DNB Working Paper

No. 403 / November 2013

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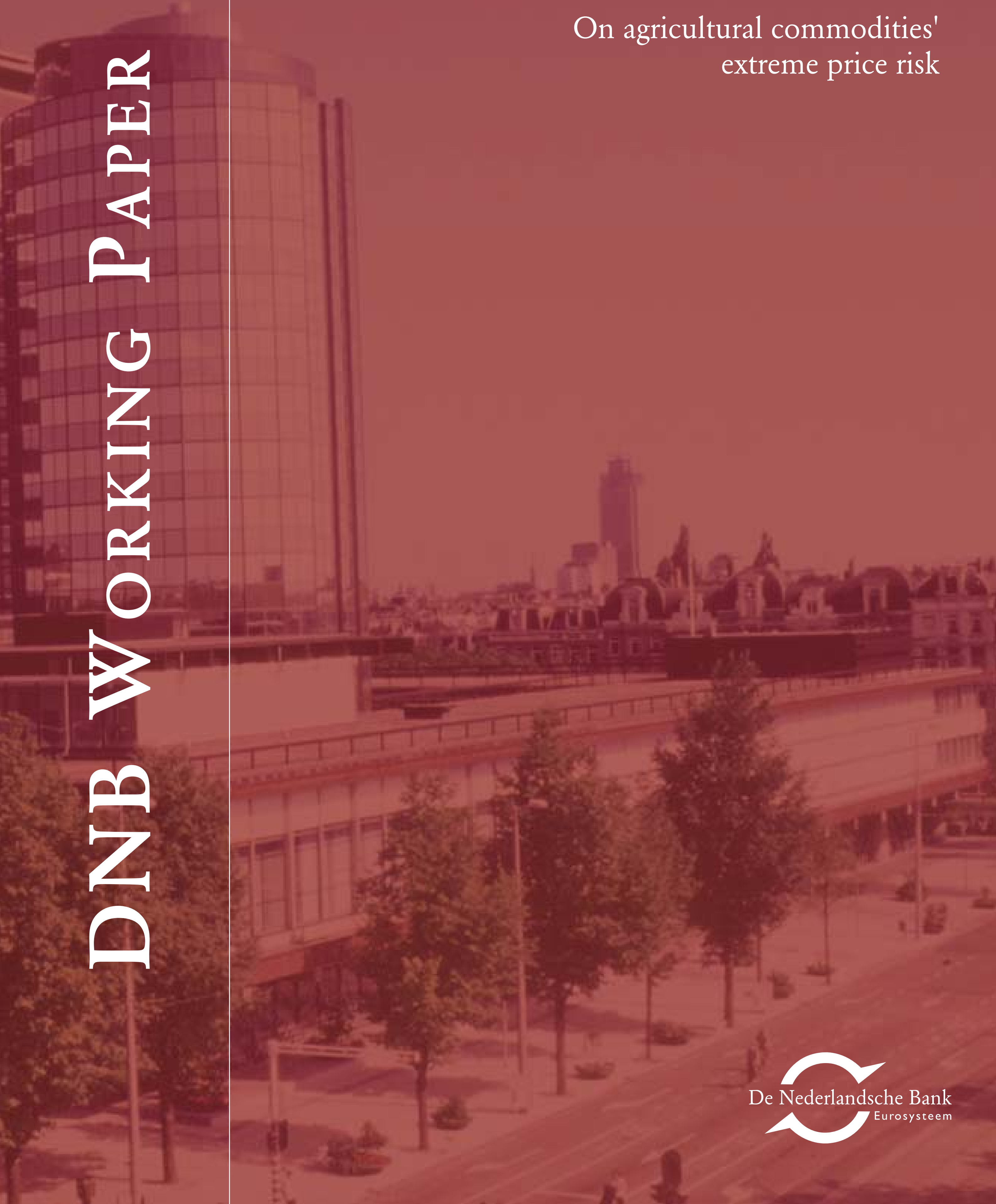

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Eurosysteem

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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

Working Paper No. 403

November 2013

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On Agricultural Commodities' Extreme Price Risk*

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November 21, 2013

Abstract: Price risk is among the most substantial risk factors for farmers. Through a two-sector general equilibrium model, we describe how fat tails in agricultural prices may occur endogenously as a result of productivity shocks. Using thirty years of daily futures price data, we show that the returns of all agricultural commodities in our sample closely follow a power law in the tail of their distributions. We apply Extreme Value Theory to estimate the size and likelihood of the highest losses a farmer may encounter. Back-testing verifies the validity of these risk measurement methods.

Keywords: Agricultural commodities, extreme value theory, heavy tails, risk management.

JEL Classification Numbers: C14, Q11, Q14.

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*Comments and suggestions from Pierre M. Lafourcade, Job Swank and Chen Zhou are gratefully acknowledged. Maarten van Oordt is grateful for the hospitality shown by the Economics department of the Erasmus School of Economics. Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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1 Introduction

The severe drought in the US during the summer of 2012 coincided with price increases of corn, soybeans and other field crops by more than 50%. Figure 1 provides an illustration of this period. The upper panel shows the contract price for the purchase of one bushel of corn or soybeans delivered at the end of 2012. The bottom panel shows the accumulated precipitation in the Primary Corn and Soybean Belt from March 2012 onwards.[1] Prices remained relatively stable during the first few months. The level of precipitation was not much different from its level in previous years and remained between its historical lower and upper quartiles. However, June and July were months with exceptionally low levels of precipitation.[2] As this drought was prolonged and growing in severity, prices started to increase rapidly. Prices somewhat flattened during the late summer, but only after the amount of rainfall in August and September returned to a level close to its historical average.

Extreme movements in agricultural commodity prices are anything but uncommon. For instance, between August 2007 and March 2008, the price of wheat almost doubled. However, before the end of 2008, the wheat price had returned to its original level. Another example is the price of corn, which fell a massive 55% in the second half of 2008 alone. These whopping figures illustrate the highly volatile behavior of agricultural commodity prices.

Strong price movements hurt many of the industry's stakeholders, including producers, traders and processing firms. Also, the economic prosperity of many developing countries often depends on the price development of raw material commodities. More than fifty countries depend on only three or fewer commodities for more than half their total exports.[3] As a result these countries are very vulnerable to price decreases and volatility, see e.g., Deaton (1999) or Balagtas and Holt (2009).

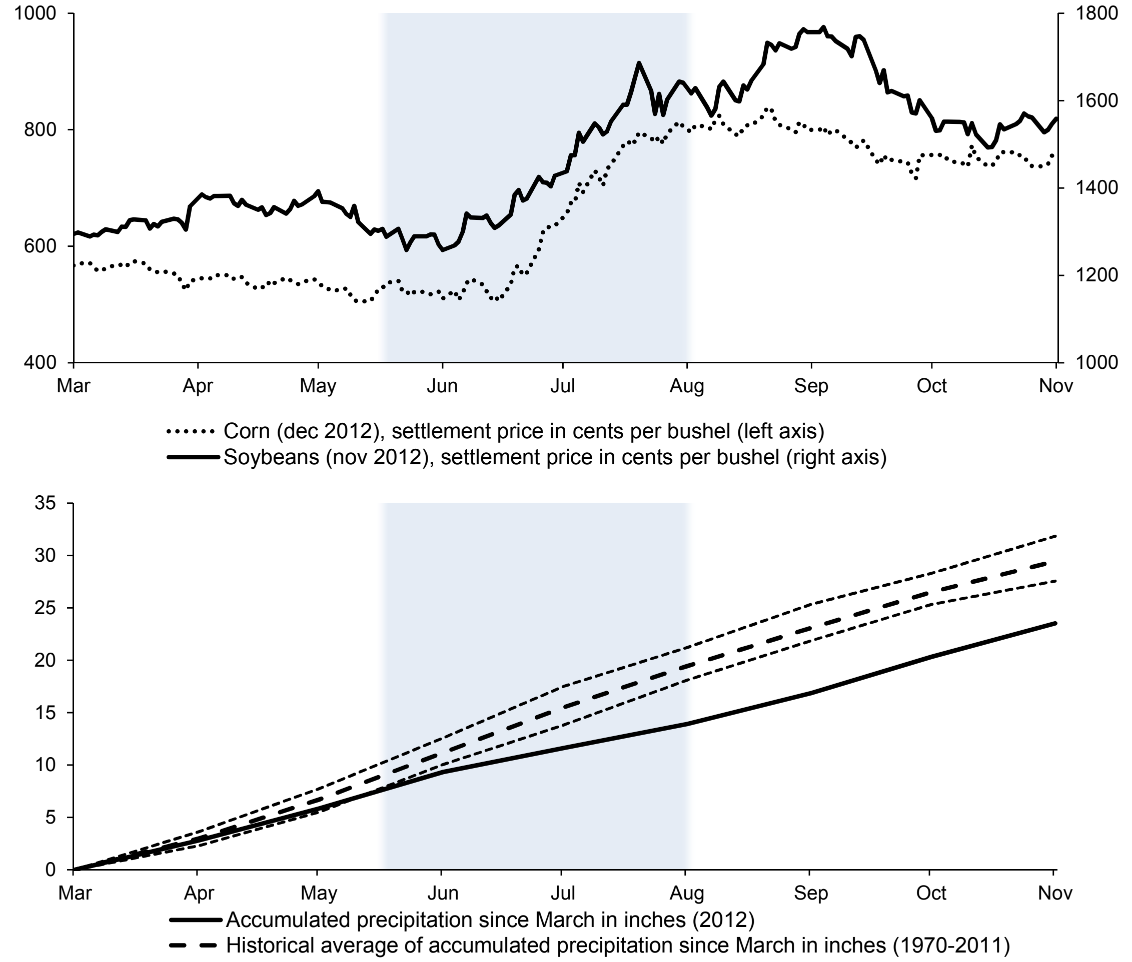
In the same vein, individual farmers are also highly vulnerable to commodity price risk. All price decreases translate fully and directly into a loss of income for the farmer. In this study we take the perspective of an individual U.S. farmer when analyzing agricultural commodities price risk. Nevertheless, many of the risk management findings in this study are also relevant to other affected parties.

