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M. Demertzis* and A.F. Tieman** ***

* Contact information: m.demertzis@dnb.nl, De Nederlandsche Bank, Research Department, P.O. Box 98, 1000 AB, Amsterdam, The Netherlands, tel: +31 (0) 20 524 2016, fax: +31 (0) 20 524 2529.
** International Monetary Fund, Washington D.C.
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De Nederlandsche Bank NV
Research Department
P.O. Box 98
1000 AB AMSTERDAM
The Netherlands

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ABSTRACT

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M. Demertzis and A.F. Tieman

We provide a framework for analysing the choice between optimal and robust rules in the presence of paradigm uncertainty in monetary policy. We thus provide for two issues: first, we discuss the conditions of uncertainty that render a robust rule a preferable substitute to optimal rules and second, we show how the degree of risk aversion increases the desirability of robust rules.

Keywords: Model Uncertainty, Monetary Policy, Optimal Policy Rules.

JEL Codes: E52, E58, C70

SAMENVATTING

De afweging tussen robuuste en optimale regelgeving in het monetaire beleid

M. Demertzis en A.F. Tieman

Wij beschrijven een analytisch raamwerk voor de keuze tussen optimale en robuuste monetaire beleidsregels indien de centrale bank onzeker is over het ware onderliggende model. Daarbij komen twee kwesties aan de orde: ten eerste analyseren wij de onzekerheidstoestand waarin robuuste regels te verkiezen zijn boven optimale, en ten tweede tonen wij aan dat naarmate het beleid risicoschuerer is, de wenselijkheid van robuuste regelgeving toeneemt.

Trefwoorden: model onzekerheid, monetair beleid, optimale beleidsregels

JEL Codes: E52, E58, C70
1 INTRODUCTION

Often the problem of model uncertainty in an optimisation framework is dealt with by weighing the optimal outcomes with the respective probabilities in the form of an appropriately adjusted utility function (Holtham and Hughes Hallett, 1992, Frankel and Rockett, 1988). Alternatively, and if the losses of applying the wrong rule are considerable, policy makers often prefer to use rules which, although not optimal given the true economic model, perform relatively well across the spectrum of different possible models (Levin et al, 2001). Robustness is therefore, associated with the wish to reduce the risks of applying a totally inappropriate rule. Robust rules are therefore, rules that “... in the face of paradigm uncertainty are derived from procedures that maintain the distinctiveness of the two paradigms and yet integrate analysis of the losses that rules give in each of the paradigms”, Gerdesmeier et al (2002).

In this paper we provide for two issues: First, given the existence of at least one robust rule, we show for which levels of uncertainty one should apply the robust rule and when to revert back to using the optimal rules (section 3.1). Second, we show that the desire to apply a robust rule is linked to risk aversion such that as the latter increases, the likelihood of applying a given robust rule also increases (section 3.2).
2 OPTIMAL VERSUS ROBUST RULES: A MODEL

Assume a Central Bank (CB) that optimises the following loss function:

$$\min_r L = E (x'Qx)$$

s.t $M$

with respect to its instrument, the interest rate $r$, and subject to the economic model prevailing, $M$. Vector $x$ represents the objectives, inflation and output, as deviations from their targets and $Q$ is a preference matrix of the Central Bank with the weights on the diagonal and zeros everywhere else. Function $L$ is continuous and twice differentiable and $M$ is a system of equations that define the constraining model. The optimisation procedure produces a rule with respect to the instrument $r$

$$r(.) = \arg \min_r L$$

The system is characterised by uncertainty in the sense that a number of different models are candidates to explain the economy ex post. Which economic model will in fact prevail however, is not known to the Central Bank. Suppose therefore, that the economy operates either as model $M = A$ or as model $M = B$ and in a static world no learning takes place. If model $A$ is assumed, the optimal rule is $r_A$; if however, model $B$ is assumed to be correct then the optimal rule is $r_B$. The question that arises is which rule to apply given the uncertainty of which model will prevail. The sequence of events is such that the CB needs to take a decision as to which rule to implement first, and only after is the true model of the economy actually revealed.

![Figure 1: Timing of Events](attachment:image.png)
This implies therefore, that the decision made may actually turn out to be the wrong one. In a world where the economy can be described by only two alternative models, four different outcomes can therefore occur *ex post*:
- $r_A$ rule is implemented and $A$ occurs, incurring losses $L_{AA} \geq 0$
- $r_A$ rule is implemented and $B$ occurs, incurring losses $L_{BA} > L_{AA}$
- $r_B$ rule is implemented and $A$ occurs, incurring losses $L_{AB} > L_{BB}$
- $r_B$ rule is implemented and $B$ occurs, incurring losses $L_{BB} \geq 0$

