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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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Robust monetary policy under shock uncertainty

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Abstract

We assess the robustness of monetary policy under shock uncertainty based on a novel empirical method. Shock uncertainty arises from the inability to observe the output gap in real time, by which the contribution of supply and demand shocks to inflation is unknown. We apply our method in a medium-scale Dynamic Stochastic General Equilibrium (DSGE) model to the recent inflation surge in the US. We find that robust monetary policy aimed at limiting extreme welfare losses under shock uncertainty should neither be too strong nor too mild, given the probability that supply shocks are a dominant driver of economic fluctuations. An overly strong response to inflation in supply driven scenarios is associated with large tail losses due to adverse output dynamics.

JEL classification: E52, E58, D81

Keywords: Monetary policy, Inflation, Policy-making under risk and uncertainty

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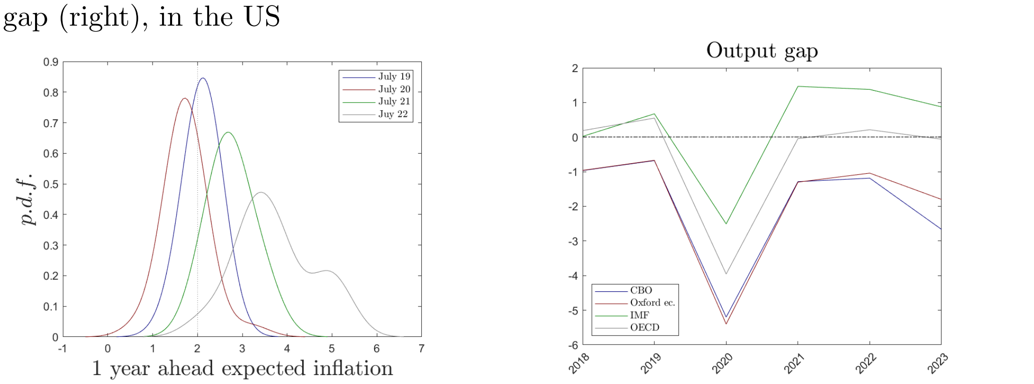
1 Introduction

The recent series of demand and supply shocks hitting the global economy created large uncertainty about the inflation outlook. The economy faced a mixture of negative supply shocks – like the Covid-19 pandemic and the surge in energy prices - and positive demand shocks, driven by fiscal spending and pent-up consumer demand. The shocks resulted in soaring inflation. Monetary policymakers faced considerable challenges to understand the nature of the shocks and their impact on the economy, which complicated their policy reactions. Waller (2023) emphasizes that central bankers have to deal with elevated uncertainty regularly.

Powell (2023) pointed out the need for a risk management approach of monetary policy in order to assess the impact of the shocks and hedge against unforeseen risks. Schnabel (2023a) mentions that high uncertainty can also be observed in financial markets, particularly in market-based measures of inflation compensation. This is amongst others reflected in an increased dispersion of inflation expectations, following the shocks that hit the US economy between 2019 and 2022 (Figure 1, left panel).

Figure 1: Distributions of inflation expectations (left) and estimates of the output gap (right), in the US



The left panel shows probability density functions (p.d.f.) of the one year ahead expected inflation in the US (p.d.f. on vertical axis; inflation in percentage on horizontal axis), source: Consensus Economics. The right panel includes output gap estimates of different institutions on an annual basis.

This unprecedented combination of shocks in the last years have made projections of inflation and output highly uncertain, also because model relationships based on past regularities broke down (Bobeica and Hartwig, 2021). In this context, Schnabel (2023b) refers to model and parameter uncertainty, as well as policy mistakes due to the reliance of policymakers on model outcomes. The right panel of Figure 1 illustrates the uncertainty of output gap estimates. In contrast to the inflation rate, which is an observable variable, different institutions disagree markedly on estimates of the output gap. These discrepancies have important policy implications. According to the IMF estimate of the output gap, the US economy seems to be dominated by demand shocks, with high inflation and a positive output gap, requiring tight monetary policy. In contrast to that, the Congressional Budget Office output gap estimate implies that supply shocks likely drive the high inflation, since the output gap is negative (around -2.8%). The latter suggests that an overly tight monetary policy would be counter-productive, since it can provoke a recession (Jiménez et al. 2022).

The contradictions of these estimates introduce the key issue that we address in this paper: if it is not possible to observe (or reliably estimate) the output gap, then it is not possible to identify the true combination of shocks that drives observed inflation. Thus, monetary policymakers suffer from missing information due to shock uncertainty. Our research question is which monetary policy reaction, aimed at inflation and output stabilization, is most robust to shock uncertainty. We define robustness as the ability of the central bank to limit adverse tail outcomes, while achieving an inflation rate sufficiently close to target.

Model simulations suggest that supply and demand factors played broadly similar roles in the increase of inflation, as shown by Gonçalves and Koester (2022) or Ascari et al. (2023). Their research is based on VAR models and related assumptions about shock identification. However, the precise sources of the inflation surge are hard to identify given the unprecedented nature of the shocks. This makes the inflation process fundamentally uncertain. According to Knight (1921), such epistemic uncertainty relates to events with unknown or objectively unmeasurable probabilities. This contrasts to measurable uncertainty or risk, which can be quantified based on known probability distributions of events. In case of fundamental uncertainty however, the data distribution is unknown, either intrinsically or because of practical limitations. This goes with an unknown event space and indeterminate outcomes.

Structural macroeconomic models, such as Dynamic Stochastic General Equilibrium (DSGE) models, are not tailored to analyze fundamentally uncertain shocks, since the models are calibrated or estimated on past regularities and shocks that are expected or have a known distribution. DSGE models usually take into account deterministic simulations or stochastic simulations. In case of deterministic simulations, agents have perfect foresight and shocks are entirely expected. The assumption of fully anticipated shocks is relaxed in models with bounded rationality, related to backward looking agents, adaptive expectations and learning (see Woodford, 2013 for an overview). In stochastic simulations, agents in the model behave as if expected future shocks have zero mean; this is the certainty equivalence property. The uncertainty about the shocks is expressed in confidence intervals based on the known distribution of the shock. Hence, stochastic shock simulations deal with aleatory uncertainty, in the sense that randomness is based on a known distribution of shocks.

Our approach is different as it takes into account epistemic uncertainty by assuming that the distribution of shocks is unknown. We develop a novel empirical method that generates combinations of supply and demand shocks causing the observed high inflation. The relative contribution of the shocks to inflation is unknown, while output gap estimates are uncertain. The shock combinations represent the central bank's 'informed guess', which is shaped solely by the structure of the economy, via the *shock marginal rate of substitution (SMRS)* as defined in section 2, plus observed inflation and past information. The methodology can be applied to any DSGE model, and generalized to consider any number of observables and shocks in appendix A. This is our main contribution to the literature.

Our paper is divided into two parts, first, we apply our methodology in a small-scale New Keynesian model in which we isolate the effects of shock uncertainty. We conclude that monetary policy tightening in response to soaring inflation should be moderate, neither too mild nor too strong if there is fundamental uncertainty about the nature of the shocks. The intuition is that if the tightening is too mild, and the unobserved true shock combination is largely demand-driven (i.e. the true output gap is positive), the economy will overheat too much for too long. On the other hand, if monetary tightening is strong and inflation is driven by supply factors (i.e. the true output gap is negative), the policy response will cause a deep recession. This result is in line with Brainard's attenuation principle (1967), which implies that under uncertainty the central bank should respond to shocks more cautiously and in smaller steps than in conditions without uncertainty due to a fundamental lack of information.

In the second part, we draw some policy conclusions in the context of the 2022 inflation surge in the US economy. We apply our method to the model of Smets and Wouters (2007) (SW, henceforth) in a calibrated exercise built to match the US inflation peak in 2022Q2. It shows that there is a high probability that the shock combination that drove the inflation peak is roughly equally driven by supply and demand shocks, in line with previous research, however there is a non-negligible chance (close to 5%) that inflation is almost entirely driven by supply factors. In that case, a strong central bank response to inflation generates large losses, implying that a moderate response is desirable.

Our approach relates to Giannoni (2007), who simulates the effects of a variety of exogenous shocks, instead of a single exogenous innovation, with a New Keynesian model. He distinguishes efficient from inefficient supply shocks and considers uncertainty about the relative importance of each shock, as well as uncertainty about the persistence in the shock processes. In disagreement with our results, he finds that the robust optimal policy rule requires the interest rate to respond more strongly to fluctuations in inflation and the output gap than in the absence of uncertainty. Another related paper is Grassi et al. (2016), who select shocks in a DSGE model to avoid the estimation bias arising from imposing non-observable shocks. These are generated by non-existing exogenous processes. Based on the SW model, they find that selecting instead of imposing shocks substantially affects the values of the parameters that drive the propagation in the model. Our approach relates to theirs, as we also generate shocks instead of inferring them from the data.