[1]The National Climatic Data Center uses the expression Primary Corn and Soybean Belt to specify a large agricultural belt around (and including) Illinois, Indiana and Iowa. It covers parts of Alabama, Arkansas, Kansas, Kentucky, Louisiana, Minnesota, Mississippi, Missouri, Michigan, Nebraska, North Dakota, Ohio, South Dakota, Tennessee and Wisconsin.

[2]The area-weighted precipitation in the Primary Corn and Soybean Belt in June and July 2012 was 4.6 inches (cumulative), or almost half the historical average of 8.3 inches between 1970 and 2011. Since 1895, such low levels of precipitation in June and July were only recorded in 1936 and 1988, with 3.6 and 4.5 inches, respectively.

[3]Based on the UNCTAD 1995 Commodity Yearbook. We refer to Bidarkota and Crucini (2000) for an extensive analysis of the relationship between the terms of trade of developing nations and world prices of internationally traded primary commodities.

Figure 1: Futures prices and precipitation in 2012



Daily prices of futures contracts for delivery at the end of 2012 come from Datastream. Monthly area-weighted precipitation data for the Primary Corn and Soybean Belt come from the U.S. Climate Divisional Database and are obtained from the National Climatic Data Center of the National Oceanic and Atmospheric Administration. The bandwidth around the dashed line for the historical average corresponds to the historical lower and upper quartile.

A good understanding of the most extreme commodity returns is instrumental in any commodity risk management application. Small changes in revenues should not affect the farmer much. Large price drops however, may result in bankruptcy. Using a standard two-sector macro model, we describe how fat tails in agricultural commodity prices may occur endogenously. In our general equilibrium model, commodity price spikes occur as a result of factor productivity shocks, due to e.g., hurricanes, diseases and droughts. These shocks feed through the system, rendering the equilibrium price distribution fat-tailed.

As the typical farmer is risk-averse, see e.g., Coyle (2007), he is inclined to hedge his price risks. He may for instance remove his price risk by entering into forward or futures contracts. Alternatively, he may choose to hedge only against large price declines by buying out-of-the-money put options, thereby retaining the potential profit from sudden price increases. In order to decide upon the optimal risk mitigation strategy, the farmer needs a risk quantification methodology.[4] He should be able to answer questions like: How likely is a 10% fall in the corn price over the next two days? How likely is a price change that may result in bankruptcy? Is either the corn or the wheat price more likely to experience extreme price movements? What is the expected size of the maximum loss due to price risk during the next decade?

Such questions may be addressed by the use of Extreme Value Theory (EVT). This technique is particularly suited for estimating the likelihood of extreme returns when the probability distribution functions are non-normal. Mills (1927) was one of the first to discuss the non-normality of commodity returns as he reported higher kurtosis, implying more extreme returns. Mandelbrot (1963a) modeled the returns in the spot market for cotton by means of the stable distributions to capture the heavy-tail phenomenon. More recently, Ai et al. (2006) also discuss the non-normality of commodity returns, which they find to be characterized by frequent price jumps and fat tails. We refer to Kat and Oomen (2006), and Wang and Tomek (2007) for thorough studies of the time series properties of agricultural product prices.

Over the last few years, the popularity of EVT to assess the risk of an extreme event has increased considerably. For example, EVT has been used to examine the severity of stock market crashes, the pricing of catastrophic loss risk in reinsurance or the extent of operational risk in banks.[5] EVT is particularly suitable for analyzing extremely rare events when sample sizes are too small for determining the probability, extent or cause of the extreme returns using conventional statistical techniques. The semi-parametric EVT approach exploits the functional regularities that probability distributions display far from the center.

Interestingly, in spite of its growing recognition, application of EVT in agricultural price risk management has so far been sparse in the academic literature. Kofman and De Vries (1990) estimate the tail distribution parameters for potato futures. Matia et al. (2002) estimate the parameters of a large number of general commodities and find the tails to be fat. Their article provides no risk management applications, however. Krehbiel and Adkins (2005) apply EVT to four complex NYSE energy futures contracts to estimate various risk measures. Even so, their analysis is limited to oil and gas contracts, whose return distribution may be very different from those of renewable agricultural commodities. More recently, Morgan et al. (2012) use EVT on weekly data to estimate three different tail risk measures for corn and soybeans. Their thorough study is evidence of the growing interest in this topic.

The main contribution of this article is twofold. The first contribution is to show how the heavy-tailedness of agricultural prices may arise endogenously in an economic model.

[4]For a discussion on the use of derivatives to hedge commodity price risk we refer to Lu and Neftci (2009).

[5]Based on the ECB June 2006 Financial Stability Review.

The second contribution is the use of EVT to measure the extreme price risk of nine different agricultural commodities. We use a back-testing procedure to provide empirical evidence on the accuracy of the proposed risk measures. We show that the non-normality of the return distribution strongly influences the level of the risk measures. Our empirical estimates provide a good indication of the size of the risks as measured by widely used and easily interpretable risk measures. This study provides farmers and other stakeholders with a reliable toolset to quantify their price risks and to answer the above questions.

The remainder of this article is as follows. Section 2 provides a model in which commodity price spikes arise endogenously, as a result of productivity shocks. Section 3 discusses how to apply EVT to estimate the Value-at-Risk and Expected Shortfall risk measures. The data are described in Section 4. Empirical estimates of the distributions' tails are presented in Section 5. Value-at-Risk and Expected Shortfall estimates, as well as back-testing results are provided in Section 6. We conclude in Section 7.

2 Theory

Mandelbrot (1963a,b), using Houthakker's cotton price series, is probably the first who documented that the tails of the distribution of logarithmic commodity price changes diminish by a power instead of an exponent, as is the case under the more common (log)normal assumption. If the tail of a distribution diminishes by a power, then the probability of variable $\tilde{x}$ exceeding threshold u, if u is large, is distributed as:

$$\Pr(\tilde{x} > u) \sim Cu^{-\alpha}, \tag{1}$$

where $C > 0$ and $\alpha > 0$ are, respectively, the *scale* and the *shape* parameter. The distribution is named after Pareto who discovered that the tail of the income distribution follows a power law. Distributions with tails that obey the functional form in (1) are classified as *heavy-tailed.* Tails which follow a power law are in the end always fatter than tails that decrease by an exponent.

To explain the heavy-tail nature, Mandelbrot advances that the physical world is full of heavy-tailed phenomena, which may trigger the heavy-tailedness of commodity price changes.[6] But how a power law may arise endogenously in this type of market has not been investigated.

