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Abstract

We analyse the optimal composition of a federal or supra-national committee. The representation of regional (national) entities in federal committees is typically motivated by their superior knowledge of local conditions. Using this argument, we formally model the optimal composition of a committee. Our results indicate that a region’s representation in a federal committee depends on its ability to reduce uncertainty about the state of the economy. Furthermore, correlation of regional uncertainty increases the value of information. This induces synergy effects, which result in higher optimal representation in the committee.

Keywords: Optimal composition of a committee, currency union, information uncertainty, regional representation, EMU enlargement, monetary institutions

JEL Codes: E50, E58, F33
1 Introduction

The aim of this paper is to investigate how individual countries or regions of a currency area should be represented in the body taking monetary policy decisions (henceforth ‘monetary policy committee’, MPC). Monetary policy decisions are often made in a highly uncertain environment, where the overall state of the economy can not be fully assessed at the moment interest rate decisions are taken (Goodfriend, 1999). To reduce this uncertainty, information on the economic situation in the monetary area is required. We motivate how Governors from regional central banks – by providing local information – can contribute to reduce overall uncertainty and thereby allow a better assessment of the state of the economy in a currency area. Then, we investigate the extent to which individual regions or countries should be part of the monetary policy committee (MPC) in a situation where the uncertainties regarding the state of the respective economies are not equally distributed within the currency area.1

The model is formulated in terms of a monetary decision-making body, but its implications extend beyond the monetary sphere. Representation of regional entities in federal states or supra-national commissions (such as the European Commission) can be motivated by the regional information they provide. Their aim to safeguard ‘national’ or ‘regional’ interests (and to make sure that they are adequately reflected in the final decision) is not fundamentally different from the regional uncertainties introduced in our model: they are the result of uncertainty surrounding the economic situation of individuals. Thus our paper also provides a rationale for contribution of lower branches of government in federal or supra-national decision-making.

This paper is linked to different strands of the literature. Olson and Zeckhauser (1966) examine the workings of international organisations, emphasising the public good character of the services they supply. Casella (1992) analyses the desirability to join a monetary union. Although the setup of the model is very different (no modelling of information uncertainties), Casella’s results are comparable to ours in that small countries may exert larger influences than is warranted by their size. Von Hagen and Stippel (1994) do not focus on the relative weight of different countries in a monetary policy committee, but evaluate alternative distributions of power over monetary policy between the member and the administrative center of the union. Hefeker (2003) develops a model based on structural differences across members of a currency union and finds that it can be beneficial to limit the representation of regions if their economic structure is too different from the rest of the currency area.

To preview the conclusions, it may be preferable for countries associated

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1 We do not propose a specific rotation scheme or a voting procedure for the MPC, but under general conditions we evaluate the optimal composition of such a body if information asymmetries arise.
with a high degree of uncertainty to have a higher representation in the MPC than is justified based on their economic weight. This is because the likelihood of making an error in judging these countries’ performance decreases with the availability of regional information. If the difficulties in assessing a country’s performance are correlated between countries, it is important that these countries are represented in the MPC, but which one of those countries sends a delegate is arbitrary.

We proceed as follows. In the next section we summarize the main reasons for regional representation within a monetary policy committee. Section 3 contains a theoretical model of information asymmetries and their effect on the optimum composition of the monetary policy committee. In section 4 we show that information uncertainty could offer a rationale for the ECB’s newly adopted rotation scheme. Finally, we summarize the main findings in the last section.

2 Regional representation in the formulation of monetary policy

European monetary policy is carried out by the Eurosystem, consisting of the European Central Bank (ECB) and the National Central Banks (NCBs) of the countries participating in the single currency. The supreme body is the Governing Council, which comprises members of the ECB’s Executive Board and the Governors of all participating NCBs. At present EMU comprises 12 member states, but EMU enlargement might result in a union of 27 or more members. This could increase the number of NCB Governors up to a point where it might not be efficient to grant voting rights to all Governors in all meetings. Therefore, the ECB has recently adopted a rotation scheme for the NCB Governors, where countries have different voting frequencies. In a situation where all EU acceding countries will have joined EMU, the 8 smallest countries together comprise 1% of the euro area in terms of GDP, but according to the rotation scheme in the ECB Governing Council their weight will be considerably larger: 38%. How can we explain this apparent discrepancy?

Under perfect information, regional representation of central banks would not be necessary: one single central bank would collect all relevant information and monetary policy decisions would be taken in Washington (US FED) and Frankfurt (ECB), respectively, without any regional representation. However, in the real world such perfect information may not be available:

‘The growing dispersion of economic activity increases the value of local information that Reserve Bank presidents bring to the Federal Open Market Committee. ... Personal contacts are particularly

\[2\text{Details on the ECB’s rotation scheme can be found in European Central Bank (2002).}\]
valuable in periods of financial crisis when it is especially difficult to
know what is happening in certain sectors. Reserve banks tend to
specialize in knowledge concerning industries concentrated in their
respective districts.3

If a central bank features regional representation, the composition of the
monetary policy committee of the central bank has to strike a balance between
political and economic considerations (Akhtar and Howe, 1991): on the one
hand, a purely political approach aims at mirroring society’s composition of the
currency area, setting up a monetary policy committee proportional to popu-
lation weights, party preferences, etc. On the other hand, membership in the
monetary policy committee, purely based on efficiency considerations, reflects
the idea that regional representation can improve the accuracy of information
about the state of the economy. In such a case representation of regions or
national member states is related to the information content provided by their
representatives. ‘Over-representation’ of small countries or regions can be de-
sirable if their economic situation is relatively more uncertain than the rest.

We abstract from any political, regional or national considerations regarding
the voting behavior of the members of the MPC.4 Instead, we take a purely eco-

3 The model

3.1 Informational uncertainty and regional representation

Aggregate statistics (e.g. provided by a central statistical office) aim at depicting
the economic situation. However, data is often only available with a certain
lag (GDP data, for instance, exist only on quarterly basis), whereas monetary
policy decisions require up-to-date information. By the time decisions have to
be taken, other sources of information are needed to decrease the uncertainty.5

Uncertainty about the state of the economy essentially results from not
knowing the personal situation of every inhabitant. Perfect information is iden-
tical to knowing each inhabitant’s economic situation. However, usually the number of inhabitants is so large that collecting such perfect information would be very costly. To overcome this uncertainty, ways have to be found to aggregate individual information. A natural starting point is the city level. Suppose each city nominates one representative, who knows all inhabitants of his city personally. He collects all individual information and is thus able to provide an accurate description of the state of the economy in his city. The more local representatives share information in the monetary committee, the more accurate the description of the state of the national economy that can be derived and the likelihood of making a judgement error decreases.

