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* Views expressed are those of the author and do not necessarily reflect official positions of De Nederlandsche Bank.

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Securitization and the dark side of diversification*

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Abstract Diversification by banks affects the systemic risk of the sector. Importantly, Wagner (2010) shows that linear diversification increases systemic risk. We consider the case of securitization, whereby loan portfolios are sliced into tranches with different seniority levels. We show that tranching offers nonlinear diversification strategies, which can reduce the failure risk of individual institutions beyond the minimum level attainable by linear diversification, without increasing systemic risk.

Keywords: Securitization, Diversification, Systemic risk, Risk management, Tranching.

JEL Classification Numbers: G11, G21.

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1 Introduction

We study the relation between systemic risk and diversification by financial institutions through securitization. We investigate this issue in a framework where the securitization of loan portfolios is explicitly modeled as a diversification strategy. That is, financial institutions are allowed to structure securities on loan portfolios into a junior and a senior tranche. This framework is used to answer two questions. First, does diversification through securitization provide better diversification strategies than linear diversification? Second, what is the relation between diversification through securitization and systemic risk?

Although it is generally known that diversification usually decreases the riskiness of individual institutions, the relation between diversification and systemic risk is not-so common knowledge. Banking literature finds that diversification increases the similarity among institutions. It therefore tends to increase the probability of joint failures or systemic crises, which is the dark side of diversification. Shaffer (1985, 1994) and Ibragimov et al. (2010) find this result for the case of full diversification, or full risk sharing. Wagner (2010) establishes the result that any degree of diversification increases the probability of joint failures.

The theoretical result on the dark side of diversification is based on linear diversification strategies. That is, financial institutions diversify asset holdings by exchanging shares in their projects. Although this result has several important applications, it does not apply unreservedly to securitization. The reason is that the securities on loan portfolios are usually sliced in tranches with different seniority levels. The payoff to those tranches is nonlinear in the return of the underlying loan portfolio. Diversification through securitization is therefore different from linear diversification.

Interestingly, taking tranching into account turns out to have substantial impact, from both a microprudential and a macroprudential point of view. From a microprudential point of view, tranching facilitates a decrease in the probability of individual failures beyond the minimum level that could be achieved by linear diversification strategies. From a macroprudential point of view, securitization through diversification may help to avoid the dark side of diversification. In contrast to the linear diversification result, we find that diversification through securitization does not necessarily increase the probability of systemic failures.

The difficulty with linear diversification is that losses and profits are shared among investors always. Therefore, if two banks share the ownership of two loan portfolios, the losses generated by one portfolio may trigger the insolvency of both banks. This may happen even if the other portfolio performs relatively well. With tranching such a scenario can be avoided, because the different seniority levels of tranches determine the order of payment. Suppose that each bank owns the junior tranche of one portfolio and the senior tranche of the other portfolio. If the maximum payoff to a senior tranche is set sufficiently high, then any return of a portfolio above this threshold will guarantee the solvency of the bank owning the senior tranche. Nevertheless, risk sharing is still in place, because any return on top of this threshold benefits the owner of the junior tranche and may counterbalance potential losses on the other portfolio.

However, the benefits of securitization do come at a cost. The analysis reveals that structuring claims on loan portfolios into different seniority classes introduces nonlinear effects in the financial system. If financial institutions choose to follow the optimal strategy, even a small unanticipated confidence shock may strongly increase the risks in the financial system due to those nonlinearities. Such a shock may rise the level of both individual and systemic risks beyond the level without any diversification.

Many studies on systemic risk focus on contagion through, for example, the interbank market, the payment system or through asset prices. In contrast, the present results do not depend on contagion. Whether a bank fails does not depend on other banks. In line with the mechanism of Wagner (2010), the conclusions follow automatically from the similarity in risk exposures. This channel has been the focus of several other studies. Acharya and Yorulmazer (2007) and Acharya (2009) model an increase in joint failure risk if financial institutions invest in similar projects. De Vries (2005) discusses the frequency of joint tail events in case of risk sharing. In a setting with more than two institutions, Slijkerman et al.

(2012) show that mergers (as a form of diversification) do not necessarily increase the risk of all institutions failing jointly. Zhou (2010) investigates the relation between the similarity in portfolio holdings of other institutions and the expected number of other institutions that face financial distress simultaneously. Allen et al. (2012) discuss how the frequency of joint failures can be affected by the interaction between asset commonality and the network of cross holdings. The contribution of the present paper is to show that the positive relation between risk sharing and joint failures may breakdown in a world with tranching.

Although the focus of this study is on the securitization of loan portfolios, the conclusions in this paper can be applied to several other fields. The conclusions on the benefits and dangers of securitization directly apply to the syndicated loan business. The 'loan portfolios' can be given a much broader interpretation, such as regions or business lines. In this context, it is notable that the hierarchy in payoffs due to seniority classes also arises in the distinction between debt and equity. Among financial institutions, obtaining an equity stake can be regarded as obtaining a junior claim on the assets of another firm, while providing an interbank loan can be regarded as obtaining a senior claim. Finally, the result on the improvement of risk management due to tranching can be applied by any two investors who wish to avoid losses beyond a certain level.

The remainder of the paper is organized as follows. Section 2 discusses the model and introduces the possibility of tranching. Section 3 reports the model results. Section 4 discusses the dark side of diversification through securitization. Section 5 discusses several generalizations. Section 6 concludes. The formal proofs are postponed to the Appendix.

2 Model

Before modeling the securitization of loan portfolios, we first give a short description of the general framework. We use the framework of Wagner (2010). In this framework two banks each manage one unit of funds from risk neutral investors. The share of deposits is denoted by d and the remaining share of funds is equity capital. The investment opportunities are given by assets X and Y. We will refer to these assets as loan portfolios. The gross returns on the loan portfolios, x and y, are identical and independently distributed with density function $\phi(\cdot)$ which has full support on $[0, s]$, with $s > d$. The joint density function is denoted by $\phi(x, y)$ if independence is not assumed.

There are three periods. At date 1, the two banks make an investment decision. At date 2, the returns on the loan portfolios x and y are revealed. If the returns are not sufficient to cover the deposits, d, then a bank run will occur on that particular bank. Consequently, the loan portfolio must be liquidated. It is possible to sell the loan portfolio at a discount c if no run occurs on the other bank. However, if a systemic crisis occurs, that is, if a bank run occurs on both banks, then the loan portfolios must be liquidated at depressed prices. In that case the portfolios of both banks are sold at a larger loss cq ($q > 1$). The loan portfolios mature at date 3.

Banks maximize their value by maximizing the expected return to the risk neutral providers of funds. Denote the probability of a systemic crisis by π^S and the probability of a run on bank i only by π_i^I. Further, denote the bank's value if not liquidated by v_i. Then the expected value, W_i, can be written as

$$W_i = E[v_i] - c(\pi_i^I + q\pi^S), \tag{2.1}$$

where the last part reflects the expected default costs. Total welfare in the economy equals to the sum of the expected returns of both banks

$$W_1 + W_2 = E[v_1] + E[v_2] - c(\pi_1^I + \pi_2^I + 2q\pi^S). \tag{2.2}$$

Diversification in the original framework is restricted towards linear diversification strategies. That is, banks are allowed to diversify asset holdings by constructing linear combinations of the loan portfolios X and Y. Each bank i is allowed to invest a share of their funds, $r_i \in [0,1]$, in the other bank's loan portfolio.[1] Hence, bank i's portfolio return, if not liquidated, is given by

$$v_1(x,y) = (1-r_1)x + r_1 y \tag{2.3}$$

$$v_2(x,y) = (1-r_2)y + r_2 x. \tag{2.4}$$

The restriction towards linear diversification is covered in these equations, because the bank's portfolio returns are defined as linear combinations of returns x and y.

