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\* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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# Firm Entry, Endogenous Markups and the Dynamics of the Labor Share of Income<sup>1</sup>

Andrea Colciago<sup>a</sup> and Lorenza Rossi<sup>b</sup>

a: Dutch National Bank and University of Milano Bicoccab: University of Pavia, Department of Economics

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#### Abstract

Recent U.S. evidence suggests that the response of labor share to a productivity shock is characterized by *countercyclicality* and *overshooting*. These findings cannot be easily reconciled with existing business cycle models. We extend the Diamond-Mortensen-Pissarides model of search in the labor market by considering strategic interactions among an endogenous number of producers, which leads to countercyclical price markups. While Nash bargaining delivers a countercyclical labor share, we show that countercyclical markups are fundamental to address the overshooting. On the contrary, we find that real wage rigidity does not seem to play a crucial role for the dynamics of the labor share of income.

JEL classification: E24, E32, L11.

Keywords: Labor Share Overshooting, Endogenous Market Structures, Search and Matching Frictions.

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#### 1 Introduction

Figure 1, shows the dynamics of the labor share, the average product of labor and the real wage to a one standard deviation orthogonalized productivity innovation for the U.S. in the period 1954.I–2004.IV, as reported by Rios-Rull and Santaeulàlia-Llopis (2010). The response of the labor share is characterized by *countercyclicality* and *overshooting*. In response to the shock the labor share falls on impact, and then shows an hump-shaped response, overshooting its long-run level after five quarters, and peaking at the fifth year at a level larger in absolute terms than the initial drop. Seven years after the peak the labor share is still half-way toward its steady state value.

To account for the response of the labor share to a technology shock displayed in the figure a model should satisfy two desiderata. The first one is that the impact increase in the real wage must be lower than that of average labor productivity. The second one is the presence of a persistent wedge between average labor productivity and real wages, such that the response of the latter is smoother and more inertial with respect to that of the former. The first property implies a countercyclical labor share, while the second one is necessary for overshooting.

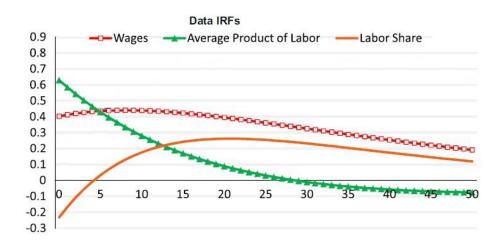


Figure 1: Empirical IRFs of wages, average product of labor, and labor share to productivity innovations in the U.S. Percentage deviations from long run averages. Source: Rios-Rull and Santaeulàlia-Llopis (2010).

In this paper we build on Colciago and Rossi (2011) to develop a theory of the joint dynamics of the labor share and technology shocks which satisfies both desiderata and replicates the countercyclicality and the overshooting of the labor share.

As argued by Rios-Rull and Santaeulàlia-Llopis (2010), standard

business cycle models cannot explain these empirical regularities. The RBC model implies that the real wage and labor productivity move identically, so that the labor share of income displays no cyclical dynamics. The conventional Diamond-Mortensen-Pissarides model (DMP model, henceforth) of search in the labor market with Nash bargaining explains the countercyclicality of the labor share in response to a productivity shock, but cannot address the overshooting.<sup>2</sup> While the overshooting of the labor share is still unexplained, targeting the dynamics of the labor share in DSGE estimated models can help the identification of relevant parameters.

We outline a DMP model with Nash Bargaining and Endogenous Market Structures. Market structures are said to be endogenous since both the number of producers and price markups are determined in each period. The model features firms' entry à la Bilbiie, Ghironi and Melitz (2012) (BGM 2012, henceforth) and oligopolistic competition between producers as in Jaimovich and Floetotto (2008) and Colciago and Etro (2010). Nash bargaining allows to replicate the countercyclicality of the labor share, while the key ingredient to replicate the overshooting result is the countercyclicality of price markups originating from strategic interactions between an endogenous number of producers. To build intuition, consider the effect of a technology shock. The latter creates profits opportunities which attract firms into the market. This strengthens competition and, via strategic interactions, reduces persistently the price markup. A persistently lower markup acts as a shifter of the standard marginal product of labor and creates a wedge between average labor productivity and the real wage. Specifically, a lower markup pushes the real wage schedule above the average productivity of labor for several periods. Besides being consistent with the dynamics displayed in Figure 1, this leads to the overshooting of the labor share.

