Subjective Housing Beliefs, Falling Natural Rates and the Optimal Inflation Target

Klaus Adam   Oliver Pfaeuti   Timo Reinelt

University of Mannheim & CEPR

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Introduction

- Subjective beliefs an important contributor to HP fluctuations

  Kaplan, Mitman Violante (2020), Piazzesi and Schneider (2009)

- Housing price beliefs deviate from Full Info RE:

  Case, Shiller & Thompson (2012), Armona, Fuster & Zafar (2018), Kuchler & Zafar (2019)

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  - Understand monetary policy implications of subj. HP beliefs

Preview of Main Findings

- Households' subjective housing beliefs in Michigan survey data:

  - capital gain beliefs revised too sluggishly (by a factor 3 - 4)

  - beliefs initially undershoot, followed by delayed overshooting

  - exp. capital gain procyclical, actual gains countercyclical (NEW)

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- Simple model featuring (weak) capital gain extrapolation replicates

  - housing price behavior: large & persistent swings

  - observed patterns of beliefs deviations

  (quantitatively!)

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Insert capital gain extrapolation into

- MP model with housing sector & lower bound constraint

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- Ramsey optimal MP

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Investigate the optimal MP implications of falling natural rates

- In line with data: larger housing price fluctuations
  => belief-driven housing price fluctuations amplified
  (absent under RE)

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- Lower bound constraint more relevant:
  average inflation $\Uparrow$ under Ramsey optimal MP: 1% $\Uparrow$ for 3% $\Downarrow$

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- Lower bound constraint more relevant:
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- Inflation effect absent with rat. HP expectations:
  Subj. housing beliefs quant. important for Ramsey optimal MP

Structure of Talk

1. Deviations from rational HP expectations in survey data
2. Simple housing model: quantitative match
3. Full monetary policy model & policy implications

Subjective Housing Beliefs: Data

- Survey of Consumers, University of Michigan (2007 - 2021)
- 1-year-ahead house price expectations
- Use mean and median expectations
- Case/Shiller Price Index for actual house prices

Subjective Housing Beliefs: Sluggish Updating

- Following Coibion & Gorodnichenko (2015):

$$q_{t+4} - E_t^{\mathcal{P}}\left[q_{t+4}\right] = a^{CG} + b^{CG} \cdot \left(E_t^{\mathcal{P}}\left[q_{t+4}\right] - E_{t-1}^{\mathcal{P}}\left[q_{t+3}\right]\right) + \varepsilon_t.$$

where q_t is nominal/real housing price.

Under FIRE $b^{CG} = 0$, but find $b^{CG} > 0$

Subjective Housing Beliefs: Sluggish Updating

Table:

	Mean Expectations	Median Expectations
Nominal HP		
$\widehat{b}^{CG}$	2.22^{***}	2.85^{***}
	(0.507)	(0.513)
Real HP		
$\widehat{b}^{CG}$	2.00^{***}	2.47^{***}
	(0.332)	(0.366)

Subjective Housing Beliefs: Wrong Cyclicality

Following Adam, Marcet & Beutel (2017):

$$E_t^{\mathcal{P}}\left[\frac{q_{t+4}}{q_t}\right] = a + c \cdot PR_{t-1} + u_t \qquad (1)$$

$$\frac{q_{t+4}}{q_t} = \mathbf{a} + \mathbf{c} \cdot PR_{t-1} + \mathbf{u}_t. \qquad (2)$$

- With FIRE: $H_0 : c = \mathbf{c}$
- We find $c > 0$ and $\mathbf{c} < \mathbf{0}$

Subjective Housing Beliefs: Wrong Cyclicality

Table:

	$\hat{c}$ (in %)	$\hat{\mathbf{c}}$ (in %)	p-value $H_0 : c = c$ (small sample corrected)
Nominal HP			
Mean	0.033	-0.102	0.000
	(0.008)	(0.007)	
Median	0.014	-0.102	0.000
	(0.001)	(0.007)	
Real HP			
Mean	0.030	-0.113	0.000
	(0.017)	(0.009)	
Median	0.010	-0.113	0.000
	(0.004)	(0.009)	