- INSERT FIGURE 1 -

Without diversification, a bank run occurs whenever a return below d occurs on the bank's own loan portfolio. This is represented for bank 1 by the dashed area in Figure 1, panel (a). Bank 2 has been added in Figure 1, panel (b). The set of outcomes under which a systemic crisis occurs is represented by the double dashed square. If banks are allowed to do linear diversification, then banks can turn the border at which no bank run occurs in point (d,d), see Figure 1, panel (c). The probability of a run on bank 1 depends on the probability mass of the dashed area, and can be obtained by taking the surface integral of $\phi(x,y)$ over the dashed area. The probability of a run on bank 1 may decrease due to linear diversification.[2] However, the probability of a joint failure will increase simultaneously, because the set of outcomes which correspond to a systemic crisis strictly increases due to diversification. Diversification increases the double dashed square in Figure 1, panel (b), by two triangles, see Figure 1, panel (d). This is the core mechanism behind the dark side of diversification in the framework of Wagner (2010).

[1] Wagner (2010) restricts without loss of generality $r_i \in [0, 1/2]$. In the new model this restriction is not without loss of generality.

[2] With linear diversification this is for example the case if $\phi(\cdot)$ denotes a uniform distribution on interval $[0, s]$ with $s > 2d$.

2.1 Model Innovation

The restriction towards linear diversification does not allow financial institutions to securitize loan portfolios into tranches with different seniority levels. Because the return from investment in tranches is nonlinear in the return on the underlying loan portfolios, it is not possible to mimic investment in tranches by investment in linear combinations of the underlying portfolios. The innovation is to introduce tranching to the model.

The feature of tranching loan portfolios is modeled as follows. When making the investment decision, banks 1 and 2 securitize their loan portfolios into a senior and a junior tranche. Banks set the maximum return to the senior tranche at $k \in [0, s]$. At date 3, the senior tranches are served first, up to this maximum k. The junior tranches receive the residual of the return on the underlying loan portfolio. The junior tranches thus receive a nonzero payment, only if the return on the underlying portfolio is sufficient to honor the maximum payment k to the senior tranches. Hence, for securities on portfolio X, the return on the senior tranche equals to $x^s(k) = \min\{x, k\}$, while the return on the junior tranche is given by $x^j(k) = \max\{x - k, 0\}$. Equivalently, for portfolio Y we have $y^s(k) = \min\{y, k\}$ and $y^j(k) = \max\{y - k, 0\}$. Banks are assumed to keep the junior tranche on their own balance sheet, but choose to exchange fraction r_i of the senior tranches. Because the underlying portfolios have equal return distributions, the return distributions of the two senior tranches on X and Y with the same maximum payment k must also be the same. Finally, the proportion of the senior tranche that is not exchanged, $(1 - r_i)$, remains on bank i's balance sheet.[3]

Following this securitization process, bank i's portfolio return, if not liquidated, is given by

$$v_1(x, y) = (1 - r_1)x + r_1 \cdot \max\{x - k, 0\} + r_1 \cdot \min\{y, k\}, \tag{2.5}$$

$$v_2(x, y) = (1 - r_2)y + r_2 \cdot \max\{y - k, 0\} + r_2 \cdot \min\{x, k\}. \tag{2.6}$$

[3] Note that holding both a junior and a senior tranche yields the same return as the underlying portfolio: $x^j(k) + x^s(k) = \max\{x - k, 0\} + \min\{x, k\} = x$ for any k.

Banks maximize their expected value by setting $k \in [0, s]$ and $r_i \in [0, 1]$. Following Wagner (2010), we focus on symmetric equilibria, such that tranching boils down to a redistribution of the cash flows from loan portfolios X and Y among bank 1 and 2.

It is important to notice that linear diversification is captured by equations (2.5) and (2.6) as a special case. If banks set the maximum payoff to the senior tranche equal to the maximum return on the loan portfolio, i.e. if $k = s$, then the portfolio returns in equations (2.3) and (2.4) are obtained. In this case, the junior tranche pays zero always, while the senior tranche simply replicates the return of the original loan portfolio. In equation (2.5) and (2.6) this follows from the fact that the max operators return zero always, while the min operators mimic the underlying portfolio return, if the level of k is set at the maximum possible realization of x and y.

The junior tranche is assumed to remain on the balance sheet of the originator by assumption. No formal justification is given in this model. However, this assumption may be justified by potential asymmetric information. It has been shown that the strategy of keeping junior and selling senior tranches may arise as an optimal structure in case of potential asymmetric information between the originator of the securities and the buyer of the securities, see DeMarzo and Duffie (1999) and DeMarzo (2005).

2.2 Tranching and bank runs

Because bank runs generate liquidation costs, banks in the model try to avoid bank run outcomes by optimizing their investment strategy, that is by setting $k \in [0, s]$ and $r_i \in [0, 1]$. Given the investment strategy, the set of bank run outcomes can be obtained for both banks. This is done by deriving a 'no bank run' border, $\bar{y}_i(x)$, which provides the minimum return y to prevent a run on bank i given the realization of x. The surface below the function $\bar{y}_i(x)$ on the xy-plane represents the set of bank run outcomes. The probability of a bank run depends on the amount of probability mass in the area with bank run outcomes and can be obtained by taking the corresponing surface integral over the joint density function, $\phi(x, y)$.

The 'no bank run' border, $\bar{y}_i(x)$, is derived by setting the return given no default equal to the level of deposits, $v_i(x, y) = d$, and solving for y. Because of the noncontinuous max and min operators, it will be convenient to write the solution in three cases that each represent a possible class of investment strategies. Due to space considerations, $\bar{y}_i(x)$ is given for bank 1 only.[4]

Class 1: conditional on $k < d$ we have

$$\bar{y}_1(x) = \begin{cases} \infty, & \text{if } x \in [0, d), \\ (d + r_1 k - x)/r_1, & \text{if } x \in [d, s]. \end{cases} \tag{2.7}$$

Class 2: conditional on $k \geq d$ and $r_1 k < d$ we have

$$\bar{y}_1(x) = \begin{cases} \infty, & \text{if } x \in [0, (d - r_1 k)/(1 - r_1)), \\ (d + r_1 x - x)/r_1, & \text{if } x \in [(d - r_1 k)/(1 - r_1), k), \\ (d + r_1 k - x)/r_1, & \text{if } x \in [k, s]. \end{cases} \tag{2.8}$$

Class 3: conditional on $k \geq d$ and $r_1 k \geq d$ we have

$$\bar{y}_1(x) = \begin{cases} (d + r_1 x - x)/r_1, & \text{if } x \in [0, k), \\ (d + r_1 k - x)/r_1, & \text{if } x \in [k, s]. \end{cases} \tag{2.9}$$

The set of bank run outcomes within Class 1 is illustrated for bank 1 by the dashed area in Figure 2, panel (a).[5] In comparison with the dashed area in Figure 1, panel (a), the set of bank run outcomes within Class 1 is increased relative to the no diversification case. The intuition is as follows. Within Class 1, bank 1 sets the maximum amount that senior tranches receive below the amount of deposits on its balance sheet, i.e. $k < d$. Consequently, the senior tranches start to share in the losses, i.e. receive less than the promised amount,

[4]Because of symmetry, the 'no bank run' border for bank 2, $\bar{y}_2(x)$, can be obtained by mirroring $\bar{y}_1(x)$ in $y = x$.

[5]The notation $\bar{y}_i(x) = \infty$ is used to specify that no return y would be sufficient to avoid a run on bank i (given the support of the density function $[0, s]$).

k, only after losses generated by the junior tranches are sufficient to trigger a run on bank 1. In other words, by selling the senior tranche, bank 1 overcomes tail risks on loan portfolio X, that only materialize if a run on bank 1 would already occur. Consequently, bank runs are not avoided. On the contrary, for $r_1 > 0$, the bank is also exposed to a senior tranche on loan portfolio Y. This makes bank 1 vulnerable for bad realizations of y. This vulnerability is represented by the additional dashed triangle in Figure 2, panel (a).

- INSERT FIGURE 2 -

Within Class 2 and Class 3, bank 1 sets the maximum amount that senior tranches receive larger than or equal to the amount of deposits on the balance sheet, i.e. $k \geq d$. However, within Class 2, the maximum return of the investment in senior tranches on loan portfolio Y is not sufficient save bank 1 from a failure for every possible return on loan portfolio X. The maximum amount received from investment in senior tranches of loan portfolio Y is smaller than the amount of deposits, that is $r_1 k < d$. This is why the minimum return $\bar{y}_1(x)$ in equation (2.8) equals infinity for low realizations of x. For sufficiently low returns on loan portfolio X, a run on bank 1 occurs, regardless of the return on portfolio Y. In Figure 2, panel (b), this is represented by the dashed rectangle along the y-axis. In contrast, in Class 3, any adverse return on loan portfolio X can be absorbed by a sufficiently high return on loan portfolio Y, see Figure 2, panel (c).

