THE IMPACT OF UNCERTAINTY AND CERTAINTY SHOCKS*

Yves S. Schüler†

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Abstract

I identify uncertainty shocks within a novel Bayesian quantile VAR, unifying Bloom's (2009) two identification steps into one. I find that uncertainty shocks are at least three times more important for the macroeconomy relative to the two-step identification. Exactly opposite to uncertainty shocks, I identify certainty shocks and show that uncertainty and certainty shocks differ. While uncertainty shocks persistently depress real activity (explaining up to 25.8% of its fluctuations), certainty shocks temporarily raise real activity before subsequently suppressing it, erasing almost entirely its previous gains (explaining around 0.4% of its fluctuations). Furthermore, while uncertainty shocks occur, for instance, at the Bloom (2009) dates of uncertainty, events associated with certainty shocks are often reminiscent of irrational exuberance. The distinction between uncertainty and certainty shocks suggests that a range of studies is at risk of recovering incorrect dynamic responses to uncertainty shocks.

Keywords: Uncertainty shocks · Tail risks · Irrational exuberance · Bayesian quantile VAR

JEL-**Codes**: C32 · E44 · G01

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†Deutsche Bundesbank, Research Centre; e-mail address: yves.schueler@bundesbank.de

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"Le doute n'est pas un état bien agréable, mais l'assurance est un état ridicule."

[Uncertainty is an uncomfortable position, but certainty is a ridiculous one.]

(Voltaire, 1785, p. 418)

1 INTRODUCTION

Major economic and political shocks like Black Monday or 9/11 dramatically increase uncertainty. Bloom (2009) finds that these uncertainty shocks that lead to a strong increase in stock market volatility have a severe negative impact on the real economy.

In this paper, I explicitly model Bloom's (2009) seminal idea of identifying uncertainty shocks to the upper tail of stock market volatility, unifying his two identification steps into one.[1] I do so by proposing a novel Bayesian quantile vector autoregressive (BQVAR) framework, both for identifying uncertainty shocks to the tails of financial variables – stock market volatility and returns – and for tracing their impact on the real economy. In this, the BQVAR framework captures a transmission channel from financial markets to the real economy that Bloom's (2009) two-step identification is missing.

Additionally, I extend Bloom's (2009) seminal idea by identifying and analysing *certainty shocks* within the BQVAR framework. Certainty shocks – in exact opposition to uncertainty shocks – lead to a strong fall in conditional stock market volatility. The contrasting juxtaposition of uncertainty and certainty shocks is important, because a range of studies empirically analyses the impact of uncertainty shocks, assuming that both positive and negative shocks result in the same impact, just in opposite directions.[2] Based on such assumption, researchers formulate and validate theories as well as provide recommendations for regulators, such as the role of monetary policy in offsetting the impact of uncertainty shocks.

My results suggest an important role for the transmission channel of uncertainty shocks from financial markets to the real economy. Uncertainty shocks identified within the BQVAR framework lead to a persistent rise in stock market volatility that is not captured through Bloom's two-step identification. Consequently, uncertainty shocks identified in this paper have a much stronger negative impact than Bloom's uncertainty shocks. For instance, six months after a shock, real activity declines overall by -1.2% when hit by the former and only -0.3% by the latter shock. Furthermore, Bloom's uncertainty shocks explain only 1/5 of the variation in stock returns explained by the uncertainty

[1] In a first step, Bloom (2009) constructs a dummy variable of uncertainty shocks that takes a value of one for time periods in which stock-market volatility is more than 1.65 standard deviations above the Hodrick-Prescott detrended ($\lambda = 129,600$) mean of the stock market volatility series. In a second step, he employs the dummy variable within a VAR framework to structurally identify the impact of uncertainty shocks on the real economy.

[2] See, for instance, Bachmann, Elstner and Sims (2013); Bekaert, Hoerova and Lo Duca (2013); Colombo (2013); Jurado, Ludvigson and Ng (2015); Ludvigson, Ma and Ng (2018); Caldara, Fuentes-Albero, Gilchrist and Zakrajšek (2016); Scotti (2016); Baker, Bloom and Davis (2016); Leduc and Liu (2016); Basu and Bundick (2017); Gorodnichenko and Ng (2017)

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shocks of this paper (8.1% vs. 39.9%, over the first half-year after a shock).

Moreover, I conclude that the identification of uncertainty shocks needs to distinguish between uncertainty and certainty shocks. The impacts of uncertainty and certainty shocks on the real economy differ strongly. For instance, uncertainty shocks lead to a strong and persistent decline in growth of real economic activity, accounting for up to 25.8% of its fluctuations. In contrast, certainty shocks temporarily raise economic activity but suppress it thereafter, erasing almost entirely its previous gains. Certainty shocks account for only up to 0.4% of fluctuations in growth of real economic activity. Assuming that positive and negative shocks have the same impact, I show that the dynamic responses of the macroeconomy differ to the analysis of uncertainty and certainty shocks. Additionally, the importance of uncertainty shocks is severely underestimated; e.g. 7.5% vs. 25.8% for explaining fluctuations in growth of real economic activity.

To strengthen my finding that uncertainty and certainty shocks are very different shocks, I externally validate the identified shocks analysing their correspondence with an uncertainty-to-certainty ratio that I construct in the spirit of Baker et al. (2016). The ratio indicates the number of newspaper articles reflecting words such as "uncertainty", "fear", or "panic" relative to the number of articles reflecting words such as "certainty", "confidence", or "euphoria". Furthermore, I scan for relevant newspaper headlines during months of shocks to compare the events that are associated with uncertainty and certainty shocks respectively. First, I show that the uncertainty-to-certainty ratio increases for periods of uncertainty shocks and decreases for periods of certainty shocks. This reveals that i) periods of shocks can be externally validated and ii) the two types of shocks differ because each one distinctly correlates with the ratio. Second, I find that the events associated with uncertainty and certainty shocks differ. Uncertainty shocks tend to be driven by fundamental shocks, such as the Bloom (2009) dates of uncertainty. They also relate to fears, i.e. expectations, about future fundamental shocks like worries about a slowing economy (March 2001).[3] In contrast, certainty shocks are only occasionally driven by fundamental shocks such as sudden peace hopes in Vietnam (August 1968). They are rather associated with increases or even records on stock markets, like the Dow surpassing 1,000 first time in history (November 1972). These events are not clearly linked to specific fundamental shocks and as such are reminiscent of irrational exuberance. Such interpretation is actually in line with the observed impact of certainty shocks, where stock returns and growth in real activity first increase but then undershoot, largely wiping out previous gains. Furthermore, this is consistent with the view that certainty induces heuristic information processing, implying agents that base their judgements on superficial cues, such as records in stock indices (see Tiedens and Linton (2001) and references therein).[4] From a policy perspective, this provides an argument in favour of caution state-

[3]See Appendix C for further details on the newspaper headlines.

[4]Also consistent with the results is that uncertainty provokes systematic information processing, implying agents that adhere to fundamental shocks.

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ments or even policy actions by regulators during phases of irrational exuberance, possibly revealed through the analysis of certainty shocks.

Finally, exploiting the BQVAR framework, I add to the recent discussion on downside risks to the macroeconomic outcome (see, for instance, Adrian, Boyarchenko and Giannone (2019)). I find that only uncertainty shocks persistently impact on the downside risk of the macroeconomy; certainty shocks do so only temporarily. Furthermore, uncertainty shocks account for up to 41.8% of fluctuations, for instance, in the lower tail of economic activity growth, while certainty shocks account for only up to 3.9%. Since policy makers increasingly focus on downside risks to the macroeconomy, for instance within the Federal Open Market Committee (FOMC) statements, the elevated importance of uncertainty shocks stresses the need to carefully incorporate different scenarios.

This paper relates to a vast literature discussing the impact of uncertainty shocks. The asymmetric effects of uncertainty have been subject to analysis in other studies as well, however, in importantly different ways. For instance, based on the seminal idea of Jurado et al. (2015), i.e. to measure uncertainty through the unforecastable component, Rossi and Sekhposyan (2015) derive a positive and negative uncertainty index by splitting forecast errors that undershoot or overshoot the conditional mean forecast. I differ by considering strong increases and reductions in conditional volatility, i.e., tracking the asymmetric impact of changes in second moments. In a series of studies, Caggiano, Castelnuovo and Groshenny (2014), Caggiano, Castelnuovo and Nodari (2014), Caggiano, Castelnuovo and Pellegrino (2017), and Caggiano, Castelnuovo and Figueres (2017) employ non-linear VAR models and find that uncertainty shocks have different effects over the business cycle and at the zero lower bound. Furthermore, Popp and Zhang (2016), Allesandri and Mumtaz (2014), and Mumtaz and Theodoris (forthcoming) find that the impact of uncertainty shocks has declined over time and differs over the financial cycle. While previous research is seminal in pointing out the time-dependent impact of uncertainty, I disentangle the effects of strong increases in uncertainty versus strong reductions. I show that these are not specifically related to different phases in the business or financial cycle. To the best of my knowledge, this is the first study to identify uncertainty shocks and certainty shocks, compare related events, and track their impact on the macroeconomy, including downside and upside risks.

Furthermore, the present analysis relates to studies in finance. For instance, Segal, Shaliastovich and Yaron (2015) construct a good and bad uncertainty index and show that the former predicts an increase and the latter a decrease in future economic activity. Similarly to Rossi and Sekhposyan (2015), the authors split positive and negative innovations to macroeconomic growth, thus not considering the differing effects between sudden increases or decreases in second moments. The present paper also links to recent studies by Bekaert, Engstrom and Ermolov (2015) and Bekaert and Engstrom (2017), who discuss bad environments and good environments modelled through a non-Gaussian asymmetric

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volatility model. The authors support the notion that there are times during which negative or positive shocks are more prominent as evidenced by consumption growth.

The structure of the paper is as follows. Section 2 introduces the BQVAR framework. Section 3 discusses empirical issues, such as the data and the pseudo structural analysis. Subsequently, the impact of uncertainty and certainty shocks is analysed in Section 4 and the external validation exercise provided in Section 5. Section 6 concludes.

2 BAYESIAN QUANTILE VECTOR AUTOREGRESSION

To address the research questions of this paper, I introduce a Bayesian quantile vector autoregressive (BQVAR) framework that allows me to identify pseudo structural shocks to the conditional quantiles of economic variables and trace their impact on the system. The reduced form quantile VAR is

$$\mathbf{y}_t = \boldsymbol{\nu}_{\boldsymbol{\tau}} + \sum_{i=1}^{p} \mathbf{A}_{\boldsymbol{\tau},i}\mathbf{y}_{t-i} + \mathbf{v}_t, \text{ for } t = 1, \dots, T, \tag{1}$$

where $\mathbf{y}_t = (y_{1t}, y_{2t}, \dots, y_{dt})'$ is a $(d \times 1)$ vector of endogenous variables, $\boldsymbol{\nu}_{\boldsymbol{\tau}}$ is a $(d \times 1)$ vector of intercepts at quantile values $\boldsymbol{\tau} = (\tau_1, \dots, \tau_d)'$, $\mathbf{A}_{\boldsymbol{\tau},i}$ for $i = 1, \dots, p$ denotes the matrix of lagged coefficients of size $(d \times d)$ also at quantile values $\boldsymbol{\tau} = (\tau_1, \dots, \tau_d)'$, and $\mathbf{v}_t = (v_{1t}, v_{2t}, \dots, v_{dt})'$ is a $(d \times 1)$ vector of error terms, with $Q_{\tau_j}(v_{jt}|\mathcal{F}_{t-1}) = 0$, where Q_{τ_j} refers to the τ_j-th quantile and $\mathcal{F}_{t-1}$ to the information set including information up to time period $t-1$. Furthermore, let $j \in \{1, \dots, d\}$ and d be the number of variables.

In this model, each equation reflects the conditional quantile of one endogenous variable. Such a setup represents a special case of the multivariate regression quantile model (VAR for VaR) proposed by White, Kim and Manganelli (2015), who provide – in a frequentist setting – the asymptotic theory for this class of models. It represents a special case, as the QVAR model does not include lagged values of the conditional quantiles of the endogenous variables. Similar to the methodology of White et al. (2015), the QVAR allows for each equation to be estimated at a possibly different quantile. Recently, Chavleishvili and Manganelli (2019) propose a frequentist quantile VAR for forecasting and stress testing.

2.1 Pseudo structural analysis

I propose identifying pseudo structural disturbances by the Cholesky decomposition of a dependence measure in the spirit of Koenker and Portnoy (1990) and Blomqvist (1950) that tracks co-exceedences. It is written as

$$\boldsymbol{\Omega}_{\boldsymbol{\tau}} = (\omega_{jk}) \equiv \frac{\mathrm{E}[\psi_{\tau_j}(v_{jt})\psi_{\tau_k}(v_{kt})]}{f_{v_{jt}}(0)f_{v_{kt}}(0)}, \tag{2}$$

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where $\psi_{\tau_j}(v_{jt}) \equiv \tau_j - \mathbb{1}(v_{jt} < 0)$, with $\mathbb{1}$ being the indicator function and $j,k \in \{1,\dots,d\}$. Furthermore, $f_{v_{jt}}(0)$ denotes the pdf of v_{jt} evaluated at 0.

Equation 2 allows to identify pseudo structural disturbances, $\boldsymbol{\varepsilon}_{t|\boldsymbol{\tau}} = (\varepsilon_{1t|\boldsymbol{\tau}},\dots,\varepsilon_{dt|\boldsymbol{\tau}})'$, by the Cholesky decomposition $\boldsymbol{\Omega}_{\boldsymbol{\tau}} = P_{\boldsymbol{\tau}}P_{\boldsymbol{\tau}}'$ that leads to

$$\boldsymbol{\varepsilon}_{t|\boldsymbol{\tau}} = P_{\boldsymbol{\tau}}^{-1}\psi_{\boldsymbol{\tau}}(\mathbf{v}_t), \tag{3}$$

where $\psi_{\boldsymbol{\tau}}(\mathbf{v}_t) = (\psi_{\tau_j}(v_{1t}),\dots,\psi_{\tau_d}(v_{dt}))'$. Assuming that

$$\breve{Q}_{\boldsymbol{\tau}}(\mathbf{y}_t|\boldsymbol{\varepsilon}_{t|\boldsymbol{\tau}},\mathcal{F}_{t-1}) \equiv \boldsymbol{\nu}_{\boldsymbol{\tau}} + \sum_{i=1}^{p}\mathbf{A}_{\boldsymbol{\tau},i}\mathbf{y}_{t-i} + P_{\boldsymbol{\tau}}\boldsymbol{\varepsilon}_{t|\boldsymbol{\tau}} \tag{4}$$

enables me to define the pseudo quantile impulse response function (PQIRF), for example given a structural shock $\varepsilon_{jt|\boldsymbol{\tau}} = 1$ and assuming the other shocks to be zero:

$$\text{PQIRF}_{\boldsymbol{\tau}}(h,\varepsilon_{jt|\boldsymbol{\tau}}=1,\mathcal{F}_{t-1}) = \breve{Q}_{\boldsymbol{\tau}}(\mathbf{y}_{t+h}|\varepsilon_{jt|\boldsymbol{\tau}}=1,\mathcal{F}_{t-1}) - Q_{\boldsymbol{\tau}}(\mathbf{y}_{t+h}|\mathcal{F}_{t-1}), \tag{5}$$

where, for instance,

$$\text{PQIRF}_{\boldsymbol{\tau}}(0,\varepsilon_{1t|\boldsymbol{\tau}}=1,\mathcal{F}_{t-1}) = P_{\boldsymbol{\tau}}\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix}, \tag{6}$$

$$\text{PQIRF}_{\boldsymbol{\tau}}(1,\varepsilon_{1t|\boldsymbol{\tau}}=1,\mathcal{F}_{t-1}) \equiv \mathbf{A}_{\boldsymbol{\tau},1}P_{\boldsymbol{\tau}}\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix}, \text{ and so on.} \tag{7}$$

2.2 The multivariate Laplace distribution for multiple equation quantile regression

In this section I introduce the multivariate Laplace distribution that I use to carry out the Bayesian estimation of the coefficient matrix $\mathbf{A}_{\boldsymbol{\tau}} = (\boldsymbol{\nu}_{\boldsymbol{\tau}},\mathbf{A}_{\boldsymbol{\tau},1},\dots,\mathbf{A}_{\boldsymbol{\tau},p})'$ for fixed quantile values $\boldsymbol{\tau}$.