Below we develop a small standard macro model with an agricultural sector to study agricultural prices in equilibrium. The equilibrium agricultural price distribution is the

[6]See e.g., Newman (2005) and Salvadori et al. (2007) for a number of natural hazards that follow a power law distribution, including the magnitude of earthquakes, the volume of air-fall material from volcanic eruptions, various drought measures, flood levels, and the scale of wars. Several of the above events influence agricultural prices in one way or another. Spikes or sudden drops in prices can be triggered by, for instance, a drought or bumper crop.

result of factor productivity shocks that feed through the system. In themselves such shocks need not be (although they may be) heavy-tailed. This is the extra kick that our economic analysis provides. We show how the power law spikes observed in agricultural commodity prices can arise endogenously in the economy. The model describes how adverse productivity shocks, such as drought and hurricanes, affect the tail distribution of commodity prices.

2.1 Model

We use a standard off-the-shelf two-sector macro model.[7] The agricultural sector is modeled as the competitive sector. The other sector produces differentiated goods in the spirit of Dixit and Stiglitz (1977) (subsequently referred to as DS1977). Exogenous shocks affect the productivity of both sectors. In the agricultural sector, these shocks can be best thought of as changes in weather and other natural hazards. For the differentiated goods sector, which also captures the services industry, the shocks mostly represent changes in productivity.

The macro literature has focused almost exclusively on the DS1977 specification for the differentiated goods demand, see e.g., Walsh (2008). The familiar DS1977 specification with endogenous labor supply derives from the following utility function

$$U = Z^{1-\theta}\left[\frac{1}{n}\sum_{i=1}^{n} Q_i^{\rho}\right]^{\theta/\rho} - \frac{1}{1+\gamma}L^{1+\gamma}, \tag{2}$$

where Z is the competitive good, the Q_is are the differentiated goods and L is labor. To guarantee concavity and allow for zero demand for a particular Q_i, the parameter ρ is constrained to $\rho \in (0,1)$. We envision the Z good to be the staple of agricultural produce, while the Q_i goods capture the production of other goods and services. Parameter $\theta \in (0,1)$ determines the relative importance of the other goods and services to the agricultural produce in the consumer's consumption bundle. The higher the level of θ, the smaller the share of income the consumer is willing to spend on agricultural goods. Parameter γ is the inverse of the Frisch (1959) elasticity of labor supply. In general, the higher the level of γ, the less responsive labor supply will be to changes in the wage rate.

The budget constraint reads

$$wL + \Pi(Q) = qZ + \frac{1}{n}\sum_{i=1}^{n} p_i Q_i, \tag{3}$$

where w is the wage rate and q, p_i are the goods prices, while $\Pi(Q)$ are the profits received

[7]See Ardeni and Freebairn (2002) for a discussion on the interaction between agricultural prices and the macro economy.

from the differentiated goods sector.[8]

For the supply side we assume Ricardian technologies for all the goods, where

$$Z = BN, \tag{4}$$

and

$$Q_i = AN_i. \tag{5}$$

Here A and B are the productivity coefficients while N and N_i are the respective labor inputs. Both A and B are random variables. In the case of variable A, these are the familiar supply side total factor productivity shocks. In the case of the agricultural sector, variable B captures the random element in agricultural productivity inherited from nature. We assume that the market for the agricultural product is perfectly competitive. The producer of the differentiated product exploits his pricing power, but ignores his pricing effect on consumer income $wL + \Pi(Q)$ and on the price index of the differentiated goods,

$$P = \left(\frac{1}{n} \sum_{i=1}^{n} p_i^{\rho/(\rho-1)} \right)^{\frac{\rho-1}{\rho}}. \tag{6}$$

Finally, to determine the price level we assume a simple quantity type relation

$$M = wL. \tag{7}$$

2.2 Equilibrium price distribution

With the above preparations, we can now obtain the implications for the equilibrium prices.

Proposition 1 *The prices of the differentiated goods are*

$$p_i = M \frac{1/\rho^{\theta/\gamma+1}}{A \left(\theta^\theta \left(1-\theta\right)^{1-\theta} A^\theta B^{1-\theta} \right)^{1/\gamma}}. \tag{8}$$

For the agricultural good the price is

$$q = M \frac{1/\rho^{\theta/\gamma}}{B \left(\theta^\theta \left(1-\theta\right)^{1-\theta} A^\theta B^{1-\theta} \right)^{1/\gamma}}. \tag{9}$$

[8] The quantities of the differentiated goods, the Q_i, are normalized by the number of differentiated goods, n. This notation is analogous to the common continuous good notation often used in theoretical macro literature.

Proof. See Appendix A. ■

Most macro models consider shocks to M, A and B. Let us focus on the natural shocks to B.[9] Assuming M and A to be constant, we can write the price of the agricultural good as

$$q(B) = \Theta B^{-\frac{1+\gamma-\theta}{\gamma}}, \tag{10}$$

where

$$\Theta = M \frac{1/\rho^{\theta/\gamma}}{\left(\theta^{\theta}\left(1-\theta\right)^{1-\theta} A^{\theta}\right)^{1/\gamma}}.$$

For illustrative purposes we assume that B follows a beta distribution (we relax this assumption later):

$$\Pr\left\{B < t\right\} = t^{\beta} \tag{11}$$

on $[0, 1]$ and $\beta > 0$. Consider the implication for the price distribution of the agricultural product. Denote the randomness in q by $\tilde{q}$. Then

$$\begin{aligned} \Pr\left\{\tilde{q} > u\right\} &= \Pr\left\{\Theta B^{-\frac{1+\gamma-\theta}{\gamma}} > u\right\} \\ &= \Pr\left\{B < \Theta^{\frac{\gamma}{1+\gamma-\theta}} u^{-\frac{\gamma}{1+\gamma-\theta}}\right\} \\ &= \Theta^{\frac{\beta\gamma}{1+\gamma-\theta}} u^{-\frac{\beta\gamma}{1+\gamma-\theta}}, \end{aligned} \tag{12}$$

with support on $[\Theta, \infty)$. The distribution of equilibrium prices in equation (12) has the same functional form as the heavy-tailed distribution in equation (1). But naturally, this result is subject to the qualification that it crucially relies on the restrictive assumption of the beta distribution for shocks from nature in (11).

We proceed by relaxing the assumption of the beta distribution for B in (11). More specific, we derive a general condition on the density function of B, $f_B(B)$, such that the equilibrium prices follow a heavy-tailed distribution if this condition holds. This condition is provided in the following proposition.

Proposition 2 *Suppose that the distribution and density of the agricultural productivity coefficient B are continuous. Given the price-productivity relation for agricultural*

[9] For our results to hold it is not necessary to assume a constant A and M. For example, the heavy-tailedness of the equilibrium price distribution due to natural shocks is preserved if the productivity of the differentiated sector does not collapse completely, which implies that the support of A is bounded away from zero. Further, the heavy-tailedness of the equilibrium price distribution is not affected if the distribution of M has exponential tails, such as the lognormal distribution.

products in equation (10)*, we have that*

$$\Pr(\tilde{q} > u) \sim \mathcal{L}(u)u^{-\alpha} \text{ as } u \to \infty, \tag{13}$$

with

$$\alpha = \xi \frac{\gamma}{1 + \gamma - \theta}, \tag{14}$$

if

$$\lim_{s \downarrow 0} \frac{w f_B(sw)}{f_B(s)} = w^{\xi} \text{ with } \xi \in \mathbb{R}^+. \tag{15}$$

Proof. See Appendix B. ■

Whether the condition on the distribution of B in (15) is satisfied depends on the shape of the density of the productivity shocks, $f_B(B)$, for values close to zero. The reason is that high price levels occur in periods of low agricultural productivity, such as severe droughts. The distribution of those low productivity levels determine the shape of the distribution function for extremely high prices. For the beta distribution in (11), which we considered for illustrative purposes, the condition in (15) is satisfied with $\xi = \beta$. The lower parameter β in (11), the more severe are the shocks resulting in productivity levels close to zero, and the heavier is the tail of the agricultural prices.

Corollary 1 *The shape of the distribution of the agricultural productivity coefficient B close to zero determines whether the equilibrium price distribution is heavy-tailed. The more slowly the density of B converges to zero for extremely low productivity levels, i.e., the lower ξ, the heavier is the tail of the agricultural price distribution, i.e., the lower is α.*

It is not difficult to verify that a broad range of distribution functions with positive support satisfy the condition in Proposition 2 with different values for ξ. For instance, the standard uniform distribution and the exponential distribution satisfy the condition in (15) with $\xi = 1$, the Chi-squared distribution with k degrees of freedom satisfies the condition with $\xi = k/2$, the Gamma distribution with shape parameter k satisfies the condition with $\xi = k$, and the (heavy-tailed) Burr (Type XII) distribution with parameters (c, k) satisfies the condition with $\xi = c$. All these distributions would result in a heavy-tailed equilibrium price distribution. However, not every possible distribution yields heavy-tailed prices in the macro-economic framework. An example of a popular exception is the lognormal distribution: Its limit in (15) converges to 0.[10] Seriously

[10]The statistical distribution of crop yields has been the topic of a wide body of literature, see e.g., Nelson and Preckel (1989), Moss and Shonkwiler (1993), Just and Weninger (1999), Atwood et al. (2003), Ramírez et al. (2003), Harri et al. (2009) and Koundouri and Kourogenis (2011).

low levels of agricultural productivity are too rare under the lognormal distribution to generate a heavy tail among the occurrences of high agricultural prices in the model.