Figure 2, panel (b) and (c), both report a kink in the downward slope of the 'no bank run' border at $x = k$. The kink can be explained as follows. The value of k determines at which level fluctuations in returns are shared among holders of the senior tranche. For $x > k$, fluctuations entirely hit the bank who originated portfolio X and holds the junior tranche. The steeper slope for $x \geq k$, means that a decrease in the return on portfolio X has to be offset by a larger increase in the return on portfolio Y. For $x < k$, fluctuations in returns are shared among all holders of the senior tranche. This is why the downward slope is less steep for $x < k$. In Figure 2, panel (b), the slope remains less steep until the point that the 'no bank run' border hits the value $y = k$ for some low value of x. At this point, the senior tranche on loan portfolio Y pays the maximum amount. Consequently, higher returns on loan portfolio Y will no longer offset further losses on loan portfolio X. The difference between Class 2 and Class 3 is whether this point occurs before or after $x = 0$.

3 Results

Systemic crises cannot be avoided by linear diversification. Wagner (2010) shows that linear diversification cannot decrease the set of outcomes with a systemic crisis beyond the square to the left and below point (d, d) in Figure 1, panel (a). This result turns out to be robust if tranching loan portfolios is allowed.

Proposition 1 *If banks keep the junior tranches of their loan portfolios, as in equations* (2.5) *and* (2.6)*, then the probability of a systemic failure cannot be decreased by tranching or linear diversification.*

In all panels reported in Figure 2, the (not increasing) 'no bank run' border passes through point (d, d). Consequently, for any securitization strategy, the square to the left and below point (d, d) is captured by the set of outcomes in which a run on bank 1 occurs. From symmetry, the same holds for bank 2. Hence, outcomes that result in a systemic failure under the no diversification case reported in Figure 1, panel (b), also result in a systemic failure under tranching or linear diversification.

Although tranching and linear diversification do not decrease the probability of systemic failures, they may decrease the probability of a run on each individual bank. The diversification strategy that minimizes the probability of individual bank failures is a strategy that structures the loan portfolio in a junior and a senior tranche.

Proposition 2 *The probability of a run on bank i is minimized if and only if bank i sets the maximum payoff to the senior tranche equal to the level of deposits and exchanges its senior tranches entirely, i.e. $k = d$ and $r_i = 1$.*

Corollary 1 *If bank i follows the diversification strategy with tranching in Proposition 2, then the probability of a run on bank i is strictly smaller than the minimum bank run probability attainable by any linear diversification strategy.*

We consider the strategy in Proposition 2 for bank 1. Following equation (2.9), the no bank run border for bank 1 under the strategy in Proposition 2 is given by

$$\bar{y}_1^*(x) \begin{cases} d, & \text{if } x = [0, d), \\ 2d - x, & \text{if } x = [d, s]. \end{cases}$$

The probability to have bank 1 insolvent under the strategy in Proposition 2, is given by

$$\pi_1^* = \int_0^d \int_0^d \phi(x)\phi(y)\, dy\, dx + \int_d^{\min\{2d,s\}} \int_0^{2d-x} \phi(x)\phi(y)\, dy\, dx, \tag{3.1}$$

which is the probability of observing an outcome in the dashed area in Figure 3, panel (a). The first and second double integral in equation (3.1) do correspond to respectively the dashed square and the dashed triangle in the set of bank run outcomes.

- INSERT FIGURE 3 -

From the Figure 3, panel (a) it is not hard to see why Corollary 1 holds. The set of bank run outcomes under the strategy in Proposition 2 comprises a dashed square and a single triangle. In case of full risk sharing, i.e. $k = s$ and $r_i = 0.5$, the no bank run border is given by the dashed downward sloping 45°-line through point (d, d). Consequently, the set of bank run outcomes under full risk sharing comprises the dashed square and two triangles. These two triangles contain twice the probability mass of the single triangle from the strategy in Proposition 2. Alternative linear diversification strategies can be considered by drawing no bank run borders through point (d, d) with different downward slopes. Under each alternative linear diversification strategy the set of bank run outcomes covers at least one triangle entirely and covers the other triangle partially.

It is further notable that there are always nonnegative diversification benefits of tranching possible. In contrast, in some cases every degree of linear diversification increases the probability of bank runs.[6] The reason is that the tranching strategy in Proposition 2 can shrink the set of bank run outcomes relative to the no diversification case without introducing new bank run outcomes.

In case of linear diversification, the probability of simultaneous bank failures increases by any degree of diversification. This result cannot be generalized to tranching. The probability of systemic failures is not increased by the strategy proposed in Proposition 2.

Proposition 3 *If both banks follow the strategy in Proposition 2, i.e.* $k = d$ *and* $r_1 = r_2 = 1$*, then the probability of a systemic failure is not larger than in the absence of diversification, i.e.* $r_1 = r_2 = 0$*.*

From Figure 3, panel (b) it is not hard to see that the probability of simultaneous bank runs is not increased by the strategy in Proposition 2. This result brings us to the optimal strategy for the banks.

Theorem 1 *The optimal investment strategy is to set the maximum payoff to the senior tranches equal to the level of deposits, i.e.* $k = d$*, while exchanging the senior tranches entirely, i.e.* $r_1 = r_2 = 1$*.*

Proposition 2 guarantees that the liquidation costs due to individual failures, excluding the additional liquidation costs due to systemic failures, are minimized by the strategy in Theorem 1. Proposition 3 guarantees that this strategy also minimizes the that the additional liquidation costs due to systemic crises. Consequently, all liquidation costs are minimized. Because the expected returns excluding liquidation costs are not affected by the investment strategies, Theorem 1 is proven. □

[6]For example, if $\phi(\cdot)$ denotes a uniform distribution on interval $[0, s]$ with $d < s < 2d$, then every degree of linear diversification increases the probability of a bank run. With the tranching strategy in Proposition 3.1, the bank run area would in this case be characterized by a dashed square and a triangle that is truncated on the right side for values above s.

4 Securitization and confidence shocks

The results show how securitization provides a new optimal diversification strategy that is not prone to the dark side of diversification. Nevertheless, the analysis also reveals that tranching loan portfolios introduces nonlinear outcomes to the financial system. This section shows that even a small adverse shock to depositor confidence may be very destabilizing due to these nonlinearities.

The shock is modeled as follows. After banks set the strategy at date 1, an unanticipated shock to depositors confidence occurs. Due to this adverse confidence shock, ϵ, depositors will run if the return on assets at date 2 is smaller than $\delta = d + \epsilon$. Hence, δ denotes the new minimum return that would avoid a bank run. Due to the confidence shock, it may happen that depositors run on a solvent bank, i.e. if $\delta > d$. Further, to quantify the results, we assume that x and y are uniformly distributed over the interval $[0, s]$, with $s > \bar{\delta}(1+\sqrt{2q-1})$ and where $\bar{\delta}$ denotes the maximum value of δ. Nevertheless, the instability due to tranching does not depend on this distributional assumption.