Proposition 1. Assuming that

$$\mathbf{v}_t \sim \mathcal{L}_d(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}},\mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}'), \tag{8}$$

where $\mathcal{L}_d$ denotes the general multivariate Laplace distribution, one can estimate the coefficient matrix $\mathbf{A}_{\boldsymbol{\tau}}$ for fixed quantile values $\boldsymbol{\tau}$, using $\mathbf{m}_{\boldsymbol{\tau}}$ and the diagonal elements of $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ defined as

$$\mathbf{m}_{\boldsymbol{\tau}} = (m_j) = \frac{1-2\tau_j}{\tau_j(1-\tau_j)} \quad \text{and} \quad \text{diag}(\boldsymbol{\Sigma}_{\boldsymbol{\tau}}) = (\sigma_{jj}^2) = \frac{2}{\tau_j(1-\tau_j)}, \tag{9}$$

and $\mathbf{B} = \text{diag}(b_1,\dots,b_d)$ reflects a positive definite matrix of size $(d \times d)$.

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Proof. There exists a univariate Laplace distribution that is employed for single equation quantile regression (see e.g., Koenker and Machado (1999) or Yu and Moyeed (2001)). Since each component of a general multivariate Laplace admits a univariate representation (Kotz, Kozubowski and Podgórski, 2001, Remark 6.3.2, p. 247), I generalize restrictions derived in the univariate case to the multivariate one.

To begin with, the univariate Laplace distribution employed for single equation quantile regression is

$$f_\tau(\eta_t) = \tau(1-\tau)\exp\{-\rho_\tau(\eta_t)\}, \text{ where } \rho_\tau(\eta_t) = \begin{cases} \eta_t \cdot \tau & \text{, if } \eta_t \geq 0 \\ \eta_t \cdot (\tau - 1) & \text{, if } \eta_t < 0. \end{cases} \tag{10}$$

[5]

Given that the characteristic function of a general univariate Laplace in Kotz et al. (2001) is defined by

$$\Psi_{\eta_t}(s) = \frac{1}{1 + \frac{1}{2}\sigma^2 s^2 - ims}, \tag{11}$$

where $m \in \mathbb{R}$, $\sigma \geq 0$, i is the imaginary unit, and s an arbitrary real number, the following restrictions on the parameters of the characteristic function can be derived:

$$\begin{aligned}
\Psi_{\eta_t}(s) &= E[\exp(is\eta_t)] && (12)\\
&= \int_{-\infty}^{\infty} \exp(is\eta_t) f_\tau(\eta_t) d\eta_t && (13)\\
&= \int_{-\infty}^{0} \tau(1-\tau)\exp(is\eta_t + (1-\tau)\eta_t)d\eta_t + \int_{0}^{\infty} \tau(1-\tau)\exp(is\eta_t - \tau\eta_t)d\eta_t && (14)\\
&= \tau(1-\tau)\left(\frac{1}{is + (1-\tau)} + \frac{1}{\tau - is}\right) && (15)\\
&= \frac{1}{1 + \frac{1}{\tau(1-\tau)}s^2 - i\frac{1-2\tau}{\tau(1-\tau)}s}, && (16)
\end{aligned}$$

or more specifically:

$$m = \frac{1-2\tau}{\tau(1-\tau)} \quad \text{and} \quad \sigma^2 = \frac{2}{\tau(1-\tau)}. \tag{17}$$

To extend the above result to the multivariate setting note that, following Kotz et al. (2001), the characteristic function of a general multivariate Laplace is defined as

$$\Psi_{\boldsymbol{\eta}_t}(\mathbf{s}) = \frac{1}{1 + \frac{1}{2}\mathbf{s}'\boldsymbol{\Sigma}\mathbf{s} - i\mathbf{m}'\mathbf{s}}, \tag{18}$$

where $\boldsymbol{\eta}_t \in \mathbb{R}^d$, $\mathbf{m} \in \mathbb{R}^d$, $\boldsymbol{\Sigma}$ is a $(d \times d)$ nonnegative definite symmetric matrix, and $\mathbf{s}$ is a $(d \times 1)$ vector of arbitrary real numbers. Thus, the elements of $\mathbf{m}$ and the diagonal elements of $\boldsymbol{\Sigma}$ have to fulfill the following criteria

$$m_j = \frac{1-2\tau_j}{\tau_j(1-\tau_j)} \quad \text{and} \quad \sigma_{jj}^2 = \frac{2}{\tau_j(1-\tau_j)}. \tag{19}$$

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While the diagonal elements of $\mathbf{\Sigma}_{\boldsymbol{\tau}}$ are restricted, the off-diagonal elements of $\mathbf{\Sigma}_{\boldsymbol{\tau}}$ are not restricted. These control the covariances between the univariate asymmetric Laplace distributions. The covariances can be decomposed into the product of the unrestricted correlations and the restricted variances, i.e., $\rho_{lk}\sigma_{\tau_l}\sigma_{\tau_k}$, where $l,k \in \{1,\ldots,d\}$ and $\sigma_{\tau_j} = \sqrt{\frac{2}{\tau_j(1-\tau_j)}}$. In this way, $\mathbf{\Sigma}_{\boldsymbol{\tau}}$ may be decomposed to yield

$$\mathbf{\Sigma}_{\boldsymbol{\tau}} = \mathbf{S}_{\boldsymbol{\tau}}\mathbf{R}\mathbf{S}_{\boldsymbol{\tau}}, \tag{20}$$

where $\mathbf{R}$ denotes the correlation matrix with ones on the diagonal and ρ_{lk} as off diagonal elements and $\mathbf{S}_{\boldsymbol{\tau}} = \text{diag}(\sigma_{\tau_1},\ldots,\sigma_{\tau_d})$.

Finally, the quantile restrictions lead to a Laplace distribution with a variance that is, besides the correlation structure in $\mathbf{R}$, completely defined through $\boldsymbol{\tau}$.[6] To this end, let $\mathbf{B}$ denote a scaling parameter that is defined by $\mathbf{B} = \text{diag}(b_1,\ldots,b_d)$. Following Kotz et al. (2001, p. 254) it holds that

$$\mathbf{v}_t = \mathbf{B}\boldsymbol{\eta}_t \sim \mathcal{L}_d(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}}, \mathbf{B}\mathbf{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}'). \tag{21}$$

□

Proposition 2. Due to a mixture representation of the multivariate Laplace (Kotz et al., 2001, p. 246), which is given by

$$\mathbf{v}_t = \mathbf{B}\mathbf{m}_{\boldsymbol{\tau}} w_t + \sqrt{w_t}\mathbf{B}\mathbf{\Sigma}_{\boldsymbol{\tau}}^{1/2}\mathbf{z}_t, \tag{22}$$

one can use commonly known results for estimation as

$$\mathbf{y}_t | \mathbf{A}_{\boldsymbol{\tau}}, \mathbf{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, w_t, \mathcal{F}_{t-1} \sim \mathcal{N}_d, \tag{23}$$

where w_t denotes a standard exponential random variable ($w_t \sim \mathcal{E}(1)$) and $\mathbf{z}_t$ a d-dimensional standard multivariate normal random variable ($\mathbf{z}_t \sim \mathcal{N}_d(\mathbf{0}, \mathbf{I}_d)$), with $\mathbf{I}_d$ being an identity matrix of dimension d. Additionally, let $\mathbf{\Sigma}_{\boldsymbol{\tau}}^{1/2}$ represent the square root matrix $\mathbf{\Sigma}_{\boldsymbol{\tau}}$ that yields $\left(\mathbf{\Sigma}_{\boldsymbol{\tau}}^{1/2}\right)\left(\mathbf{\Sigma}_{\boldsymbol{\tau}}^{1/2}\right)' = \mathbf{\Sigma}_{\boldsymbol{\tau}}$.

Proof. The result of Equation (22) allows me to rewrite Equation (1) to yield

$$\mathbf{y}_t = \boldsymbol{\nu}_{\boldsymbol{\tau}} + \sum_{i=1}^{p} \mathbf{A}_{\boldsymbol{\tau},i}\mathbf{y}_{t-i} + \mathbf{B}\mathbf{m}_{\boldsymbol{\tau}} w_t + \sqrt{w_t}\mathbf{B}\mathbf{\Sigma}_{\boldsymbol{\tau}}^{1/2}\mathbf{z}_t. \tag{24}$$

It follows that the conditional distribution of $\mathbf{y}_t$ given $\mathbf{A}_{\boldsymbol{\tau}}$, $\mathbf{\Sigma}_{\boldsymbol{\tau}}$, $\mathbf{B}$, w_t, and $\mathcal{F}_{t-1}$ is normal. The first two conditional moments of $\mathbf{y_t}$ are given by:

$$\text{E}[\mathbf{y}_t | \mathbf{A}_{\boldsymbol{\tau}}, \mathbf{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, w_t, \mathcal{F}_{t-1}] = \boldsymbol{\nu}_{\boldsymbol{\tau}} + \sum_{i=1}^{p} \mathbf{A}_{\boldsymbol{\tau},i}\mathbf{y}_{t-i} + \mathbf{B}\mathbf{m}_{\boldsymbol{\tau}} w_t = \boldsymbol{\mu}_{\boldsymbol{\tau},t} \tag{25}$$

$$\text{V}[\mathbf{y}_t | \mathbf{A}_{\boldsymbol{\tau}}, \mathbf{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, w_t, \mathcal{F}_{t-1}] = w_t\mathbf{B}\mathbf{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}' = w_t\mathbf{\Sigma}_{\boldsymbol{\tau}\star}, \tag{26}$$

[6] The variance of the multivariate Laplace with quantile restrictions is given by $\mathbf{m}_{\boldsymbol{\tau}}\mathbf{m}_{\boldsymbol{\tau}}' + \mathbf{\Sigma}_{\boldsymbol{\tau}}$.

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where $\boldsymbol{\Sigma}_{\boldsymbol{\tau}\star} = \mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}'$. Thus, it holds that

$$\mathbf{y}_t | \mathbf{A}_{\boldsymbol{\tau}}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, w_t, \mathcal{F}_{t-1} \sim \mathcal{N}_d(\boldsymbol{\mu}_{\boldsymbol{\tau},t}, w_t \boldsymbol{\Sigma}_{\boldsymbol{\tau}\star}). \tag{27}$$

□

2.3 Posteriors

This section introduces the conditional posterior distributions of $\boldsymbol{\alpha}_{\boldsymbol{\tau}}$, $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}, w_t$, and $\mathbf{B}$, where $\boldsymbol{\alpha}_{\boldsymbol{\tau}}$ denotes the column vector vec$(\mathbf{A}_{\boldsymbol{\tau}})$ of size $(d(dp+1) \times 1)$. To ease the exposition, I first cast the VAR model in compact form:

$$\mathbf{y} = (\mathbf{I}_d \otimes \mathbf{X})\boldsymbol{\alpha}_{\boldsymbol{\tau}} + (\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}} \otimes \mathbf{I}_T)\mathbf{w} + \left(\mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{1/2} \otimes \mathbf{W}^{1/2}\right)\mathbf{z}, \tag{28}$$

where $\mathbf{y} = \text{vec}(\mathbf{y}_1, \ldots, \mathbf{y}_T)'$ is a $(Td \times 1)$ vector of observations, $\mathbf{X} = (\mathbf{x}_1', \ldots, \mathbf{x}_T')'$ is a $(T \times (dp+1))$ matrix, where $\mathbf{x}_t = (1, \mathbf{y}_{t-1}', \ldots, \mathbf{y}_{t-p}')$ represents a $(1 \times (dp+1))$ vector, $\mathbf{w} = (w_1, \ldots, w_T)'$ is a $(T \times 1)$ vector and $\mathbf{W} = \text{diag}(\mathbf{w})$ reflects a $(T \times T)$ diagonal matrix. Thus, $\mathbf{W}^{1/2} = \text{diag}(\sqrt{w}_1, \ldots, \sqrt{w_T})$. $\mathbf{z} = \text{vec}(\mathbf{z}_1, \ldots, \mathbf{z}_T)$ denotes a $(Td \times 1)$ vector of multivariate standard normal random variables.

2.3.1 Conditional posteriors of $\boldsymbol{\alpha}_{\boldsymbol{\tau}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$

The prior is assumed to be of an independent normal-inverse Wishart ($\mathcal{IW}$) type:[7]

$$\boldsymbol{\alpha} \sim \mathcal{N}(\underline{\boldsymbol{\alpha}}, \underline{\mathbf{V}}) \quad \text{and} \quad \boldsymbol{\Sigma} \sim \mathcal{IW}(\underline{\boldsymbol{\Sigma}}, \underline{\nu}). \tag{29}$$

Prior times likelihood yields the standard posterior probability density functions:[8]

$$\boldsymbol{\alpha}_{\boldsymbol{\tau}} | \mathbf{y}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, \mathbf{w} \sim \mathcal{N}(\overline{\boldsymbol{\alpha}}_{\boldsymbol{\tau}}, \overline{\mathbf{V}}_{\boldsymbol{\tau}}) \quad \text{and} \quad \boldsymbol{\Sigma}_{\boldsymbol{\tau}} | \mathbf{y}, \boldsymbol{\alpha}_{\boldsymbol{\tau}}, \mathbf{B}, \mathbf{w} \sim \mathcal{IW}(\overline{\boldsymbol{\Sigma}}_{\boldsymbol{\tau}}, \overline{\nu}), \tag{30}$$

where

$$\overline{\mathbf{V}}_{\boldsymbol{\tau}} = [\underline{\mathbf{V}} + ((\mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}')^{-\mathbf{1}} \otimes (\mathbf{X}'\mathbf{W}^{-\mathbf{1}}\mathbf{X}))]^{-1} \tag{31}$$

$$\overline{\boldsymbol{\alpha}}_{\boldsymbol{\tau}} = \overline{\mathbf{V}}_{\boldsymbol{\tau}}[\underline{\mathbf{V}}^{-\mathbf{1}}\underline{\boldsymbol{\alpha}} + ((\mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}')^{-1} \otimes \mathbf{X}'\mathbf{W}^{-\mathbf{1}})(\mathbf{y} - (\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}} \otimes I_T)\mathbf{w})] \tag{32}$$

and

$$\overline{\nu} = \underline{\nu} + T \tag{33}$$

$$\overline{\boldsymbol{\Sigma}}_{\boldsymbol{\tau}} = \underline{\boldsymbol{\Sigma}} + (\mathbf{B}')^{-1}(\mathbf{Y} - \mathbf{X}\mathbf{A}_{\boldsymbol{\tau}} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}})')'\mathbf{W}^{-1}(\mathbf{Y} - \mathbf{X}\mathbf{A}_{\boldsymbol{\tau}} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}})')(\mathbf{B})^{-1}. \tag{34}$$

[7] All prior distributions are assumed to be independent of the remaining parameters. For instance, I assume for the prior of $\boldsymbol{\alpha}$ that $f(\boldsymbol{\alpha}|\boldsymbol{\Sigma}, \mathbf{B}, w_t) = f(\boldsymbol{\alpha})$. As indicated, priors do not necessarily depend on the chosen quantiles $\boldsymbol{\tau}$.

[8] The decomposition of $(\mathbf{Y} - \mathbf{X}\mathbf{A}_{\boldsymbol{\tau}} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}})')'\mathbf{W}^{-1}(\mathbf{Y} - \mathbf{X}\mathbf{A}_{\boldsymbol{\tau}} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}})')$ into $(\mathbf{Y} - \mathbf{X}\hat{\mathbf{A}}_{\boldsymbol{\tau}} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}})')'\mathbf{W}^{-1}(\mathbf{Y} - \mathbf{X}\hat{\mathbf{A}}_{\boldsymbol{\tau}} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}})')$ and $(\mathbf{A}_{\boldsymbol{\tau}} - \hat{\mathbf{A}}_{\boldsymbol{\tau}})\mathbf{X}'\mathbf{W}^{-\mathbf{1}}\mathbf{X}(\mathbf{A}_{\boldsymbol{\tau}} - \hat{\mathbf{A}}_{\boldsymbol{\tau}})$ also holds in this context.