For the purpose of this paper we assume that the state of the economy is fully characterised by its inflation rate, $\pi^{\text{est}}$. The statistical office of the currency area supplies an estimate of the state of the economy, but by the time this information becomes public, the economic situation of each individual might have changed, so that $\pi^{\text{est}}$ can deviate from the true state of the economy, $\pi^{\text{true}}$. In other words, $\pi^{\text{est}}$ may contain an observation error, $\varepsilon$, which by definition is identical to the sum of the individual observations errors (i.e. individual deviations from $\pi^{\text{est}}$):$^8$

$$
\varepsilon = \pi^{\text{est}} - \pi^{\text{true}} = e_{11} + ... + e_{1v_1} + e_{21} + ... + e_{2v_2} + ... + e_{n1} + ... + e_{nv_n} \quad (1)
$$

where $v_i$ is the number of inhabitants in city $i$ and $n$ is the number of cities.$^9$ We assume that all individual observation errors $e_{ij}$ have zero mean and are uncorrelated.$^{10}$ In a first-best world the monetary policy committee would invite all citizens, thereby removing all informational uncertainty. However, as we show below it might be efficient not to exceed a certain number of representatives, so the information needs to be aggregated. If the number of cities is large, a selection of representatives must even be made between the different cities.

We assume that each city-representative possesses more up-to-date local information than the national statistical office. After bundling all individual information he knows the true state of ‘his’ regional economy (i.e. he knows

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$^6$Alternatively, the country could be divided into ‘regions’, ‘counties’, ‘districts’ etc.

$^7$Clearly, the central bank could also care about other variables. In that sense, inflation should be regarded as a container variable, e.g. comprising other indicators or representing a certain relationship between growth and inflation.

$^8$To keep the model simple the uncertainty of every individual enters with the same weight. Implicitly, this assumes that wealth is equally distributed, otherwise wealthier people should be given a larger weight (due to their larger economic weight they exhibit stronger influence on aggregate inflation). An alternative approach would be to base uncertainty on GDP weights. Qualitatively, both approaches lead to similar results.

$^9$In all equations time indices are skipped for the sake of clarity.

$^{10}$Later we assume that within a currency union national observations errors can be correlated (see section 3.4)
$e_1 + \ldots + e_n = e_i$. If all local representatives were present in the MPC meeting, the observation error made by the statistical office could be estimated as the sum of all errors at the city level, i.e.:

$$
\varepsilon = \sum_{i=1}^{n} e_i
$$

(2)

As an individual errors are uncorrelated, $\varepsilon$ has the following variance:

$$
\text{Var}(\varepsilon) = \sum_{i=1}^{n} \text{Var}(e_i)\quad \text{and} \quad \text{Var}(e_i) = \begin{cases} 
\sigma_i^2; & \text{if } i \text{ is not present} \\
0; & \text{if } i \text{ is present}
\end{cases}
$$

(3)

where $n$ is the number of cities and $e_i$ the observation error at the city level. The more representatives are present, the lower the variance of the error $\varepsilon$ (i.e. $\text{Var}(\varepsilon)$ declines in the number of regional representatives). Intuitively, this occurs because by providing certainty about the state of the economy in his city, the regional representative reduces a part of the potential error with respect to the overall state of the economy.

Regarding the value of getting more information by inviting one additional representative (i.e. the ‘returns to extra information’) we have to make an assumption about the variance of the error terms, $\text{Var}(e_i)$. Starting from the idea that the economic situation of each individual is equally uncertain, two cases can be distinguished.

- If the country is divided in such a way that each city consists of the same number of inhabitants, inviting an additional delegate reduces the uncertainty by a fixed amount: Each representative provides information about the same number of inhabitants, the returns to having one extra person in the monetary policy committee is constant. Such a situation is depicted by the straight line in figure 1.

Conceptually, very strict assumptions are needed to justify why the returns to information should continue constantly (e.g. same number of inhabitants per city, same degree of uncertainty at the individual level etc.). Therefore, this case will not further be analysed. Instead, a situation where not all cities are of equal size seems more realistic.

- If the numbers of inhabitants differ across cities, the reduction in uncertainty from having a person from a large city exceeds the reduction in information from having a representative from a small city: in the former case the economic situation of a large group of people becomes known, whereas in the latter case the representative only provides information about a few individuals.\textsuperscript{11}

Formally, this implies that the observation

\textsuperscript{11}One could additionally assume that the informational uncertainty regarding some cities is
Figure 1: Constant and decreasing returns to information

errors have different variances.

If the error terms have different variances the return to inviting one extra representative is, \textit{a priori}, unclear. But by ordering all cities according to the variance of the error term, the returns to information are decreasing: the city with the highest uncertainty sends the first representative, then – proceeding along this ordering – each consecutive representative removes a smaller share of uncertainty than his (her) predecessor. Thus, the returns to inviting more people are decreasing in the number of representatives already present. Analytically:

\[ \text{Var}(\varepsilon) = f(r), \text{ such that } f'(r) < 0 \text{ and } f''(r) > 0, \]

where \( r \) is the number of representatives. This case is depicted in the concave curve in figure 1. We call this case ‘decreasing returns to information’. To solve the model algebraically, a functional form for the decreasing returns is needed. Although several functional forms fulfil these criteria, we have decided to use the following exponential specification:

\[ \text{Var}(\varepsilon) = \sigma^2 e^{-\lambda r}. \]  \hspace{1cm} (4)

The shape of the graph depends on the intrinsic uncertainty, \( \sigma^2 \), which determines the intersection of the graph with the y-axis, and how fast an additional representative can reduce the uncertainty, \( \lambda \). The latter parameter can be regarded as a proxy for the degree of agglomeration (or efficiency for representatives gathering information): if country 1 has more inhabitants than country 2, higher than for others, e.g. because they are more prone to external shocks or because of their geographical location etc. For clarity we have refrained from introducing economic weights at the city level, as qualitatively this idea is captured by differences in variances, \( \sigma_i^2 \).
but both are divided into the same number of cities, the uncertainty decreases faster for country 1 (i.e. \( \lambda_1 > \lambda_2 \)).

Having established a negative relationship between regional representation and uncertainty about the state of the economy, we proceed by showing that efficient local representation typically not implies full representation of all inhabitants. Then, we examine the optimal composition of the monetary policy committee, by looking first at optimal regional representation at the national level. Then we analyse the case of two countries forming a monetary union.

### 3.2 Why full representation is not optimal

In our model, the benefits from local representation are given by the reduction in economic uncertainty: depending on the number of local MPC members, uncertainty decreases. This comes, however, at a cost: The more local representatives are invited, the more difficult it will become to process the information, the more lengthy the discussions and costs for travels of the delegates etc. increase. Although the exact relationship of these costs are unknown, it seems safe to assume (i) that increasing the number of representatives yields costs and (ii) that these costs rise exponentially.\(^{12}\) In other words, a function relating the number of committee members \( m \) to costs of local representation should fulfil the following criteria:

\[
\text{Costs}(m) = f(m), \text{ such that } f'(m) > 0 \text{ and } f''(m) > 0.
\]

In figure 2 we plot two functions for costs and benefits of regional representation. The functional form we have chosen here is somewhat arbitrary, but the message of the graph is clear: more local representation is beneficial, but only up to a certain point. For \( m > \overline{m} \) the costs of involving more regional representatives exceed the benefits, which is why full representation – i.e. inviting every citizen – is typically not beneficial.