Aggregate real wages are characterized by an high degree of persistence. Hall (2005), inter alia, points out that real wage rigidity is a feature needed to account for a number of labor market facts. For this reason we study the effect of real wage rigidity on the dynamics of the labor share. Introducing real wage rigidity in the DMP framework with constant markups is not sufficient to match the empirical evidence on the dynamics of the labor share in response to a technology shock. We find that augmenting our framework with (a limited degree of) real

 $<sup>^{2}</sup>$ Chois and Rios-Rull (2008), consider alternative search and matching models with Nash bargaining and show that none of these models can replicate the labor share overshooting. Further, Rios - Rull and Santaeulàlia-Llopis (2010), notice that the departure from a Cobb-Douglas technology is a necessary but not sufficient condition to get the labor share overshooting.

wage rigidity does not alter the previous findings, and allows a better matching of the amplitude of the labor share overshooting observed in the data.

To the best of our knowledge we are the first to present a model addressing the overshooting of the labor share through countercyclical markups. Hornstein (1993) augments the neoclassical growth model with increasing return to scale, a fixed number of firms and constant markups. He finds a labor share that is half as volatile as what is observed in the data, but does not address the overshooting. Also, the role of real wage rigidities for the dynamics of the labor share had not been explored yet.

Choi and Rios-Rull (2010) obtain the overshooting considering a model with putty-clay technology, decentralized non-competitive wage setting (bilateral Nash bargaining) and an aggregate technological shock that has a stronger effect for newer hires. The technology process that we adopt is, instead, fully standard. Shao and Silos (2011) also consider an economy with costly entry of firms and a frictional labor market. However, their model is characterized by monopolistic competition between small firms and by constant price markups. In their framework the overshooting is due to the countercyclical value of vacancies. Nevertheless, this condition is difficult to test empirically. On the contrary, our transmission mechanism is well supported by the empirical evidence. Bils (1987), Rotemberg and Woodford (2000) and Galì et al. (2007) forcefully document price markup countercyclicality.

The remainder of the paper is organized as follows. Section 2 provides a decomposition of the labor share of income. Section 3 outlines the model economy. Section 4 is devoted to calibration. Section 5 contains the main results. Section 6 concludes. Technical details are left in the Appendix.

#### 2 The labor share and its components

Independently of the specification of the model considered, the labor share is defined as  $ls_t = \frac{w_t H_t}{Y_t} = w_t \mathcal{A}_t$ , where  $H_t$  are total hours worked and  $\mathcal{A}_t = \frac{Y_t}{H_t}$  is the average productivity of labor. In log-deviations

$$\widehat{ls}_t = \widehat{w}_t - \left(\widehat{y}_t - \widehat{H}_t\right) = \widehat{w}_t - \widehat{\mathcal{A}}_t,\tag{1}$$

where a hat over a variable denotes the log-deviation from the steady state. Equation (1) simply states that the log-deviation of the labor share is the difference between the log-deviation of the real wage and that of the average labor productivity. In the standard RBC model the real wage equals the marginal product of labor. In log-deviation this amounts to

$$\hat{w}_t = \hat{y}_t - \hat{H}_t = \widehat{\mathcal{A}}_t \tag{2}$$

As a result the labor share is constant and does not deviate from its steady state, that is  $\hat{ls}_t = 0$ . Equations (1) and (2) suggest that in order to obtain a non constant labor share the allocative role of the real wage has to be broken.

In the search and matching framework this is obtained through Nash bargaining. The latter implies that workers and firms split the total surplus originating from a match. The equilibrium real wage maximizes the joint surplus of the parties and depends on their relative bargaining power. Thus, in the aftermath of a productivity increase just a fraction of the latter is distributed to workers. Differently from the standard RBC model, this implies that the real wage rises by less than the increment in labor productivity. Hence, Nash Bargaining helps explaining the countercyclicality of the labor share.

