DNB Working Paper

No. 685 / May 2020

Monetary Policy, Productivity, and Market Concentration

Andrea Colciago and Riccardo Silvestrini

DeNederlandscheBank

EUROSYSTEEM

Monetary Policy, Productivity, and Market Concentration

Andrea Colciago and Riccardo Silvestrini*

* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

Working Paper No. 685

May 2020

De Nederlandsche Bank NV
P.O. Box 98
1000 AB AMSTERDAM
The Netherlands

Monetary Policy, Productivity, and Market Concentration*

Andrea Colciago[1] and Riccardo Silvestrini[2]

[1] De Nederlandsche Bank and University of Milano-Bicocca
andreacolciago@gmail.com
[2] Erasmus School of Economics and Tinbergen Institute
silvestrini@ese.eur.nl

May , 2020

Abstract. This paper builds a New Keynesian industry dynamics model for the analysis of macroeconomic fluctuations and monetary policy. A continuum of heterogeneous firms populates the economy, markets are imperfectly competitive and nominal wages are sticky. An expansionary monetary policy shock triggers a response in labor productivity. By reducing borrowing costs, the shock initially attracts low productivity firms in the market. As a result, aggregate productivity decreases on impact. It then overshoots its initial level since, after the initial over-crowding, competition cleanses the market from low productivity firms. The overshooting amplifies the response of the main macroeconomic variables to the shock. A high ex-ante degree of market concentration partially impairs the transmission of monetary policy by disrupting the entry and exit mechanism.

Keywords: Market Concentration · Monetary Policy · Competition · Productivity
JEL Classification: D42, E52, E58, L16

*This paper was partly written while Silvestrini was visiting De Nederlandsche Bank

1 Introduction

Over the last few decades, we observed structural shifts in both labor and product markets on both sides of the Atlantic. The degree of worker unionisation shrank in a number of industries. At the same time there was a secular rise in the degree of globalization and automation in the workplace, together with a fall in the labor share of income. Structural changes in the goods market have been no less relevant. Grullon, Larkin, and Michaely (2019), among other, argue that the vast majority of US industries experienced a broad growth in profit rates, sales concentration, and price-cost margins. Decker Ryan A. and Miranda (2015) point out that these upward trends have come along with persistent drops in firm entry rates. De Loecker, Eeckhout, and Unger (2019) find that markups have risen in both North America and Europe. These changes may have influenced the pricing and provision of goods and services in the economy, the Phillips curve, and through this channel the transmission of monetary policy. In this paper we study how competition and market power affect firm dynamism and labor productivity and, through these channels, the transmission of monetary policy. Notice that modern models of the business cycle do not usually address or incorporate the determinants and evolution of labor productivity. The vast majority of such macroeconomic models take the pattern of labor productivity and the extent of competition between market competitors as given. As such, they can address neither the relationship between policy measures and productivity, nor the one between competition and productivity. On the other hand, growth theories postulate that technology is endogenous because it relies on the decision to invest in research and development by individual firms. Theories of industrial organization typically predict that innovation should decline with competition, while empirical work finds that this relationship has an inverted U shape. The literature at the intersection between competition, innovation and economic policy, usually focuses on the long-run. A recent example of this approach is the work by Chu, Cozzi, Furukawa, and Liao (2019), who study how inflation affects innovation and international technology transfer. One notable exception to the long run approach is the analysis by Moran and Queralto (2018). They propose a framework where monetary policy influences firms' incentives to develop and implement innovations. As in their work, in this study we take a business cycle perspective.

Our framework builds upon Ghironi and Melitz (2005)'s model, amended to allow for nominal rigidities to provide a framework where real and nominal disturbances affect the evolution of labor productivity, and where the extent of competition affects the incentives for the economy to invest in the development of new products. Just as real and monetary shocks may lead firms to adjust the scale of production, they also create opportunities to introduce new goods in the market, as lower costs or higher demand raise the profitability of new product lines. Those incentives depend on the degree of competition and market concentration. Final goods markets and labor markets are imperfectly competitive. Additionally, nominal wages are sticky. Firms face an initial uncertainty concerning their future productivity when making an investment decisions to enter the market. Firm entry is subject to sunk product development costs, which investors pay in expectation of future profits. Firms enter the market up the point where the value of their newly created product equals its sunk cost. As a result, and as argued by Bilbiie, Ghironi, and Melitz (2012), this makes the framework conceptually close to a variety-based endogenous growth models, which abstracts from growth to focus on business cycles. After entry, firms' production depends on their productivity levels. As in Clementi and Palazzo (2016) firms' productivity is the product of a common and of an idiosyncratic component. The former is driven by a persistent aggregate stochastic exogenous processes, while the latter is assigned once and forever upon paying the sunk entry cost. Firms face fixed production costs and their variable inputs costs must be financed by short term loans. Given aggregate conditions, only a subset of firms will be able to break even on their costs and continue production. Firms with idiosyncratic productivity levels below a specific threshold will be forced to interrupt production and stay inactive until production becomes profitable again.[3] The interplay between firms dynamics and aggregate shocks determines the composition of active product lines and thus the level of labor productivity, which becomes endogenous. Exogenous shocks to the common component of productivity, or to the interest rate on short terms loans, induce firms to enter and exit production, thus altering the labor productivity in the economy. As shown by Axtell (2001), the distribution of firms size is very fat tailed and the typical economy is dominated by a few very large firms. Using data on the entire population of tax-paying firms in the United States, he shows that a Zipf distribution characterizes firm sizes: the probability a firm is larger than size s is inversely proportional to s. Those results hold for data

[3] In an extension we consider an alternative exit process, where firms characterized by idiosyncratic productivity below the specified threshold exit the market immediately, as in Melitz (2003).

from multiple years and for various definitions of firm size. Note that, in our framework, the size distribution is endogenously determined by the productivity distribution. The Schumpeterian hypothesis suggests that concentration is positively related to innovation. Concentration, however, may also slow down firms' dynamics and, through this channel, it can affect the response of the economy to shocks. For these reasons, we study the dynamic of labor productivity under alternative degree of initial market concentration in the economy. An expansionary monetary policy shock that lower the real interest rates translates into a higher discount factor leading to a higher discounted value of future profits. For a given entry cost this increases entry. Hence, by reducing the real interest rate, a temporary monetary shock will lead to more investment in new products. This amounts to the transmission channel of monetary policy in the presence of endogenous entry described by Bergin and Corsetti (2008), who stress the analogy in the monetary transmission channel between a model with entry and standard models without entry but with investment in physical capital. In the presence of heterogeneous firms with fixed operational costs, and a working capital constraint, a monetary policy shock spurs additional effects. A reduction in the real rate of interest entails lower variable costs through the working capital constraint. As a result, the idiosyncratic productivity threshold that allows firms to break-even on their costs goes down: firms with a lower idiosyncratic productivity level will be able to stay in the market with positive profit. This has two effects that are absent in the case of firms with homogeneous technology. The first one is that firms with low productivity will find it convenient to operate, and will stay active in the market. The second one is that firms that before the shock where prevented to enter the market will now find it convenient to do so. The additional entry will boost investment demand and output with respect to the case of homogeneous firms described by Bergin and Corsetti (2008), leading to an additional output effect of the initial shock. Further, the monetary policy easing introduces a non-trivial dynamics in aggregate labor productivity. The latter decreases on impact, and, after few periods, overshoots its initial level. This can be explained as follows. A lower interest rate attracts firms with lower idiosyncratic productivity in the market, leading to an impact reduction in the endogenous component of aggregate labor productivity. The reason for the overshooting is a competition effect. The crowding of the market following the decrease in the cutoff increases competition. As a result individual demand for each firm shrinks together with individual profits. Increased competition means that only the most productive firms are able to stay actively in the market, counteracting the initially negative effect of the shock on productivity. As a result, the endogenous component of productivity overshoots, inducing the same dynamics in aggregate labor productivity, which persistently rises above its initial level.

This results are broadly consistent with those in the banking literature. Cetorelli (2014) finds that, when credit market conditions are relatively favorable, entering firms are less productive on average. This is consistent with the transmission mechanism of monetary policy just described.

A similar reasoning applies to a positive shock to the common productivity component. The latter reduces firms marginal costs of production. As a result, the idiosyncratic productivity threshold that allows firms to break even on their costs is reduced. Firms with lower productivity will enter the market and at the same time firms that would have otherwise gone out of business will be able to continue production. For this reason, the average idiosyncratic level of productivity of operating firms goes down on impact, counteracting the initial effect of the shock to the common component of technology on labor productivity. As the effects of the shock to the exogenous component of labor productivity disappears, average idiosyncratic productivity goes back to its initial level. However, before doing so, it persistently overshoots its steady state value. The reason for this is again a competition effect: the crowding of the market following the decrease in the cutoff increases competition. The overshooting of endogenous component of labor productivity leads aggregate labor productivity to grow faster that its exogenous component after some quarters from the shock, boosting the response of the main macroeconomic variables to the shock with respect to those obtained under the case of homogeneous firms.

Turning to the effect of the degree of market concentration on the transmission of monetary policy, we find that a higher degree of concentration dampens the dynamics just described. In a concentrated market, characterized by the presence of few large firms, variations in the productivity threshold will affect a smaller share of firms, leading to smaller flows of entry and exit with respect to the case of a less concentrated market. This affects both the impact effect on output, and the endogenous variation in labor productivity spreading from firm entry and exit. Thus, a higher degree of concentration in the market for final goods partially impairs the transmission of monetary policy by disrupting the entry and exit mechanism. Since the number of firms is predetermined and cannot rise in the same period in which the shock occurs, endogenous labor productivity tends to generate some persistence in response to monetary shocks, though these particular effects are not large in the calibrated version of the model.