Subj. Beliefs: Underreaction & Delayed Overshooting

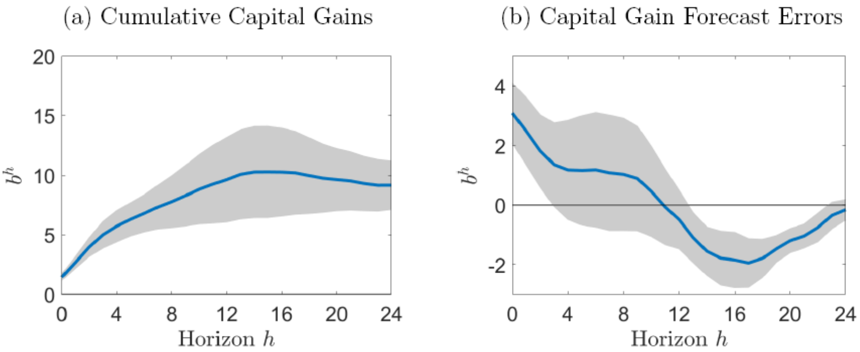
Local projections of the form

$$X_{t+h} = a^h + b^h \cdot \frac{q_{t-1}}{q_{t-2}} + u_t^h, \tag{3}$$

For l.h.s. variable X_{t+h} we consider:

- cumulative housing capital gain q_{t+h}/q_{t+1}
- 1-year ahead forecast error, $q_{t+h+4}/q_{t+h} - E^{\mathcal{P}}_{t+h}[q_{t+h+4}/q_{t+h}]$

Subj. Beliefs: Underreaction & Delayed Overshooting



Structure of Talk

1. Deviations from rational HP expectations in survey data
2. **Simple housing model: quantitative match**
3. Full monetary policy model & policy implications

Simple Model of Subj. Housing Beliefs

(Risk-neutral) household:

$$\max_{\left\{C_t \geq 0, D^{\max} \geq D_t \geq 0, D_t^R \geq 0\right\}_{t=0}^{\infty}} \quad E_t^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \left[C_t + \xi_t^d \left(D_t + D_t^R \right) \right]$$

$$s.t. \quad : \quad C_t + \left(D_t - (1-\delta) D_{t-1} \right) q_t + R_t D_t^R = Y_t \text{ for all } t \geq 0,$$

Two first-order conditions:

$$R_t = \xi_t^D$$
$$q_t = \xi_t^D + \beta(1-\delta) E_t^{\mathcal{P}} q_{t+1}$$

Capital Gain Extrapolation form Bayesian Learning

Households perceive capital gains to evolve as

$$\frac{q_t}{q_{t-1}} = b_t + \varepsilon_t$$

with

$$b_t = b_{t-1} + \nu_t$$

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With conjugate prior beliefs, it follows:

$$E_t^{\mathcal{P}}\left[\frac{q_{t+1}}{q_t}\right] \equiv \beta_t$$

where

$$\beta_t = \min\left\{\beta_{t-1} + \frac{1}{\alpha}\left(\frac{q_{t-1}}{q_{t-2}} - \beta_{t-1}\right), \beta^U\right\}$$

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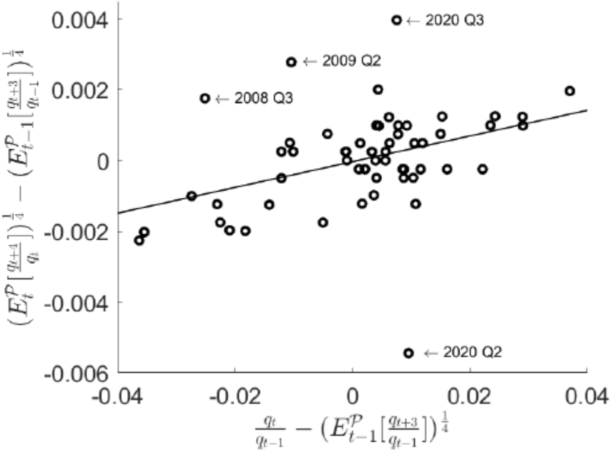
Capital Gain Extrapolation in Survey Data



Equilibrium Housing Price

The equilibrium house price is given by

$$q_t = \frac{1}{1 - \beta(1-\delta)\beta_t} \xi_t^D$$

- driven by fundamental shocks ξ_t^D and subjective beliefs β_t

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- driven by fundamental shocks ξ_t^D and subjective beliefs β_t
- lower natural rate/higher β => more volatile house prices
- Simple model gets very far in terms of *quantitatively* matching