As follows from Proposition 2, the shape parameter of the tail of the distribution of agricultural prices not only depends on the distribution of productivity shocks, but also on the preference parameters θ and γ.

Corollary 2 *The greater the share of agricultural produce in consumption, i.e., the higher $1-\theta$, and the higher the elasticity of labor supply, i.e., the higher $1/\gamma$, the heavier is the tail of the distribution of agricultural prices, i.e., the lower is α.*

Given the distribution function of productivity shocks, it follows from equation (14) that a high value of $1-\theta$ results in a low shape parameter of the equilibrium price distribution of agricultural goods, α, and hence in a fatter tail. This finding has the following intuition. The importance of the share of the agricultural good in the consumption bundle of the agents is represented by $1-\theta$, see equation (2). The larger the role of the agricultural good for the agents' utility, the more extreme price reactions one may expect if supply falls. This is reflected in a fatter tail of the equilibrium price distribution, i.e., a lower α.

It also follows from equation (14) that a high value of parameter γ results in a high value of shape parameter α. Adverse technology shocks have a dual effect on the output of the competitive sector. First, given the amount of labor used, an adverse technology shock in the competitive sector directly reduces output. Second, low productivity decreases the equilibrium amount of labor used in the competitive sector, which further reduces output. The inverse of the labor supply elasticity γ determines the sensitivity of the wage rate to changes in the amount of labor used in production. With a higher value of γ, a reduction in the amount of labor results in a lower drop of the wage rate, which translates in a smaller change in the equilibrium amount of labor. Therefore, the change in production of the competitive good is smaller for high values of parameter γ, which results in thinner tails of the equilibrium price distribution, i.e., a higher α.

3 Empirical Methodology

The previous section discussed the plausibility of fitting the tail distribution of changes in food prices to a power law. Next, we apply EVT to determine the parameters of the power law.

3.1 Fitting the power law

As a first step we calculate n returns, R_t, from the observed prices as $(P_t - P_{t-1})/P_{t-1}$. Secondly, the returns are ordered from high to low: $X_1 \geq \cdots \geq X_n$. The number of returns in the tail of the distribution is set equal to k. This implies that X_{k+1} approaches the threshold, which is the minimum u for which the distribution in (1) applies. All returns above this threshold are assumed to be distributed by a power law. Next, we estimate the shape and the scale parameter (α and C) by following Hill (1975):

$$\frac{1}{\hat{\alpha}} = \frac{1}{k}\sum_{j=1}^{k} \ln \frac{X_j}{X_{k+1}} \tag{16}$$

and

$$\hat{C} = \frac{k}{n} X_{k+1}^{\hat{\alpha}}, \tag{17}$$

where equation (16) is generally referred to as the Hill estimator.[11]

Whereas the concept and the estimation of the parameters are straightforward, the choice of k is not. The optimum depends on the sample size T and the tail-thickness α; the further one moves out into the tails, the better becomes the Pareto approximation of those tails. However, this reduces the number of observations available for estimation and increases the uncertainty of the estimate.

In practice, one may resort to visual inspection of the so-called Hill plots to determine the optimal level of threshold k. The number of observations included in the tail, k, is plotted along the x-axis. For each number of observations, k, the Hill estimate for α is calculated. The optimal threshold is selected from the region in which the Hill estimate for α is more or less stable, see also Drees et al. (2000).

Standard errors of the shape parameters can be obtained from a bootstrap procedure. De Haan et al. (1994) demonstrate the asymptotic normality for the Hill estimator and the VaR estimate for independent and identically distributed (iid) returns. The asymptotic normality of the Hill estimator also holds in the presence of serial dependence, see e.g., Drees (2008). Following Hartmann et al. (2006), we refrain from assumptions on the specific dependence structure and apply a bootstrap procedure with fixed block length and 10,000 replications. Following Hall et al. (1995), we set the optimal block length equal to $n^{1/3} \approx 20$.

3.2 Risk measures

Both risk measures introduced in this subsection measure the probability of extremely low returns (or extremely high returns in case of a short position), also called 'tail risks'. The first one, Value-at-Risk (VaR), is one of the most widely used risk measures in financial risk modeling. VaR plays an important role in the safety-first framework developed by Roy (1952) and Telser (1955). Agents with the safety-first principle of Telser (1955)

[11]This procedure describes how a power law is fitted to the right tail of the return distribution. To fit a power law to the left tail multiply the return series with -1.

in their utility functions maximize their expected return, while limiting the probability that a loss larger than some disaster level occurs at some admissible level p. Basically, such agents maximize their expected return under a VaR constraint with probability p.

The VaR (in terms of returns) is simply defined as a quantile estimate. After fitting a power law distribution to the data, the VaR is estimated by inverting the power law in equation (1). To derive a VaR estimator, the parameter estimates for C and α in equations (16) and (17) are substituted into the power law, which gives $\Pr(\tilde{x} > u) \sim \frac{k}{n} X_{k+1}^{\hat{\alpha}} u^{-\hat{\alpha}}$. Following the definition of VaR, $VaR(p)$ can be considered as the threshold u which is exceeded with probability p. Hence, we replace u by $VaR(p)$ and $\Pr(\tilde{x} > VaR(p))$ by p. After rewriting, the following VaR estimator is obtained:

$$\widehat{VaR}(p) = X_{k+1} \left(\frac{k}{np} \right)^{1/\hat{\alpha}}. \tag{18}$$

One of the shortcomings of VaR is that it contains no information on the size of the losses beyond the $p\%$ worst case. A risk measure which overcomes this problem is the Expected Shortfall (ES). The $ES(p)$ measures the expected amount one loses in the $p\%$ worst cases. In case of a power law tail, the estimated Expected Shortfall is a multiplication of VaR and a constant, which depends on the shape parameter only, see also Danielsson et al. (2006):

$$\lim_{p \downarrow 0} \frac{ES(p)}{VaR(p)} = \frac{\alpha}{\alpha - 1}. \tag{19}$$

4 Data

In many financial risk management applications it is common practice to use relatively high-frequency return series. For our purpose, daily data are preferable over weekly or monthly data for two reasons. First, the use of high frequency data, implying more observations, improves the quality of the parameter estimates. Second, the choice for daily data corresponds to the time required to hedge a farmer's price risk on the financial markets, which can typically be accomplished within one day.

In this study we employ futures prices instead of spot prices. One reason for this choice is the lack of reliable high frequency commodity spot prices. In general, historical daily commodity spot prices contain a high number of zero returns and are more likely to be affected by bid-ask bounces due to a lack of liquidity. By contrast, for many agricultural commodities, futures contracts are highly liquid and exchange-traded instruments for which reliable historical high frequency data is available.

Futures contracts for delivery at a particular date are usually traded for a relatively short period, ranging from several months to several years. To obtain long-term futures returns series, or so-called *continuous* series, we therefore need to combine consecutive data from several futures contracts, see e.g., De Roon et al. (2000). We take considerable effort to construct high-quality continuous futures return series. Our procedure is as follows. First, we download daily open interest and price series of all available futures contracts from Datastream for each commodity. Those time series are available over a period of 34 years: from January 1979 until December 2012. Subsequently, daily returns are calculated for all futures price series. Finally, we construct the continuous futures returns series from the individual return series. In January 1979 we start with the futures contract that has the largest open interest. For each day we include its returns in the new continuous series until six weeks before the contract's last trading day. At this date we switch to the futures contract with the largest open interest and a later last trading day. Again we include the returns until six weeks before the last trading day and repeat the last step. This procedure results for each commodity in a continuous futures returns series with 8,870 daily observations from, on average, 164 different futures contracts.

