Default Cycles

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13 of November, 2018 De Nederlandsche Bank

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Motivation

- Corporate default rates and credit spreads (all rated - Aaa) are countercyclical.
- Yet, the links between the two are non-trivial:
  - Volatility of spreads is not accounted for by variations in expected default losses.
  - Spreads do not predict default rates perfectly.

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  (e.g. Duffie et al. 2009, Giesecke et al. 2011, Gilchrist/Zakrajsek 2012)
- Fundamentals and non-fundamentals for credit and spreads?
- How do they matter for the macroeconomy?

This paper

- Tractable macro model with endogenous firm default.
- Self-fulfilling beliefs in credit conditions (sunspots):
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- Self-fulfilling beliefs in credit conditions (sunspots):
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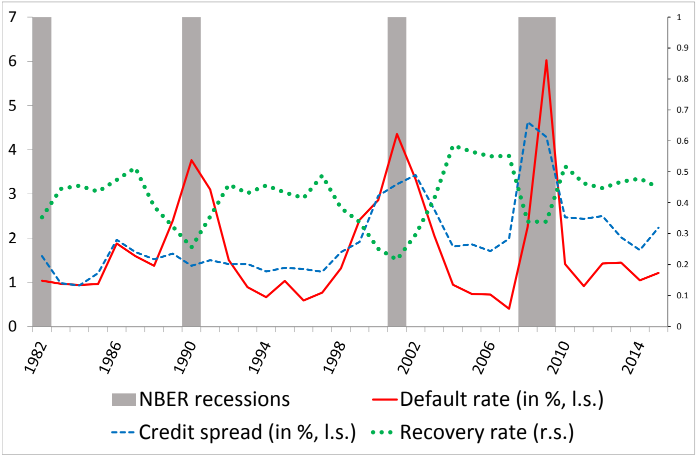
- Equilibrium is indexed by the variance of beliefs shocks
- Solve the model around the (indeterminate) risky steady state - zero excess bond premium determins the variance of beliefs
- Also consider other financial shocks to: excess bond premium and recovery (liquidity) correlated with expectations
- All three shocks account for close to 2/3 of U.S. output growth volatility, 1982–2016.

U.S. corporate bonds



Literature

Default, spreads and the business cycle

Bernanke, Gertler & Gilchrist 1999, Christiano, Motto & Rostagno 2014, Miao & Wang 2010, Gomes & Schmid 2012, Gourio 2013, Khan, Senga & Thomas 2016

Sunspots and credit market frictions

Azariadis, Kaas & Wen 2016, Harrison & Weder 2013, Benhabib & Wang 2013, Liu & Wang 2014, Gu, Mattesini, Monnet & Wright 2013

Self-fulfilling sovereign default

Calvo 1988, Lorenzoni & Werning 2013, Cole & Kehoe 2000, Conesa & Kehoe 2015, Aguiar, Amador, Farhi & Gopinath 2013

Outline

1. Illustrative example of indeterminacy
2. Macroeconomic model
3. Quantitative analysis

Example

- Firms with preferences

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[(1-\beta)\log c_t - \mathbf{1}_{\{\text{defaulting}\}}\eta_t\right]$$

where η_t is a default utility cost:

$$\eta_t = \begin{cases} 0 & \text{with prob. } p \\ \Delta > 0 & \text{with prob. } 1-p \end{cases}$$

- Default $\Rightarrow$ No access to credit.

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- Default $\Rightarrow$ No access to credit.
- Linear technology with return Π.
- Competitive risk-neutral investors with outside return $\bar{R} < \Pi$.
- Investors offer standard debt contracts (b, R).

Firm's problem

- Let $V(\omega)$ be the value of a firm with net worth ω.

$$V(\omega) = \max_{s,(R,b)} (1-\beta)\log(\omega - s) \\ + \beta \mathbb{E} \max \left\{ V[\Pi(s+b) - Rb], V^d[\Pi(s+b)] - \eta \right\}$$

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- Verify: $V(\omega) = \log(\omega) + \bar{V}$ and $V^d(\omega) = \log(\omega) + \bar{V}^d$.
- Write $v \equiv \bar{V} - \bar{V}^d$ for the surplus value of credit market access (*expected credit conditions*).

