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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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Abstract

In pursuing its mandate, a central bank assumes financial risks through its monetary policy operations. Central bank capital is a critical tool in mitigating these risks. We investigate the concept of central bank capital as a mechanism for risk-sharing with its shareholder. Adopting an option pricing framework, we explore the setting where the central bank commits to distributing dividends when its capital is robust, while the shareholder may be called upon to recapitalize the bank during adverse economic conditions, with negative capital. Our analysis dissects the trade-offs inherent in these options, seeking a mutually beneficial agreement that disincentivizes deviation for either party. This equilibrium is essential for safeguarding the independence and credibility of the central bank in executing monetary policy effectively.

JEL classification: G13; G32; E58
Keywords: Capital; Central Bank; Contingent Claim Analysis, Risk Management; Shareholder; Stackelberg Games

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1 Introduction

Owned typically by the government, central banks operate within a framework that involves accepting financial risks to implement monetary policy. Central bank capital can play a key role in absorbing these risks. A key question then arises: how much capital should a central bank have relative to the risks it bears in monetary policy operations and other activities such as reserve management and lending of last resort? One perspective suggests that maintaining capital is irrelevant for a central bank, as long as it can continue to issue legal tender and receive unconditional fiscal support from the government (Reis, 2015a; Hall and Reis, 2015). However, from a different perspective, adequate capital levels are essential for a central bank to establish independence and to maintain policy credibility. From this standpoint, capital serves as an instrument ensuring effective policy implementation (Stella, 2005; Klüh and Stella, 2008; Benigno, 2020; Wessels and Broeders, 2023).

The question of what the adequate levels of central bank capital are is becoming increasingly pertinent as central banks worldwide report losses due to rapid changes in monetary policy stance (Bell et al., 2023; Belhocine et al., 2023). Furthermore, quantitative easing is evolving into a standard tool of monetary policy, consequently heightening financial risks on central banks' balance sheets. Central banks are therefore compelled to meticulously evaluate capital adequacy, given that these novel monetary policy instruments are riskier and may yield lower profitability in contrast to conventional tools. A prolonged period of losses can impair their capital positions and may ultimately undermine their ability to fulfill their mandate. In this context, the literature on central banking underscores the importance of fiscal support from the government in the form of recapitalization (Del Negro and Sims, 2015; Benigno and Nisticò, 2020). However, capital policy as a contract between the central bank and its shareholder is still widely unexplored.

In this study, we examine this contract as a financial risk-sharing agreement between the central bank and its shareholder.[1] This arrangement entails mutual acceptance of obligations. First, the central bank distributes dividends to the shareholder during periods of robust capitalization. Second, the shareholder, often implicitly, pledges fiscal support by committing to recapitalize the bank, should asset values drop significantly relative to liability values, jeopardizing the credibility of its monetary policy framework.

To analyze the risk-sharing arrangement we employ models that are typically used for credit risk and equity valuation (Merton, 1974; Leland, 1994; Black and Cox, 1976). Through this approach, we delve into the dynamics between the central bank and its shareholder, enabling a comprehensive evaluation of the risks assumed by each party. From the central bank's perspective, the obligation to pay dividends in the future, conditional on having sufficient buffers, is a short call option on the central bank's assets. The recapitalization guarantee, again from the central bank's perspective, is a long put option on its assets. The value of equity is the result of a bargaining problem for the central bank and the shareholder on the key features of these two options. In this fashion, we translate Merton's model, commonly used to evaluate default risk, into a context where insolvency is defined not in terms of the ability to pay liabilities, but in terms of the ability to keep up with policy objectives. We take a three-step approach in our analysis.

First, we introduce an economic definition of central bank equity. In contrast to capital defined in accounting terms as the value of total assets minus liabilities, we define equity as accounting capital minus the value of the dividend-payment option plus the value of the recapitalization option. These options are implicitly defined, as they do not appear in the central bank's reported balance sheet, and they are not traded on any market. As one would expect, then, holding everything else fixed, a higher value of central bank assets

[1]Without loss of generality, we consider a scenario where the sole shareholder is the government, a common assumption reflecting the prevailing reality in many cases, as the state often holds exclusive ownership. It is noteworthy, however, that in practical terms, the ownership structure of central banks varies. While few central banks are privately owned, there are exceptions to the prevailing norm. Stocks of central banks in Belgium, Greece, Japan, South Africa, Switzerland and Turkey are traded on a stock exchange. Moreover, in certain instances such as in the US, the central bank is partially owned by commercial banks, adding an additional layer of complexity to their ownership dynamics, see, e.g., Archer and Moser-Boehm (2013). The European Central Bank is fully owned by the national central banks in the euro area, with no governments directly backing it.

means a higher value of the dividend option and a lower value of the recapitalization option. Conversely, lower asset values lower the value of the dividend option and increase the value of the recapitalization option. Second, we explore the risk-sharing implications of the capital policy, where the central bank and its shareholder establish the terms for dividend distribution and recapitalization. In our setting, these terms are defined by the strike prices (or 'strikes') of the implicit options, which dictate when the options are exercised. Third, we formalize the negotiation process wherein the central bank and its shareholder establish terms for dividend distribution and recapitalization via a Stackelberg game. The goal is to derive a contract from which neither party would have incentives to deviate. Such an agreement ensures the central bank's independence while simultaneously securing a commitment from the shareholder to provide fiscal support when necessary.

Following this three-step approach, we first find that the dynamics of a central bank's accounting capital and the economic value of its equity are remarkably different. The dividend and recapitalization options smooth the impact of changes in asset value on equity. Second, we demonstrate that capital policy can have implications about the risk-appetite of central banks, depending on the institutional setting. For example, an undercapitalized central bank with fiscal backing may be incentivized to enhance the value of its equity by taking on more risk. Conversely, a well-capitalized central bank without strong fiscal backing may exhibit hesitancy in engaging in unconventional monetary policy, as it fears that doing so could deplete its capital position, while any surpluses would flow to the shareholder in the form of dividends. Third, using our framework, we demonstrate that the capital policy in equilibrium depends on how decision rights are allocated between the shareholder and the central bank. A negotiation, for instance, in which the central bank sets the recapitalization conditions, and the shareholder sets the dividend conditions, will result in a relatively high recapitalization threshold and a low dividend payment threshold. This means high protection against capital shortfalls, as recapitalization is more likely to be triggered. However, it also implies less ability to rebuild capital by retaining profits, as, in return, the shareholder would require a lower dividend payment threshold to agree on the arrangement. These findings underscore the necessity of well-designed institutional frameworks to align the risk incentives of the central bank and its shareholders, ensuring stability and resilience in the financial system.

The central bank's capital structure problem is distinct in several notable ways that are worth discussing before setting up our model. First, the unique ability of a central bank to print money and create reserves ensures that it will never run out of liquidity in its own currency. As a result, there are no bankruptcy proceedings in the conventional sense for central banks and having negative capital is completely feasible. However, in that case, a central bank that experiences a large drain on its capital, may need to resort to issuing reserves to service the costs on its ongoing liabilities. This inevitably would affect its ability to curb inflation and as a result would jeopardize its credibility. That is why fiscal support in the form of recapitalization is needed to safeguard its credibility.[2]

At the same time, the asset-liability structure of a central bank is very specific, and has evolved in recent years. While liabilities are typically short-term, assets are either short-term, when created through lending operations or through the central bank's standing facilities, or long-term and subject to repricing risk, when created through quantitative easing or other asset purchase programs. This exposes the central bank to specific asset-liability risk mismatches. Unlike commercial banks, however, central banks do not hedge these risks, as that would go against the monetary stance which requires purchasing these assets in the first place. In this paper we abstract from the maturity dimension of interest rate risk, and model it through a generic risky asset representing purchase programs. This allows us to keep the model abstract and tractable, while still retaining the key features relevant for valuing the options implicit in the balance sheet.

[2]There is some heterogeneity in the academic literature on what central bank insolvency entails precisely. Reis (2015b) covers a number of definitions from the point of view of valuing the central bank's promise to generate seigniorage while keeping in line with its resource constraint. Similarly, there are different ways to approach central bank credibility and independence. Blinder (2000) provides an overview and relates credibility to inflation bias - the tendency of central banks to lose policy credibility and generate excess inflation when they lack independence from the fiscal authority. Here, without engaging in these discussions in detail, we use the terms insolvency, credibility, and independence in the context of the balance sheet of the central bank as a tool in effectively support its inflation objectives.

In terms of the related literature, our paper contributes to the debate on central bank capital and fiscal support (Reis, 2015a; Del Negro and Sims, 2015; Hall and Reis, 2015; Benigno, 2020) by showing how dividend and recapitalization arrangements between the central bank and government are made. We also relate to the growing literature on the effect of non-conventional monetary policy and central bank losses (Chiacchio et al. (2018); Benigno and Nisticò (2020); Goncharov et al. (2023)), and to the literature on central bank independence (Barro and Gordon (1983); Eijffinger et al. (1996); Binder (2021)). Further, the model presented in this paper relates to the literature on commercial bank recapitalization (DeMarzo and Sannikov, 2006; Décamps et al., 2011; Peura and Keppo, 2006; Moreno-Bromberg and Rochet, 2018), albeit with substantial differences given the specificity of the central bank operations and objectives discussed earlier.

The structure of this chapter is as follows. In Section 2, we introduce the assumptions that underpin our analysis of the risk-sharing arrangement between the central bank and the shareholder. Section 3 outlines the model used to determine the equilibrium capital policy in potential negotiations between the central bank and the shareholder. Next, in Section 4, we present numerical results derived from the optimal policy. In Section 5, we explore several real-world cases of capital policy implementation. Finally, Section 6 provides concluding remarks.

2 Model

We develop a static, one-period model in the spirit of Merton (1974). We assume that during the period under examination the central bank will not implement new monetary policy measures and the amount of banknotes in the economy is fixed. Our approach is further based on a number of simplifying assumptions which keep the focus of the analysis on risk-sharing arrangement between the central bank and the shareholder:

- All information is public and becomes available according to a standard filtration $\mathcal{F}_t$ generated by a Brownian motion W_t.

- Markets are complete and frictionless. The central bank's assets and liabilities are readily tradeable and liquid.

- We assume partial equilibrium dynamics. This means that the development of asset prices and the risk-free rate are purely exogenous and any shocks introduced by the central bank's policies before the start of the period have already been priced in by markets.

- At time t the central bank and the shareholder agree on the capital policy. Similarly to the standard Merton model, any uncertainty resolves at time T and the European-type options embedded in the central bank's capital structure mature.

- There is no discretionary expansion or contraction of the central bank's balance sheet during the analysis period. Any central bank intervention has happened beforehand and is already priced into the interest rates. The money supply in the form of banknotes is fixed.

- The shareholder keeps its commitment. Thus, we abstract from the risk of political interference once the agreement is agreed upon, and assume that the state has deep pockets and is able and willing to fulfill the recapitalization commitment in all states of the world.

2.1 The Central Bank Balance Sheet

Based on these assumptions, and under the additional presumption that assets and liabilities on the balance sheet are valued on a marked-to-market (MTM) basis, we can now describe the stylized model of the central bank.

Assets (A_t)		Liabilities (L_t)	
Lending operations	M_t	Banknotes	N
Asset purchase program	P_t	Reserves	R_t
		Capital	B_t

Table 1: Central bank's accounting (MtM) balance sheet

The central bank utilizes open market operations and asset purchase programs to implement its monetary policy. Total assets can be categorized into two main components: lending operations (M_t), and asset purchase programs (P_t).[3] Total assets are then defined as

$$A_t = M_t + P_t \tag{1}$$

The evolution of lending operations M is given by

$$dM_t/M_t = rdt \tag{2}$$

where r is the risk-free rate. The risk-free nature of r is derived from the fact that loans to commercial banks are well collateralized through high-quality assets or through haircuts which are applied whenever collateral risk needs to be managed.

The value of the purchase programs evolves with the price changes of the risky assets under the physical risk measure as

$$dP_t/P_t = (r + \lambda\sigma)dt + \sigma dW_t \tag{3}$$

where λ is the price of risk, or risk premium, per unit of asset volatility σ, and dW_t is a Brownian increment.

