy implies that the output and inflation gaps tend to deviate most from the policy target over the simulation horizon. At the end of the simulation period ($t = 10$) policy strategy 4 (responsive) generates the lowest loss (around 0.75), followed by policy strategy 1 (historical reaction function) and strategy 3 (modestly responsive).

Figure 6. Loss functions model 1

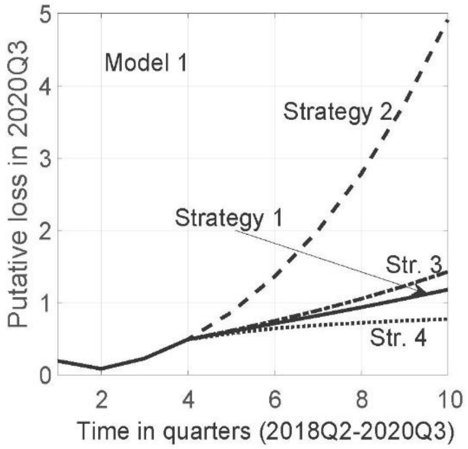
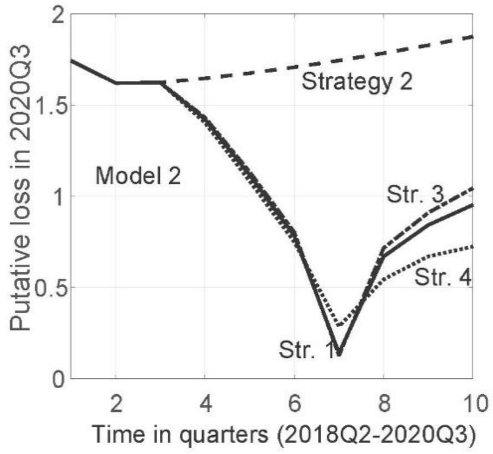


Figure 7. Loss functions model 2



5.2. Augmented Taylor rule: Model 2

In model 2, the Taylor rule is augmented with financial variable *f*. So in this model the central bank reacts to the inflation gap, the output gap as well as to financial developments, captured by the credit spread variable *f*. Column 2 in Table 1.B in Annex 1 provides evidence that the ECB indeed reacted to financial developments with monetary policy in the in-sample period. The coefficient of variable *f* is significantly negative, meaning that the interest rate (or shadow rate) is reduced in response to a rising spread and vice versa. The coefficients of the other variables in the augmented model have similar signs as in the benchmark model. As in model 1, in the out-of-sample simulations the value of r^* is kept constant at its 2018Q3 level.

If the central bank would follow the historical reaction function (strategy 1) in model 2, the shadow rate i_t jumps from around -3% in $t = 1$ and $t = 2$ to nearly 1% at $t = 3$, after which it gradually declines to negative values at the end of the simulation horizon (Figure 8). The real rate gap $\hat{r}$ follows a similar pattern. The output and inflation gap both converge to zero over the simulation horizon. The outcomes of $\hat{y}_t$ and $\hat{\pi}_t$ in strategy 3 (modestly responsive) are similar to those in strategy 1 (Figure 10). If the central bank would apply a responsive strategy (strategy 4) the shadow rate and real rate gap jump to somewhat higher levels at the first out-of-sample quarter ($t = 3$) compared to strategies 1 and 3, but thereafter both i_t and $\hat{r}$ decline more strongly than in the other two strategies (Figure 11). This outcome differs from the outcomes of i_t and $\hat{r}$ in model 1, strategy 4, in which both variables stabilize at the end of the simulation horizon . The explanation for this difference is that in model 2 the central bank also reacts to the financial variable (*f*) and more strongly so in strategy 4 than in the other strategies. Since in the out-of-sample period variable *f* increases above its mean (except in strategy 2), the central bank reduces its policy or shadow rate in response to *f*, as follows from the augmented Taylor rule in eq. (3b).

This response is stronger in strategy 4 (responsive) than in the other policy strategies, which explains that i_t and $\hat{r}$ decline relatively more strongly in strategy 4 as the credit spread rises above its mean.