For an arbitrary minimum return to avoid bank runs, δ, the expected default costs in the absence of diversification are given by

$$c(\pi_1^I + \pi_2^I + 2q\pi^S) = 2c\frac{s\delta - \delta^2}{s^2} + 2cq\frac{\delta^2}{s^2}. \tag{4.1}$$

Further, following Proposition 1 of Wagner (2010), the social optimal strategy with linear diversification is $r^* = 1/(1+\sqrt{2q-1})$. Under the optimal linear diversification strategy the expected default costs are given by

$$c(\pi_1^I + \pi_2^I + 2q\pi^S) = 2c\left(q + \sqrt{2q-1}\right)\frac{\delta^2}{s^2}. \tag{4.2}$$

Finally, if banks follow the optimal investment strategy with tranching in Theorem 1, the expected default costs are given by

$$c(\pi_1^I + \pi_2^I + 2q\pi^S) = \begin{cases} 2c\frac{k\delta-(1/2)\delta^2}{s^2} + 2cq\frac{\delta^2}{s^2}, & \text{if } \delta \in [0,k], \\ 2c\frac{s\delta-\delta^2}{s^2} + 2cq\frac{\delta^2}{s^2} + c(2q-1)\frac{k^2}{s^2}, & \text{if } \delta \in (k,s]. \end{cases} \tag{4.3}$$

From equation (4.3) follows that the level of expected default costs strongly depends on the maximum payoff to the holders of the senior tranche, k, if banks follow the optimal tranching strategy. From the equation follows a jump in the expected default costs if the unanticipated adverse confidence shock increases the minimum return to avoid bank runs above the maximum payoff to the senior tranche, i.e. if $\delta > k$. This is worrying, because the optimal investment strategy with tranching prescribes banks in the model to set $k = d$. Consequently, with the optimal tranching strategy, even a small adverse confidence shock, ϵ, may trigger the jump in expected default costs.

The origin of the jump in expected default costs is caused by a shift in the class of investment strategies. The adverse confidence shock pushes the investment strategies of the banks from Class 3, equation (2.9), into Class 1, equation (2.7). The strategies in Class 1 are known to result in a larger set of bank run outcomes than in the case of no diversification, see also Figure 2, panel (a). Consequently, the expected default costs in the case of diversification through securitization increase to a level beyond the expected default costs in the absence of diversification, see equation (4.1) and (4.3). Hence, after a adverse confidence shock, the situation in a banking system with diversification through securitization could become worse than the situation in a banking system without any diversification.

- INSERT FIGURE 4 -

These findings are illustrated in Figure 4, panels (a)-(c). For different levels of δ, the panels report the probability that each bank fails, the probability of a systemic failure and the expected default costs. The panels report the results for the no diversification strategy (dashed line), the optimal linear diversification strategy (dotted line) and the optimal tranching strategy (solid line). From the figure follows that the change of risks due to a change in confidence is quite smooth in case of no diversification or linear diversification.

The optimal tranching strategy achieves a lower probability of bank failures in panel (a) without increasing the probability of systemic crises in panel (b). Without adverse confidence shock, it therefore obtains the lowest possible expected default costs if $\delta = k = 0.9$. However, if a small confidence shock happens such that a bank runs are triggered more quickly, i.e. if $\delta > k = 0.9$, then both the individual and systemic failure probabilities of the optimal tranching strategy rise to a level beyond that without any diversification. As a consequence the expected default costs rise above the level of both other strategies.

5 Discussion

5.1 Dependence

The probability distributions of x and y are assumed to be independent in the model. In practice, it may be hard to find two assets for which this is the case. Nevertheless, the Propositions and the Theorem do not depend on this assumption. Formally, it is sufficient to assume the symmetry of the return distribution, i.e. $\phi(x, y) = \phi(y, x)$, and

$$\phi(x, y) > 0 \text{ for } \{(x, y) : 0 \geq x \geq s, 0 \geq y \geq s\}.$$

The second condition requires that all combinations of outcomes of x and y below or equal to s may occur. This condition guarantees the *strict* optimality of the optimal strategy. Further, it is not necessary to assume that the return distribution is bounded from above (i.e. $s \to \infty$). Therefore, many popular distributions satisfy the two conditions on the return distribution, including for example the bivariate (log)normal distribution with a correlation coefficient $-1 < \rho_{x,y} < 1$ and equal marginals, i.e. $\sigma_x = \sigma_y$.

5.2 More assets

The diversification effect of tranching is not limited by the number of assets in the model. Although the model is based on two assets, X and Y, the diversification strategy with tranching can be applied on two portfolios that are formed out of any (even) number of assets. Suppose there are four assets, $X_1, ..., X_4$. Consider the following two portfolios: $X = X_1 + X_2$ and $Y = X_3 + X_4$. Let each bank hold a junior tranche on one portfolio and a senior tranche on the other portfolio. From both an individual and macroprudential point of view, this strategy performs better than full diversification, in which both banks hold 50% of each portfolio, which boils down to both banks holding 50% of each asset.

5.3 Uncertainty

Banks in the model have precise knowledge on the minimum return necessary to avoid bank runs. Based on this knowledge, banks can calibrate the maximum payoff to the senior tranches perfectly. In practice, banks do not possess this knowledge and therefore the calibration of the tranches will be imperfect. This raises the question whether diversification through securitization remains better than linear diversification if banks do not know precisely when bank runs occur.

From the theory follows that it is optimal to set the maximum payoff to the senior tranches equal to the minimum return necessary to avoid bank runs, i.e. $k = \delta$. Further, Section 4 shows that accidentally setting $k < \delta$ is disastrous. If the precise value of δ is uncertain, banks could, of course, set k above the perceived level of δ as a precautionary measure. However, would such a tranching strategy still perform better than the optimal linear diversification strategy? To shed light on this question we compare the expected default costs of the optimal linear diversification strategy with those of a tranching strategy where banks set $k > \delta$ and $r_1 = r_2 = 1$. We start from the distributional assumption in Section 4. From comparing equation (4.2) and equation (4.3) follows that the tranching strategy has lower expected default costs if the maximum return to the senior tranche is set such that

$$k < \left(\frac{1}{2} + \sqrt{2q-1}\right)\delta. \tag{5.1}$$

Hence, provided that $q > 1$, the tranching strategy remains better than the optimal linear diversification strategy if banks set k at a level considerably above the minimum return that avoids bank runs, δ. Consequently, the optimality of the tranching strategy does not depend on a perfect calibration. Interestingly, the better performance of the tranching strategy is especially robust with relatively high liquidation costs in case of systemic crises, i.e. a high q. The underlying reason is that the linear diversification strategy increases the probability of systemic crises relative to the no diversification case, while the tranching strategy with $k \geq \delta$ does not. Therefore, tranching may be more important in a system with relatively high liquidation costs in case of systemic crises.

It is further notable that the result provides evidence that tranching is not always optimal. Sometimes banks could be better of by choosing a linear diversification strategy. This is especially the case if runs on very solvent banks may occur, i.e. if bank have to anticipate very high values of δ. From a policy point of view, this may serve as an additional argument to promise liquidity support in case of a run on very solvent banks. Providing liquidity support to very solvent banks in case of bank runs neutralizes the impact of exceptional high values of δ relative to d. The knowledge that liquidity support is provided in case of a run on a very solvent bank may therefore encourage banks to choose a better diversification strategy with tranching ex ante. Consequently, besides avoiding the liquidation of solvent banks ex post, promising liquidity assistance to very solvent banks may also prevent financial institutions from becoming insolvent.

6 Concluding remarks

Securitization of loan portfolios gained in importance during the last decades. Securitization decreased the entrance cost of investment in other loan types for financial institutions.

Investment in specific business or geographical areas became available without opening an entire new loan business. Securitization thus catalyzed diversification by offering new prospects to diversify asset holdings. However, did securitization also amplify the dark side of diversification? According to the conclusions drawn in the present paper this is not necessarily the case. Nevertheless, the results in this paper show how securitization by financial institutions may have cast a shadow of its own on the stability of the financial system and that of individual institutions.

Several interesting directions remain unexplored in the current paper. One direction is to analyze individual and systemic risk due to diversification through securitization in a system with more than two banks. This extension may trigger potential network effects or may draw attention to the default costs in case of a partial systemic failure. Our final remark concerns the similar payoff structure of tranches and options. Under additional assumptions it may be possible to derive the prices of tranches from an option pricing framework. These prices could be useful in a study on potential contagion in the financial system due to tranching.

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Appendix A: formal proofs

A.1 Derivation of the no bank run border $\bar{y}_1(x)$

The set of possible returns (x, y) to have bank i solvent, is derived from solving $v_i(x, y) \geq d$. Following (2.5), for bank 1 we must have

$$(1 - r_1)x + r_1 \cdot \max\{x - k, 0\} + r_1 \cdot \min\{y, k\} \geq d.$$

The solution is given by the union of the following four subsets:

Subset A, is defined on $x \in [0, k)$ and $y \in [0, k]$, and is the set where $y \geq \frac{d}{r_1} - \frac{x - r_1 x}{r_1}$.