We define the share of risky assets in the balance sheet as $\omega = P_t/A_t$. The evolution of total assets can then be written as

$$\begin{aligned} dA_t/A_t &= dM_t/A_t + dP_t/A_t \\ &= (1-\omega)dM_t/M_t + \omega dP_t/P_t \\ &= (r + \omega\lambda\sigma)\, dt + \omega\sigma dW_t. \end{aligned}$$

Liabilities consist of a fixed amount of banknotes N and reserves R_t. Reserves are deposits held by commercial banks at the central bank. Total liabilities are then given by

$$L_t = N + R_t. \tag{4}$$

[3] We ignore for the purposes of this study other central bank activities that may require capital such as reserve management and acting as the lender of last resort.

The central bank forgoes interest costs through its privilege of issuing banknotes, while on the reserves it pays the risk-free rate. This implies that total liabilities grow only through the interest rate that has to be paid on reserves

$$dL_t = rR_t dt. \tag{5}$$

We define the level of the central bank's capital in accounting terms as the net asset position:

$$B_t = A_t - L_t. \tag{6}$$

As a result, changes in capital can be written as:

$$\begin{aligned} dB_t &= dA_t - dL_t \\ &= dM_t + dP_t - dR_t \\ &= \underbrace{(M_t + P_t - R_t) r dt}_{\text{Seigniorage}} + \underbrace{P_t \lambda \sigma dt}_{\text{Risk Premium}} + \underbrace{P_t \sigma dW_t}_{\text{Financial Risk}} . \end{aligned} \tag{7}$$

From Equation (7), it follows that the change in capital comes from the central bank earning risk-free seigniorage profits by issuing banknotes, the risk premium obtained by investing in the risky asset, and the asset risk that is injected into its balance sheet as a result of holding the purchase program assets.

It is worthwhile discussing some caveats of our model before we proceed further. First, we simplify reality in a number of ways to enhance the tractability of the model. We treat the asset purchase programs P_t as a homogeneous asset class with a single risk driver. In reality, these programs can consist of long-term government bonds, commercial bonds, asset-backed securities, covered bonds, or even equity, as is the case with the Bank of Japan.[4] Second, we do not discuss the motivations behind the asset purchases, which in practice may come as a response to an extreme market shock.[5] From that point of

[4]Charoenwong et al. (2021) discuss the Bank of Japan's asset purchase programs. For implementation in the U.S., see Di Maggio et al. (2020). Benigno et al. (2023), among others, discuss the purchase programs initiated by the ECB in response to the Covid-19 pandemic shock.

[5]Central banks have utilized large-scale asset purchase programs, commonly known as Quantitative Easing (QE), since the Global Financial Crisis. These programs, such as the ECB's Public Sector Purchase Programme (PSPP), are designed to ease the monetary stance by reducing interest rates. However, central banks may also initiate asset purchase initiatives in response to various triggers. For instance, programs like the ECB's Securities Markets Programme (SMP) were there to ensure sufficient depth and liquidity in dysfunctioning segments of the sovereign bond markets and the Pandemic Emergency Purchase Programme (PEPP) was introduced in response to the Covid-19 pandemic to support the monetary transmission process by alleviating extreme financial market distress.

view, the asset price log-normality assumption embedded in the Merton model may seem troublesome. It is, in theory, possible to extend the risk dynamics of our model, for instance, by incorporating stochastic variance to address market turbulence or stochastic interest rates to reflect term-structure risk. However, we refrain from doing so here, as we are not concerned with the triggers of asset purchases or their interaction with asset dynamics, a research question posed, for instance, in Broeders et al. (2023). In our model, the size of asset purchases is exogenous and appears in the central bank's balance sheet as a legacy from previously fulfilled policy. Here, we aim to emphasize the option-type structure of central bank capital, which remains conceptually consistent regardless of the underlying risk factor dynamics.

Furthermore, we assume that the central bank's balance sheet is continuously marked to market, while in reality assets may also be accounted for as held to maturity (HTM). This is the case, for instance, of bonds held under the asset purchase programmes of the Eurosystem. Conceptually, however, the MTM and the HTM accounting differ only with respect to the timing of the recognition of profits and losses. To provide an example, an upward shift in the yield curve will immediately be recognized as a loss under MTM accounting by revaluing all bonds held on the asset side of the balance sheet; at the same time, losses will be recognized only gradually under HTM accounting when cash outflows associated with funding costs (interest rates on reserves) increase potentially above the cash inflows from the HTM assets (coupons received on the bonds held to maturity).

2.2 Capital Policy

Capital acts as a buffer to absorb the risks taken by the central bank to implement monetary policy. We examine the concept of central bank capital as the result of a risk-sharing arrangement with the shareholder. In our paper, the central bank and its shareholder agree on a capital policy for the duration of one period. This policy has two features that reflect the risk sharing arrangement. First, the central bank and the shareholder agree on the parameters of the dividend policy. If the bank is well capitalized it will pay a dividend. Concretely, any assets above a multiple κ_C of liabilities are paid in the form of a dividend to the shareholder. Second, the central bank and the shareholder agree on the parameters of a recapitalization. Following the premises of structural finance, we assume that at time T the central bank needs to be recapitalized by the shareholder if its assets fall below a multiple of κ_Π of its liabilities, i.e. if $A_T \leq \kappa_\Pi L_T$. Below this threshold, the central bank is in jeopardy of not being able to fulfill its monetary policy objectives and therefore it needs to be recapitalized by the shareholder.

We take the perspective of the central bank. From this perspective the future dividend payment is represented by a short call option on the central bank's assets. The pay-off at maturity T of the call option is given by

$$C_T = \max\{0, A_T - \kappa_C L_T\} \tag{8}$$

The option to recapitalize is a long put option on the central bank's assets. We explore the case in which the central bank is recapitalized so that all liabilities L_T are covered with sufficient assets A_T. This entails that the shareholder injects $L_T - A_T$ to recapitalize the bank.

The pay-off at maturity T of the put option is given by:

$$\Pi_T = \max\{0, L_T - A_T\} \tag{9}$$

Note that the put option is triggered, whenever $A_T \leq \kappa_\Pi L_T$. This setting cannot be analyzed using regular options but rather "gap options" (Hull, 2012). Hence, the strike that triggers the option is not the strike that determines the pay-off.

It is important to note that the implementation of recapitalization can vary in practice. The most straightforward approach involves a bond transfer directly from the shareholder to the central bank's balance sheet in return for an equivalent capital stake in the bank. However, recapitalization can also manifest itself in subtler forms. For instance, the central bank might record losses as deferred assets, essentially creating a claim on the shareholder without an immediate cash transfer, as exemplified by the approach taken by the US Federal Reserve (Carpenter et al., 2018). Gradually, these deferred assets are mitigated by retaining future central bank earnings, essentially meaning that the shareholder recapitalizes the central bank by forfeiting future dividends. In our context, the specific assets utilized for recapitalizing the central bank have no relevance, as we do not model the post-maturity developments.

So far we have used capital to denote the central bank's net assets in accounting terms, without the two options on its implicit balance sheet. We now introduce the term "equity" to refer to the central bank's net assets including the two options. Factoring in these options, we can write the value of equity at time t as the sum of capital and the recapitalization option minus the dividend option

$$E_T = B_T + \Pi_T - C_T \tag{10}$$

The options are implicit as they do not appear on the central bank's MtM accounting balance sheet. Nevertheless, we can now present the central bank's economic balance sheet including the implicit options as in Table 2.

Table 2: Central bank's balance sheet including the implicit options

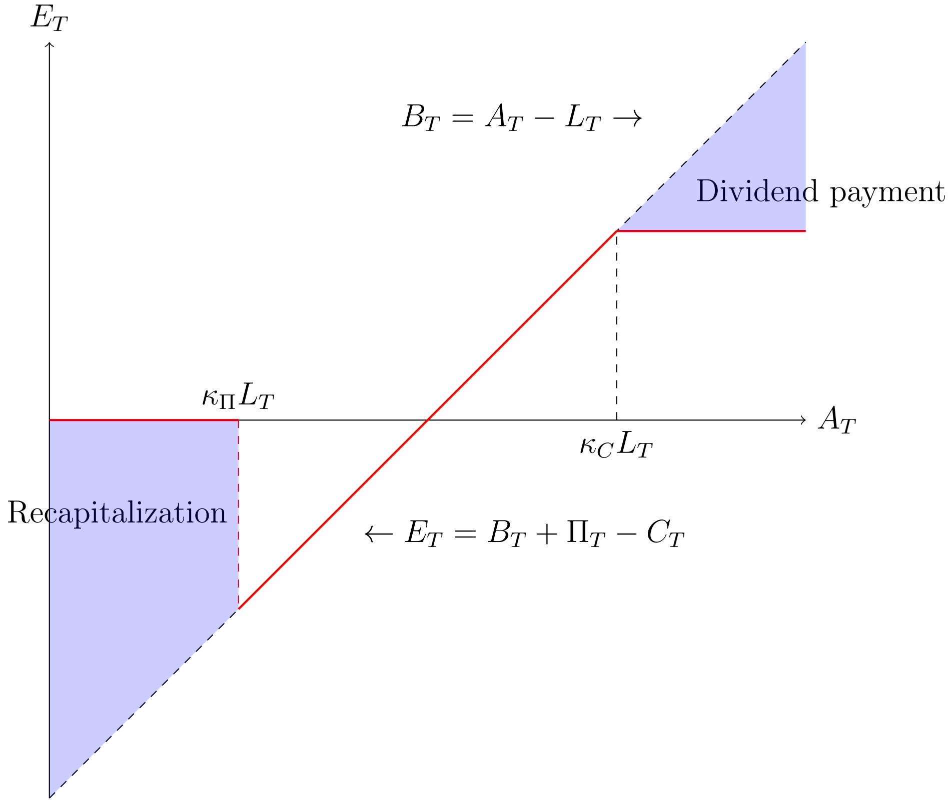
Assets		Liabilities	
Lending operations	M_t	Banknotes	N
Asset purchase program	P_t	Reserves	R_t
Recapitalization option	Π_t	Dividend option	C_t
		Equity	E_t

Based on this, we can determine the value of equity at the end of the period. The thick red line in Figure 1 shows the equity value E_T as a function of the value of the central bank's assets A_T at time T. Note that the red line is discontinuous at the strike of the put option ($\kappa_\Pi L_T$) due to the fact that the option's exercise and the payoff are disconnected. The black dashed line shows capital B_T as a function of assets at maturity. Equity and the capital overlap in between the option multiples.

2.3 Valuation of Central Bank Equity

At time t, we define the value of central bank equity as the current value of net assets plus the market value of the embedded recapitalization option, net of the market value

Figure 1: Central bank equity payoff at maturity



of the contingent obligation to pay dividends:[6]

$$E_t = A_t - L_t + \Pi_t - C_t. \tag{11}$$

Conceptually, the idea of valuing equity through the options embedded on a firm's balance sheet goes back to Merton (1974), who relates the value of a firm's equity to the value of the implicit call option that the shareholders have on its assets. It is important to note that the capital structure of a central bank diverges from that of a typical firm for two key reasons. First, whereas shareholders of a firm benefit from limited liability and can choose to let the firm default if recapitalization is not projected to yield profits, such recourse is unavailable for central banks. The shareholder of a central bank is more inclined to recapitalize it, given the role of the central bank in the economy, without the option of default as an exit strategy. In fact, the central bank's equity resembles an

[6]Formally, the value of equity can be seen as the discounted expected value of the central bank's payoff at period T, defined as $E_t = e^{-r_t(T-t)}\mathbb{E}_t^{\mathbb{Q}}\left(A_T - L_T + \Pi_T - C_T\right)$ where $\mathbb{E}_t^{\mathbb{Q}}(.)$ denotes an expectation under the risk-neutral distribution of asset returns. See Shreve (2000) for details on risk-neutral asset pricing.

option strategy referred to by traders as a "collar", in which the investor holds a long out-of-the money call and a short out-of-the money put on the same underlying asset.

Second, unlike a commercial enterprise, the central bank can display negative equity on its balance sheet without creditors demanding back their claims. This resilience is sustained by the trust held by counter-parties, who recognize that the central bank's liabilities are ultimately backed by its shareholder. Furthermore, the shareholder has no incentive to disengage, even in situations where the central bank's assets fall below its liabilities. This commitment stems from the central bank's pivotal institutional role in society to offer price or exchange rate stability.