A related strand of the literature assumes fundamental uncertainty about deep model parameters or key variables. Orphanides and Williams (2007) for instance assume that the central bank has imperfect knowledge about the natural rate of interest and unemployment, which may be related to structural economic changes. They examine the performance and robustness of monetary policy rules to this uncertainty and conclude that a more aggressive response to inflation and greater policy inertia (i.e.

a higher degree of interest rate smoothing) would be optimal compared to a situation with perfect knowledge. Ben-Haim and Van den End (2022) follow a similar approach based on a small-scale New Keynesian model. They find that a monetary policy strategy which sufficiently responds to inflation and the output gap is more robust against natural rate uncertainty than a response function equivalent to an estimated Taylor rule. Dennis et al. (2009) come to a similar conclusion based on a robust control method that supports decision making under model uncertainty. They find that if the central bank is uncertain about the persistence of inflation as modeled by a DSGE model, the optimal response is to be more aggressive in response to shocks.

The rest of this paper is structured as follows. In section 2 we theoretically show how uncertainty plays a role in the dynamics of inflation and output in a small-scale New Keynesian model. Section 3 presents the methodology developed to simulate the shock scenarios that form the central bank's guess, which is applied to the New Keynesian model in section 4, where we also assess robust monetary policy. In section 5 we apply our methodology to SW to the recent inflation surge in the US. Section 6 discusses the results in the wider context of policy responses to fundamental uncertainty, after which section 7 concludes.

2 Theoretical framework

We start our analysis focusing on a small scale forward looking NK model in the spirit of Gali (2008), augmented with a preference and a mark up shock as in Bhattarai et al. (2018),

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa Y_t + \kappa\gamma_\mu\mu_t \tag{1}$$

$$Y_t = E_t(Y_{t+1}) - \sigma(i_t - E_t(\pi_{t+1}) + \varphi_t - E_t(\varphi_{t+1})) \tag{2}$$

$$i_t = \phi_\pi\pi_t + \phi_Y Y_t \tag{3}$$

Where Y_t, π_t and i_t represent output, the inflation rate and the nominal policy rate as deviations from their respective steady state values. Furthermore, μ_t and φ_t represent a mark-up and a preference shock, which both follow an AR(1) process with persistence parameters ρ_μ and ρ_φ. Shock innovations follow a zero-mean distribution, and we don't impose further assumptions on its functional form. Also $\gamma_\mu = \frac{1}{\sigma^{-1}+\phi}$. $E(\cdot)$ is the expectations operator, assumed to be rational, which implies that agents understand the structure of the economy and know the composition of shocks. Furthermore, σ and ϕ represent the intertemporal elasticity of substitution and the Frisch elasticity of labor supply. The central bank reacts with full and credible commitment following a standard Taylor Rule that is known by the agents.

This set of assumptions does not mean that inflation expectations cannot de-anchor, or drift away from target, though this will then be based on the underlying economic conditions and model relations. We are able to isolate the impact of shock uncertainty from assumptions about expectation formation, which in itself would add an additional layer of uncertainty. This distinguishes our approach from for instance Orphanides and Williams (2007), who assume that the central bank is uncertain about the formation of expectations by economic agents.

Lastly, to determine optimal policy, which minimizes welfare losses in the context of uncertainty, we specify the following canonical loss function.

$$\mathcal{L} = \sum_{j=0}^{\infty} (\pi_{t+j}^2 + \lambda_y Y_{t+j}^2) \tag{4}$$

Where λ_y is the relative weight of the output gap in the loss function. Here we assume that the policymaker aims to limit squared deviations from the steady state of inflation and output, optimizing household welfare, as in the canonical New Keynesian model.

Central bank uncertainty is modeled in a similar way as in Giannoni (2007), in which the central bank faces the issue of choosing a policy to commit to, with imperfect information. In the approach presented in our paper, the central bank fully understands the structure of the model, however in period t the information set includes solely π_t. This implies that the central bank is not able to trace back μ_t and φ_t since Y_t is unknown.

The central bank cannot observe Y_t by assumption, which is problematic when determining monetary policy. Nonetheless, it is possible to use its knowledge about the true structure of the economy in the following way. Given the rational expectations assumption, after a preference shock the model has a solution of the form $Y_t = Z_\varphi^y \varphi_t$ and $\pi_t = Z_\varphi^\pi \varphi_t$ [1], where Z represents different model parameters. This implies that the one period rational expectation of any of these three variables x is $E_t(x_{t+1}) = Z_\varphi^x \rho_\varphi \varphi_t$. By substitution, one can derive the following expressions:

$$Z_\varphi^y = \frac{\sigma(1-\rho_\varphi)(1-\beta\rho_\varphi)}{(1-\beta\rho_\varphi)(1-\rho_\varphi+\sigma\phi_y)+\sigma\kappa(\phi_\pi-\rho_\varphi)} \tag{5}$$

$$Z_\varphi^\pi = \frac{\sigma\kappa(1-\rho_\varphi)}{(1-\beta\rho_\varphi)(1-\rho_\varphi+\sigma\phi_y)+\sigma\kappa(\phi_\pi-\rho_\varphi)} \tag{6}$$

The same holds for a mark-up shock μ_t, which yields the following coefficients Z_μ^y and Z_μ^π:

$$Z_\mu^y = \frac{-\sigma\kappa(\phi_\pi-\rho_\mu)}{(1-\beta\rho_\mu)(1-\rho_\mu+\sigma\phi_y)+\sigma\kappa(\phi_\pi-\rho_\mu)}\gamma_\mu \tag{7}$$

$$Z_\mu^\pi = \frac{\kappa(1-\rho_\mu+\sigma\phi_y)}{(1-\beta\rho_\mu)(1-\rho_\mu+\sigma\phi_y)+\sigma\kappa(\phi_\pi-\rho_\mu)}\gamma_\mu \tag{8}$$

These four coefficients contain all necessary information to understand the dynamics of the economy. Considering that the model is linearized, a given observation $\bar{\pi}_t$ of inflation can be explained for any combination of shocks (μ_t, φ_t) such that:

[1] Note in the monetary policy setting of (3), the interest rate rule if fully endogenous since we don't consider monetary policy shocks.

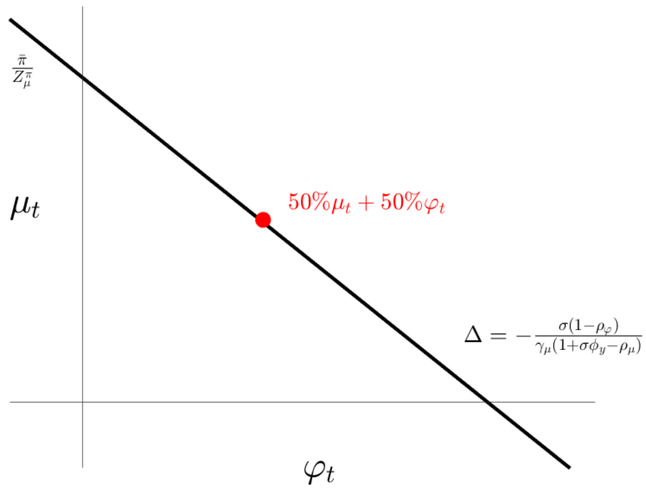
$$\bar{\pi}_t = Z^{\pi}_{\varphi}\varphi_t + Z^{\pi}_{\mu}\mu_t \tag{9}$$

Re-writing (9) and assuming for simplicity but without loss of generality that $\rho_\mu = \rho_\varphi$:

$$\mu_t = \left(\frac{(1-\beta\rho_\varphi)(1-\rho_\mu+\sigma\phi_y)+\sigma\kappa(\phi_\pi-\rho_\mu)}{\gamma_\mu\kappa(1+\sigma\phi_y-\rho_\mu)}\right)\bar{\pi}_t - \left(\frac{\sigma(1-\rho_\varphi)}{\gamma_\mu(1+\sigma\phi_y-\rho_\mu)}\right)\varphi_t \tag{10}$$

The term multiplying $\bar{\pi}_t$ broadly determines the magnitude of the shocks, whereas $\frac{\sigma(1-\rho\varphi)}{\gamma_\mu(1+\sigma\phi_y-\rho_\mu)}$ can be defined as the *shock marginal rate of substitution (SMRS)*, which determines by how much φ_t has to increase for a marginal decrease in μ_t in such a way that both shocks together still explain $\bar{\pi}_t$. Broadly speaking, one can infer that the *SMRS* is determined by the persistence of the shocks (via the expectations channel) and by the magnitude in which supply shocks affect the natural output (γ_μ). In section 3 we explain how the *SMRS* is a key source of information that, under the assumption that shocks have a finite variance, can help understand (not pin down) potential sources of observed inflation. Within the information set of the central bank, any shock combination that fulfills the restriction (10) is a potential candidate to explain $\bar{\pi}_t$. The infinite number of shock combinations that explain a certain observation of inflation lies on the line formed by this function, which for a positive $\bar{\pi}_t$ looks as follows:

Figure 2: Shock drivers of $\bar{\pi}_t$ as determined by the shock marginal rate of substitution



Graphical representation of (10). The points on the black line represent shock combinations that explain a given inflation observation ($\bar{\pi}_t$), while the red dot shows the point in which $\bar{\pi}_t$ is equally explained by supply and demand factors.