We suppose model $A$ occurs with *ex ante* probability $q$ and model $B$ with *ex ante* probability $(1-q)$. This probability $q$ is fixed and exogenously given, but unknown to the Central Bank. Then, under risk-neutrality, the disutility of applying each of the rules is given by the expected losses:

$$U_{r_A}(q) = \mathbb{E}[L(r_A)] = qL_{AA} + (1-q)L_{BA}$$
$$U_{r_B}(q) = \mathbb{E}[L(r_B)] = qL_{AB} + (1-q)L_{BB}$$

(1)

(2)

Naturally the decision as to which one to implement is based on which of the two rules produces the lowest expected losses. This however, leaves the possibility of having applied the wrong rule, incurring *ex post* losses $L_{AA}$ or $L_{BA}$. And if these losses happen to be sufficiently large, then having applied the correct rule *ex post* becomes much more imperative. This is the reason that policy makers often search for alternative rules, which although not optimal given the true economic model that actually prevails, have nevertheless the property of performing reasonably well across the spectrum of possible models.

Similarly to above, losses incurred when applying such a rule, say $r_C$, are:
- $r_C$ rule is implemented and $A$ occurs, incurring losses $L_{AC} > L_{AA}$
- $r_C$ rule is implemented and $B$ occurs, incurring losses $L_{BC} > L_{BB}$.

The expected disutility of using rule $r_C$ is now

$$U_{r_C}(q) = \mathbb{E}[L(r_C)] = qL_{AC} + (1-q)L_{BC}$$

(3)

Rule $r_C$ will only be implemented if for some probabilities, $r_C$ produces smaller losses than both $r_A$ and $r_B$. We label this property robustness in the given probability space.

We denote $q^*$ the probability for which the Central Bank is indifferent between applying $r_A$ or $r_B$.

**Definition 1.** A rule $r_C$ is considered robust vis-à-vis any optimal rules, when, two conditions hold:

1. Evaluated at $q^*$, $U_{r_C}(q^*) < U_{r_A}(q^*) = U_{r_B}(q^*)$ and,

$$|U'_{r_C}(q)| < |U'_{r_A}(q)| \text{ and } |U'_{r_C}(q)| < |U'_{r_B}(q)| \quad \forall \quad q \in [0, 1].$$

The first condition ensures that there is a range of probabilities for which it is optimal to apply $r_C$ and
the second that the losses incurred are relatively invariant to the degree of uncertainty by comparison to the other two rules. Figure 2 provides a graphical representation of the definition and plots the expected values of the three rules in terms of the unknown probability $q$.

![Figure 2: Expected Losses - Risk Neutrality](image)

It follows that for a robust rule the following hold:

$$L_{A,A} \ < \ L_{A,C} \ < \ L_{A,B}$$
$$L_{B,B} \ < \ L_{B,C} \ < \ L_{B,A}$$

and from the second condition of the definition we have that,

$$|L_{A,C} - L_{B,C}| \ < \ |L_{B,A} - L_{A,A}| \quad (4)$$
$$|L_{A,C} - L_{B,C}| \ < \ |L_{A,B} - L_{B,B}| \quad (5)$$

In what follows we provide a framework that links the desirability of robust rules to the policy maker’s degree of risk aversion.
3 POLICY RULES AND RISK PREFERENCES

3.1 Ex-ante Losses with risk neutrality

Without loss of generality, we normalize $L_{A,A} = L_{B,B} = 0$. Furthermore, the value of $q$ for which the Central Bank is indifferent between $r_C$ and $r_A$ ($r_B$) is $q_A$ ($q_B$). Looking again at Figure 2, we note that from (1) and (2) we have $U_{r_A}' = c_A > 0$ and $U_{r_B}' = c_B < 0$. The derivative $U_{r_C}'$ is also constant, but its sign maybe positive, negative or zero depending on which model, $A$ or $B$, is more favourable to rule $r_C$. Note that the range $(q_B, q_A)$ in Figure 2 shows the range of ex ante probabilities for which it is worth applying the robust rule $r_C$. The decision therefore, which rule to apply under risk neutrality is given by the following scheme:

\[
\begin{align*}
0 \leq q < q_B & \quad U_{r_A}(q) > U_{r_C}(q) > U_{r_B}(q) \quad \Rightarrow \quad \text{apply } r_B \\
q_B < q \leq q^* & \quad U_{r_A}(q) \geq U_{r_B}(q) > U_{r_C}(q) \quad \Rightarrow \quad \text{apply } r_C \\
q^* \leq q < q_A & \quad U_{r_B}(q) \geq U_{r_A}(q) > U_{r_C}(q) \quad \Rightarrow \quad \text{apply } r_C \\
q_A < q \leq 1 & \quad U_{r_B}(q) > U_{r_C}(q) > U_{r_A}(q) \quad \Rightarrow \quad \text{apply } r_A
\end{align*}
\]

while at $q_A$ ($q_B$) the CB is indifferent between $r_A$ ($r_B$) and $r_C$.