In reality, determination of the maximum number of representatives in the monetary policy committee, \( m \), is a political decision. It might coincide with the \( \overline{m} \) of figure 2, but it need not. For the rest of our paper, we therefore simply assume that the number of committee members has been set exogenously to \( m \).

### 3.3 Regional representation in its simplest form

We assume the monetary authority has a single objective, i.e. to minimise the deviations of the true inflation rate \( \pi^{true} \) from a given target inflation rate \( \pi^* \).

\(^{12}\)I.e. it seems realistic that a committee’s ‘capacity for efficient and timely decision-making’ (ECB, 2002) suffers more from an increase in number of committee members from 100 to 200 than if they rise from 5 to 10.
We assume it can control inflation directly.\footnote{This assumption is frequently found in the literature, see e.g. Rogoff (1985).} The loss function is given by:

\[
L = E((\pi^{true} - \pi^*)^2) = (E(\pi^{true}) - \pi^*)^2 + Var(\pi^{true})
\]

As we can see from equation (5) the loss function of the monetary policy committee is the sum of two parts: the first part refers to the optimal rule for monetary policy decision-making, the second to informational uncertainty.

**Proposition 1** The optimal rule for monetary policy (i.e. the first part in eq. 5) is invariant to the composition of the MPC.

**Proof.** *Ex ante*, \(E(e_i) = 0\) for all \(e_i\). Of course, \(E(e_i) \neq 0\) *ex post*, i.e. after all representatives have revealed their local information, but the composition of the MPC is invariant to that. ■

This implies that to minimise equation (5) we can concentrate on selecting regional representation in such a way that the likelihood of making an error in the assessment of the current state of the economy is minimised. This reduces the loss function to:

\[
\bar{L} = Var(\pi^{true}) = Var(\pi^{ext} - \varepsilon) = Var(\varepsilon)
\]

Minimising this loss function yields a straightforward solution, as all local representatives are ordered according to their uncertainty: the best strategy is to pick the first representative from the city with highest uncertainty (i.e. with

![Figure 2: Costs and benefits of regional representation](image-url)
the highest number of inhabitants), and to proceed down the list to cities with lower probability of making a judgement error.

3.4 Optimal regional representation in the monetary policy committee of a currency union of two countries

3.4.1 A central bank’s loss function in a currency union

Next, we investigate the optimal solution if two countries\(^{14}\) decide to form a currency area. We retain the idea that aggregate statistics of both countries can be subject to the information uncertainties sketched above.

In the previous section we have assumed that deviations from the aggregate forecast are uncorrelated.\(^{15}\) Between countries, making such an assumption is less realistic: as we cannot exclude the possibility that e.g. due to international trade links the economic situation between countries might be correlated, we allow for the possibility that observation errors between countries are correlated. In that case the regional representatives may not be aware of the fact that the local information they provide actually not only contains news about the state of the economy in their own country, but is also affected by the economic situation in the other country.

In other words, the regional representative continues to provide regional information. However, the nature of the information they provide changes: the observation errors are actually influenced by what is happening in both countries. To introduce correlation between \(\varepsilon_1\) and \(\varepsilon_2\) (the observation errors of country 1 and 2, respectively) we assume that they are linear combinations of two underlying (unobservable and uncorrelated) country-specific disturbances, \(\eta_1\) and \(\eta_2\).\(^{16}\) Analytically,

\[
\begin{align*}
\varepsilon_1 &= \alpha \eta_1 + (1 - \alpha) \eta_2 \\
\varepsilon_2 &= (1 - \beta) \eta_1 + \beta \eta_2.
\end{align*}
\]

We say a country’s observation error is correlated with the rest of the currency area if know this country’s state of the economy can provide information useful to judge other country’s economic situation. Note that e.g. a decrease in \(\alpha\) implies that the country-specific disturbance of country 2 has become more

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\(^{14}\)In what follows we use the terms ‘currency union’ for the currency area and ‘countries’ for regions, districts etc. Of course all results apply equally to supra-national and federal institutions.

\(^{15}\)A possible explanation why this assumption seems realistic is that if any pattern would exists between cities within a country, the national statistical office would take this pattern into account and correct its estimation accordingly.

\(^{16}\)This way of introducing correlation does not imply model inconsistency in the forecasts supplied by the statistical office. Instead, it simply reflects that fact that economic spillovers between countries need not, but might occur.
important in the observation error of country 1 but not vice versa, or simply speaking: the influence of country 2 on the observation error of country 1 increases, e.g. because country 2 is a large country and country 1 is a small one. Note finally that for $\alpha = \beta = 1$ the observation errors of both countries are uncorrelated.\footnote{17}

We assign economic weights to the countries participating in the currency union, $a_1$ and $a_2$, reflecting differences in size. The union-wide true and estimated inflation rates, $\pi^{true}_{CU}$ and $\pi^{est}_{CU}$, respectively, become the weighted averages of the national indices:

$$\pi^{est}_{CU} = a_1\pi^{est}_1 + a_2\pi^{est}_2$$

$$\pi^{true}_{CU} = a_1\pi^{true}_1 + a_2\pi^{true}_2 = a_1(\pi^{true}_1 - \varepsilon_1) + a_2(\pi^{true}_2 - \varepsilon_2). \quad (10)$$

As before the MPC aims at minimizing deviations of the true union-wide inflation rate from its target level $\pi^*$. Its loss function now becomes:

$$L = E((\pi^{true}_{CU} - \pi^*)^2) \quad (11)$$

In analogy to equation (5) the loss function of the supra-national monetary policy committee, given by eq. (11), can be split into a component referring to the information uncertainty and a component referring to the optimal rule for monetary policy decision-making. Again we can reduce the information problem to minimizing the informational uncertainty:

$$\bar{L} = Var(\pi^{true}_{CU}) = Var(a_1\varepsilon_1 + a_2\varepsilon_2) \quad (12)$$

The question who should be represented in the monetary policy committee can be answered in two steps: first, a choice has to be made regarding the country of origin, second within each country representatives have to be chosen. The solution to the second question has been outlined in the previous section, in what follows we explore the division of committee members with regard to their country of origin. In other words, we are interested in an optimal solution for $r_1$, the number of committee members from country 1, on the basis of which the share from country 2 can be calculated as $(m - r_1)$ representatives.