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2.3.2 Conditional probability density function of the latent variable w_t

***Proposition* 3.** The conditional probability density of w_t is proportional to

$$f(w_t|\mathbf{y_t}, \mathbf{A}_{\boldsymbol{\tau}}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, \mathcal{F}_{t-1}) \propto w_t^{-d/2} \exp\left(-\frac{1}{2}\left(a_{\boldsymbol{\tau},t} w_t^{-1} + b_{\boldsymbol{\tau}} w_t\right)\right), \tag{35}$$

with $a_{\boldsymbol{\tau},t} = (\mathbf{y}_t - \boldsymbol{\nu}_{\boldsymbol{\tau}} - \sum_{i=1}^{p} \mathbf{A}_{\boldsymbol{\tau},i}\mathbf{y}_{t-i})'(\mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}')^{-1}(\mathbf{y}_t - \boldsymbol{\nu}_{\boldsymbol{\tau}} - \sum_{i=1}^{p} \mathbf{A}_{\boldsymbol{\tau},i}\mathbf{y}_{t-i})$ and $b_{\boldsymbol{\tau}} = 2 + \mathbf{m}_{\boldsymbol{\tau}}'\boldsymbol{\Sigma}_{\boldsymbol{\tau}}{}^{-1}\mathbf{m}_{\boldsymbol{\tau}}$. This implies that w_t, conditional on the latter parameters, is proportional to a generalized inverse Gaussian with the following parameters:[9]

$$w_t|\mathbf{y}_t, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, \mathbf{A}_{\boldsymbol{\tau}}, \mathcal{F}_{t-1} \sim \mathcal{GIG}\left(-d/2+1, a_{\boldsymbol{\tau},t}, b_{\boldsymbol{\tau}}\right). \tag{36}$$

Proof. Let $w \sim \mathcal{E}(1)$ and $\mathbf{y} \sim \mathcal{L}_d(\mathbf{m}, \boldsymbol{\Sigma})$. In order to show that the kernel of the $f(w|\mathbf{y})$ is proportional to that of a generalized inverse Gaussian distribution, recall that the conditional density is obtained through

$$f(w|\mathbf{y}) = \frac{f(\mathbf{y}|w)f(w)}{f(\mathbf{y})}. \tag{37}$$

It has been shown that $f(\mathbf{y}|w)$ has a multivariate normal pdf, i.e.,

$$f(\mathbf{y}|w) = (2\pi)^{-d/2}|w\boldsymbol{\Sigma}|^{-1/2} \exp\left(-\frac{1}{2}(\mathbf{y}-\mathbf{m}w)'(w\Sigma)^{-1}(\mathbf{y}-\mathbf{m}w)\right) \tag{38}$$

Furthermore, $f(w) = \exp(-w)$. Neglecting $f(\mathbf{y})$ and the invariant terms of $f(\mathbf{y}|w)$,

$$\begin{aligned} f(w|\mathbf{y}) &\propto w^{-d/2} \exp\left(-\frac{1}{2}(\mathbf{y}-\mathbf{m}w)'(w\boldsymbol{\Sigma})^{-1}(\mathbf{y}-\mathbf{m}w) - w\right) &(39)\\ &= w^{-d/2} \exp\left(-\frac{1}{2}\left(\frac{\mathbf{y}'\boldsymbol{\Sigma}\mathbf{y}}{w} - \mathbf{y}'\boldsymbol{\Sigma}\mathbf{m} - \mathbf{m}'\boldsymbol{\Sigma}\mathbf{y} + w\mathbf{m}'\boldsymbol{\Sigma}\mathbf{m}\right) - w\right) &(40)\\ &\propto w^{-d/2} \exp\left(-\frac{1}{2}\left((\mathbf{y}'\boldsymbol{\Sigma}\mathbf{y})w^{-1} + (2+\mathbf{m}'\boldsymbol{\Sigma}\mathbf{m})w\right)\right). &(41) \end{aligned}$$

The probability density function of a generalized inverse Gaussian denoted by $\mathcal{GIG}(\lambda, \chi, \psi)$, with $\lambda = -(d/2)+1$, is given by

$$f(x|\lambda, \chi, \psi) = \frac{(\psi/\chi)^{\lambda/2}}{2K_\lambda(\sqrt{\chi\psi})} x^{\lambda-1} \exp\left\{-\frac{1}{2}(\chi x^{-1} + \psi x)\right\}, \tag{42}$$

where $K_\lambda(\cdot)$ reflects the modified Bessel function of the second kind.

Hence,

$$f(w|\mathbf{y}) \propto \mathcal{GIG}(-d/2+1, \mathbf{y}'\boldsymbol{\Sigma}\mathbf{y}, 2+\mathbf{m}'\boldsymbol{\Sigma}\mathbf{m}). \tag{43}$$

□

[9] There are several algorithms available for the generation of random numbers from a generalized inverse Gaussian. I apply the one proposed by Devroye (2012) as it is computationally fast.

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2.3.3 *Conditional posterior of* $\mathbf{B}$

I assume a noninformative prior for $\mathbf{B}$, i.e. let

$$f(\mathbf{B}) = \text{const.} \tag{44}$$

The conditional posterior of $\mathbf{B}$ then follows the likelihood of a $\mathcal{L}_d(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}\star})$, where $\boldsymbol{\Sigma}_{\boldsymbol{\tau}\star} = \mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}'$. Following Kotz et al. (2001), it is given by:

$$f(\mathbf{B}|\mathbf{y}, \boldsymbol{\alpha}_{\boldsymbol{\tau}}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}) \propto \prod_{t=1}^{T} \frac{2\exp\left((\mathbf{y}_t - \boldsymbol{A}_{\boldsymbol{\tau}}'\mathbf{x}_t')'\boldsymbol{\Sigma}_{\boldsymbol{\tau}\star}^{-1}\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}}\right)}{(2\pi)^{d/2}|\boldsymbol{\Sigma}_{\boldsymbol{\tau}\star}|^{1/2}} \left(\frac{(\mathbf{y}_t - \boldsymbol{A}_{\boldsymbol{\tau}}'\mathbf{x}_t')'\boldsymbol{\Sigma}_{\boldsymbol{\tau}\star}^{-1}(\mathbf{y}_t - \boldsymbol{A}_{\boldsymbol{\tau}}'\mathbf{x}_t')}{2 + \mathbf{m}_{\boldsymbol{\tau}}'\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{-1}\mathbf{m}_{\boldsymbol{\tau}}}\right)^{(-d/2+1)}$$
$$K_{(-d/2+1)}\left(\sqrt{(2 + \mathbf{m}_{\boldsymbol{\tau}}'\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{-1}\mathbf{m}_{\boldsymbol{\tau}})((\mathbf{y}_t - \boldsymbol{A}_{\boldsymbol{\tau}}'\mathbf{x}_t')'\boldsymbol{\Sigma}_{\boldsymbol{\tau}\star}^{-1}(\mathbf{y}_t - \boldsymbol{A}_{\boldsymbol{\tau}}'\mathbf{x}_t'))}\right), \tag{45}$$

where $K_{(-d/2+1)}(\cdot)$ reflects the modified Bessel function of the second kind of order $-d/2+1$.

2.4 *Metropolis-within-Gibbs sampler*

The sampling of the $\boldsymbol{\alpha}_{\boldsymbol{\tau}}$ coefficients and w_t is straightforward using a Gibbs sampler. More involved are the draws of the correlations contained in $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ and the scaling factors in $\mathbf{B}$, on which I elaborate in the following.

For the first case, I propose using the conditional posterior of $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ and standardizing each draw.[10] Note that $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ may be decomposed to yield

$$\boldsymbol{\Sigma}_{\boldsymbol{\tau}} = \mathbf{S}_{\boldsymbol{\tau}}\mathbf{R}\mathbf{S}_{\boldsymbol{\tau}}, \tag{46}$$

where $\mathbf{R}$ denotes the correlation matrix with ones on the diagonal and ρ_{lk} as off diagonal elements and $\mathbf{S}_{\boldsymbol{\tau}} = \text{diag}(\sigma_{\tau_1}, \dots, \sigma_{\tau_d})$. Following this, each draw of $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ can be rearranged as

$$\mathbf{R} = \mathbf{S}_{\boldsymbol{\tau}}^{-\mathbf{1}}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{S}_{\boldsymbol{\tau}}^{-\mathbf{1}}. \tag{47}$$

What this achieves is that the diagonal elements of $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ remain unchanged. This is important because quantile restrictions on the Laplace distribution have to remain fixed to obtain a consistent posterior for $\boldsymbol{\alpha}_{\boldsymbol{\tau}}$. Having drawn the new correlation matrix $\mathbf{R}$ the covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ can be updated using equation (46).

In the case of $\mathbf{B}$, the posterior probability density function is rather complicated as the matrix appears both in the mean and the variance of the conditional distribution of $\mathbf{y_t}$ (e.g. Equation (27)). Thus, for the draw of $\mathbf{B}$ I propose to use a random walk Metropolis-Hasting (MH) algorithm that just requires that the conditional posterior probability density function can be evaluated (see Chib

[10] The other option would be to use a Metropolis-Hasting algorithm and sample the off-diagonal elements of $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$. A Gibbs sampler, however, is preferred as every draw is accepted; thus convergence is faster. Simulation studies have shown that both options provide similar estimates.

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and Greenberg (1995)). In contrast to the Gibbs sampler, not every draw is accepted using the MH algorithm. At each draw an acceptance probability is calculated and compared to a random draw of a uniform random variable to decide on its acceptance. If not accepted, the previous draw is taken as the new draw. The acceptance probability is derived as in the following. Given a new draw of $\mathbf{B}$, called $\mathbf{B}^*$, and the last draw $\mathbf{B}^{(n-1)}$, where $n \in \{1, \ldots, N\}$, it is

$$\alpha_{\text{MH},\mathbf{B}}(\mathbf{B}^{(n-1)}, \mathbf{B}^*) = \min \left[\frac{f\left(\mathbf{B}^*|\mathbf{y}, \boldsymbol{\alpha}_{\boldsymbol{\tau}}^{(n)}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{(n)}, \mathbf{w}^{(n)}\right)}{f\left(\mathbf{B}^{(n-1)}|\mathbf{y}, \boldsymbol{\alpha}_{\boldsymbol{\tau}}^{(n)}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{(n)}, \mathbf{w}^{(n)}\right)}, 1 \right]. \tag{48}$$

[11]

I calibrate the acceptance probability to be between 0.2 and 0.5.

In the following, the algorithm is depicted for the case when draws of the scaling parameters are carried out jointly. This, of course, can be broken down into separate steps to ease the calibration of the acceptance rate. Furthermore, a random walk MH algorithm may be carried out using any symmetric distribution in the innovation part. This paper assumes a normal distribution.[12]

Algorithm 1 Bayesian Quantile VAR

A. Define prior distribution for $\boldsymbol{\alpha}_{\boldsymbol{\tau}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ and set starting values $\boldsymbol{\alpha}_{\boldsymbol{\tau}}^0, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}^0$ and $\mathbf{B}^0$. Set variance of the random walk innovation used in the MH step, c.
B. Repeat for $n = 1, 2, \ldots, N$

1. Gibbs Step 1: For $t = 1, \ldots, T$: Draw $w_t^{(n)}|\mathbf{y}_t, \boldsymbol{\alpha}_{\boldsymbol{\tau}}^{(n-1)}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{(n-1)}, \mathbf{B}^{(n-1)}$
2. Gibbs Step 2: Draw $\boldsymbol{\alpha}_{\boldsymbol{\tau}}{}^{(n)}|\mathbf{y}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{(n-1)}, \mathbf{B}^{(n-1)}, \mathbf{w}^{(n)}$
3. Gibbs Step 3: (i) Draw $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{(n)}|\mathbf{y}, \boldsymbol{\alpha}_{\boldsymbol{\tau}}^{(n)}, \mathbf{B}^{(n-1)}, \mathbf{w}^{(n)}$; (ii) Calculate $\mathbf{R}^{(n)} = \mathbf{S}_{\boldsymbol{\tau}}^{-1}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{(n)}\mathbf{S}_{\boldsymbol{\tau}}^{-1}$; (iii) Set $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{(n)} = \mathbf{S}_{\boldsymbol{\tau}}\mathbf{R}^{(n)}\mathbf{S}_{\boldsymbol{\tau}}$
4. MH Step 1: (i) Draw $\mathbf{v}_{**} \sim \mathcal{N}(\mathbf{0}, c\mathbf{I_d})$; (ii) Calculate $(b_1^*, \ldots, b_d^*)' = \left(b_1^{(n-1)}, \ldots, b_d^{(n-1)}\right)' + \mathbf{v}_{**}$; (iii) Evaluate $\alpha_{\text{MH},\mathbf{B}}$; (iv) Draw $u_{**} \sim \mathcal{U}(0,1)$; (v) **If** $u_{**} \leq \alpha_{\text{MH},\mathbf{B}}$ set $\left(b_1^{(n)}, \ldots, b_d^{(n)}\right)' = (b_1^*, \ldots, b_d^*)'$; (vi) **else** set $\left(b_1^{(n)}, \ldots, b_d^{(n)}\right)' = \left(b_1^{(n-1)}, \ldots, b_d^{(n-1)}\right)'$

3 Empirical issues

In this section, I detail the data and then turn to the discussion of the Bayesian estimation setup and the pseudo structural analysis.

[11] In the depiction of the acceptance probabilities the draws of the other variables are also used as conditioning variables. Variables at draw (n) or $(n-1)$ are chosen in line with the algorithm presented in this section; however, they may of course vary according to the ordering in the sampler used.

[12] In practice, I draw the elements of $\mathbf{B}$ separately. The scaling parameter c_d, i.e. for each draw is adjusted automatically in order to satisfy the acceptance ratio mentioned of 0.2 and 0.5.

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3.1 Data

The monthly data set spans the period from 1968-04 to 2015-04. The sample is chosen such that the different proxies of uncertainty employed in this study – as described below – can be analyzed over the same time period.[13] Furthermore, only stationary transformations of the series are included, i.e. first (log) differences for all variables but the uncertainty proxies as the derivation of conditional quantiles for trending variables makes no sense:[14] it would imply an ordering over time. Next, I describe the macroeconomic variables employed in this study, second, the proxies of uncertainty, and third, an uncertainty-to-certainty ratio used for validation.

To investigate the effects of uncertainty and certainty shocks, I include variables that are used in similar analyses to ensure comparability.[15] Specifically, the choice of variables for the monthly quantile VAR is related to Jurado et al. (2015) and Caldara et al. (2016). I include a subset of their variables that measure real economic activity, private consumption, inflation, interest rates, and equity markets.

Economic activity is measured by growth in real manufacturing industrial production (Δq), private consumption by growth in real personal consumption expenditure (PCE, Δc), inflation by growth in the PCE deflator (Δp), interest rates by percentage point changes in the effective federal funds rate (Δi), and equity markets by the return of the S&P500 index (r).

Proxies of uncertainty

For comparative purposes, I identify structural shocks using five different proxies of uncertainty; two measures are employed in the main analysis, the results of the others are presented in Appendix D.

The main proxy of uncertainty employed in the BQVAR is the Chicago Board of Options Exchange VXO index expanded by the actual volatility of the S&P500 to cover the long sample (u_v). This reflects Bloom's (2009) proxy of uncertainty, on which basis he constructs a dummy variable (u_{Bloom}) capturing exogenous shocks to uncertainty. The author uses the latter dummy to shed light on the impact of uncertainty shocks. Thus, for comparative purposes I benchmark my findings based on the Bloom's proxy of uncertainty to an uncertainty shock identified using Bloom's dummy variable.

The set of other proxies includes a measure of forecast dispersion that exploits the Philadelphia Fed's Business Outlook Survey as proposed by Bachmann et al. (2013) (u_{fd}). It is based on expectations of future developments in general business activity. The other two proxies are argued to capture fundamental financial and macroeconomic uncertainty, focussing on the time-varying variance of the

[13]The uncertainty measure proposed by Bachmann et al. (2013) restricts the earliest date of the sample and the indices of macroeconomic and financial uncertainty by Jurado et al. (2015) and Ludvigson et al. (2018) the most recent.

[14]Including (log) levels of the series would only make sense if variables share a stochastic trend. This possibility is excluded by taking first (log) differences.

[15]All variables, their transformations, and their times series plots are presented in Appendix A.

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u.

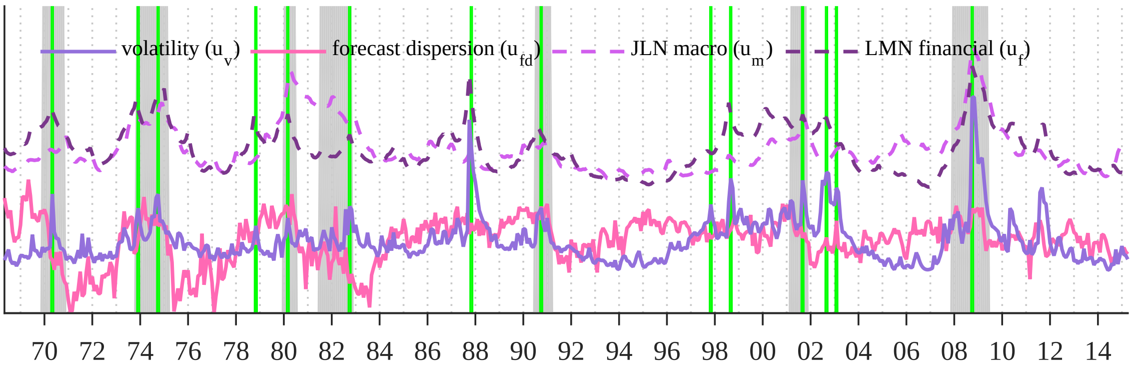

Figure 1: Proxies and dates of uncertainty

Notes: Uncertainty proxies are normalized to have unit variance. Grey shaded areas mark NBER recession periods. Green vertical lines indicate the Bloom (2009) dates of uncertainty.

unforecastable component of a large set of variables. They have been suggested by Ludvigson et al. (2018) (LMN,u_f) and Jurado et al. (2015) (JLN, u_m).[16] While I compare shocks identified on the basis of u_{fd} within the BQVAR framework, I benchmark the impact of other two proxies by considering a regular VAR. The reason is that u_f and u_m already reflect the outcome of an empirical exercise and, thus, measure conditional uncertainty identified at the mean.