A recent and growing literature, inspired by the work of Melitz (2003), Bilbiie et al. (2012), and Jaimovich and Floetotto (2008), among others, studies how the extensive margin of firm entry and product variety can contribute to our understanding of the business cycle. Bilbiie, Ghironi, and Melitz (2007), Bergin and Corsetti (2008), and Lewis and Poilly (2012), study the monetary transmission mechanism in the presence of an extensive margin of investment. Bilbiie, Fujiwara, and Ghironi (2014), show that deviations from long-run stability of product prices are optimal in the presence of endogenous producer entry and product variety in a sticky-price model with monopolistic competition. Bilbiie (2020, forthcoming) shows that entry restores monetary neutrality in the New Keynesian model with rigid prices. Additionally, he shows that nominal wage rigidity, and frictions to the entry process, as in our framework, re-establish non-neutrality. Etro and Rossi (2015) depart from monopolistic competition to consider a framework with nominal rigidities and Bertrand competition between and endogenous number of producers. They find that strategic interactions imply a form of endogenous price stickiness through price complementarities. Papers closely related to ours are Clementi and Palazzo (2016) and Hamano and Zanetti (2017). The aspects that differentiate our paper from those of the aforementioned authors pertain to both assumptions and message. Concerning assumptions, we develop a New Keynesian model by considering nominal wage rigidities and a working capital constraint. Both Clementi and Palazzo (2016) and Hamano and Zanetti (2017) consider real models. Concerning the message, we argue that aggregate shocks, and in particular monetary policy shocks, affect the composition of the pool of active firms over the business cycle, and, through this channel, the dynamics of labor productivity and ultimately the response of macroeconomic aggregates to those shocks. Further, we point out that market concentration alters the monetary transmission mechanism by diminishing the allocative power of the process of entry and exit. Clementi and Palazzo (2016) point out that entry and exit propagate the effects of aggregate shocks. Hamano and Zanetti (2017) find that product heterogeneity and the persistence of technology shocks play a crucial role for the cyclicality of product turnover. The remainder of the paper is organized as follows. Section 2 introduces the model with monopolistic competition, nominal rigidities and heterogeneous firms. Section 3 describes the model equilibrium under a Pareto distribution for the size of firms. Section 4 outlines the main findings, and Section 5 concludes. Technical details are left to the Appendix.

2 The Model

2.1 Firms and Competition

As customary, we assume a production structure with two layers: a fictitious, perfectly competitive, final good producer and a monopolistically competitive sector producing intermediate goods. Firms operating in the intermediate goods sector are subject to entry and exit, and are characterized by heterogeneous levels of productivity. In what follows we will refer to firms producing intermediate goods as firms or as producers. The process of entry and exit in the intermediate goods sector are described below. There is a continuum of potential entrants of mass N_t. Prior to entry firms must draw their individual productivity level, z, from a p.d.f. g(z) with a positive support. The idiosyncratic productivity level z remains unchanged over the lifetime of a firm. In order to draw their productivity level, firms must pay an entry cost, that we describe below. Every firm produces an imperfectly substitutable good $y_t(z)$, which is then aggregated into the bundle Y_t by a final good producer. The latter operates in a perfectly competitive environment with the following constant elasticity of substitution (CES) production function:

$$Y_t = \left[\int_0^\infty N_t y_t(z)^{\frac{\theta-1}{\theta}} g(z)dz\right]^{\frac{\theta}{\theta-1}} \tag{1}$$

where $\theta > 1$ is the elasticity of substitution between intermediate goods. The producer of the final good takes prices as given and chooses the quantities of intermediate goods as to maximize its profits. This generates the demand for intermediate good z and the price of the final good, which are, respectively

$$y_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\theta} Y_t, \tag{2}$$

and

$$P_t = \left[\int_0^\infty N_t p_t(z)^{1-\theta} g(z)dz\right]^{\frac{1}{1-\theta}} \tag{3}$$

Intermediate inputs are produced by a continuum of monopolistic firms indexed by the idiosyncratic productivity level z. The production is linear in labor, and reads as

$$y_t(z) = Z_t z L_t(z) \tag{4}$$

where Z_t is the exogenous, and common to all firms, aggregate level of productivity, and $L_t(z)$ is the labor input used by firm z. The common component of technology is assumed to follow a first order autoregressive process given by $log\left(\frac{Z_t}{Z}\right) = \rho_z log\left(\frac{Z_{t-1}}{Z}\right) + \varepsilon_{z,t}$, where $\rho_z \in (0,1)$, Z is the steady state value of Z_t, and $\varepsilon_{z,t}$ is a white noise disturbance, with zero expected value and standard deviation σ_z. The labor input is defined as a CES aggregator of differentiated labor inputs indexed by $j \in [0,1]$. Formally the labor inputs reads as $L_t(z) = \left(\int_0^1 \left(l_t^j(z)\right)^{\frac{\theta_w-1}{\theta_w}} dj\right)^{\frac{\theta_w}{\theta_w-1}}$, where $\theta_w > 1$ is the elasticity of substitution between labor inputs. Firm-z's demand for labor type j and the aggregate nominal wage index are, respectively $L_t^j(z) = \left(\frac{w_t^j}{w_t}\right)^{-\theta_w} L_t(z)$ and $W_t = \left[\int_0^1 \left(W_t^j\right)^{1-\theta_w} dj\right]^{1/(1-\theta_w)}$. We assume that firms pay their workers before the production takes place. In order to do so firms borrow at the beginning of each period a fraction $0 \leq \alpha_W \leq 1$ of their wage bill from financial intermediaries, which are reimbursed at the end of the period at the gross risk-free interest rate R_t. Of course, if $\alpha_W = 1$ the entire wage bill must be payed in advance, if $\alpha_W = 0$ the model simplifies to a standard monopolistic competition optimization problem. Cost minimization shows that nominal marginal costs amounts to

$$MC(z) = (1 + \alpha_w (R_t - 1)) \frac{W_t}{Z_t z}$$

Intermediate goods producers face a monopolistically competitive environment, and maximize their period nominal profits by choosing the price $p_t(z)$. Firm z´s maximization problem reads as

$$\max_{p_t(z)} \quad p_t(z)y_t(z) - MC_t y_t(z) - F_{x,t}$$

subject to constraints 2 and 4. Notice that $F_{x,t}$ is a fixed cost of production in nominal terms. Profit maximization delivers the optimal relative price $\rho_t(z) = \frac{p_t}{P_t}$ as

$$\rho_t(z) = \frac{\theta}{\theta - 1} mc_t(z) \tag{5}$$

where $mc_t(z)$ denotes real marginal costs. Using the definition of real marginal costs provided above leads to:

$$\rho_t(z) = \frac{\theta}{\theta - 1} \left(1 + \alpha_w (R_t - 1)\right) \frac{w_t}{Z_t z}$$

where w_t is the real wage. Notice that we can write individual real profits as

$$d_t(z) = \rho(z) y_t(z) - \frac{\theta - 1}{\theta} \rho_t(z) y_t(z) - f_{x,t}$$

where $f_{x,t}$ denotes the fixed cost of production in real terms. Using the individual production function and the demand function for for good z, that is 2, we can finally write the real profits of the firm with productivity z as a function of aggregate output Y_t as follows:

$$d_t(z) = \frac{1}{\theta} \rho_t(z)^{1-\theta} Y_t - f_{x,t} \tag{6}$$

2.2 Entry and Exit

As mentioned above, prior to entry firms pay an entry costs in order to draw their individual productivity level, z, from a p.d.f. g(z). In order to draw their productivity level, firms must pay a sunk entry cost $f_{e,t}$ in terms of units of output. Firms enter the market up to the point where the cost of entry is equal to the value of the firm. The latter is determined by the expected value of future profits. Since the idiosyncratic productivity is unknown ex-ante, the firm value is computed considering the average productivity level. In other words the firm value is the discounted value of future profits of the firm with average productivity, or, in short, the average firm. Notice that the average firm's value is computed considering only the productivity of firms active in the market. Free entry condition is, hence:

$$\frac{f_{e,t}}{Z_t} = \tilde{e}_t \tag{7}$$

where $\tilde{e}_t$ is the value of the average firm operating in period t. For a firm to continue its production, profits must be non-negative. Following Melitz (2003), we define as the cut-off productivity level, z_t^c, the minimal level of idiosyncratic productivity such that individual profits are non-negative. Algebraically, the latter can be identified setting real individual profits, $d_t(z)$, equal to zero

$$\frac{1}{\theta} \left[\rho_t(z)\right]^{1-\theta} Y_t = f_{x,t}$$

Using the optimal price together with the definition of marginal costs delivers

$$\left[\frac{\theta}{\theta - 1} \left(1 + \alpha_w (R_t - 1)\right) \frac{w_t}{Z_t z}\right] = \left(\frac{f_{x,t}}{Y_t}\right)^{\frac{1}{1-\theta}}$$

which results in the cut-off productivity level

$$z_t^c = \left(\frac{1}{\theta}\right)^{\frac{1}{1-\theta}} \frac{\theta}{\theta - 1} \left(1 + \alpha_w (R_t - 1)\right) \frac{w_t}{Z_t} \left(\frac{f_{x,t}}{Y_t}\right)^{\frac{1}{\theta-1}} \tag{8}$$

Notice that z_t^c is affected by the magnitude of the fixed cost of production, $f_{x,t}$, by aggregate economic conditions, by fluctuations in the common exogenous productivity, Z_t, and, importantly, by the interest rate. This implies that the monetary policy stance affects the minimum productivity of the firms which can stay in the market, and through the channel, as we show below, the whole productivity distribution of active firms.

A higher Z_t implies lower variable costs of production for any given idiosyncratic productivity level, making it easier to break-even for individual firms. In this sense Z_t can be regarded as an aggregate productivity level: it affects the goods-production and the firm-creation technologies, as well as cut-off productivity. A higher fixed costs of production and/or a higher interest rate require a higher idiosyncratic productivity level to break-even, leading to a larger z_t^c. Obviously with zero fixed cost of production the cut-off level is nil.

What happens to firm z when its idiosyncratic level of productivity falls below the cut-off level, that is when $z < z_t^c$? In our baseline scenario, firms for which $z < z_t^c$ become inactive, ready to start producing again once market conditions or exogenous changes in aggregate productivity or a change in monetary policy lead to a reversal in the sign of the inequality. In this setting firms leave the market when hit by an exogenous death shock.

In the appendix we propose a an alternative exit process where whenever $z < z_t^c$, firm z must leave the market, as in Melitz (2003). Under both specifications there is a one time period to build, i.e. a one period lag between the decision to enter the market and the beginning of production. This period represents the amount of time required to set up production facilities. In the following, we spell out the details of the two exit processes.

2.3 Operative Firms and Exit

The number of firms in the market evolves according to:

$$N_t = (1-\delta)(N_{t-1} + N_{t-1}^e) \tag{9}$$

where N_{t-1} is the mass of firms in the market in period t-1 while N_{t-1}^e is the mass of entrants between periods t-1 and t. However, due to the fixed costs of production, not all N_t firms have non negative profits, but just those which productivity level z is above the cut-off productivity z_t^c. Operative firms, $N_{o,t}$, are formally defined as:

$$N_{o,t} = N_t Pr[z > z_t^c] = [1 - G(z_t^c)]N_t \tag{10}$$

where G(z) is the cumulative distribution function of g(z): $G(z) = \int_0^z g(x)dx$.