\footnote{17}We make the simplifying assumption that $\alpha$ and $\beta$ take values between 0.5 and 1.
3.4.2 The optimal monetary policy committee

Using equation (4) the variance of the judgement errors in the two countries can be expressed as follows:

\[ \text{Var}(\varepsilon_1) = \sigma_1^2 e^{-\lambda_1 r_1} \]
\[ \text{Var}(\varepsilon_2) = \sigma_2^2 e^{-\lambda_2 (m-r_1)} \] (13) (14)

From equation (8) and (9) it follows that

\[ \tilde{L} = \text{Var}(a_1 (\alpha \eta_1 + (1-\alpha)\eta_2) + a_2 ((1-\beta)\eta_1 + \beta \eta_2)) \]
\[ = (a_1 \alpha + a_2 (1-\beta))^2 \text{Var}(\eta_1) + (a_1 (\alpha - 1) - a_2 \beta)^2 \text{Var}(\eta_2) \]
\[ = w_1 \sigma_1^2 e^{-\lambda_1 r_1} + w_2 \sigma_2^2 e^{-\lambda_2 (m-r_1)}, \] (17)

where

\[ w_1 = \frac{(a_1 \alpha + a_2 (1-\beta))^2}{\alpha^2 \beta^2 - (1-\alpha)^2 (1-\beta)^2} \]
\[ w_2 = \frac{(a_1 - a_2 (1-\beta))^2}{\alpha^2 \beta^2 - (1-\alpha)^2 (1-\beta)^2} \] (18) (19)

Equation (17) basically states that loss is given by the weighted, scaled uncertainty in each country. Minimizing loss with respect to \( r_1 \) requires

\[ \frac{\partial \tilde{L}}{\partial r_1} = -\lambda_1 w_1 \sigma_1^2 e^{-\lambda_1 r_1} + \lambda_2 w_2 \sigma_2^2 e^{-\lambda_2 (m-r_1)} = 0. \] (20)

In the optimum the marginal reduction in uncertainty in both countries (\( MRU_i \)) must be equal. The optimal solution for the shares of both countries in the monetary policy committee is given by:

\[ r_1 = \frac{\lambda_2}{\lambda_1 + \lambda_2} m + \frac{1}{\lambda_1 + \lambda_2} \ln \left( \frac{\lambda_1 w_1 \sigma_1^2}{\lambda_2 w_2 \sigma_2^2} \right), \] (21)
\[ r_2 = \frac{\lambda_1}{\lambda_1 + \lambda_2} m - \frac{1}{\lambda_1 + \lambda_2} \ln \left( \frac{\lambda_1 w_1 \sigma_1^2}{\lambda_2 w_2 \sigma_2^2} \right) = m - r_1. \] (22)

---

Note that the variances of the country-specific disturbances can be expressed in terms of the (known) variances of observation errors, \( \varepsilon_1 \) and \( \varepsilon_2 \):

\[ \text{Var}(\eta_1) = \frac{\sigma_1^2}{D_1} \text{Var}(\varepsilon_1) - \frac{(1-\alpha)^2}{D_1^2} \text{Var}(\varepsilon_2) \] (15)
\[ \text{Var}(\eta_2) = \frac{\sigma_2^2}{D_1} \text{Var}(\varepsilon_2) - \frac{(1-\beta)^2}{D_1^2} \text{Var}(\varepsilon_1) \] (16)

whereby \( D_1 = \alpha^2 \beta^2 - (1-\alpha)^2 (1-\beta)^2 \).
In the simplest case, both countries are fully symmetric regarding the (weighted) economic uncertainty and the returns to information ($a_1 \sigma_1 = a_2 \sigma_2$ and $\lambda_1 = \lambda_2$). In that case the optimal representation is $r_1 = r_2 = \frac{1}{2}m$. If the informational uncertainties between the countries are not correlated ($\alpha = \beta = 1$) from equation (21) and (22) we can derive the following propositions for a given $m$:

**Proposition 2** A higher degree of agglomeration leads to an increase in representation in the monetary policy committee.

**Proof.** Assume —without loss of generality— that the reduction in uncertainty in country 2 is faster than in country 1, i.e. $\lambda_2 = \lambda + \theta$, where $\lambda = \lambda_1$, $\lambda > 0$ and $\theta > 0$. Eq. (21) is reduced to:

$$r_1 = \frac{\lambda + \theta}{2\lambda + \theta}m + \frac{1}{2\lambda + \theta} \ln \left( \frac{1}{\lambda + \theta} \right)$$

and $\frac{\partial r_1}{\partial \theta} > 0$. ■

Simply speaking, a higher degree of agglomeration corresponds to dividing the same number of inhabitants in a country over less, but larger cities. Then, as the city size increases, the information provided by each city representative becomes more important. In the optimum $MRU_1$ should be equal to $MRU_2$, therefore relatively more representatives from country 1 are needed.

**Proposition 3** A higher (weighted) degree of informational uncertainty leads to an increase in representation in the monetary policy committee.

**Proof.** Formally, assuming $\lambda_1 = \lambda_2 = \lambda$, we allow for different degrees of uncertainty in both countries (scaled by the economic weight), i.e. $a_1^2 \sigma_1^2 = a$ and $a_2^2 \sigma_2^2 = a + \phi$, $\phi > 0$. Then,

$$r_1 = \frac{1}{2}m + \frac{1}{2\lambda} \ln \left( \frac{a}{a + \phi} \right)$$

and $\frac{\partial r_1}{\partial \phi} < 0$. ■

Note that differences in weighted uncertainty can result from (i) different economic weights or (ii) because the assessment of a country’s economic situation is relatively more difficult. Lastly, we investigate how these results change if we allow for nonzero correlation between the two countries.

**Proposition 4** A country’s representation in the monetary policy committee is increasing if the information its representative provide help to judge the economic situation in other countries.

**Proof.** The first derivative of $r_1$ is decreasing in $\beta$, the first derivative of $r_2$ is increasing in $\alpha$, i.e.

$$\frac{\partial r_1}{\partial \beta} < 0 \text{ and } \frac{\partial r_2}{\partial \alpha} > 0.$$
Intuitively, if $\beta$ decreases the informational uncertainty in country 2 becomes more dependent on the state of country 1’s country-specific disturbance $\eta_1$. The information provided by representatives of country 1 becomes more valuable, as it also allows judging the state of country 2’s economy. More delegates from country 1 are needed and the share of country 2 in the MPC decreases. If $\alpha$ increases, country 1 becomes more ‘independent’. The information its delegates provide become less valuable, since they are only relevant to judge country 1. Consequently, country 1’s share in the MPC declines.

3.4.3 Graphical analysis of the two-country case

Graphical analysis of the results yields some additional insights. For our example we assume two equally uncertain countries ($\sigma^2_1 = \sigma^2_2 = 1$) of different size. We start by assuming both countries are uncorrelated ($\alpha = \beta = 1$) and to keep the graphs simple, we chose the parameters for the $a_i$’s and $\lambda_i$’s such that the first derivative of the variance of the error term has the value of 1 at point 0. In the graphs uncertainty declining faster in country 2, which implies that country 1 divided into more cities than country 2 ($\lambda_1 < \lambda_2$).\textsuperscript{19}

In figure 3 we plot the marginal reduction in uncertainty ($MRU_i$) provided by each representative for the two countries as a function of $r_i$, i.e. the first derivative of $Var(\varepsilon_i) = a_i e^{-\lambda_i r_i}$.\textsuperscript{20} Suppose that the total number of committee members has been set at 20 and following a political decision both countries send exactly 10 representatives. Is this representation optimal?