Our method has an important advantage compared to Datastream's procedure to construct continuous futures series. By calculating returns prior to constructing the continuous series, no returns are calculated over price observations from two different futures series. Therefore, our series represents the return that investors could achieve by rolling over futures contracts as opposed to the continuous Datastream series which includes price jumps due to changes in the underlying futures series. The extreme returns in our series thus represent genuine financial risks to market participants.[12]

From all commodity futures traded in the United States, the following seven crop commodities and two animal commodities are investigated: Corn, oats, soybeans, wheat, cotton, sugar, orange juice, live cattle and lean hogs.[13]

5 Empirical tail estimates

Table 1 reports the descriptive statistics of the daily futures return series. A quick overview of the data confirms the non-normality of the returns. Six out of nine series contain at least one observation with a distance of at least six standard deviations from the mean. The probability of such a return occurring equals about 2.0×10^{-9} under the assumption of normality, or roughly once every 2 million years.[14] The other three series contain at least one observation with a distance of five standard deviations from

[12]In addition, shifts in the roll-over date often occur in the Datastream continuous series. To give an extreme example: the second largest daily price fall during the last 30 years in the unadjusted Datastream series for cotton (NCTCS00) is caused by a delayed roll-over date. The return of -26.3% is caused by the difference between 113.6, which is the price for delivery in July 1995 listed on the 4th of July, 1995, and 83.75, which is the price for delivery in October 1995 listed on the 5th of July, 1995. Such extreme observations may distort the assessment of the actual tail of the risk distribution.

[13]See Appendix C for details on the selection process of the commodities. The continuous futures returns are available from the corresponding author on request.

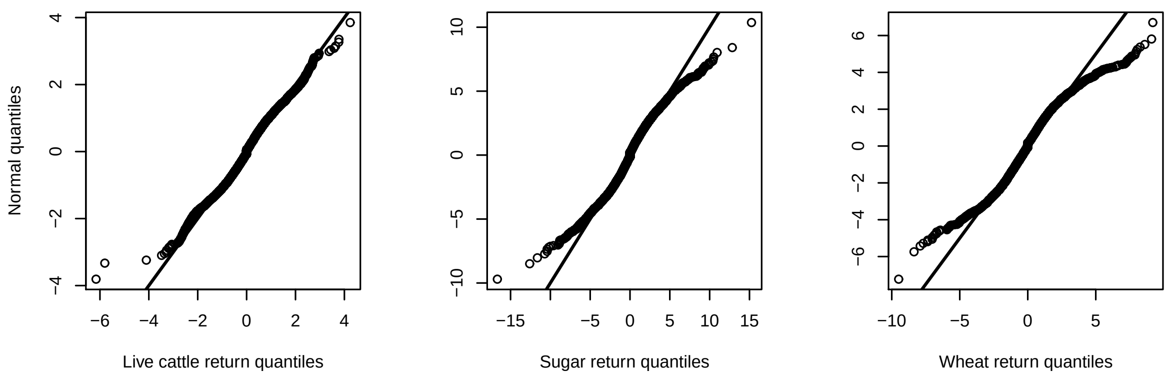
[14]If $\tilde{x} \sim N(0,1)$, then $\Pr(\tilde{x} \leq -6) \approx 1.0 \times 10^{-9}$.

Table 1: Descriptive statistics

Commodity	Mean	St.dev.	Min.	Min. Date	Series	Max.	Max. Date	Series
Corn	-0.01	1.4	-7.6	2009-06-30	1009	9.0	2009-09-15	0310
Cotton	0.00	1.4	-6.9	2012-06-21	1012	7.2	2008-12-08	0309
Oats	-0.01	1.7	-11.3	2005-03-31	0705	11.1	2005-03-30	0705
Soybeans	0.01	1.4	-7.1	2009-07-07	1009	6.9	1999-08-02	1099
Wheat	-0.01	1.6	-9.5	2009-01-12	0309	9.2	2008-10-29	0309
Lean hogs	-0.01	1.4	-6.7	1998-12-11	0399	7.1	1998-12-14	0399
Live cattle	0.01	0.9	-6.2	2003-12-30	0304	4.2	1989-07-19	1089
Orange juice	0.00	1.7	-12.8	2010-01-11	0310	16.3	2006-10-12	0307
Sugar	0.02	2.3	-16.7	1988-07-26	1088	15.3	1985-07-26	1085

Note: The first two columns report the mean and the standard deviation of the daily returns series. The other columns report the minima and maxima of the return series, the date of these observations and the code of the futures series in which these were observed. The code of the futures series refers to the month of delivery with format MMYY.

Figure 2: QQ-plots of agricultural commodity returns.



QQ-plots of the daily returns of futures contracts for live cattle, sugar and wheat against a normal distribution with the same mean and standard deviation.

the mean, an observation that would happen roughly once every 7 thousand years under the assumption of a normal distribution.

As an illustration, Figure 2 reports QQ-plots of three arbitrarily chosen daily return series (live cattle, sugar and wheat). The non-normality is strongly confirmed by QQ-plots of the return series against the normal distribution. Only the daily return series for live cattle seems to be an exception. Except for a few extreme tail observations, the distribution of the live cattle return series is generally quite close to the normal distribution.

Table 2 documents the estimated tail parameters. Unreported Hill plots show that the Hill estimates are relatively stable when a total of 150 tail observations are used, i.e., approximately 1.75% of all observations. The value of most shape parameters is estimated to be around 4. The most risky commodities with respect to the shape parameter are wheat, with an estimate of around 3.6 for the left tail, and orange juice, with an estimate

Table 2: Tail parameter estimates.

Commodity	Left tail			Right tail		
	Shape	(s.e.)	Scale	Shape	(s.e.)	Scale
Corn	4.16	0.37	2.67	4.31	0.33	4.25
Cotton	4.78	0.46	6.24	4.63	0.39	5.36
Oats	4.65	0.34	10.71	4.20	0.32	5.75
Soybeans	4.48	0.35	4.22	4.68	0.36	4.82
Wheat	3.60	0.28	1.52	3.84	0.29	2.86
Lean hogs	6.63	0.79	47.65	5.67	0.54	12.26
Live cattle	8.23	1.23	11.64	7.87	0.77	7.62
Orange juice	4.36	0.35	7.57	3.62	0.28	2.63
Sugar	4.58	0.36	34.74	4.17	0.34	16.83

Note: The first columns on the left and on the right side report the estimated shape parameter from equation (16) for, respectively, the left and right tail. The second columns report the corresponding standard error from the bootstrap procedure described in Section 5. The third columns report the estimated scale parameter from equation (17). Each tail consists of 150 observations, or approximately 1.75% of 8,609 observations.

of around 3.6 for the right tail. Live cattle and lean hogs are the commodities with the highest estimates for the shape parameters, implying thinner tails. For live cattle this finding is not remarkable: The QQ-plots already showed that the live cattle futures return distribution is quite similar to the thin-tailed normal distribution.

5.1 Alternative data frequencies

In this subsection, the shape parameters are tested for sensitivity to changes in the data frequency. In principle, the estimated shape parameters should be robust for changes in the data frequency in case of independent and identically distributed returns.[15] Nevertheless, volatility clustering or daily price limits may result in different tail behavior for different data frequencies. To this end, two-day returns and weekly returns are calculated from the daily return series. In order to test for equality of the shape parameters, we estimate the shape parameters from the two-day returns and weekly returns. Subsequently, we calculate the following t-statistic

$$T = \frac{\hat{\alpha}_1 - \hat{\alpha}_2}{\hat{\sigma}(\alpha_1 - \alpha_2)}, \tag{20}$$

[15]Mandelbrot (1963b) shows that power law distributions are invariant with respect to the shape parameter under several basic transformations. The shape parameter is invariant with regard to summation of random variables, mixing random variables with different scale parameters and selection of maxima. It follows that the power law distribution is independent of data frequency choices, distribution mixture assumptions and missing data. As a consequence, sample-specific data problems are unlikely to affect the observed shape parameter.

where $\hat{\sigma}(\alpha_1 - \alpha_2)$ denotes the standard deviation of the difference between the shape parameters estimated from two different frequencies, and where the t-statistic converges to a standard normal distribution under the null hypothesis of equal shape parameters. The standard deviation, $\hat{\sigma}(\alpha_1 - \alpha_2)$, is obtained from a block bootstrap procedure, in which each bootstrapped sample is obtained from daily returns, but subsequently also transformed into lower frequency returns to calculate the two-day or weekly shape parameters.

Table 3, panels (a) and (b) report the results of the robustness test for changing the data frequency. The differences in the shape parameters between daily and two-day return distributions in panel (a) are statistically significant at the 5% level for two commodities. The tails of the live cattle return distribution and the lean hogs return distribution have significantly lower shape parameters for two-day returns. In panel (b) we test whether the shape parameter changes significantly if one extends the data frequency further from two-day returns to weekly returns. We do not find significant differences between the shape parameters of two-day and weekly returns. Apparently, the issues that potentially cause differences between the daily and two-day parameter estimates in our sample do not play a large role if one turns to estimation at lower data frequencies.