However, in the reminder we show that in the standard DMP framework with Nash bargaining the real wage remains always below average labor productivity along the cycle. This goes against the evidence reported in Figure 1 and, importantly, prevents the standard DMP model from addressing the overshooting of the labor share.

In order to reproduce the overshooting, the real wage should display a smoother and more inertial dynamics than labor productivity. The countercyclical and inertial dynamics of price markup which characterizes our approach delivers this mechanism.

#### 3 The model

#### **3.1** Labor and Goods Markets

There are two main building blocks in the model: oligopolistic competition with endogenous entry in the goods market and search and matching frictions in the labor market. In this paragraph we outlay their main features.

As in Colciago and Etro (2010) the economy features a continuum of sectors, or industries, on the unit interval. Sectors are indexed with  $j \in (0, 1)$ . Each sector j is characterized by different firms  $i = 1, 2, ..., N_{jt}$ producing the same good in different varieties. At the beginning of each period  $N_{jt}^e$  new firms enter into sector j, while at the end of the period a fraction  $\delta \in (0, 1)$  of market participants exits from the market for exogenous reasons.

The labor market is characterized by search and matching frictions, as in Andolfatto (1996) and Mertz (1995). A fraction  $u_t$  of the unit mass population is unemployed at time t and searches for a job. Firms producing at time t need to post vacancies in order to hire new workers. Unemployed workers and vacancies combine according to a CRS matching function and deliver  $m_t$  new hires, or matches, in each period. The matching function reads as  $m_t = \gamma_m (v_t^{tot})^{1-\gamma} u_t^{\gamma}$ , where  $\gamma_m$  reflects the efficiency of the matching process,  $v_t^{tot}$  is the total number of vacancies created at time t and  $u_t$  is the unemployment rate. The probability that a firm fills a vacancy is given by  $q_t = \frac{m_t}{v_t^{tot}}$ , while the probability to find a job for an unemployed worker reads as  $z_t = \frac{m_t}{u_t}$ . Firms and individuals take both probabilities as given. Matches become productive in the same period in which they are formed. Each firm separates exogenously from a fraction  $1 - \rho$  of existing workers each period, where  $\rho$  is the probability that a worker stays with a firm until the next period.

As a result a worker may separate from a job for two reasons: either because the firm where the job is located exits from the market or because the match is destroyed. Since these sources of separation are independent, the evolution of aggregate employment,  $L_t$ , is given by  $L_t = (1 - \delta) \rho L_{t-1} + m_t$ . Thus, the number of unemployed workers searching for a job at time t is  $u_t = 1 - L_{t-1}$ .

#### **3.2** Households and Firms

Using the family construct of Mertz (1995) we can refer to a representative household consisting of a continuum of individuals of mass one. Members of the household insure each other against the risk of being unemployed. The representative family has lifetime utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \ln C_{jt} dj - \chi L_t \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right\} \quad \chi, \varphi \ge 0$$
 (3)

where  $\beta \in (0, 1)$  is the discount factor and the variable  $h_t$  represents individual hours worked. Note that  $C_{jt}$  is a consumption index for a set of goods produced in sectors  $j \in [0, 1]$ , defined as

$$C_{jt} = N_{jt}^{\frac{1}{1-\varepsilon}} \left[ \sum_{i=1}^{N_{jt}} C_{jt}(i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$
(4)

where  $C_{jt}(i)$  is the production of firm *i* of this sector, and  $\varepsilon > 1$  is the elasticity of substitution between the goods produced in each sector.<sup>3</sup> The distinction between different sectors and different goods within a sector allows to realistically separate limited substitutability at the aggregated level, and high substitutability at the disaggregated level. The

<sup>&</sup>lt;sup>3</sup>The term  $N_{jt}^{\frac{1}{1-\varepsilon}}$  in (4) implies that there is no variety effect in the model. However, allowing for a variety effect would not change our results.

family receives real labor income  $w_t h_t L_t$  and profits from the ownership of firms. Further, we assume that unemployed individuals receive an unemployment benefit b in real terms, leading to an overall benefit for the household equal to  $b(1 - L_t)$ . This is financed through lump sum taxation by the government. Notice that the household recognizes that employment is determined by the flows of its members into and out of employment according to

$$L_t = (1 - \delta) \varrho L_{t-1} + z_t u_t \tag{5}$$

Households choose how much to save in riskless bonds and in the creation of new firms through the stock market according to standard Euler and asset pricing equations.<sup>4</sup>