Figure 8. Outcomes model 2, strategy 1

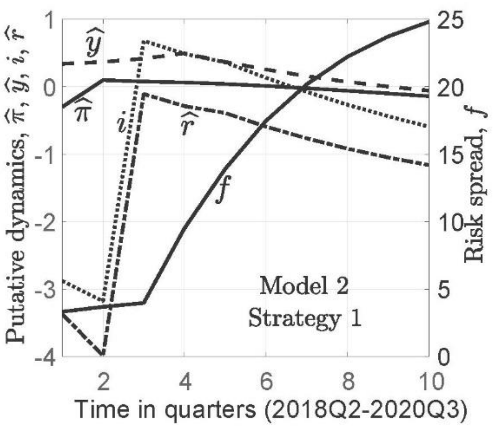
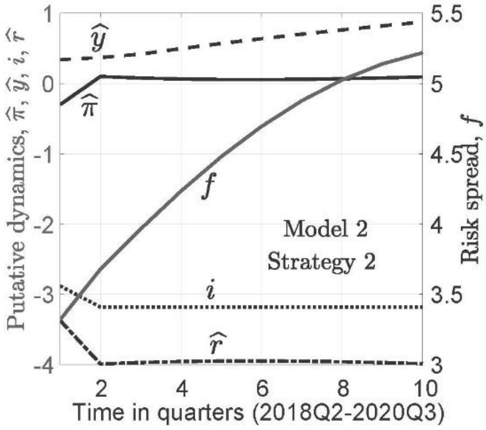


Figure 9. Outcomes model 2, strategy 2



Note: $\hat{y}_t$ and $\hat{\pi}_t$ are the output and inflation gaps, *i* is the nominal shadow rate, $\hat{r}$ is the real rate gap and *f* is the financial variable (the difference between *i* and $\hat{r}$ equals $r_t^* + \hat{\pi}_t$ (see Table 1.A in Annex 1).

Figure 10. Outcomes model 2, strategy 3

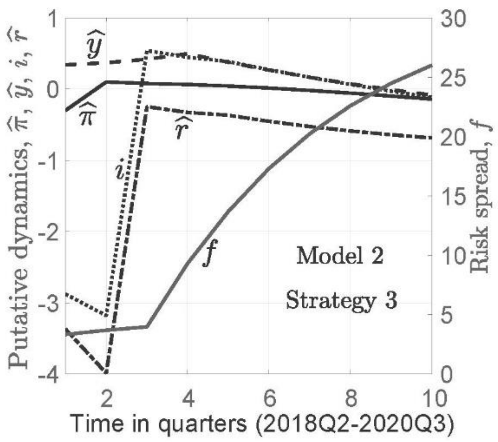
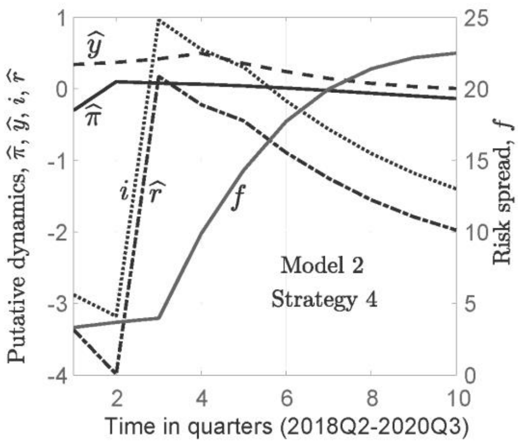


Figure 11. Outcomes model 2, strategy 4



Note: $\hat{y}_t$ and $\hat{\pi}_t$ are the output and inflation gaps, *i* is the nominal shadow rate, $\hat{r}$ is the real rate gap and *f* is the financial variable (the difference between *i* and $\hat{r}$ equals $r_t^* + \hat{\pi}_t$ (see Table 1.A in Annex 1).

Figure 7 compares the loss functions of the four policy strategies produced by model 2; these are the outcomes of eq. (5b). They differ clearly from the loss functions of model 1 in Figure 6, which is due to the influence of the financial variable *f*. In the beginning of the simulation horizon variable *f* is below its mean, by which it has an upward effect on the loss in model 2. In the course of the out-of-sample period variable *f* increases towards its mean and this point is reached in $t = 7$ when policy strategies 1, 3 and 4 are applied. As a result, the loss is at its lowest level at $t = 7$. Thereafter, *f* rises above its mean, which causes the responsive policy strategy (4) to be most successful. Hence, at the end of the simulation horizon ($t = 10$) policy strategy 4 (responsive) generates the lowest loss (around 0.7), followed by policy strategy 1 (historical response) and strategy 3 (modestly responsive). Not surprisingly, the loss in strategy 2 is largest, because following a passive strategy implies that the output gap, inflation gap and financial variable tend to deviate most from their policy targets.