Note that the values of this function that intersect the vertical and horizontal axes represent the unique single shocks that yield observed inflation without further interference from the other shock.

The values on the line correspond to shock combinations with various shares of mark-up and preference shocks, that together attain $\bar{\pi}_t$. The red dot represents a scenario in which inflation is equally driven by supply and demand factors, therefore shock combinations to the left of this point represent a state of the world dominated by supply effects and vice versa for points at the right.

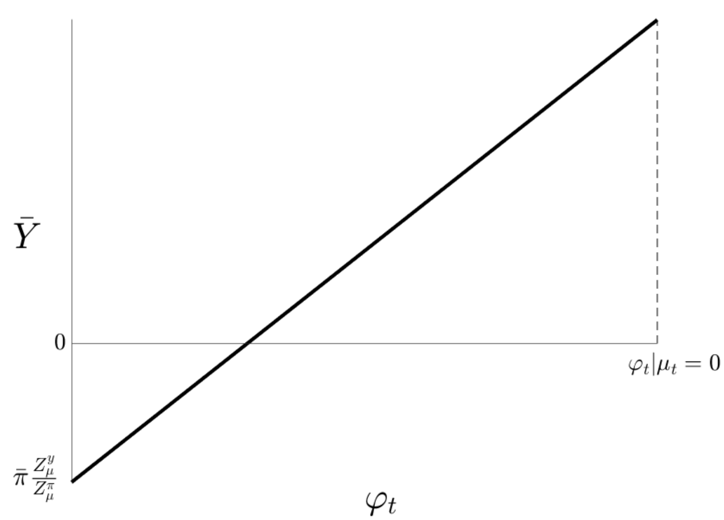
Y_t is by assumption unknown, however, the central bank knows that (10) holds, given the observed value of $\bar{\pi}_t$. The shock combinations, that determine a certain observed inflation rate $\bar{\pi}_t$, have significantly different effects on the output gap. Similar to (9), the impact of any combination of shocks on the output gap can be expressed as a linear combination of the shocks and the coefficients defined before,

$$\bar{Y}_t = Z^y_\varphi \varphi_t(\bar{\pi}_t) + Z^y_\mu \mu_t(\bar{\pi}_t) \tag{11}$$

Where $\bar{Y}_t$ represents the realized value of the output gap generated by the pair of shocks $(\varphi_t(\bar{\pi}_t), \mu_t(\bar{\pi}_t))$ that satisfies the constraint (10). Substituting and simplifying we derive the following expression, summarized in Figure 12,

$$\bar{Y}_t = \left(Z^y_\varphi - \frac{\sigma(1-\rho_\varphi)}{\gamma_\mu(1+\sigma\phi_y - \rho_\mu)} \right) \varphi_t + \bar{\pi}_t \frac{Z^y_\mu}{Z^\pi_\mu} \tag{12}$$

Figure 3: Illustrative example of output gap movements as a function of φ_t, fulfilling the constraint defined by eq.(12)



Graphical representation of (12).

Intuitively, (12) determines that the output gap will be negative if the observed level of inflation $\bar{\pi}_t$ is solely explained by a mark-up shock (note that Z^y_μ is negative), and vice versa if inflation is driven by a preference shock. The higher the weight of the demand shock that drives inflation, the higher the output gap, becoming positive eventually. This stylized framework helps to point out how the sources of inflation matter, since the trade-off between inflation and output gap stabilization depends on the direction of the shock effects on inflation and output.

Hence, assessing the source of inflation is crucial for the central bank. Theoretically, any combination of shocks that satisfies restriction (10) is a potential scenario that explains the observed level of inflation. Nonetheless, the policy implications differ markedly according to equation (12). These two restrictions summarize the information set of the central bank, which involves a large degree of uncertainty considering that welfare is defined by equation (4). In the next section we introduce our novel approach which helps the central bank to make an informed guess about the potential sources of inflation.

3 Methodology

Due to the lack of information the central bank cannot statistically pin down the true shock combination that determines the observed inflation rate. Therefore it must assess the potential welfare losses of its reaction function (i.e. the parameters of the Taylor rule) without knowing whether inflation is determined by supply and/or demand factors. We define robust monetary policy to shock uncertainty as the policy parameters that mitigate the risk of extreme welfare losses. In this section we explain how the central bank can use its knowledge about the structure of the economy, provided by the *SMRS*, and how it can cope with uncertainty to provide an informed guess of the potential range of shock scenarios. This helps to determine the optimal policy stance.

As shown in the previous section, any shock combination that fulfills equation (10) is a potential scenario to explain $\bar{\pi}_t$. Observing the structure of the economy implies that the central bank observes the *Shock Marginal Rate of Substitution* as defined above, which is a function of the structural parameters of the model. Following (10), note that if the $SMRS \to \infty$ (i.e. a very steep slope in Figure 2), the relative size of a supply shock necessary to explain inflation is significantly larger than demand shocks, and vice versa if the *SMRS* tends to approach 0. This implies that, for shock innovations with finite variance, the *SMRS* by itself is somewhat informative in providing a general sense of what can be the main driver of the observed fluctuations of inflation and output. However it is not sufficient information for point estimates.

To introduce this information in the central bank's guess of the potential drivers of inflation, we employ the algorithm of Sims and Zha (2012) by minimizing the following function,

$$MIN_{\zeta_t} \quad F_t(\pi_t) = (\bar{\pi}_t - \pi_t)^2 \tag{13}$$

Where function $F(\pi_t)$ is a standard loss function (not to be confused with 4) used to minimize the distance between observed inflation $\bar{\pi}_t$ at time t, and the inflation rate (π_t) determined by the shock vector $\zeta_t = [\varphi_t, \mu_t]'$. Note that the gradient of $F(\pi_t)$ depends on the structure of the economy, π_t is determined directly by equation (9), and for a simple model as the one described in section 2 can be derived analytically.[2]

We exploit the property that, since ζ_t is composed of more shocks than observables included in $F(\pi_t)$, there is an infinite amount of shock combinations ζ_t^* that will yield $F(\pi_t) = 0$. Particularly, these shock combinations by construction are represented by the line (9). Note that if the number of shocks in ζ_t is the same as the number of observables in (13), all state variables of the model are observed and therefore the result of the algorithm is a standard shock decomposition.

The key contribution of our paper comes from applying the Sims and Zha (2012) algorithm to the uncertainty in the central bank information set. The algorithm, formalized in (13), runs K iterations until it finds a (either local or global) minimum of $F(\pi_t)$, starting at an initial guess in which both shocks are zero.[3] In each iteration k the vector $\zeta_{t,k-1}$ is updated with the gradient of (13), found by the algorithm. This is formalized by the following iterative process,

$$\begin{pmatrix} \mu_{t,k} \\ \varphi_{t,k} \end{pmatrix} = \begin{pmatrix} \mu_{t,k-1} \\ \varphi_{t,k-1} \end{pmatrix} + \begin{pmatrix} \Delta\mu_{t,k} \\ \Delta\varphi_{t,k} \end{pmatrix} \tag{14}$$
$$\text{with} \quad \mu_{t,0} = \varphi_{t,0} = 0$$

Where $\Delta\mu_{t,k}$ is the gradient of (13) evaluated in $\mu_{t,k-1}$ (the same procedure is applied to $\Delta\varphi_{t,k}$). After K iterations, the algorithm converges to a solution that, by construction of this particular exercise, minimizes $F(\pi_t)$, selecting a point on the line plotted in Figure 2.