Each firm i in sector j produces a good with a linear production function. We abstract from capital accumulation issues and assume that labor is the only input. Output of firm i in sector j is then:

$$y_{jt}(i) = A_t n_{jt}(i) h_{jt}(i)$$
 (6)

where  $A_t$  is the, common to all sectors, total factor productivity at time t,  $n_{jt}(i)$  is firm *i*'s time t workforce and  $h_{jt}(i)$  represent hours per employee. Since each sector can be characterized in the same way, in what follows we will drop the index j and refer to the representative sector.

#### **3.3** Endogenous Market Structures

Following BGM (2012) we assume that new entrants at time t will only start producing at time t+1. Given the exogenous exit probability  $\delta$ , the average number of firms per sector,  $N_t$ , follows the equation of motion:

$$N_{t+1} = (1 - \delta)(N_t + N_t^e)$$
(7)

where  $N_t^e$  is the average number of new entrants at time t. In each period, the same nominal expenditure for each sector  $EXP_t$  is allocated across the available goods according to the direct demand function:

$$y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} \frac{Y_t}{N_t} = \frac{p_t(i)^{-\varepsilon}}{P_t^{1-\varepsilon}} \frac{EXP_t}{N_t} \quad i = 1, 2, ..., N_{jt}$$
(8)

where  $P_t$  is the price index

$$P_{t} = N_{jt}^{\varepsilon - 1} \left[ \sum_{i=1}^{N_{t}} \left( p_{t} \left( i \right) \right)^{1 - \varepsilon} \right]^{\frac{1}{1 - \varepsilon}}$$

$$\tag{9}$$

<sup>&</sup>lt;sup>4</sup>These conditions are in the Appendix.

such that total expenditure,  $EXP_t$ , satisfies  $EXP_t = \sum_{j=1}^{N_t} p_t(j)y_t(j) =$ 

 $P_t Y_t.{}^5\,$  Inverting the direct demand functions, we can derive the system of inverse demand functions

$$p_t(i) = \frac{y_t(i)^{-\frac{1}{\varepsilon}}}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}} EXP_t \qquad i = 1, 2, ..., N_{jt}$$
(10)

which will be useful for the derivation of the Cournot equilibrium. Period t real profits of an incumbent producer are defined as

$$\pi_t(i) = p_t(i) y_t(i) - w_t(i) n_t(i) h_t(i) - \kappa v_t(i)$$
(11)

where  $w_t(i)$  is the real wage paid by firm  $i, v_t(i)$  represents the number of vacancies posted at time t and  $\kappa$  is the output cost of keeping a vacancy open. The value of a firm is the expected discounted value of its future profits

$$V_t(i) = E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} \pi_s(i)$$
(12)

where  $\Lambda_{t,t+1} = (1 - \delta) \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1}$  is the households' stochastic discount factor which takes into account that firms' survival probability is  $1 - \delta$ . Incumbent firms which do not exit from the market have a time t individual workforce given by

$$n_t(i) = \varrho n_{t-1}(i) + v_t(i) q_t \tag{13}$$

Under different forms of competition between firms we obtain prices satisfying:

$$\frac{p_t(i)}{P_t} = \mu(\varepsilon, N_t) mc_t(i)$$
(14)

where  $\mu(\theta, N_t) > 1$  is the markup depending on the degree of substitutability between goods,  $\varepsilon$ , and on the number of firms,  $N_t$ , and  $mc_t(i)$ is the real marginal cost. In the remainder of this section we characterize this mark up under Bertrand and Cournot competition taking strategic interactions into account.

<sup>&</sup>lt;sup>5</sup>The demand of the individual good and the price index are the solution to the, usual, consumption expenditure minimization problem.