3.2 Ex-ante Losses with risk-aversion

Expected disutility $U_i(q)$ in the previous section, is a linear function of probability $q$, reflecting risk neutrality on the part of the Central Bank. We consider next the case when the Central Bank is risk-averse and label the risk-averse expected disutility of the Central Bank as $\tilde{U}_i(q)$, a non-linear function of $q$. The definition of risk aversion (in terms of losses), $\mathbb{E}(qX + (1-q)Y) < q\mathbb{E}(X) + (1-q)\mathbb{E}Y$ implies that $\tilde{U}_{r_A}(q)$, $\tilde{U}_{r_B}(q)$ and $\tilde{U}_{r_C}(q)$ are concave. The properties of the functions can therefore be summarised as:

\[
\begin{align*}
\tilde{U}_{ri}(q) > U_{ri}(q) & \quad \text{for } q \in (0,1) \\
\tilde{U}_{ri}(q) = U_{ri}(q) & \quad \text{for } q = 0,1 \\
\tilde{U}_{ri}(q)'' < 0 & \quad \text{for } q \in (0,1)
\end{align*}
\]

for $i = A, B, C$. In relation to the robust rule $r_C$, it is also the case that

\[
\begin{align*}
\tilde{U}_{r_A}(q) - U_{r_A}(q) & > \tilde{U}_{r_C}(q) - U_{r_C}(q) \\
\tilde{U}_{r_B}(q) - U_{r_B}(q) & > \tilde{U}_{r_C}(q) - U_{r_C}(q)
\end{align*}
\]

(6) (7)

\[1\] Note that $U_{ri}'$ is short for $\frac{dU_{ri}}{dq}$, equal to a constant. Similarly for the other rules.
which holds $\forall q \in (0, 1)$ and follows from (4) and (5). This is necessarily the case because for given risk aversion, the degree of concavity is an increasing function in the distances, $(L_{A,B} - L_{A,A})$, $(L_{B,A} - L_{B,B})$ and $(L_{C,A} - L_{C,B})$ for each of the rules respectively. This reflects the range of losses across the probability spectrum when each rule is applied to either of the two models. By definition this is lesser for the robust rule.

**Proposition 1:** The higher the degree of risk aversion, the larger the range of probabilities for which the Central Bank will choose to apply the robust rule.

**Proof 1:** Figure 3 plots the functions $\tilde{U}_i(q)$ and $U_i(q)$ for $i = A, B, C$.

![Figure 3: Utility with Risk Aversion](image)

It suffices to show that (for continuous functions) if

\begin{align}
\tilde{U}_{rb}(q) - \tilde{U}_{rc}(q) &> 0, \text{ for } q \geq q_B \quad \text{(8)} \\
\tilde{U}_{rc}(q) - \tilde{U}_{rc}(q) &> 0, \text{ for } q \leq q_A \quad \text{(9)}
\end{align}

then $\tilde{U}_{rb}(q)$ and $\tilde{U}_{rc}(q)$ cross to the left of $q_B$ and $\tilde{U}_{rc}(q)$ and $\tilde{U}_{rc}(q)$ cross to the right of $q_A$. We show that (8) holds and therefore, (9) will hold by analogy. We re-write the left-hand side of (8) as follows:

\[
\tilde{U}_{rb}(q) - \tilde{U}_{rc}(q) = [\tilde{U}_{rb}(q) - U_{rb}(q)] - [\tilde{U}_{rc}(q) - U_{rc}(q)] + [U_{rb}(q) - U_{rc}(q)]
\]
But we know that,

\[ U_{rb}(q) - U_{rc}(q) \geq 0, \quad \text{for } q \geq q_B \text{ (by definition)} \]
\[ \bar{U}_{rb}(q) - U_{rb}(q) > \bar{U}_{rc}(q) - U_{rc}(q) \quad \forall q \quad \text{from (6)} \]

Therefore, it follows that since \( \bar{U}_{rb}(q_B) = \bar{U}_{rc}(q_B) \), we have that \( q_{B'} < q_B \) and inequality (8) holds. Similarly we can show that \( q_{A'} > q_A \) and therefore, \( q_A - q_B < q_{A'} - q_{B'} \). This implies that the robust rule \( r_C \) will produce lower ex-ante expected losses for a greater spectrum of probabilities. Furthermore, as the degree of risk aversion increases, so does the concavity of the utility functions, increasing the discrepancy between magnitudes in (6) and (7).²

² Note that for risk loving policy makers, the results turn on their head and a robust rule becomes attractive for a smaller probability range compared to when they are risk indifferent. In other words, \( q_A - q_B > q_{A'} - q_{B'} \). Proof available on request.
4 DISCUSSION/CONCLUSION

Our objective was to show that the search for robust rules is dependent on the expected losses from applying optimal rules to incorrect models. We also show that this becomes even more eminent for risk averse policy makers. Our analysis does not provide an indication as to whether such rules exist or what their features would be\(^3\). We only show which types of rules can be characterised as robust and under which circumstances they become desirable.

\(^3\) For further information on what the properties of robust rules should be, see Rudenbusch and Svensson (1999) and Batini and Haldane (1999), amongst others.
REFERENCES