Subset B, is defined on $x \in [0, k)$ and $y \in (k, s]$, and is the set where $x \geq \frac{d - r_1 k}{1 - r_1}$.

Subset C, is defined on $x \in [k, s]$ and $y \in [0, k]$, and is the set where $y \geq \frac{d}{r_1} - \frac{x - r_1 k}{r_1}$.

Subset D, is defined on $x \in [k, s]$ and $y \in (k, s]$, and is the set where $x \geq d$.

We separate the solution into three cases: case 1 in which $k < d$, case 2, in which $k \geq d$ and $r_1 k < d$ and case 3, in which $k \geq d$ and $r_1 k \geq d$. These three cases refer to the three different classes of investment strategies in the paper.

Case 1: subset A is empty. To see this, we rewrite the condition as $r_1(y - x) \geq d + x$. In subset A, $y - x$ is bounded by k, and because we have $k < d$, the condition is never satisfied with $r_1 \in [0, 1]$. Also, B is empty. Rewriting the condition gives $(1 - r_1)x \geq d - r_1 k$, which is never satisfied for $x < k < d$. The condition for subset C can be rewritten as $r_1 y \geq d - x + r_1 k$. For $y \leq k$, the condition cannot be satisfied, unless we have $x \geq d$. If $x \geq d$, then the border of the set is described by $y \geq \frac{d}{r_1} - \frac{x - r_1 k}{r_1}$ (until we have $y \geq 0$ for $x \geq d + r_1 k$). Within subset D the condition is $x \geq d$. Hence, equation (2.7) provides the minimum return y that is necessary to have bank 1 solvent given return x for case 1.

Case 2: in subset A, rewriting the condition gives $x \geq (d - r_1 k)/(1 - r_1)$. Because we have $r_1 k < d$, it follows from the condition that subset A is empty for $x \in [0, \frac{d - r_1 k}{1 - r_1})$, while subset A is nonempty for $x \in [\frac{d - r_1 k}{1 - r_1}, k)$ and has the left border $y \geq \frac{d}{r_1} - \frac{x - r_1 x}{r_1}$. The left border of subset B is described by $x \geq \frac{d - r_1 k}{1 - r_1}$ because $r_1 k < d$. Setting the condition for subset C equal to zero gives $x = d + r_1 k$. Hence, if $k \leq d + r_1 k$, then the left border of C is described by $y \geq \frac{d}{r_1} - \frac{x - r_1 k}{r_1}$

for $x \in [k, d + r_1 k)$. Alternatively, if $k > d + r_1 k$, the condition on subset C is not binding. The condition for subset D is not binding because $k \geq d$. Hence, for case 2, the return $\bar{y}_1$ is given by equation (2.8).

Case 3: following the derivation in case 2, we have that subset A is nonempty for $x \in [0, k)$, because the condition $x \geq (d - r_1 k)/(1 - r_1)$ is always satisfied provided that $r_1 k \geq d$. The left border of subset A is described by $y \geq \frac{d}{r_1} - \frac{x - r_1 x}{r_1}$. The conditions on B and D are not binding because respectively $r_1 k \geq d$ and $d < k (\leq x)$. For subset C, the derivation of the left border is the same as under case 2. Hence, for case 3, the return $\bar{y}_1$ is given by equation (2.9).

A.2 Proof of Proposition 1

The no tranching and no diversification case is given by $r_1 = r_2 = 0$. For $r_1 = 0$ the minimum return to have bank 1 solvent, $\bar{y}_1(x)$, is given by either equation (2.7) or (2.8). Given $r_1 = 0$, both equations give $\bar{y}_1(x) = \infty$ for $x \in [0, d)$ and $\bar{y}_1(x) \leq 0$ for $x \in [d, s]$. Hence, if $r_1 = 0$, bank 1 is solvent if and only if $x \geq d$. Conversely, if $r_2 = 0$, bank 2 is solvent if and only if $y \geq d$. Hence, without tranching or diversification a system failure occurs if and only if both $x < d$ and $y < d$, see also the double dashed area in Figure 1, panel (b).

When allowing for tranching and diversification, first we have that $\bar{y}_1 = \infty$ if $x \in [0, d)$ for any $k \in [0, d)$ and $r_1 \in [0, 1]$ from equation (2.7). Second, from equations (2.8) and (2.9) follows that $\bar{y}_1(d) = d$ for any $k \in [d, s]$ and $r_1 \in [0, 1]$. Note that $\bar{y}_1(x)$ in equations (2.8) and (2.9) are non increasing functions in x for any $k \in [d, s]$ and $r_1 \in [0, 1]$. Combining the three gives $\bar{y}_1(x) \geq d$ if $x \in [0, d)$ for any $k \in [0, s]$ and $r_1 \in [0, 1]$. Consequently, a run on bank 1 must occur if both $x < d$ and $y < d$ for any $k \in [0, s]$ and $r_1 \in [0, 1]$. From symmetry along the 45-degree line, $y = x$, a run on bank 2 occurs if both $y < d$ and $x < d$ for any $k \in [0, s]$ and $r_2 \in [0, 1]$. Consequently, irrespective of the tranching or diversification strategy, a system failure occurs if both $x < d$ and $y < d$. $\square$

A.3 Proof of Proposition 2

Proposition 2 is proven if $\pi_1 > \pi_1^*$ for any $k \in [0, s]$ and $r_1 \in [0, 1]$, except $k = d$ and $r_1 = 1$. The probability of a bank run for each strategy can be obtained by taking the surface integral of $\phi(x, y)$ over the area below the no bank run border

$$\pi_1 = \int_0^\infty \int_0^{\bar{y}_1(x)} \phi(x, y)\, dy\, dx. \tag{A.1}$$

From Proposition 1 follows that all strategies associate the square of outcomes with both x and y below d with a run on bank 1. Therefore, the double integral can be written out as

$$\pi_1 = \int_0^d \int_0^d \phi(x, y)\, dy\, dx + \int_d^\infty \int_0^{\bar{y}_1(x)} \phi(x, y)\, dy\, dx + \int_0^d \int_d^{\bar{y}_1(x)} \phi(x, y)\, dy\, dx \tag{A.2}$$

$$=: I_A^* + I_B + I_C. \tag{A.3}$$

Similarly, the optimal strategy can be written as

$$\pi_1^* = \int_0^d \int_0^d \phi(x, y)\, dy\, dx + \int_d^{2d} \int_0^{2d-x} \phi(x, y)\, dy\, dx \tag{A.4}$$

$$=: I_A^* + I_B^*. \tag{A.5}$$

Further, the symmetry of the distribution function, $\phi(x, y) = \phi(y, x)$, gives

$$\pi_1^* = \int_0^d \int_0^d \phi(x, y)\, dy\, dx + \int_0^d \int_d^{2d-x} \phi(x, y)\, dy\, dx \tag{A.6}$$

$$=: I_A^* + I_C^*, \tag{A.7}$$

with $I_B^* = I_C^*$. To prove $\pi_1 > \pi_1^*$ for a certain strategy, it is sufficient to prove either $I_B > I_B^*$ or $I_C > I_C^*$. From a comparison of (A.2) with (A.4) and (A.6), it follows that it is sufficient to prove respectively either

$$\bar{y}_1(x) > 2d - x \quad \text{for} \quad d < x < 2d, \tag{A.8}$$

or

$$\bar{y}_1(x) > 2d - x \quad \text{for} \quad 0 < x < d. \tag{A.9}$$

To prove Proposition 2, it is necessary to do this for any $k \in [0, s]$ and $r_1 \in [0, 1]$, except $k = d$ and $r_1 = 1$. This describes the general line of the proof.

- INSERT FIGURE 5 -

First, we consider $k < d$, i.e. the strategies in Class 1. The corresponding no bank run border is given in equation (2.7). For $x \leq d$ we have $\bar{y}_1(x) = \infty > 2d - x$, which proves $k < d$ yields $\pi_1 > \pi_1^*$ via (A.9). An illustration of this case is provided in Figure 5, panel (a).