The value of the embedded call and put payoffs can then be determined through the well-known Black and Scholes relations as follows:

$$C_t = N(d_{1,C})A_t - N(d_{2,C})\kappa_C L_t^* \tag{12}$$

$$\Pi_t = N(-d_{2,\Pi})L_t^* - N(-d_{1,\Pi})A_t \tag{13}$$

where

$$d_{1,i} = \frac{1}{\omega\sigma\sqrt{T-t}}\left[\ln\left(\frac{A_t}{\kappa_i L_T}\right) + \left(r + \frac{1}{2}(\omega\sigma)^2\right)(T-t)\right] \tag{14}$$

$$d_{2,i} = d_1(\kappa_i) - \omega\sigma\sqrt{T-t}. \tag{15}$$

The indicator $i = \{C, \Pi\}$ refers to the call and the put option, respectively. Furthermore, L_t^* is the terminal value of liabilities, discounted at the risk-free rate or $L_t^* = (R_T + N)e^{-r(T-t)} = Ne^{-r(T-t)} + R_t$. Note that the risk on the asset side of the balance sheet is driven by the fraction ω that the central bank invests in the risky assets. Therefore we scale the volatility factor in the Black and Scholes pricing formula accordingly $(\omega\sigma)$.[7]

It is important to note that the valuation of the put option follows a modification of the standard Black and Scholes formula. When the option is triggered, the shareholder

[7] To verify our formulas, recall the well-known Black and Scholes relation. For example, the price of a call option with strike K and underlying value S_t can be written as $C_t = N(d_1)S_t - N(d_2)Ke^{-rt}$ In our case, specifically, the strike becomes $K = \kappa_C L_T = \kappa_C (N + R_t)$, where $R_t = R_t e^{r(T-t)}$. Then, we can absorb the discount factor into L_t^*, such that $Ke^{r(T-t)} = \kappa_C \left(Ne^{-r(T-t)} + R_t\right) = \kappa_C L_t^*$. The discount factor is canceled out from the reserves portfolio R_t, but is maintained on the banknote liabilities. This also makes intuitive sense, as the issuance of banknotes allows a central bank to finance its assets thorough a cost-free liability.

does not pay the difference between the asset value and the strike price, but rather pays a higher amount such that capital is replenished back to zero.Equation (13) reflects this feature through the fact that the strike term in front of $N(-d_{2,\Pi})$ is not equal to $\kappa_\Pi L_t^*$ (as it would have been in case of a plain-vanilla option) but equal to L_t^*. This derivative can be priced as a gap-type option, which disconnects the option trigger κ_Π from the pay-out level. We assume that the central bank is recapitalized up to 100% of its liabilities when assets drop below the trigger level $\kappa_\Pi L_T$.[8]

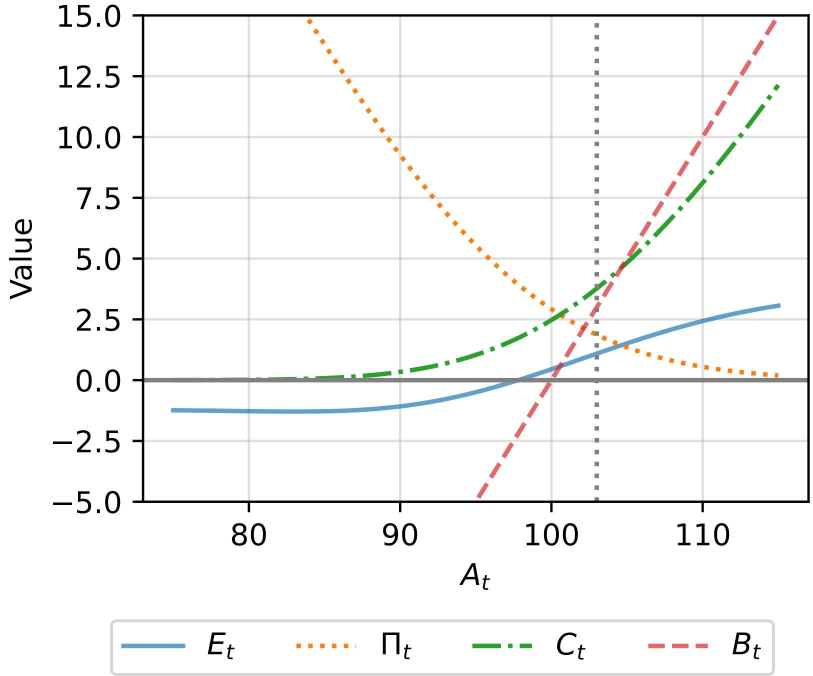
We are now able to determine the value of the policy options, which consequently influences the valuation of the central bank's equity. To initiate the calibration process of the model, we set the total liabilities at a standardized level of $L_t = 100$, while total assets will vary. We set the dividend and recapitalization policy parameters ('strike multiples') for call and put options at $\kappa_C = 1.05$ and $\kappa_\Pi = 0.90$, respectively. The remaining numerical values are presented in Annex A.1. Figure 2 shows how the value of equity E_t varies for different asset levels A_t. The figure also shows capital B_t and the value of the dividend option C_t and recapitalization option Π_t. Since any losses beyond the $\kappa_\Pi L_T$ are (more than fully) covered by the shareholder, the equity value is higher than capital for low asset values. Conversely, for high asset values the equity is below capital, as the former takes into account future dividend payments. Therefore the dynamics of a central bank's accounting capital and the economic value of its equity are remarkably different. The dividend and recapitalization options smooth the impact of changes in asset value on equity.

2.4 Policy Parameters

We now turn to the problem at hand and assume that the central bank and the shareholder negotiate the capital policy by setting the key parameters of the dividend option and the recapitalization option. Negotiations can help align the interests of the central bank with

[8]See for example Hull (2012) for a formal derivation of the Black and Scholes pricing function of gap options.

Figure 2: Equity value as a function of asset value



Note. This figure shows the value of equity (E_t), the dividend option (C_t), the recapitalisation option (Π_t) and accounting capital (B_t), as a function of the central bank's asset value. The dotted vertical line shows the initial asset value ($A_t = 103$). The dividend and recapitalization policy parameters for the call and put options are $\kappa_C = 1.05$ and $\kappa_\Pi = 0.90$, respectively. The remaining parameter values are given in Annex A.1.

those of the government and the public. After all, central bank capital comprises public funds.[9]

In our setting, this means that the central bank and the government agree on a particular set of policy parameters for the dividend and recapitalization policy $\{\kappa_C, \kappa_\Pi\}$. We are therefore interested in how the values of the dividend option and the recapitalization option change with respect to these key policy design parameters.

Intuitively, a higher strike, κ_C, lowers the price of the call option as it raises the probability that the call will expire worthless and the shareholder will receive no dividend. Similarly, a higher strike of the put, κ_Π, increases the chances of recapitalization, so it increases the value of the put. Mathematically, we can show this by evaluating the first derivatives of the option values with respect to the strike parameters as follows (cf. Annex

[9]In a broader context, the government, as a shareholder, may seek to ensure that the central bank's policies regarding dividends and recapitalization are in line with economic goals and fiscal responsibilities.

A.3 for details):

$$C_{\kappa_C} \equiv \frac{\partial C_t}{\partial \kappa_C} = -N(d_{2,C})L_t^* < 0;$$
$$\Pi_{\kappa_\Pi} \equiv \frac{\partial \Pi}{\partial \kappa_\Pi} = (\kappa_\Pi - 1)L_t^* N(-d_{2,\Pi})\frac{\partial d_{2,\Pi}}{\partial \kappa_\Pi} > 0 \text{ for } \kappa_\Pi < 1 \tag{16}$$

3 Equilibrium Capital Policy: Theory

So far, we have treated the two main policy parameters, the strike price or threshold of the dividend payment option, κ_C, and the strike price of the recapitalization option, κ_Π, which together govern the capital policy, as given. Now, we turn to the problem of setting these two parameters. For that, we will rely on a bargaining game. First, we examine the payoffs and objectives of the central bank and the shareholder as functions of the dividend and recapitalization thresholds. Second, we establish a number of constraints that the two thresholds need to satisfy. Third, we provide the equilibrium solution in the form of a Stackelberg game to coordinate the actions of the shareholder and the central bank.

3.1 Payoffs and Objectives

When analyzing payoffs and objectives, it is clear that any transfers between the central bank and the shareholder represent a reallocation of existing assets and are therefore a zero-sum game in valuation terms. However, the assets may serve different purposes for each entity, with the central bank needing capital for effective implementation of monetary policy and independence and the government seeking revenue for public expenditures.

To simplify the analysis, we assume that both the central bank and the shareholder are risk-neutral, implying that agents do not require additional compensation for taking risk and only care about maximizing their expected payoff. This enables us to sidestep the conversation about the degree of risk aversion exhibited by each agent, and our estimates will ultimately be unaffected by the quantification of risk premiums.

The central bank's payoff at the end of the period will be a function of the accumulated buffers, net of any dividends paid out to the shareholder in good states of the world and net of any recapitalization funds received in bad states of the world. In that sense, it is equivalent to the value of central bank equity defined in Equation (10). Since the choice of the policy parameters governs the value of the contingent transfers from and to the central bank, this payoff can be written as a function of κ_C and κ_Π. With risk-neutrality in mind, the beginning of period discounted expected payoff becomes:

$$\rho_{CB}(\kappa_C, \kappa_\Pi) \equiv B_t + \Pi_t - C_t. \tag{17}$$

The central bank prefers to have a high strike price for the dividend option and also a high strike price for the recapitalization option. This is illustrated in Figure 3 by the arrows in the bottom left corner. Technically, this follows from the central bank's objective function which is increasing in both strike prices, as we have that

$$\frac{\partial \rho_{CB}}{\partial \kappa_C} = -C_{t,\kappa_C} > 0$$
$$\frac{\partial \rho_{CB}}{\partial \kappa_\Pi} = \Pi_{t,\kappa_\Pi} > 0$$

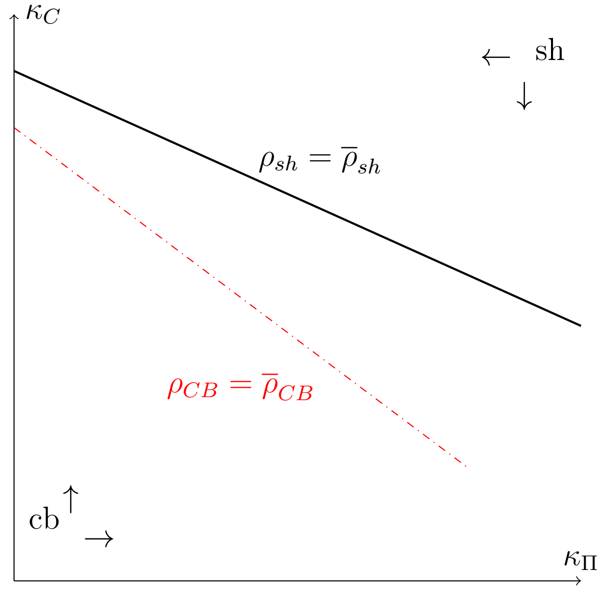
where C_{t,κ_C} and Π_{t,κ_Π} are the partial derivatives defined in Equation (16) of the call and the put pricing function.

To illustrate the trade-offs between the dividend payment and recapitalization thresholds, we conceptualize the idea of an *indifference curve.* We do that by plotting on a curve the combinations of the two thresholds between which each party, the central bank or respectively the shareholder, are indifferent. In Figure 3, each line presents the combinations of the policy parameters that deliver a given equity value for the central bank ($\overline{\rho}_{CB}$) and a given payoff for the shareholder ($\overline{\rho}_{sh}$).