In a general sense, Sims and Zha (2012) is designed to minimize any numerical function with a gradient descent approach.[4] This algorithm estimates the gradient of the function via quasi-Newton methods and BFGS update, implying that it is able to accurately estimate the gradient of $F(\pi_t)$, even with a high degree of curvature. Our use of the Sims and Zha (2012) algorithm can be applied to any DSGE model, regardless of the size and complexity, since it is able to estimate the gradient when analytical solutions are untractable. It can also include a larger central bank information set as detailed in appendix A.

This exercise allows the central bank to use the information given by the *SMRS*. Furthermore, we extend this approach in order to account for uncertainty. This is done by a Monte Carlo experiment in which we run the above-mentioned version of Sims and Zha (2012) N times. In each simulation n,

[2]Precisely:

$$\left[2Z_\varphi^\pi(\bar{\pi}_t - Z_\varphi^\pi\varphi_t - Z_\mu^\pi\mu_t), \quad 2Z_\mu^\pi(\bar{\pi}_t - Z_\varphi^\pi\varphi_t - Z_\mu^\pi\mu_t)\right]$$

[3]This condition responds to economic intuition. If the gradient of a shock is null then it implies that the shock has no implications in terms of inflation dynamics. Thus, the best guess for that hypothetical shock is 0.

[4]This algorithm is used in the literature for different purposes, such as for finding policy and value functions, or for calibrating parameters via impulse response matching.

the estimated gradient is interacted with a random variable d_n that is governed by a $Uniform(0,1)$ distribution in the following setting,

$$\begin{pmatrix} \mu_{t,k} \\ \varphi_{t,k} \end{pmatrix} = \begin{pmatrix} \mu_{t,k-1} \\ \varphi_{t,k-1} \end{pmatrix} + \begin{pmatrix} \Delta\mu_{t,k} \cdot (1-d_n) \\ \Delta\varphi_{t,k} \cdot d_n \end{pmatrix} \tag{15}$$
$$\text{with} \quad \mu_{t,0} = \varphi_{t,0} = 0$$

In each iteration k of simulation n, the algorithm computes the gradient of $F(\pi_t)$, which we then multiply by different random numbers d_n or $1 - d_n$. This implies that the combination of shocks, once the algorithm found the solution (ζ_n^*), is tilted towards either a mark-up or a preference shock dominated scenario, depending on d_n. Note that in a random draw in which $d_n = 1$ we find the shock combination $[\varphi_t, \mu_t] = [\bar{\pi}_t/Z_\varphi^\pi, 0]$, thus, a state of the world in which inflation is fully determined by a demand shock. On the contrary, with a random draw of $d_n = 0$, we locate $[\varphi_t, \mu_t] = [0, \bar{\pi}_t/Z_\mu^\pi]$. The Monte Carlo exercise therefore yields a distribution of paired shocks that lie on the line defined by (10) and so explaining $\bar{\pi}_t$.

With a sufficiently large N we are able to find a guess of the range of shock combinations in (10) considering that the iterative process described in (15) tilts the results towards a particular region of Figure 2, according to the structure of the economy and d_n. For a large enough N, it is possible to map the entire relevant region of the shocks, determined by the sign of the gradient of (13). For an observation of a positive inflation rate this implies a strictly positive supply or demand shock. This is another piece of information as proceeded by the *SMRS*.

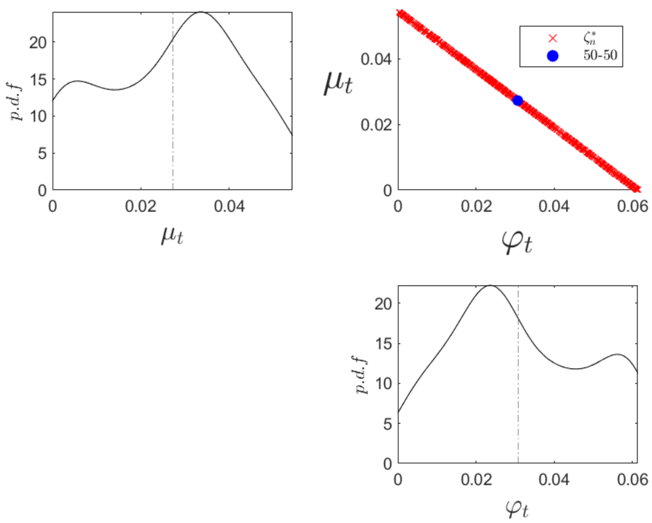
The entire set of ζ_n^* shock combinations constitute the central bank's informed guess. This set of combinations includes the information given by the structure of the model, as well as uncertainty by considering the full range of potential weights of supply and demand forces to explain inflation. This way we find N shock combinations that first incorporate the information of the economy via the *SMRS*, and also account for uncertainty given that the shock combinations found are potential scenarios. This way the central bank has the capacity to analyze what monetary policy is robust to uncertainty by doing a welfare analysis on the full range of shock combinations N (Appendix A elaborates on the conditions and the procedure to enrich the exercise proposed in this section with more variables, shocks and larger scale models). In the next section we apply the methodology, based on the small-scale New Keynesian model described in section 2, to understand the use of the algorithm for monetary policy.

4 Shock analysis and robust policy responses

In this section we analyze the outcome of the shock exercise through the lens of the small-scale New Keynesian model described in section 2, to assess the robustness of monetary policy under uncertainty. We compute the ζ_n^* shock combinations and analyze the distribution of potential losses under N potential scenarios. The robustness of monetary policy, which aims at minimizing tail risks instead of the average expected loss, is assessed, taking into account the drivers of potential extreme welfare losses.

To understand the results of the proposed method, we consider a standard calibration of the model taken from Gali (2008) and determine the potential causes of an observed positive inflation rate $\bar{\pi}_t > 0$.[5] We find the set of shock combinations ζ_n^* as plotted on the downward sloping red line in Figure 4.

Figure 4: Simulated shock combinations based on small-scale New Keynesian model and a standard calibration



ζ_n^* for the New Keynesian model and a standard calibration ($N = 1000$) and $\bar{\pi}_t = 2\%$. The red x shows the N combinations of shocks ζ_n^*, while the blue dot 50-50 represents the point in which inflation is equally driven by supply and demand factors, which is also reflected in the dotted vertical lines in the left and lower panels.

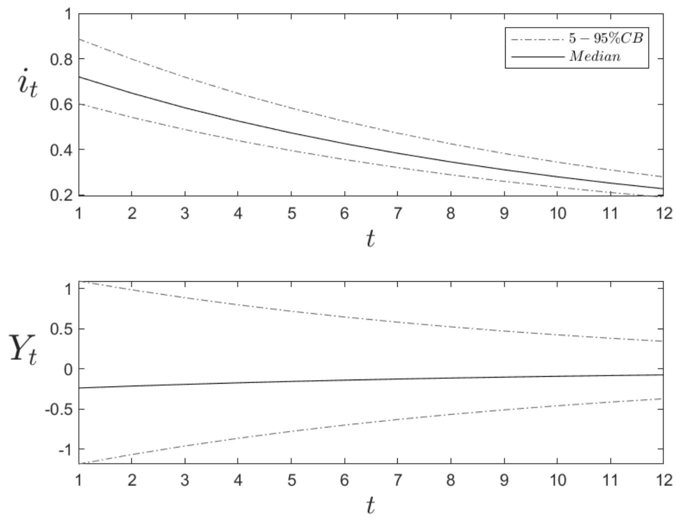
For this particular model and calibration the algorithm finds that most shock scenarios are supply driven (the mass of red dots on the downward sloping line in the top right panel is concentrated at the left of the blue dot). There is also a large mass of demand driven scenarios, which are however 25% less likely than supply dominated scenarios. Without imposing the restriction that both shocks must be inflationary, the algorithm finds shocks that meet this property, while being evenly spread across the positive domain. Intuitively, in this stylized example, the *SMRS* determines that supply shocks have to be slightly smaller in size compared to demand shocks to explain inflation dynamics and therefore supply shocks are more likely according to Sims and Zha (2012). Furthermore, in this particular case the distribution of shocks is bimodal and considerably evenly spread around the domain, which implies that the model is not very informative about the potential causes of inflation. In a more complex setting such as the case study discussed in section 5, we show that with more information the

[5] $\sigma = 1, \kappa \approx 0.05, \phi = 1, \rho_\varphi = \rho_\mu = 0.9, \beta = 0.99, \phi_\pi = 1.5, \phi_y = 0.5/4$

algorithm can be highly informative.