Second, we consider the case $r_1 \in [0, 1/2)$ and $k \geq d$. In this case, the no bank run border is given by either (2.8) or (2.9). Following these equations, for $x < d$ and $k \geq d$, we either have $\bar{y}_1(x) = \infty$ or $\bar{y}_1(x) = (d + r_1 x - x)/r_1$. Hence, for $x < d$ we have $\bar{y}_1(x) \geq (d + r_1 x - x)/r_1$. For the proof we thus need for $x < d$

$$(d + r_1 x - x)/r_1 > 2d - x \tag{A.10}$$

$$d + r_1 x - x > 2r_1 d - r_1 x \tag{A.11}$$

$$(1 - 2r_1)d > (1 - 2r_1)x. \tag{A.12}$$

It follows from inequality (A.12) that $\bar{y}_1(x) > 2d - x$ is implied by $x < d$ provided that $r_1 \in [0, 1/2)$. This proves $r_1 \in [0, 1/2)$ and $k \geq d$ yields $\pi_1 > \pi_1^*$ via (A.9). An illustration of this case with $k \geq d + r_1 k$ is given in Figure 5, panel (b). Figure 5, panel (c) provides an illustration of this case for $k < d + r_1 k$.

Third, when concentrating on Class 3, i.e. $k \geq d$ and $r_1 k \geq d$, we first consider the case $k \geq 2d$ and $r_1 \in [1/2, 1]$. From equation (2.9) we have $\bar{y}_1(x) = (d + r_1 x - x)/r_1$ for $0 \leq x < 2d$. It follows from inequality (A.12) that $\bar{y}_1(x) > 2d - x$ is implied by $d < x < 2d$ provided that $r_1 \in (1/2, 1]$. This proves that Class 3 with $k > 2d$ and $r_1 \in (1/2, 1]$ yields $\pi_1 > \pi_1^*$ via (A.8). Figure 5, panel (d) provides an illustration of this case. We left out the special case $r_1 = 1/2$. In this case we have $\bar{y}_1(x) = 2d - x$ for $0 < x < 2d$. Since we have $\bar{y}_1(x) = 2d - x$ for $d < x < 2d$ and $\bar{y}_1(x) = 2d - x > d$ for $0 < x < d$, this case also yields $\pi_1 > \pi_1^*$ ($I_B = I_B^*$ and $I_C > 0$). To summarize, Class 3 with $k > 2d$ and $r_1 \in [1/2, 1]$ yields $\pi_1 > \pi_1^*$.

Fourth, we consider Class 3, $k \geq d$ and $r_1 k \geq d$, with $k < 2d$ and $r_1 \in [1/2, 1]$. It follows from equation (2.9) and inequality (A.12) that $\bar{y}_1(x) \geq 2d - x$ for $d < x < k$, where the inequality holds with equality only if $r_1 = 1/2$. From equation (2.9) we have $\bar{y}_1(x) = (d + r_1 k - x)/r_1$ for $k \leq x < 2d$. Hence, for $k \leq x < 2d$ we have to prove

$$(d + r_1 k - x)/r_1 \geq 2d - x.$$

This is implied by $r_1 k \geq d$. Substituting $r_1 k \geq d$ and further manipulating gives

$$(2d - x)/r_1 \geq 2d - x \tag{A.13}$$

$$2d - x \geq r_1(2d - x), \tag{A.14}$$

which holds with equality only if $r_1 = 1$ and $r_1 k = d$ or $k = d$. Via (A.8), this proves that Class 3 with $d \leq k < 2d$ and $r_1 \in [1/2, 1]$ yields $\pi_1 > \pi_1^*$, unless $r_1 = 1$ and $k = d$. Figure 5, panel (e) provides an illustration of this case. Note that setting $r_1 = 1$ and $k = d$ corresponds to the strategy in Proposition 2.

Finally, we consider Class 2, $k \geq d$ and $r_1 k < d$, with $r_1 \in [1/2, 1]$. Figure 5, panel (f) provides an illustration of this case. The proof that this case is suboptimal follows a slightly different line. For this case we prove that $\bar{y}_1(x) > 2d - x$ for $0 < x < a$ and $d < x < 2d - a$, where $a = \frac{d - r_1 k}{1 - r_1}$. This is sufficient to prove $\pi_1 > \pi_1^*$. To see this, consider the following inequality

$$\int_0^a \int_d^{2d-x} \phi(x, y)\, dy\, dx > \int_0^a \int_{2d-a}^{2d-x} \phi(x, y)\, dy\, dx = \int_{2d-a}^{2d} \int_0^{2d-x} \phi(x, y)\, dy\, dx, \tag{A.15}$$

where the inequality holds because $a = \frac{d - r_1 k}{1 - r_1} < d$ and where the equality holds due to the symmetry of $\phi(x, y)$. Further, consider

$$\int_d^{2d-a} \int_0^{2d-x} \phi(x, y)\, dy + \int_{2d-a}^{2d} \int_0^{2d-x} \phi(x, y)\, dy = \int_d^{2d} \int_0^{2d-x} \phi(x, y)\, dy = I_C^*. \tag{A.16}$$

From the first double integral in (A.15) and the first double integral in (A.16) follows that $I_B + I_C > I_C^*$ if $\bar{y}_1(x) > 2d - x$ for $0 < x < a$ and $d < x < 2d - a$. The first part of the proof follows straightforward from equation (2.8), which gives $\bar{y}_1(x) = \infty$ for $x < \frac{d-r_1k}{1-r_1}$. Consequently, we must have $\bar{y}_1(x) > 2d - x$ for $0 < x < \frac{d-r_1k}{1-r_1}$. The second part of the proof is more complicated. From equation (2.8) and inequality (A.12) with $r_1 \in (1/2, 1]$ we have that $\bar{y}_1(x) > 2d - x$ for $d < x < k$. Following (2.8) we need for $k \leq x < 2d - a$

$$(d + r_1k - x)/r_1 > 2d - x \tag{A.17}$$

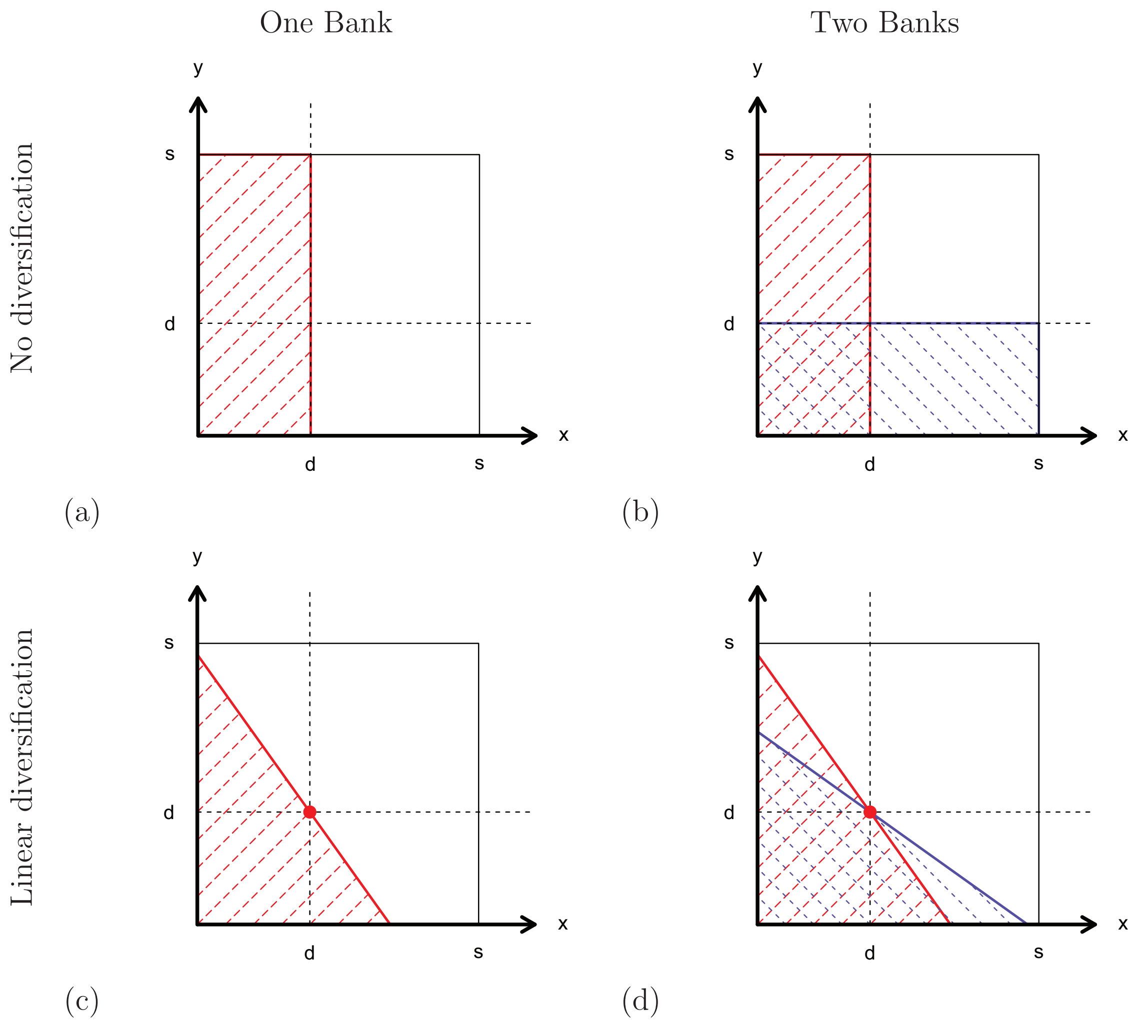
$$d - 2r_1d + r_1k > x - r_1x \tag{A.18}$$