2.4 Households

Consider a representative agent with utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[logC_t\left(i\right) - \chi \frac{L_t\left(i\right)^{1+1/\phi}}{(1+1/\phi)} \right] \quad , \chi, \phi \geq 0 \tag{11}$$

where $\beta \in (0,1)$ is the discount factor, $L_t\left(i\right)$ are hours worked and $C_t\left(i\right)$ is the consumption of the final good. In each time period t, agents can purchase any desired state-contingent nominal payment A_{t+1} in period $t+1$ at the dollar cost $E_t \Lambda_{t,t+1} A_{t+1}$, where $\Lambda_{t,t+1}$ denotes the stochastic discount factor between period $t+1$ and t. Representative agent enjoys labour and dividend income. The household maximizes equation (11) by choosing hours of work and how much to invest in state-contingent asset and in risky stocks x_{t+1}.

As mentioned above, we assume a continuum of differentiated labor inputs indexed by j ∈ [0, 1]. As in Schmitt-Grohé and Uribe (2005), each household supplies each possible type of labor input. Wage-setting decisions are made by labor type specific unions indexed by $j \in [0, 1]$. Given the wage W_t^j fixed by union j, agents stand ready to supply as many hours to the labor market j, l_t^j , as required by firms, that is:

$$L_t^j = \left(W_t^j / W_t\right)^{-\theta_w} L_t^d \tag{12}$$

where L_t^d is the total labor demanded by firms. Agents are distributed uniformly across unions; hence, aggregate demand for labor type j is spread uniformly across the households. It follows that the individual quantity of hours worked, $L_t(i)$, is common across households, and we denote it as L_t. This must satisfy the time resource constraint $L_t = \int_0^1 L_t^j dj$. Combining the latter with equation (12) we obtain that total hours worked by the households are $L_t = L_t^d \int \left(W_t^j/W_t\right)^{-\theta_w} dj$. The labor market structure rules out differences in labor income between households without the need to resort to contingent markets for hours. The common labor income is given by $L_t = L_t^d \int W_t^j \left(W_t^j/W_t\right)^{-\theta_w} dj$.

The timing of investment in the stock market is as in Bilbiie et al. (2012) and Chugh and Ghironi (2011). At the beginning of period t, the household owns x_t shares of a mutual fund of the N_t firms in the market in period t. The period-t value of the portfolio can be expressed as the product between the value of the average firm $\tilde{e}_t$ and the existing mass of firms N_t. During period t, the household purchases x_{t+1} shares in a fund of these N_t firms as well as the N_t^e new firms created during period t, to be carried into period $t+1$. The value of total stock market purchases is thus $\tilde{e}_t x_{t+1}(N_t + N_t^e)$. At the very end of period t , a fraction of these firms disappears from the market. Since the household does not know which firms will disappear from the market, it finances the continued operations of all incumbent firms as well as those of the new entrants. Following the production and sales of varieties in the imperfectly competitive goods markets, firms distribute dividends to households. Notice that just operative firms distribute dividends, thus dividends received by a household can be written as $x_t \tilde{d}_t N_{o,t}$. A fraction of the funds of the household is deposited to financial intermediaries that offer loans to firms. In equilibrium, a nominal amount of $W_t L_t^d$ must be gathered to finance the total wage bill. The deposit yields a gross interest rate R_t, which is return to households at the end of each period t in a lump sum fashion.

The flow budget constraint of the household reads as:

$$c_t + \tilde{e}_t \left(N_t + N_t^e\right) x_{t+1} + E_t r_{t,t+1} a_{t+1} = L_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\theta_w} dj + N_t x_t \tilde{e}_t + N_{o,t} \tilde{d}_t x_t + \frac{a_t}{\pi_t} + \left(R_t - 1\right) w_t L_t^d \quad (13)$$

The household maximizes (11) subject to (13) and the time resource constraint. The F.O.C. with respect to c_t is $\lambda_t = \frac{1}{c_t}$. Using this condition and the F.O.C. with respect to L_t^s, one can obtain:

$$\chi \left(L_t^s\right)^{\frac{1}{\phi}} c_t = \frac{w_t}{\tilde{\mu}_t} \quad (14)$$

Note that $\tilde{\mu}_t$ creates a wedge between the marginal rate of substitution between consumption and labor and the marginal rate rate of transformation, lowering the provision of labor. This wedge is originated by the frictions in the labor markets.

The F.O.C. for a_{t+1}, using again the expression for λ_t found above, is:

$$E_t r_{t,t+1} = \beta E_t \left[\left(\frac{c_t}{c_{t+1}}\right) \frac{1}{\pi_{t+1}}\right] \quad (15)$$

Finally, by combining the F.O.C.s for c_t and for x_{t+1}, the Euler equation for assets can be written. The latter reads as:

$$\tilde{e}_t = \beta \left(1 - \delta\right) E_t \left[\left(\frac{c_t}{c_{t+1}}\right) \left(\tilde{e}_{t+1} + \left(1 - G\left(z_{t+1}^c\right)\right) \tilde{d}_{t+1}\right)\right] \quad (16)$$

The Euler equation and the Euler equation for assets, that is equations (15) and (16) respectively, serve together as a no-arbitrage condition. Note, indeed, that we can define $\frac{(1-\delta)\left(\tilde{e}_{t+1} + \left(1 - G\left(z_{t+1}^c\right)\right)\tilde{d}_{t+1}\right)}{\tilde{e}_t}$ as $\frac{R_t}{\pi_{t+1}}$.The no-arbitrage condition is, thus: $R_t = 1/E_t r_{t,t+1}$.

2.5 Unions

Wages are reset as in Calvo (1983): in every labor market, the union faces a constant probability $1 - \tilde{\alpha}$ of being able to re-optimize the wage. In this setting, we consider that non-optimized wages are not indexed to inflation and, thus, there is no updating. A type j real wage optimized in period t and not re-optimized for s periods is, hence: $w_{t+s}^j = \frac{\tilde{w}_t^j}{\prod_{k=1}^s \pi_{t+k}}$. When the union is able to re-optimize the wage for sector j, its objective is to maximize the relevant part of the household's Lagrangian:

$$E_t \sum_{s=0}^{\infty} \left(\beta\tilde{\alpha}\right)^s \left\{ \lambda_{t+s} L_{t+s}^d \int_0^1 w_{t+s}^j \left(\frac{w_{t+s}^j}{w_{t+s}}\right)^{-\theta_w} dj - \frac{\lambda_{t+s} w_{t+s}}{\tilde{\mu}_{t+s}} L_{t+s}^d \int_0^1 \left(\frac{w_{t+s}^j}{w_{t+s}}\right)^{-\theta_w} dj \right\} \quad (17)$$

Due to the symmetry of labor markets when resetting in period t, the latter can be rewritten as:

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_w} L^d_{t+s} (w_{t+s})^{\theta_w} \lambda_{t+s} \left\{ (\tilde{w}_t)^{1-\theta_w} \left(\prod_{k=1}^{s} \frac{1}{\pi_{t+k}}\right) - \left(\frac{\chi \left(L^s_{t+s}\right)^{\frac{1}{\phi}}}{\lambda_{t+s}}\right) (\tilde{w}_t)^{-\theta_w} \right\}$$

The F.O.C.s with respect to $\tilde{w}_t$ reads as:

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_w} L^d_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\theta_w} \lambda_{t+s} \left[\tilde{w}_t \frac{(\theta_w - 1)}{\theta_w} \left(\prod_{k=1}^{s} \frac{1}{\pi_{t+k}}\right) - \left(\frac{\chi \left(L^s_{t+s}\right)^{\frac{1}{\phi}}}{\lambda_{t+s}}\right) \right] = 0$$

Following Schmitt-Grohé and Uribe (2005) let:

$$f^1_t = E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_w - 1} L^d_{t+s} (w_{t+s})^{\theta_w} \lambda_{t+s} \tag{18}$$

and

$$f^2_t = E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_w} L^d_{t+s} (w_{t+s})^{\theta_w} \lambda_{t+s} \chi_{t+s} \tag{19}$$

where:

$$\left(\frac{\chi \left(L^s_{t+s}\right)^{\frac{1}{\phi}}}{\lambda_{t+s}}\right) = \chi_{t+s} \tag{20}$$

As a result, the F.O.C. for wage setting can be written as:

$$\tilde{w}_t = \frac{\theta_w}{(\theta_w - 1)} \frac{f^2_t}{f^1_t} \tag{21}$$

2.6 Central Bank

The Central bank follows a Taylor rule for the nominal interest rate: the log deviation of the nominal interest rate from its target value depends on its own lag deviation and on the log deviations of inflation and output from their targets. Thus:

$$\ln\left(\frac{R_t}{R}\right) = \alpha_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \alpha_Y \ln\left(\frac{Y_t}{Y}\right) + \alpha_R \ln\left(\frac{R_{t-1}}{R}\right) + \varepsilon_{R,t} \tag{22}$$

where variables without time subscript denote steady state values. The monetary policy shock, $\varepsilon_{R,t}$, has unit variance and zero mean. For simplicity we assume that the steady state gross inflation rate equals one.