6. Info-gap approach

6.1. Background

This section introduces info-gap as a risk management strategy to deal with fundamental uncertainty about the natural rate of interest in monetary policy. As a starting point, we assume that for the central bank r_t^* is an unobservable variable for which it has a model-based series of values. Given a sequence of r_t^* values together with other historical data, dynamic equations, (1), (2), (3a) or (3b) are used to predict the future inflation gap and output gap. These predictions underlie the loss function in eq.(5a) or (5b). The monetary policymaker wants the value of the predicted loss (expressed in terms of deviations from target) to be small, and in any case less than a critical value: a largest acceptable loss. Similarly, the central bank would like to know what values of critical loss are realistic and how confident it can be that these values won't be exceeded. Based on that it would like to explore the policy implications of these questions.

The challenge, however, is that r_t^* is highly uncertain and can deviate either above or below the model-based values by an unknown amount. By nature, the deviation of r_t^* is persistent, or slowly changing, and the rate of change is unknown. That is, its actual value over a time interval of T quarters in the future may differ, perhaps greatly, from the anticipated values for that interval. r_t^* can either increase or decrease and can even be negative.

The relevant question for the central bank is which monetary policy strategy (historical reaction, inert, modestly responsive, or responsive) is most robust to uncertainty in the magnitude and temporal behavior of the deviation of r_t^* from its modelled value, while assuring that the loss is acceptable.

6.2. Info-gap model of uncertainty

To address this question with an info-gap approach, we suppose that r_t^* varies over a future interval of T quarters according to the following relation, which is a generator model of future values of r^* with uncertain parameters:

$$r_{1+j}^* = (1+\varepsilon j) r_1^*, \quad j = 0 \ \ldots \ T-1 \tag{6}$$

Thus r_1^* is the first future value of r^*. r_{1+j}^* changes by a fraction ε of r_1^* each quarter, and the sign of r_{1+j}^* can change at most once during the future interval. r_{1+j}^* increases over time if and only if r_1^* and ε are both positive or both negative.

The first future value, r_1^*, and the rate of change, ε, are both unknown. Regarding r_1^*, the policymaker knows an estimated value based on historical data and modeling up to and including time 2018Q3, denoted $\tilde{r}_1^*$. However, the policymaker does not know the fractional error of the true (or more realistic) value, r_1^*, from this estimate. That is, the ratio $|(r_1^* - \tilde{r}_1^*)/\tilde{r}_1^*|$ is of unknown magnitude. Likewise, regarding ε, the policymaker knows a positive scale parameter $\tilde{\varepsilon}$ that determines an anticipated typical range of ε values, but the true (or truly realistic) value of ε is unknown. The uncertainty in r_1^* and ε is represented by the following info-gap model of uncertainty, which is an unbounded family of nested sets of r_1^* and ε values:

$$U(h) = \left\{ r_1^*, \varepsilon : \quad \left| \frac{r_1^* - \tilde{r}_1^*}{\tilde{r}_1^*} \right| \le h, \quad \left| \frac{\varepsilon}{\tilde{\varepsilon}} \right| \le h \right\}, \quad h \ge 0 \tag{7}$$

At any value of *h*, the set *U(h)* contains an interval of possible values of r_1^* and an interval of possible ε values. The value of *h* is unknown, so the info-gap model of uncertainty is an unbounded family of nested sets, *U(h),* for all $h \ge 0$. As *h* increases, these sets become more inclusive, which endows *h* with its meaning as an horizon of uncertainty.

6.3. Robustness and policy prioritization

The dynamic equations, together with historical data, enable us to calculate the loss function for each policy strategy and both economic models 1 and 2, as a function of the unknown natural rates r_{1+j}^* over *T* future quarters. Let $L(k, m, t, r_1^*, \varepsilon)$ denote the loss at the end of future quarter *t* when using policy strategy *k* and model *m*, if the deviant natural rates are determined by r_1^* and ε according to eq.(6). Let L_c denote the largest acceptable loss in terms of missing the inflation, output and financial stability objectives. The relevant question for the central bank is how small a loss it can be confident of not exceeding. Formally, the performance requirement for strategy *k* with model *m* is:

$$L(k, m, t, r_1^*, \varepsilon) \le L_c \tag{8}$$

at the end of some future quarter *t*.