The implications for the policy interest rate and the output gap are shown in Figure 5. Note that the range of the interest rate outcomes is fairly stable and narrow. However, the shock scenarios have a wide ranging impact on the output gap (as explained in Figure 3).

Figure 5: Simulated paths of interest rate and output gap in the small-scale New Keynesian model based on the shock combinations in Figure 4



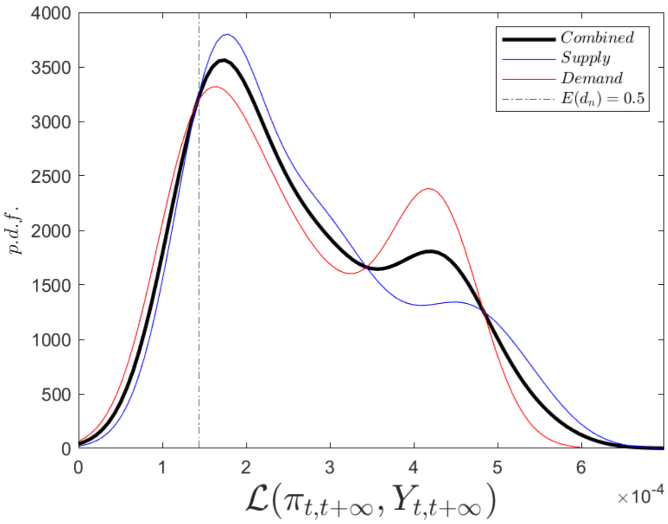
Variables expressed in p.p. deviation from the steady state.

If the central bank was able to observe the shock combination, the optimal Taylor rule response depends on whether the shock scenario is supply or demand driven. In demand driven states of the world, it is optimal to react aggressively to cool down the economy as fast as possible. However, if the economy is hit by supply shocks, the central bank has to balance its response between the objectives of inflation and output gap stabilization to limit welfare losses. We further explore these policy considerations by analyzing the distribution of welfare losses associated with the shock combinations, shown in Figure 6. We distinguish “supply” from “demand” states of the world by identifying the main source of inflation in each ζ_n^*.[6]

The loss distribution displays bimodality (assuming the baseline Taylor rule with $\phi_\pi = 1.5$). The biggest (left) hump is associated with shock combinations that are almost equally driven by supply and demand shocks and the hump at the right tail by scenarios mainly driven by one of the shocks. The intuition is that shocks that are almost equally driven by supply and demand have an output gap closer to zero than scenarios that are dominated by one type of shock. Thus, the left hump corresponds

[6]According to this classification, if a given ζ_n^* inflation is explained infinitesimally more by μ_t than φ_t, then it is considered a supply driven state of the world, and vice versa.

Figure 6: Loss ($\mathcal{L}$) distribution generated by the shock combinations identified in Figure 4 based on the small-scale New Keynesian model



The red line reflects the loss distribution of shock combinations mainly driven by demand shocks, and the blue line the loss distribution of supply dominated shock scenarios. The thick black line shows the loss distribution considering all scenarios together. The vertical dotted line $E(d_n = 0.5)$ determines the expected scenario, which is equally driven by a supply and demand shock.

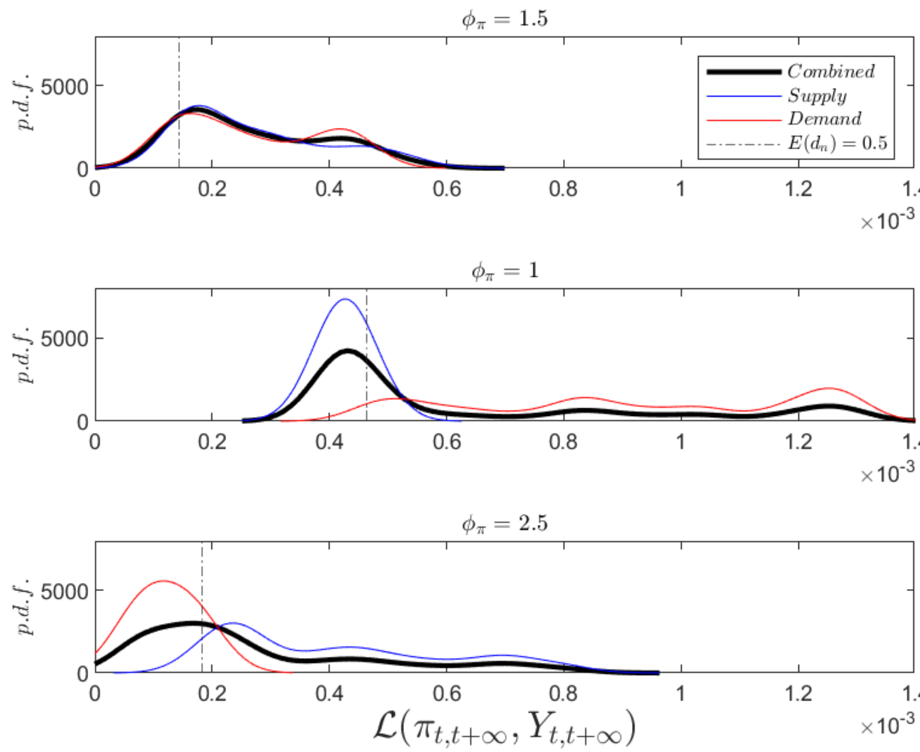
to shock combinations close to the expected shock combination $E(d_n)$, whereas the right hump, with bigger losses, is mainly driven by one type of shock. While scenarios dominated by demand shocks are more likely to generate larger losses (red line), the far right tail of the loss distribution is driven by supply dominated shock scenarios (blue line).

For optimal monetary policy this generates a trade-off. On the one hand, the central bank would like to react stronger to inflation to minimize the demand driven right tail of the loss distribution. On the other hand, if the true shock combination is supply driven, a strong increase of the policy interest rate would make the output gap more negative and so increases welfare losses. One could also argue that the central bank must reduce tail losses, which in this case are driven by states of the world in which supply shocks (μ_t) explain almost all observed inflation. Following this logic, the central bank should react mildly to high inflation to limit tail losses, at the risk of incurring losses in demand driven states of the world.

To assess the robustness of monetary policy to shock uncertainty, we take the loss distribution in period $t + 1$, which is influenced by different policy responses to inflation. Particularly, we look at the case of a milder ($\phi_\pi = 1$) and stronger ($\phi_\pi = 2.5$) response to inflation by the central bank.

It shows that if the central bank would react mildly to inflation ($\phi_\pi = 1$ in the middle panel),

Figure 7: Loss distribution generated by the shock combinations identified in Figure 4 for different ϕ_π



The red lines are the shock combinations mainly driven by demand shocks (i.e. Monte Carlo simulations with $d_n > 0.5$), and the blue lines are the supply dominated shock scenarios. The loss distributions $E(d_n = 0.5)$ relate to the expected scenario, which is equally driven by a supply and demand shock.

the entire loss distribution shifts to the right (i.e. higher losses for each possible scenario). There are demand dominated scenarios in which losses are more than twice as high as in the case of supply dominated scenarios. Supply shocks can generate large losses as well with a mild monetary policy response compared to the baseline response, but the loss distribution of supply shocks looks Gaussian and does not have a fat right tail.

In case the central bank reacts strongly to inflation ($\phi_\pi = 2.5$), the loss distribution of demand dominated shock scenarios is centered towards its mean, while overall losses are limited (lower panel).[7] However, in the case of supply driven scenarios, the losses can be more than eight times higher than in demand driven scenarios. This is because reacting strongly to inflation in case of supply shocks more likely worsens the output gap.

[7] A similar result is found in the learning literature, where under expectations based on adaptive learning, a stronger reaction to inflation is deemed necessary (Gaspar et al., 2011). There, uncertainty amounts to learning about the true law of motion which in some sense implies uncertainty about shocks.