Then, we can show that for a fixed payoff level $\overline{\rho}_{CB}$ the indifference curve of the central bank will be downward sloping. Formally, an indifference curve is constructed by finding the marginal change in κ_C compensated by a corresponding change in κ_Π that leaves the payoff of the corresponding party fixed. Mathematically, we have:

$$d\rho_{CB} = \frac{\partial \rho_{CB}}{\partial \kappa_C} d\kappa_C + \frac{\partial \rho_{CB}}{\partial \kappa_\Pi} d\kappa_\Pi = 0$$
$$\implies \frac{d\kappa_C}{d\kappa_\Pi} = \frac{\partial \rho_{CB}/\partial \kappa_C}{\partial \rho_{CB}/\partial \kappa_\Pi}\bigg|_{\rho_{CB}=\overline{\rho}_{CB}} < 0$$

Figure 3: Indifference curves in terms of policy parameters κ_Π and κ_C



Similarly, the shareholder's profit function at time t is defined by the value of the call option on the central banks assets, net of the value of the recapitalization put

$$\rho_{sh}(\kappa_C, \kappa_\Pi) = C_t - \Pi_t \tag{18}$$

Again, we can show that the period t value of the shareholder payoff is decreasing in both strike levels, as we have that $\frac{\partial \rho_{sh}}{\partial \kappa_C} < 0$ and $\frac{\partial \rho_{sh}}{\partial \kappa_\Pi} < 0$. Consequently, the slope of the indifference curve for the shareholder will also be negative, as illustrated with the red dashed line in Figure 3. In optimizing its payoff, the shareholder will aim to shift its indifference curve down and to the left. In other words, the central bank will try to maximize the strike levels of the two options, while the shareholder, mirroring the central bank's objective, will try to minimize them.

3.2 Central Bank and Shareholder Constraints

Before the central bank and the shareholder can negotiate and decide on the capital policy, they need to factor in that the choice of the policy parameters $\{\kappa_C, \kappa_\Pi\}$ is restricted in practice. We therefore define the domain of admissible values for the central bank's equity. There are four main dynamics that determine this domain.

3.2.1 The central bank's credibility constraint at the start of the policy period

First, the central bank should *ex ante* be adequately financed to function as a credible and effective monetary authority (Bolt et al., 2023). This means that a central bank can have negative equity, but it should have a minimum amount of assets relative to its liabilities. We define η as a proportion of the central bank's liabilities as this implicit distress boundary beyond which the policy objectives of the central bank are jeopardized. Then, we can write formally the lower bound condition on the central bank equity as

$$B_t + \Pi_t - C_t \geq \eta L_t. \tag{19}$$

It is generally understood that a central bank can continue to operate with negative equity as long as credible fiscal support is in place. Therefore, for the purposes of our analysis going forward, we assume that $\eta < 0$.

In the context of our paper, we assume that the distress boundary η is known and fixed. In practice, it is likely to be determined by factors such as the perceived ability of the central bank to maintain inflation and economic stability, policy consistency, transparency, and independence from political interference. To abstract away from any dynamics beyond the scope of this paper, we refrain from theorizing the foundations of η and accept the existence of a lower bound as given. Nevertheless, the literature offers various approaches to defining the point at which a central bank breaks; a situation of policy insolvency. Bolt et al. (2023) use a global games approach to determine the point in terms of economic fundamentals at which public trust in the currency backed up by the central bank is undermined. Reis (2013, 2015b) discusses different types of central bank insolvency in terms of the resource constraint that a central bank faces and its ability to generate seigniorage over time balanced by the limit on money demand from the public. The inverse relationship between money demand and inflation induces a Laffer-type curve with a maximum level of acceptable inflation, beyond which the public will reduce its money holdings, thus refusing to fund the central bank (Cagan, 1956; Easterly et al., 1995).

In our interpretation, the critical threshold for a central bank lies in its ability to uphold its policy objectives, particularly in ensuring price stability. Note, however, that an equilibrium capital policy may suggest the need for earlier recapitalization. To address this nuanced dynamic, we advocate for a separation between the strike price of the recapitalization option (κ_Π) and the point where the central bank is no longer credible (η) in the next subsection.

3.2.2 The central bank's credibility constraint at the end of the policy period

Second, the central bank should be adequately financed at the end of the period. To that end we have to ensure that recapitalization is triggered at levels of assets where the central bank is still credible, and not below the credibility threshold η. For the central bank credibility constraint to hold at this trigger point, we must ensure that the level of accounting capital at time T just before recapitalization is such that $B_T = A_T - L_T \geq \eta L_T$. As indicated in Figure 1, recapitalization is triggered at level of assets $A_T = \kappa_\Pi L_T$. This implies:

$$\kappa_\Pi L_T - L_T \geq \eta L_T$$

or

$$\kappa_\Pi \geq \eta + 1. \tag{20}$$

This implies a lower bound for κ_Π which ensures that given recapitalization, it occurs at a point that can still save the central bank's credibility.[10]

3.2.3 The shareholder's participation constraint

Third, from the shareholder's point of view, and in order for the capital policy to be enforceable, we need to ensure that the value of the dividend option exceeds the value of

[10]Note that this condition is compatible with Equation (19) - since the call option in this scenario is worthless, we can write:

$$B_T + \Pi_T - C_T \geq \eta L_T$$

Taking expectations under the risk-neutral measure $\mathbb{Q}$ and discounting both sides at the risk-free rate brings us to Equation (19). See Shreve (2000) for details on risk-neutral asset pricing.

the recapitalization option by a minimum amount of θ, scaled by the size of liabilities:

$$C_t - \Pi_t \geq \theta L_t. \tag{21}$$

Take for instance the minimum level $\theta = 0$. This signifies that the shareholder is only willing to support the central bank if the present discounted value of future dividends equals the present discounted value of any recapitalization. A positive θ indicates that the shareholder demands to receive more dividend payments from seigniorage profits than it is prepared to provide in backing.

3.2.4 Additional threshold conditions

To complete our set of constraints, we argue that it only makes sense to pay dividends if assets are higher than liabilities ($A_t \geq L_T$). At the same time, it only makes sense to pay dividends to the shareholder in states of the world when the central bank is not being recapitalized. We thus constrain the trigger of the recapitalization option below an asset-liability ratio of 100%. As a result, we have[11]

$$\kappa_C \geq 1 \tag{22}$$

$$\kappa_\Pi \leq 1. \tag{23}$$

Figure 4 combines all the constraints and shows the colored region of feasible combinations of the policy parameters $\{\kappa_\Pi, \kappa_C\}$.

3.3 Equilibrium Solution

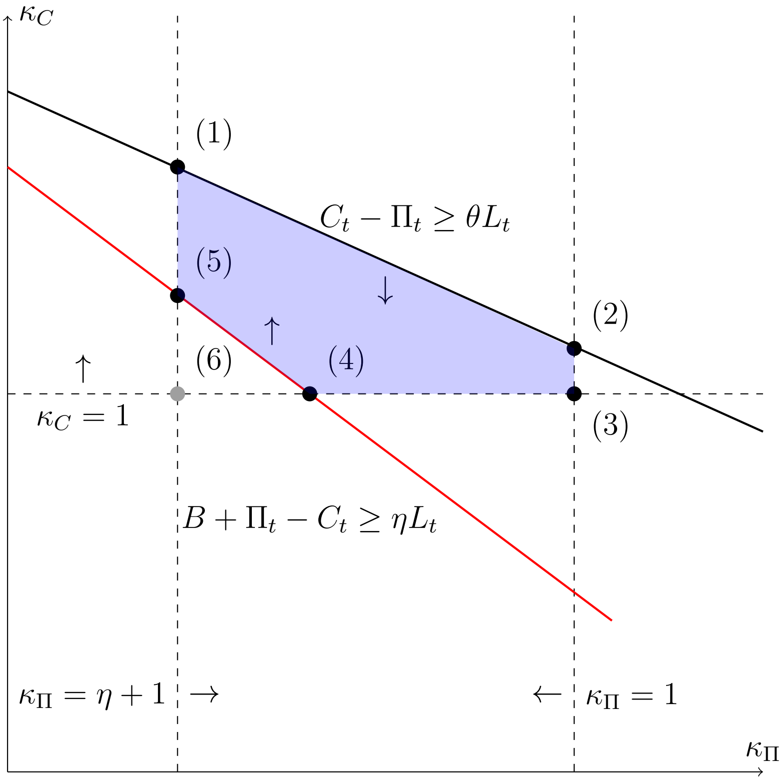
Building on the preceding section, we now establish the negotiation dynamics which determine the central bank's capital levels. First, we look at a situation in which one of the two parties holds all the decision power over both the recapitalization and the

[11] We ignore in Equation (20) that the credibility constraint holds in period T by construction when the call is exercised. If we relax the assumption that $\eta < 1$, however, we need to make sure also that κ_C does not break the viability constraint in period T when the call is exercised, such that with $A_T > \kappa_C L_T$:

$$A_T - L_T - (A_T - \kappa_C L_T) > \eta L_T.$$

This would then imply a condition $\kappa_C \geq \eta + 1$ which is stricter than $\kappa_C \geq 1$ when η is assumed to be positive.

Figure 4: Constraints and feasible combinations of policy parameters



dividend-payment parameter and the other party complies given its constraints. Then, we look at a situation where the decision of each parameter is allocated to one player, and each player acts strategically in a Stackelberg style leader-follower game-theoretic setting.

3.3.1 One player dominates the decision

First, assume that one player can set both policy parameters. In this scenario, we have two options: either the central bank possesses all bargaining power and the shareholder must accept the arrangement, or the shareholder sets the parameters to maximize its payoff, and the central bank is required to accept the conditions. Formally, we can write the optimization problem for both cases as:

$$\begin{aligned} &\max_{\kappa_C, \kappa_\Pi} \{\rho_i(\kappa_C, \kappa_\Pi)\}, \text{ subject to} \\ &\kappa_C, \kappa_\Pi \in \mathcal{A} \end{aligned} \tag{24}$$

where $i = \{cb; sh\}$ is the indicator of the payoff function of the player who dominates the decision process with the objectives of the two players being defined as $\rho_{cb} = B_t + \Pi_t - C_t$ while $\rho_{sh} = C_t - \Pi_t$. $\mathcal{A}$ is then the set of feasible outcomes outlined in Section 3.2. We now look at the two corner solutions that are possible in terms of negotiation power. For that we look at Figure 4 that shows the colored area or combinations of the policy parameters $\{\kappa_C, \kappa_\Pi\}$ that the central bank and shareholder can choose from.

First, if the central bank sets both policy parameters, it will aim to decrease the value of the dividend option and to increase the value of the recapitalization option as much as possible. As a result, it will provide a solution that exactly satisfies the shareholder's minimum required payoff, such that $C_t - \Pi_t = \theta L_t$. Substituting this in the central bank's objective function, we get that at the optimum, the payoff of the central bank is $\rho_{cb} = B_t - \theta L_t$. As a result, any combination of κ_c and κ_Π that lies on the shareholder's minimum participation requirement is optimal when it also satisfies the rest of the constraints. So, graphically, the solution is any point between and including points (1) and (2) in Figure 4.

Second, if instead the shareholder is capable of setting the two policy parameters it will try to increase the value of the dividend option and reduce the value of the recapitalization option. This means it will set κ_C and κ_Π at any point between (4) and (5) in Figure 4. On this line segment the central bank can just fulfill its credibility constraint. Note that in this stylized example, we have assumed that the central bank's credibility constraint (the red line in Figure 4) lies above the point where $\kappa_C = 1$ and $\kappa_\Pi = \eta + 1$. This depends solely on the parametrization of the model. If that is not the case and the central bank credibility constraint is not binding, the shareholder will instead simply pick point (6), setting $\kappa_C = 1$, the lowest possible value, and $\kappa_\Pi = \eta + 1$, the lowest possible recapitalization trigger threshold at which the central bank would still be credible (Cf. Section 3.2.2).

In reality, each of the two parties may be unwilling to grant the other the full control over setting the threshold parameters. Therefore, we will see in the next section how the equilibrium changes if instead the rights over each separate policy parameter are split between the two counterparties, such that each player can set one policy parameter in a way that optimizes its payoff.

3.3.2 Each player sets one policy parameter

In this section, players are endowed with the responsibility of sequentially choosing one policy parameter each, akin to a Stackelberg game. In this strategic setting, the initial player takes the lead by making the first move, operating under the assumption that its decision will be observed and factored into the subsequent moves of the second player. The equilibrium is contingent on the identity of the initial mover and the extent of authority each player wields in determining a specific parameter. The power that the leader has in a Stackelberg game relative to the follower is akin to the power that a counterparty has to set the terms in a negotiation.