Optimal debt contract

Maximize borrower utility s.t. investors' participation constraint

$$\max_{(R,b)} \mathbb{E} \max \left\{ \log[\Pi(s+b) - Rb] + v, \log[\Pi(s+b)] - \eta \right\} \quad \text{s.t.}$$

$$\bar{R}b = \begin{cases} Rb & \text{if } \log[\Pi(s+b) - Rb] + v \geq \log[\Pi(s+b)] \ , \\ (1-p)Rb & \text{if } \log[\Pi(s+b)] > \log[\Pi(s+b) - Rb] + v \\ & \geq \log[\Pi(s+b)] - \Delta \ , \\ 0 & \text{else.} \end{cases}$$

(No default / partial default / default with certainty)

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Proposition 1

Under some condition, there exists $\bar{v} > 0$ s.t. the optimal contract has no default if $v \geq \bar{v}$ and partial default if $v < \bar{v}$. ▸ Details

Stationary equilibria

In steady state, the value difference $v^* = V - V^d$ satisfies

$$v^* = f(v^*) \equiv \begin{cases} \beta \log\left[\frac{\bar{R}}{\bar{R} - \Pi(1 - e^{-v^*})}\right] & \text{if } v^* \geq \bar{v} \, , \\ \beta\left\{ \log\left[\frac{\bar{R}}{\bar{R} - \Pi(1-p)(1 - e^{-v^* - \Delta})}\right] - (1-p)\Delta \right\} & \text{if } v^* < \bar{v} \, . \end{cases}$$

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Proposition 2

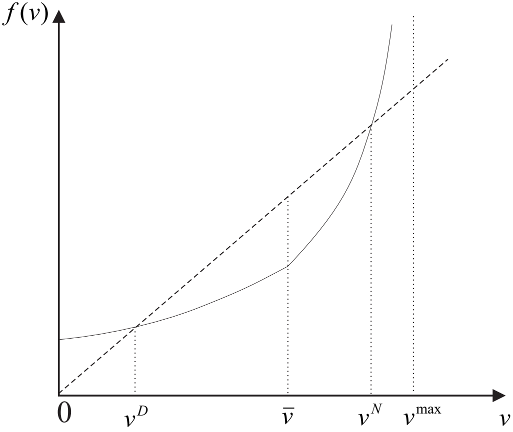
Under some condition, there are two stationary credit market equilibria $v^D < v^N$ s.t. default rates and interest spreads are positive at v^D and zero at v^N. ▸ Details

Multiple stationary equilibria

$\Pi/\bar{R}$ in a certain range for coordination failures of lenders

Too large: no default; Too small: default for sure

sunspot default cycles / indeterminacy



Macroeconomic model

- Firm owners with the same preferences, producing

$$y = (z_t k_t)^{\alpha} (A_t l_t)^{1-\alpha}$$

- Exogenous A_t with trend growth μ_t^A
- z_t is idiosyncratic:

$$z_t = \begin{cases} z^H & \text{with prob. } \pi \\ z^L & \text{with prob. } 1-\pi \end{cases}$$

- Default costs η has cdf $G(.)$
- Competitive real wage w_t.
- Hand-to-mouth workers supply ℓ_t such that $w_t / A_t = \kappa \ell_t^{\nu}$.

Credit market: contract (R_t, θ_t)

- Creditors recover a random fraction λ_t of net worth.Defaulter loses collateral and access to credit (return w/ prob. ψ).

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- Lenders' zero-profit condition:

$$\bar{R}_t(1+\Phi_t) = \mathbb{E}_t\Big\{(1-G(\tilde{\eta}_{t+1}))R_t + G(\tilde{\eta}_{t+1})\lambda_{t+1}\frac{1+\theta_t}{\theta_t}\Pi_t z^H\Big\},$$

  $\tilde{\eta}_{t+1}$: ex-post default threshold. Φ_t: excess bond premium
- Default threshold

$$\tilde{\eta}_{t+1} = \log\left[\frac{(1+\theta_t)(1-\lambda_{t+1})\zeta}{1+\theta_t(1-\rho_t)}\right] - v_{t+1}$$