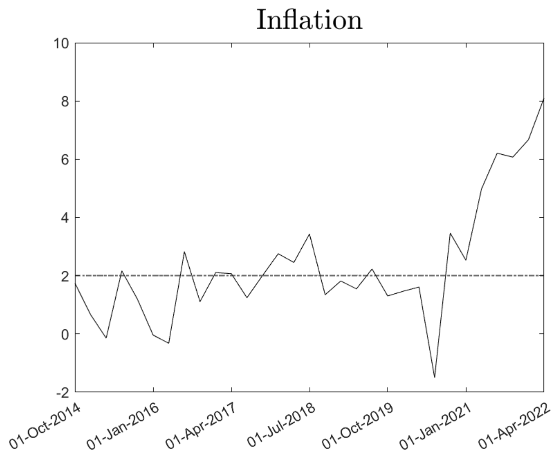
The results imply that robust monetary policy aimed at preventing extreme losses should react neither too aggressively nor too mildly to inflation. Although a strong response to inflation is optimal in the subset of demand driven scenarios, the right tail of the loss distribution of combined supply and demand shock scenarios is fat and long, implying that an aggressive response to inflation is not robust optimal. This holds in particular for supply driven scenarios. In the next section we use the intuition from this exercise to extract conclusions about robust monetary policy responses to the inflationary spiral in the US in the aftermath of the pandemic.

5 Application to the US economy

5.1 Initial conditions

In this section we apply the algorithm to the US economy to analyze the inflation surge in the context of shock uncertainty. Since 2020, the US and other advanced economies have faced a series of unprecedented inflationary shocks, which led to an increase in inflation substantially above target. The GDP deflator in the US peaked at 8.8% in 2022Q2 (see left panel of Figure 8).

Figure 8: Observed inflation measured as annualized change of the US GDP deflator



We simulate shock scenarios with the method presented in section 3, based on the SW model. This is a medium-scale log-linearized DSGE model, estimated for the US. The model equations pin down the dynamics of capital formation, consumption, investment, the rental rate of capital, labor, wages, inflation, and the value of the capital stock. The model dynamics are driven by supply shocks (technology, wage and price mark-up), demand shocks (risk premium, government spending), and a monetary policy shock. The mark-up and preference shocks affect the dynamics of the model through the Phillips and IS curves, equations (16) and (17) respectively:

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 \pi_{t+1} - \pi_3 (mpl_t - w_t) + \mu_t \tag{16}$$

$$c_t = c_1 c_{t-1} + (1 - c_1)c_{t+1} + c_2(l_t - l_{t+1}) - c_3(i_t - \pi_{t+1}) + c_3(\varphi_t - \varphi_{t+1}) \tag{17}$$

In (16) inflation depends on past and expected future inflation and on mark-up shock μ_t. The effect of mark-up shocks on the economy is determined by the Calvo price adjustment parameter which has a rather strong influence on π_3. In (17), consumption c_t depends on past and expected future consumption, expected growth in hours worked (l_t, l_{t+1}), the real interest rate gap, and a preference shock φ_t. A positive preference shock (demand shock) increases consumption and exerts upward pressure on real factor prices, and thus on inflation via an increase of marginal costs. In order to simplify and better understand the driving forces of our results we assume that the mark-up and preference shock both follow an AR(1) process, instead of an ARMA(1,1) process as in SW.

To assess whether the response of the US Federal Reserve (Fed) was robust in an environment of shock uncertainty, we calibrate the model on the recent macroeconomic outlook. Based on a Kalman filter we estimate the initial conditions of the US economy in 2022Q1, which we then use to run the algorithm detailed in section 3 to explain the observed inflation peak in 2022Q2 by scenarios of mark-up and preference shocks.[8] To run the Kalman filter, we use as calibration the posterior parameter estimates and the same observables as in SW except for the short-term interest rate, which we proxy by the shadow rate of Wu and Xia (2016), with quarterly observations for the 1991Q1- 2022Q1 period. The shadow rate also reflects the effect on monetary conditions of Quantitative Easing by the Fed. The Kalman filter estimates the unobserved variables as well.[9] In this setting we find an estimate of the output gap $Y_{2022Q1} = -2.66\%$. Furthermore, our definition of $F(\cdot)$ in (13) considers only inflation as an observable variable, as in section 3.1.

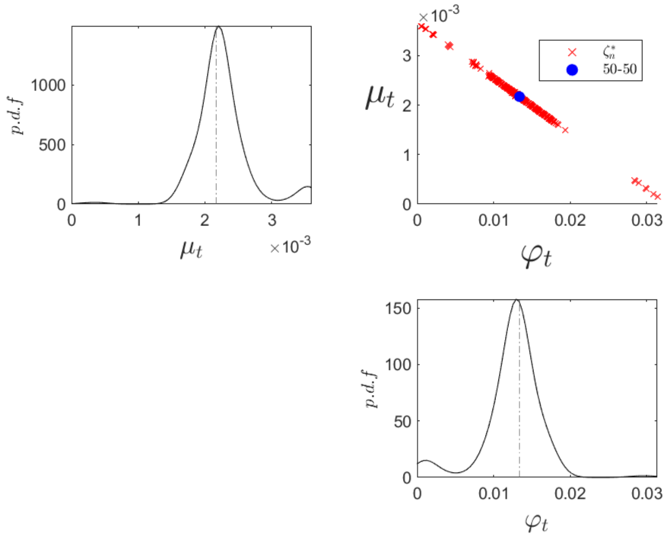
5.2 Shock scenarios and robust monetary policy

In this section, we use the algorithm defined in section 3 to determine the shock combinations that define the central bank's informed guess, conditional on posterior estimates of the parameters in the SW model. The initial conditions of the model at $t-1$ follow from the Kalman filter. Figure 9 includes the full range of ζ_N^* potential shock drivers of peak inflation (π_{2022Q2}), showing that most scenarios are composed of an even mixture of supply and demand shocks (most mass of the red dots on the downward sloping line in the top right panel - i.e. the SW equivalent of (10) - is centered around the blue dot).

[8] For simplicity and comparability with the stylized example of section 2 we limit our analysis to these two shocks only, although we could have chosen more shocks, up to all of the ones modeled in SW, to get a more comprehensive picture of inflation drivers.

[9] For the Kalman filter we assume that all shocks are MA. So we achieve that there is no influence of other shocks beyond the mark-up and preference shock in driving the economy when we apply the algorithm. So we fully isolate the effect of the selected shocks.

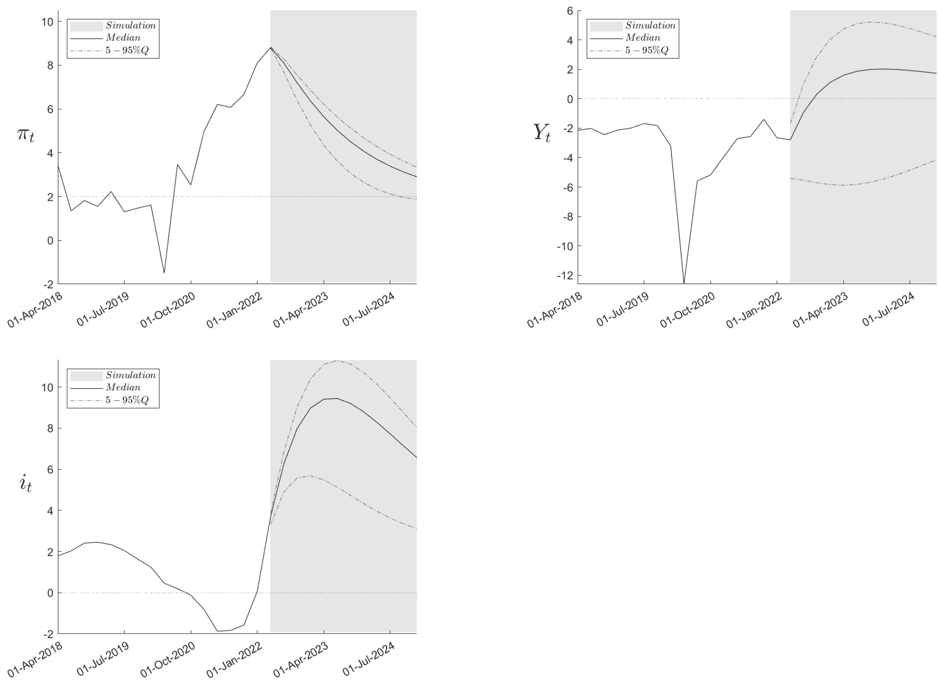
Figure 9: Simulated shock combinations explaining the inflation rate in 2022Q2, based on SW



ζ_n^* for the SW model ($N = 1000$) explaining observed peak inflation in 2022Q2) (ie GDP deflator under the estimated initial conditions following from the Kalman filter estimates for 1991Q1-2022Q2). The red x show the N combinations of shocks ζ_n^*, while the blue 50-50 dot is the point in which inflation is equally driven by supply and demand factors, which is also reflected in the dotted vertical lines in the left and lower panels.