The conventional methodology of backward induction can be applied to solve the game. First, we determine the reaction function of the follower with respect to any of the first mover's potential choices. Then, given that reactions are common knowledge, the leader picks out of the follower's reaction function the best choice with respect to his own payoff. In all cases, the players can choose only from the feasible region.

To illustrate, assume that the central banks is a first mover in setting the recapitalization parameter (κ_Π), while the shareholder follows by setting the dividend policy parameter (κ_C). Then, the solution approach that is used to find the equilibrium point can be separated into two main steps: first we solve the follower's optimal problem (in this case the shareholder) and then solve the leader's optimization. We can formalize this as:

1. First, based on any κ_Π within the reach of the central bank, we find the optimal choice of κ_C for the shareholder. This provides the shareholder's optimal reaction function which we denote as $\tilde{\kappa_C}(\kappa_\Pi)$. The function represents the choice of κ_C, which maximizes the shareholder's payoff function, given a κ_Π. Mathematically,

this is the solution of the following optimization:

$$\tilde{\kappa}_C(\kappa_\Pi) = \arg\max_{\kappa_C \in \mathcal{A}} \rho_{sh}(\kappa_C, \kappa_\Pi).$$

2. Then, given the shareholder's reaction function, we derive the leader's optimal choice, i.e. the κ_Π which maximizes its payoff, given that the shareholder picks κ_C according to the reaction function established in the previous step. Formally, this is:

$$\tilde{\kappa}_\Pi = \arg\max_{\kappa_\Pi \in \mathcal{A}} \rho_{cb}(\tilde{\kappa}_C(\kappa_\Pi), \kappa_\Pi).$$

The final equilibrium is then the point $\{\tilde{\kappa}_\Pi, \tilde{\kappa}_C(\tilde{\kappa}_\Pi)\}$.

The same algorithm can also be used in the other cases, simply adjusting the payoff functions and the leader and the follower, and adjusting the decision parameter over which each party has control. Using this approach, we can now distinguish a number of settings for the Stackelberg game. The equilibrium solution will depend on which party is the first mover and which party has authority over which of the two decision parameters:

- The central bank is the first mover and sets the policy parameter for recapitalization, (κ_Π), with the idea that the shareholder will choose the dividend policy parameter, denoted as κ_C, optimally in response. There are two steps in this procedure. First, we determine the reaction function of the shareholder, who will choose the lowest feasible value for κ_C when confronted with κ_Π by the central bank. The solid black line segments on the lower left edge of the feasible region in Figure 5a illustrate this reaction function. Second, with the knowledge of this reaction function, the optimal choice for the central bank as a first mover is to choose κ_Π at the level of solution point (3).

- The shareholder is the first mover and sets κ_Π. Then the central bank will set the dividend policy parameter κ_C optimally as a response. Again, we can find the equilibrium solution through backward induction. First, we determine the reaction function of the second mover, the central bank, which gives $\kappa_C(\kappa_\Pi)$. In Figure 5b this is the black line between points (1) and (2), which provides the optimal response of the central bank for any potential choice of κ_Π by the shareholder. Now, determining the optimal κ_Π by the shareholder, it is important to remember that the line segment (1) to (2) actually lies on a shareholder indifference curve (Cf. Section 4.1). This means that the shareholder will be indifferent between any point on the reaction function of the central bank, so the equilibrium outcome is any point on the line segment between (1) and (2).

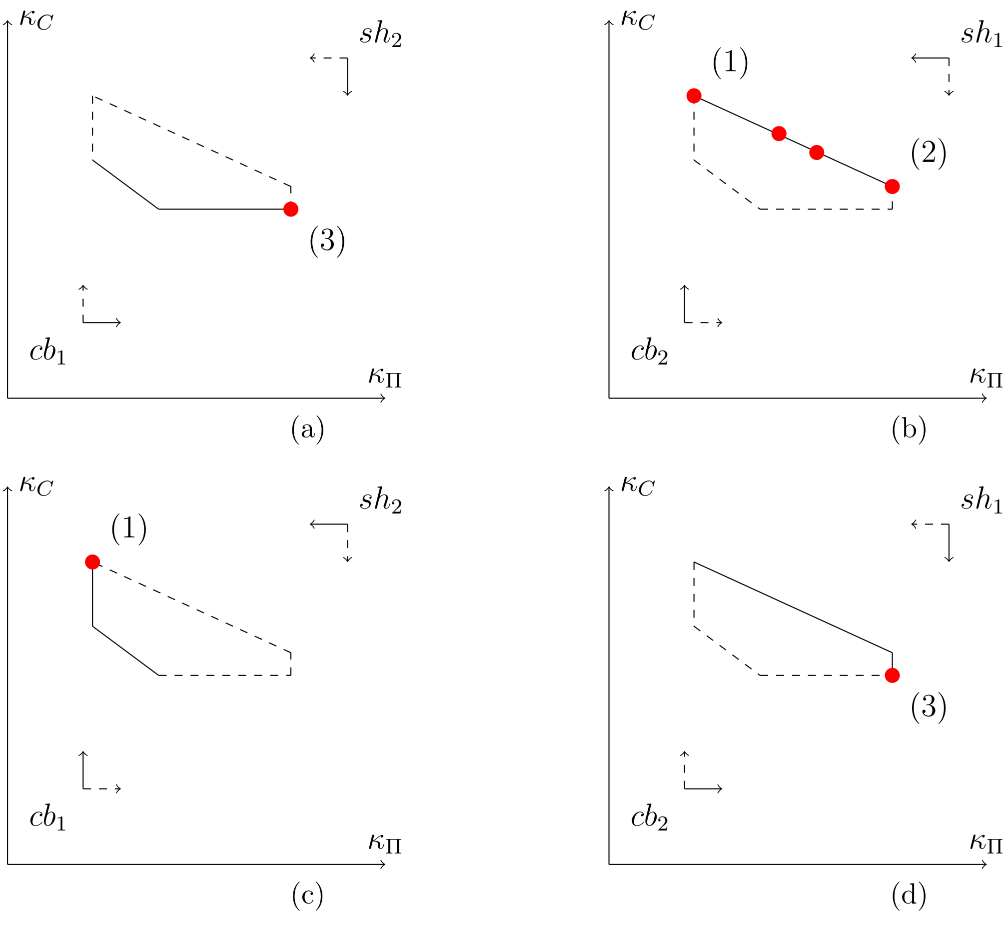
- The central bank is the first mover to set κ_C with the knowledge that the shareholder will set κ_Π optimally as a response. The reaction function of the second-mover $\kappa_\Pi(\kappa_C)$ consists in the solid black line segments representing the lower-right edge of the feasible region in Figure 5c. Then the optimal choice for the central bank is choosing κ_C at the level of point (1), which indicates the equilibrium.

- The shareholder sets κ_C first, and the central bank optimally sets κ_Π as a response. The reaction function of the second mover consists of the solid black line segments on the upper-right edge of the feasible region in Figure 5d. With the knowledge of the reaction function, the shareholder picks the highest feasible value for κ_Π, and the equilibrium is at point (3).

Table 3 summarizes all possible equilibria. As illustrated, what is more important in the process of setting the recapitalization conditions is not so much which party moves first in the game, but rather how the decision rights over the policy parameters are allocated. If the central bank has power over the dividend policy (κ_C), the equilibrium ends up with a relatively low level of the capitalization threshold (top left on Figure 5b and Figure 5d) and a relatively high value of the dividend threshold. In economic terms, this means that the central bank does not have much protection against negative buffers as the recapitalization option is less likely to be exercised. However, it can also accumulate a high capital without paying dividends to the shareholder, as the dividend option is also less likely to be triggered.

In contrast, if the central bank has the power over the recapitalization policy (κ_Π) the equilibrium has a relatively high recapitalization threshold and low dividend threshold (bottom right in Figure 5a and Figure 5d). In that case, the central bank has better protection against negative buffers as the recapitalization option is more likely to be triggered. However, it is also more likely to pay dividend to the shareholder. Moreover, at the equilibrium point (3) in Figure 4, we have that $\kappa_\Pi = \kappa_C = 1$. In this case, the position of the shareholder in the recapitalization is similar to a long position in the net assets of the central bank: any positive buffers are paid out to the shareholder and any negative buffers are recapitalized. One can also see that through the put-call parity: the payoff for the shareholder becomes $C_t - \Pi_t = A_t - L_t^*$.

Figure 5: Stackelberg solutions



Note. This set of figures shows the reaction function of the second-mover and the equilibrium in the Stackelberg game. The subscripts indicate whether the party is a first or a second-mover. The arrows indicate the direction in which the player wants to move the two parameters of the game, with the solid arrow indicating the variable over which the player has decision power.

Table 3: Stackelberg Equilibrium Combinations

First mover	Central bank (CB)		Shareholder (SH)	
Who sets which policy				
- Dividend policy	SH	CB	SH	CB
- Recapitalization policy	CB	SH	CB	SH
Equilibrium impact on thresholds				
- Dividend payment threshold	Lower	Higher	Lower	Higher
- Recapitalization threshold	Higher	Lower	Higher	Higher
Equilibrium Point	(3)	(1)	(3)	(1) to (2)

3.4 Risk-Shifting Implications

The setting presented so far enables us to assess how risks are distributed between the central bank and its shareholder. An intriguing question arises: once the capital thresholds are set, and abstracting from policy objectives, does the central bank have an incentive to take more risk onto its balance sheet, thereby transferring risk toward the shareholder, who ultimately provides the downside protection? Similarly, does a central bank have an incentive to reduce risk-taking to limit the upside dividend potential for the shareholder?

To study the risk-shifting implications, we look at how the economic value of equity changes when the share of risky assets on its balance sheet increases. Notably, the central bank has the authority to determine the risk profile of its assets by engaging in purchase programs rather than traditional lending operations. In our model this is captured through the weight of risky assets (ω). We can then examine this question by looking at how equity value changes through the repricing of the dividend and recapitalization options, when the share of risky assets changes.

In option pricing, the sensitivity to risk is commonly referred to as "vega" and can be evaluated through the partial derivatives of the Black-Scholes option valuation functions. We can thus show that for a given strike level, the vega for both the call and the put options is given by:

$$\nu_t = \frac{\partial C_t}{\partial(\omega\sigma)} = \frac{\partial \Pi_t}{\partial(\omega\sigma)} = A_t\sqrt{T-t}N'(d_{1,i}) > 0 \quad (25)$$

with $i = \{C, \Pi\}$ being an indicator for the call or the put option.[12]

To analyze this, note that the values of the dividend and the recapitalization options are increasing in the asset's return volatility, which is determined by the share of risky assets ω. Intuitively, a higher share of risky assets increases the asset return volatility on the balance sheet of the central bank, which increases the likelihood that one of the two options will be exercised. Furthermore, in Figure 6a we show how vega for the call and the put options in our model changes as a function of how deep in the money each of the options is.[13] In our setting the moneyness, or how close the assets currently are to the strike parameters, is determined by the value of capital B_t, the 'underlying'. It follows from the figure that the vega is non-linear in the underlying, and in each case it peaks at the corresponding strike. As the strike of the put option is lower than the strike of the call (by construction the recapitalization threshold is lower than the dividend payment threshold), the call vega is shifted to the right. This implies that with high central bank capitalization the dividend payment option is close to or in the money and will be more sensitive to risk than the recapitalization option, which in that case is out of the money. The opposite happens for an undercapitalized bank.

