Bank Risk within and across Equilibria
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Itai Agur *

* Views expressed are those of the author and do not necessarily reflect official positions of De Nederlandsche Bank.
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Abstract

This paper models a financial sector in which there is a feedback between individual bank risk and aggregate funding market problems. Greater individual risk taking worsens adverse selection problems on the market. But adverse selection premia on that market push up bank risk taking, leading to multiple equilibria. The model identifies shifts among equilibria as a function of parameter shocks. Measures that reduce individual bank default risk within an equilibrium can actually make the system as whole more sensitive to shocks. Risks may thus seem small and market risk premia low precisely when the system as whole is most fragile.

Keywords: Bank risk, Wholesale funding, Adverse selection, Financial crisis, Liquidity

JEL Classification: G01, G21.

*Email: i.agur@dnb.nl. Phone: +31 20 5245701. Postal address: De Nederlandsche Bank, PO Box 98, 1000 AB Amsterdam, The Netherlands. I am grateful to Enrico Perotti, Charles Calomiris, Eric van Wincoop, Viral Acharya, Xavier Freixas, Luc Laeven, Enrique Mendoza, Chen Zhou, Maria Demertzis, Razvan Vlahu, Christian Castro, Roland Meeks and Iman van Lelyveld and to audiences at the University of Amsterdam, the Dutch Central Bank and the Bank of England for comments.
1 Introduction

The market for subprime mortgages formed only a fraction of banks’ total exposures. Yet, its meltdown brought the entire sector to its knees and led, in particular, to a protracted freeze on banks’ funding market. How can a financial system seem so stable, with low default rates and low funding costs on the market before the crisis, and simultaneously be so vulnerable to shocks? This paper provides a model to differentiate between the concepts of risk within an equilibrium (individual bank default risk) and across equilibria (the likelihood that a shock leads to market breakdown). It shows that risk within and across equilibria can move in opposite directions and that it is possible for default rates to be lowest when the system as a whole is most sensitive to shocks.

The mechanism is based on a feedback loop between an individual bank’s risk taking and the extent of asymmetric information on the wholesale market for unsecured bank funding. That market is plagued by adverse selection as both banks that have genuine liquidity shortfalls and those that are insolvent but want to gamble for resurrection demand funds. Financiers do not know the soundness of a given bank, but do know the probability that they face a sound bank. The greater is banks’ risk taking ex-ante, the larger the share of unsound banks on the market ex-post. If banks could cooperate they would limit their risk taking behavior in order to subsequently face lower adverse selection premia. However, in an uncooperative game there is a free-rider problem as each individual bank does not internalize how its risk taking affects aggregate premia, and therefore takes too much risk. Moral hazard in our model thus arises through cross-sectional externalities, rather than through externalities to society.

The feedback mechanism arises as follows. Firstly, when banks have greater incentives to take risk, adverse selection premia on their funding market rise. Secondly, the higher are banks’ funding costs, the greater banks’ incentives to take risk. This comes about through a charter value effect: when funding is more expensive, banks’ equity is worth less, and they are more inclined to play risky strategies that pay off much if successful, but externalize most losses to creditors if unsuccessful. Both of these channels are quite well established in the
The idea that bank risk feeds into risk premia on the unsecured funding market is modelled by Freixas, Parigi and Rochet (2004) and Heider, Hoerova and Holthausen (2009). And the effect of charter values on bank risk has been a common feature in the banking literature since Keeley (1990). It is the combination of the channels that is new to the paper, and that sparks non-linearity in the model. An example of the game dynamics is given in figure 1.

Figure 1: A shift to breakdown

In this figure the solid line is the reaction function of a bank’s monitoring effort, \( q_i \), at given monitoring effort of other banks, \( q \). The effort to monitor borrowers directly determines the bank’s default probability in the model and is, therefore, the inverse of bank risk taking. The reaction function slopes upward because greater monitoring effort of other banks lowers market risk premia, thereby raising charter values and each individual bank’s monitoring incentives. That is, banks’ optimal monitoring levels are complementary. An interior Nash equilibrium occurs where the reaction function crosses the 45 degree line, as depicted in the left panel of the figure. At this point no bank deviates, given the play of other banks. Outside of the interior equilibrium the arrows portray the direction of play, which evolves towards one of two corner equilibria, one at maximal monitoring and one at zero monitoring. The latter lies within the shaded area, which is the breakdown zone, where adverse selection problems are so severe that the funding market freezes.

The difference between the left and the right panels of figure 1 captures our main story.
Initially, the financial sector can be at one of the "good" equilibria in the left panel, namely the interior equilibrium or the top-right corner, where individual bank default risk is small or zero. However, it does not take a large parameter shock - such as a decline in asset returns - to push the reaction function beyond the 45 degree line, where the breakdown equilibrium is unique. This is shown in the right panel.

Figure 1 helps to highlight the differences between the "traditional" modelling of multiple equilibria in banking since Diamond and Dybvig (1983) and our work. Firstly, shifts between equilibria can be related to parameters, rather than sunspots, which makes it possible to discuss the size of a shock needed to induce an equilibrium shift. The only other paper we know of that relates shifts between banking equilibria to parameter shocks is Goldstein and Pauzner (2005). For a given set of fundamentals they use a global games technique to solve for a unique equilibrium. If parameters are drawn from known distributions, the model can be used to compute the ex-ante probability of a panic-based bank run. Similarly, for such distributions our model would yield the likelihood of a funding market freeze.