#### 3.3.1 Bertrand Competition

Each firm chooses  $p_t(i)$ ,  $n_t(i)$  and  $v_t(i)$  to maximize  $\pi_t(i)+V_t(i)$ , taking as given the price of the other firms in the sector. The problem is subject to two constraints, namely equation (8) and (13).<sup>6</sup> The symmetric Bertrand equilibrium generates an equilibrium markup

$$\mu_t^P(\varepsilon, N_t) = \frac{\varepsilon \left(N_t - 1\right) + 1}{(\varepsilon - 1) \left(N_t - 1\right)}$$
(15)

The markup  $\mu_t^P$  is decreasing in the degree of substitutability between products  $\varepsilon$ , with an elasticity  $\epsilon_{\varepsilon}^P = \varepsilon N_t / (1 - \varepsilon + \varepsilon N_t)(\varepsilon - 1)$ . Moreover, the markup vanishes in case of perfect substitutability:  $\lim_{\varepsilon \to \infty} \mu^P(\theta, N_t) =$ 1. Finally, the markup is decreasing in the number of firms, with an elasticity  $\epsilon_N^P = N / [1 + \varepsilon (N - 1)] (N - 1)$ . Notice that the elasticity of the markup to entry under competition in prices is decreasing in the level of substitutability between goods, and it tends to zero when the goods are approximately homogenous. When  $N_t \to \infty$  the markup tends to  $\varepsilon / (\varepsilon - 1)$ , the traditional one under monopolistic competition. As well known, strategic interactions between a finite number of firms lead to a higher markup than under monopolistic competition.

#### 3.3.2 Cournot Competition

In this case firms maximize  $\pi_t(i) + V_t(i)$  choosing their production  $y_t(i)$  beside  $n_t(i)$  and  $v_t(i)$ , taking as given the production of the other firms. The profit maximization problem is constrained by the inverse demand function (10) and by equation (13). The symmetric Cournot equilibrium generates a equilibrium markup

$$\mu^{Q}(\varepsilon, N_{t}) = \frac{\varepsilon N_{t}}{(\varepsilon - 1) (N_{t} - 1)}.$$
(16)

First of all notice that for a given number of firms, the markup under competition in quantities is always larger than the one obtained under competition in prices.<sup>7</sup> Further, also in this case the markup is decreasing in the degree of substitutability between products  $\varepsilon$ , with an elasticity  $\epsilon_{\varepsilon}^{Q} = 1/(\varepsilon - 1)$ , which is always smaller than  $\epsilon_{\varepsilon}^{P}$ : higher substitutability reduces markups faster under competition in prices. In the Cournot equilibrium, the markup remains positive for any degree of substitutability, since even in the case of homogenous goods, we have  $\lim_{\varepsilon \to \infty} \mu^{Q}(\varepsilon, N_t) = N_t/(N_t - 1)$ . The markup  $\mu^{Q}(\varepsilon, N_t)$  is decreasing

<sup>&</sup>lt;sup>6</sup>Details concerning the firm maximization problem under Bertrand and Cournot competition are in the Appendix.

<sup>&</sup>lt;sup>7</sup>This is well known for models of product differentiation (see for instance Vives, 1999).

and convex in the number of firms with elasticity  $\epsilon_N^Q = 1/(N-1)$ , which is decreasing in  $N_t$  (the markup decreases with entry at an increasing rate) and independent from the degree of substitutability between goods. Since  $\epsilon_N^Q > \epsilon_N^P$  for any number of firms or degree of substitutability, entry decreases markups faster under competition in quantities compared to competition in prices, a result that will impact on the relative behavior of the economy under the two forms of competition. Only when  $N_t \rightarrow \infty$  the markup tends to  $\varepsilon/(\varepsilon - 1)$ , which is the traditional markup under monopolistic competition.

#### **3.4** Entry and Job creation

We assume that entry requires a fixed cost  $\psi$ , which is measured in units of output. Define  $V_t^e$  as the value at time t of a prospective entrant. Given our timing assumption, the latter represents the value of a firm which will start producing at time t+1. In each period the level of entry is determined endogenously to equate the value of a prospective entrant to the entry cost

$$V_t^e = \psi \tag{17}$$

Profits maximization implies the following Job Creation Condition (JCC)

$$\frac{\kappa}{q_t} = \left(\frac{1}{\mu_t^j} - \frac{w_t}{A_t}\right) A_t h_t + \varrho E_t \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}}$$

The JCC equates the real marginal cost of hiring a worker, the left hand side, with the marginal benefit, the right hand side. Importantly, the marginal benefit depends positively on the ratio  $\frac{1}{\mu_t^J}$  (with J equal either to P or to Q), which is a positive function of the number of firms in the market,  $N_t$ . Stronger competition leads to a lower mark up which stimulates demand by consumers and hence has a positive effect on output and ultimately on employment.