$$d + \frac{r_1}{1-r_1}(k - d) > x. \tag{A.19}$$

Note that the condition on x in A.19 corresponds to the $x < 2d - a$ with $a = \frac{d-r_1k}{1-r_1}$. Hence the condition in (A.17) holds for $k \leq x < 2d - a$. Consequently, the second part $\bar{y}_1(x) > 2d - x$ for $d < x < 2d - a$ is also proven. Thus Class 2 with $r_1 \in [1/2, 1]$ yields $\pi_1 > \pi_1^*$.□

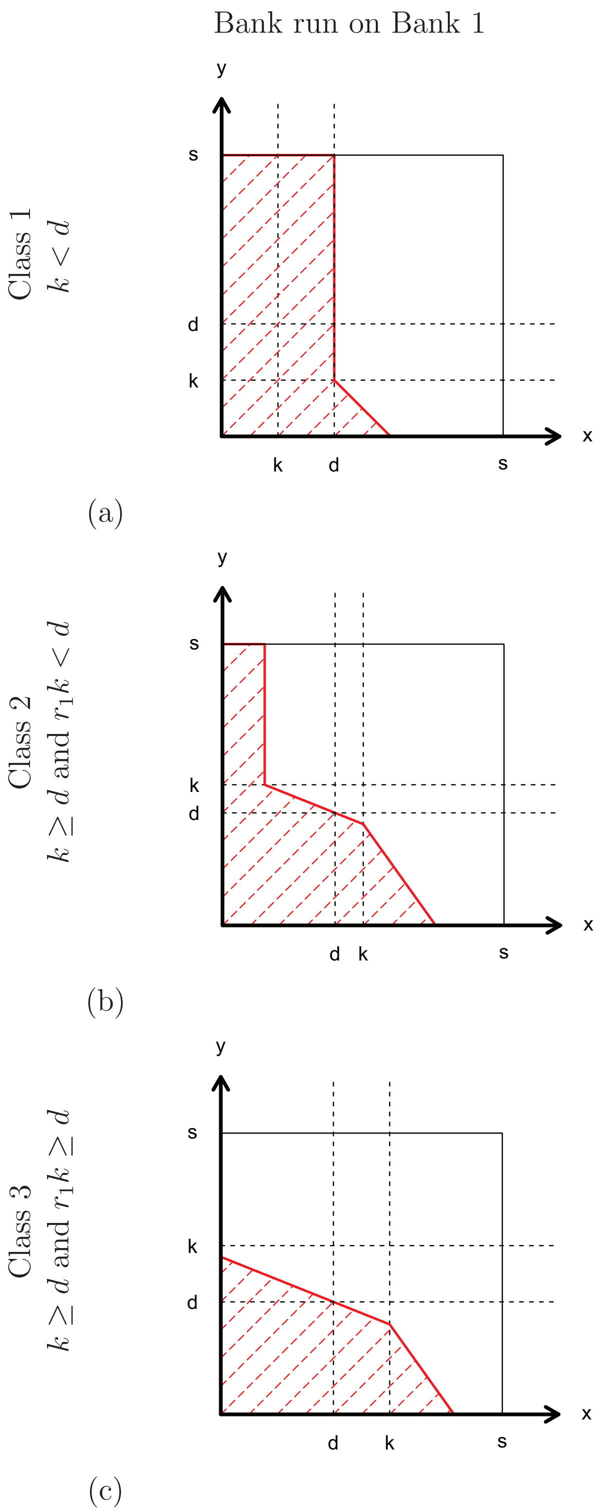
Figures

Figure 1: No diversication and linear diversification



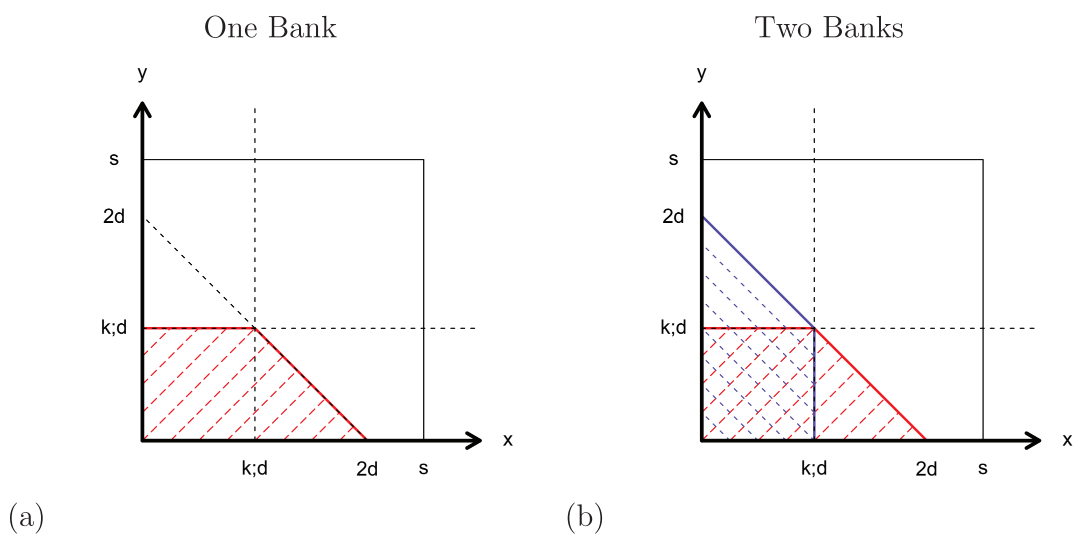
Note: the dashed areas in panel (a) and (c) give the possible outcomes for which a run on bank 1 would occur under respectively no diversification and linear diversification. Bank 2 has been added in panel (b) and (d). The double dashed areas report the outcomes that correspond to joint failures.

Figure 2: The three classes of investment strategies with tranching



Note: the dashed areas in panel (a), (b) and (c) give the possible outcomes for which a run on bank 1 occurs for each of the three classes of possible investment strategies described in equations (2.7), (2.8) and (2.9).