2.7 Aggregation

Using equation (12), we can express the labor demanded to the household in period t in each market in which the wage has been reset in period t as:

$$\tilde{L}_t = \left(\frac{\tilde{W}_t}{W_t}\right)^{-\theta_w} L^d_t$$

For the law of large numbers, this occur with probability $1-\tilde{\alpha}$, or, equivalently, in a fraction $1-\tilde{\alpha}$ of markets: define, hence, $L_{t,t} = (1-\tilde{\alpha})\,\tilde{L}_t$. On the other hand, the amount of labor supplied to the markets without reoptimization for s periods is:

$$L_{t,s} = (1-\tilde{\alpha})\,(\tilde{\alpha})^s \left(\frac{\tilde{W}_{t-s}}{W_t}\right)^{-\theta_w} L^d_t$$

Summing across all the possible waiting periods s we obtain:

$$L_{t,t-s} = (1-\tilde{\alpha}) \sum_{s=1}^{\infty} (\tilde{\alpha})^s \left(\frac{\tilde{W}_{t-s}}{W_t} \right)^{-\theta_w} L_t^d$$

Combining these definitions we can write:

$$L_t^s = \int_0^1 L_t^j dj = \int_0^1 [L_{t,t} + L_{t,t-s}]\, dj = L_{t,t} + L_{t,t-s} = (1-\tilde{\alpha}) \sum_{s=0}^{\infty} (\tilde{\alpha})^s \left(\frac{\tilde{W}_{t-s}}{W_t} \right)^{-\theta_w} L_t^d = \tilde{s}_t L_t^d \tag{23}$$

where $\tilde{s}_t$ measures the resource cost, induced by the wage dispersion implied by Calvo wage setting, which entails an inefficiently large labor supply with respect to the the one that is required for production. The first can be written recursively as:

$$\tilde{s}_t = (1-\tilde{\alpha}) \left(\frac{\tilde{w}_t}{w_t} \right)^{-\theta_w} + \tilde{\alpha} \left(\frac{w_{t-1}}{w_t} \right)^{-\theta_w} \pi_t^{\theta_w} \tilde{s}_{t-1} \tag{24}$$

Using the definition of wage of the economy as $W_t = \left[\int_0^1 \left(W_t^j \right)^{1-\theta_w} dj \right]^{1/(1-\theta_w)}$ it is easy to show that:

$$w_t^{1-\theta_w} = (1-\tilde{\alpha}) \tilde{w}_t^{1-\theta_w} + \tilde{\alpha} \left(\frac{w_{t-1}}{\pi_t} \right)^{1-\theta_w} \tag{25}$$

In equilibrium, the representative household holds the entire portfolio of firms and the trade of state-contingent asset trade is nil. Setting, thus, $x_{t+1} = x_t = 1$ and $a_{t+1} = a_t = 0$, the following resource constraint can be obtained:

$$c_t + N_t^e \tilde{e}_t = R_t w_t L_t^d + N_{o,t} \tilde{d}_t \tag{26}$$

Finally, by the definition of the aggregate demand, Y_t is consumed and used to cover fixed costs of production and of entry:

$$Y_t = c_t + N_{o,t} f_{x,t} + N_t^e f_{e,t} \tag{27}$$

3 Equilibrium with a Pareto Productivity Distribution

To obtain tractable results, a Pareto distribution is assumed for the function $g(z)$ with parameters $zmin$ and κ. This assumption simplifies considerably several equilibrium conditions and allows to compute analytical solutions. Following Melitz (2003), a special average productivity $\tilde{z}_t$ is defined over the operating firms. This productivity summarizes all the relevant information of the model, since the entire economy is isomorphic to one populated by a mass of homogeneous firms $N_{o,t}$ with productivity $\tilde{z}_t$. Using the properties of the Pareto distribution, this productivity can be written easily as a function of the cut-off productivity:

$$\tilde{z}_t = \left[\frac{1}{1 - G\left(z_t^c\right)} \int_{z_t^c}^{\infty} z^{\theta-1} g(z) dz \right]^{\frac{1}{\theta-1}} = \nu z_t^c \tag{28}$$

where $\nu = \left[\kappa / \left(\kappa - (\theta - 1)\right)\right]^{1/(\theta-1)}$. Given that only some firms are active in the market, the aggregate price P_t can be written as:

$$P_t = \left[\frac{1}{1 - G\left(z_t^c\right)} \int_{z_t^c}^{\infty} p_t(z)^{1-\theta} N_{o,t} g(z) dz \right]^{\frac{1}{1-\theta}}$$

Substituting the optimal price $p_t(z)$ and dividing both sides by the aggregate price P_t one can obtain:

$$1 = N_{o,t}^{\frac{1}{1-\theta}} \frac{w_t R_t}{Z_t} \frac{\theta}{\theta - 1} \left[\frac{1}{1 - G\left(z_t^c\right)} \int_{z_t^c}^{\infty} z^{\theta-1} g(z) dz \right]^{\frac{1}{1-\theta}}$$

Using the definition of the special average productivity $\tilde{z}_t$ and the definition of the optimal real price $\rho_t(z)$, the following equilibrium condition can be obtained:

$$N_{o,t}^{\frac{1}{\theta-1}} = \frac{\theta}{\theta - 1} \frac{w_t R_t}{Z_t \tilde{z}_t} = \tilde{\rho}_t$$

where $\tilde{\rho}_t$ is the optimal real price set by the firm with the productivity $\tilde{z}_t$.

Furthermore, note that, due to the properties of the Pareto distribution:

$$1 - G\left(z_t^c\right) = \left(\frac{zmin}{z_t^c} \right)^{\kappa}$$

In the following subsections I summarize the equilibrium conditions for both specifications.

4 Business Cycle Analysis

This section has two purposes. First, we wish to understand how shocks to monetary policy and to the common component of technology propagate through the economy by altering the composition of firms in the market. Second we aim at understanding whether market concentration affect firms' dynamics and through this channel the response of the economy to monetary and real shocks.

In Section 4.1 we will consider the effects of heterogeneity across firms at shaping the impulse response to shocks of the main macroeconomic variables. We will do this by comparing the IRFs delivered by our model to that of a model characterized by homogeneous firms as in Bilbiie et al. (2012). When running this comparison we will also evaluate the role played by the working capital constraint. Section 4.2 will, instead consider the role played by the initial degree of market concentration by comparing IRFs under alternative value of the shape parameter of the Pareto distribution for the idiosyncratic technology levels.

Calibration of structural parameters is as follows. The time unit is meant to be a quarter. The discount factor, β, is set to the standard value for quarterly data 0.99, while the rate of business destruction, δ, equals 0.025 to match the US empirical level of 10% job destruction a year. The value of χ is such that steady state labour supply is equal to one in our baseline case. The elasticity of the marginal disutility of labor take a value of 4. We set the steady state level of the common component of productivity, Z, equal to 1. Elasticity of substitution between goods is set to $\theta = 6$, which implies a price markup of 20% as estimated for the US by Christiano, Eichenbaum, and Evans (2001). The same value is assigned to the elasticity of substitution across labor types, θ_w. As for the fixed operation costs and entry costs, we follow Ghironi and Kim (2018) and calibrate the ratio $\frac{f_e}{f_x}$ following the evidence reported by Collard-Wexler (2013), who find that the ratio of entry costs to fixed production costs, $\frac{fe}{fx}$, is around 4.5. As emphasized by Ghironi and Kim (2018), changing the entry cost while maintaining the same ratio $\frac{fe}{fx}$ does not alter any of the impulse responses. We therefore set f_e to 1. For similar reasons, we normalize $zmin$ to 1. We will, instead, consider two alternative degrees of initial market concentration. Our baseline calibration features a low initial degree of concentration by setting $\kappa = 10$. We compare impulse response functions obtained under the baseline degree of concentration to that obtained under the case of a higher initial degree of concentration by setting $\kappa = 6$. The parameter χ denoting the utility cost of hours worked is set, with no loss of generality, such that steady state hours equal 1 under the baseline parametrization of our model and it is held constant across all experiments, as well as the elasticity of the marginal disutity of labor, ϕ that we set to 4. Similarly the parameters of the interest rate rule are held constant across experiments and are set to the customary values of $\alpha_\pi = 1.5$, $\alpha_Y = 0.1$ and $\alpha_R = 0.8$. We calibrate the productivity process as in King and Rebelo (1999), with persistence $\rho_z = 0.979$ and standard deviation $\sigma_z = 0.0072$.

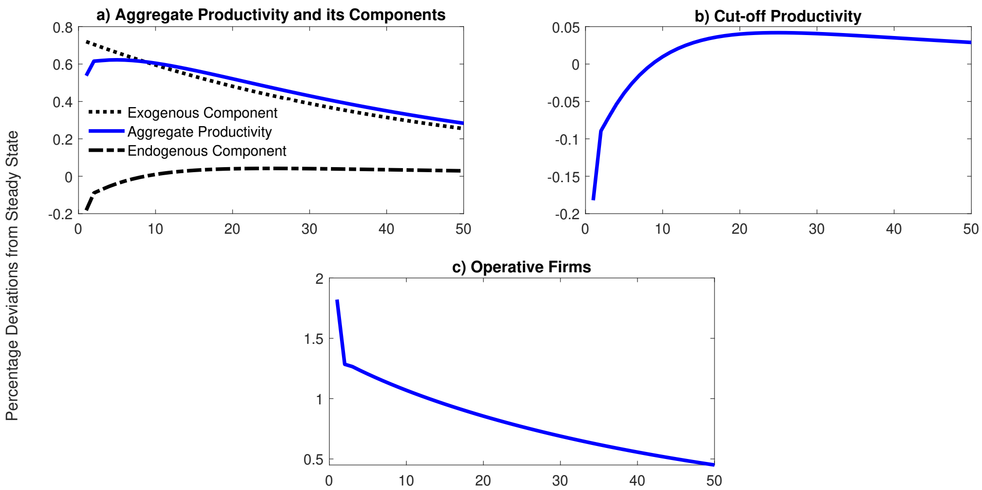
4.1 The Role of Heterogeneity

Figure 1 depicts percentage deviations from the steady state of labor productivity in response to a 1 s.d. shock to Z_t. In figure 2, we compare the IRFs delivered by our model in response to a shock to Z_t to that entailed by a model characterized by homogeneous firms as in Bilbiie et al. (2012). Notice that in the latter case aggregate labor productivity coincides with Z_t.

Figure 1 decomposes the response of labor productivity into the response of its exogenous and endogenous components. The solid line in Panel a) refers to the dynamics of labor productivity, that is $Z_t\widetilde{z}_t$, the dotted line refers to the exogenous component, that is Z_t, common across the two configurations under analysis, while the dashed line refer to the endogenous component, that is $\widetilde{z}_t$. The latter is depressed for several periods in the aftermath of the shock. An increase in the common component of technology implies that even firms with lower idiosyncratic productivity will break even on their costs. This will entail the entry of firms will lower productivity with respect to those already in the market, leading to a lower average productivity in the economy. For this reason labor productivity does not increase as much as Z_t on impact.

As the effects of the shock to Z_t dissolve, average idiosyncratic productivity goes back to its initial level. However, before doing so, it persistently overshoots its initial level. The reason for the overshooting is that the crowding of the market following the decrease in the cutoff increases competition. As a result individual demand for each firm shrinks together with individual profits. Increased competition means that just more productive firms will be able to stay actively in the market. The overshooting of $\widetilde{z}_t$ also leads labor productivity to grow faster that its exogenous component after some quarters from the shock. Consistently

Fig. 1: Response of the exogenous and endogenous components of labor productivity to a TFP shock



with this description, panel b) shows that in response to the shock the cutoff productivity becomes lower on impact and then overshoots its initial level.