All of the mentioned measures of uncertainty are portrayed in Figure 8. Obviously, the Bloom (2009) dates of uncertainty (green vertical lines, reflecting the ones in Bloom's dummy variable) align with local maxima that are visible in the u_v series as they are constructed on the basis of the latter. Local maxima in the u_v series are also reflected in the LMN measure of financial uncertainty, u_f. In contrast, the index of macroeconomic uncertainty (u_m) does not correspond closely to u_v. In general, the index of forecast dispersion u_{fd}, does not seem to correspond to the fundamental uncertainty indices as well as the Bloom (2009) dates. Contemporaneous correlation between the indices is as follows: u_v is correlated with u_{fd}, u_m, and u_f of magnitude 0.2, 0.46, and 0.76 respectively. u_{fd} is correlated with u_m and u_f of size 0.1 and 0.19 respectively. Correlation of u_m with u_f is 0.57. That is, the correlation of u_v with u_f is strongest, while that of u_{fd} with u_m is the lowest. In sum, the proxy of uncertainty employed in the main part of the paper relates well to the Bloom dates of uncertainty as well as the measure of fundamental financial uncertainty, but not to the other proxies.

[16] The authors propose several forecast horizons for their measures. In this study, I use the instantaneous indices, i.e., a forecast horizon of one, when comparing these indexes to my identified contemporaneous uncertainty shocks. The 3-month horizon indices u_f and u_m are considered in the robustness exercise, in which I identify uncertainty shocks from a VAR on the basis of these indicators. This reflects the choice of the authors when analyzing the impact of uncertainty shocks.

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An external measure of periods of uncertainty versus periods of certainty

To externally validate the statistically identified uncertainty and certainty shocks, I construct a measure that I call the *uncertainty-to-certainty ratio* in the spirit of Baker et al. (2016) starting in 1985 (see Figure 5 in Section 5).[17] The ratio reflects the number of newspaper articles with uncertainty narratives relative to the number of newspaper articles with certainty narratives within a given month. It thus tracks whether, from one month to another, there has been a relative increase or decrease in uncertainty narratives over certainty narratives.

As keywords for determining the number of articles with uncertainty narratives I use: uncertainty, uncertain, fear, concern, panic, worry, doubt, and low. On the other hand, as keywords for determining the number of articles with certainty narratives I employ: certainty, certain, trust, faith, confidence, euphoria, hope, and high.

In constructing this index, I deviate in two important aspects from Baker et al. (2016): First, I ensure that articles representing uncertainty (certainty) exclude any keywords of certainty (uncertainty). Second, I specifically consider articles that deal with stock markets. Overall, the index represents changes in uncertainty narratives relative to certainty narratives that are related to both economic issues and stock markets.

For more details on the construction of the ratio, please see Appendix B.

3.2 Estimation setup

Throughout the study I consider non-informative priors, so that the data is allowed to drive the estimation of the parameters. The priors are

$$\boldsymbol{\alpha} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{d(pd+1)} \cdot 10) \qquad \text{and} \qquad \boldsymbol{\Sigma} \sim \mathcal{IW}(d, \mathbf{I}_d),$$

where $\boldsymbol{\alpha} = \text{vec}(\mathbf{A})$. Due to the decision to use non-informative prior information, I am required to specify the model parsimoniously. In this light, I choose a lag length of three.[18]

The Metropolis-within-Gibbs sampler is set up with 15000 draws where 5000 are discarded as burn-in-draws. The convergence of the parameters is assured using trace plots and the conditional quantile conditions ($Q_{\tau_j}(v_{jt}|\mathcal{F}_{t-1}) = 0$) are checked using the median values of the posterior.

3.3 Pseudo structural analysis

For the identification of the pseudo structural shocks, first, the reduced form BQVAR is estimated. Second, the derived dependence matrix in Equation (2) is decomposed using a Cholesky decomposition, which implies a recursive structure of the shocks.

[17] Only for that period a large list of newspaper magazines is available on Factiva.

[18] For comparative purposes this lag structure is imposed for the analysis of different uncertainty proxies as well.

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The ordering of the Wold causal chain for the standard variables follows the one of monetary policy VARs (see, e.g., Leeper, Sims and Zha (1996) or Christiano, Eichenbaum and Evans (1999)). This entails arranging the real sector first (economic activity, consumption, prices), then specifying the policy sector (interest rate). I subsequently introduce the proxy of uncertainty and, lastly, I add the index of equity markets. This order entails that the uncertainty proxy may be affected contemporaneously by shocks in the real and policy sector. However, an uncertainty/certainty shock does not affect the real or policy sector in the month of the shock, while it does affect equity markets. This is in line, for instance, with the identification used in Jurado et al. (2015).

Using this ordering, I identify an uncertainty and a certainty shock. An uncertainty shock is identified as a positive shock to the right tail ($\tau_5 = 0.9$, high state of conditional volatility) of u_v that is allowed to contemporaneously affect the left tail of stock returns ($\tau_6 = 0.1$, low conditional returns). I motivate this assumption by my goal of identifying uncertainty shocks that have a negative impact on the real economy, similar to the events captured by Bloom's (2009) dates of uncertainty. Among others, Segal et al. (2015), Bekaert et al. (2015), and Bekaert and Engstrom (2017) report that there may be shocks increasing volatility, which however improve economic conditions ("good uncertainty"), for instance, as shown by the increase in volatility during the technological boom in the US prior to 2000. I aim to exclude these shocks with my identification scheme. Exactly opposite, I identify certainty shocks, i.e. as shocks to the left tail of u_v ($\tau_5 = 0.1$, low state of conditional volatility) that possibly impact the right tail of stock returns ($\tau_6 = 0.9$, high conditional returns)

The impact of the two shocks on the remaining variables is considered at the median ($\tau_1 = \tau_2 = \tau_3 = \tau_4 = 0.5$), which can be argued to be similar to considering the effects within a regular VAR, i.e., at the mean.

Additionally, I exploit the BQVAR framework to analyse the impact of uncertainty shocks and certainty shocks on the downside and upside risks of the macroeconomy. To this end, I study the impact of shocks to the left and right tail of the macroeconomic variables, considering the impacts on the development of their 0.1 and 0.9 quantiles, i.e., $\tau_1 = \tau_2 = \tau_3 = \tau_4 = 0.1$ and $\tau_1 = \tau_2 = \tau_3 = \tau_4 = 0.9$ respectively. Similar analyses have gained recent attention, as downside as well as upside risks to the macroeconomy have become prominent when analyzing the role financial conditions in shaping real outcomes (see Adrian et al. (2019)).

To ensure comparability of the pseudo structural impulse responses across shocks, I normalize the size of shocks to one standard deviation of the proxy of uncertainty. Furthermore, as indicated by Equation (5), throughout the analysis I trace the effect of a shock on the conditional quantile of a variable that was assumed at the moment of the shock. Lastly, the pseudo forecast error variance decomposition exercise uses the un-normalized decomposed dependence matrix as presented in Equation (2).

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4 THE IMPACT OF UNCERTAINTY AND CERTAINTY SHOCKS

The pseudo impulse responses of an uncertainty shock and a certainty shock are shown in Figure 2. For comparative purposes, the impulse responses of a Bloom uncertainty shock and of an uncertainty shock identified within a regular VAR that I call "mean uncertainty shock" are portrayed in Figure 3. The (pseudo) forecast error variance decompositions of all four shocks are set forth in Table 1. It shows the average variance explained by the various uncertainty shocks in the first and the second half of the first year after a shock.

First, I discuss the impact of uncertainty and certainty shocks, which I subsequently compare to Bloom uncertainty shocks and mean uncertainty shocks. Briefly I mention the impact of uncertainty shocks identified on the basis of other proxies mentioned in Section 3.

Uncertainty shock

An uncertainty shock leads to a persistent rise of the 0.9 conditional quantile of the proxy of uncertainty. After six months, the level of uncertainty is similar to the level at the impact of the shock. The shock leads to a persistent decline on stock markets. One month past the shock, I find a strong decline of 6 percentage points (p.p.). After a quick recovery – until month three – the shock leads to a persistent decline of stock returns. Overall, i.e. considering the cumulated impact, there is a decline in the stock index of -14.6% six months after the shock.

The real economy suffers in a similar fashion. Growth in economic activity steadily declines in response to an uncertainty shock. After six months, the overall, i.e. cumulated fall in economic activity is about -1.2%. The effect on consumption growth is negative as well, yet less severe. After six months, the overall reduction amounts to -0.3%. In contrast, there is only a small effect on inflation, for instance, leading to an overall decline of about -0.05% two months after the shock. The monetary authority tries to counter the strong fall in activity by lowering interest rates. After six months, I find an overall reduction in interest rates by -0.3 p.p.

The pseudo forecast error variance decompositions of the uncertainty shock paint a similar picture. An uncertainty shock is most important for explaining variation in economic activity (8.6%) and the interest rates (8.2%) in the first half-year after the shock. For consumption, the shock only accounts for 2.6% of the variation; 1.8% in the case of inflation. Clearly, the variation explained is the highest, first, for the proxy of uncertainty (93.7%) and, second, for stock returns (39.9%). In the second half-year after the shock, its importance increases, for instance, for economic activity, reaching 25.8%.

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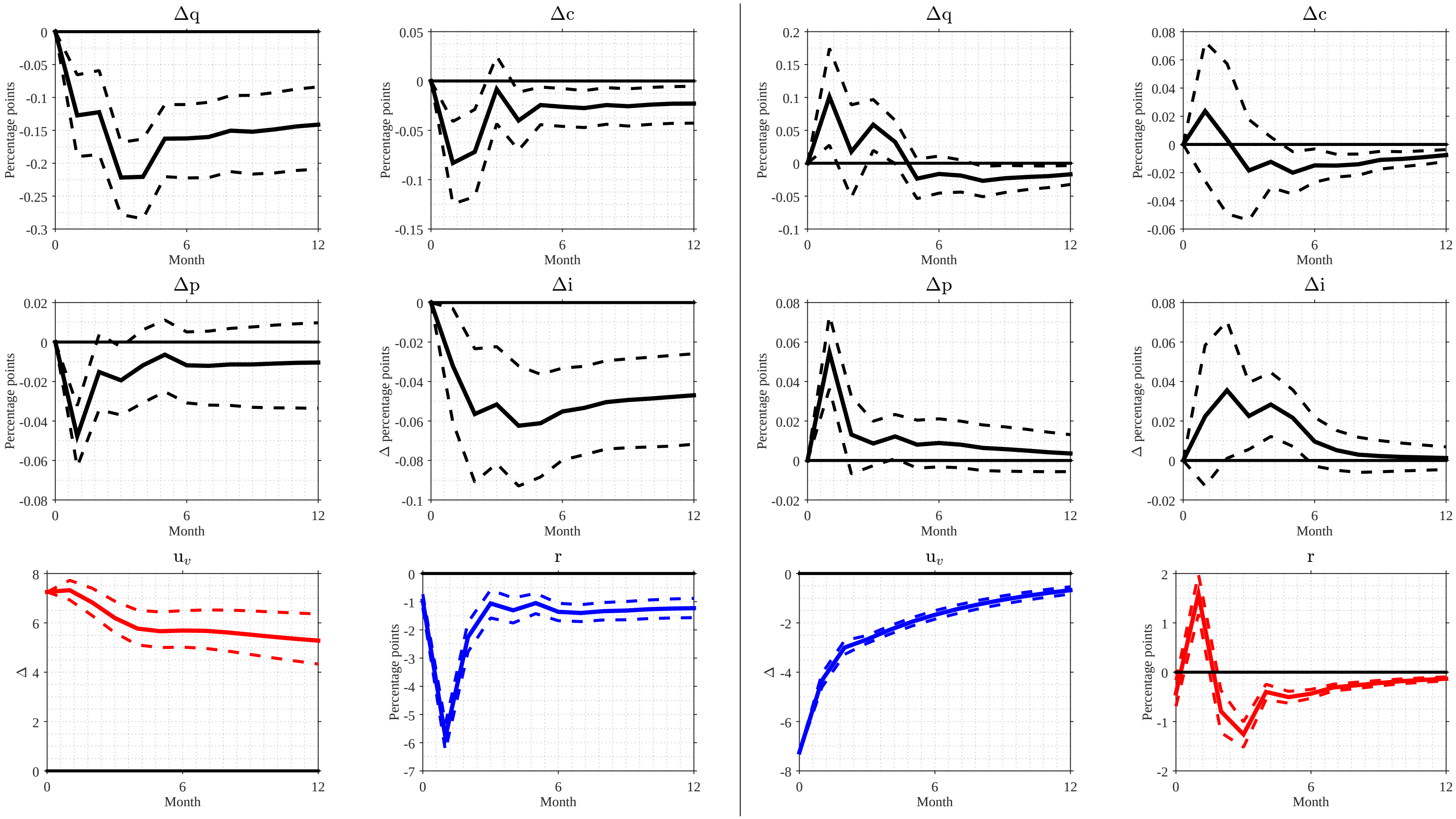

Figure 2: The impact of an uncertainty shock (left panel) and a certainty shock (right panel)

Notes: The panels depict the pseudo impulse responses of the median of the macroeconomic variables (Δq, Δc, Δp, Δi) and of the quantiles of the financial variables (u_v, r) to an uncertainty and certainty shock. For identifying uncertainty shocks, u_v and r are considered at their 0.9 and 0.1 conditional quantiles respectively; for identifying certainty shocks at their 0.1 and 0.9 conditional quantiles. Solid lines refer to the median impulse responses and the dashed lines correspond to posterior 68% probability bands. Δq denotes growth in industrial production; Δc is growth in consumption , Δp inflation, Δi changes in the interest rate, u_v the proxy of uncertainty, and r stock returns.

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Table 1: Variance explained by various shocks

Months	Δq	Δc	Δp	Δi	u_v	r
Uncertainty shock						
0–6	8.6	2.6	1.8	8.2	93.7	39.9
7–12	25.8	5.7	3.0	25.7	79.7	49.0
Certainty shock						
0–6	0.3	0.0	0.7	0.5	93.4	2.8
7–12	0.4	0.1	0.7	0.8	87.2	4.1
Bloom uncertainty shock						
0 – 6	2.8	0.4	0.4	2.8	94.2	8.1
7 – 12	3.7	0.6	0.8	3.9	93.1	8.4
Mean uncertainty shock						
0–6	3.8	1.0	1.8	0.6	96.2	20.8
7–12	7.4	1.4	1.8	1.4	94.7	23.3

Notes: The table shows the pseudo forecast error variance of an uncertainty and certainty shock for the median of the macroeconomic variables (Δq, Δc, Δp, Δi) and for the quantiles of the financial variables (u_v, r). For identifying uncertainty shocks, u_v and r are at their 0.9 and 0.1 conditional quantiles respectively; for identifying certainty shocks at their 0.1 and 0.9 conditional quantiles. Furthermore, the table shows the forecast error variance decomposition of a Bloom uncertainty shock, identified on the basis of the Bloom (2009) dummy variable. The mean uncertainty shock is identified within a regular VAR model, i.e. reflecting the impact at the mean. Δq denotes growth in industrial production; Δc is growth in consumption , Δp inflation, Δi changes in the interest rate, u_v the proxy of uncertainty, and r stock returns.

Certainty shock

A certainty shock leads to a temporary decline in the proxy of uncertainty, in contrast to an uncertainty shock. On impact, stock returns do not move significantly; however, they increase by about 1.6 p.p. one month after the shock and decline thereafter. After six months, there is an overall, i.e. cumulated, decline of 2.3% of the stock index value. In a similar fashion, the real economy responds temporarily positively then negatively to the certainty shock. Specifically, activity growth increases by 0.1 p.p. one month after the shock, but becomes insignificant for the second month. Considering the positive and significant response in month three and the insignificant response thereafter, the overall effect on activity six months after the shock amounts to an increase of about 0.16%. At month eight, the response then becomes significantly negative (-0.03 p.p.) and slowly converges to zero. Thus, the initial gains in real activity are subsequently reduced. After 12 months – cumulating only the significant responses – the gain in activity amounts to only 0.05%. Consumption growth is not significantly affected in the first few months after the shock. There is, however, a reduction five months after the shock by about 0.02 p.p. Similarly to economic activity growth, it then slowly converges to zero. Inflation increases one month after the shock by 0.05 p.p. The response becomes insignificant in the fourth month after the shock. The monetary authority responds to the certainty shock with a time lag by raising interest rates from month two until month five by 0.11 p.p overall. After month six the interest rates respond insignificantly to the shock.