Secondly, our model distinguishes between stable and unstable equilibria. Stable equilibria occur when the arrows outside an equilibrium point back towards that equilibrium (a possibility not shown in figure 1). In figure 1 the unique interior equilibrium is unstable, which means that any small perturbation in banks’ expectations about each other’s play leads to an equilibrium shift. This is, in some sense, a "smaller" psychological shock than the traditional sunspot that leads agents to immediately re-coordinate on a different equilibrium (Benhabib and Farmer, 1999). Alternatively, one could say that the arrows show how a sunspot shift "arises" from small perturbations. This relates the paper to the literature on complex systems. Though a general definition of what constitutes complexity is elusive (Rosser, 1999), two elements that are commonly associated with it are extreme sensitivity to parameters and unstable equilibria (Rosser, 1999, p. 173), both of which are present in our model.

In terms of bank regulation, our model indicates that measures geared at limiting risk

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1 Instead of a parameter shock, it also possible for an institutional change, such as international financial integration, to trigger an equilibrium shift. See Freixas and Holthausen (2005) and Boissay (2010).
within a given equilibrium, may not succeed at increasing the resilience of the system as a whole. This is related, yet quite distinct, to the argument that capital requirements can backfire, because they reduce bank charter values, thereby making banks take more risk instead of less (Flannery, 1989; Blum, 1999; Hellmann, Murdock and Stiglitz, 2000). Our point rather focuses on the existence of multiple margins by which to measure a policy’s "success". Intra and inter equilibrium risk can move in opposite directions, so that even if capital requirements do work in the desired direction on individual default risk, they can fail to safeguard the system. In our model the best regulatory policies, in the sense that they unambiguously improve systemic resilience, are to be found in the realm of counterparty transparency. If financiers have a clearer view of which banks are solid, adverse selection problems ease. Ways to achieve this could be by greater regulatory transparency on bank data, investing in high quality stress tests, or prompt closure of banks that the regulator knows are insolvent.

The next section briefly discusses the institutional setup of the wholesale funding market and the related literature on it. Section 3 presents the model. Section 4 derives the equilibria, on which section 5 performs comparative statics. Section 6 discusses issues related to the anticipation and insurability of shocks. Finally, section 7 considers the policy implications.

2 The wholesale funding market

On the market for wholesale bank funding a key distinction is that between the unsecured and the secured (collateralized) market. In the eurozone prior to the crisis the unsecured market was about twice the size of the secured market, although since then secured funding has been on the rise (Alloway, 2011). Both forms of bank funding experienced a freeze during the crisis (Heider, Hoerova and Holthausen, 2009; Allen, Carletti and Gale, 2009), with lenders withdrawing first briefly in the summer of 2007 and subsequently for a longer time after the collapse of Lehman Brothers in 2008. Another distinction is between the type of financier: banks versus non-banks. There is the interbank market in which banks lend to each other, but there is also a wider wholesale market in which non-banks, such as corporations and
institutional investors, act as lenders. We do not know of a study stating the relative shares of bank and non-bank lenders worldwide, but according to De Haan and Van den End (2011) they are roughly of equal size in the Dutch wholesale funding market.

Our model focusses on the unsecured segment of the wholesale market, and is closely related to that of Freixas, Parigi and Rochet (2004). They show that because adverse selection arises when insolvency and illiquidity are indistinguishable to financiers, Lender of Last Resort intervention can be efficient. The reason is that the authority can lend at a penalty rate that discourages insolvent banks from pretending they are only illiquid. The difference to our paper is that bank solvency is exogenous in their model. Hence credit risk premia do not feed back into the decisions of banks, and the type of effects we analyze do not arise in their work.

We, like Freixas, Parigi and Rochet, model only the endogeneity of borrowers on the wholesale market. The financiers are exogenous and provide an infinitely elastic supply of funds at the zero expected profit rate. Instead, Heider, Hoerova and Holthausen (2009) model an interbank market in which not only borrowing but also lending by banks is endogenous. They identify three regimes: one with low interbank rates and full participation; another where adverse selection pushes up rates and the best quality borrowers leave; and the last regime involves liquidity hoarding and market breakdown.

Adverse selection is not the only reason that an unsecured interbank market could break down, however. For instance, in Acharya, Gromb and Yorulmazer (2008) it is the monopoly power of liquidity surplus banks that makes them underprovide lending when it is most needed, in order to induce borrowing banks to sell their assets to them at depressed prices. Freezes on the secured segment of the market are modelled by Allen, Carletti and Gale (2009), whose mechanism is based on incomplete markets and the implied uninsurability of aggregate shocks. Another approach to the interbank market is through network models (Van Lelyveld and Liedorp, 2006; Nier et al., 2008). These focus not on market freezes but rather on contagion within the system when one bank fails.

3 Model

We assume a mass of identical, risk-neutral banks, indexed by $i \in [0,1]$, whose asset side initially consists of one project. That project yields a total return $R > 1$ if it succeeds. Whether the project succeeds depends on its funding and the behavior of its entrepreneur. Firstly, the project needs a certain amount of funding to be of any value. One can think of this as fixed development costs on the part of the project’s entrepreneur, who can only make his business profitable if he receives a minimum amount of initial capital, which is 1. If the bank provides the project with funding less than this amount then the best the entrepreneur can do is invest the principal in a risk-free technology which yields zero net return.