Fig. 2: Response of the economy to a TFP shock, comparison between homogeneous and heterogeneous firm models

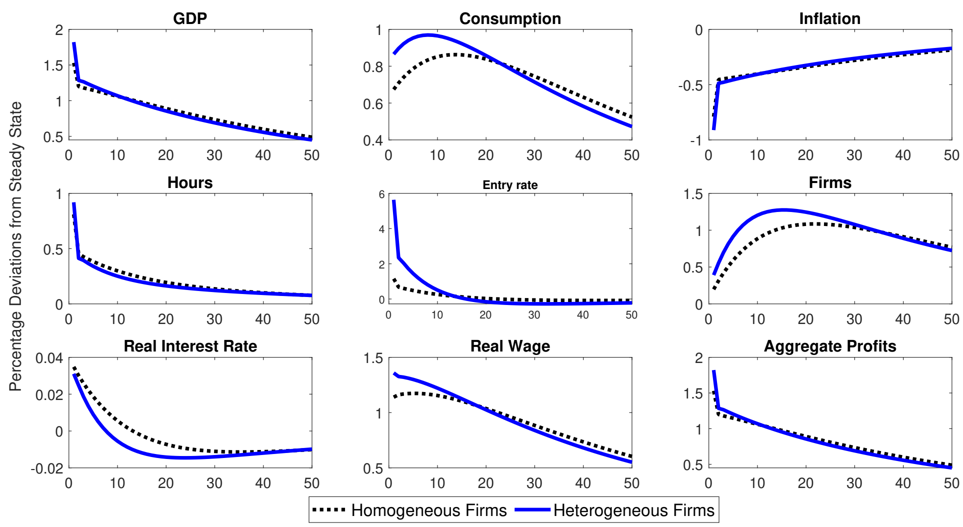
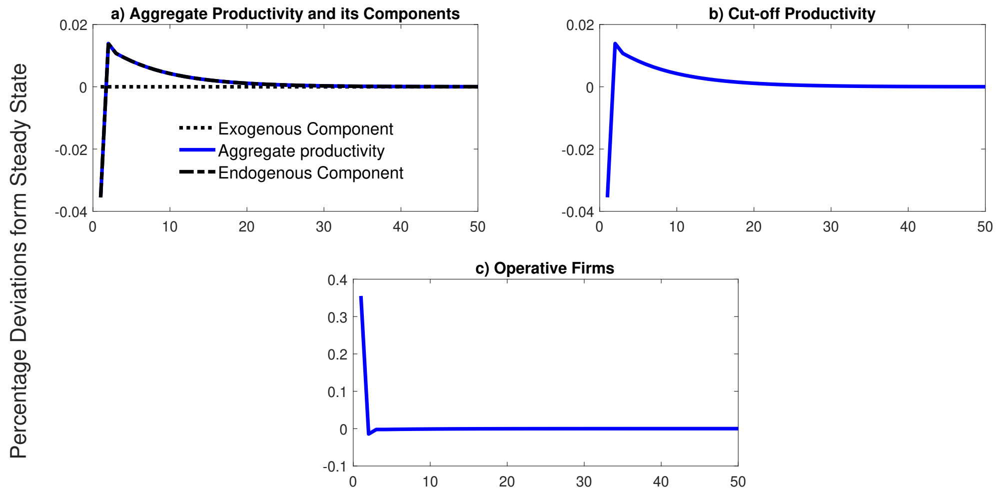


Figure 2 displays the response of other main macroeconomic variables to the shock. Solid lines refer to our economy with firms characterized by idiosyncratic productivity and an endogenous number of active firms, while dotted lines to the same economy but with homogeneous firms. Heterogeneity amplifies the effects of the shock on both GDP and consumption.

The reason is again due to the dynamics of the cutoff productivity and its effect on firms dynamics. As the cutoff productivity goes down, more firms, although less productive, will enter the market leading to a higher investment and thus to higher demand. There is thus an additional effect on the entry rate with respect to the case where firms are homogeneous. In the latter case the increase in entry rate is uniquely driven by the increase in expected profits due to higher productivity. There is no extra investment coming from a change in the composition of firms in the market.

Figure 3 and 4 are are the counterpart of figure 1 and 2 in the case of an expansionary monetary policy shock. We assume that the size of the shock is one percent of the steady state interest rate. The impact response of both the nominal and the real interest rate will differ across our economy and the economy with homogeneous firms due to their different structure.

Fig. 3: Response of the exogenous and endogenous components of labor productivity to a monetary policy shock



Panel a) in Figure 3 shows that the response of labor productivity to the shock is entirely endogenous, indeed the latter is uniquely due to a change in $\widetilde{z}_t$, the average productivity of operating firms. The latter decreases on impact, driving down aggregate labor productivity, $Z_t\widetilde{z}_t$. The reason is that a lower interest rate reduces variables costs for the firms in the market by easing the working capital constraint. As a result, firms with lower idiosyncratic productivity will break even on their costs. This will entail the entry of firms will lower productivity with respect to those already in the market, leading to a lower average productivity in the economy. As the effects of the shock disappear, average idiosyncratic productivity goes back to its initial level. However, before doing so, it persistently overshoots its initial level. The reason for the overshooting is, as in the earlier case relative to the expansionary technology shock, coming from a competition effect. The crowding of the market following the decrease in the cutoff increases competition. As a result individual demand for each firm shrinks together with individual profits. Increased competition means that just more productive firms will be able to stay actively in the market. The overshooting of $\widetilde{z}_t$ induces the same dynamics in aggregate labor productivity, which persistently rises above its steady state level. Consistently with this description, panel b) shows that in response to the shock the cutoff productivity becomes lower on impact and then overshoots its initial level.

Fig. 4: Response of the economy to a TFP shock, comparison between homogeneous and heterogeneous firm models

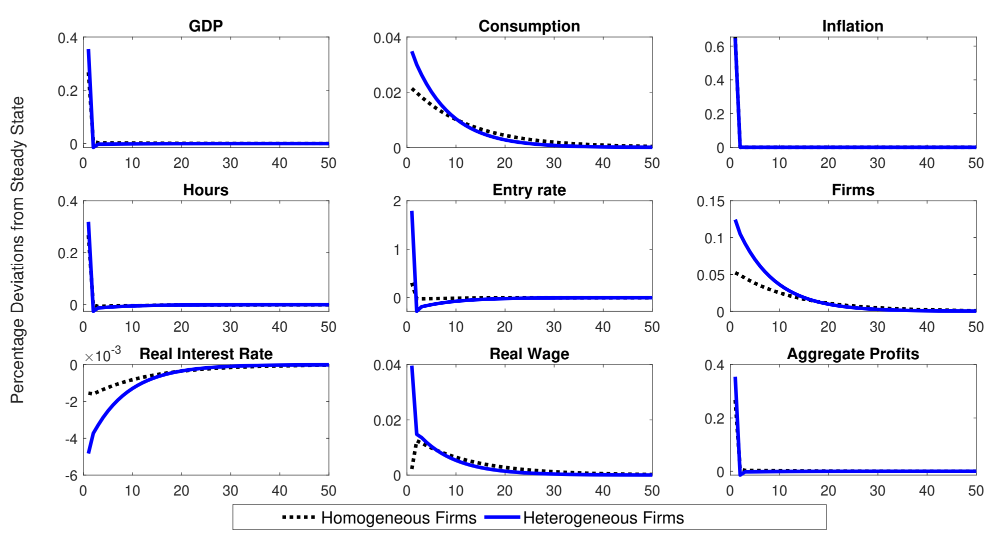


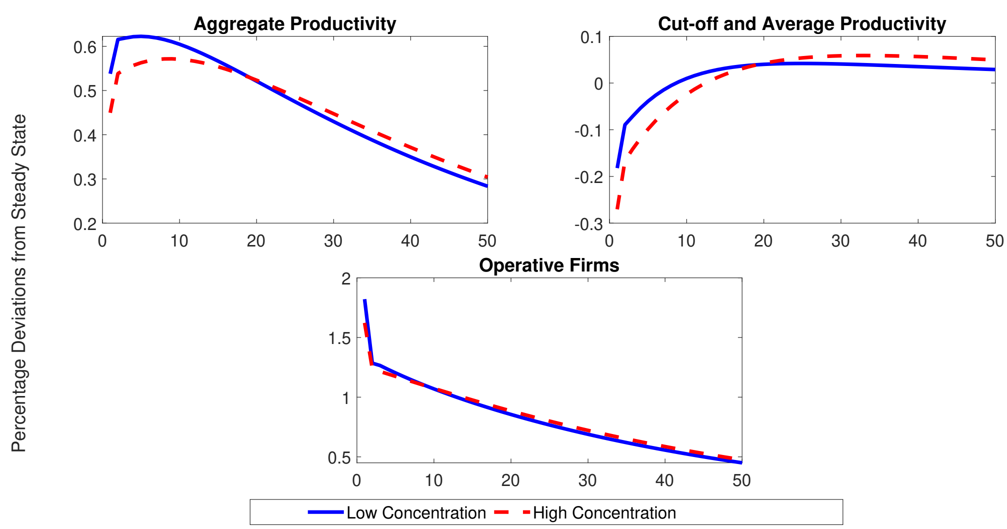
Figure 4 shows the implication of endogenous productivity dynamics on the main macroeconomic variables. With respect to the case of homogeneous firms, heterogeneity amplifies the effects of the shock. This is so for two reasons. The first one is additional output demand coming from additional entry, as firms with low productivity will also be able to enter the market and break-even on their cost in the aftermath of the shock. The second, an more relevant for our purposes, is due to the overshooting of labor productivity. While labor productivity in the homogeneous case does not react to the shock, it does in the heterogeneous firms framework, adding to the expansionary effect of the initial policy shock.

To sum up, this section shows that entry and exit of heterogeneous firms amplifies and propagate the effects of aggregate shocks. This is due to the effect of shocks on aggregate labor productivity. The dynamics of the latter is obtained by aggregating an exogenous, common to all firms, component and an endogenous one. Due to a competition effect, the endogenous component persistently overshoots its exogenous component in response to expansionary shocks, boosting the effects of the initial shocks on aggregate variables. In the next section we analyze the effect of the degree of market concentration for the propagation of aggregate shocks on firms and productivity dynamics.