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- Optimal contract maximizes borrower utility s.t. the banks' zero profit condition and the default threshold condition

General equilibrium

- Credit market expectations

$$v_t = \beta \pi \mathbb{E}_t \Big\{ \log \Big(\zeta (1+\theta_t)(1-\lambda_{t+1}) \Big) - \tilde{\eta}_{t+1}(1 - G(\tilde{\eta}_{t+1})) - \int_{-\infty}^{\tilde{\eta}_{t+1}} \eta \, dG(\eta) + (1-\psi-\pi) v_{t+1} \Big\}$$

- beliefs: ε^b_{t+1} and $\mathbb{E}_t[\varepsilon^b_{t+1}] = 0$ added to v_{t+1}

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- Credit market equilibrium (f_t = fraction with credit market access)

$$z^L\Pi_t = \bar{R}_t, \quad f_t\pi\theta_t \leq (1-\pi) \;.$$

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- Aggregate dynamics of net worth, capital stock and f_t. ▸ Details

Fundamental and belief shocks

$\eta \sim N(\mu, \sigma)$: a non-linear $G(.)$

$$\log(1+\Phi_t) - \log(1+\Phi) = \rho_\Phi \left[\log(1+\Phi_{t-1}) - \log(1+\Phi)\right] + \varepsilon_t^\Phi,$$
$$\log(1-\lambda_t) - \log(1-\lambda) = \rho_\lambda \left[\log\left(1-\lambda_{t-1}\right) - \log\left(1-\lambda\right)\right] + \varepsilon_t^\lambda + \chi_\lambda^\phi \varepsilon_t^\phi,$$
$$\log(1+\mu_t^A) - \log(1+\mu^A) = \rho_A \left[\log\left(1+\mu_{t-1}^A\right) - \log\left(1+\mu^A\right)\right] + \varepsilon_t^A,$$
$$\varepsilon_t^b = \chi^\Phi \varepsilon_t^\Phi + \varepsilon_t^s,$$

ε_t^Φ excess bond premium (EBP) shocks
ε_t^λ recovery shocks
ε_t^A shocks to productivity growth
ε_t^b belief shocks / ε_t^s pure sunspot shocks
shocks are mean zero with variance σ_j^2

Calibration

- 2 risky steady states (RSS).
- An illustration of the zero profit condition

$$\frac{1+\Phi_t}{\Delta_t} = \mathbb{E}_t\left[1 - G(\tilde{\eta}_{t+1})\left(1 - \frac{\lambda_{t+1}}{\xi_t}\right)\right]$$

- Denote $\mathbb{E}_t[\tilde{\eta}_{t+1}] = \tilde{\eta}_t^e$. The RHS becomes

$$1 - \left[G(\tilde{\eta}_t^e) + \frac{G''(\tilde{\eta}_t^e)\sigma_b^2}{2}\right]\left(1 - \frac{1-(1-\lambda_t)^{\rho_\lambda}(1-\lambda)^{1-\rho_\lambda}e^{\frac{\sigma_\lambda^2}{2}}}{\xi_t}\right)$$

Calibration (cont)

- Calibrate to match the U.S. 1982–2016 targets. $\rightarrow$ *indeterminate* (lower default) risky steady state
- Explore the role of shocks to recovery rate λ_t, EBP Φ_t, credit expectations (beliefs ε_t^b), and productivity μ_t^A.
- MLE using recovery rate, credit spreads, default rate, and output (per capita) growth.