6 Risk estimates

This Section discusses the empirical results of the risk estimates. Table 4 reports the 0.1% and 0.01% VaR and Expected Shortfall estimates, following equations (18) and (19). The 0.1% VaR is expected to be exceeded about once every four years, and the 0.01% VaR about once every 40 years, or about once during a farmer's entire career.

The relevant information for farmers with respect to risk management is contained in the left tail of the distribution, shown in Table 4, panel (a). We find that sugar has the highest price risk of all commodities studied. Once every forty years the sugar price is expected to fall by more than 16.2% within a one-day period. The safest commodity, in terms of price development, appears to be live cattle. Once every forty years the price of live cattle is expected to fall by more than 4.1% within a one-day period.

In the introduction the question was posed what the likelihood is of a price change that could result in the farmer's bankruptcy. It is possible to answer this question with the results in Sections 5 and 6. Suppose a farmer who grows wheat and who would have serious solvency problems after a 15% fall in the wheat price from its current level. What is the probability that an unhedged farmer will default from one day to the next?

To answer this question, we apply equation (1) to the left tail, and substitute the absolute value of the threshold return for u. Subsequently, the parameter estimates from Table 2 for C and α are substituted into equation (1). Since the farmer stands to be hurt by price falls (as opposed to price increases), the parameters for the left

Table 3: Tail parameter estimates with other data frequencies.

Panel (a): Two-day returns

Commodity	Left tail				Right tail			
	Shape	(s.e.)	Scale	t-stat	Shape	(s.e.)	Scale	t-stat
Corn	3.47	0.33	3.44	-1.69*	3.56	0.42	5.53	-1.53
Cotton	4.05	0.37	9.57	-1.21	4.35	0.41	17.57	-0.51
Oats	4.15	0.40	22.13	-1.06	3.81	0.35	13.06	-0.70
Soybeans	3.91	0.39	6.58	-1.45	3.99	0.48	7.14	-0.53
Wheat	3.80	0.42	6.61	0.52	3.57	0.34	6.35	-0.38
Lean hogs	5.01	0.59	41.92	-2.19**	5.00	0.44	24.35	-0.77
Live cattle	4.89	0.49	3.70	-3.06***	4.80	0.45	3.52	-3.54***
Orange juice	4.10	0.41	20.78	-0.56	3.18	0.32	4.55	-1.00
Sugar	4.43	0.45	96.43	-0.37	3.97	0.42	47.04	-0.37

Panel (b): Weekly returns

Commodity	Left tail				Right tail			
	Shape	(s.e.)	Scale	t-stat	Shape	(s.e.)	Scale	t-stat
Corn	3.26	0.44	11.55	-0.39	3.95	0.51	71.31	1.53
Cotton	3.90	0.57	29.76	-0.24	2.97	0.51	6.80	-1.45
Oats	4.24	0.56	192.64	0.13	3.84	0.82	84.10	0.23
Soybeans	3.48	0.45	16.64	-0.85	3.42	0.47	14.44	-1.30
Wheat	3.83	0.56	30.68	0.05	3.25	0.48	15.86	-0.55
Lean hogs	4.17	0.66	53.06	-1.39	4.12	0.57	41.40	-0.95
Live cattle	4.16	0.71	10.17	-1.19	4.59	0.66	21.58	-0.17
Orange juice	3.69	0.48	49.69	-0.66	2.74	0.41	8.98	-0.76
Sugar	5.11	0.74	3133.97	0.99	3.75	0.77	145.42	1.00

Note: Shape parameters are estimated from equation (16) for the left and right tail of two-day returns in panel (a) and weekly returns in panel (b). For two-day (weekly) returns we set k=100 (k=50). Reported standard errors (s.e.) are generated by the bootstrap procedure described in Section 5. Scale parameters are calculated from equation (17). For the two-day estimates in panel (a) we provide t-statistics for testing against the null hypothesis of equal shape parameters in the daily return and two-day return data, see equation (20). For the weekly estimates in panel (b) we provide t-statistics for testing the null hypothesis of equal shape parameters in the two-day return and weekly return data. Significance at the 10%, 5% and 1% level is denoted by respectively *, ** and ***.

Table 4: Risk estimates

Panel (a): Left tail

Commodity	Probability level: 0.10%				Probability level: 0.01%			
	VaR	Min.	Max.	ES	VaR	Min.	Max.	ES
Corn	6.65	5.83	7.48	8.75	11.56	9.14	13.99	15.22
Cotton	6.22	5.68	6.76	7.87	10.07	8.44	11.70	12.73
Oats	7.35	6.66	8.03	9.36	12.05	10.13	13.97	15.35
Soybeans	6.44	5.75	7.13	8.29	10.77	8.91	12.63	13.86
Wheat	7.65	6.68	8.63	10.59	14.50	11.49	17.52	20.08
Lean hogs	5.08	4.52	5.64	5.98	7.19	5.83	8.55	8.47
Live cattle	3.12	2.80	3.44	3.55	4.13	3.34	4.92	4.70
Orange juice	7.77	6.98	8.57	10.09	13.19	10.88	15.50	17.12
Sugar	9.78	8.75	10.81	12.51	16.17	13.28	19.06	20.68

Panel (b): Right tail

Commodity	Probability level: 0.10%				Probability level: 0.01%			
	VaR	Min.	Max.	ES	VaR	Min.	Max.	ES
Corn	6.96	6.26	7.67	9.07	11.89	9.83	13.94	15.48
Cotton	6.39	5.71	7.06	8.15	10.50	8.58	12.43	13.39
Oats	7.86	7.02	8.70	10.32	13.60	11.18	16.02	17.85
Soybeans	6.13	5.56	6.70	7.79	10.02	8.37	11.68	12.75
Wheat	7.93	6.94	8.92	10.71	14.43	11.35	17.50	19.50
Lean hogs	5.27	4.70	5.83	6.39	7.91	6.52	9.29	9.60
Live cattle	3.11	2.94	3.28	3.57	4.17	3.74	4.61	4.78
Orange juice	8.79	7.69	9.88	12.14	16.59	13.12	20.07	22.92
Sugar	10.32	9.03	11.60	13.57	17.92	14.22	21.62	23.58

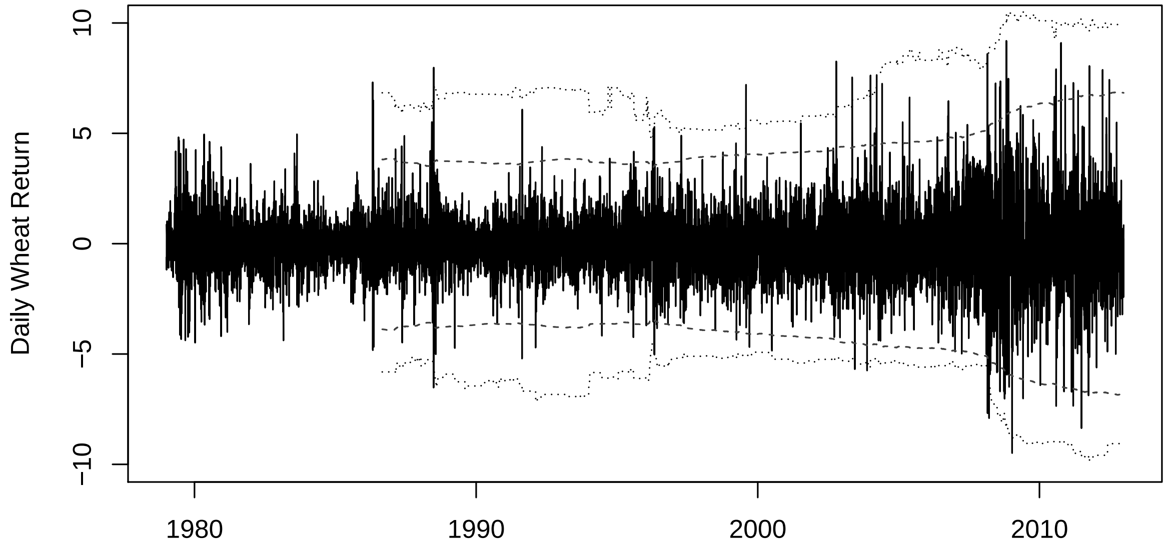
Note: Value-at-Risk (VaR) estimates for the left tail in panel (a) and the right tail in panel (b) are calculated from equation (18) for the 0.10% and the 0.01% probability level. We also provide the 95% confidence bands (Min.; Max.) of the VaR estimates. The Expected Shortfall (ES) estimates are calculated from equation (19).

tail are employed. The outcome of equation (1) gives the probability of insolvency: $\Pr(\tilde{x} > u) \approx 1.52 \times 15^{-3.60} \approx 8.87 \times 10^{-5}$. The inverse of this number yields the number of days in which at least one price fall of more than 15% is expected to occur. The outcome is around 11,750 weekdays. Hence, with approximately 261 weekdays per year, we expect to see such a large fall in the price of wheat once every 45 years.