Secondly, the bank faces the risk that the entrepreneur runs off with the money without developing the project at all. That is, the entrepreneur shirks. To prevent this from happening the bank must exert monitoring effort, $q_i \in [0,1]$, with associated non-pecuniary cost $c q_i^2$, $c \geq 0$ experienced by bank management. We assume that the probability of making the entrepreneur work on the project is directly proportional to this monitoring effort. In particular, we say that $q_i$ is the probability of the bank possessing a "sound" - that is, potentially valuable - project. With probability $1 - q_i$ bank monitoring fails, the entrepreneur shirks and the project is unsound, and therefore certain to be of no value, regardless of the amount invested in it. For simplicity we will refer to sound-project banks as "sound banks" and unsound-project banks as "unsound banks". Overall, the gross payoff structure of the project looks as follows:

<table>
<thead>
<tr>
<th>Funding</th>
<th>Unsound quality</th>
<th>Sound quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funding &lt; 1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Funding = 1</td>
<td>0</td>
<td>$R$</td>
</tr>
</tbody>
</table>

The realization of project soundness is private knowledge revealed only to the bank: only the bank’s loan officers can observe whether the project’s entrepreneur has shirked or not. It is the combination of funding needs and the inability of outsiders to distinguish between banks with sound and unsound projects that forms the basis of the adverse selection problem
in our model. The idea that a sound project loses value under funding difficulties relates to the broader notion of fire sales (Shleifer and Vishny, 2011) and forced early termination (Diamond and Dybvig, 1983), which are well established in the literature. That unsound banks have an appetite for funding, in spite of the fact that their initial project is worthless, is because of the possibility to invest the funds in a risky gamble. They can "gamble for resurrection" because they have nothing to lose if the gamble fails, but everything to gain on the upside.

A Gambles

Banks have the possibility of investing an amount between 0 and 1 in a gamble after observing the soundness of their project. If successful then the gamble multiplies the amount invested in it by $x$, but if unsuccessful then the funds invested in the gamble are lost. The probability that it succeeds is $\rho$, where $0 < \rho < \frac{1}{x}$. Thus, the gamble has a negative present value: it destroys value because the expected payoff is smaller than the principal invested ($\rho x < 1$).

Naturally, creditors (whose precise role is discussed below) will take this into consideration ex-ante and no lending would ever occur to a bank that is known to gamble. That is, there exists no interest rate at which a financier is willing to lend to a gambling bank. Since a bank is protected by limited liability upon default, the most financiers could request is to be paid the entire proceeds of the gamble if it succeeds, namely $x$. But even at an interest rate $r = x$ the expected gross return on a loan to the bank is $\rho r = \rho x < 1$: the loan is loss-making in expectation. However, because of the information asymmetry, when sound banks use the borrowed funds productively then they cross-subsidize the gambles of unsound banks, and hence gambles can occur in equilibrium.
B Funding

Table 2: Timing of the game

<table>
<thead>
<tr>
<th>Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Banks set monitoring effort</td>
</tr>
<tr>
<td>2. Project soundness realizes</td>
</tr>
<tr>
<td>3. Banks decide on gambles</td>
</tr>
<tr>
<td>4. Bank funding</td>
</tr>
<tr>
<td>5. Projects and gambles pay out</td>
</tr>
</tbody>
</table>

As shown in table 2, we model banks’ funding stage as occurring after the asset side stages. This is important for our setup because banks must first be sorted into types (on project quality and gambles) before they turn to the funding market, in order for adverse selection to arise there. The need to separate over different parts of bank decisions is seen elsewhere in the literature. For instance, in Dell’Ariccia, Laeven and Marquez’s (2010) paper on the relationship between monetary policy and bank risk taking, there is a separation between the decision stages on asset risk and leverage. Similarly, in the literature on bank competition, the competition on the market for borrowers (banks’ asset side) has to precede competition on the market for depositors (banks’ liability side), otherwise equilibria may fail to exist (Stahl, 1988).

Banks enter the game with internal funds worth $e < 1$, i.e. internal equity. We assume that these internal funds have been previously committed to the bank’s project. Thus if the bank is hit by a negative solvency shock, that is, it becomes unsound, then it loses its equity value.

Since internal funds are less than 1, if banks wish to give their initial project the full funding it requires to generate return $R$, they need to obtain additional funds. We assume that they can only do this by turning to the market for wholesale debt funding. That is, we abstract from deposit-insured retail deposits as well as from the issuance of additional external equity.\(^3\)

\(^3\)Abstracting from the issuance of external equity is quite common in the literature (Thakor, 1996; Acharya, Mehran and Thakor, 2010). Essentially, this is a reduced form for asymmetric information arguments that make external equity a relatively expensive form of finance (Myers and Majluf, 1984).
Over the past decades wholesale funding has become an increasingly prevalent form of bank finance, and its importance in banks’ activities has been highlighted by the recent financial crisis (Brunnermeier, 2009; Allen, Carletti and Gale, 2009; Diamond and Rajan, 2009). We focus on this type of financing because it reacts to bank riskiness, unlike largely insured retail deposits, and because it is this wholesale market that experienced a freeze during the recent crisis, rather than insured retail deposits on which relatively few runs were seen.

The funding market is assumed to be perfectly competitive and its participants are risk neutral. Though financiers do not observe the asset structure of a given bank, they do know the parameters of the model and can compute the probability that they are facing a sound or an unsound bank. They accordingly charge banks a fair risk premium. Without loss of generality, we set the risk-free rate to zero, so that the gross market borrowing rate for banks, \( r \), is purely a reflection of banks’ default risk.