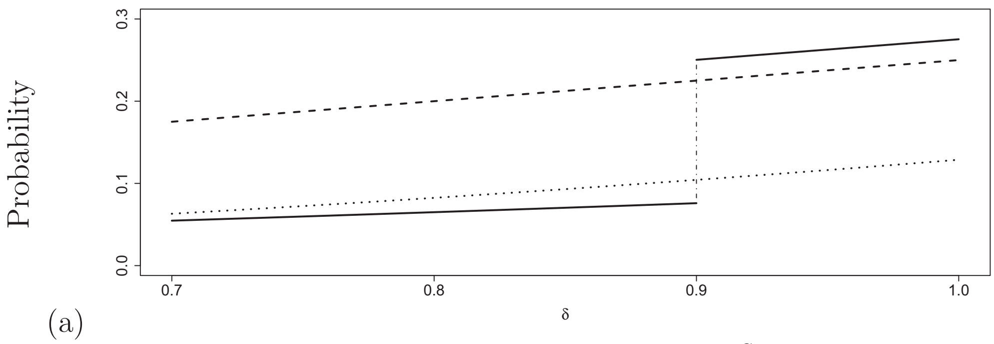
Figure 3: Diversification through securitization



Note: the dashed area in panel (a) gives the possible outcomes for which a run on bank 1 occurs under the optimal investment strategy that minimizes the probability of runs on bank 1. Bank 2 has been added to the figure in panel (b). The double dashed area reports the set of outcomes that correspond to joint failures. The double dashed area is not increased from the no diversification case in Figure 1, panel (b).

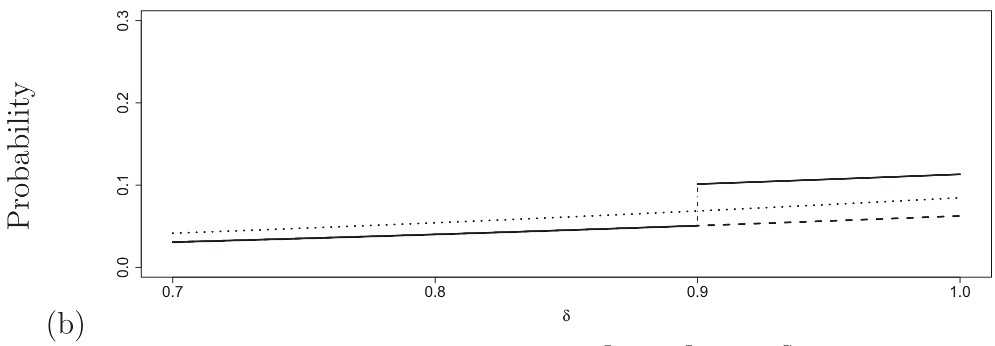
Figure 4: Diversification outcomes after a confidence shocks

Probability of a run on each bank ($\pi_i^I + \pi^S$)



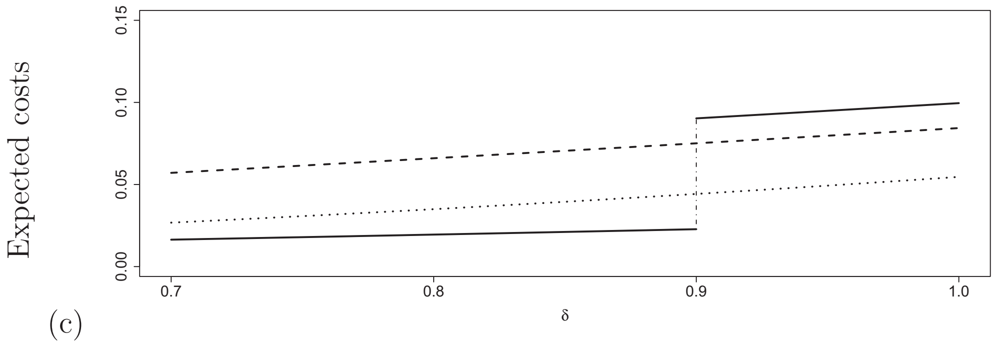
(a)

Probability of a system failure (π^S)



(b)

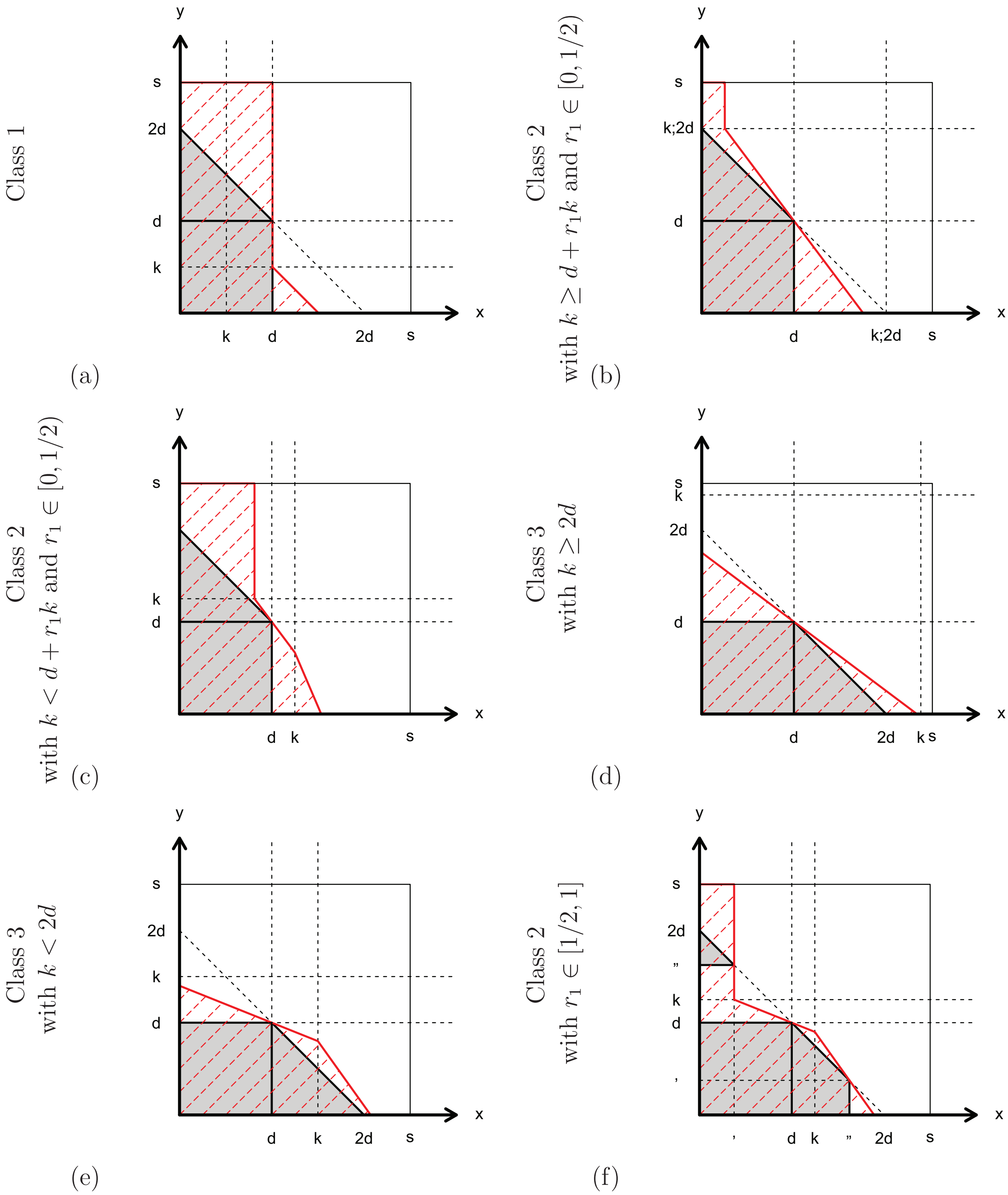
Expected failure costs ($c\pi_1^I + c\pi_2^I + cq\pi^S$)



(c)

Note: the panels report the outcomes of different investment strategies given the minimum return on assets necessary to avoid a bank run, δ. The solid (dotted) lines report the results of the optimal strategy with (without) tranching. The dashed line panel reports the result if no tranching or diversification is applied. Panel (a) reports the failure probability of each bank, panel (b) reports the probability of a systemic failure and panel (c) reports the expected default costs. The parameter choices are $c = 15\%$, $cq = 22.5\%$, $d = 0.9 (= k)$ and $s = 4$.

Figure 5: Comparison with the optimal strategy



Note: the figures help to interpret the integrals in the formal proof of Proposition 2. The dashed areas report the set of bank run probability outcomes given a certain investment strategy. The faded area reports a set of outcomes that has the same probability mass as the optimal investment strategy.

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