When considering policy *k* and model *m*, the robustness question is: what is the largest horizon of uncertainty, *h*, up to which the central bank is guaranteed that the loss at time *t* will be acceptable? Formally, the robustness of policy strategy *k* with model *m* at time *t*, with uncertain deviation of the future r^*, is defined as:

$$\hat{h}(k,m,t,L_c) = \max\left\{h: \left(\max_{(r_1^*,\varepsilon)\in U(h)} L(k,m,t,r_1^*,\varepsilon)\right) \le L_c\right\} \tag{9}$$

Note that the robustness does not depend on either r_1^* or ε, both of which are uncertain. The robustness does depend on the estimated values of these variables, $\tilde{r}_1^*$ and $\tilde{\varepsilon}$. The value of the robustness is the greatest fractional error in these estimates up to which the loss is acceptable. Thus, if the robustness is large then the dependence on the estimated values is small, and the loss is acceptable even if these estimates err greatly.

The central bank prefers policy strategy *k* with model *m*, over policy strategy *j* with model *n*, if and only if the former is more robust:

$$(k,m) \succ (j,n) \quad \text{if and only if} \quad \hat{h}(k,m,t,L_c) > \hat{h}(j,n,t,L_c) \tag{10}$$

Note that the preference may depend on the critical value of the loss, L_c. That is, there may be a preference reversal: the preference may change as the policymaker alters the critical value of the loss. This is expressed by crossing of the corresponding robustness functions when $\hat{h}(k,m,t,L_c)$ is plotted vs. L_c, as we will show subsequently.

7. Robust policy strategies

7.1. Three properties of the robustness function

Figure 12 shows the robustness function, $\hat{h}(k,m,t,L_c)$, vs. the critical loss, L_c, based on model 1 for each of the 4 monetary policy strategies. The last observed quarter is 2018Q3, and the loss is evaluated at the end of 2020Q3, 8 quarters in the future. Figure 13 is an expanded version showing strategies 1, 3 and 4. As priors for the info-gap analysis, the central bank assumes that the estimated first future value of the natural rate $\tilde{r}_1^*$ equals the last known historical observation (0.71, at 2018Q3) and that its rate of change, $\tilde{\varepsilon}$, equals 0.2. For illustration, this choice implies that $-2 \le \varepsilon j \le 2$ in eq.(7) at the 10th time step (j = 10 predicts the 8th quarter into the future), when h = 1. This section explores the robustness to these highly uncertain estimates.

Note that each robustness curve in Figure 12 sprouts off of the horizontal axis at a specific value of critical loss at which the robustness against uncertainty equals zero. For instance, the curve for strategy 2 (inert or passive strategy) intersects the L_c axis at 4.91. This means that robustness is zero for strategy 2 with this critical value of loss. A basic theorem of info-gap theory asserts, in the present context, that the value at which the robustness curve reaches the horizontal axis is precisely the predicted putative value of the loss. That is, if one adopts $L(k, m, 8, \tilde{r}_1^*, \tilde{\varepsilon})$, the putative best estimate of the loss at the 8th future quarter, as the critical value L_c, then the robustness to uncertainty in r_1^* and ε is precisely zero. This is the **zeroing** property: A best estimate has no robustness to uncertainty in the data and models upon which that estimate is based. This property means that this value of the loss (based on best estimates of the model) is not informative for the performance of the policy strategy if the central bank takes the uncertainty about the natural rate into account.

The second thing to note in the robustness curves of Figures 12 and 13 is that they are monotonically increasing functions. The positive slope expresses a **trade off**: greater robustness (a favorable property) is obtained only by accepting greater critical loss (which is not favored). For instance, in Figure 13 we see that the robustness of strategy 4 (responsive) is nil at a critical loss of 0.77, while the robustness $(\hat{h})$ equals 2.5 when the critical loss is 2.03. A robustness of 2.5 means that r_1^* can deviate fractionally from its estimated value, $\tilde{r}_1^*$, by a factor of 2.5, and ε can take any value in the interval $[-2.5\tilde{\varepsilon}, +2.5\tilde{\varepsilon}]$, without the loss exceeding the value 2.03. The zeroing property asserts that strategy 4 (with model 1) has no robustness for its putative loss of 0.77, while the trade off property shows that the robustness $(\hat{h})$ equals 2.5 only for the larger loss of 2.03.

The third property to observe is that, as a result of the trade off of robustness vs. loss, one can think of the slope of the robustness curve as a **cost of robustness**. High slope entails low cost of robustness. Figure 12 shows that the robustness curves for strategies 1, 3 and 4 are rather steep, compared with the curve for strategy 2 (inert). This means that enhancing the robustness of strategy 2 requires greater increase of the critical loss than for the other strategies. For instance, we mentioned before that increasing the robustness of strategy 4 (with model 1) from 0 to 2.5, requires an increase of 1.26 in the critical loss (from 0.77 to 2.03). For strategy 2, the same increase in robustness requires an increase of 6.79 in critical loss (from 4.91 to 11.7).