Note that the distribution of supply shocks has a fat right tail, also reflected in the low mass at the right end of the top right panel. This suggests that it is unlikely that demand shocks alone were the main driver of the inflation surge and that it is more obvious that inflation was driven by a roughly equal share of supply and demand shocks. Furthermore, the considerable mass of supply driven shocks implies that an overly strong reaction by the central bank likely leads to adverse welfare dynamics. This finding is key for our assessment of the robustness of monetary policy to shock uncertainty, as we discuss below. Figure 10 shows the median path of key model variables, based on the simulated shock scenarios, accompanied by the 5 and 95 quantiles of the simulated path distribution implied by ζ_N^*.

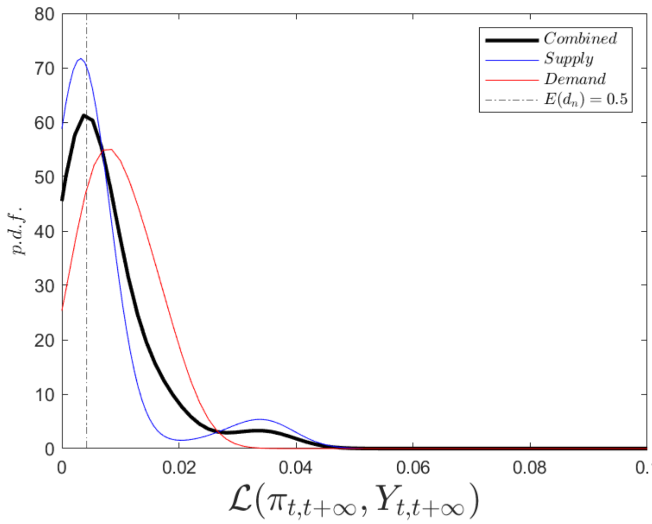
Figure 10: Simulated path of the US economy based on the shock combinations shown in Figure 9



Inflation is the annualized quarterly change of the US GDP deflator. The output gap is the percentage deviation from steady state value, estimated up to 2022Q1 with the Kalman filter used to determine the initial conditions. The interest rate is the Wu and Xia (2016) shadow Federal funds rate (period averages). The shadow area of each plot represents the simulated paths conditional on the shock combinations shown in Figure 9.

The persistence of inflation varies from one state of the world n to another. Inflation is well behaved as the quantiles of the distribution do not indicate excess dispersion. Compared to inflation, the ranges of the interest rate and output gap outcomes are much wider. The median interest rate shows a continuous increase until the end of 2023 when it peaks. The median interest rate is high relative to the lower quantile, because the median value of the output gap becomes positive during the simulation horizon, to which the central bank responds according to the Taylor rule, indicated by the rather large median response of the interest rate. Nonetheless, there is also a probability that the output gap remains negative for an extended period of time, and might not even reach the estimated 2022Q1 value (-2.66%) before 2025Q1. These states are reflected in the lower band of the range of output gap outcomes. Such conditions present a risk for hawkish monetary policy, as we show below.

Figure 11: Loss ($\mathcal{L}$) distribution generated by the shock combinations identified in Figure 9 for SW

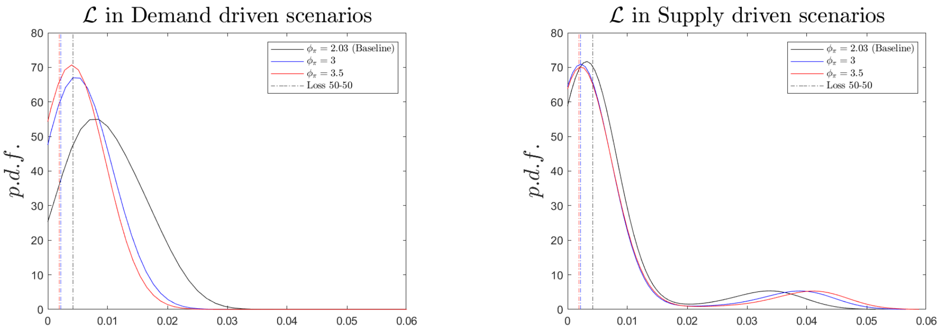


Demand shock driven scenarios represent the subset of shock combinations ζ_N^* with $d_n > 0.5$ and supply shock driven scenarios with $d_n \leq 0.5$. The dotted line represents the realized loss $E(d_n = 0.5)$ under the expected scenario the expected scenario.

The loss distributions following from the simulations for the US economy with the SW model display similar features as the ones generated by the small-scale New Keynesian model in Figure 6. For the largest mass of shock combinations, welfare losses are relatively low. However, there is bimodality in the distribution of supply shock driven scenarios. Supply driven states likely generate lower welfare losses than demand dominated ones, although the supply loss distribution has a small fat tail at the right. This tail corresponds to the shock combinations that are explained almost entirely by the mark-up shock, as shown in Figure 9. Contrary to the findings in the small-scale New-Keynesian model, demand shocks do not present a tail risk in the simulations with the SW model. This implies that the information taken into account in this exercise (via the Kalman filter initial conditions, observed inflation in 2022Q2, the model structure of SW and the estimated parameter values) means that inflation is unlikely dominated by demand factors. This provides evidence that the implied *SMRS*, together with the included data, reveal key facts that are relevant for monetary policy under fundamental uncertainty.

To assess the robustness of monetary policy, we repeat the exercise conducted in section 4, with the SW model, for the robust Taylor rule responses (ϕ_π) summarized in Figure 7. The outcomes are used to assess the impact of the central bank's response to inflation on the loss distribution. Particularly, we conduct experiments of more aggressive responses to inflation, both for demand and for supply dominated shock scenarios. The resulting loss distributions are shown in the following figure.

Figure 12: Loss ($\mathcal{L}$) distribution of the shock combinations in Figure 9 differentiated by the main inflationary driver for different ϕ_π



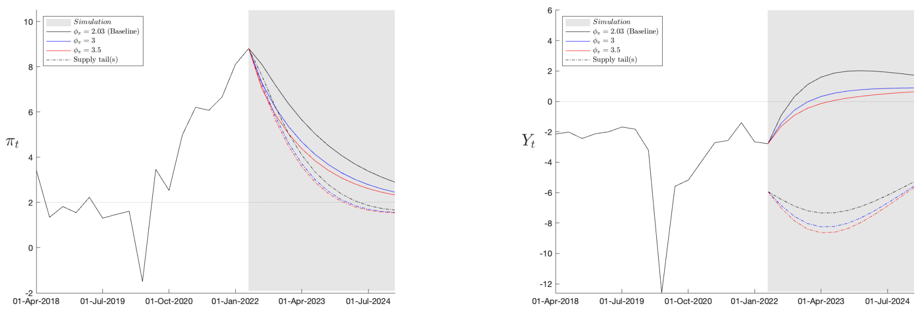
Loss distributions associated with the main driver of the scenario being a mark-up shock (left panel) or preference shock (right panel), for different values of ϕ_π. Demand shock driven scenarios represent the subset of shock combinations ζ_N^* with $d_n > 0.5$ and supply shock driven scenarios with $d_n \leq 0.5$.

In the demand driven shock scenarios, a stronger response to inflation lowers welfare losses (left panel). Even more, increasing ϕ_π reduces the tail losses for demand dominated shock combinations. It implies that responding more strongly to inflation is robust optimal in demand driven states of the world. This intuition does not hold for supply driven states (right panel). If in those states ϕ_π is raised, the tail of the loss distribution shifts to the right. When the mix of supply and demand shocks is close to 50%, increasing ϕ_π marginally improves welfare. The intuition behind this last finding is that whenever there is a significant share of preference shocks driving the economy, even if supply forces dominate, reacting more strongly to inflation has little impact. Nonetheless, if the output gap is deeply negative (i.e. the mark-up shock heavily dominates) then increasing ϕ_π is detrimental since it lowers the output gap even further.

These findings imply that the central bank should take into account the tail losses associated with a strong response to inflation. Robust monetary policy that aims at limiting tail losses should particularly be careful to respond strongly to inflation in supply driven scenarios. Although such a response reduces the mode of the loss distribution (the red and blue distributions shifting to the left in the left panel of Figure 12) and so improves the expected policy performance, it also goes with an increased tail risk of extreme losses.

To further explain the results, Figure 13 shows the median simulated path for the key US economic variables, considering different Taylor rule responses to inflation. It distinguishes the full sample of shocks pairs ζ_N^* (solid lines) from those that are heavily driven by supply (dotted lines), meaning the subset that satisfies $\zeta_N^*|\mu_n \geq 3 \cdot 10^{-3}$.