As shown by Colciago and Rossi (2011), a positive technology shock leads to entry of new firms and thus to an increase in  $\frac{1}{\mu_t^J}$ . In equilibrium, since hiring depends on the current and expected future values of the marginal product of labor, this boosts hiring and employment with respect to a model with constant markups.

The JCC is common across firms, independently of their period of entry. Thus, the optimal hiring policy of new producers, i.e. firms which at time t are producing for the first time and have no initial workforce, consists in posting as many vacancies as required to reach the size of firms which started production in earlier periods. This has two implications. The first one is that the size-gap between new producers and incumbent firms is closed in a single period. The second one is that new producers grow faster than more mature firms. This is consistent with the U.S. empirical evidence discussed in Haltiwanger et al. (2010), which suggests that a start-up creates on average more new jobs than an incumbent firm. Given vacancy posting is costly, new producers will suffer lower profits and pay lower dividends in their first period of activity with respect to firms which entered into the market in earlier periods. This is consistent with the evidence on the financial behavior of firms discussed by Cooley and Quadrini (2001).

#### 3.5 Bargaining over Wages and Hours

In the Appendix it is shown that Nash wage bargaining results in the following wage equation

$$w_{t} = (1 - \eta) \frac{b}{h_{t}} + \eta \frac{1}{\mu_{t}^{J}} A_{t} + (1 - \eta) \chi C_{t} \frac{h_{t}^{1/\varphi}}{1 + 1/\varphi} + \frac{\eta \kappa}{(1 - \delta)} \frac{1}{h_{t}} E_{t} \Lambda_{t, t+1} \theta_{t+1},$$
(18)

where  $\mu_t^J$  is the markup function,  $\theta_t = \frac{v_t^{tot}}{u_t}$  is the tightness of the job market and the parameter  $\eta$  reflects the relative bargaining power of workers. The wage shares costs and benefits associated to the match. The worker is rewarded for a fraction  $\eta$  of the firm's revenues and savings of hiring costs and compensated for a fraction  $1 - \eta$  of the disutility he suffers from supplying labor and the foregone unemployment benefits. The direct effect of competition on the real wage is captured through the term  $\eta \frac{1}{\mu_t^2} A_t$ , which represents the share of the marginal revenue product (MRP) which goes to workers. As discussed above, entry leads to an increase in the ratio  $\frac{1}{\mu_t^j}$  and hence in the MRP. Thus, everything else equal, stronger competition shifts the wage curve up. This result is similar to that in Blanchard and Giavazzi (2003), who find a positive effect of competition on the real wage. Hours are set to maximize the joint surplus of the match. This is obtained when the marginal rate of substitution between hours and consumption equals the MRP of labor, that is

$$\chi C_t h_t^{1/\varphi} = \frac{1}{\mu_t^J} A_t. \tag{19}$$

Stronger competition leads to an increase in hours bargained between workers and firms for the same reasons for which competition positively affects the wage schedule.

#### 3.6 Aggregation and Market Clearing

Considering that the individual workforce,  $n_t$ , is identical across producers leads to

$$L_t = n_t N_t \tag{20}$$

To obtain aggregate output notice that  $P_t Y_t = \sum_{i=1}^{N_t} p_t y_t = N_t p_t y_t$ , further given  $\frac{p_t}{P_t} = 1$  and the individual production function it follows that

$$Y_t = N_t y_t = A_t L_t h_t = A_t H_t \tag{21}$$

where  $H_t$  is the amount of total hours worked. As a consequence  $A_t$ amounts to average labor productivity, which is assumed to follow a first order autoregressive process given by  $\ln (A_t/A) = \rho_A \ln (A_{t-1}/A) + \varepsilon_{At}$ , where  $\rho_A \in (0, 1)$  and  $\varepsilon_{At}$  is a white noise disturbance, with zero expected value and standard deviation  $\sigma_A$ .