- forecast error patterns

- housing price behavior:

Quantitative Performance: Housing Prices

Simple Calibration:

- $\delta = 0.03/4$ and $\rho_{\xi} = 0.99$ from Adam & Woodford (2021)
- $\frac{1}{\alpha} = 0.007$ from Adam et al. (2016) for stock prices and β^U to match maximum value of PR
- β to match average natural rate of 0.75% and σ_{ξ}^2 to match PR volatility

Quantitative Performance: Housing Prices

	Data	Subjective Belief Model
$std\ (PR_t)$	8.6	8.6
$corr(PR_t, PR_{t-1})$	0.99	0.99
$std(q_t / q_{t-1})$	0.06	0.04
$corr(q_t / q_{t-1}, q_{t-1} / q_{t-2})$	0.97	0.94

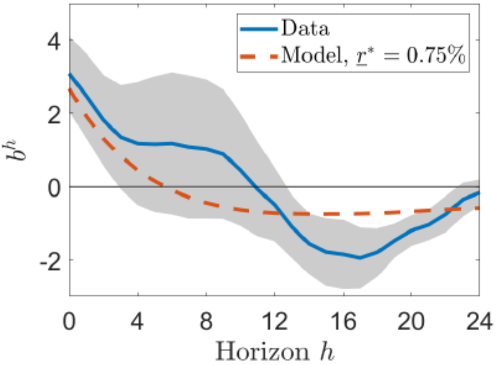
Quantitative Performance: Housing Prices

	Data	Subjective Belief Model	RE Housing
$std\,(PR_t)$	8.6	8.6	2.69
$corr(PR_t, PR_{t-1})$	0.99	0.99	0.99
$std(q_t/q_{t-1})$	0.06	0.04	0.003
$corr(q_t/q_{t-1}, q_{t-1}/q_{t-2})$	0.97	0.94	-0.01

Quantitative Performance: Housing Prices

	Subj. Belief Model	Data	
		Mean	Median
Sluggishness b^{CG}	2.09	1.68	2.12
		(0.355)	(0.394)
exp. cap gain c	0.03	0.030	0.010
		(0.172)	(0.043)
actual cap gain $\mathbf{c}$	-0.063	-0.113	-0.113
		(0.009)	(0.009)

Initial Underreaction & Delayed Overshooting



Lessons Learned from the Simple Model

- Weak extrapolation =>
  - large & persistent housing price swings
  - patterns of expectational deviations
- Subj. belief dynamics important for HP volatility: 2/3 of std(PR)
- Low real/natural interest rates => larger housing price volatility

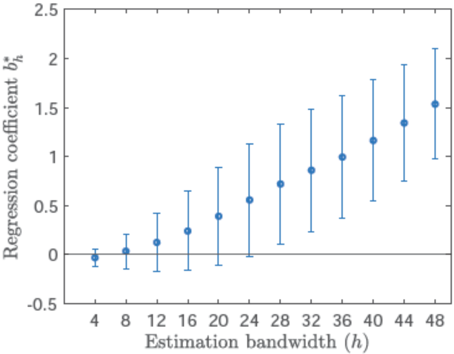
Housing Price Volatility Natural Rates: Unites States

- Consider local projections of the form

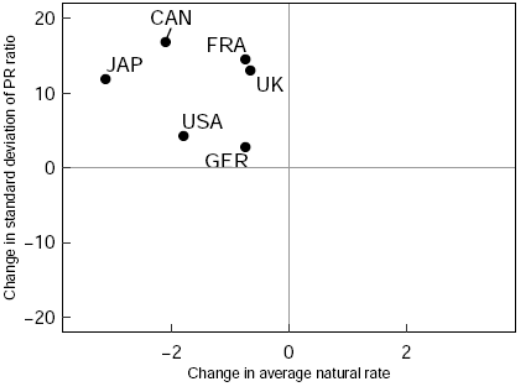
$$Std(PR_{t-\frac{h}{2}}, \ldots, PR_{t+\frac{h}{2}}) = a_h^* - b_h^* \cdot r_t^* + u_{t,h}, \qquad (4)$$

- Coefficient b_h^* has causal interpretation under standard assumptions

Housing Price Volatility Natural Rates: Unites States



Δ PR Vola vs. Δ Natural Rate (Pre-/Post-1990)