The pseudo forecast error variance decompositions suggest a rather minor role of the certainty shock for explaining the variance of the macroeconomic variables. Within the first half-year after the shock, it explains on average 0.3% of fluctuations in activity growth, 0% in consumption growth,

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0.7% in inflation and 0.5 in interest rate changes. Furthermore, while it explains about 2.8% of the variation in stock returns, it accounts for 93.4% of fluctuations in uncertainty. These figures remain similar for the second half-year after the shock.

Uncertainty versus certainty shock

Comparing the uncertainty to the certainty shock, the findings suggest that uncertainty shocks have far more severe consequences for the macroeconomy than certainty shocks. Uncertainty shocks account for strong declines in economic activity of -1.2% and explain about 8.6% of its fluctuations six months after the shock. Certainty shocks – scaled to the same initial impact on the proxy of uncertainty as the uncertainty shock – lead to only a modest increase in economic activity by 0.16% over the same time period. They only account for 0.3% of the variance of economic activity growth. In a similar vein, uncertainty shocks mark by far more important events for the monetary authority than certainty shocks. While uncertainty shocks account for about 8.2% of the variance in interest rate changes, the same figure only amounts to 0.5% for certainty shocks. Still, certainty shocks exhibit significant effects on the real economy, the monetary authority, and stock markets.

Comparison to Bloom uncertainty shock, mean uncertainty shock, and others

The Bloom uncertainty shock has a weaker impact overall than the uncertainty shock. The dummy variable capturing uncertainty shocks misses the persistent effect of uncertainty on financial markets. That is, the Bloom shock leads to a one-off effect on the dummy of uncertainty that induces a sudden decline on stock markets, however, accompanied with a rapid recovery. After six months the cumulated impact of the shock on the stock market amounts to a small decline of only -0.9%. The shock also explains only 8.1% of the variation in stock returns in the first half-year of the shock, relative to 39.9% in the case of the uncertainty shock. In consequence, the impact on the real economy is weaker and less persistent relative to the uncertainty shock. For instance, the impact on real activity already becomes insignificant in the fifth month after the shock. Thus, after six months the overall decline only amounts to about -0.3%, and up to that month it explains only 2.8% of variation. A similar pattern is observed for consumption. The impact on interest rates is temporary but strong (-0.17p.p. after three months), and it explains about 2.8% of fluctuations, still less than the uncertainty shock (8.2%). Inflation, in contrast to the uncertainty and certainty shocks, suffers a persistent impact.

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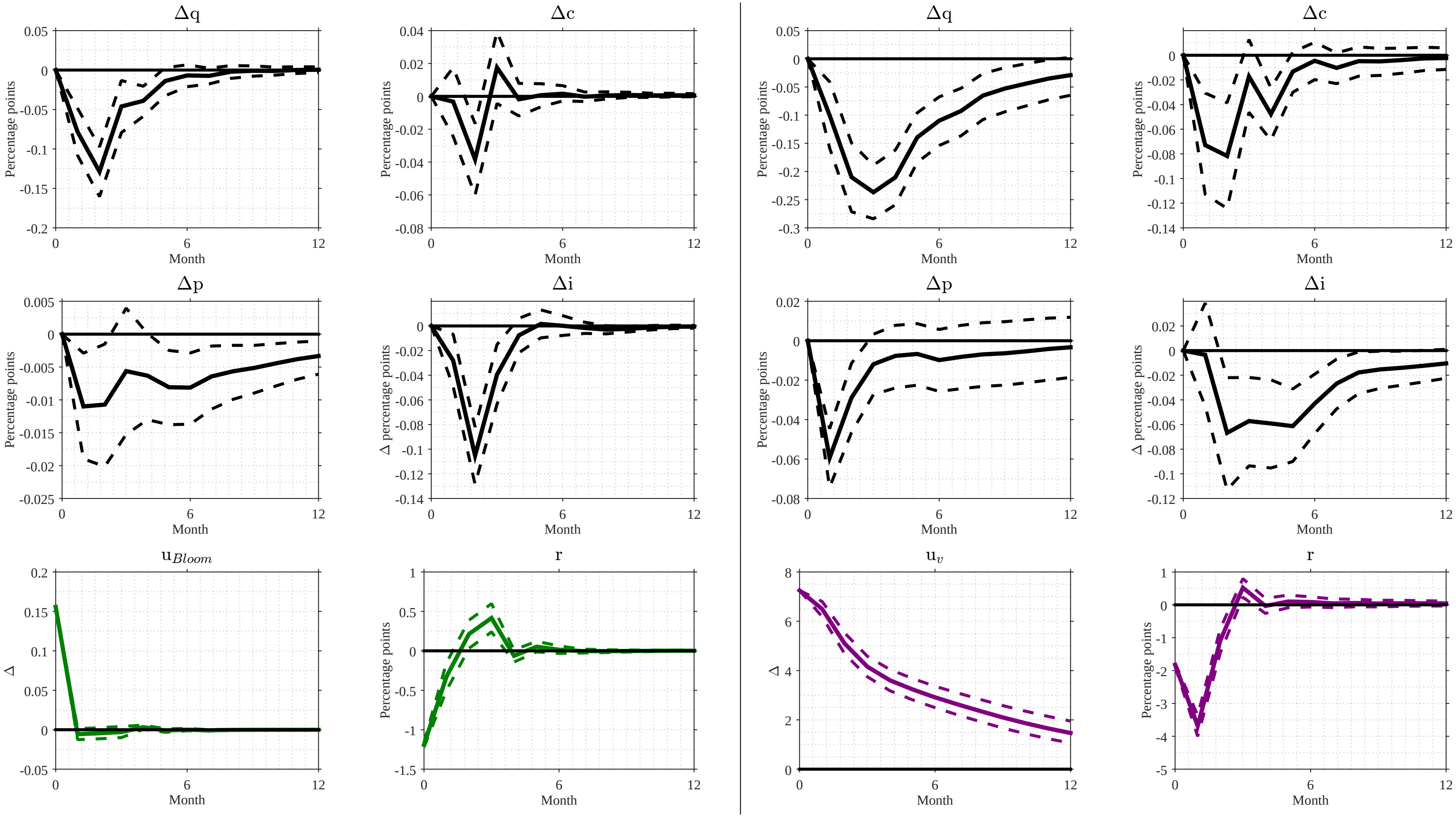

Figure 3: The impact of a Bloom uncertainty shock (left panel) and a mean uncertainty shock (right panel)

Notes: The panels depict the impulse responses of the macroeconomic variables (Δq, Δc, Δp, Δi) and of the financial variables (u_v, r) to a Bloom uncertainty shock and a mean uncertainty shock, identified within a regular VAR. Solid lines refer to the median impulse responses and the dashed lines correspond to posterior 68% probability bands. Δq denotes growth in industrial production; Δc is growth in consumption, Δp inflation, Δi changes in the interest rate, u_v the proxy of uncertainty, and r stock returns.

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The mean uncertainty shock appears to be a shock, whose impact ranges mid-way between the impact of an uncertainty shock and a certainty shock. This suggests that a mean uncertainty shock mixes uncertainty and certainty shocks, underestimating the impact of the former and overestimating the impact of the latter. The persistence of the impact of the shock on the proxy of uncertainty lies in between the persistence observed for the uncertainty shock and certainty shock. While being more strongly affected on impact, stock returns recover in the third month after the shock. Counting the significant responses, after six months I observe a cumulated loss of about 6.1%, which is in between the decline observed with the uncertainty shock (-14.6%) and the certainty shock (-2.3%). In response to a mean uncertainty shock, the real economy suffers losses. Specifically, real activity falls by 1% until month six after the shock and consumption by 0.2%. These responses are very similar to those obtained through the uncertainty shock (-1.2% and -0.3%). While the monetary authority reacts in a similar fashion to the case of the uncertainty shock (both -0.3p.p. after six months), prices respond more strongly (-0.09% vs. -0.05% after two months). The forecast error variance decomposition paints a similar picture, as the importance of the mean uncertainty shock for real activity, consumption, or stock markets lies between the values obtained for the uncertainty and certainty shocks; within the first half-year: 3.8% vs. 8.6% and 0.3%; 1% vs. 2.6% and 0.0%; 20.8% vs. 39.9% and 2.8%). Still, a mean uncertainty shock and an uncertainty shock are found to be similarly important for explaining fluctuations in inflation (1.8% vs. 1.8%).

The other proxies analyzed in this study (see Appendix D) suggest three further findings: i) Not all proxies of uncertainty generate an asymmetric impact. I conclude this, because the measure of forecast dispersion predicts that shocks to the upper and lower tail have similar effects. ii) The measure of fundamental macro uncertainty is as important for real activity as the mean uncertainty shock, but does not reflect an important shock for stock markets. Thus, it measures a different type of uncertainty shock. iii) Fundamental financial uncertainty is not important for explaining fluctuations in stock markets or for the macroeconomy.

4.1 The impact of uncertainty/certainty shocks on the tails of macroeconomic variables

In this section, I shed light on the impact of uncertainty and certainty shocks on the downside risk, i.e. 0.1 quantile, and upside risk, i.e. 0.9 quantile, of the macroeconomic variables. The pseudo impulse responses are shown in Figure 4 and the pseudo forecast error variance decompositions are summarised in Table 2. The responses of the 0.9 quantiles of the macroeconomic variables are in red and the responses of the 0.1 quantiles are in blue. Since both u_v and r are assumed to be in the same quantiles when analysing the downside and upside risks, I mark the responses with blue (red) dotted lines that represent the responses relevant for the downside (upside) risk.

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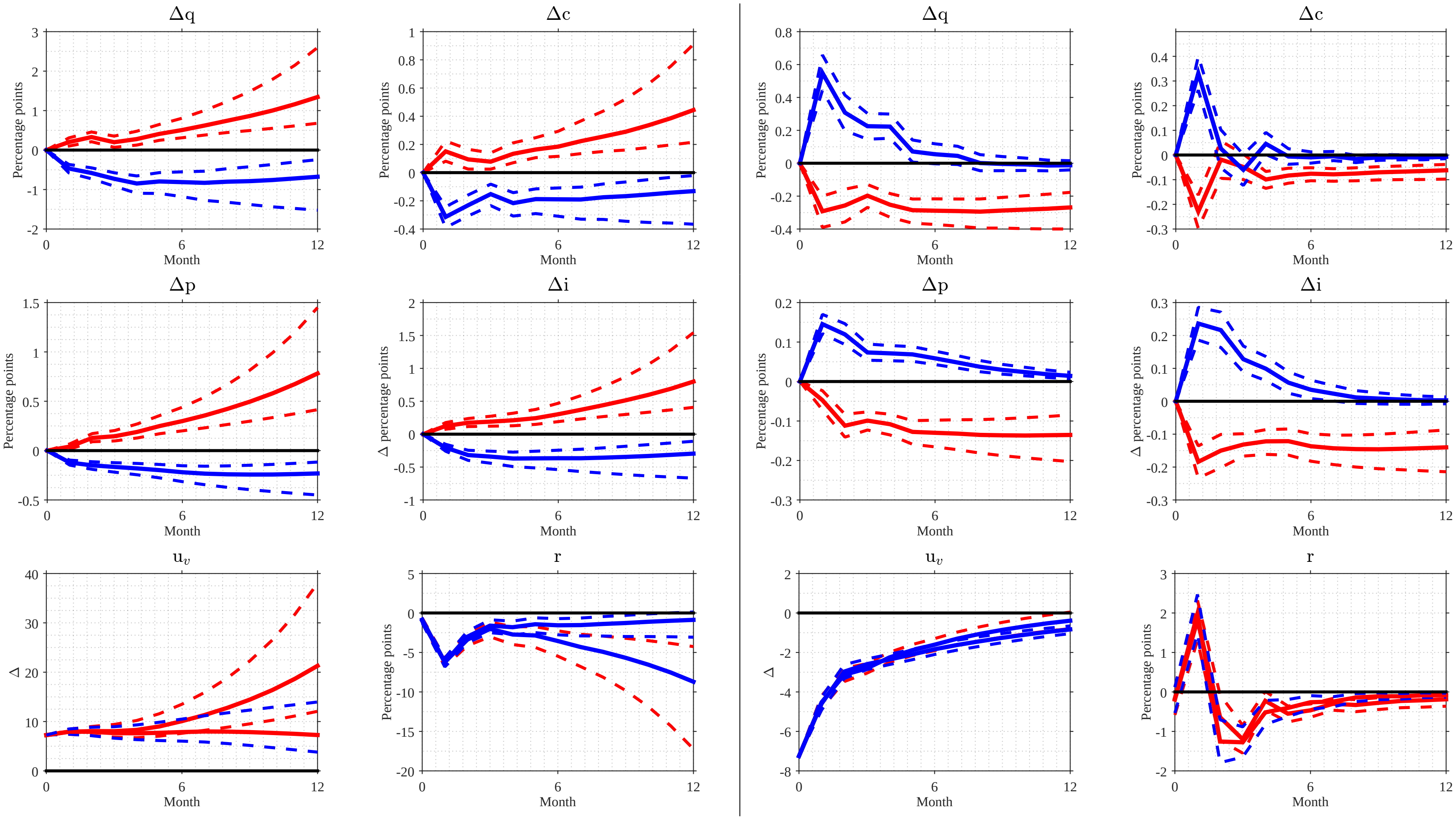

Figure 4: The impact of uncertainty (left panel) and certainty (right panel) shocks on the lower and upper tails of macroeconomic variables

Notes: The panels depict the pseudo impulse responses to an uncertainty and certainty shock to the 0.1 quantile and 0.9 quantile of the macroeconomic variables (Δq, Δc, Δp, Δi). For identifying uncertainty shocks, u_v and r are at their 0.9 and 0.1 conditional quantiles respectively; for identifying certainty shocks, they are at their 0.1 and 0.9 conditional quantiles. Solid lines refer to the median impulse responses, and the dashed lines correspond to posterior 68% probability bands. In the left panel, responses of u_v and r with blue (red) dotted lines represent the responses when considering the 0.1 (0.9) quantile of macroeconomic variables. In the right panel, responses of u_v and r with red (blue) dotted lines represent the responses when considering the 0.9 (0.1) quantile of macroeconomic variables. Δq denotes growth in industrial production; Δc is growth in consumption, Δp inflation, Δi changes in the interest rate, u_v the proxy of uncertainty (stock market volatility), and r stock returns.

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Uncertainty shock

An uncertainty shock leads to similar initial responses for u_v and r in both the downside risk and the upside risk analyses. Differences only emerge in later months. For instance, considering the 0.1 quantile, the median of the response of u_v slowly returns to zero. In contrast, considering the 0.9 quantile, the median of u_v diverges from zero.

Table 2: Variance in the lower and upper quantiles of macroeconomic variables explained by uncertainty and certainty shocks

Months	Δq	Δc	Δp	Δi	u_v	r
Uncertainty shock						
0.1 quantile - downside risk						
0–6	25.3	15.6	25.4	17.6	89.5	42.2
7–12	41.8	25.8	48.4	36.1	71.2	45.6
0.9 quantile - upside risk						
0–6	11.1	6.0	13.5	10.0	85.9	43.8
7–12	36.0	25.6	37.9	33.6	62.0	47.0
Certainty shock						
0.1 quantile - downside risk						
0–6	3.2	2.8	5.4	2.5	89.6	3.6
7–12	3.9	3.3	8.3	3.4	82.0	4.8
0.9 quantile - upside risk						
0–6	2.6	1.3	2.0	1.9	91.7	2.9
7–12	6.1	2.3	5.4	4.2	80.6	4.1

Notes: Table shows the forecast error variance of an uncertainty and certainty shock for the upper and lower quantiles of the macroeconomic and financial variables. Δq denotes growth in industrial production; Δc is growth in consumption, Δp inflation, Δi changes in the interest rate, u_v the proxy of uncertainty (stock market volatility), and r stock returns. For more details on the variables, please see Appendix A.

The tails of the real economy respond permanently, but in opposite directions, to the uncertainty shock. For growth in economic activity, the downside risk increases as the 0.1 quantile is significantly lowered by about -0.5 p.p. one month after the shock. In contrast, the upside risk increases as the 0.9 quantile is raised by 0.2 p.p. Thus, the downside risk increases by a factor of more than two relative to upside risk. The downside risk of consumption growth increases by lowering the 0.1 quantile by -0.3 p.p. and the upside risk increases by elevating the 0.9 quantile by 0.2 p.p. one month after the shock. For inflation, the effects are -0.1 p.p. and 0.04 p.p. for the 0.1 and 0.9 quantile respectively; for changes in interest rates -0.2 p.p. changes and 0.1 p.p. changes. Explained by the differing dynamics in u_v, the median responses of the 0.9 quantiles diverge from zero, while the median responses of the 0.1 quantiles slowly converge to zero.