Figure 13: Simulated median paths of the US economy, disaggregated to either include the full range of shock scenarios (solid lines) or the supply drive tail (dotted lines)



The solid lines represent the median outcome of the simulated inflation and output gap as a function of ϕ_π. The dotted lines represent the median path of each variable, considering only shock combinations that are largely driven by supply factors (i.e. $\mu_t \geq 3 \cdot 10^{-3}$).

The left panel shows that inflation drops when ϕ_π is raised. In each scenario, the inflation outcomes, including the tail outcomes, become lower when the monetary policy response to inflation is stronger. This desired effect of a stronger response to inflation also holds for the output gap in the median scenario (right panel). In that case, raising ϕ_π reduces the output gap, consistent with the loss distributions analyzed above. However, in the supply driven tail outcomes the output gap is deeply negative on impact. Hence, reacting more strongly to inflation leads to an even more negative output gap. This adversely affects welfare even if inflation becomes close to target at the beginning of 2024.

In conclusion, our analysis of welfare losses and robust monetary policy suggests that, under the central bank's best guess of the drivers of observed inflation, it is not possible to reject the hypothesis that supply shocks are the dominant source of fluctuations in the US economy. Therefore, a risk management approach for robust monetary policy calls for moderate policy responses, given the potential tail risks.

6 Discussion

The desired policy rule would be different if the central bank had prior information. The central bank can have optimistic expectations about the state of the economy, or have a particular information set that allows it to determine the shock combination, and decide to react accordingly. Hence, if there is certainty that the shock combination will yield limited losses (e.g. when inflation is mainly driven by a demand shock, which supports output) then it would by definition be optimal to implement an aggressive inflation focused policy rule to achieve a better performance in terms of a lower welfare loss.

However, if the shock scenario is dominated by supply factors and subsequent losses are known to be concentrated at the right side of the loss distribution, a very strong inflation response is less obvious.

In practice, it is unlikely that a policymaker has useful prior information in a situation where unprecedented shocks create fundamental uncertainty. Such conditions call for a risk management approach. This strategy consists of implementing a policy rule that aims to prevent tail losses; not because the policymaker is certain that high losses will materialize (most likely via supply dominated shock scenarios), but to avoid the small probability of large losses. Such a risk management strategy will be desirable even if it implies that the best possible performance becomes less likely (as reflected by the mode of the loss distribution being located more to the right). However, a risk management approach would prefer robustness to extremely costly dynamics more than achieving the (small) possibility in the best possible state of the world.

The literature covers different risk management strategies that deal with uncertainty in monetary policy. Most of these strategies deal with measurable uncertainty or risk. For instance Brainard's attenuation principle mentioned in the introduction. It would call for policy gradualism and less aggressive responses to economic shocks. There are also risk management strategies that call for a more aggressive response by the central bank, for instance if there is uncertainty about the persistence of the rate of inflation (Tetlow, 2018). More aggressive policy measures could then be needed to prevent an adverse shock from destabilizing inflation expectations.

A well-known strategy for managing Knightian uncertainty is robust control. Robust control insures against the maximally worst outcome (min-max) as defined by the policymaker (see Hansen et al., 2006; Hansen and Sargent, 2008, and Williams, 2007). Dennis et al. (2019) and Olalla and Gómez (2011) apply robust control to a New Keynesian model to study the effect of model uncertainty in monetary policy. Typically, policies derived through min-max are more aggressive compared to those derived under no uncertainty. Orphanides and Williams (2007) add that a higher degree of interest rate smoothing would be optimal under fundamental uncertainty. Simulating different smoothing parameters is beyond the scope of our paper, as we stick to the parameter estimates of the SW model.

Similar to the robust control approach, we define a robust policy as a response that limits tail losses. We add to this the trade-off between robustness against uncertainty and the performance of a policy rule in terms of delivering inflation outcomes that are sufficiently close to target. Robust control does not take into account this trade-off. However, our results demonstrate that this trade-off is relevant, since containing extreme adverse losses may imply that the best possible performance is less likely.

In this regard, our approach is in line with info-gap theory, which defines that a robust risk management strategy aims at a loss to be small, and in any case less than a critical value: the largest acceptable loss (Ben-Haim et al., 2018). Similarly, the central bank would like to know what values of the critical loss are realistic and how confident it can be that these values won't be exceeded. Based on that information, the most robust reaction function can be selected. The loss distributions that we simulate under shock uncertainty allow for such a selection method, since it generates distributions of the target variables.

7 Conclusion

We analyze the effects of shock uncertainty on a high inflation economy, taking into account that the output gap is unobserved. In such conditions, the central bank cannot pin down the true cause of observed inflation and takes a risk management approach to limit potentially large welfare losses.

Our main contribution to the literature is that we develop a new empirical method that allows the central bank to make an informed guess about the potential causes of inflation, using the existing knowledge about the economic dynamics, observed inflation, and the structure of the economy. The latter is rationalized via the *Shock marginal rate of substitution*, which summarizes the structural information about how different shocks affect inflation.

We apply our method first in a small-scale New Keynesian model, by which we are able to isolate the effects of shock uncertainty, to find robust monetary policy responses, that aims at limiting extreme welfare losses. We find that the central bank response should neither be too strong nor too mild under uncertainty. If the response by the central bank is very aggressive, it runs the risk of deepening the recession, in case of supply shock dominated inflation scenarios.

Based on the intuition from the small-scale model, we then study the role of shock uncertainty in the US inflationary surge of 2022. Using Smets and Wouters (2007), in a calibrated exercise to mimic the state of the US economy at the peak of the inflation surge, we find that there is a small (close to 5%) but non-negligible probability that inflation is almost entirely driven by supply shocks. We conclude that robust monetary policy to shock uncertainty in such conditions does not respond overly strong to high inflation, given the large downside risks to output and welfare.

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Appendix

A Generalization of the algorithm

In section 3 we described the algorithm that defines the "best guess" of the central bank, which determines its information set. The algorithm was explained for the case in which function (13) only includes observed inflation, and the vector of shocks ζ_t includes only the mark-up and demand shocks. However this algorithm, and therefore the approach we propose to define the information set of the central bank can be generalized in order to be applied to larger DSGE models, and to incorporate a larger number of shocks in ζ_t. Equation (13) is generalized as follows,

$$MIN_{\zeta_t} \quad F_t(x_{t,1}, ..., x_{t,j}) = \sum_{j=1}^{J} \omega_j (\bar{x}_{t,j} - x_{t,j})^2 \tag{18}$$

Where the function $F(\cdot)$ is the loss function used to minimize the distance between a vector of observables $\bar{x}$ at time t, and the values of the variables x_t determined by the shock vector ζ_t using any model. Note that in order to implement our methodology, $\bar{x}$ must leave out at least one state variable of the model, and ζ_t must contain at least one more shock than observables. ω represents the vector of relative importance of minimizing the distance of each variable, decided by the researcher in an informed way, which also tilts the final distribution of shocks in a certain direction.

Note that by defining the loss function like this, we are not introducing any restrictions in the total number of state variables in the model to be used to apply this algorithm. In order to take our approach and implement it the only necessary condition is to include at least one more shock in ζ_t than the number of observables in $\bar{x}$. Thus, the best guess of the central bank in order to implement optimal monetary policy can be computed for every DSGE model regardless of the particular rigidities that it encompasses.

Furthermore, equation (15) is generalized as follows,

$$\zeta_{t,k} = \zeta_{t,k-1} + D_n \Delta_k \tag{19}$$

with

$$D_n = \begin{bmatrix} D_n^1 \\ \vdots \\ D_n^P \end{bmatrix} = \begin{bmatrix} \frac{d_n^1}{\sum_1^P d_n^p} \\ \vdots \\ \frac{d_n^P}{\sum_1^P d_n^p} \end{bmatrix} \qquad \text{where:} \quad d_n^p \rightarrow U[0,1]$$

Where P is the number of shocks in ζ_t, and D_n is the vector of random numbers bounded by $[0,1]$ that alters the direction of the Monte Carlo exercise n towards a particular direction. If $D_n^p = 1$ then the shock combination found will be fully driven by shock p, implementing the same intuition as in the previous section. Δ_k is the gradient of function (18) at $\zeta_{t,k-1}$.

As discussed in the previous section, the structure of the model is implicitly defining Δ_k. This implies that the N shock combinations found is determined, first by the structural information encoded in the Multidimensional *Shock Marginal Rate of Substitution*, plus the uncertainty approach introduced by the Monte Carlo exercise governed by D_n.

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