4.2 The role of Market Concentration

In this section, we aim at understanding whether market concentration affect firms' dynamics and through this channel the response of the economy to a 1 s.d. shock to the common component of technology, Z_t. To this end we compare the IRFs under the baseline degree of market concentration, that implied by a Pareto shape parameter $\kappa = 10$, to those obtained under a higher initial degree of concentration, that obtained setting a Pareto shape parameter $\kappa = 6$. Panel a) of Figure 5 shows the response of aggregate labor productivity, $Z_t\widetilde{z}_t$, while the dashed line refers to the endogenous component, that is $\widetilde{z}_t$.

Fig. 5: Response of the components of labor productivity to a TFP shock, comparison between low and high concentration models



As we described earlier an increase in the common component of technology implies that even firms with lower idiosyncratic productivity will break even on their costs. This will entail the entry of firms will lower productivity with respect to those already in the market, leading to a lower average productivity in the economy. For this reason labor productivity does not increase as much as Z_t on impact. What matters for the comparison under analysis is that in the case of an initially higher market concentration the reaction of the endogenous component of productivity to the shock is stronger. The shock creates profits opportunities for firms with lower productivity, however in the case of higher concentration there are few small firms around the threshold productivity level. For a non-negligible measure of firms to enter the market the cutoff productivity must drop more in this case that in a case with low market concentration. As a result the average productivity of firms suffers a stronger reduction in the case of high market concentration, as displayed in panel c), and a similar pattern is passed over to aggregate labor productivity. Notice however that, for the same reason that explains a larger in magnitude drop in the cut-off productivity, the number of operative firms experiences a larger variation in a low concentration environment.

Fig. 6: Response of the economy to a TFP shock, comparison between low and high concentration models

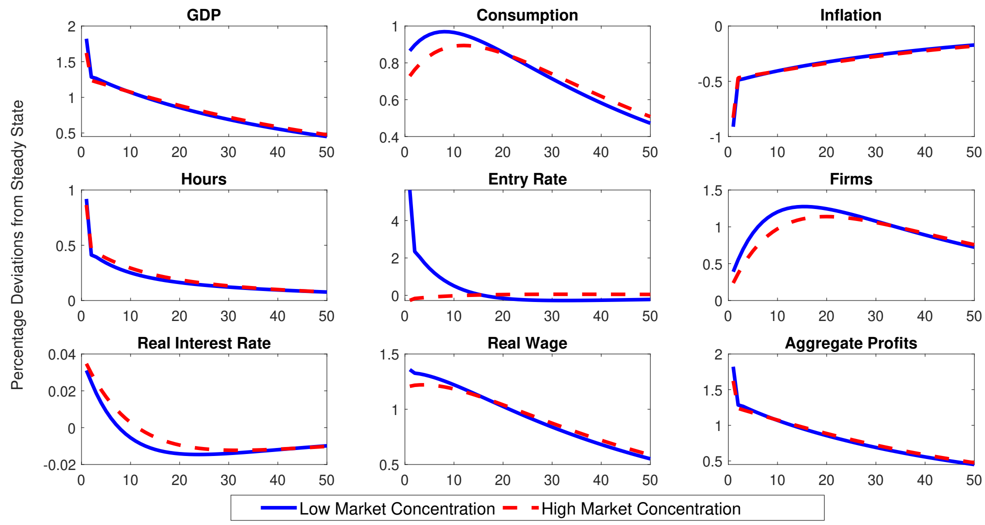
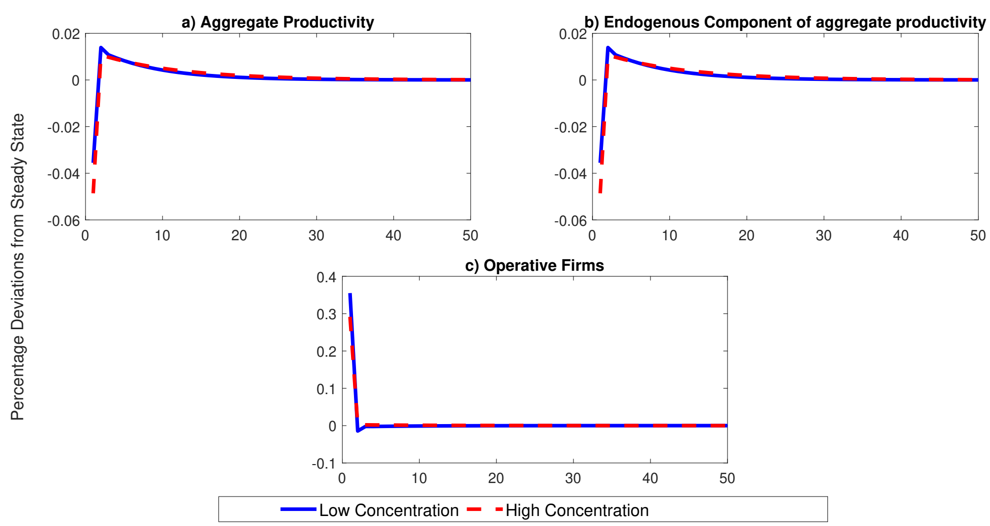


Figure 6, where lines have the same definition as in Figure 5, shows that high concentration slows firms dynamics. The entry rate of new firms in response of the shock increases much more under the baseline scenario. A similar stronger reaction described output and consumption. In other words, under an initially high market concentration, the response to the shock is dampened due to a sizeable reduction in endogenous component of aggregate labor productivity.

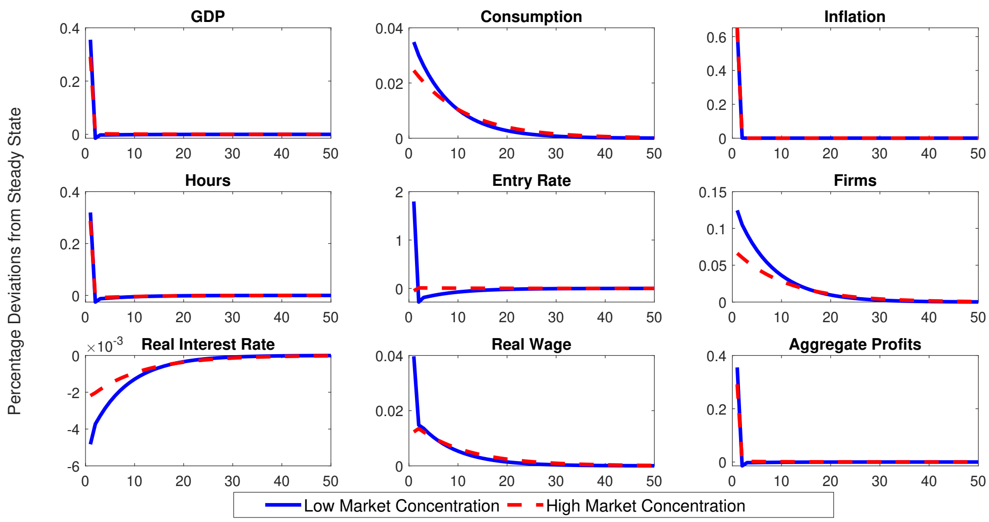
Fig. 7: Response of the components of labor productivity to a monetary policy shock, comparison between low and high concentration models



Figures 7 and 8 display the responses of the main macroeconomic variable to an expansionary monetary policy shock, of size equal to 1 percent of the steady state interest rate, under the two alternative degrees of initial market concentration. Labor productivity decreases on impact and then overshoots its initial level. As described in the earlier case, the overshooting is due to a competition effect. This is the case under both scenarios, although, as one would expect, the overshooting is slightly more sizeable when competition is more intense. As in the previous case, dynamics are more sizeable under the baseline case.

A higher degree of concentration in the market for final goods partially impairs the transmission of monetary policy by disrupting the entry and exit dynamics of firms.

Fig. 8: Response of the economy to a monetary Policy shock, comparison between low and high concentration models



5 Conclusions

We build and industry dynamics model that allows for nominal rigidities and other empirically relevant frictions. Final goods markets and labor markets are imperfectly competitive. Additionally, nominal wages are sticky. Firms face initial uncertainty concerning their future productivity when making an investment decisions to enter the market. Firms' entry is subject to sunk product development costs, which investors pay in expectation of future profits. Firms enter the market up the point where the value of their newly created product equals its sunk cost. As a result, this makes the framework conceptually close to a variety-based endogenous growth models, which abstracts from growth to focus on business cycles.

In this framework real and nominal disturbances and the extent of competition affect firms' incentive to invest in the development of new products and ultimately labor productivity. Expansionary monetary policy shocks affect the composition of firms in the market, and through this channel labor productivity. Specifically an expansionary shock initially allows less productive firms to enter the market, depressing labor productivity on impact. After few periods, however, productivity overshoots its initial level due to a competition effect. The crowding of the market following the decrease in the cutoff increases competition. As a result individual demand for each firm shrinks together with individual profits. Increased competition means now that only the most productive firms will be able to stay actively in the market, counteracting the initially negative effect of the shock on productivity. As a result the endogenous component of productivity overshoots, inducing the same dynamics in aggregate labor productivity, which persistently rises above its initial level boosting the initial effect of the policy shock. Similar reasoning applies to the transmission of a shock to the exogenous component of labor productivity.

We find that the effects of both nominal and real shocks are dampened under a high degree of concentration in the market for final goods. In particular, concentration leads to smaller flows of entry and exit with respect to the case of a less concentrated market. Thus, a higher degree of concentration in the market for final goods partially impairs the transmission of monetary policy by disrupting the entry and exit mechanism.