\textsuperscript{19}The selection of the parameters does not critically influence our results.

\textsuperscript{20}Note that on the vertical axis we plot $-MRU_i$, i.e. the negative values of $\frac{\partial Var(\varepsilon_i)}{\partial r_i}$, as the first derivative is, of course, negative. This change was purely made for expositional clarity and does not further affect our results.
Figure 3 shows that the marginal reduction in uncertainty for both countries is not equal if an equal number of representatives comes from both countries. As the reduction in uncertainty by the last representative of country 2 is smaller than by the respective colleague from country 1, the aggregated uncertainty in the currency union could easily be further reduced if the 10th representative from country 2 would be replaced by an 11th delegate of country 1. Finally, note that the total reduction in uncertainty about country 1 is given by the surface $ABCD_1$, whereas the total reduction in uncertainty about country 2 equals $ABCD_2$.

Figure 4 indicates how the optimal solution is derived graphically. In the optimum the marginal reduction of uncertainty in both countries has to be equal, as implied by eq. (20). To find this solution we aggregate the marginal reduction in uncertainty horizontally. The left part of figure 4 displays the marginal reduction in uncertainty provided by each representative, the right part – obtained by horizontal aggregation of two curves in the left part of the figure – shows the marginal reduction of uncertainty for the entire currency union, depending on the size of the committee $m$ ($m = r_1 + r_2$). For any committee size, a horizontal line between the two graphs shows the optimal representation shares for the two countries. In that case we see that the optimal representation entails that country 1 sends 12 representatives and country 2 sends 8.

We can also evaluate the impact of non-zero correlation graphically. The graphs change as follows (the derivations are given in appendix A.1):

1. $\frac{\partial(-MRU_1)}{\partial \beta} < 0$: the decline in $\beta$ shifts the $MRU_1$ to the right.

2. $\frac{\partial(-MRU_2)}{\partial \beta} < 0$, the $MRU_2$ also shifts to the right. Note, however, that $\left|\frac{\partial(-MRU_1)}{\partial \beta}\right| > \left|\frac{\partial(-MRU_2)}{\partial \beta}\right|$, which implies that $MRU_1$ shifts more to the

---

21 Horizontal aggregation of marginal reduction in uncertainty can be compared to aggregating demand curves in standard microeconomic theory. Analytically, horizontal aggregation can be done by first solving all $\text{Var}(\epsilon_i)$’s for $r_i$, taking the sum of $r_i$’s and solve for $m$ (note that $m = \sum r_i$).
Figure 5: Positive correlation (a decrease in $\beta$) in the two-country case

right than $MRU_2$.

Figure 5 shows the effect of a decrease in $\beta$. We see that for both countries the MRU-graphs shift to the right, but the shift of the $MRU_1$ graph is larger. In the right part $MRU_1$ and $MRU_2$ are again horizontally aggregated to obtain the graph $MRU_{CU}$. Repeating the same exercise as before we now see that as a result of this shift the share of country 1 increases, whereas the share of country 2 decreases. Intuitively, if country 1 also influences the informational uncertainty for country 2, country 1 representatives become relatively more important. By similar analysis it is possible to analyse the effects of an decrease in the scaled uncertainty, i.e. variation of $a_i$ or $\sigma_i^2$.

3.5 The monetary policy committee in larger currency unions

Our findings for the two-country case can be extended to larger currency areas in a straightforward way. Assume a currency union of three countries. All countries can influence each other, i.e.

\[ \begin{align*}
\varepsilon_1 &= \alpha_1 \eta_1 + \alpha_2 \eta_2 + (1 - \alpha_1 - \alpha_2) \eta_3 \\
\varepsilon_2 &= \beta_1 \eta_1 + \beta_2 \eta_2 + (1 - \beta_1 - \beta_2) \eta_3 \\
\varepsilon_3 &= \gamma_1 \eta_1 + \gamma_2 \eta_2 + (1 - \gamma_1 - \gamma_2) \eta_3,
\end{align*} \]

(23-25)

whereby the $\alpha_i$, $\beta_i$ and $\gamma_i$’s are all between 0 and 1. Proceeding along the same lines as in the previous section, we can express the loss function as follows:

\[ \begin{align*}
\bar{L} &= Var(a_1 \varepsilon_1 + a_2 \varepsilon_2 + a_3 \varepsilon_3) \\
&= w_1 \sigma_1^2 e^{-\lambda_1 r_1} + w_2 \sigma_2^2 e^{-\lambda_2 r_2} + w_3 \sigma_3^2 e^{-\lambda_3 (m-r_1-r_2)},
\end{align*} \]

(26)
where the formulas for the weights $w_1$, $w_2$, $w_3$ are given in appendix A.2. Minimizing the above loss function with respect to $r_1$ and $r_2$ yields the optimal representation of the three countries:

$$
\begin{align*}
    r_1 &= C \left( \lambda_2 \lambda_3 m + \lambda_3 \ln \frac{\lambda_1 w_1 \sigma_2^2}{\lambda_2 w_2 \sigma_2^2} + \lambda_2 \ln \frac{\lambda_1 w_1 \sigma_2^2}{\lambda_3 w_3 \sigma_3^2} \right) \tag{27} \\
    r_2 &= C \left( \lambda_1 \lambda_3 m + \lambda_1 \ln \frac{\lambda_2 w_2 \sigma_2^2}{\lambda_3 w_3 \sigma_3^2} + \lambda_3 \ln \frac{\lambda_2 w_2 \sigma_2^2}{\lambda_1 w_1 \sigma_1^2} \right) \tag{28} \\
    r_3 &= C \left( \lambda_1 \lambda_2 m + \lambda_1 \ln \frac{\lambda_3 w_3 \sigma_3^2}{\lambda_2 w_2 \sigma_2^2} + \lambda_2 \ln \frac{\lambda_3 w_3 \sigma_3^2}{\lambda_1 w_1 \sigma_1^2} \right) \tag{29}
\end{align*}
$$

where $C = \frac{1}{\lambda_3 \lambda_2 + \lambda_3 \lambda_1 + \lambda_1 \lambda_2}$. As before in the simplest case all three countries are uncorrelated and fully symmetrical and as a result, all countries should be equally represented in the monetary policy committee (i.e. $r_1 = r_2 = r_3 = \frac{1}{3}m$). Variation of the parameters of the model confirm the previous results of the two-country model.