So a preliminary conclusion is that not only the putative loss of the passive strategy 2 is greater than for the other strategies, the cost of robustness is greater for strategy 2 as well. This suggests that an inert or passive strategy is inferior to the other monetary policy strategies. Keep in mind that the cost of robustness is a different property from the putative loss, precisely because trade off and zeroing are distinct properties of the robustness function. In the next section we will show that a policy strategy can have greater (worse) putative loss, but lower (better) cost of robustness than other strategies. This will result in crossing of the robustness curves, with profound policy implications.

Figure 12. Robustness curves model 1 (all four policy strategies)

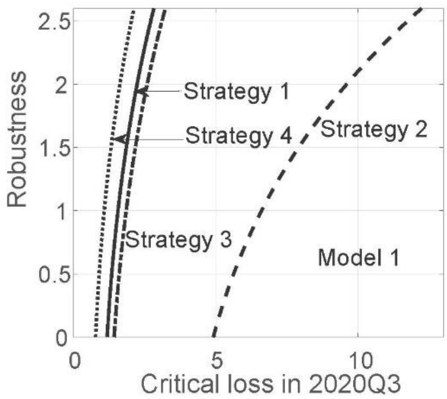
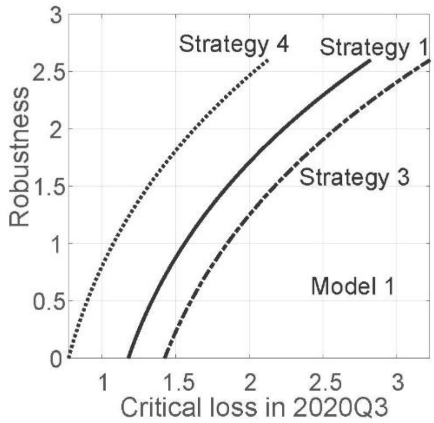


Figure 13. Robustness curves model 1 (strategies 1, 3, 4)



7.2. Robust dominance

Figures 12-15 show the robustness curves for all four strategies and both models. They indicate that strategy 4 (responsive response) is more robust than all other strategies, in both models, for all values of critical loss shown. In the sense of the preference relation of eq.(10), strategy 4 is **robust dominant**: preferred over all other strategies for each model (as indicated by the relative position of the robustness curve of strategy 4, which is above the curves of the other strategies). Furthermore, the robust ranking is the same in all 4 figures: strategy 4 is preferred over strategy 1 (historical reaction), which is preferred over strategy 3 (modestly responsive), which is preferred over strategy 2 (inert). While the robustness curves for policy strategies 1, 3 and 4 seem quite similar in Figures 12 and 14, the expanded versions in Figures 13 and 15 show that the robustnesses are quite different, resulting from the relatively low cost of robustness of these curves. So based on the robustness curves, we conclude that a responsive monetary policy strategy is superior to the other monetary policy strategies. This implies that, given the economic state on which the model estimations are based, a vigorous response to deviations from the inflation, output and financial stability objectives is preferred.

The robust prioritization of the strategies is similar to the putative prioritization (indicated by the crossing of the robustness curves with the horizontal axis). This means that a putative outcome-optimizing central bank would have the same prioritization of the strategies as a robust-satisficing central bank. However, the prioritization is based on a different reasoning, which leads to different anticipations about the future. The optimizer prefers strategy 4 because the models predict that it has the best outcome. The robust-satisficer prefers strategy 4 because it is most robust to error in the models. This implies that though the prioritization of the strategies is the same, the anticipation of future outcomes is different. The outcome optimizer assumes it is a reasonable anticipation that the models predict a loss of 0.6 (in Figure 13 for example). The robust-satisficer, however, takes into account that the robustness to uncertainty of future loss being 0.6 is zero, so there is little confidence in achieving it.

Figure 14. Robustness curves model 2 (all four policy strategies)