Appendix

Appendix A1: Wage Setting

The first order condition for wage setting can be written as follows:

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_w} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \left\{ (\tilde{w}_t)^{1-\theta_w} \left(\prod_{k=1}^{s} \frac{1}{\pi_{t+k}}\right) - \left(\frac{\chi \left(L_{t+s}^s\right)^{\frac{1}{\phi}}}{\lambda_{t+s}}\right) (\tilde{w}_t)^{-\theta_w} \right\}$$

The F.O.C.s with respect to $\tilde{w}_t$ reads as:

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_w} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \left[(1-\theta_w)(\tilde{w}_t)^{-\theta_w} \left(\prod_{k=1}^{s} \frac{1}{\pi_{t+k}}\right) + \theta_w \left(\frac{\chi \left(L_{t+s}^s\right)^{\frac{1}{\phi}}}{\lambda_{t+s}}\right) (\tilde{w}_t)^{-\theta_w-1} \right] = 0$$

or

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_w} L_{t+s}^d \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\theta_w} \lambda_{t+s} \left[\tilde{w}_t \frac{(\theta_w - 1)}{\theta_w} \left(\prod_{k=1}^{s} \frac{1}{\pi_{t+k}}\right) - \left(\frac{\chi \left(L_{t+s}^s\right)^{\frac{1}{\phi}}}{\lambda_{t+s}}\right) \right] = 0$$

For simplicity, define:

$$\left(\frac{\chi \left(L_{t+s}^s\right)^{\frac{1}{\phi}}}{\lambda_{t+s}}\right) = \chi_{t+s} \tag{29}$$

such that:

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_w} L_{t+s}^d \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\theta_w} \lambda_{t+s} \left[\tilde{w}_t \frac{(\theta_w - 1)}{\theta_w} \left(\prod_{k=1}^{s} \frac{1}{\pi_{t+k}}\right) - \chi_{t+s} \right] = 0 \tag{30}$$

The latter is equivalent to:

$$\frac{(\theta_w - 1)}{\theta_w} \tilde{w}_t E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_w - 1} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} = E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_w} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s}\chi_{t+s}$$

Define, as in Schmitt-Grohé and Uribe (2005):

$$f_t^1 = E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_w - 1} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \tag{31}$$

and

$$f_t^2 = E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_w} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s}\chi_{t+s} \tag{32}$$

As a result, the F.O.C. for wage setting can be written as:

$$\tilde{w}_t = \frac{\theta_w}{(\theta_w - 1)} \frac{f_t^2}{f_t^1} \tag{33}$$

we can write the condition recursively after considering that

$$f_t^1 = L_t^d (w_t)^{\theta_w} \lambda_t + \beta\tilde{\alpha} E_t (\pi_{t+1})^{\theta_w - 1} L_{t+1}^d (w_{t+1})^{\theta_w} \lambda_{t+1} + ...$$

while

$$f_{t+1}^1 = L_{t+1}^d (w_{t+1})^{\theta_w} \lambda_{t+1} + \beta\tilde{\alpha} E_{t+1} (\pi_{t+2})^{\theta_w - 1} L_{t+2}^d (w_{t+2})^{\theta_w} \lambda_{t+2} + ...$$

As a result:

$$f_t^1 = L_t^d \left(w_t\right)^{\theta_w} \lambda_t + \tilde{\alpha}\beta E_t \pi_{t+1}^{\theta_w - 1} f_{t+1}^1$$

Similarly we can write

$$f_t^2 = L_t^d \left(w_t\right)^{\theta_w} \lambda_t \chi_t + \tilde{\alpha}\beta E_t \pi_{t+1}^{\theta_w} f_{t+1}^2$$

Recalling the definitions of χ_t we get:

$$f_t^1 = L_t^d w_t^{\theta_w} \frac{1}{c_t} + \tilde{\alpha}\beta E_t \pi_{t+1}^{\theta_w - 1} f_{t+1}^1 \tag{34}$$

and

$$f_t^2 = L_t^d w_t^{\theta_w} \chi \left(L_t^s\right)^{\frac{1}{\phi}} + \tilde{\alpha}\beta E_t \pi_{t+1}^{\theta_w} f_{t+1}^2 \tag{35}$$

Appendix A2: Analytical Equilibrium - Form 1

The set of endogenous variables $\left\{c_t, L_t^d, L_t^s, \tilde{s}_t, N_{t+1}, N_t^e, N_{o,t}, \tilde{\mu}_t, \tilde{e}_t, \tilde{\rho}_t, \tilde{d}_t, \tilde{z}_t, z_t^c, \tilde{w}_t, w_t, Y_t, f_t^1, f_t^2, R_t, \pi_t\right\}_{t=0}^{\infty}$ and exogenous series $\{Z_t, f_{e,t}, f_{x,t}\}_{t=0}^{\infty}$ is a competitive equilibrium for the economy if the following equations are satisfied:

$$\tilde{e}_t = \beta\left(1-\delta\right) E_t \left[\left(\frac{c_t}{c_{t+1}}\right)\left(\tilde{e}_{t+1} + \left(\frac{zmin}{z_{t+1}^c}\right)^{\kappa} \tilde{d}_{t+1}\right)\right]$$

$$1 = \beta E_t \left[\left(\frac{c_t}{c_{t+1}}\right)\frac{R_t}{\pi_{t+1}}\right]$$

$$\tilde{e}_t = f_{e,t}$$

$$N_t = \left(1-\delta\right)\left(N_{t-1} + N_{t-1}^e\right)$$

$$\tilde{\rho}_t = \frac{\theta}{\theta-1}\frac{w_t R_t}{Z_t \tilde{z}_t}$$

$$c_t + N_t^e \tilde{e}_t = R_t w_t L_t^d + N_{o,t}\tilde{d}_t$$

$$\tilde{d}_t = \frac{1}{\theta}\tilde{\rho}_t^{1-\theta} Y_t - f_{x,t}$$

$$\tilde{w}_t = \frac{\theta_w}{\left(\theta_w - 1\right)}\frac{f_t^2}{f_t^1}$$

$$f_t^1 = L_t^d w_t^{\theta_w}\frac{1}{c_t} + \tilde{\alpha}\beta E_t \pi_{t+1}^{\theta_w - 1} f_{t+1}^1$$

$$f_t^2 = L_t^d w_t^{\theta_w} \chi \left(L_t^s\right)^{\frac{1}{\phi}} + \tilde{\alpha}\beta E_t \pi_{t+1}^{\theta_w} f_{t+1}^2$$

$$1 = \tilde{\rho}_t^{1-\theta} N_{o,t}$$

$$Y_t = c_t + N_{o,t} f_{x,t} + N_t^e f_{e,t}$$

$$w_t^{1-\theta_w} = (1-\tilde{\alpha})\tilde{w}_t^{1-\theta_w} + \tilde{\alpha}\left(\frac{w_{t-1}}{\pi_t}\right)^{1-\theta_w}$$

$$L_t^s = \tilde{s}_t L_t^d$$

$$\tilde{s}_t = \left(1-\tilde{\alpha}\right)\left(\frac{\tilde{w}_t}{w_t}\right)^{-\theta_w} + \tilde{\alpha}\left(\frac{w_{t-1}}{w_t}\right)^{-\theta_w} \pi_t^{\theta_w}\tilde{s}_{t-1}$$

$$\ln\left(\frac{R_t}{R}\right) = \alpha_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \alpha_Y \ln\left(\frac{Y_t}{Y}\right) + \alpha_R \ln\left(\frac{R_{t-1}}{R}\right)$$

$$\chi\left(L_t^s\right)^{\frac{1}{\phi}} c_t = \frac{w_t}{\tilde{\mu}_t}$$

$$\tilde{z}_t = \nu z_t^c$$

$$z_t^c = \frac{\theta^{\frac{\theta}{\theta-1}}}{\theta-1}\frac{w_t R_t}{Z_t}\left(\frac{f_{x,t}}{Y_t}\right)^{\frac{1}{\theta-1}}$$

$$N_{o,t} = \left(\frac{zmin}{z_t^c}\right)^{\kappa} N_t$$

Appendix A3: Analytical Equilibrium - Form 2

The set of endogenous variables $\left\{c_t, L_t^d, L_t^s, \tilde{s}_t, N_{t+1}, N_t^e, \delta_t, \tilde{\mu}_t, \tilde{e}_t, \tilde{\rho}_t, \tilde{d}_t, \tilde{z}_t, z_t^c, \tilde{w}_t, w_t, Y_t, f_t^1, f_t^2, R_t, \pi_t\right\}_{t=0}^{\infty}$ and exogenous series $\{Z_t, f_{e,t}, f_{x,t}\}_{t=0}^{\infty}$ is a competitive equilibrium for the economy if the following equations are satisfied:

$$\tilde{e}_t = \beta(1-\psi) E_t (1-\delta_{t+1}) \left[\left(\frac{c_t}{c_{t+1}} \right) \left(\tilde{e}_{t+1} + \tilde{d}_{t+1} \right) \right]$$

$$1 = \beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right) \frac{R_t}{\pi_{t+1}} \right]$$

$$\tilde{e}_t = f_{e,t}$$

$$N_t = (1-\delta_t)(1-\psi)\left(N_{t-1} + N_{t-1}^e\right)$$

$$\tilde{\rho}_t = \frac{\theta}{\theta-1} \frac{w_t R_t}{Z_t \tilde{z}_t}$$

$$c_t + N_t^e \tilde{e}_t = R_t w_t L_t^d + N_t \tilde{d}_t$$

$$\tilde{d}_t = \frac{1}{\theta} \tilde{\rho}_t^{1-\theta} Y_t - f_{x,t}$$

$$\tilde{w}_t = \frac{\theta_w}{(\theta_w - 1)} \frac{f_t^2}{f_t^1}$$

$$f_t^1 = L_t^d w_t^{\theta_w} \frac{1}{c_t} + \tilde{\alpha}\beta E_t \pi_{t+1}^{\theta_w - 1} f_{t+1}^1$$

$$f_t^2 = L_t^d w_t^{\theta_w} \chi \left(L_t^s\right)^{\frac{1}{\phi}} + \tilde{\alpha}\beta E_t \pi_{t+1}^{\theta_w} f_{t+1}^2$$

$$1 = \tilde{\rho}_t^{1-\theta} N_t$$

$$Y_t = c_t + N_t f_{x,t} + N_t^e f_{e,t}$$

$$w_t^{1-\theta_w} = (1-\tilde{\alpha})\tilde{w}_t^{1-\theta_w} + \tilde{\alpha} \left(\frac{w_{t-1}}{\pi_t} \right)^{1-\theta_w}$$

$$L_t^s = \tilde{s}_t L_t^d$$

$$\tilde{s}_t = (1-\tilde{\alpha}) \left(\frac{\tilde{w}_t}{w_t} \right)^{-\theta_w} + \tilde{\alpha} \left(\frac{w_{t-1}}{w_t} \right)^{-\theta_w} \pi_t^{\theta_w} \tilde{s}_{t-1}$$

$$\ln\left(\frac{R_t}{R}\right) = \alpha_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \alpha_Y \ln\left(\frac{Y_t}{Y}\right) + \alpha_R \ln\left(\frac{R_{t-1}}{R}\right)$$

$$\chi \left(L_t^s\right)^{\frac{1}{\phi}} c_t = \frac{w_t}{\tilde{\mu}_t}$$

$$\tilde{z}_t = \nu z_t^c$$

$$z_t^c = \frac{\theta^{\frac{\theta}{\theta-1}}}{\theta-1} \frac{w_t R_t}{Z_t} \left(\frac{f_{x,t}}{Y_t} \right)^{\frac{1}{\theta-1}}$$

$$\delta_t = 1 - \left(\frac{zmin}{z_t^c} \right)^{\kappa}$$

Appendix B

In this Appendix it is shown how to express the labor demanded in each market j as a function of the total labor demanded L_t^d. First of all, note that the ratio between individual quantities produced by two different firms can be expressed just as a function of the ratio of the relative productivities:

$$\frac{y_t(z)}{y_t(\tilde{z})} = \frac{\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t z}\right)^{-\theta} Y_t}{\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t \tilde{z}}\right)^{-\theta} Y_t} = \left(\frac{z}{\tilde{z}}\right)^{\theta}$$