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Full Monetary Policy Model

Textbook New Keynesian model

+ housing sector with endogenous housing supply
+ subjective housing price beliefs
+ effective lower bound constraint on nominal rates
+ falling natural rates of interest

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+ housing sector with endogenous housing supply
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+ falling natural rates of interest

- Full model still matches forecast error patterns & HP behavior
- Study Ramsey optimal monetary policy in this framework
- Derive closed-form 2nd order approximation of Ramsey problem

Full Monetary Policy Model

New insights generated by the full model:

Lower average levels of the natural rate

=> increased HP fluctuations

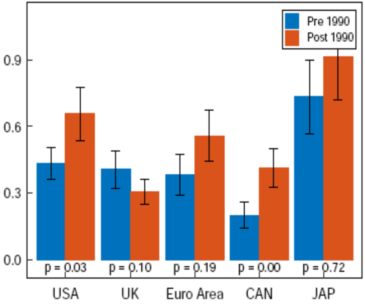
=> increased volatility of natural rate

=> lower bound become more stringent than under RE

=> average inflation increases by more as natural rate falls

Rising Std. Deviation of the Natural Rate

Figure 2: Volatility of Natural Rates



Source: Holston et al. (2017) and Fujiwara et al. (2016) (natural rate estimates). The black lines denote the 90%-confidence bands. The reported p-values are for the null hypothesis that volatility has not changed from pre to post 1990.

Optimal Policy with Lower Bound Constraint

$$\max_{\{\pi_t, y_t^{gap}, i_t \geq \underline{i}\}} -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left(\Lambda_\pi \pi_t^2 + \Lambda_y \left(y_t^{gap}\right)^2 + \Lambda_q \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)^2 \right)$$

$s.t.:$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y y_t^{gap} + \underbrace{\kappa_q}_{<0} \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) + u_t$$

$$y_t^{gap} = \lim_T E_t y_T^{gap} - \varphi E_t \sum_{k=0}^{\infty} \left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE} \right)$$

$$\underbrace{-\frac{C_q}{C_Y}}_{>0} \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)$$

+Equation(s) determining $\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)$

Rational housing expectations: $\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) = 0$

Optimal Policy with Lower Bound Constraint

- The natural rate closing the output gap under subjective beliefs

$$r_t^n \equiv r_t^{n,RE} \underbrace{-\frac{1}{\varphi}\frac{C_q}{C_Y}}_{>0} \left(\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) - E_t\left(\widehat{q}_{t+1}^u - \widehat{q}_{t+1}^{u*}\right)\right)$$

- More volatile housing prices => more volatile natural rate!

Housing Prices and Subjective Beliefs

- Bayesian learning about housing price in marginal utility units q_t^u

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  Can meaningfully discuss optimal policy design!

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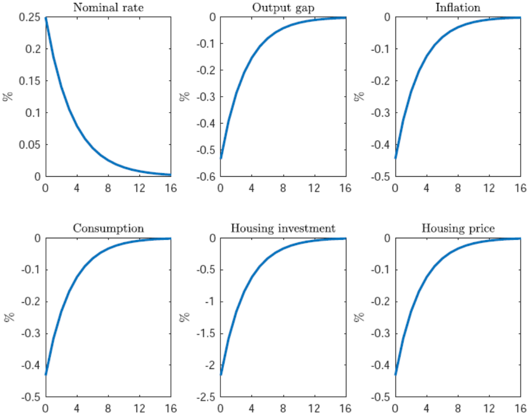
- Bayesian learning about housing price in marginal utility units q_t^u
- Belief dynamics empirically plausibleand theoretically attractive
- Belief dynamics do not depend not on monetary policy:
  Can meaningfully discuss optimal policy design!
- At the same time:
  - MP does affect housing demand & housing prices
  - natural rate (& other fundamentals) affect belief dynamics

Impulse Response to MP Shock: Calibrated Model



Model Calibration

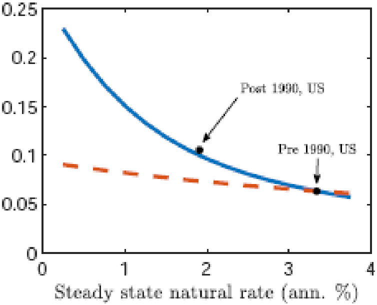
- Calibrate the model to match the pre-1990
  (1) average natural rate
  (2) volatility of the natural rate
  (3) volatility of price-to-rent ratio