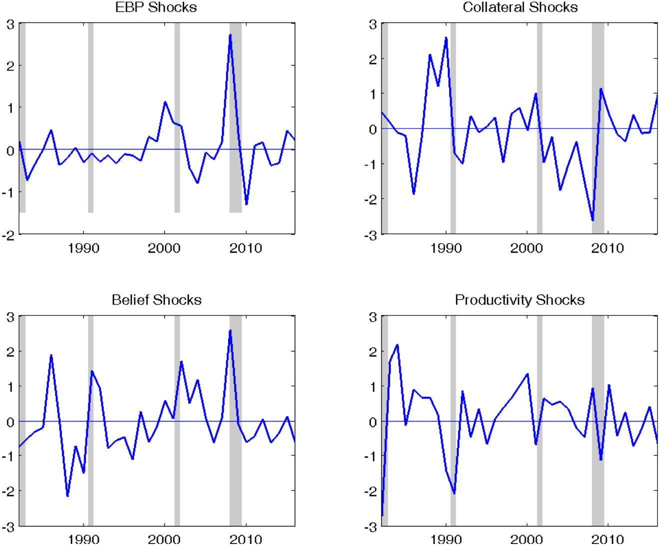
Directed calibrated parameters

Parameter	Value	Explanation/Target
α	0.33	Capital income share
δ	0.10	Depreciation rate
μ^A	1.72%	Trend growth
κ	2.38	Labor supply $\ell = 0.25$
ν	0.67	Macro labor supply elasticity $1/\nu = 1.5$
π	0.20	Constrained firms (Almeida et al. 2004)
ψ	0.10	10-year default flag
ζ	0.85	15% default loss (Davydenko et al. 2012)
Φ	0.00	0 steady-state EBP

Estimated parameters (steady state and dynamics)

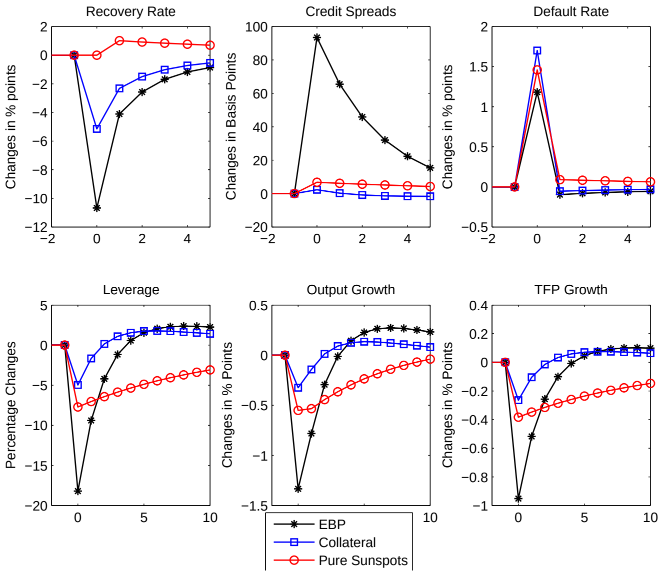
Para	Value	Explanation	Target / T stat (std err)
β	0.96	Discount factor	Capital-output ratio 200%
λ	0.20	Recovery parameter	Recovery rate 41.74%
σ_b	3.42%	Std. dev. of belief shocks	Credit spread 2%
z^H	1.13	High productivity	Debt-output ratio 82%
z^L	0.79	Low productivity	Average productivity $\tilde{z} = 1$
μ	-0.23	Mean of η	Default rate 1.58%
σ	7.31%	Std. dev. of η	Leverage $\theta = 2.1$
σ_s	2.69%	Std. dev. of pure sunspots	$\sigma_b^2 = \sigma_s^2 + (\chi_b^\Phi)^2 \sigma_\Phi^2$
ρ_Φ	0.73	Persistence of EBP	Estimated: 6.22 (0.12)
ρ_A	0.25	Persistence of productivity	Estimated: 1.23 (0.20)
ρ_λ	0.58	Persistence of collateral	Estimated: 6.55 (0.09)
σ_Φ	0.0087	Std. dev. of EBP	Estimated 10.27 (0.0009)
σ_A	0.0334	Std. dev. of productivity	Estimated 7.81 (0.0043)
σ_λ	0.0313	Std. dev. of collateral	Estimated 11.63(0.0027)
χ_b^Φ	2.4279	Spill over to beliefs variation	Estimated 3.54 (0.69)
χ_λ^Φ	0.0650	Spill over to collateral	Estimated 5.80 (0.01)

Estimated smoothed shocks



Impulse responses

An ↑ default ↓ lending and ↑ ex-post recovery. Small movements in spreads.