Going back to our original question, we can evaluate the sensitivity of the equity value to changes in ω as:

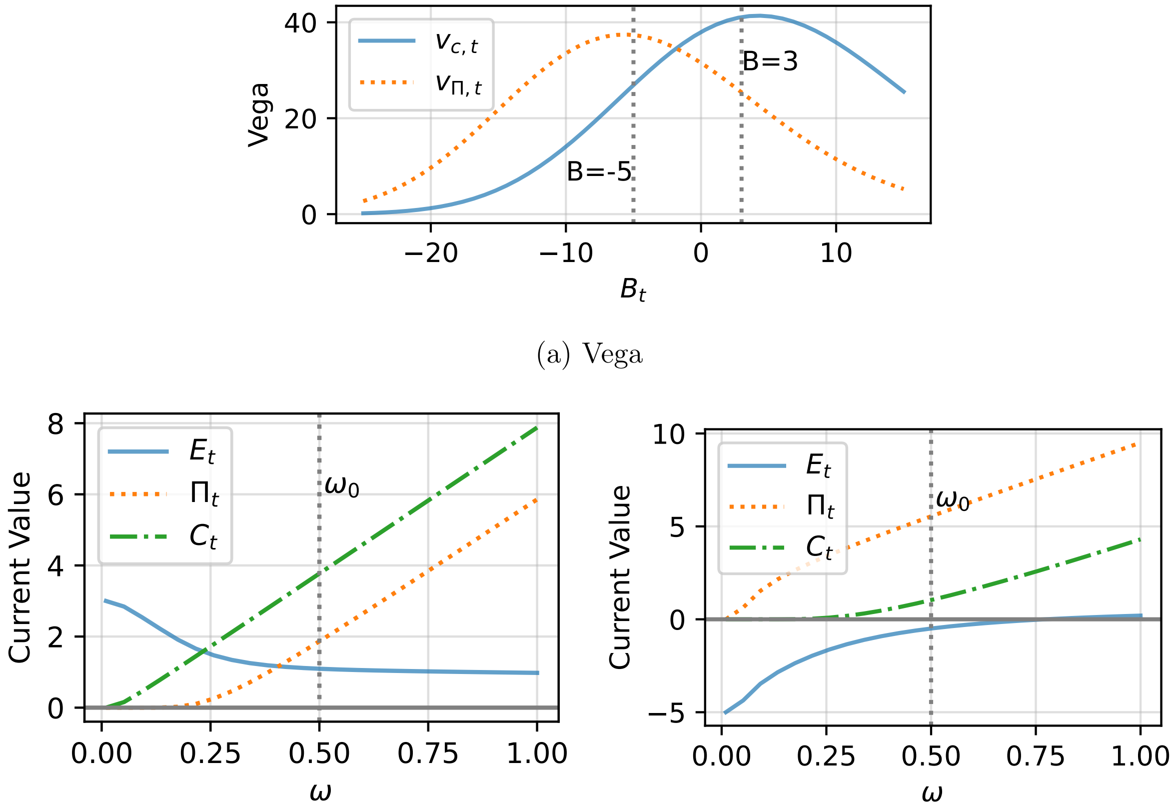
$$\frac{\partial E_t}{\partial(\omega\sigma)} = -\frac{\partial C_t}{\partial(\omega\sigma)} + \frac{\partial \Pi_t}{\partial(\omega\sigma)}.$$

We then identify two separate situations graphically outlined in Figure 6a through the two bell-shaped curves representing the vegas of the embedded call and put options. First, assuming that the central bank is well capitalized (in this example with capital $B_t = 3$), the recapitalization option (the put) will be further out-of-the-money than the dividend payment option (the call). As a result, the call's vega will dominate over the put's vega. The result is shown in Figure 6b, where we show the impact of changing the risk profile for a well-capitalized central bank. In this case increasing the assets' risk

[12]Strictly speaking, the vega is defined as the partial derivative of the option value with respect to σ in the literature. The ω and σ in our setting, however, are multiples of each other, and ω governs the degree of risk that the central bank takes.

[13]See Annex A.1 for details on the model calibration.

Figure 6: Option and equity value as a function of risk



(a) Vega

(b) Value at $B_t/L_t = 3\%$

(c) Value at $B_t/L_t = -5\%$

Note. This set of figures shows the value of the embedded call and put options in the implied balance sheet of the central bank, and the value of the implied equity, as a function of the share of risky assets, holding everything else fixed. Figure (a) shows the situation for a well-capitalized bank with an initial accounting capital of $B_t = 3$; Figure (b) shows an under-capitalized bank with an accounting capital of $B_t = -5$; Figure (c) shows the vegas of the two options given $\kappa_\Pi < \kappa_C$.

profile by investing more in risky assets increases the value of the call option (dashed curve) more than value of the put option (dotted line), and thus reduces the equity value. Ergo, the central bank has no incentive to shift risks towards the shareholder.

However, if the central bank is under-capitalized ($B_t = -5$), as illustrated in Figure 6c, the recapitalization option will be closer to at-the-money, and the value of the put will increase faster with the level of asset risk than that of the call. As a result, in that case, the central bank would be able to increase the value of its equity by taking more risk.

On an intuitive level, when considering a central bank with ample capital reserves, an increase in asset risk tends to raise the probability of distributing dividends more significantly than the likelihood of requiring recapitalization. Conversely, an undercapitalized central bank exhibits the opposite effect: heightened asset risk increases the chances of breaching the recapitalization threshold and necessitating an infusion from its shareholder. Consequently, overly capitalized central banks may not fully utilize the expansion of their balance sheets, such as through an Asset Purchase Program, as any surplus generated must be allocated to shareholder payouts. Conversely, undercapitalized central banks might find advantages in assuming higher risk levels due to the potential value associated with the recapitalization option.

This suggests that without adequate institutional frameworks in place, a central bank operating with insufficient capital might be inclined to respond excessively to its policy objectives, banking on the assurance of a shareholder bailout. Conversely, an excessively capitalized central bank might be tempted to be too conservative in its actions, aware that any surplus during prosperous periods would mostly benefit the shareholder.

As the central bank has authority over the composition of the assets on its balance sheet and can take on financial risk through the use of outright asset purchases, our findings stress the importance of sound risk management practices. More broadly, while the central bank is entitled to operational independence from the shareholder for optimally achieving its policy objectives, we highlight the need for democratic accountability and transparency in justifying the choice of tools which may potentially exhaust its capital. In the short term, the central bank functions autonomously from the government to pursue its monetary policy goals. However, in the long run, the government exercises control over the central bank's operations and role through its authority to appoint board members and enact legal or statutory amendments (Wessels and Broeders, 2023).

Note that potential risk shifting is not inherently contradictory to the central bank's mandate. For example, macroeconomic factors may compel a central bank to pursue unconventional monetary policy decisions regardless of its capital position. This effectively transfers risk from the central bank's capital to its shareholder. Moreover, any undesirable risk-shifting incentives that are not welfare-improving are likely to be tempered in the presence of well-functioning institutions which ensure central bank accountability.

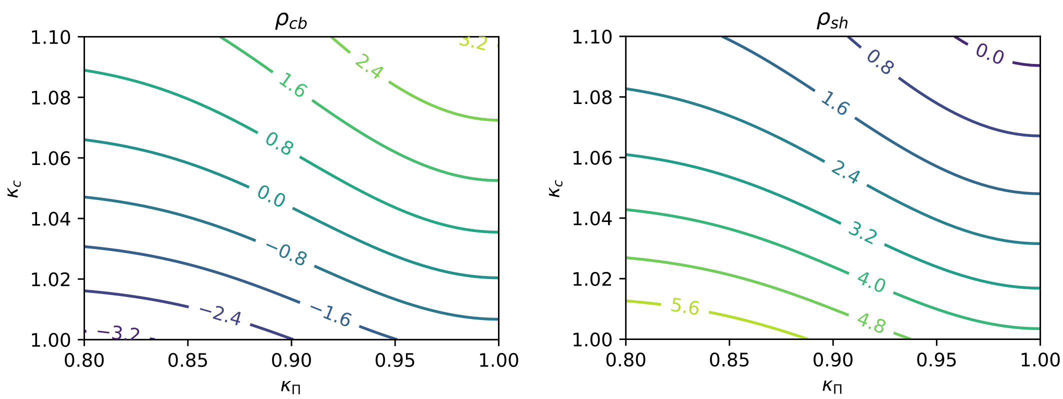
For the sake of brevity and clarity in this paper, we defer the modeling of this penalty to future work.

4 Equilibrium Capital Policy: Numerical Evaluation

Now, that we have outlined the main features of the model, we proceed to evaluate numerically the equilibrium solution for the capital policy under the initial conditions outlined in Annex A.1. In the following sections we first look at the properties of the payoff functions of the two players in the model and outline the feasible region they face. We then solve numerically for the equilibrium capital structure of the central bank using the backward induction approach outlined earlier. Finally, we verify the sensitivity of the solution to variations in the participation threshold of the central bank, and to the central bank's initial capital buffers.

4.1 Indifference Curves

Figure 7: Indifference Curves



(a) $\rho_{CB}(\kappa_C, \kappa_\Pi)$ (b) $\rho_{sh}(\kappa_C, \kappa_\Pi)$

Note. This set of figures shows the indifference curves underlying the risk-neutral objective functions of the central bank and shareholder risk-neutral objective curves.

We defined in Section 4.1 the payoff functions of each counterparty in the model. Figure 7 shows the indifference curves for the central bank and the shareholder. Each line presents combinations of the policy parameters that deliver a given equity value for the central bank (Figure 7a) and a given payoff for the shareholder (Figure 7b). The numbers along the lines represent the payoff along each indifference curve based on the given calibration. As we indicated analytically earlier, we can verify quantitatively here that the central bank will aim to move in a north-west direction, while the shareholder will move east-south to maximize its payoff.

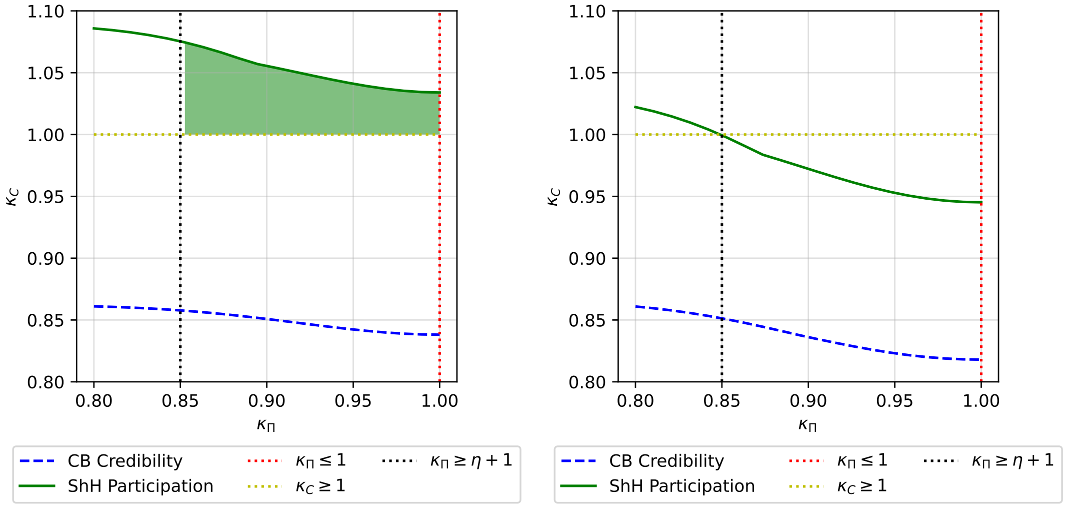
4.2 Feasible Region for the Policy Parameters

In Figure 8 we plot the feasibility region in terms of combinations of the policy parameters $\{\kappa_C, \kappa_\Pi\}$ for the model calibration in Annex A.1. In Panel (a) we assume that the central bank has an initial accounting capital B_t equal to 3% of the current liabilities. This is consistent with a central bank that is relatively well capitalized and potentially close to the dividend payment region.

The central bank's credibility condition is given by the blue line. All combinations of $\{\kappa_C, \kappa_\Pi\}$ that are on or above this curve satisfy this constraint. The shareholder's participation constraint is given by the solid green curve. All combinations of $\{\kappa_C, \kappa_\Pi\}$ that are on or below this curve, satisfy the participation constraint. Further, the constraint that the central bank only pays dividends if assets are higher than liabilities ($\kappa_C \geq 1$) is given by the green dotted line and the constraint that the central bank only pays dividends when it is not being recapitalized ($\kappa_C \geq \kappa_\Pi$) is given by the red dotted line. The feasible region is now given by the green area. Note that the central bank's credibility condition is not affecting the feasible region.