- Do this for the RE model and the Subj. Belief model

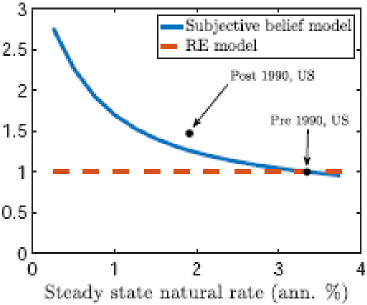
- What happens as natural rate falls to post-1990 average (or lower):
  - increase in the discount factor β
  - may reflect lower steady-state growth

Model: Non-targeted Moments

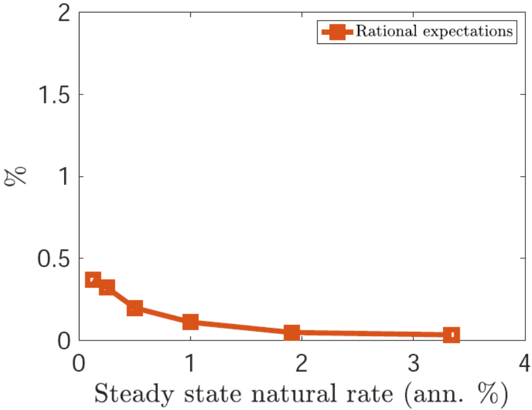
(a) Standard deviation of price-to-rent ratio (relative to corresponding m)ean



(b) Standard deviation of the natural rate relative to case with $\underline{r}^{n,RE} = 3.34\%$



Average Inflation under Optimal Monetary Policy



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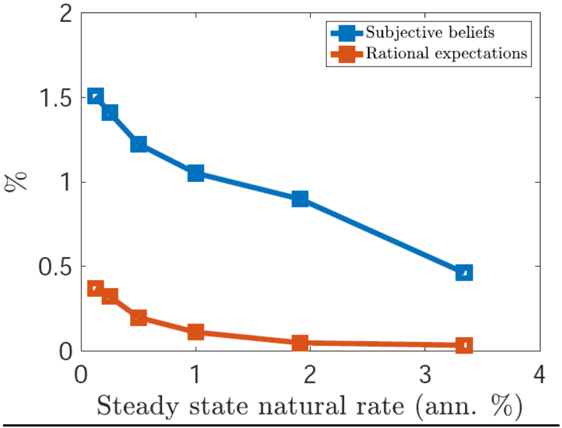

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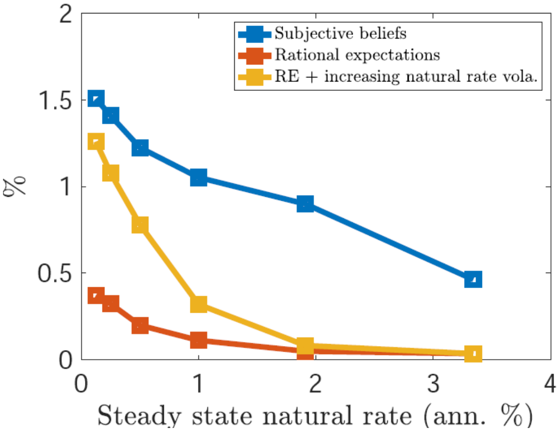


Conclusions

- MP implications of falling natural rates critically depend on the nature of housing price expectations
- Observed forecast errors & housing price behavior is consistent with capital gain extrapolation
- Justifies targeting higher inflation as natural rate falls than under RE
- Alternative approaches:

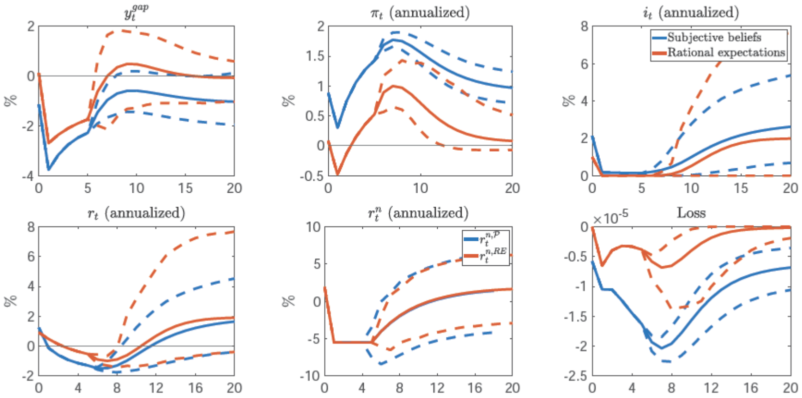
  - teach HHs how to make more accurate predictions