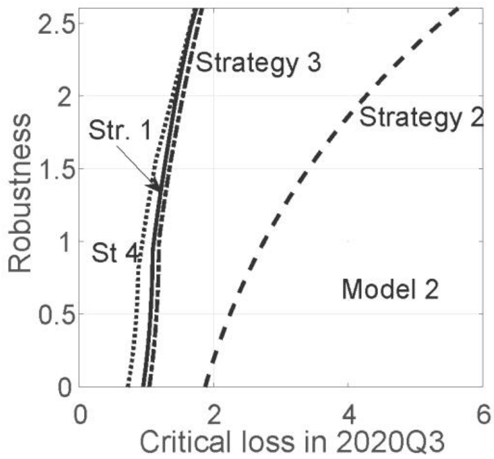
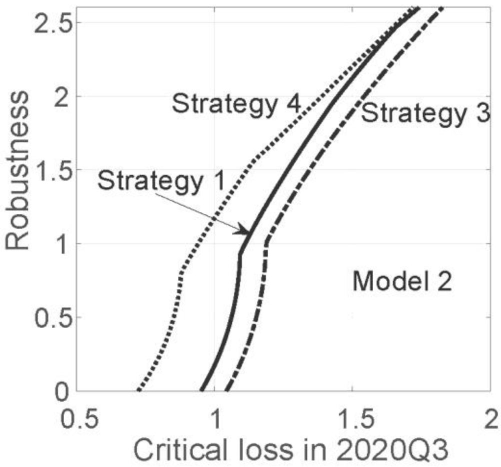


Figure 15. Robustness curves model 2 (strategies 1, 3, 4)



7.3. Preference reversal

The robust dominance shown in Figures 12-15 is a result of the specific numerical realization in this case, when the loss is evaluated at the end of the final future quarter. Let us now consider the *cumulative* loss over all future quarters, defined as:

$$L_{\text{cum}} = \sum_{t=t_1}^{t_2} L(k, m, t, r_1^*, \varepsilon) \tag{11}$$

where t_1 is the first future quarter, 2018Q4, and t_2 is the 5th future quarter, 2019Q4.

We consider the robustness of strategies 1, 3 and 4 with model 2, when the performance requirement in eq.(8) is applied to the cumulative loss during 5 quarters into the future, rather than the final loss after 8 quarters as before, resulting in the robustness curves of Figures 16 and 17. All other parameters are the same. Figure 16 is an expanded version of the lower portion of Figure 17.

Based on the predictions 5 quarters ahead, the horizontal intercepts in Figure 16 show that the cumulative putative loss of strategy 1 (historical reaction) is less than for strategies 3 and 4, and that strategy 4 (responsive) has the greatest putative loss. Thus strategy 1 sprouts from the horizontal axis further to the left than the other strategies, and strategy 1 has greater robustness to a low critical loss. This differs from the outcome shown in Figure 13, in which policy strategy 4 has the lowest putative loss. Since the cumulative putative losses of the strategies are quite similar (cumulative losses of about 4.98, 5.12 and 5.15 for strategies 1, 3 and 4, respectively), an optimizing central bank might be indifferent between the strategies. However, for a robust-satisfizing central bank these outcomes have zero robustness to uncertainty and therefore are a weak basis for prioritization.

However, also in the case of 5 quarters ahead predictions, the cost of robustness for strategy 4 is lower than for strategies 1 and 3, so strategy 4's curve is steeper and it crosses the other curves. Hence, the responsive strategy is robust-preferred over the other strategies, according to eq.(10), for critical loss greater than 5.2, while it is robust-dominated at lower critical loss by the other policy strategies. However, the differences between the curves do not seem to be substantial, which suggests that the central bank is robust-indifferent between the strategies. This is the same conclusion as for the optimizing central bank, but for a different reason. As noted earlier, trade off and zeroing are distinct properties of the robustness function, and in the present case there is a **reversal of preference** between the strategies as the value of critical loss changes.

Figure 16. Robustness curves model 2 (cumulative loss, strategies 1, 3, 4)

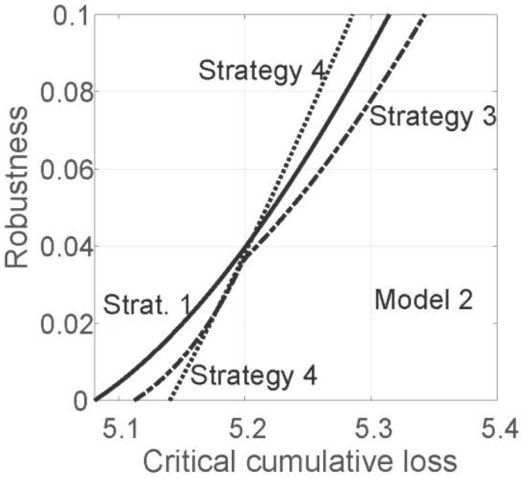
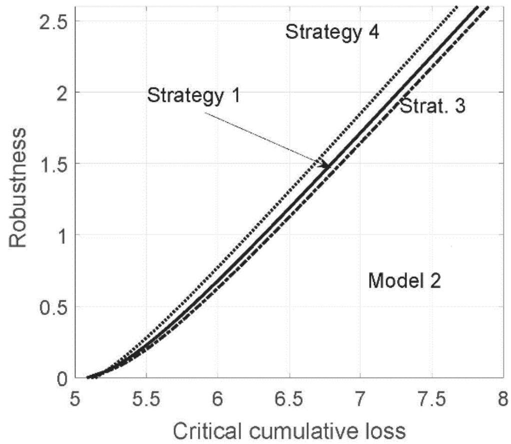