The pseudo forecast error variance decompositions support the notion that uncertainty shocks are more important for explaining the variance of the 0.1 quantiles of macroeconomic variables than the 0.9 quantiles. In the first six months, uncertainty shocks account for 25.3% of the fluctuations in the 0.1 quantile of economic activity growth and 11.1% of the fluctuations in the 0.9 quantile. For consumption growth it is 15.6% vs. 6%, for inflation 25.4% vs. 13.5%, and for interest rate changes 17.6% vs. 10.0%. In the second half of the first year, the importance, however, becomes similar.

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While the uncertainty shock explains more of the fluctuations in u_v in the 0.1 quantile (89.5%) than in the 0.9 quantile (85.9%), it is the opposite – albeit only marginally – for stock returns (43.3% vs. 43.8%), for instance, in the first half-year after the shock.

Certainty shock

A certainty shock leads to similar initial responses for u_v and r when considering downside and upside risks. Even in later months, differences remain marginal. A certainty shock leads to a temporary decrease in u_v. Furthermore, stock returns initially increase and drop thereafter, converging back to zero.

The tails of the real economy respond to the certainty shock in opposite directions. Interestingly, the effects on the 0.1 quantiles are only temporary, while highly persistent for the 0.9 quantiles. For instance, the initial response, i.e. one month after the shock, to a certainty shocks leads to a reduction in the downside risk of economic activity growth as the 0.1 quantile increases by 0.5 p.p. This effect, however, is temporary and becomes insignificant after month six. In contrast, upside risk diminishes by -0.3 p.p. one month after the shock, but persistently. The initial responses are 0.3 p.p. vs. -0.2 p.p. for the 0.1 and 0.9 quantile of consumption growth respectively, 0.1 p.p. vs. -0.04 p.p. for inflation, and 0.2 p.p. changes vs. -0.2 p.p. changes for interest rates.

The pseudo forecast error variance decompositions indicate an importance of certainty shocks for the tails of the macroeconomic variables between 1.3% and 8.3%. A certainty shock explains more variance of the fluctuations of the 0.1 quantiles than of the 0.9 quantiles in the first half of the year after the shock, though less for some variables in the second half of the year, supporting the notion of permanent effects on the 0.9 quantile. For instance, a certainty shock explains 3.2% of fluctuations of the 0.1 quantile in economic activity growth and 2.6% of the 0.9 quantile. These figures are 2.8% and 1.3% for consumption growth, 5.4% and 2.0% for inflation, and 2.5% and 1.9% for interest rate changes. The variance explained is larger for the 0.9 quantile in the second part of the first year after the shock, for instance, for economic activity growth (0.1: 3.9% vs. 0.9: 6.1%).

Uncertainty shock versus certainty shock

Uncertainty shocks permanently affect both downside and upside risks, while certainty shocks permanently affect only the upside risks. Specifically, an uncertainty shock increases the downside risk and reduces the upside risk of macroeconomic variables permanently. Certainty shocks only reduce the upside risk permanently. Downside risks are reduced for several months only. Furthermore, uncertainty shocks move the 0.1 quantile relative to the 0.9 quantile more strongly than the certainty shocks.

Similar to before, it is important to note that uncertainty shocks account for a large percentage of variance of the tails of the macroeconomic variables. Certainty shocks do not account for much

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variance in relative terms. Still, their impact is found to be significant and more important for the tails of macroeconomic variables than for the centre of the distribution.

5 UNDERSTANDING UNCERTAINTY AND CERTAINTY SHOCKS

In the case of the uncertainty shocks, of interest are the innovations that fall above their estimated conditional quantile at 0.9, i.e. the right tail exceedances (upward spikes of $\varepsilon_{5t|\boldsymbol{\tau}}$; red in Figure 5). In the case of the certainty shocks, of interest are the innovations that fall below their estimated conditional quantile at 0.1, i.e. the left tail exceedances (downward spikes of $\varepsilon_{5t|\boldsymbol{\tau}}$; blue in Figure 5). Exceedances are of interest as they mark the strongest 10% of shocks, being informative about the location of the upper and lower quantiles and, thus, determining the dynamic responses of the system.

To understand uncertainty shocks and certainty shocks, and potential differences, I relate those exceedances, first, to the uncertainty-to-certainty ratio (Figure 5, black line), second, to collected newspaper headlines (Appendix C), and third, to the u_m and u_f proxies of uncertainty (Figure 7).

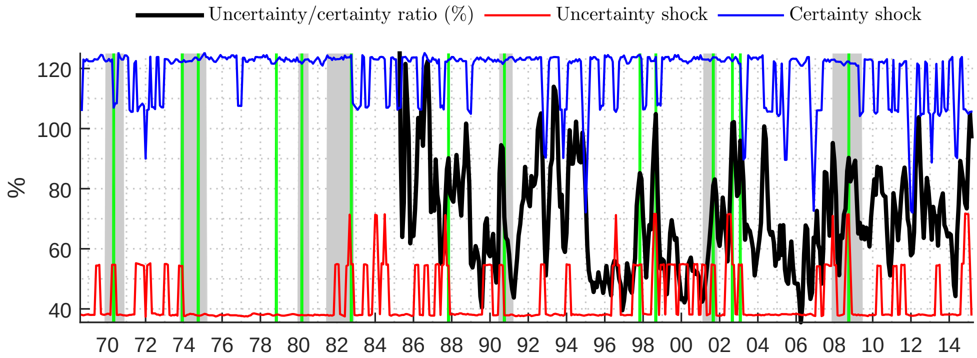

Figure 5: Uncertainty-certainty ratio and the uncertainty / certainty shocks

Notes: The uncertainty-to-certainty ratio indicates the number of newspaper articles with uncertainty narratives relative to the number of articles with certainty narratives during a specific month. For more details on the ratio, please see Section 5 and Appendix B. The uncertainty-to-certainty ratio and the structural shocks are smoothed using a three-month moving average. The transformation is applied in order to reduce noise that is present in the two series. Green lines reflect Bloom (2009) dates of uncertainty and the grey areas NBER recession periods.

At first glance, Figure 5 suggests that spikes that mark exceedances do not necessarily relate to business cycle fluctuations, as they occur over the entire sample and not only during NBER recession periods (grey area). Furthermore, exceedances occur in tandem with changes in the uncertainty-to-certainty ratio. To stress this result more formally, Figure 6, which is discussed below, summarises the developments of the uncertainty-to-certainty ratio around the specific dates of exceedances. In this figure, the solid line *"first exceedances"* shows the developments of the ratio around dates of exceedances, which were not preceded by other left or right tail exceedances in the month before. This allows me to focus on the arguably most relevant events, as consecutive exceedances have a

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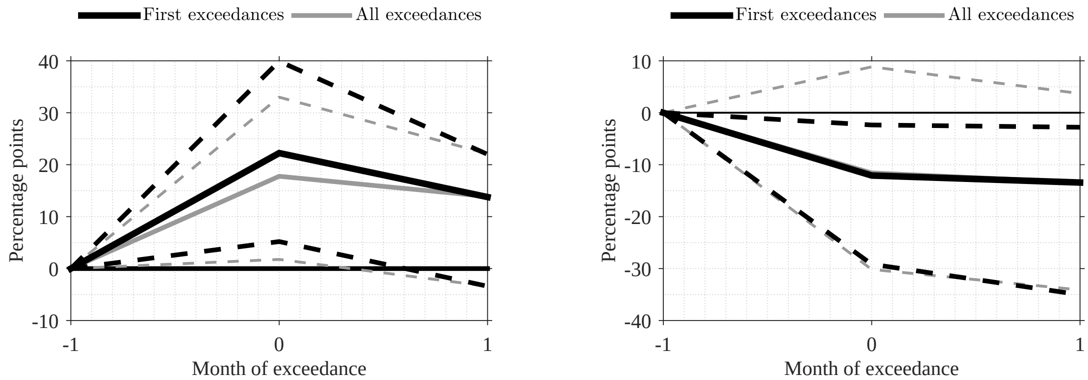
high chance of being related to the same event.[19] Note that, for the same reason, the collection of newspaper headlines focusses specifically on months of *first exceedances*.

What are uncertainty shocks?

Periods of exceedances of the uncertainty shock series go hand in hand with increases in the uncertainty-to-certainty ratio (see Figure 6a). That is to say, during months of exceedances, there exists a relative increase in uncertainty narratives relative to certainty narratives. The figure suggests that the ratio rises by about 22 percentage points, considering the median development around *first exceedances*. Considering all dates of exceedances, the median response is 18 percentage points. Periods of exceedances of the uncertainty shock series go hand in hand with increases in the uncertainty-to-certainty ratio (see Figure 6a). That is, during months of exceedances, there exists a relative increase in uncertainty narratives relative to certainty narratives. The figure suggests that the ratio rises by about 22 percentage points, considering the median development around *first exceedances*. Considering all dates of exceedances, the median response is 18 percentage points.



(a) Uncertainty shock (b) Certainty shock

Figure 6: Uncertainty-to-certainty ratio around dates of exceedances

Notes: -1 (1) indicates month prior (past) to the exceedance. The solid line is the median development of the ratio around dates of exeedances. Dotted lines mark the 75% and 25% quantiles. First shock means that the exceedances shown are not preceeded by other left or right tail exceedances in the month before.

Uncertainty shocks can be associated with events that are commonly thought of as fundamental shocks. Events reflect, for instance, the downgrading of the U.S. credit rating (August 2011) or the uncertainty over the political future of President Richard Nixon (November 1973). I also find that environmental shocks may lead to a sudden increase in uncertainty, such as the nuclear crisis in Japan (March 2011). While these events reflect fundamental shocks, fears about future fundamental shocks seem to be relevant as well. The exceedance in October 1989, during which the worst drop on stock markets occurred after Black Monday 1987, is strongly accounted for by fears regarding future

[19] Within the category *first exceedances*, 46 out of 56 are uncertainty shocks and 33 out of 56 are certainty shocks.

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real economic outcomes. Another example marks the exceedance in August 1999, the month in which Alan Greenspan indicated concerns over the highly valued stock market that sent down stocks sharply. Analysts remarked that Greenspan's concerns carried extra weight during an otherwise quiet session. Clearly, this event exacerbated already existing fears, as suggested, for instance, by another newspaper article entitled "When the bubble bursts. . .".

Furthermore, exceedances of the uncertainty shock series occur on all Bloom (2009) dates of uncertainty shocks, except for the dates of the Franklin National financial crisis shock (October 1974), the OPEC II oil price shock (November 1978), and Afghanistan war / Iran hostages shock (March 1980) (see Figure 5). For this specific validation exercise, it is important to keep in mind that the identified uncertainty shocks mark innovations, while the Bloom (2009) dates refer to the unconditional developments. Given that the Bloom dates of uncertainty are identified on the basis of the same proxy, they must be accounted for by developments in the other series included in the BQVAR or be part of the remaining shock series which, however, are not subject to analysis in this paper.

Finally, the exceedances coincide with jumps in uncertainty as captured by the LMN financial index (see Figure 7).They correlate more weakly with the JLN macro uncertainty measure.

What are certainty shocks?

The uncertainty-to-certainty ratio declines during months of exceedances of the certainty shock (see Figure 6b). This indicates that there exits a relative increase in certainty narratives over uncertainty narratives. During dates of *first exceedances*, the median decrease in the ratio is -12 percentage points. Considering all dates of exceedances, the median decline is also around -12 percentage points.

The analysis of newspaper articles suggests that exceedances of the certainty shocks are often associated with strong increases or records in stock indices. However, record-breaking major indices do not provide fundamental news and only irrationally increase investors' confidence. For instance, in December 2005 an exceedance relates to the event that the S&P 500 index reached new highs for the year. Even more suggestive are months of exceedances, such as November 1972, during which the Dow hit 1,000 for the first time in history. Few of the exceedances also relate to fundamental shocks. For instance, in August 1968 an exceedance relates to sudden peace hopes for Vietnam, or in June 2005 to a sudden decline in oil prices.

While by construction the exceedances of the certainty shock do not relate to the Bloom (2009) dates, interestingly, they partially correspond to sudden decreases in the LMN and JLN proxies of uncertainty. For instance, the sudden decrease of the LMN financial uncertainty proxy around 2012 coincides with left tail exceedances, or the lowest point of the LMN measure prior to the global financial crisis also corresponds to left tail exceedances, indicating that certainty shocks are somehow reflected in these aggregate measures of uncertainty.

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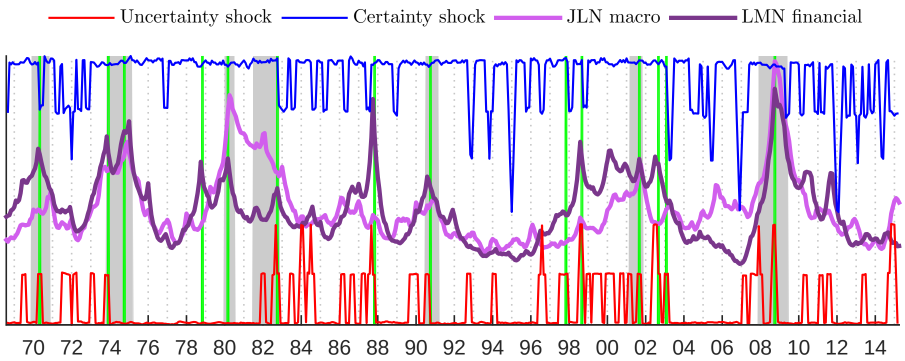

Figure 7: Uncertainty and certainty shocks related to other measures of uncertainty

Notes: Structural shocks are smoothed using a three-month moving average. Green vertical lines reflect the Bloom (2009) dates of uncertainty shocks, and the grey areas NBER recession dates.

Uncertainty shock versus certainty shock

As the above analysis suggests, uncertainty shocks and certainty shocks relate to very different events. First, the uncertainty-to-certainty ratio distinctly moves for each of the shocks. Second, I find that uncertainty shocks are often associated with fundamental shocks or fears about fundamental shocks, whereas certainty shocks are not. Rather, events are reminiscent of irrational exuberance.

It is interesting to note that findings in the field of social psychology may provide an explanation for the differing events related to uncertainty and certainty shocks (see Tiedens and Linton (2001) and references therein). Here, it is argued that uncertainty provokes systematic information processing, implying agents that only adhere to fundamental shocks. In contrast, certainty induces heuristic information processing, implying that agents base their judgements on superficial cues, such as records in stock indices. From a policy perspective, this provides justification for cautionary statements or policy actions by regulators during phases of certainty shocks.

6 Concluding Remarks

Within a novel Bayesian quantile VAR (BQVAR) framework, I identify uncertainty shocks and certainty shocks building on the seminal study by Bloom (2009).

The BQVAR framework allows me to model a transmission channel from financial markets to the real economy that Bloom's (2009) identification cannot. My results suggest an important role for this transmission channel. Uncertainty shocks identified within the BQVAR framework lead to a persistent rise in stock market volatility that is not captured through Bloom's identification. As a consequence, Bloom's uncertainty shock underestimates the importance of uncertainty shocks.

Furthermore, I show that uncertainty and certainty shocks have very different impacts on the real economy and that a regular VAR mixes these two shocks. This is an important result, as several

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studies assume linearity when identifying uncertainty shocks.

In line with the difference in impacts, I find that events related to the uncertainty and certainty shocks differ. While uncertainty shocks relate to fundamental shocks or fears about future fundamental shocks, certainty shocks do so only rarely. Most often, certainty shocks cannot be clearly linked to fundamental shocks. Events related to these shocks tend to be reminiscent of irrational exuberance. This is consistent with the view that certainty induces heuristic information processing (Tiedens and Linton (2001)). As certainty shocks significantly impact on the real economy, these findings stress the importance of regulators cautioning markets during phases of irrational exuberance identified, for instance, through the analysis of uncertainty shocks.

Finally, I find that uncertainty shocks strongly and importantly increase the downside risks of the real economy and should thus be carefully incorporated into scenario analyses regularly undertaken by policy makers.

Paths for future research are ample. Other shocks commonly identified in a linear way could also have strong non-linear consequences for the real economy. Furthermore, the analysis of downside and upside risk seems to be a promising avenue to characterise the importance of financial markets for the real economy. Future research should expand in this type of analysis identifying various financial shocks.