  - reverse unfavorable macro trends causing fall in average natural rate

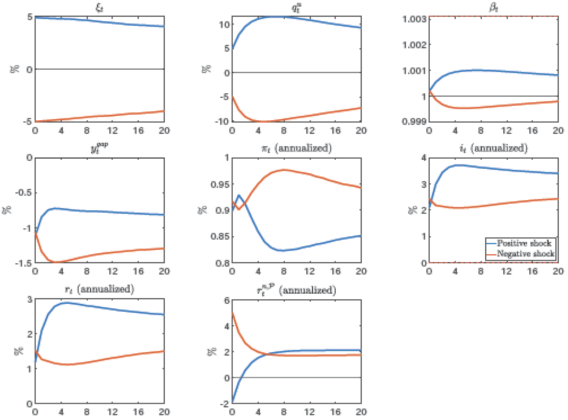
Impulse Repsonse Analysis for ZLB Event

- Start economy in period 0 at ergodic mean of state variables
- 6 quarters negative natural rate that puts RE economy to ZLB & no other shocks
- After quarter 6: all shocks move gain according to their stochastic laws of motion
- Show the mean response: average over all path
- Show the 1% and 99% percentile of the response distribution
- Put the same shocks into the subjective belief model

Impulse Repsonse Analysis for ZLB Event

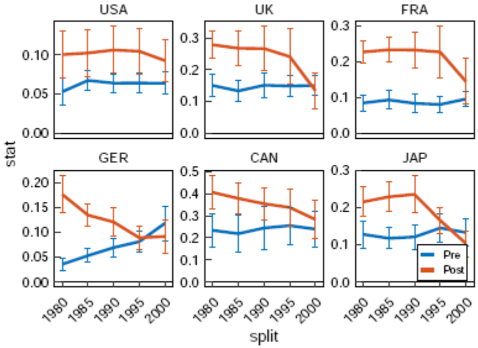


(Asymmetric) Leaning Against Housing Demand Shocks



Std. Deviation of the Price-to-Rent Ratio

(a) Standard Deviation of the Price-to-Rent Ratios for Different Sample Splits.



NK Model with Housing

- Representative HH:

$$\max E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \left[\tilde{u}(C_t; \xi_t) - \int_0^1 \tilde{v}(H_t(j); \xi_t) dj + \tilde{\omega}(D_t + D_t^R; \xi_t) \right]$$

$s.t.:$

$$C_t + B_t + (D_t - (1-\delta) D_{t-1})\, q_t + k_t + R_t D_t^R =$$

$$\tilde{d}(k_t; \xi_t) q_t + \int_0^1 w_t(j) H_t(j) dj + \frac{B_{t-1}(1+i_{t-1})}{\Pi_t} + \frac{\Sigma_t + T_t}{P_t}$$

- $\mathcal{P}$: subjective housing price beliefs, otherwise rational beliefs

NK Model with Housing

- Model formulated in terms of growth-detrended variables
- Discount rate β jointly captures:

  - pure time preference rate $\widetilde{\beta} \in (0, 1)$

  - the steady-state growth rate g_c of consumption

$$\beta \equiv \widetilde{\beta} \frac{\tilde{u}_C(C(1 + g_c))}{\tilde{u}_C(C)},$$

- Growth rate g_c falls $=>$ discount rate β increases
- Decline in growth & natural rate captured via increase in β

NK Model with Housing

- Internally rational households & firms (Adam&Marcet (JET, 2011))
- HHs choose $\left\{C_t, H_t(j), D_t, D_t^R, k_t, B_t\right\}_{t=0}^{\infty}$ to maximize utility subject to the budget constraints
- Beliefs about variables beyond their control given by $\mathcal{P}$: $\left\{P_t, w_t(j), q_t^u, R_t, i_t, \Sigma_t / P_t, T_t / P_t\right\}$, where

  $q_t^u \equiv q_t \tilde{u}_C(C; \xi_t)$ is housing price in marginal utility units