1. **A higher degree of agglomeration leads to an increase in representation in the monetary policy committee:** This can e.g. be seen by assuming that the coefficient for the reduction in uncertainty in countries 1 and 2 is equal and smaller than in country 3 (we set $\lambda_1 = \lambda_2 = \lambda$ and $\lambda_3 = \lambda + \theta$, $\lambda > 0$ and $\theta > 0$). Then,

$$
\begin{align*}
    r_1 = r_2 &= \frac{\lambda + \theta}{3 \lambda + 2 \theta} m + \frac{1}{3 \lambda + 2 \theta} \ln \frac{\lambda}{\lambda + \theta} \quad \text{and} \quad \frac{\partial r_1}{\partial \theta} = \frac{\partial r_2}{\partial \theta} > 0,
\end{align*}
$$

provided that $m$ is sufficiently large.\(^{22}\)

2. **A higher (weighted) degree of informational uncertainty leads to an increase in representation in the monetary policy committee:** Assume, for instance, higher uncertainty for country 3, i.e. we set $a_1^2 \sigma_1^2 = a_2^2 \sigma_2^2 = a$ and $a_3^2 \sigma_3^2 = a + \phi$, $\phi > 0$. Then,

$$
\begin{align*}
    r_1 = r_2 &= \frac{1}{3} m + \frac{1}{3 \lambda} \ln \frac{a}{a + \phi} \quad \text{and} \quad \frac{\partial r_1}{\partial \phi} = \frac{\partial r_2}{\partial \phi} < 0.
\end{align*}
$$

The intuition behind this result is as follows: Country 1 and country 2 are identical and can be regarded as one big country. This reduces the problem of finding the optimal representation share for all three countries to a two-country problem, i.e. finding the optimal representation for country 1+2 and country 3. Proposition 3 continues to hold; note also that since country 1 and 2 are identical the optimal shares of country 1 and country 2 are equal.

\(^{22}\)The following condition must hold: $m > \frac{1}{X} \frac{3 \lambda + 2 \phi}{\lambda + \phi} + \frac{2}{3} \ln \frac{\lambda}{\lambda + \phi}$. 

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Lastly, we can refine proposition 4:

**Proposition 4a** A country’s representation in the monetary policy committee increases if the information its representatives provide help to judge the economic situation in other countries. An increase in correlation between certain countries leads to a decrease in representation of countries not correlated with this group, a decrease in correlation leads to an increase in representation of the rest of the currency area.

**Proof.** As an example, we investigate a case where the observation errors between country 1 and 2 are correlated, but independent of the observation error of country 3 (analytically, the parameters in equations (23)-(25) are subject to the following restrictions: \( \alpha_1 + \alpha_2 = \beta_1 + \beta_2 = 1 \) and \( \gamma_1 = \gamma_2 = 0 \)). The derivatives of \( r_i \) with respect to \( \alpha_1 \) and \( \beta_1 \) are:

\[
\begin{align*}
\frac{\partial r_1}{\partial \alpha_1} &> 0 \text{ and } \frac{\partial r_1}{\partial \beta_1} > 0, \\
\frac{\partial r_2}{\partial \alpha_1} &< 0 \text{ and } \frac{\partial r_2}{\partial \beta_1} < 0, \\
\frac{\partial r_3}{\partial \alpha_1} &> 0 \text{ and } \frac{\partial r_3}{\partial \beta_1} < 0,
\end{align*}
\]

provided that \( \lambda_3 \) is large enough relative to \( \lambda_1 \) and \( \lambda_2 \).\(^{23}\) The intuition is as follows:

**Increasing correlation** (\( \beta_1 \) increases) implies that the country-specific error of country 1 becomes more important for country 2. The share of country 1 representatives increases and the share of country 2 in the MPC decreases. The combined share of country 1+2 increases, as – due to the increase in the correlation of the observation errors between those two countries – the ‘value’ of their representatives increases. Consequently, the share of country 3 in the MPC declines.

**Decreasing correlation** (\( \alpha_1 \) increases) implies that the informational uncertainty in country 1 depends more heavily on the country-specific disturbance \( \eta_1 \) and the representatives from country 2 can provide less information about country 1. The share of country 2 decreases, the share

\(^{23}\) More specifically, the following conditions must hold:

- If \( \lambda_3 > \lambda_1 \):
  \[
  \frac{\partial r_1}{\partial \alpha_1} > 0 \text{ if } \lambda_3 > \frac{-2(\beta_1 - 1)\beta_1 (1 - \alpha_1) - 3\alpha_1^2 - 2\alpha_1^2 \beta_1 + 2\alpha_1 \beta_1}{(3\beta_1^2 + 2\alpha_1 \beta_1 - \alpha_1)(\beta_1 + 4\alpha_1) \beta_1 + 5\alpha_1 \beta_1 - 3\alpha_1 \beta_1 + 4\alpha_1 \beta_2 - \alpha_1} \lambda_2 \text{ and}
  \]

- If \( \lambda_3 < \lambda_1 \):
  \[
  \frac{\partial r_1}{\partial \beta_1} < 0 \text{ if } \lambda_3 > \frac{-1 + 2\alpha_1 \beta_1 - \alpha_1}{3\beta_1^2 + 2\alpha_1 \beta_1 + 2\beta_1^2 - \alpha_1} \lambda_1.
  \]
of country 1 increases slightly, but not sufficiently to offset the decrease of country 2 ($\frac{\partial \beta_2}{\partial \alpha_1} < \frac{\partial \beta_1}{\partial \alpha_1}$). The overall effect of the representation of country 1+2 is negative, as country 3 gains additional representational weight. Simply speaking, as the relationship between country 1 and 2 becomes less narrow, the value of the information provided by their representatives declines, which is why relatively more representatives from country 3 should be invited.

3.5.1 Graphical analysis of the effect of nonzero correlation in the three-country case

We can also analyse the case of nonzero correlation between two or more countries graphically. Figure 6 illustrates the example given above graphically by displaying the effect of an increase in correlation between country 1 and 2 on the representation of country 3 (i.e. $\beta_1$ increases): The left graph displays the aggregated MRU of the countries 1 and 2, the middle graph shows MRU$_3$ and the right graph the MRU in the entire currency union. Country 1 and 2 can be treated as one big country: the share of representatives from the first two countries is given by $r_{1+2}$, whereas country 3 sends $r_3$ representatives to fill the MPC of $m$ members.

Now assume that $\beta_1$ increases, i.e. the observation error of country 2 can be related to the country-specific error of country 1. The graphs shift as follow (see appendix A.3):

- $\frac{\partial(-MRU_1)}{\partial \beta_1} > 0$ and $\frac{\partial(-MRU_2)}{\partial \beta_1} > 0$.\textsuperscript{24} The graph $-MRU_{1+2}$ shifts to the right.

- $\frac{\partial(-MRU_3)}{\partial \beta_1} = 0$.

\textsuperscript{24}But as before $\left|\frac{\partial(-MRU_1)}{\partial \beta_1}\right| > \left|\frac{\partial(-MRU_2)}{\partial \beta_1}\right|$, consequently the share of country 2 representatives declines (recall in figure 5).
The aggregated \( MRUCU \) of the currency union will also shift to the right, reflecting the shift in the ‘big’ country 1+2. Keeping the number of representatives constant at \( \hat{m} \), we see that the share of representatives from the countries 1+2 increases to \( \frac{r_{1+2}}{\hat{m}} \) due to the nonzero correlation, whereas country 3 will lose representatives (from \( r_3 \) to \( \frac{r_3}{\hat{m}} \)). This occurs because the marginal gain from inviting an additional representative from the first two countries exceeds the gains an increase in \( r_3 \).