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A Data

Table 3: Data sources, descriptions, and transformations

Variable	**Symbol**	**Base Variable**	**Transformation**	**Code/Source**
Real economic activity growth	Δq	real manufacturing industrial production	log differences	IP.B00004.S (FRB)
Real consumption growth	Δc	real personal consumption expenditures	log differences	DPCERC1 (BEA)
Inflation	Δp	personal consumption expenditure price deflator	log differences	DPCERG3 (BEA)
Change in interest rates	Δi	effective federal funds rate	differences	USFEDFUN (DS)
Stock returns	r	S&P 500	log differences	S&PCOMP (DS)
Stock market volatility	u_v	S&P 500 and since 1986 VXO index	-	see Bloom (2009)
Forecast dispersion in business outlook	u_{fd}	Philadelphia Fed's Business Outlook Survey	-	see Bachmann et al. (2013)
JLN macroeconomic uncertainty ($h = 1$)	u_m	-	-	Jurado et al. (2015)
LMN financial uncertainty ($h = 1$)	u_f	-	-	Ludvigson et al. (2018)

Notes: FRB denotes Federal Reserve Board, BEA Bureau of Economic Analysis, and DS Datastream. All data are seasonally adjusted when necessary.

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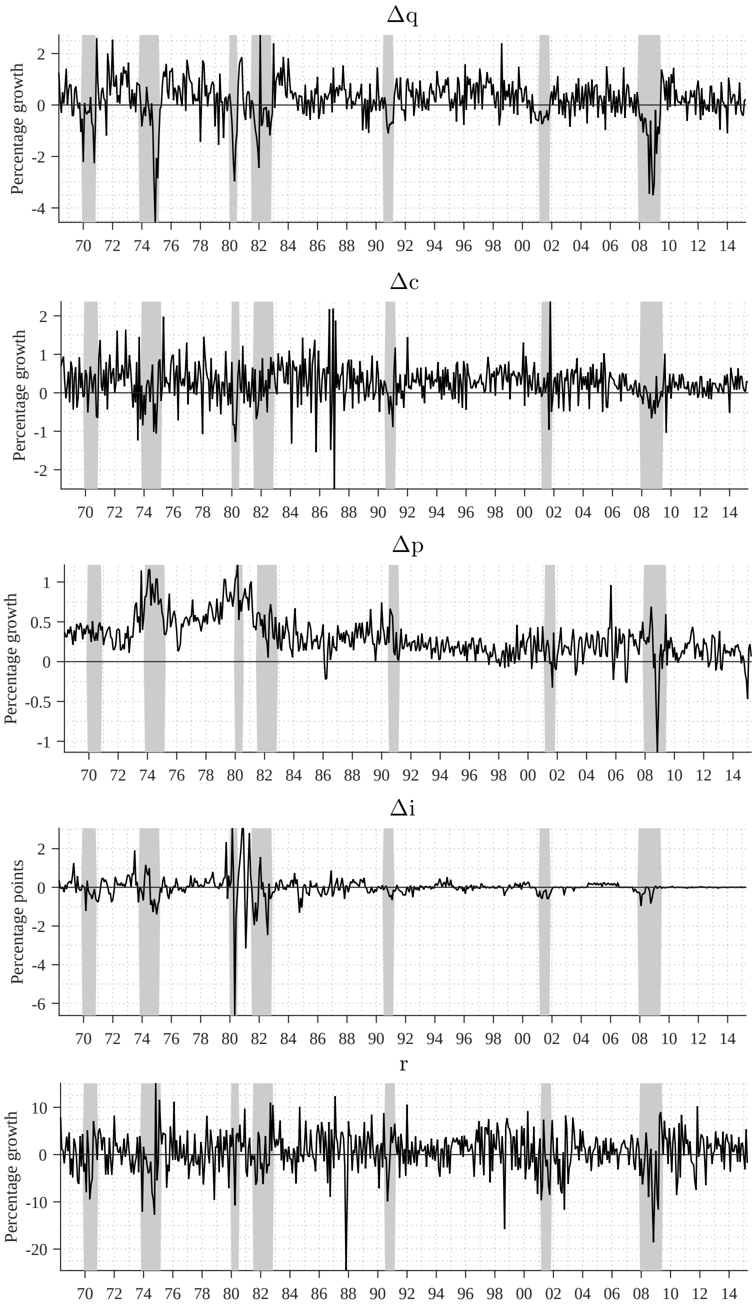

Figure 8: Macro variables and stock returns

Notes: Grey shaded areas mark NBER recession periods.

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B CONSTRUCTING THE UNCERTAINTY-TO-CERTAINTY RATIO

This section summarizes the construction of the uncertainty-to-certainty ratio. As indicated, I closely follow Baker et al. (2016).

Let x_t^{UC} and x_t^{C} denote the number of newspapers related to uncertainty and certainty in month t respectively. The index, X_t, reflects the ratio of the two:

$$X_t = \frac{x_t^{\mathrm{UC}}}{x_t^{\mathrm{C}}} \cdot 100.$$

In determining the number of newspaper articles, I exploit Factiva and search ten different U.S. newspaper magazines. These magazines are: LA Times, USA Today, Chicago Tribune, Washington Post, Boston Globe, Wall Street Journal, Miami Herald, Dallas Morning News, Houston Chronicle, and San Francisco Chronicle.

The search terms for capturing uncertainty are: (economy OR economic) AND ("stock market" OR "stock markets" OR "stock index" OR "stock indexes" OR "stock indices" OR S&P OR "Standard & Poor") AND (uncertainty OR uncertain OR fear OR concern OR panic OR worry OR doubt OR low) NOT (certain OR certainty OR trust OR faith OR confidence OR euphoria OR hope OR high).

The search terms for capturing certainty are: (economy OR economic) AND ("stock market" OR "stock markets" OR "stock index" OR "stock indexes" OR "stock indices" OR S&P OR "Standard & Poor") AND (certain OR certainty OR trust OR faith OR confidence OR euphoria OR hope OR high) NOT (uncertainty OR uncertain OR fear OR concern OR panic OR worry OR doubt OR low).

I deviate from Baker et al. (2016), as I specifically focus on news related to stock markets and, furthermore, that I require that specific words do not appear.

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C NEWSPAPER HEADLINES ASSOCIATED WITH THE UNCERTAINTY AND CERTAINTY SHOCKS

Table 4, Table 5, and Table 6 show events that I find during months of uncertainty shocks, i.e. during times of exceedances. Here, I focus on shocks that that were not preceded by another uncertainty/certainty shocks in the month before. The same holds for Table 7 and Table 8 that show events that occurred during months of exceedances of the certainty shock series.

All tables refer to a representative newspaper article of the indicated month, describing an event that contributed to the uncertainty and certainty shock respectively. Additional keywords that describe the events are included in occasions, where I find that the headline is not sufficiently informative about the underlying event. I classify the type of the event into several categories: economic, environmental, oil, political, terror, and war.

The search of newspaper headlines was done using the New York Times online archives and Factiva considering the ten newspapers stated in Appendix B. I consulted The New York Times online archives in order to cover the time period prior to 1985. The search phrase was "Dow Jones" OR "Standard and Poor*" OR "Stock Market", so as to screen news related to stock market news, in line with the identification of shocks.

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Table 4: Uncertainty shocks

Headline (date), *newspaper* (or Bloom (2009))	*Additional keywords*	Date of shock	Type
Stocks slip to a '69 low (1969, July 10), *The New York Times*	*Lack of peace progress in Vietnam; growing tension in Middle East; concerned about widely predicted slowdown*	1969-07	War, Economic
Cambodia and Kent State (Bloom (2009))		1970-05	War
Stock prices drop across wide front (1971, August 4), *The New York Times*	*Also: Nixon's Wage and Price Controls*	1971-08	Economic
Dow Industrials off 11.24 to 814.91, lowest of '71 (1971, November 12), *The New York Times*	*New pricing standards; strength of the growing bearish sentiment*	1971-11	Economic
Bewildered market skids 16.85 points (1973, February 15), *The New York Times*	*Most massive decline in nearly 20 months; dollar devaluation*	1973-02	Economic
Wide uncertainty on Nixon and oil stirs decline (1973, November 6), *The New York Times*	*Uncertainty over political future of President Nixon; oil shortages induced by the Middle East conflict*	1973-11	Political, Oil
Stocks retreat on rate fears (1982, January 26), *The New York Times*	*Renewed fears of rising interest rates*	1982-01	Economic
Dark days on Wall Street (1982, August 12), *The New York Times*; monetary cycle turning point (first volatility: Bloom (2009))	*Cruel economic news from all sides*	1982-08	Economic
Monetary cycle turning point (Bloom (2009))		1982-10	Economic
Foreign debt worries send Dow down by 10.21 (1983, July 8), *The New York Times*; Rate fear and earnings (1983, July 11), *The New York Times*	*Fear of rising interest rates; disappointing earnings*	1983-07	Economic
Stocks fall broadly, Dow off 8.48 (1984, January 31), *The New York Times*	*A 50-point retreat in the last nine trading days; concerns over direction of interest rates, the slowing economy and the huge Federal budget deficit*	1984-01	Economic
Industrials fall to 15 1/2-month low on worry about third world debt (1984, June 15), *The Wall Street Journal*	*Concern about repayment of debt*	1984-06	Economic
Economic indicators fall 0.8% (1984, August 30), *The Washington Post*	*Indicator declined for two consecutive months*	1984-08	Economic
International tensions drive market lower (1986, March 26), *Los Angeles Times*	*Libya conflict*	1986-03	War
Stock prices fall by record amount in busiest session (1986, September 12), *The New York Times*	*Growing concern about the nation's economy; worries about interest rates*	1986-09	Economic
Markets battered again; Dow off 34 (1987, April 15), *The Washington Post*	*Continuing fall of the dollar; persistent U.S. trade deficit*	1987-04	Economic
Black Monday (first volatility: Bloom (2009))		1987-10	Economic

Note: Bloom (2009) dates of uncertainty refer to the maximum volatility dates unless otherwise indicated.

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Table 5: Uncertainty shocks (cont'd)

Headline (date), *newspaper* (or Bloom (2009))	*Additional keywords*	Date of shock	Type
Dow falls 190 (1989, October 14), *Los Angeles Times*	*Drop is worst since '87 crash stock market: the selloff is fueled by fears that takeover fever is cooling, and that record prices will moderate*	1989-10	Economic
Basic correction: unlike October dives, this stock market fall is due to fundamentals — as profits fall and rates rise many buyers stand aside and look for the bottom — a 250-point drop in 3 weeks (1990, January 26), *The Wall Street Journal*		1990-01	Economic
Middle East woes batter markets Dow plunges on surge in anxiety (1990, August 7), Chicago Tribune	*Tensions in Middle East; oil prices continued to skyrocket*	1990-08	War, Oil
The Dow's sell-off could signal a correction (1992, October 5), *The Wall Street Journal*	*Sick economy; election growing more uncertain; economic turmoil abroad among biggest trading partners*	1992-10	Economic, Political
Fed's move jolts stock and bond markets (1994, February 5) *The New York Times*	*Monetary cycle turning point; first increase in short-term interest rates in five years*	1994-02	Economic
Stock indexes slide to six-month lows markets (1996, July 24), *Los Angeles Times*; Stock markets skid on worry about profits, interest rates (1996, July 16), *The Washington Post*	*Technology sector leads retreat amid concerns that the pace of business will falter during the remainder of the year; stock market took one of its sharpest dives ever*	1996-07	Economic
Broad stock sell-off signals change in market — cyclical issues take command (1997, August 18), *The Wall Street Journal*	*Second largest decline ever of Dow; weaker dollar; rising interest rates; outlook of lower-than-expected earnings*	1997-08	Economic
Asian Crisis (Bloom (2009))		1997-11	Economic
Russian, LTCM default (Bloom (2009))		1998-08	Economic
Crisis is deepening in Brazil markets (1999, January 15), *The New York Times*	*Brazil's economic crisis*	1999-01	Economic
Greenspan's remarks send down 108; yields jump (1999, August 28), *Los Angeles Times*; When the bubble bursts... (1999, August 18), *The Wall Street Journal*	*Concerns over the highly valued stock market*	1999-08	Economic
Markets shaken as economic statistics fan inflation fears (2000, January 29), *The New York Times*	*Three major market indexes prepared to post their worst performance for January in a decade or more; rising short-term rates and fear over interest rate rise*	2000-01	Economic
Stock market in steep drop as worried investors flee; Nasdaq has its worst week	*One of the worst weeks in the history of United States markets; higher-than-expected inflation*	2000-04	Economic

Note: Bloom (2009) dates of uncertainty refer to the maximum volatility dates unless otherwise indicated.

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Table 6: Uncertainty shocks (cont'd)

Headline (date), *newspaper* (or Bloom (2009))	*Additional keywords*	Date of shock	Type
Volatile world events cause investor flight from stocks (2000, October 13), *The New York Times*	*Oil prices surging; turmoil in middle east escalating*	2000-10	War, Oil
Markets plunge in wide sell-off; Nasdaq falls 6% (2001, March 13), *The New York Times*	*Worries about slowing economy; declining corporate earnings*	2001-03	Economic
9/11 terrorist attack (Bloom (2009)))		2001-09	Terror
Year's first half is worst in 32 years; June 24-28, 2002 (2002, June 30) *The Washington Post*	*Economic conditions; corporate accounting scandals*	2002-06	Economic
Worldcom Enron (Bloom (2009))		2002-09	Economic
Gulf War II (Bloom (2009))		2003-02	War
Asia and Europe stocks follow Wall Street (2007, March 14), *The New York Times*	*Concerns spread about the consequences of loose lending practices in the United States housing market*	2007-03	Economic
Impact of mortgage crisis spreads — Dow tumbles 2.8% as fallout intensifies; moves by central banks (2007, August 10), *The Wall Street Journal* (Credit crunch: first volatility: Bloom (2009))	*BNP Paribas decides to suspend three hedge funds focused on US mortgages*	2007-08	Economic
It's official. Wall Street correction — industrials, S&P 500 drop 10% from highs as recession fears grow (2007, November 27), *The Wall Street Journal*		2007-11	Economic
Global stocks plunge as U.S. crisis spreads; sell-offs in all major exchanges (2008, January 22), *The Washington Post*		2008-01	Economic
Lehman Brothers' collapse (Bloom (2009))		2008-09	Economic
Stocks plunge on fears of a spreading European crisis (2010, May 21), *The New York Times*		2010-05	Economic
Shares fall amid concerns about Japan (2011, March 13), *The New York Times*	*Nuclear crisis in Japan*	2011-03	Environmental
Stock market plummets after historic downgrade of U.S. credit rating (2011, August 9), *The Washington Post*	*S&P downgrades U.S. credit rating*	2011-08	Economic
Markets extend slide over Fed concerns, poor economic news from China (2013, June 21), *The Washington Post*	*Investors are anxious the Fed will pull back on stimulus and unnerved by weak economic data from China; biggest one-day drop since 2011*	2013-06	Economic
Steep sell-off spreads fear to Wall Street (2014, October 16), *The New York Times*	*Fear that governments and central banks have failed to anticipate the recent weakening in the global economy; particularly in Europe; Vix to its highest level since 2011*	2014-10	Economic

Note: Bloom (2009) dates of uncertainty refer to the maximum volatility dates unless otherwise indicated.