### C Bank maximization

Let us define \( d = 1 - e \) as the amount a sound bank needs to borrow so that, if invested in its project, it yields return \( R \). We assume the following parameter restriction:

\[
x > \frac{R}{d}
\]

This means that the return on a successful gamble has to be sufficiently large. When this is the case, we can show that unsound banks will always choose to gamble, and that by backward induction a bank’s stage 1 optimization problem is given by:

**Lemma 1** At stage 1 a bank maximizes to \( q_i \)

\[
E [\Pi_i] = \begin{cases} 
q_i (R - rd) + (1 - q_i) \rho d (x - r) - cq_i^2 & \text{if } R - rd \geq e + \max \{ \rho d (x - r), 0 \} \\
q_i (e) - cq_i^2 & \text{otherwise}
\end{cases}
\]

**Proof.** In the appendix. ■
Here the condition
\[ R - rd \geq e + \max \{ \rho d (x - r), 0 \} \]  

is what we term the no-breakdown condition, because if it is not satisfied then the funding market freezes. That is, as shown in the proof of Lemma 1, when this condition is not satisfied then financiers realize they face only gambling banks and provide no funds. In this case banks can do no better than to optimize over \( q_i(e) - cq_i^2 \) which is merely about monitoring the entrepreneur so as to prevent him from running off with the funds already given to him. Instead, if condition (2) holds then the bank earns \( R - rd \) if it has a sound project, while it receives an expected return from gambling of \( \rho d (x - r) \) if it has an unsound project. Note that in condition (2) we have made the tie-breaking assumption that if banks are indifferent between alternatives then they choose to invest in their project (hence the weak inequality).

D Breakdown threshold

Definition 1 Funding market breakdown occurs when there exists no interest rate at which financiers are willing to lend to banks.

We can provide a full analytical solution for the no-breakdown condition in (2) if we replace for the endogenous interest rate, \( r \). But the existence of that interest rate depends on whether or not breakdown occurs. We solve this with the following procedure:

Step 1 Derive the interest rate assuming that the no-breakdown condition holds.

Step 2 Solve for the no-breakdown condition and the equilibrium given that interest rate.

Step 3 Verify if the no-breakdown condition indeed holds.

Assuming that the no-breakdown condition holds, the fair market rate for bank funding is given by
\[ r = \frac{1}{q^* + (1 - q^*) \rho} \]
where $q^*$ is the equilibrium monitoring effort of banks. In equilibrium, with probability $q^*$ financiers get their money back for sure, while with probability $(1 - q^*)$ they face a gambling bank and get their money back only with probability $\rho$.

**Lemma 2** Given $r$ as in (3) the no-breakdown condition (2) can be written to:

$$q^* > \tilde{q} = \max \left\{ \frac{1 - \rho x}{x(1 - \rho)}, \frac{d}{R - 1 + d(1 - \rho x)} - \frac{\rho}{1 - \rho} \right\} > 0$$

(4)

**Proof.** In the appendix. ■

In the form of (4) the no-breakdown condition says that the equilibrium monitoring effort of banks, $q^*$, must be above a certain threshold, $\tilde{q}$. If it falls below that threshold then adverse selection problems become too severe and the funding market freezes. The determinants of that threshold are quite intuitive. For instance, the threshold falls when asset returns ($R$) increase or leverage ($d$) decreases, because this raises the attractiveness of the sound project as compared to a gamble. However, the gambling parameters $x$ and $\rho$ have ambiguous effects on the breakdown threshold. For instance, a higher $\rho$ raises banks’ charter values (ceteris paribus $r$ declines in (3)), which reduces gambling incentives, but also increases the expected return on gambling relative to sound investments (the right-hand side of equation (2)), which works in the opposite direction.

**E Reaction function**

If the no-breakdown condition holds, then by Lemma 1 a bank solves

$$\max_{q_i} \left\{ q_i (R - rd) + (1 - q_i) \rho d (x - r) - cq_i^2 \right\}$$

(5)

The solution to this, $q_i^*$, is

$$q_i^* = \frac{R - d [r (1 - \rho) + \rho x]}{2c}$$

(6)
and replacing for $r$ from equation (3) this becomes

$$q_i^* = \frac{1}{2c} \left[ R - d \left( \frac{1 - \rho}{q^* (1 - \rho) + \rho x} \right) \right]$$ (7)

Equation (7) is a bank’s reaction function, reflecting how at given behavior of other banks, $q^*$, an individual bank best responds. When asset returns $(R)$ increase, then there is a higher return on monitoring, since a successful project is worth more. The opposite is true for higher leverage $(d)$, while for a larger $c$ monitoring is more expensive and so banks do less of it. Only the probability of success of the gamble $(\rho)$ enters ambiguously, with the same type of countervailing effects discussed before.

Lemma 3 A bank’s optimal monitoring effort is increasing and concave in other banks’ monitoring effort: $\frac{\partial q_i^*}{\partial q^*} > 0$ and $\frac{\partial^2 q_i^*}{\partial (q^*)^2} < 0$.

Proof.

$$\frac{\partial q_i^*}{\partial q^*} = \frac{1}{2c} \frac{(1 - \rho)^2 d}{[q^* (1 - \rho) + \rho]^2} > 0$$ (8)

and

$$\frac{\partial^2 q_i^*}{\partial (q^*)^2} = -\frac{1}{c} \frac{(1 - \rho)^3 d}{[q^* (1 - \rho) + \rho]^3} < 0$$ (9)

Intuitively, this means that when other banks monitor more a given bank has greater incentive to monitor: there is a cross-sectional complementarity in monitoring effort. The reason is that when $q^*$ increases, the funding rate, $r$, falls which raises a given bank’s charter value. That is, the profits from a fully funded sound project $(R - rd)$ increase, which strengthens banks’ incentives to make sure that their projects are sound by monitoring the entrepreneurs more intensely.

Moreover, the marginal effect on the funding rate is strongest when monitoring effort is low. For higher levels of monitoring the impact on $r$ flattens out. This what explains the concavity of $q_i^*$ in $q^*$.
4 Equilibria

Definition 2 An equilibrium occurs when \( q_i^* = q^* \): a bank’s reaction function is such that for a given monitoring effort of other banks, \( q^* \), its optimal monitoring effort, \( q_i^* \), is identical.