In Panel (b) we assume a lower capital B_t equal to -2% of liabilities. Lower capital shifts down the shareholder's participation constraint (the solid green curve). This occurs as, ceteris paribus, lower buffers make the put option held by the central bank more expensive. In order to compensate, the shareholder would demand lower strikes for the call and/or the put in order to get back to its reservation value set in Equation (19) by θL_t. In this case, the curve is shifted down so much that the feasible region is a single point at $\kappa_C = 1$ and $\kappa_\Pi = 0.85$.

Figure 8: Feasible Region



(a) $B_t/L_t = 3\%$ (b) $B_t/L_t = -2\%$

Note. This set of figures shows the feasible region (shaded are) with (a) positive buffers; and (b) negative buffers, holding everything else fixed.

4.3 Equilibrium

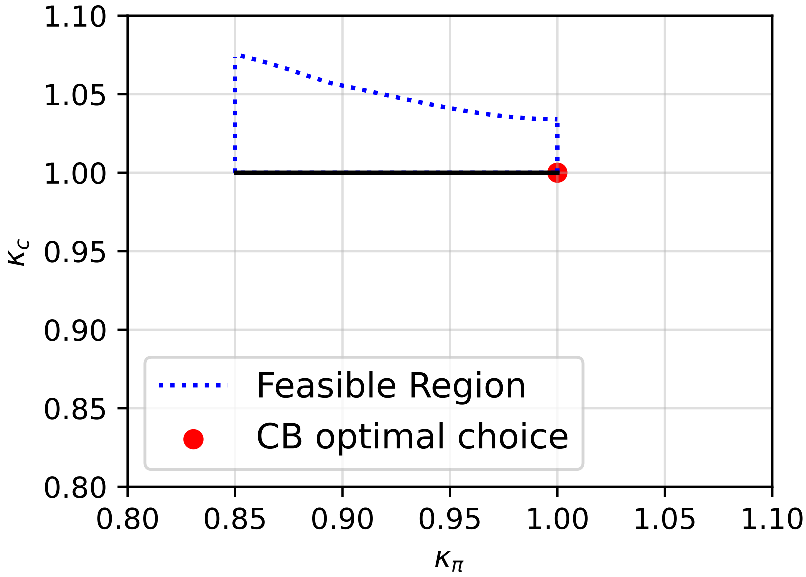
Figure 9 illustrates the solution of the Stackelberg game. We examine in particular the setting which is most in line with central bank independence: the central bank is the first mover and it has the authority to set the recapitalization threshold, κ_Π. In response, the shareholder sets an optimal threshold for paying dividends, κ_C. As outlined in Section 3.2, there are no informational frictions between the two counterparties, so in setting κ_Π the central bank already anticipates the optimal reaction function of the shareholder.

Under this arrangement, we find that the shareholder will stick to a demand of keeping the dividend payment threshold at 100% of the assets. In equilibrium, then, the central bank will also demand that it is recapitalized if liabilities fall below 100%.

4.4 Sensitivity Analyses

The equilibrium is influenced by many factors. In the following subsection, we look at how the equilibrium capital policy changes when some of the key input parameters are varied. The game setting is again in line with the arrangement discussed in Section 4.3.

Figure 9: Stackelberg Equilibrium Solution

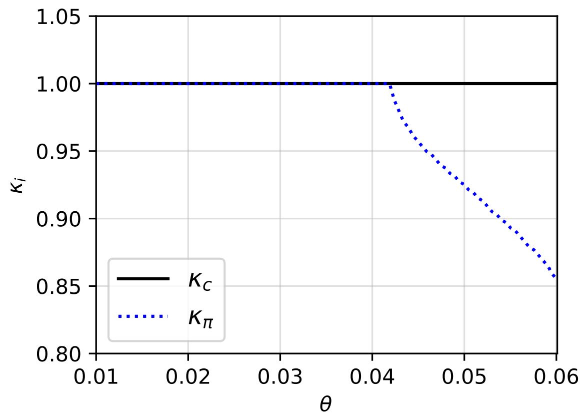


Note. This figure shows the edge of the feasible region (dotted), the optimal reaction function of the shareholder as a Stackelberg follower (solid line), and the final optimal solution with the central bank being a Stackelberg leader (single dot).

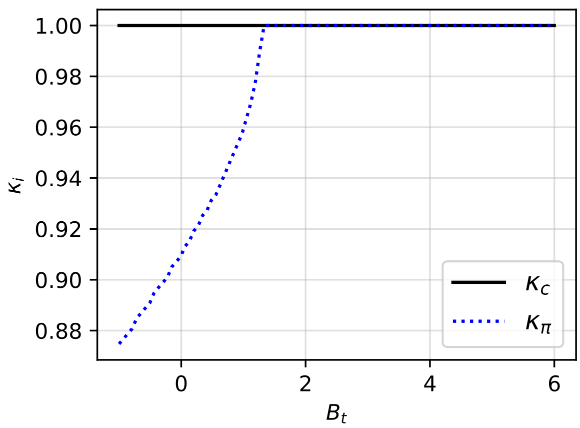
First, we evaluate the sensitivity of the solution to the participation threshold θ, a parameter which is not directly observable in reality. At the same time, it shifts the shareholder's participation constraint and thus can affect the equilibrium results. A higher θ implies a stronger position of the shareholder, and thus a stricter participation constraint. In that case, as the shareholder becomes stronger, its participation curve (cf. Figure 4) will shift down, demanding more value from the call option or less value from the put option, or equivalently lower strikes for each of the options. As a result, the feasible region shrinks, and the central bank is left with fewer options to move the solution point to the utmost right direction in the κ_C region. As the lowest possible value for κ_C is fixed at 100%, the central bank can only agree to reduce the value of the strike if that becomes needed. Figure 10a then shows the equilibrium solutions for the two parameters. As θ increases, at some point the shareholder and the central bank will agree on a κ_Π that will be below 100%. If θ increases further, there may be no solution left in the feasible region, leaving the two parties without agreement.

Second, we can evaluate, under the same setting, the impact of the initial capital B_t on the Stackelberg equilibrium. As indicated in Section 4.2, higher B_t shifts down

Figure 10: Stackelberg Equilibrium (CB chooses first)



(a)



(b)

Note. This set of figures shows that a stricter shareholder's participation constraint affects the choice of optimal recapitalization and diffident payment thresholds. In this game, the central bank moves first, picking the optimal value of κ_Π. In the first chart, we vary θ; in the second, we vary the current level of buffers.

the shareholder participation curve. Then, as soon as it becomes a binding constraint, the equilibrium condition will also change as indicated in Figure 10b. Since κ_C is at its minimum possible value of 100% already, it does not change.

5 Policy Evaluation

In this paper, we present a generalization of the capital policy. In practice, central banks around the world have heterogeneous capital policies. In this section, we first discuss four cases, and show how they fit into our model: the Federal Reserve System (Fed), the Bank of England (BoE); Sveriges Riksbank; and De Nederlandsche Bank (DNB). Next, we consider the policy implications of recapitalization.

5.1 Capital Policies in Practice

We begin by considering the **Fed**. Recapitalization is not explicitly a component of the legislative framework established with the US Treasury. There is, however, a ceiling to the surplus that the Fed can hold.[14] The Fed's income statement presents these cumulative payments to the US Treasury as "*Earnings remittances to the Treasury*". In

[14] "*The Federal Reserve Act requires that aggregate Federal Reserve Bank surplus does not exceed a cap. After providing for the costs of operations, payment of dividends, and the amount necessary to maintain*

practice, these remittances are paid weekly and the annual report states the sum of all weekly payments over the year. If earnings are not sufficient to provide for the cost of operations, payment of dividends, and maintaining a surplus, negative remittances will be booked in the Fed's balance sheet as a deferred asset.[15] The deferred asset position will be reduced and eventually cancelled out by future earning remittances. The working assumption of this framework is that the net present value of future remittances to the US Treasury is positive through the Fed's seigniorage income and will at some point offset the deferred asset.

The **BoE**, on the other hand, has an explicit memorandum of understanding with its shareholder, the UK Treasury, which sets a capital target level of GBP 3.5 billion, which is deemed sufficient to allow the BoE to perform its monetary policy and financial stability functions. Similarly to our stylized setting presented earlier, the BoE memorandum establishes a floor and a ceiling level of capital (GBP 0.5 billion and GBP 5.5 billion respectively). If the BoE's capital falls below the floor, the UK Treasury is committed to providing capital injections. At the same time, a flexible dividend-sharing arrangement is established, which states that when capital falls below the target, but remains above the floor, the BoE retains any net income completely. When the capital is above the target, but below the ceiling, half of the net income is retained and half is distributed to the UK Treasury. When the capital is above the ceiling, the net income is completely transferred to the UK Treasury (Bank of England. 2018).

The floor and ceiling framework established by the BoE is captured very well by our setting presented in Section 2. In fact, using the notation of our model, we can combine the value of the central bank liabilities as of 2023 and the size of the floor to establish that[16]

$$\kappa_{\Pi} = \frac{\text{Floor} + L_T}{L_T} = \frac{0.5 + 1,070}{1,070} = 1.0005.$$

the surplus cap, any residual net earnings will be remitted to the US Treasury". Cf. Federal Reserve Board, 2024.

[15] See for example Table 6 in Federal Reserve Board, 2023.

Similarly, we can derive the strike price of the dividend option, κ_C, from the ceiling capital:

$$\kappa_C = \frac{\text{Ceiling} + L_T}{L_T} = \frac{5.5 + 1,070}{1,070} = 1.005.$$

Similarly, **Sveriges Riksbank** has an explicit capital framework in place specifying the rules for recapitalization. Effective from 2023, the Sveriges Riksbank Act sets a target level for its capital at the level SEK 60 billion and a baseline at SEK 40 billion. The target level of SEK 60 billion is intended as an appropriate upper limit and is adjusted upwards by inflation each year.

Furthermore, the Act requires the Riksbank to petition the Swedish parliament for capital restoration through recapitalization if it falls below a minimum threshold of SEK 20 billion. This means that in years in which the Riksbank reports capital lower than the target level, any profit shall be retained to replenish it. In years in which the Riksbank reports capital above the target level, part of the profit is retained to adjust the target capital to inflation growth, but the excess profit is distributed to the state. If the capital falls below the minimum threshold, the Riksbank is obliged to request recapitalization from the state. While the act also foresees certain exceptions to this general rule, in principle, the state then has to restore capital up to the baseline level (Sveriges Riksbank, 2023).

Again, we can easily map the Riksbank's capital framework onto our stylized model. Using 2022 annual report data leads to $\kappa_\Pi = \frac{20+1{,}437}{1{,}437} = 1.01$. By setting the target capital as the threshold for dividend payouts, we obtain $\kappa_C = \frac{60+1{,}437}{1{,}437} = 1.04$.

DNB also has an explicit capital policy in place with target capital of EUR 9.4 billion, which is also linked to GDP and grows yearly by 3.9%. The annual growth rate is revised every five years and currently represents a ten year moving average of the growth rate of Dutch GDP. There is no explicit recapitalization level. There are, however, two relevant floors: first, dividend payments are forbidden if capital falls below EUR 0.5 billion (the

[16]Remember that the put option is triggered at $\underline{A}_T = \kappa_\Pi L_T$. We can then derive the strike price of the recapitalization option, κ_Π, by noting that the recapitalization threshold value of assets, $\underline{A}_T$, is the sum of BoE floor capital and the value of liabilities.

issued capital); second, EU legislation mandates that Member States must ensure their national central banks maintain sufficient financial resources and adequate 'net equity' to fulfill their duties. Net equity encompasses various elements, including capital, the general reserve fund, revaluation accounts, any accumulated losses from previous periods, and any profit or loss for the current year. Should the net equity fall below the statutory capital or even turn negative, government intervention is required to inject the central bank with capital, ensuring that it reaches at least the level of statutory capital within a reasonable period of time. Using DNB's 2022 target capital as the threshold above which dividends can be paid in full, we obtain $\kappa_C = \frac{9.4+480}{480} = 1.0195$. κ_Π can be approximated by assuming that recapitalization would occur if capital falls below the issued capital. In this case, using 2022 annual report data we obtain $\kappa_\Pi = \frac{0.5+480}{480} = 1.001$.