NK Model: Household Optimality Conditions

- Set of standard FOCs: labor-leisure choice, cons. Euler EQ
- 3 new optimality conditions:

  Optimal housing demand : $q_t^u = \xi_t^d + \beta(1-\delta) E_t^{\mathcal{P}} q_{t+1}^u$

  Optimal housing investment: $k_t = \left(A_t^d q_t^u \frac{C_t^{\tilde{\sigma}-1}}{\bar{C}_t^{\tilde{\sigma}-1}} \right)^{\frac{1}{1-\tilde{\alpha}}}$

  Purchase vs. renting margin: $\xi_t^d = R_t \tilde{u}_C(C_t, \xi_t)$

NK Mode: Optimal Price Setting by Firms

- Supply side standard:

  - differentiated goods with Calvo price stickiness $\alpha \in (0, 1)$

  - Dixit-Stiglitz aggregation

- Standard firm FOCs for optimal reset price: Phillips curve

- New feature: wage/marginal costs depend on housing prices

Structure of Presentation

1. New Keynesian model with housing & lower bound constraint
2. **Optimal policy problem & economic mechanisms**

Nonlinear Optimal Policy Problem

$$\max_{\{Y_t, q_t^u, p_t^*, w_t(j), P_t, \Delta_t, i_t \geq 0\}} E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t, q_t^u; \xi_t)$$

$$\begin{aligned}
\left(\frac{p_t^*}{P_t}\right)^{1+\eta(\phi-1)} &= \frac{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} \frac{\eta \phi w_T(j)}{\eta - 1} \left(\frac{Y_T}{A_T}\right)^{\phi} \left(\frac{P_T}{P_t}\right)^{\eta\phi+1}}{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T}(1-\tau_T) Y_T \left(\frac{P_T}{P_t}\right)^{\eta}} \\
(P_t / P_{t-1})^{\eta-1} &= (1 - (1-\alpha)(p_t^* / P_t)^{1-\eta}) / \alpha \\
\Delta_t &= h(\Delta_{t-1}, P_t / P_{t-1}) \\
w_t(j) &= \lambda \frac{\bar{H}_t^{-\nu}}{\bar{C}_t^{\tilde{\sigma}^{-1}}} \left(\frac{Y_t}{A_t}\right)^{\phi\nu} C\left(Y_t, q_t^u, \xi_t\right)^{\tilde{\sigma}^{-1}} \left(\frac{p_t^*}{P_t}\right)^{-\eta\phi\nu} \\
\tilde{u}_C(C(Y_t, q_t^u, \xi_t); \xi_t) &= \lim_{T \to \infty} E_t^{\mathcal{P}} \left[\tilde{u}_C(C_T; \xi_T) \beta^T \prod_{k=0}^{T-t} \frac{1 + i_{t+k}}{P_{t+k+1} / P_{t+k}} \right] \\
q_t^u &= \xi_t^d + \beta(1-\delta) E_t^{\mathcal{P}} q_{t+1}^u
\end{aligned}$$

Optimal Policy with Lower Bound Constraint

- Can derive insightful LQ approx. to nonlinear policy problem

- Helps understanding stabilization trade-offs for output & inflation

Model Calibration

Parameter	Value	Source/Target
		Preferences and technology
β	0.9917	Average U.S. natural rate pre 1990
φ	1	Adam and Billi (2006)
κ_y	0.057	Adam and Billi (2006)
$\frac{\Lambda_y}{\Lambda_\pi}$	0.007	Adam and Billi (2006)
κ_q	-0.0023	Adam and Woodford (2020)
$\frac{C_q}{C_Y}$	-0.29633	Adam and Woodford (2020)
s^d	15%	Adam and Woodford (2020)
δ	0.03/4	Adam and Woodford (2020)
		Exogenous shock processes
ρ_{r^n}	0.8	Adam and Billi (2006)
σ_{r^n}	0.2940% (RE)	Adam and Billi (2006)
	0.1394% (subj beliefs)	
ρ_{ξ^d}	0.99	Adam and Woodford (2020)
σ_{ξ^d}	0.0233 (RE)	Std. dev. of price-to-rent ratio pre 1990
	0.0165 (subj. beliefs)	
		Subjective belief parameters
α	1/0.007	Adam et al. (2016)
β^U	1.0031	Max percent deviation of PR-ratio from mean