A one-shot, non-cooperative Nash equilibrium is defined by the absence of an incentive to deviate by any of the players. This can only occur at a point where, when other banks invest \( q^* \) in monitoring, a bank’s individually optimal monitoring is the same (interior equilibrium) or cannot be changed farther in the desired direction (corner solution).

Proposition 1 There are five possible constellations of Nash equilibria:

<table>
<thead>
<tr>
<th>I. One equilibrium: breakdown</th>
</tr>
</thead>
<tbody>
<tr>
<td>II. Two equilibria: one interior, one breakdown</td>
</tr>
<tr>
<td>III. Two equilibria: one high-end corner, one breakdown</td>
</tr>
<tr>
<td>IV. Three equilibria: two interior, one breakdown</td>
</tr>
<tr>
<td>V. Three equilibria: one interior, one high-end corner, one breakdown</td>
</tr>
</tbody>
</table>

Proof. In the appendix. ■

We give the intuitions graphically in the text. In the proof it is shown that at \( q^* = 0 \) it always holds that \( q_i^* < 0 \). Graphically this means that, when plotting a bank’s optimal monitoring (\( q_i^* \)) against \( q^* \), a bank’s reaction function always starts below the 45 degree line. Moreover, by Lemma 3 we know that the reaction function is increasing and concave. One
possibility is that the reaction function is as in figure 2:

Figure 2: Only breakdown

Here the dotted line is the 45 degree line, while the solid line is an individual bank’s reaction function. An interior equilibrium can only occur at a point where the reaction function crosses the 45 degree line. In this figure that does not happen. That is, within the entire domain of $q^* \in [0, 1]$ there is no crossing point. We can now show why breakdown is the only outcome. The black arrows on the reaction function give the direction in which the game evolves. For any given play of other banks, an individual bank will set a lower monitoring effort than they do. The fact that the reaction function is below the 45 degree line is the visual representation of this: it means that at any given $q^*$ a bank’s optimal monitoring $q^*_i$ is smaller than $q^*$. Since all banks are identical, all lower the monitoring effort until the point where breakdown occurs: $q^* < \hat{q}$. This is the shaded area in the figure. Here adverse selection problems are too severe for the funding market to operate, and no lending takes place. Banks then only optimize using their initial internal funds, as shown in Lemma 1. Thus figure 2 represents case I in the
If the bank’s reaction function does cross the 45 degree line, then it can do so once, as in figure 3, or twice, as in figure 4. In figure 3 there is a single interior equilibrium. This equilibrium is unstable, however: the arrows point outwards. Any perturbation in banks’ expectations about each other, and dynamics move out of the equilibrium. The reason is that to the right of the equilibrium the reaction function is above the 45 degree line, which means that an individual bank sets higher monitoring effort than other banks, for any given effort of other banks, and hence the only outcome is the high-end corner with maximum monitoring ($q^* = 1$). And to the left of the interior equilibrium, the reaction function is below the 45 degree line, so that dynamics are as in figure 2, and breakdown ensues. Hence, figure 3 depicts case V in the proposition.
Instead, if the reaction function crosses the 45 degree line twice, then there is also a stable interior equilibrium. This is shown in figure 4, where the arrows outside of the rightmost equilibrium point back towards that equilibrium. Breakdown remains feasible, however. If banks’ initial belief is that other banks set monitoring effort below the level of the leftmost interior equilibrium, then the arrows point towards the shaded area, and adverse selection freezes the funding market. Figure 4 is case IV in the proposition.

In figures 2-4 the breakdown threshold, \( \hat{q} \), is depicted as being relatively low. When that threshold is higher then it pushes some of the interior equilibria out of existence. This is depicted in figures 5 and 6, which are just figures 3 and 4 with a higher breakdown threshold. These represent cases III and II in the proposition, respectively. Even though in both of these figures the only non-breakdown equilibrium is stable, this does not eradicate breakdown, because if banks’ expectations about each other start out in the shaded area, then they never exit from it: the belief of breakdown (that is, a bank assumes that other banks set monitoring effort below \( \hat{q} \)) is self-fulfilling, because it makes banks switch what they optimize over (Lemma 1).
5 Comparative statics

The equilibria of the game are primarily of interest because of the insight they can give into the concepts of financial sector resilience. In particular, the potential for a trade-off between risk within and across equilibria. We highlight this using quantitative examples. These examples are not intended to be calibrations. We do discuss the realism of the assumed parameter values, however, to give some feeling for the numbers.

A Parameter shock

In our first example banks’ debt ratio is 92.5% \((d = 0.925)\) (i.e. banks have 7.5% own capital) and the recovery rate on loans to unsound banks is 50% \((\rho x = 0.5)\). Our exercise consists of reducing the return on sound bank assets from 10% to 8% \((R = 1.10 \text{ to } R = 1.08)\), and we use the free parameter \(c\) (monitoring costs, on which there is little empirical "feeling") to match the initial survival probability of banks at 99% \((q^* = 0.99)\). This is near the value banks were required to achieve in their Value at Risk models by regulation under Basel II (99.5%), and a capitalization of 7.5% is also in line with Basel minima. However, the assumed return on assets and the recovery rate are higher than what would be expected empirically. For instance, realistic recovery rates for loans to distressed banks are in the order of 35% (Altman and Kishmore, 1996). We need these somewhat more positive figures in order to generate non-breakdown equilibria. In a sense this compensates for the quantitatively "harsh" features of the model, such as the fact that initial assets are worthless if a solvency shock occurs (table 1), and the complete inability of financiers to disentangle bank types, which keeps funding rates for sound banks relatively high.