Figure 17. Robustness curves model 2 (cumulative loss, strategies 1, 3, 4)



7.4. How much robustness is enough?

Eq.(10) establishes a criterion for preference between policy options based on the robustness to uncertainty. For example, in Figure 15 we see that, with model 2, strategy 4 (responsive) is preferred over strategy 1 (historical reaction) which is preferred over strategy 3 (modestly responsive), for the range of critical loss shown. At critical loss of 1.2, the robustnesses ($\hat{h}$, along the vertical axis) of strategies 4, 1 and 3 are 1.66, 1.32 and 1.06, respectively. Strategy 4 is robust preferred, but the central bank may raise the question whether robustness against natural rate uncertainty of 1.66 is enough. This is a judgment for which various conceptual tools are available, such as analogical reasoning or consequence severity (Ben-Haim, 2006). Without going into much detail, the basic intuition devolves from the info-gap model of uncertainty, eq.(7). With regard to r^*, robustness of 1.66 means that the fractional error of the estimated value can be as large as 1.66 without violating the corresponding performance requirement on the loss. In other words, this is a 166% robustness with respect to r^*, and similarly for robustness with respect to ε. Contextual understanding and subjective judgment is needed to decide if this is adequate.

Suppose, hypothetically, that the central bank's understanding is that the natural rate, r^*, could deviate from its historical value by 100%, or 200%, or perhaps even more. Without knowing, of course, the degree of error, the central bank would likely conclude that robustness of 1.66 is inadequate. In other words, the conclusion from Figure 15 would be that none of these policy alternatives can be relied upon to keep the loss below the value of 1.2 (the critical loss in strategy 4 if $\hat{h} = 1.66$). One response would be to seek other policy alternatives. Alternatively, the central bank could consider whether a greater critical loss is acceptable. For instance, if loss as large as 1.7 is acceptable, then Figure 15 shows robustnesses for strategies 4, 1 and 3 taking values of 2.56, 2.52 and 2.34, respectively. The central bank may judge that 250% robustness is adequate, (though perhaps not entirely satisfactory because it has to accept a critical loss of 1.7), and decide to use strategy 4 (or strategy 1 whose robustness is very nearly the same).

The judgment of how much robustness against natural rate uncertainty is enough for the central bank, like most judgments of safety and reliability, is difficult and subjective. It may for instance depend on the inflation process and developments in the economy. If these are driven by shocks beyond the control of the central bank (e.g. global or demographic shocks that drive the natural rate), it cannot perfectly control inflation, and especially headline inflation. A greater tolerance of deviations of inflation from the objective or a longer horizon over which it is expected to be achieved could make communication easier about what monetary policy can deliver and so contribute to the central banks' credibility. In such circumstances, the central bank might accept a higher critical loss (in terms of deviations from the policy objectives) to improve the robustness of its policy strategy. The robustness curves provide quantitative support for such deliberations. Nonetheless, the robust prioritization in eq.(10) is unambiguous, and if a decision must be made from among a specified set of options, that decision is clear. The policymaker should always keep in mind, though, that the greatest available robustness may not be a solid guarantee of success. Shocks that go beyond the robustness can still happen, but within this horizon the maximum possible loss associated with a certain policy strategy is acceptable. Moreover, the robust prioritization of the policy strategies that follows from the info-gap analysis is unambiguous.

8. Conclusions

Info-gap theory ranks different monetary policy strategies in terms of their robustness against natural rate uncertainty, while also achieving specified policy goals. This approach is based in part on the zeroing property of the robustness function, stating that putative predictions have no robustness and thus are a poor basis for policy selection. "Optimization" is commonly implemented by using the best available knowledge to predict outcomes of policy alternatives and to select the policy whose predicted outcome is best, also in macro models that include the (inherently uncertain) natural rate of interest. However, monetary policy strategies based on such models essentially fail in the presence of Knightian uncertainty. The zeroing property demonstrates that the putative performance of a policy strategy - measured by a loss function and best estimates of an economic model - is not informative if the central bank takes the uncertainty about the natural rate into account. As stated by the trade off property of the robustness function, only worse-than-predicted (putative) outcomes have robustness to uncertainty and the central bank must make judgments about the acceptability of this trade off for each policy strategy.