Table 4 provides an overview of these capital policies and the actual positions with recent annual report data. For the BoE, Sveriges Riksbank, and DNB the put option is out of the money, as κ_Π is below the ratio (A_t/L_t), indicating that recapitalization conditions are not yet met. However, in the case of Sveriges Riksbank, this is only the case because the government already provided recapitalization at the end of 2022. On the other hand, the call option is in the money for the BoE, as κ_C is just below the ratio (A_t/L_t). The call option is also in the money for Sveriges Riksbank, although this is a mechanical consequence of the capital injection from the government. In practice no dividend has been paid in 2022. The call option is out of the money for DNB, as κ_C is above the ratio (A_t/L_t), so DNB did not pay any dividend in 2022. We need to be aware, however, that comparing central banks is not straightforward, as they may be subject to different accounting rules governing the way assets and liabilities are valued.

5.2 Recapitalization

Although we do not explicitly model this, we want to know how recapitalization works in practice? In the event that a central bank experiences losses surpassing its capital, it will initially reflect either a negative capital on the liability side or a positive loss carried forward on the asset side of its balance sheet (Buiter, 2024). Both scenarios essentially

Table 4: Different central banks' capital policies at the end of 2022

Central bank	Imposed strike put option (κ_{Π})	Imposed strike call option (κ_C)	Actual position (A_t/L_t)
Federal Reserve	1.0*	1.0*	1.049
Bank of England	1.0005	1.005	1.0051
Sveriges Riksbank	1.01	1.04	1.0435
De Nederlandsche Bank	1.001	1.0195	1.0177

Notes: All numbers in the table are based on figures from the annual reports of the central banks, or official documentation describing the agreement between the shareholder and the central banks. In the case of the Federal Reserve, the thresholds are implicit, as recapitalization and dividend payments are virtually continuous and make use of deferred assets on the central bank's balance sheet.

signify a claim of the central bank on the government, assuming the government is the sole owner of the central bank.

Subsequently, two options become available. First, the government can directly inject funds into the central bank to settle the claim immediately. The simplest method for recapitalizing a central bank involves transferring a government bond to the central bank in exchange for an equity stake of equal value. This transaction results in an increase in the central bank's balance sheet size. Second, the central bank can reduce the loss carried forward by retaining future profits, effectively recapitalizing through the government forgoing future dividends.

It should be noted, however, that recapitalizing a central bank through government intervention can yield unintended consequences. First, it may jeopardize the central bank's independence, potentially compromising its ability to make impartial monetary policy decisions. Government influence might prioritize short-term political considerations over long-term economic stability. Second, recapitalization could strain the government's fiscal position, diverting resources from essential public services or exacerbating existing budgetary pressures. Additionally, the terms of the contract between the central bank and the shareholder are likely to resemble a repeated game setting, given that central bank management faces reelection at regular intervals or when governments change. These unintended consequences may discourage the central bank from seeking recapitalization, thereby potentially forgoing the exercise of the put option.

6 Conclusion

This paper discusses how a central bank manages financial risks through capital as a buffer for risk-taking in monetary policy operations. It explores the central bank's capital formation resulting from a risk-sharing arrangement with the government, using an option pricing perspective and the tools from finance. We investigate the trade-offs between dividend payouts in favorable states of the world and recapitalizing the central bank in unfavorable states, deriving equilibrium solutions through Stackelberg games where both parties negotiate optimal dividend and recapitalization policies. The findings of this study have several significant policy implications for central banks and their shareholders, typically governments, in managing financial risks and optimizing their capital policies.

First, understanding central bank capital as a result of a risk-sharing arrangement emphasizes the importance of effective risk management strategies. Policymakers should carefully evaluate the trade-offs associated with paying dividends during prosperous periods and holding recapitalization options for challenging economic states. This awareness can guide the formulation of robust risk-sharing agreements that strike an optimal balance between risk absorption and financial stability.

Second, we document the risk incentives embedded in the balance sheet of central banks. In accordance with option pricing theory, we observe that the value of both the dividend payment and recapitalization options rises with the level of balance sheet risk. In the context of central banking, risk primarily stems from the proportion of risky assets within monetary portfolios, such as those acquired through an asset purchase program. We find, however, that an increase in risk will have different implications for the value of equity of a well capitalized versus an undercapitalized central bank. The initial level of buffers determines which of the two options will dominate.

Building on this, our paper also highlights a coordination problem between the central bank and the government. As a next step, we look for the trade-offs and optimal arrangement in which the recapitalization and dividend-payment thresholds are set, as a result of a negotiation game. We establish the solution from which neither party has incentives to deviate. From that point of view, coordination is endogenous and is essential in negotiating dividend and recapitalization policies that align with overall economic objectives. Recognizing central bank capital as the result of a dynamic negotiation game, we highlight the ex ante arrangements contingent on changing economic conditions.

The application of the Stackelberg game provides insights into the strategic interactions between central banks and shareholders in setting policy parameters. Policymakers may consider this perspective when formulating policies, acknowledging that each counterparty's decision influences the other's strategy. This strategic awareness can lead to more informed decision-making in negotiations regarding dividend and recapitalization policies.

The framework that we outline is only an initial attempt to handle the recapitalization arrangement between the central bank and the shareholder from a finance point of view. There is ample room for further research that can build on the current paper. First, we recognize that a dynamic version of the model would be able to capture the temporal aspect of the problem, as central banks have the ability to rebuild buffers and vary the size of monetary assets over time through the ability to generate seigniorage. Furthermore, central bank capital can be put within a macro framework that would allow evaluation of the wider welfare implications of different capital policies by also capturing the relationship between inflation, monetary policy, and interest rates through the business cycle.

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A Annex

A.1 Calibration

To determine the equity value we apply the calibration given in Table 5. The model horizon, denoted as T, represents the option maturity set at one year. The bank's composition is characterized by banknotes (N), reserves (R_t), and total assets (A_t) with values of 20, 75, and 100, respectively. The accounting capital (B_t) is therefore at 5. The percentage share of the risky asset denoted as ω is set at 50%. The standard deviation of the risky asset's return (σ) is specified as 0.2, and the risk-free rate (r) is set at 5%. Additionally, dividend and recapitalization strike multiples for call and put options are set at $\kappa_C = 1.05$ and $\kappa_\Pi = 0.9$, respectively. The shareholder participation threshold (θ) and the central bank credibility threshold (η) are established at 2.5 and -15, providing a comprehensive overview of the model's parameterization.

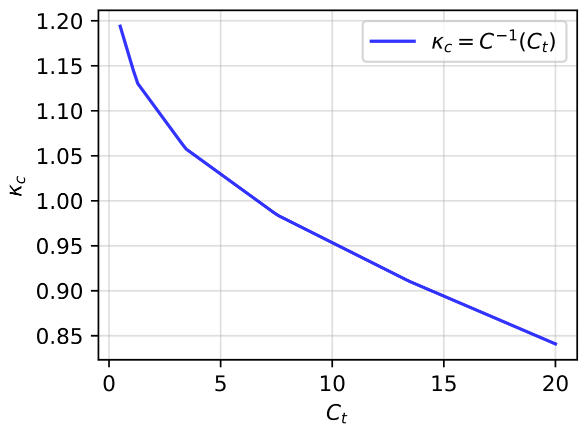
Variable	Notation	Value
Model horizon (option maturity in years)	T	1
Banknotes	N	25
Reserves	R_t	75
Total assets	A_t	103
Accounting capital	B_t	3
Share invested in the risky asset	ω	50%
Standard deviation of the risky asset's return	σ	20%
Risk-free rate	r	5%
Dividend (call) strike multiple	κ_C	1.05
Recapitalization (put) strike multiple	κ_Π	0.95
Shareholder participation threshold	θ	2.5%
Central bank credibility threshold	η	−15%

Table 5: Baseline Model Calibration

A.2 Inverse Option Price

Going forward, we need to solve for the strike multiples κ_Π and κ_C that are optimal with respect to the payoffs of each counterparty given its constraints in terms of the value of the call and the put option. For this purpose, it is useful to define the inverse function $C_t^{-1}(x)$ for any $x > 0$. It will provide the strike multiples consistent with a Black and Scholes option call price of x, holding the other parameters in the model fixed. Since the Black and Scholes call and put prices are monotonously decreasing, and increasing, respectively, with respect to the strike multiples, this allows us to define the inverse. We infer the functional form numerically through a root-finding procedure. Figure 11 plots how the inverse option values are derived for certain choices of the option multiples κ_C and κ_Π.

Figure 11: The Inverses Option Value Functions



Note. This figure shows the inverse call and put values as function of the strike multiples κ_Π and κ_C using the calibration from Table 5

We wrote the condition under which the central bank operates as

$$B_t + \Pi_t - C_t \geq \eta L_t \tag{26}$$

We need to solve for the strike values. As a result, we need to rewrite the condition in terms of the dividend strike multiple as

$$C_t \leq B_t + \Pi_t - \eta L_t$$

$$\kappa_C \geq C_t^{-1} \left(B_t + \Pi_t - \eta L_t\right)$$

where the last line follows from the fact that the value of a call option is monotonously decreasing in the strike.

We can find the line at which the above inequalities are binding numerically by examining the inverse Black and Scholes prices.

Similarly, we can rewrite the shareholder's participation constraint in Equation (21) in terms of the size of the dividend strike multiple

$$\kappa_C \leq C_t^{-1}(\Pi_t + \theta L_t)$$

where we make use of the fact that Π_t is a monotonously increasing function, while C_t is a monotonously decreasing function with respect to its strike. Again, using the numerical procedure outlined earlier, we can find the curve along which the condition is binding.

A.3 Call and Put Derivatives with respect to the Strikes

From the density of the standard normal distribution,

$$N'(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right)$$

we can easily show that $d_{1,i}$ and $d_{2,i}$ from the Black-Scholes formulas in Equation (12) and Equation (13) satisfy:

$$\begin{aligned} \ln N'(d_{1,i}) - \ln N'(d_{2,i}) &= -\ln \frac{A_t}{\kappa_i L_t^*} \\ \implies N'(d_{1,i})A_t &= \kappa_i L_t^* N'(d_{2,i}) \end{aligned}$$

Using this property and the fact that $\frac{\partial d_{1,i}}{\partial \kappa_i} = \frac{\partial d_{2,i}}{\partial \kappa_i}$ we can find the derivative of the call option price in Equation (12) with respect to the strike multiple as follows:

$$\begin{aligned} \frac{\partial C_t}{\partial \kappa_C} &= -N'(d_{1,C})A_t \frac{\partial d_{1,C}}{\partial \kappa_C} - N'(d_{2,C})\kappa_C L_t^* \frac{\partial d_{2,C}}{\partial \kappa_C} - N(d_{2,C})L_t^* \\ &= \underbrace{\left(N'(d_{1,C})A_t - N'(d_{2,C})\kappa_C L_t^*\right)}_{=0} \frac{\partial d_{1,C}}{\partial \kappa_C} - N(d_{2,C})L_t^* \\ &= -N(d_{2,C})L_t^* < 0 \end{aligned} \tag{27}$$

Similarly we can derive the derivative of the put option price in Equation (13) with respect to the strike multiple as follows:

$$
\begin{aligned}
\frac{\partial \Pi_t}{\partial \kappa_\Pi} &= -\left(N'(-d_{1,\Pi}) A_t \frac{\partial d_{1,\Pi}}{\partial \kappa_\Pi} - N'(-d_{2,\Pi}) L_t^* \frac{\partial d_{2,\Pi}}{\partial \kappa_\Pi} \right) \\
&= \left(N'(d_{1,\Pi}) A_t - N'(d_{2,\Pi}) L_t^* \right) \frac{\partial d_{1,\Pi}}{\partial \kappa_\Pi} \\
&= \underbrace{(\kappa_\Pi - 1)}_{<0} \underbrace{N'(d_{2,\Pi}) L_t^*}_{>0} \underbrace{\frac{\partial d_{1,\Pi}}{\partial \kappa_\Pi}}_{<0} > 0
\end{aligned}
\tag{28}
$$

To see the sign of the put option derivative, note that

$$
\frac{\partial d_{1,\Pi}}{\partial \kappa_\Pi} = -\left(\frac{1}{\kappa_\Pi \omega \sigma \sqrt{T-t}} \right) < 0
$$

and that by assumption $\kappa_\Pi < 1$.

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