We pin down \(x\) at its lower bound from condition (1), and then impute \(\rho\) from the value
given for the recovery rate. This gives $c = 0.048$, $x = 1.19$, and $\rho = \frac{0.5}{x} = 0.42$.

The outcome is shown in figure 7. Here the dashed vertical line is the breakdown threshold, the thick solid line is the reaction function when the return on sound assets is 10%, while the thin solid line is that reaction function when the return is 8%. With the higher return the graph is just like figure 3, with one interior equilibrium at $q^* = 0.99$. However, with the lower return the outcome is like figure 2, in which there is no interior crossing point at all. Thus, individual banks’ default risk within the initial equilibrium is minimal. Yet, a small drop in returns leads to certain market gridlock. In fact, the lower is default risk in the initial equilibrium (i.e. the closer the crossing point of the reaction function with the 45 degree line is to the top-right corner), the greater the systemic sensitivity of the funding market.

B Stable equilibria

Figures 3 and 7 represent the case of an unstable interior equilibrium. That instability refers to the effects of "psychological" perturbations in banks’ expectations about each other’s strategies, however, not to the sensitivity of the equilibrium to fundamentals. Stable equilibria can be just as sensitive to shocks in parameters as unstable equilibria. Figure 8 provides a quantitative example of a shift to breakdown starting from a stable, interior equilibrium as in figure
4. We start from parameterization $R = 1.1, d = 0.45, x = \frac{R}{d}, \rho = \frac{0.2}{x}$ and $c = 0.3$. This gives the thick solid line in figure 8 with a stable, interior equilibrium near maximum monitoring effort, and located quite far above the breakdown threshold. This parameterization is not very realistic, as can be seen from the debt ratio of 45%. We did not manage to generate a stable interior equilibrium for more plausible parameter values, however. Quantitatively it is easier to generate graphs like figure 3 than like figure 4, because the latter requires a large degree of concavity that does not easily come about. For example, in figure 7, it is not visible to the eye that the reaction function is concave.

To highlight that shocks need not only be to asset returns, we here consider an increase in leverage as a shock. In particular, an increase of the debt ratio from 45% to 50% (the thin solid line) eradicates the interior equilibrium and leads to funding market breakdown. This shows once more how intra-equilibrium default risk may be small while sensitivity across equilibria is large.

![Figure 8: Quantitative example with concavity](image)

C **Endogenous breakdown threshold**

In the above examples we did not show how the breakdown threshold, which is endogenous, shifts in response to shocks. Depicting this was of little relevance, since the shock leads to breakdown regardless of the final location of the threshold. However, there are cases in which
no shift to another equilibrium occurs, but nonetheless a shock leads to breakdown, because it pushes the threshold beyond the location of that equilibrium. Take the example in figure 8, and consider the case of a declining return on assets. If the breakdown threshold were constant, the funding market could survive a reduction of asset returns down to $R = 1.085$. But since the threshold itself shifts rightward, breakdown already occurs below $R = 1.09$, which is shown in figure 9, where the thin dashed line is the threshold after the shock.

This is not a point about the difference between risk within and across equilibria, but rather a cautioning about the "perceived" distance to breakdown within an equilibrium. If a regulator were to take the amount of stress that the market can handle as given, and focus only on how shocks affect bank profitability, he would overestimate the resilience of the financial sector.

6 Anticipation and insurability of shocks

We have implemented shocks by changing exogenous parameters and recomputing the solution to the game. Implicitly, we have assumed that shocks are ex-ante unanticipated and uninsurable. To what extent might banks be able to insure against shocks in their environment, like those affecting asset returns? The answer depends upon who provides the insurance, and how correlated the shocks are. In particular, the interbank market, which is the common
means for banks to insulate themselves from idiosyncratic shocks, cannot hedge aggregate
shocks (Allen, Carletti and Gale, 2009). Since our analysis focusses on shocks that affect the
parameters of all banks, there would need to be an insurer available outside of the financial
system, namely the central bank (or government) in its role as the Lender of Last Resort. In a
multiple equilibrium setting, the knowledge that there will be an aggregate liquidity injection
by the central bank in the case of an adverse common shock can help align expectations on
a good equilibrium (Freixas, Martin and Skeie, 2009). This differs from our setting, though,
because after the shock shifts the reaction function (as in figures 7 and 8) the bad equilibrium
is unique.

A broader point concerns the anticipation of shocks. Of particular concern might be the
fact that financiers provide funds against low rates when in a good equilibrium, and do not
impose a premium for the risk of a future bad equilibrium. Within the context of our model this
of course happens because we analyze shocks by solving the one-shot game anew for different
parameters. More interesting, however, is the fact that empirical patterns of unsecured bank
funding rates do look in line with very low and stable rates in good states and high ones
in bad states (Heider, Hoerova and Holthausen, 2009). Of course, one possibility is that
financiers attach only a tiny probability to the event of a market crisis, or, alternatively that
they genuinely do not anticipate the possibility of that event. Another option, however, is that
they do anticipate the shocks and do attach significant probabilities to them, but need not
price this in because the funding maturities are so short. With very short funding maturities,
the outcome resembles the comparative statics of a one-shot game, whereby financiers know
that if banks are now in a good state then even if the bad state hits they will still get their
money back before banks actually fail. They only refuse to fund, or demand extreme premia,
for new funding requested during the bad state. The risk of a shift away from the good state
is then borne elsewhere in the system, by bank shareholders or by the Lender of Last Resort
(and the taxpayers funding it).
7 Policy implications

What type of risks should a bank supervisor aim to contain? As the analysis in this paper suggests, limiting individual bank’s default risk does not necessarily coincide with preventing systemic failure. This conclusion fits within the debate on macroprudential regulation. To a large extent that debate has centered on fitting existing tools to macro-aspects, like imposing a systemic surcharge on capital requirements (Galati and Moessner, 2010). However, an implication of the paper is that microprudential and macroprudential regulation may genuinely conflict, rather than implying different tax bases for the same tools.