Variance decompositions

	Exogenous Shocks to			
	EBP	Collateral	Sunspot	Productivity
Credit Spreads	98.25	0.18	1.57	0
Recovery Rate	77.15	19.59	3.26	0
Default Rate	22.06	44.56	33.38	0
Output Growth	41.16	3.32	17.63	37.88
Debt-to-Output	37.73	5.77	54.25	2.26
TFP Growth	17.30	1.75	10.72	70.23

Variance decomposition: fundamentals versus expectations

	Shocks that change	
	Fundamentals	Beliefs
Credit Spreads	77.04	22.96
Recovery Rate	76.63	23.37
Default Rate	45.96	54.04
Output Growth	78.70	21.30
Debt-to-Output	78.48	21.52
TFP Growth	90.93	10.07

Conclusions

- Endogenous firm default and different financial shocks
- Self-fulfilling changes in credit market expectations important for default cycle (54%)
- The risks of beliefs play a big role for the steady state. The expectation channel accounts for about 22% variation in output growth
- Excess bond premium / collateral also important through the credit channel
- Policy targeting credit market expectations could be useful (for both the steady state and dynamics)

Proposition 1

Suppose that the parameter condition

$$\frac{(e^{\Delta}-1)(1-p)}{e^{\Delta}-1+p} < \frac{\bar{R}}{\Pi} < \frac{(e^{(1-p)\Delta}-e^{-p\Delta})(1-p)}{e^{(1-p)\Delta}-1}$$

holds. Then there exists a threshold value $\bar{v} \in (0, v^{\max})$ with $v^{\max} \equiv \log(\Pi/(\Pi-\bar{R}))$, such that

(i) If $v \in [\bar{v}, v^{\max})$, the optimal contract is $(b, R) = (b(s), \bar{R})$ with debt level and borrower utility

$$b(s) = s\frac{\Pi(1-e^{-v})}{\bar{R}-\Pi(1-e^{-v})}\ , \ \ U(s) = \log\left[\frac{\bar{R}\Pi s}{\bar{R}-\Pi(1-e^{-v})}\right]\ .$$

(ii) If $v \in [0, \bar{v})$, the optimal contract is $(R, b) = (\bar{R}/(1-p), b(s))$, with debt level and borrower utility

$$b(s)=s\frac{\Pi(1-p)(1-e^{-v-\Delta})}{\bar{R}-\Pi(1-p)(1-e^{-v-\Delta})}\ , \ U(s)=\log\left[\frac{\bar{R}\Pi s}{\bar{R}-\Pi(1-p)(1-e^{-v-\Delta})}\right]-(1-p)\Delta\ .$$

Back

Proposition 2

Suppose that parameters satisfy

$$\left(\frac{\bar{R}}{\bar{R} - \Pi(1 - e^{-\bar{v}})}\right)^{\beta} < \frac{\Pi[1 - (1 - p)e^{-p\Delta}]}{\Pi - \bar{R} + e^{(1-p)\Delta}(\bar{R} - \Pi(1 - p))}\ , \quad (1)$$

Then there are two stationary credit market equilibria $v^D < v^N$ such that default rates and interest spreads are positive at v^D and zero at v^N.

Back

Aggregate dynamics

- Net worth

$$\Omega_{t+1} = \beta z^H \Pi_t \Omega_t \left\{ (1-\pi)\bar{\rho}_t + \pi f_t \Big[(1 - G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) \right.$$

$$\left. + G(\tilde{\eta}_{t+1})(1+\theta_t)(1-\lambda_t)\zeta \Big] + \pi(1-f_t) \right\}$$

- Capital stock of productive and unproductive firms

$$K_t^H = \beta \Omega_t \pi \Big[f_t(1+\theta_t) + 1 - f_t \Big] \ , \ K_t^L = \beta \Omega_t \Big[(1-\pi) - \pi f_t \theta_t \Big]$$

- Fraction of firms with credit market access f_t

$$f_{t+1}\Omega_{t+1} = \beta z^H \Pi_t \Omega_t \left\{ (1-\pi) f_t \bar{\rho}_t \right.$$

$$\left. + \pi f_t (1 - G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) + (1-f_t)\psi[(1-\pi)\bar{\rho}_t + \pi] \right\}$$

Back