6.1 Back-testing

To examine the accuracy of the $VaR(p)$ estimates, we employ an out-of-sample back-testing procedure. In this method, the $VaR(p)$ estimates based on historical price changes are compared to the realized price changes. Thus, first $VaR_t(p)$ is estimated using a horizon of m preceding returns: $\{R_{t-m}, ..., R_{t-1}\}$. If the realized return R_t exceeds the estimated $VaR_t(p)$, then a VaR-violation is registered. The above procedure is repeated

Figure 3: Backtesting agricultural Value-at-Risk.



The spikes show the daily wheat returns. The dotted (dashed) line shows the 0.1% VaR estimates according to the power law tail (normal) distribution. The risk measure is estimated from the preceding 2,000 daily returns. From the figure it is clear that the VaR estimate from the normal distribution is exceeded at a frequency higher than 0.001.

at time $t + 1$ et cetera.

For a dataset containing n returns, the procedure is repeated $n-m$ times. According to the Value-at-Risk definition, if the $VaR(p)$ estimate is of high quality, then the proportion of VaR violations should have a value close to p. Thus, if the underlying distribution does not change over time, $\frac{1}{n-m}\sum_{t=m+1}^{n} \mathbf{1}(R_t > VaR_t(p)) = p$ holds approximately for accurate VaR estimates, where $\mathbf{1}(\cdot)$ denotes the indicator function.

The procedure is implemented as follows. The back-testing procedure is performed both under the assumption of power law tails and under the assumption of the more conventional normal distribution. The estimation 'horizon' m is set at 2,000 daily observations. Each tail is assumed to contain the most extreme 2.0% of all observations.

A visual representation of the procedure's results is given in Figure 3. The spikes show daily wheat returns. The dotted and dashed lines show the 0.1% VaR estimates from the power law tail and the normal distribution, based on the preceding 2,000 daily returns. The test procedure boils down to counting the number of spikes that exceed either the dotted or the dashed line.

The results of the test procedure are summarized in Table 5. For each commodity, the number of VaR violations is provided for three different VaR probabilities. The first and fourth columns in Table 5 show that the 1% VaR estimates from both distributions are violated too often. In other words, the 1% VaR estimates are too low for both distributions, although the error is markedly smaller for the power law tail VaR estimates.

The predictions from the normal distribution underestimate the risk when one moves further into the tail. The third and sixth columns in Table 5 show that the number of

Table 5: Back-testing agricultural Value-at-Risk.

Commodity	Left tail						Right tail					
Probability	1.00%		0.10%		0.05%		1.00%		0.10%		0.05%	
	EVT	Nrm	EVT	Nrm	EVT	Nrm	EVT	Nrm	EVT	Nrm	EVT	Nrm
Corn	130	184	3	68	2	49	130	202	4	78	1	61
Cotton	126	170	10	33	5	23	104	166	9	52	3	38
Oats	102	147	7	43	3	27	91	152	5	50	2	40
Soybeans	98	163	4	66	0	52	85	139	3	56	0	43
Wheat	108	131	8	49	2	36	105	161	8	61	3	45
Lean hogs	93	130	10	26	5	16	86	107	6	28	2	22
Live cattle	82	111	4	26	3	14	86	108	9	20	3	12
Orange juice	102	186	5	66	2	52	103	174	9	68	3	49
Sugar	73	115	4	42	2	33	70	121	3	28	0	20
Average	101.6	148.6	6.1	46.6	2.7	33.6	95.6	147.8	6.2	49	1.9	36.7
Expected	68.7	68.7	6.9	6.9	3.4	3.4	68.7	68.7	6.9	6.9	3.4	3.4

Note: The table reports the number of VaR-violations in a back-testing procedure. The columns report the number of VaR-violations for different VaR levels. The two bottom lines report respectively the expected number of VaR-violations and the average number of VaR-violations. Although the normal distribution (Nrm) and the power law tail distribution (EVT) both seem to underestimate the 1% VaR, the power law tail distribution turns out to be quite accurate for more extreme events.

0.05% VaR violations is approximately 10 times too high under the assumption of the normal distribution. However, the power law tail estimates for VaR at smaller probabilities (e.g., 0.1% and 0.05%) turn out to be quite accurate. The 0.05% VaR for the left (right) tail is, on average, exceeded by 2.7 (1.9) observations per commodity, which is not far from the expected number of violations in case of a perfectly accurate VaR prediction (i.e., 3.4). Those estimates concern the very extreme events (about 1 observation in, respectively, 4 and 8 years), which are most important from a farmer's risk management perspective.

7 Conclusion

A good understanding of extreme commodity returns is instrumental in any commodity risk management application. Especially knowledge regarding large price swings is most relevant for risk management purposes. We construct a two-sector general equilibrium model which describes how productivity shocks affect agricultural commodity prices. In our model, extreme price spikes arise endogenously as a result of productivity shocks in the agricultural sector, which results in a heavy-tailed equilibrium price distribution.

The economic literature on real business cycles reasons that productivity shocks are a dominant source of fluctuations in economic aggregates. Agricultural producers experience a relative large amount of those shocks through their exposure to weather conditions and other natural forces. Prior studies show that those shocks have a relatively large impact on agricultural commodity price behavior, see e.g., Deaton and Laroque (1992, 1996), Ai et al. (2006) and Boudoukh et al. (2007). Our study shows why such productivity shocks may result in heavy-tailed price distributions, even if they are not heavy-tailed themselves.

We build on prior work to provide further empirical evidence that agricultural commodity price returns are heavy-tailed. We use Extreme Value Theory to estimate the parameters or the power law in the tail of their distribution. These estimates are used to measure an agricultural producer's extreme price risks. We calculate Value-at-Risk and Expected Shortfall measures to provide estimates of the likelihood and size of the largest losses a farmer may encounter. Back-testing shows that this methodology is superior to risk measures based on the conventional normal distribution assumption.

A Appendix. Derivation of equilibrium prices

The first order conditions for optimality entail

$$(1-\theta)\, Z^{-\theta} n^{-\theta/\rho} \left[\sum_{i=1}^{n} Q_i^{\rho}\right]^{\theta/\rho} - \lambda q = 0,$$

$$\theta \left(\frac{Z}{\left[\sum_{i=1}^{n} Q_i^{\rho}\right]^{1/\rho}}\right)^{1-\theta} n^{-\theta/\rho} \left[\sum_{i=1}^{n} Q_i^{\rho}\right]^{\frac{1}{\rho}-1} Q_j^{\rho-1} - \lambda \frac{1}{n} p_j = 0,$$

$$-L^{\gamma} + \lambda w = 0,$$

and

$$wL + \Pi(Q) = qZ + \frac{1}{n}\sum_{i=1}^{n} p_i Q_i.$$

The first order conditions imply the familiar price and wage ratios

$$\frac{p_i}{p_j} = \frac{Q_i^{\rho-1}}{Q_j^{\rho-1}},$$

$$\frac{p_j}{q} = \frac{\theta}{1-\theta}\frac{Z}{Q_j}\frac{p_j^{\rho/(\rho-1)}}{\frac{1}{n}\sum_{i=1}^{n} p_i^{\rho/(\rho-1)}},$$

and

$$\frac{w}{q} = (qP)^{\theta}\,\frac{L^{\gamma}}{(1-\theta)^{1-\theta}\,\theta^{\theta}},$$

where the price index for differentiated goods is defined as in equation (6).