Following this reasoning, we evaluate different monetary policy strategies based on the trade-off between reaching the objectives (closing the inflation and output gap and achieving financial stability) and robustness against uncertainty in the natural rate. The optimal policy is the robust-optimal strategy that maximizes the robustness and satisfices the policy outcome. This distinguishes our approach from robust control, which aims at minimizing the loss of the worst outcome, but does not account for the choice between robustness against uncertainty and performance of a policy strategy.

The application to the euro area, over a forecast horizon of two years, shows that a monetary policy strategy which sufficiently responds to deviations from the inflation and output gap, is more robust against natural rate uncertainty than the historically followed reaction function and a modestly responsive policy strategy. An inert or passive strategy, which keeps the policy rate constant over the forecast horizon might be optimal to prevent a worse outcome (such as a recession or deflation) in terms of the robust control concept, but it turns out to be least robust in the info-gap approach when applied to the economic situation in the euro area. There are possible constellations in which the responsive policy strategy can have greater (worse) putative loss, but lower (better) cost of robustness than other strategies. This results in crossing of the robustness curves, implying that a responsive policy ultimately turns out to be the preferred strategy. The preference ranking of the monetary policy strategies also holds if the macro model is extended with a financial variable, to which the central bank reacts for financial stability reasons. The info-gap model suggests that a strategy that is responsive to credit spreads is preferred. This response should be symmetric; by relaxing monetary policy if credit spreads are relatively high (to prevent a financial crisis) and by tightening if spreads are low (to lean against potential financial excesses).

An important caveat is that the outcomes are dependent on the model set-up and on the economy and sample period on which the estimations are based. This implies that the policy strategies will not necessarily be ranked similarly in all situations in which fundamental uncertainty about r^* plays a role. The ranking could also be different if the policymaker assumes that another model would better describe the economy, for instance a more complex forward-looking new-Keynesian model instead of the simple backward-looking model that we use. Simulations based on another model will generate different dynamics, but info-gap is also applicable to rank policy strategies based on more complex models. Moreover, the consequences of fundamental uncertainty in other variables or model equations, such as the slope of the Phillips curve, could be studied. Such applications of info-gap are left for further research.

Annex 1

Table 1.A. Definition of model variables

$\hat{\pi}_t$ = inflation gap (realized inflation rate – inflation objective of 2%)

$\hat{y}_t$ = output gap (real GDP – potential GDP) / potential GDP); potential output is based on the definition of Oxford Economics.

i_t = relevant policy rate (Eonia rate until 2008, shadow rate from 2008 onward, source: Krippner)

$\hat{r}_t$ = real interest gap ($i_t - r_t^* - \hat{\pi}_t$)

f_t = financial variable (proxied by spread between 3 months Euribor over 3 months OIS rate)

$\hat{f}_t$ = deviation of financial variable, f_t, from its long-term mean, $\bar{f}$.

r_t^* = HP filtered 10 years euro average nominal government bond yield (lamba 1,600)

Table 1.B. Estimation outcomes (euro area, sample period 1995-2018q3, quarterly observations)

	Model 1	**Model 2**
	Benchmark model	Augmented model
Phillips curve		
α_1	-0.046*	-0.045***
	(0.026)	(0.017)
β_1	0.914***	0.917***
	(0.046)	(0.033)
β_2	0.081***	0.076***
	(0.016)	(0.012)
IS curve		
α_2	0.096	-0.106
	(0.114)	(0.081)
β_3	0.949***	0.945***
	(0.034)	(0.034)
β_4	-0.055**	-0.052***
	(0.022)	(0.020)
Taylor rule		
α_3	0.524***	0.563***
	(0.064)	(0.053)
β_5	1.308**	1.99***
	(0.614)	(0.349)
β_6	0.761***	0.568***
	(0.175)	(0.136)
β_7		-0.0321***
		(0.004)
Risk spread		
α_4		6.313**
		(2.370)
β_8		0.840***
		(0.041)
β_9		1.363**
		(0.541)
N	279	337
J stat	0.013	0.055

Outcomes of GMM estimation of system of equations 1, 2,3 and 4, with data for the euro area. Sample period 1990-2018q3 (quarterly observations). The model is estimated by heteroskedasticity and autocorrelation consistent GMM (HAC), applying prewhitening to soak up the correlation in the moment conditions. Standard errors in parentheses, where *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level.

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