The main example is in the realm of transparency. In contrast to monetary authorities who have placed increasing focus on transparent communication (Blinder et al., 2008), bank supervisors have traditionally tended to be weary of raising transparency about banks. Presumably, this derives from a fear of runs on certain banks. Even if runs occur on genuinely insolvent institutions, supervisors might prefer secrecy in order to implement an "orderly" unwinding or in order to forbear and hope the institution finds its way back to solvency. However, when outsiders know that the system as a whole is troubled, intransparency from the regulator essentially constitutes a transfer of reputation from healthy to unhealthy banks. When this "subsidy" becomes unbearable because there are too many fears of insolvency, the system as a whole can freeze, imposing large costs on society. The resolution of asymmetric information problems, then, is a macroprudential instrument. From the perspective of our paper, this is a type of policy that is unambiguously beneficial. Open regulatory data and assessments, frequent stress tests, and prompt closure of troubled banks might all be part of macroprudential transparency.

To some extent the paper can also be said to provide a pessimistic note on systemic crisis prevention. If asymmetric information problems are too deeply entrenched in the financial sector itself or if their resolution is too challenging to regulators from a political economy perspective, then the occasional occurrence of market breakdown may be inevitable. This can be due to shocks to fundamentals, but even in their absence unstable equilibria can lead to
breakdown in the presence of small psychological shocks. In policy terms, this means that investing in systemic crises resolution may be just as important as investing in prevention.
Appendix: Proof of Lemma 1

At stage 4 of the game depicted in table 2 banks always demand an amount of $d$ on the funding market. This is true both for banks that want to invest the funds in sound projects and for those who want to invest in gambles. By the payoff given in table 1, for a sound bank investing in its project yields zero net return unless a total funding of 1 is provided to it. Given that banks that invest in sound projects always choose to borrow $d$, so do gambling banks. Borrowing any less would immediately reveal to financiers that the bank is using the money for a gamble. As argued in the text, by $(\sup \rho r) = \rho x < 1$, there exists no interest rate at which a financier is willing to lend to a bank that is known to gamble.

As banks always demand funding $d$, then under conditions discussed below, at stage 1 a bank solves

$$
\max_{q_i} \{ q_i (R - rd) + (1 - q_i) \rho d (x - r) - cq_i^2 \}
$$

where ex-ante there is a probability of $q_i$ that the bank has a sound project and obtains a gross return of $R$ on invested funds $e + d = 1$, while it repays $r$ on borrowed funds $d$, so profits are $R - rd$. And with probability $1 - q_i$ the bank’s project is unsound, which implies that it becomes a gambler, borrowing funds $d$ on which it receives gross return $x$ and repays $r$ if the gamble is successful, so expected profits are $\rho d (x - r)$.

For this to be the case there are three participation constraints and two incentive compatibility constraints that need to be satisfied. Namely:

1. Sound banks prefer their initial project to a gamble (incentive compatibility of sound banks).

2. And the profit of investing in the initial project is positive (participation constraint of sound banks).

3. Unsound-project banks prefer the gamble to their initial project (incentive compatibility constraints).
of unsound banks).

4. And the expected profit of the gamble is positive (participation constraint of unsound banks).

5. There exists an interest rate, $r$, at which financiers are willing to fund banks (participation constraint of bank financiers).

The first two of these are summarized by the condition given in equation (2), namely

$$ R - rd \geq e + \max \{ \rho d (x - r) , 0 \} $$

since $R - rd$ is the return of borrowing $d$ and investing in the sound project, $e$ is the return if a sound bank does not borrow (invests in the risk-free technology), and $e + \rho d (x - r)$ is the expected return if a sound bank borrows and invests the funds in a gamble.

The incentive compatibility constraint of the unsound banks to gamble (point 3) is automatically satisfied once their participation constraint (point 4) is satisfied. That is, when $x > r$ then gambles are profitable in expectation, which necessarily means that they also yield more than an unsound project whose gross return is 0. But since

$$ R - rd \geq e + \max \{ \rho d (x - r) , 0 \} \Rightarrow R - rd \geq e \Rightarrow R - rd \geq 0 \Leftrightarrow \frac{R}{d} \geq r $$

then by the condition in (1) we have

$$ x > \frac{R}{d} \geq r $$

and so the condition in (2) is sufficient to ensure $x > r$. This, by the arguments given in the text, is also sufficient to satisfy the bank financiers’ participation constraint (point 5).

If the condition in (2) is not satisfied then it means that either $R - rd < e$ or $R - rd < e + \rho d (x - r)$ or both. Therefore, either a sound bank chooses to gamble or it chooses not to
borrow. In both cases, financiers then know that any bank that approaches them for funding (whether sound or unsound) wants to gamble. As previously discussed, when financiers know for certain that their funds are used to gamble then there exists no interest rate at which they are willing to lend, an implication of the fact that the gamble is value destroying. Thus, the funding market is closed, and all that banks can do is optimize with their internal funds, \( e \):

\[
\max_{q_i} \left\{ q_i (e) - c q_i^2 \right\}
\]