Then the labor supply can be written as

$$L = \left((1-\theta)^{1-\theta} \theta^{\theta} \frac{w}{q^{1-\theta} P^{\theta}} \right)^{1/\gamma}. \tag{21}$$

The competitive goods demanded can be expressed as

$$Z = (1-\theta) \frac{wL + \Pi(Q)}{q}. \tag{22}$$

The differentiated goods demanded can be expressed as

$$Q_i = \theta \frac{wL + \Pi(Q)}{p_i} \left(\frac{p_i}{P} \right)^{\rho/(\rho-1)}. \tag{23}$$

A.1 Supply

From the perfectly competitive agricultural market we have that

$$\Pi(Z) = qZ - wN = \left(q - \frac{w}{B} \right) Z = 0,$$

so that

$$q = w/B. \tag{24}$$

The differentiated goods profit function reads

$$\begin{aligned} \Pi(Q_i) &= p_i Q_i - wN_i = \left(p_i - \frac{w}{A} \right) Q_i \\ &= \left(p_i - \frac{w}{A} \right) \theta \frac{wL + \Pi(Q)}{p_i} \left(\frac{p_i}{P} \right)^{\rho/(\rho-1)}. \end{aligned}$$

The producer exploits his pricing power, but ignores his pricing effect on the price index P of the differentiated goods and the consumer income $wL + \Pi(Q)$.[16] Differentiation gives

$$\frac{\partial \Pi(Q_i)}{\partial p_i} = \frac{1}{\rho - 1} Q_i \left\{ \rho - \frac{1}{A} \frac{w}{p_i} \right\}.$$

Exploiting the pricing power therefore implies setting prices

$$p_i = \frac{w}{\rho A}. \tag{25}$$

[16] One can easily incorporate this effect as well, if desired. For two reasons we do not follow this route. One may doubt that producers take this macro effect of their pricing behavior into account. Moreover, it adds little to the insights derived form specifying the differentiated goods sector.

Hence, $P = w/\rho A$ as all prices are identical. Total profits in the differentiated goods sector equal

$$\begin{aligned}\Pi(Q) &= \frac{1}{n}\sum_{i=1}^{n}\Pi(Q_i) = \sum_{i=1}^{n}\left(1-\frac{w/p_i}{A}\right)\theta\left[wL+\Pi(Q)\right]\left(\frac{p_i}{P}\right)^{\rho/(\rho-1)} \\ &= (1-\rho)\,\theta\left[wL+\Pi(Q)\right].\end{aligned}$$

Solve for the total sectorial profits as

$$\Pi(Q) = \frac{(1-\rho)\,\theta}{1-(1-\rho)\,\theta}wL. \tag{26}$$

A.2 Equilibrium

It follows in equilibrium, after substituting the price levels into the labor supply equation (21), that

$$L = \left(\theta^{\theta}\,(1-\theta)^{1-\theta}\,A^{\theta}B^{1-\theta}\right)^{1/\gamma}\rho^{\theta/\gamma} = \varphi\rho^{\theta/\gamma}, \tag{27}$$

say, and where

$$\varphi = \left(\theta^{\theta}\,(1-\theta)^{1-\theta}\,A^{\theta}B^{1-\theta}\right)^{1/\gamma}.$$

Furthermore, from (22), (26) and (27)

$$Z = (1-\theta)\,\frac{B}{1-(1-\rho)\,\theta}\varphi\rho^{\theta/\gamma}. \tag{28}$$

Similarly, using (23), (26) and (27)

$$Q_j = \theta\frac{A}{1-(1-\rho)\,\theta}\rho\varphi\rho^{\theta/\gamma}.$$

Hence,

$$\frac{1}{n}\sum_{j=1}^{n}Q_j = \theta\frac{A}{1-(1-\rho)\,\theta}\varphi\rho^{\theta/\gamma+1}. \tag{29}$$

With the above preparations, we now derive the implications for the equilibrium prices. From (24), combined with (7) and (27), we obtain

$$q = \frac{w}{B} = \frac{M}{B}\frac{1}{L} = M\frac{1/\rho^{\theta/\gamma}}{B\left(\theta^{\theta}\,(1-\theta)^{1-\theta}\,A^{\theta}B^{1-\theta}\right)^{1/\gamma}}.$$

Similarly, using (25) combined with (7) and (27) yields

$$p_i = p = \frac{w}{\rho A} = \frac{M}{\rho A}\frac{1}{L} = M\frac{1/\rho^{\theta/\gamma+1}}{A\left(\theta^{\theta}\left(1-\theta\right)^{1-\theta}A^{\theta}B^{1-\theta}\right)^{1/\gamma}}.$$

B Appendix. Proof of Proposition 2

Given (10), we want to find the condition on the density for B such that probability distribution of the price $\tilde{q}$ follows a heavy-tailed distribution. We have that $\Pr(\tilde{q} > u) \sim \mathcal{L}(u)u^{-\alpha}$ as $u \to \infty$ if $\tilde{q}$ is regularly α-varying at infinity with $0 < \alpha < \infty$, i.e., if

$$\lim_{t\to\infty} \frac{1 - F_q(tu)}{1 - F_q(t)} = u^{-\alpha} \text{ with } \alpha \in \mathbb{R}^+, \tag{30}$$

where F_q denotes the cumulative distribution function of $\tilde{q}$, see also De Haan (1970). We thus need to find the condition such that F_q it is regularly varying at infinity. Rewriting (30) with L'Hôpital's Rule gives the condition

$$\lim_{t\to\infty} \frac{uf_q(tu)}{f_q(t)} = u^{-\alpha} \text{ with } \alpha \in \mathbb{R}^+, \tag{31}$$

where f_q denotes the density of $\tilde{q}$. Given equation (10), we have that the equilibrium price $q(B)$ is a strictly decreasing function of B for $\theta \in (0, 1)$. Therefore, by a transformation of variable we have that

$$f_q(\tilde{q}) = \left|\frac{dB(\tilde{q})}{d\tilde{q}}\right| f_B(B(\tilde{q})), \tag{32}$$

where $B(\tilde{q})$ denotes the inverse of $\tilde{q}(B)$. With the inverse of equation (10) and the derivative of the inverse of equation (10) this gives

$$f_q(\tilde{q}) = \frac{1}{\eta}\Theta^{1/\eta}\tilde{q}^{-(1/\eta+1)}f_B(\Theta^{1/\eta}\tilde{q}^{-1/\eta}), \tag{33}$$

where

$$\eta = \frac{1+\gamma-\theta}{\gamma}.$$

Hence, from (31) we seek

$$\lim_{t\to\infty} \frac{u\frac{1}{\eta}\Theta^{1/\eta}(tu)^{-(1/\eta+1)} f_B(\Theta^{1/\eta}(tu)^{-1/\eta})}{\frac{1}{\eta}\Theta^{1/\eta}(t)^{-(1/\eta+1)} f_B(\Theta^{1/\eta}(t)^{-1/\eta})} = u^{-\alpha} \text{ with } \alpha \in \mathbb{R}^+;$$

$$\lim_{t\to\infty} \frac{u^{-1/\eta} f_B(\Theta^{1/\eta} t^{-1/\eta} u^{-1/\eta})}{f_B(\Theta^{1/\eta} t^{-1/\eta})} = u^{-\alpha} \text{ with } \alpha \in \mathbb{R}^+;$$

$$\lim_{s\downarrow 0} \frac{w f_B(sw)}{f_B(s)} = w^{\eta\alpha} \text{ with } \alpha \in \mathbb{R}^+. \tag{34}$$

where $w = u^{-1/\eta}$ and $s = \Theta^{1/\eta} t^{-1/\eta}$. Hence, given (10), if the condition in (34) holds for the density function of B, we have that $\Pr(\tilde{q} > u) \sim \mathcal{L}(u)u^{-\alpha}$ as $u \to \infty$. Proposition 2 is then obtained by writing $\xi = \eta\alpha$ in the condition in (34) and using $\eta > 0$.

C Appendix. Commodity selection

In this appendix we explain how the commodities for this research are selected. The employed futures series need to satisfy two conditions: Availability and relevance from the perspective of a US farmer. Initially, our sample contains all traded commodity futures within the US. A list of 19 commodities remains after removing non-agricultural and identical commodities. Four commodities from this list are removed because of data availability (butter, milk, dry whey and rice are only available from 1996 onwards or even later). Next, six of the remaining fifteen series are removed because of low relevance. Because soybeans is included, soy meal and soybean oil are removed. Because live cattle is included, cattle feeder is removed. Because lean hogs is included, frozen pork bellies is dropped. Cocoa and coffee are removed because of relatively low relevance for US farmers. This leaves us with corn (CC.), wheat (CW.), oats (CO.), soybeans (CS.), live cattle (CLC), lean hogs (CLH), cotton (NCT), sugar (NSB) and orange juice (NJO).[17]

[17]Datastream codes between brackets.

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