4 Regional representation at the European Central Bank and the US Federal Reserve

A central implication of our model is that if within a currency area the state of the economy of certain regions is relatively more difficult to assess than for other regions, then these ‘uncertain’ regions should be overrepresented in the federal committee. In what follows we show that this model is compatible with the ‘disproportionate’ (i.e. not related to economic size) weighting of regions in the ECB Governing Council and the US FOMC.\(^{25}\)

The ECB Governing Council consists of ECB Executive Board Members and all 12 national central bank (NCB) Governors. According to the recently adopted ‘rotation scheme’, the number of NCB Governors will be limited and they will exercise their voting rights with different frequencies, depending on (a) the member country’s size and (b) the size of its financial sector. When all current and future EU members will have joined EMU, the first group votes at 80% of all ECB Governing Council meetings. Economically, this group represents 76% of the euro area. The second group, which will vote in 57% of all meetings, represents economically 23% of the euro area, whereas the third group – voting in 38% of all meetings – economically represents 1%.\(^{26}\) Judged against their economic weight small countries seem largely overrepresented.\(^{27}\)

Our model offers an explanation. Using each country’s weight and the voting frequency of the ECB’s rotation scheme, we calculate the ‘information uncertainty’ for each of the three groups, as implied by our model. Parameters

\(^{25}\)Obviously, in practice any voting scheme is a political decisions and may therefore not fully reflect economic considerations.

\(^{26}\)The exact voting frequency also depends on the total number of countries joining the euro area (European Central Bank, 2002).

\(^{27}\)In the discussion about the rotation scheme overrepresentation has been feared on the basis that coalitions of small countries might ‘dominate’ the ECB Governing Council (see e.g. Baldwin et al., 2001, Berger, 2002, and Berger and de Haan, 2002): if ‘small’ countries (small in terms of GDP weight, such as the EU acceding countries) are largely overrepresented in the ECB Governing Council, a situation might occur where a coalition of small countries (representing, say, 35% of euro area GDP) sets monetary policy for the entire union – or worse, according to their own needs, rather than the economic requirements of the entire euro area (the so-called ‘regional voting bias’, see Meade and Sheets (1999, 2002).
like the speed of the reduction in informational uncertainty are obviously difficult to measure, therefore we have simply made the following assumptions: we assume that all three groups of countries are uncorrelated and we set $\lambda_1 = \lambda_2 = \lambda_3 = 0.5$. Table 1 shows the results for the euro area, setting the implied uncertainty for the largest group at 1.0: as it turns out, under these (admittedly somewhat arbitrary) assumptions the informational uncertainties in the third groups need to be considerably higher than in the other two groups to be compatible with our model. Looking at an enlarged EMU, this seems reasonable: The assessment of the economic situation of the acceding countries (all in part of group 3) is likely to be considerably more difficult than that of the current EMU members, e.g. because their economic structure undergo severe changes, because few historical data is available and because the time series that exist are are likely to exhibit structural breaks. Then, increasing the share of small countries beyond their economic weight might be beneficial.\footnote{An alternative explanation offered by the model why representation of the first group is so high is that the observation errors of the first group are highly correlated with the second group.}

In the FOMC of the US Federal Reserve, 11 regional FED Presidents share 5 votes. They are divided as follows: New York has 1 vote, Chicago and Cleveland have 0.5 vote whereas all other presidents have 0.33 vote each (Meade and Sheets, 2002). Grouping the different FED branches in three groups of (roughly) similar economic size as done for the ECB, we see that the implied uncertainty across the smaller FED districts is somewhat smaller than in the enlarged euro area. However, also in the US small FED districts are overrepresented.

\section{Conclusions}

Our model was based on the idea that although local considerations should not influence monetary policy decision in a monetary union, local information might improve the accuracy of the judgement of the current economic situation. This is a main motivation for including regional FEDs or NCBs in the formulation of US and European monetary policy. We have investigated the implications of this argument under fairly general conditions. In fact the only relevant assumption

<table>
<thead>
<tr>
<th>ECB</th>
<th>GDP Uncertainty</th>
<th>FED</th>
<th>GDP Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>76% $\sigma^2$</td>
<td>Group 1\footnote{San Francisco, New York, Chicago, Atlanta, Richmond, Dallas}</td>
<td>73% $\sigma^2$</td>
</tr>
<tr>
<td>Group 2</td>
<td>23% 12.8$\sigma^2$</td>
<td>Group 2\footnote{Cleveland, Boston, Kansas City, Philadelphia, St. Louis}</td>
<td>24% 8.1$\sigma^2$</td>
</tr>
<tr>
<td>Group 3</td>
<td>1% 7133.1$\sigma^2$</td>
<td>Group 3\footnote{Minneapolis}</td>
<td>2% 612.3$\sigma^2$</td>
</tr>
</tbody>
</table>

Table 1: Regional representation at the ECB and the US FED
we make is that the levels of uncertainty surrounding the current economic conditions in the countries forming monetary union are (potentially) different.

Based on a simple framework we show that if the accuracy of the judgement of the local economy improves with the number of representatives from that region in the monetary policy committee, it may make sense to increase the representation of regions where the economic situation is relatively more uncertain, beyond their economic weight. The flipside of this argument is that a large economic weight of a certain country alone is not sufficient to justify a large representation in the monetary policy committee. This holds in particular if representatives from other countries can provide similar information.

We can summarise the implications as follows: the optimal representation of a region in the common monetary policy committee is increasing (i) in the uncertainty regarding the state of its economy, (ii) its economic weight (i.e. the number of inhabitants), (iii) the lower the ‘returns to information’, and (iv) its correlation with the rest of the currency area.

The main conclusions of the paper extend beyond the optimal composition of a monetary policy committee. In a federal nation state, committees at the national level typically also include regional representation. Their added-value in a committee is that they represent local interests, which – conceptually – are not different from the uncertainties about the economic state of the region used in the model. Proceeding along this reasoning, the optimal composition of any national committee of a federal nation state is not different than the composition of a monetary policy committee and should therefore follow along the lines of our model.