**Appendix: Proof of Lemma 2**

We first write the no-breakdown condition to

\[
R - rd \geq e + \max \left\{ \rho d (x - r), 0 \right\} = \begin{cases} 
R - rd \geq e + \rho d (x - r) & \text{if } x > r \\
R - rd \geq e & \text{otherwise}
\end{cases}
\]

\[
= \begin{cases} 
r \leq \frac{R - (1 - d)}{d(1 - \rho)} - \frac{\rho}{1 - \rho} x & \text{if } r < x \\
r \leq \frac{R - (1 - d)}{d} & \text{otherwise}
\end{cases}
\]

and replacing for \( r \) from (3) and rewriting

\[
= \begin{cases} 
\frac{1}{q^* + (1 - q^*) \rho} \leq \frac{R - 1 + d - \rho x}{d(1 - \rho)} & \text{if } q^* + \frac{1}{q^* + (1 - q^*) \rho} < x \\
\frac{1}{q^* + (1 - q^*) \rho} \leq \frac{R - (1 - d)}{d} & \text{otherwise}
\end{cases}
\]

where all numerators and denominators in all inequalities are positive, because \( R > 1, \rho x < 1 \) and \( \rho < 1 \). Hence we know that division and multiplication do not change the signs of the inequalities, and we can write the inequalities to \( q^* \):

\[
= \begin{cases} 
q^* \geq \frac{d}{R - 1 + d(1 - \rho)} - \frac{\rho}{1 - \rho} & \text{if } q^* > \frac{1 - \rho x}{1 - \rho} \\
q^* \geq \frac{d(1 - \rho) - \rho(R - 1)}{(R - (1 - d))(1 - \rho)} & \text{otherwise}
\end{cases}
\]
We can show, moreover, that

\[
\frac{d (1 - \rho) - \rho (R - 1)}{[R - (1 - d)] (1 - \rho)} > \frac{1 - \rho x}{x (1 - \rho)}
\]

since

\[
\Leftrightarrow \quad d (1 - \rho) x - \rho (R - 1) x > [R - (1 - d)] [1 - \rho x]
\]

\[
\Leftrightarrow \quad dx - \rho xd - \rho x R + \rho x > R - \rho x R - 1 + d + \rho x - \rho xd
\]

\[
\Leftrightarrow \quad d (x - 1) > R - 1
\]

for which, from the condition in (1) \( x > \frac{R}{d} \), it is sufficient that

\[
d \left( \frac{R}{d} - 1 \right) > R - 1
\]

And rewriting further we have

\[
\Leftrightarrow R - d > R - 1
\]

which is true by \( d < 1 \).

Thus,

\[
q^* \leq \frac{1 - \rho x}{x (1 - \rho)} \Rightarrow q^* < \frac{d (1 - \rho) - \rho (R - 1)}{[R - (1 - d)] (1 - \rho)}
\]

and hence, whenever \( q^* > \frac{1 - \rho x}{x (1 - \rho)} \) does not hold breakdown certainly follows.

This means that we can write the no-breakdown condition to

\[
q^* > \hat{q} = \max \left\{ \frac{1 - \rho x}{x (1 - \rho)}, \frac{d}{R - 1 + d (1 - \rho x)} - \frac{\rho}{1 - \rho} \right\}
\]

because if if \( q^* \leq \frac{1 - \rho x}{x (1 - \rho)} \) then breakdown follows as argued above, while if \( q^* > \frac{1 - \rho x}{x (1 - \rho)} \) then by equation (10) that no breakdown occurs if and only if \( q^* \geq \frac{d}{R - 1 + d (1 - \rho x)} - \frac{\rho}{1 - \rho} \).

Finally, by \( \rho x < 1 \) and \( \rho < 1 \) we have that \( \frac{1 - \rho x}{x (1 - \rho)} > 0 \) and, therefore, \( \hat{q} > 0 \).
Appendix: Proof of Proposition 1

Firstly, in equation (7) $q^*_i < 0$ for $q^* = 0$. To see this, replacing $q^* = 0$ write equation (7) to

$$q^*_i = \frac{1}{2c} \left[ R - d \left( \frac{1}{\rho} (1 - \rho) + \rho x \right) \right]$$

where by $\frac{1}{\rho} > x > \frac{R}{d}$ we have that

$$d \left( \frac{1}{\rho} (1 - \rho) + \rho x \right) > d \left( \frac{R}{d} (1 - \rho) + \rho \frac{R}{d} \right) = R$$

which implies $q^*_i < 0$.

This in conjunction with Lemma 3 ($q^*_i$ increasing and concave in $q^*$) implies $q^*_i$ can have either 0, 1 or 2 crossing points with $q^*$ in the domain $q^* \in [0, 1]$: 0 if $q^*_i < q^* \forall q^* \in [0, 1]$, 1 if $q^*_i > q^*$ for some $q^* \in [0, 1]$ and $q^*_i > q^*$ at $q^* = 1$, and 2 if $q^*_i > q^*$ for some $q^* \in [0, 1]$ and $q^*_i \leq q^*$ at $q^* = 1$.

If there are 0 crossing points then this means that a no-breakdown equilibrium is not feasible, and there is a unique outcome to the game, namely breakdown (case I in Proposition 1). If there is 1 crossing point then it depends whether $q^*_i = q^* > \hat{q}$ or $q^*_i = q^* < \hat{q}$. In the latter case, the crossing point is below the breakdown threshold, implying that it cannot be a no-breakdown equilibrium. The only feasible no-breakdown equilibrium is then the corner where $q^*_i > q^*$ at $q^* = 1$ (case III), because given $q^* = 1$ an individual bank sets $q^*_i = 1$ since he cannot set a higher monitoring effort. If, instead, $q^*_i = q^* > \hat{q}$ then this adds a feasible interior no-breakdown equilibrium (case V). If there are 2 crossing points, then either both (case IV), the higher (case II) or none (case I) of these may satisfy $q^*_i = q^* > \hat{q}$.  

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