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Table 7: Certainty shocks

Headline (date), *newspaper*	*Additional keywords*	Date of shock	Type
Market rallies on broad front (1968, August 13), *The New York Times*	*Peace hopes for Vietnam*	1968-08	War
Dow is up by 11.59 in heavy trading (1972, February 10), *The New York Times*	*Best level since early September*	1972-02	Economic
Stocks rebound on a broad front (1972, June 15), *The New York Times*	*Partial recovery after a prolonged decline touched off by profit taking; "technical rebound"; there was no hard news to account for the snapback*	1972-06	Economic
The Dow at 1,000 (1972, November 17), *The New York Times*	*Above 1,000 for the first time in history*	1972-11	Economic
Oil-price optimisim lifts market (1976, December 15), *The New York Times*	*Saudi Arabian oil minister had called for six-month freeze in oil prices*	1976-12	Oil
Stock prices climb briskly (1981, January 28), The New York Times	*Prospects for early decontrol of domestic crude oil price*	1981-01	Oil
Dow jumps to 1,070.55, a record (1982, December 28), The New York Times	*Signs of an economic recovery*	1982-12	Economic
Stocks gain in late rally; 2 indexes shatter records (1985, April 26), *Chicago Tribune*		1985-04	Economic
Stocks indexes end at record levels after early dip on bond weakness (1986, May 30), The Wall Street Journal		1986-05	Economic
Stocks rocket to 3rd straight record high (1987, July 31), *Houston Chronicle*		1987-07	Economic
Dow finishes year above 2,000 mark (1988, December 31), *Houston Chronicle*	*Bluechip (...) at highest levels since October 1987 crash*	1988-12	Economic
Dow industrials hit record on upbeat economic news (1993, October 29), The Wall Street Journal	*Increase in third-quarter GDP exceeded analysts' expectations*	1993-10	Economic
Dow index climbs 35 to yet another record; The Standard & Poor's 500 index also hits a new high (1993, December 28), *San Francisco Chronicle*; Industrials reach record again as recovery signs ignite cyclicals (1993, December 14), The Wall Street Journal		1993-12	Economic
Binge of stock buybacks makes 1994 a record year (1994, December 19), The Wall Street Journal	*Buybacks are important signal to investors*	1994-12	Economic
Dow industrials close above 5000 mark (1995, November 22), The Wall Street Journal	*Just nine months after crossing the 4000 barrier*	1995-11	Economic
The S.&P. 500 breaks through the 1,000 mark (1998, February 3), The New York Times; Dow industrials jump 115.09, back to a record (1998, February 11), The Wall Street Journal		1998-02	Economic
What correction? With dazzling speed, market roars back to another new high — surge puts the Dow at 9374 in a lightning reversal of autumn's doldrums — 'Nothing to get in its way' (1998, November 24), The Wall Street Journal	*Widespread euphoria*	1998-11	Economic

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Table 8: Certainty shocks (cont'd)

Headline (date), *newspaper*	*Additional keywords*	Date of shock	Type
Stocks rally, though tentative, offers hope for economy (2003, April 24), Los Angeles Times	*War in Iraq has wound down*	2003-04	Economic
Treasuries stage sharp rally on consumer price data (2004, June 16), The New York Times	*Stocks also rose; the market was priced for accelerating inflation*	2004-06	Economic
As Bush goes, so goes market — major indexes are up in month that historically is the weakest, echoing bets on a re-election (2004, September 20), The Wall Street Journal		2004-09	Political
Stocks advanced last week, ... (2005, February 28), The Washington Post	*"...powered by an upbeat report on economic growth and some consumer inflation data that didn't rattle investors."*	2005-02	Economic
Shares rally after crude oil prices decline $2 a barrel (2005, June 29), The New York Times	*Consumer confidence jumped to a three-year high*	2005-06	Oil
Volatility expectations tumble (2005, December 15), The Wall Street Journal	*S&P 500-index reached new highs for the year*	2005-12	Economic
S.&P. passes 1,400 and oil continues to slide (2006, November 19), The New York Times	*First close above 1,400 in six years*	2006-11	Economic
Shares rise, erasing Dow's loss for '09 (2009, June 13), The New York Times; Economic data push the Dow 2.6% higher (2009, June 2), The New York Times	*S.&.P 500-index best level in five months; signs of economic growth in China; stability in Europe; sings of improvement in construction and manufacturing in the United States*	2009-06	Economic
Stocks soar, but many ask why (2010, March 29), The New York Times		2010-03	Economic
S.&P. 500 reaches 2-year high as shares post modest gains (2010, December 21), The New York Times		2010-12	Economic
Stocks hit 5-month high in year-end rebound (2011, December 24), The Wall Street Journal	*Accelerating recovery; break in the latest congressional deadlock*	2011-12	Economic
Markets rally as volatility hits five-year low (2012, August 18), The Washington Post	*S&P 500 index near four-year high*	2012-08	Economic
Stocks near record highs; The S&P 500 rises for the eigth day, closing above 1,500 for the first time since 2007 (2013, January 26), Los Angeles Times		2013-01	Economic
S.&P. index surpasses high point of 2007 (2013, March 29), The New York Times		2013-03	Economic
S&P rises into the spotlight — broad index tops 1800 for the first time a day after DJIA climbs past 16000 (2013, November 23), The Wall Street Journal		2013-11	Economic
S.&P. 500-stock index closes at new high (2014, May 24), The New York Times	*Better-than-expected home sales*	2014-05	Economic

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D OHTER PROXIES OF UNCERTAINTY

In this section, I analyse the impact of shocks identified on the basis of other proxies of uncertainty. First, I use Bachmann et al.'s (2013) proxy of uncertainty and second, I benchmark my main findings to the impact of the available indices of fundamental uncertainty u$_f$ and u$_m$.

D.1 Forecast dispersion

I analyze the impact of uncertainty shocks identified using the forecast dispersion measure proposed by Bachmann et al. (2013), purely reflecting uncertainty about future business activity.[20]

Similar to the main analysis, I identify an exogenous uncertainty/certainty shock to the 0.9 and 0.1 quantiles of the proxy of uncertainty conditional on the dynamics of stock returns at their 0.1 and 0.9 quantile, respectively. I consider the impact on the median of the macroeconomic variables. The impulse response analysis is depicted in Figure 9 and the forecast error variance decomposition is summarized in Table 9.

Overall, the findings suggest that the asymmetric findings of the main analysis cannot be recovered using all proxies of uncertainty. The strong effects of uncertainty shocks do not seem to relate to elevated periods of uncertainty about future business activity captured by this proxy.

Specifically, the impulse response analysis suggests that shocks to the 0.9 and 0.1 quantiles of the proxy of uncertainty do not lead to differing effects on the macroeconomy. That is, the impulse responses have similar dynamics. An exogenous rise of one standard deviation in u$_{fd}$ – in the case of a shock to the upper quantile – leads to an initial response of economic activity growth, i.e. one month after the shock, of -0.2 p.p. Similarly, a fall of one standard deviation in u$_{fd}$ – in the case of a shock to the lower quantile – leads to an initial rise in economic activity growth by 0.2 p.p. The forecast error variance decomposition indicates that shocks to the 0.9 and 0.1 quantiles of u_{fd} explain about a similar percentage of fluctuations in the macroeconomic variables, whereas shocks to the lower quantile seem to be marginally more important. For instance, a shock to the 0.9 quantile accounts for 2.9% of fluctuations in the first half of the first year after the shock and a shock to the 0.1 quantile of 3.2%.

D.2 Fundamental macroeconomic and financial uncertainty

The impulse responses of the macroeconomic and financial uncertainty indices within a regular VAR are depicted in Figure 10 and the forecast error variance decompositions are captured in Table 9.

Overall, the impact of uncertainty shocks differs from the impact of uncertainty shocks based the LMN and JLN measures of uncertainty. Most markedly, uncertainty shocks based on the latter

[20] For more details about this measure, please see Section 3.1.

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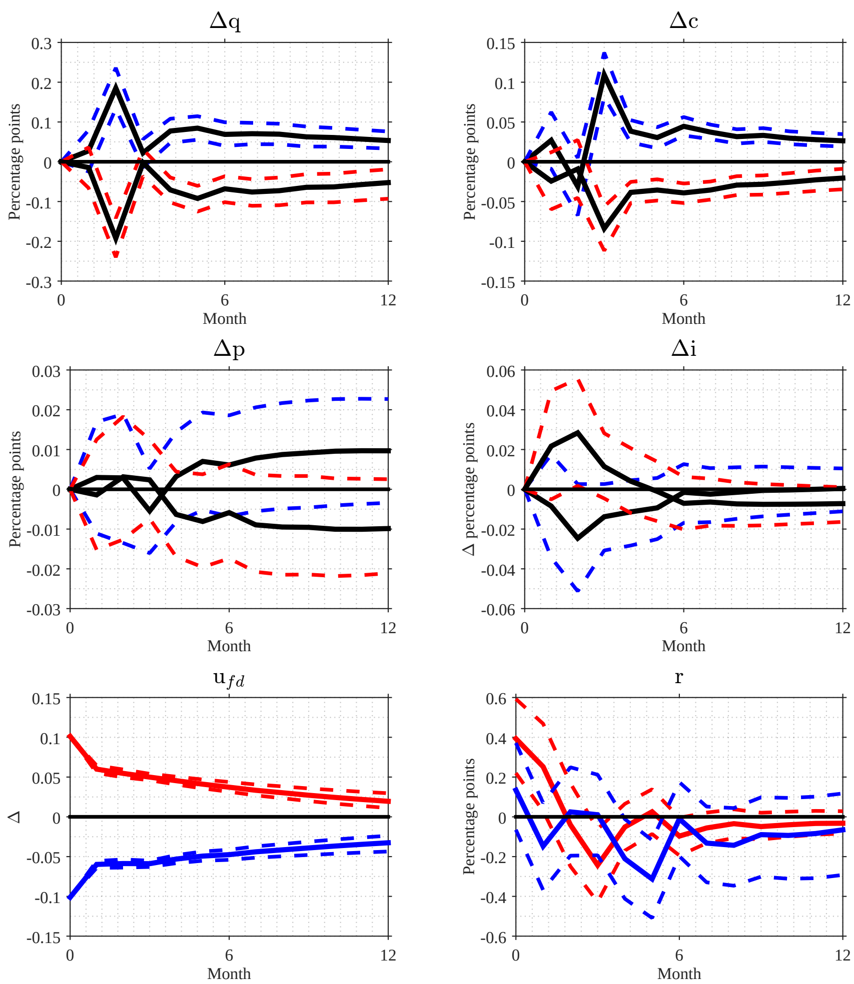

Figure 9: The impact of an uncertainty shock to the 0.1 and 0.9 quantiles of u_{fd}

Notes: The panels depict the pseudo impulse responses of a shock to the 0.1 and 0.9 quantiles of u_{fd} assuming that r is at its 0.9 and 0.1 quantile respectively. The responses of the macroeconomic variables (Δq, Δc, Δp, Δi) reflect their median responses. Solid lines refer to the median impulse responses and the dashed lines correspond to posterior 68% probability bands. Blue dashed lines mark responses to a shock to the 0.9 quantile of u_{fd} and red dashed lines to the 0.1 quantile of u_{fd}. Δq denotes growth in industrial production; Δc is growth in consumption, Δp inflation, Δi changes in the interest rate, u_{fd} the proxy of uncertainty, and r stock returns.

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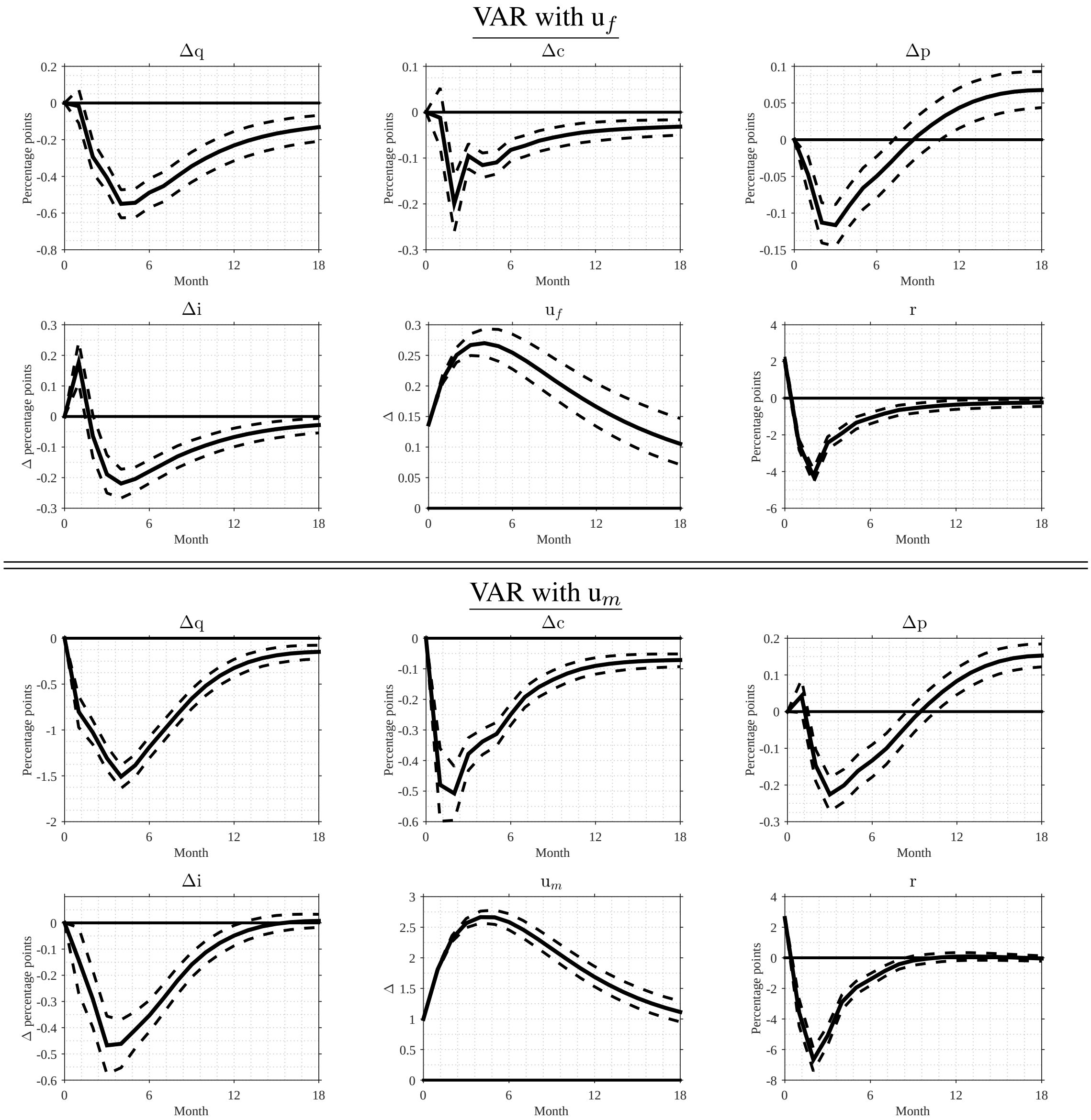

Figure 10: The impact of uncertainty shocks identified by different proxies of fundamental uncertainty (u_m and u_f) in a regular VAR

Notes: Solid lines refer to the median impulse response and the dashed lines correspond to posterior 68% probability bands. Δq denotes growth in industrial production; Δc is growth in consumption, Δp inflation, Δi changes in the interest rate, $u_{\cdot}$ the proxy of uncertainty, and r stock returns.

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Table 9: Variance explained by various uncertainty shocks

Proxy	Model	Months	Δq	Δc	Δp	Δi	$u_{.}$	r
Shock to 0.9 quantile								
u_{fd}	QVAR	0–6	2.9	1.3	0.1	1.0	88.8	0.1
u_{fd}	QVAR	7–12	5.4	3.2	0.5	1.5	64.7	0.3
Shock to 0.1 quantile								
u_{fd}	QVAR	0–6	3.2	2.2	0.1	0.8	92.0	0.3
u_{fd}	QVAR	7–12	6.9	5.0	0.4	1.2	73.2	0.4
u_f	VAR	0–6	0.8	0.2	0.5	0.9	95.8	1.3
u_f	VAR	7–12	2.8	0.4	0.8	2.6	92.1	2.1
u_m	VAR	0–6	3.2	0.9	0.7	1.3	94.0	0.2
u_m	VAR	7–12	8.1	1.7	1.4	3.7	84.6	0.4

Notes: Table shows the psuedo forecast error variance of a shock identified to the 0.1 and 0.9 quantiles of u_{fd} assuming that r is at its 0.9 and 0.1 quantile respectively. Explained variance is for fluctuations in the conditional median of the macroeconomic variables (Δq, Δc, Δp, Δi) and for the conditional quantiles of the financial variables ($u_{.}$, r). Furthermore, the table depicts the forecast error variance of shocks identified on the basis of the fundamental uncertainty indices, u_f and u_m. Δq denotes growth in industrial production; Δc is growth in consumption, Δp inflation, Δi changes in the interest rate, $u_{.}$ the proxy of uncertainty, and r stock returns.

proxies explain only a small percentage of variation in the macroeconomic time series; less than half the share of uncertainty shocks.

Specifically, in contrast to the uncertainty shocks of the main analysis, both uncertainty indices first rise in response to a shock, and only decline thereafter. In line with the uncertainty shock of the main analysis, their effect is highly persistent. Stock markets react counter intuitively by rising instantaneously, whereas the stock market instantaneously falls in response to an uncertainty shock. In this exercise, only after one month do stock markets fall. In general, the forecast error variance decomposition indicates that the two identified shocks are relatively unimportant for stock markets (0.2% for u_m and 1.3% for u_f during the first half-year after the shock).

The responses of the macroeconomic series differ from the analysis of the uncertainty shock. Differences are most pronounced using the u_f proxy, where economic activity and consumption do not react instantaneously. Furthermore, interest rates rise after one month following the shock, which is in stark contrast to the impact of uncertainty shocks. Moreover, in both scenarios inflation rises around one year after the shock, whereas the response is insignificant for uncertainty shocks around this period. Also, both shocks are, by far, less important than an uncertainty shock. For instance, the maximum variance explained pertains to the macroeconomic uncertainty shocks on output. This figure is 3.2% for the first half-year after the shock. For comparison, the uncertainty shock explains 8.6% of the variance in output over the same horizon. The result is rather similar to the importance of a mean uncertainty shock that amounts to 3.8% over that horizon.

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