\section{Appendix: Additional derivations}

\subsection{Graphical analysis: The two-country case}

In the two-country case the MRU’s are given by:

\begin{align*}
MRU_1 &= -\lambda_1 w_1 \sigma_1^2 e^{-\lambda_1 r_1} \\
MRU_2 &= -\lambda_2 w_2 \sigma_2^2 e^{-\lambda_2 (m - r_1)}
\end{align*}

The signs of the derivatives with respect to $\beta$ are determined by the sign of the derivatives of $w_i$:

\begin{align*}
\frac{\partial w_1}{\partial \beta} &= -2a_2a_1 \frac{2\alpha \beta^2 + 2\beta + \alpha - \beta^2 - 2\alpha \beta - 1}{(1 - \alpha - \beta + 2\alpha \beta)^2} \leq 0 \\
\frac{\partial w_2}{\partial \beta} &= 2a_1a_2 \frac{\alpha (2\alpha - 1)(\alpha - 1)}{(1 - \alpha - \beta + 2\alpha \beta)^2} \leq 0
\end{align*}
In the graphs we plot \((-MRU_i)\). Thus:

\[
\frac{\partial (-MRU_1)}{\partial \beta} \leq 0;
\]
\[
\frac{\partial (-MRU_2)}{\partial \beta} \leq 0; \text{ but note that } \left| \frac{\partial (-MRU_1)}{\partial \beta} \right| \geq \left| \frac{\partial (-MRU_2)}{\partial \beta} \right|.
\]

### A.2 The country weights in the three-country case

For the general solution section 3.5 the country weights \(w_i\) in the loss function are given by:

\[
w_1 = \frac{(a_1 + a_2 \beta_1 + a_3 \gamma_1)^2(\beta_2 \gamma_1 - \beta_3 - \gamma_2 + 2 \gamma_3 \beta_1)(\beta_3 \gamma_1 - \beta_2 - \gamma_3 \beta_1)}{\text{DET}} + \frac{(a_1 + a_2 + a_3 \gamma_2)^2(\beta_2 \gamma_1 - \gamma_3 - \beta_1)(\beta_3 \gamma_1 - \gamma_2 - \beta_1 + 2 \gamma_3 \gamma_1)(\beta_3 \gamma_1 - \beta_2 + \gamma_3 \beta_1)}{\text{DET}} + \frac{(a_1 + a_2 + a_3 \gamma_3)^2(\beta_2 \gamma_1 - \gamma_3 - \beta_1)(\beta_3 \gamma_1 - \gamma_3 - \beta_1 + 2 \gamma_3 \gamma_1)(\beta_3 \gamma_1 - \beta_2 + \gamma_3 \beta_1)}{\text{DET}} \tag{30}
\]

\[
w_2 = \frac{(a_1 + a_2 \beta_1 + a_3 \gamma_1)^2(\gamma_2 \gamma_1 - \gamma_2 - \alpha_2 + 2 \alpha_2 \gamma_2 + a_2 \gamma_1)(\gamma_2 \gamma_1 - \gamma_2 - \alpha_2 + 2 \alpha_2 \gamma_1)}{\text{DET}} + \frac{(a_1 + a_2 + a_3 \gamma_2)^2(\gamma_2 \gamma_1 - \gamma_3 - \beta_1)(\gamma_2 \gamma_1 - \gamma_2 - \beta_1 + 2 \gamma_3 \gamma_1)(\gamma_2 \gamma_1 - \gamma_3 - \beta_1 + 2 \gamma_3 \gamma_1)(\gamma_2 \gamma_1 - \gamma_3 - \beta_1 + 2 \gamma_3 \gamma_1)(\gamma_2 \gamma_1 - \beta_2 + \gamma_3 \beta_1)}{\text{DET}} \tag{31}
\]

\[
w_3 = \frac{-(a_1 + a_2 \beta_1 + a_3 \gamma_1)^2(\beta_2 \gamma_1 + a_2 - \beta_2 - \beta_2)(\beta_3 \gamma_1 - \beta_3 - \gamma_3 \beta_1)}{\text{DET}} + \frac{(a_1 + a_2 + a_3 \gamma_2)^2(\beta_2 \gamma_1 - \gamma_3 - \beta_1)(\beta_3 \gamma_1 - \beta_2 + \gamma_3 \beta_1)(\beta_3 \gamma_1 - \beta_2 + \gamma_3 \beta_1)(\beta_3 \gamma_1 - \beta_2 + \gamma_3 \beta_1)(\beta_3 \gamma_1 - \beta_2 + \gamma_3 \beta_1)}{\text{DET}} \tag{32}
\]

where

\[
\text{DET} = (\alpha_1 \beta_2 (1 - \gamma_1 - \gamma_2))^2 + (\beta_1 \gamma_2 (1 - \alpha_1 - \alpha_2))^2 + (\gamma_1 \alpha_2 (1 - \beta_1 - \beta_2))^2 - (\alpha_1 \gamma_2 (1 - \beta_1 - \beta_2))^2 - (\gamma_1 \beta_2 (1 - \alpha_1 - \alpha_2))^2.
\]

In the case of nonzero correlation (as investigated in the text) the weights are reduced to

\[
w_1 = \frac{\beta_1 (1 - \alpha_1) + 2 (1 - \beta_1) \beta_1 + \alpha_1 (1 - \beta_1)}{\beta_1 (1 - \alpha_1) + \alpha_1 (1 - \beta_1)},
\]
\[
w_2 = \frac{2 (1 - \alpha_1) \alpha_1 + \beta_1 (1 - \alpha_1) + \alpha_1 (1 - \beta_1)}{\beta_1 (1 - \alpha_1) + \alpha_1 (1 - \beta_1)},
\]
\[
w_3 = 1.
\]

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A.3 Graphical analysis: The three-country case

In the text we examine the following structure of the errors: 

\[ e_1 = \alpha_1 n_1 + (1 - \alpha_1) n_2; \]

\[ e_2 = \beta_1 n_1 + (1 - \beta_1) n_2; e_3 = n_3, \text{ whereby } \alpha_1 \geq 0.5, \beta_1 \leq 0.5. \]

On the basis of this error structure the weights in eq. (26) are given by:

\[ w_1 = (2\beta_1^2 a_2 - \beta_1 a_1 + 2\beta_1 a_1 \alpha_1 - 2a_2 \beta_1 - a_1 \alpha_1) \frac{a_1}{2\alpha_1 \beta_1 - \beta_1 - a_1}, \]

\[ w_2 = (-a_2 \beta_1 + 2a_1 a_2 \beta_1 - 2a_1 \alpha_1 + 2a_1 \alpha_1^2 - a_1 a_2) \frac{\alpha_1}{2\alpha_1 \beta_1 - \beta_1 - a_1}, \]

\[ w_3 = \alpha_3^2. \]

The derivatives of the weights with respect to \( \alpha_1 \) and \( \beta_1 \) are given by:

\[ \frac{\partial w_3}{\partial \alpha_1} = w_3' (\beta_1) = 0, \]

\[ \frac{\partial w_1}{\partial \beta_1} = 2a_1 a_2 \frac{2\beta_1^2 \alpha_1 + \beta_1^2 - 2a_1 \beta_1 + a_1}{(2\alpha_1 \beta_1 - \beta_1 - a_1)^2} \geq 0, \]

\[ \frac{\partial w_2}{\partial \beta_1} = -2a_2 a_1 \frac{\alpha_1 (2\alpha_1 - 1)(\alpha_1 - 1)}{(2\alpha_1 \beta_1 - \beta_1 - a_1)^2} \geq 0, \text{ but } \left| \frac{\partial w_1}{\partial \beta_1} \right| > \left| \frac{\partial w_2}{\partial \beta_1} \right|. \]

References


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