Using the definition $y_t(z) = Z_t z l_t(z)$ we can write:

$$\frac{l_t(z)}{l_t(\tilde{z})} = \left(\frac{z}{\tilde{z}}\right)^{\theta-1}$$

which gives $l_t(z)$ as a function of $\tilde{z}$, z and $l_t(\tilde{z})$. Multiplying both sides by $\frac{1}{1-G(z_t^c)} N_{o,t} g(z)$ and integrating over z we can obtain:

$$\frac{1}{1-G(z_t^c)} \int_{z_t^c}^{\infty} N_{o,t} l_t(z) g(z) dz_t = L_t^d = \frac{1}{1-G(z_t^c)} l_t(\tilde{z}) \tilde{z}^{1-\theta} N_{o,t} \int_{z_t^c}^{\infty} z^{\theta-1} g(z) dz = l_t(\tilde{z}) N_{o,t}$$

Moreover, using that $l_t^j(z) = \left(\frac{w_t^j}{w_t}\right)^{-\theta_w} l_t(z)$, we can rewrite the ratio above as:

$$\frac{l_t^j(z)}{l_t^j(\tilde{z})} = \left(\frac{z}{\tilde{z}}\right)^{\theta-1}$$

Thus:

$$\frac{1}{1-G(z_t^c)} \int_{z_t^c}^{\infty} N_{o,t} l_t^j(z) g(z) dz_t = L_t^j = \frac{1}{1-G(z_t^c)} l_t^j(\tilde{z}) \tilde{z}^{1-\theta} N_{o,t} \int_{z_t^c}^{\infty} z^{\theta-1} g(z) dz = l_t^j(\tilde{z}) N_{o,t}$$

This means that the demand constraint faced by the household can be obtained just be multiplying by the number of operating firms $N_{o,t}$ both sides of the individual demand of labor inputs, evaluated for the firm with productivity $\tilde{z}$.

References

Axtell, R. L. (2001). Zipf distribution of us firm sizes. *science*, *293*(5536), 1818–1820.
Bergin, P. R., & Corsetti, G. (2008). The extensive margin and monetary policy. *Journal of Monetary Economics*, *55*(7), 1222–1237.
Bilbiie, F. O. (2020, forthcoming). Monetary neutrality with sticky prices and free entry. *Review of Economics and Statistics*.
Bilbiie, F. O., Fujiwara, I., & Ghironi, F. (2014). Optimal monetary policy with endogenous entry and product variety. *Journal of Monetary Economics*, *64*, 1–20.
Bilbiie, F. O., Ghironi, F., & Melitz. (2007). Monetary policy and business cycles with endogenous entry and product variety [with comments and discussion]. *NBER Macroeconomics Annual*, *22*, 299–379.
Bilbiie, F. O., Ghironi, F., & Melitz, M. J. (2012). Endogenous entry, product variety, and business cycles. *Journal of Political Economy*, *120*(2), 304–345.
Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, *12*(3), 383–398.
Cetorelli, N. (2014). Surviving credit market competition. *Economic Inquiry*, *52*(1), 320–340.
Christiano, L. J., Eichenbaum, M., & Evans, C. (2001). *Nominal rigidities and the dynamic effects of a shock to monetary policy* (Tech. Rep.). National bureau of economic research.
Chu, A. C., Cozzi, G., Furukawa, Y., & Liao, C.-h. (2019). Inflation and innovation in a schumpeterian economy with north–south technology transfer. *Journal of Money, Credit and Banking*, *51*(2-3), 683–719.
Chugh, S. K., & Ghironi, F. (2011). *Optimal fiscal policy with endogenous product variety* (NBER Working Paper No. 17319). National Bureau of Economic Research.
Clementi, G. L., & Palazzo, B. (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal: Macroeconomics*, *8*(3), 1–41.
Collard-Wexler, A. (2013). Demand fluctuations in the ready-mix concrete industry. *Econometrica*, *81*(3), 1003–1037.
Decker Ryan A., R. S. J., John Haltiwanger, & Miranda, J. (2015). *Where has all the skewness gone? the decline in high-growth (young) firms in the u.s* (Working Paper No. 15-43,). Center for Economic Studies, U.S. Census Bureau.
De Loecker, J., Eeckhout, J., & Unger, G. (2019). The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics, Forthcoming*.
Etro, F., & Rossi, L. (2015). New-keynesian phillips curve with bertrand competition and endogenous entry. *Journal of Economic Dynamics and Control*, *51*, 318–340.
Ghironi, F., & Kim, J. (2018). Structural reforms in granular economies.
Ghironi, F., & Melitz, M. J. (2005). International trade and macroeconomic dynamics with heterogeneous firms. *The Quarterly Journal of Economics*, *120*(3), 865–915.
Grullon, G., Larkin, Y., & Michaely, R. (2019). Are US industries becoming more concentrated? *Review of Finance*, *23*(4), 697–743.
Hamano, M., & Zanetti, F. (2017). Endogenous product turnover and macroeconomic dynamics. *Review of Economic Dynamics*, *26*, 263–279.
Jaimovich, N., & Floetotto, M. (2008). Firm dynamics, markup variations, and the business cycle. *Journal of Monetary Economics*, *55*(7), 1238–1252.
King, R. G., & Rebelo, S. T. (1999). Resuscitating real business cycles. *Handbook of macroeconomics*, *1*, 927–1007.
Lewis, V., & Poilly, C. (2012). Firm entry, markups and the monetary transmission mechanism. *Journal of Monetary Economics*, *59*(7), 670–685.
Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, *71*(6), 1695–1725.
Moran, P., & Queralto, A. (2018). Innovation, productivity, and monetary policy. *Journal of Monetary Economics*, *93*, 24–41.
Schmitt-Grohé, S., & Uribe, M. (2005). Optimal fiscal and monetary policy in a medium-scale macroeconomic model. *NBER Macroeconomics Annual*, *20*, 383–425.

Previous DNB Working Papers in 2020

No. 662 **Carin van der Cruijsen-Knoben, Jakob de Haan and Ria Roerink**, Financial knowledge and trust in financial institutions
No. 663 **Jon Frost,** Economic Forces Driving FinTech Adoption
No. 664 **Anna Samarina and Nikos Apokoritis,** Evolution of monetary policy frameworks in the post-crisis environment
No. 665 **Christian König-Kersting, Stefan T. Trautmann and Razvan Vlahu,** Bank instability: Interbank linkages and the role of disclosure
No. 666 **Claus Brand, Gavin Goy, Wolfgang Lemke,** Natural Rate Chimera and Bond Pricing Reality
No. 667 **Joost Bats,** Corporates' dependence on banks: The impact of ECB corporate sector Purchases
No. 668 **Marc van Kralingen, Diego Garlaschelli, Karolina Scholtus and Iman van Lelyveld,** Crowded trades, market clustering, and price instability
No. 669 **Mark Mink, Rodney Ramcharan and Iman van Lelyveld,** How Banks Respond to Distress: Shifting Risks in Europe's Banking Union
No. 670 **Jasmira Wiersma, Rob Alessie, Adriaan Kalwij, Annamaria Lusardi and Maarten van Rooij,** Skating on thin ice: New evidence on financial fragility
No. 671 **Michiel Bijlsma, Carin van der Cruijsen and Nicole Jonker,** Consumer propensity to adopt PSD2 services: trust for sale?
No. 672 **Duncan van Limbergen and Robert Vermeulen,** The importance of value chains for euro area trade: a time series perspective
No. 673 **Martijn Boermans en Bram van der Kroft,** Inflated credit ratings, regulatory arbitrage and capital requirements: Do investors strategically allocate bond portfolios?
No. 674 **Andras Lengyel and Massimo Giuliodori**, Demand Shocks for Public Debt in the Eurozone
No. 675 **Raymond Chaudron, Leo de Haan and Marco Hoeberichts,** Banks' net interest margins and interest rate risk: communicating vessels?
No. 676 **Martijn Boermans and John Burger**, Global and local currency effects on euro area investment in emerging market bonds
No. 677 **Patty Duijm and Ilke van Beveren**, Product diversification as a performance boosting strategy? Drivers and impact of diversification strategies in the property-liability insurance industry
No. 678 **Richard Heuver and Ron Berndsen,** Liquidity Coverage Ratio in a Payments Network: Uncovering Contagion Paths
No. 679 **Gabriele Galati, Jan Kakes and Richhild Moessner,** Effects of credit restrictions in the Netherlands and lessons for macroprudential policy
No. 680 **Frank van der Horst, Jelle Miedema, Joshua Snell and Jan Theeuwes,** Banknote verification relies on vision, feel and a single second
No. 681 **Leonard Sabetti and Ronald Heijmans,** Shallow or deep? Detecting anomalous flows in the Canadian Automated Clearing and Settlement System using an autoencoder
No. 682 **Shaun Byck and Ronald Heijmans,** How much liquidity would a liquidity-saving mechanism save if a liquidity-saving mechanism could save liquidity? A simulation approach for Canada's large-value payment system
No. 683 **Neville Arjani and Ronald Heijmans,** Is there anybody out there? Detecting operational outages from LVTS transaction data
No. 684 **Jan Willem van den End, Paul Konietschke, Anna Samarina, Irina Stanga,** Macroeconomic reversal rate: evidence from a nonlinear IS-curve

DeNederlandscheBank

EUROSYSTEEM

De Nederlandsche Bank N.V.
Postbus 98, 1000 AB Amsterdam
020 524 91 11
dnb.nl