Aggregating the budget constraints of households we obtain the aggregate resource constraint of the economy

$$C_t + \psi N_t^e = W_t h_t L_t + \Pi_t \tag{22}$$

which states that the sum of consumption and investment in new entrants must equal the sum between labor income and aggregate profits,  $\Pi_t$ , distributed to households at time t. Goods' market clearing requires

$$Y_t = C_t + N_t^E \psi + \kappa v_t^{tot} \tag{23}$$

where  $v_t^{tot}$  is the sum of vacancies posted by new entrants and by firms which entered in earlier periods. Finally, the dynamics of aggregate employment reads as

$$L_t = (1 - \delta) \varrho L_{t-1} + q_t v_t^{tot}$$

$$\tag{24}$$

which shows that workers employed to a firm which exits the market join the mass of unemployed.

#### 4 Calibration

To solve the model described in the previous section the equations are linearized around the model's steady state.<sup>8</sup> Calibration is as follows. The discount factor,  $\beta$ , is set to 0.99. As in BGM (2012) the rate of business destruction,  $\delta$ , equals 0.025. This means roughly 10 percent of firms disappear from the market every year, independently of firm age. The entry cost is  $\psi = 1$  and held constant along the cycle. With no loss of generality, the value of  $\chi$  is such that steady state labor supply equals one. The Frisch elasticity of labor supply is  $\varphi = 1$ . The intersectoral elasticity of substitution is  $\varepsilon = 6$ , as estimated by Christiano, Eichenbaum and Evans (2005). As standard in the literature we set the

<sup>&</sup>lt;sup>8</sup>The resulting linearized system is solved using DYNARE.

steady state marginal productivity of labor, A, to 1. As Rios-Rull and Santaeulàlia-Llopis (2010) we consider a one standard deviation technology innovation, that corresponds to a one percent increase in the labor productivity, and we set  $\rho_a = 0.925$ . We set the separation rate  $\rho$  equal to 0.1, as suggested by estimates provided by Hall (1995) and Davis et al. (1996). The elasticity of matches to unemployment,  $\gamma$ , is set equal to the worker bargaining power  $\eta$  and is equal to  $\frac{1}{2}$ , as in the bulk of the literature. The efficiency parameter in matching,  $\gamma_m$ , and the steady state job market tightness are calibrated to target an average job finding rate, z, equal to 0.7 and a vacancy filling rate, q, equal to 0.9. We draw the latter value from Andolfatto (1996) and Dee Haan et al. (2000), while the former from Blanchard and Galì (2010).<sup>9</sup> Finally, we calibrate the unemployment benefit in real terms, b, such that the monetary replacement rate,  $\frac{b}{wb}$ , equals 0.60. This value is consistent with that reported in the OECD Economic Outlook of 1996 for the US. Given these parameters we can recover the cost of posting a vacancy  $\kappa$  by equating the steady state version of the JCC and the steady state wage setting equation. Notice that none of the qualitative result is affected by the calibration strategy.

#### 5 Productivity Shocks and Dynamics of the Labor share

In what follows we study the impulse response functions of the labor share and its components to a one percent increase in technology. To isolate the role of endogenous markup variability for the dynamics of the labor share we compare the performance of the models with Bertrand and Cournot competition to that of a model characterized by monopolistic competition. Under monopolistic competition firms do not interact strategically and set a constant markup over marginal costs equal to  $\mu = \frac{\varepsilon}{\varepsilon - 1}.$ 

Figure 2 shows that, on impact, the real wage increase less than average labor productivity no matter the form of competition in the goods market. As argued above, Nash bargaining delivers the countercyclicality of the labor share of income. Under monopolistic competition, after peaking on impact, the real wage returns monotonically to its initial level. Further, it remains below labor productivity in all periods so that the labor share does not overshoot.

This is not the case when the goods market is characterized by oligopolistic competition. Under both Bertrand and Cournot, the la-

 $<sup>^{9}\</sup>mathrm{A}$  job finding rate equal to 0.7 corresponds, approximately, to a monthly rate of 0.3, consistent with US evidence.

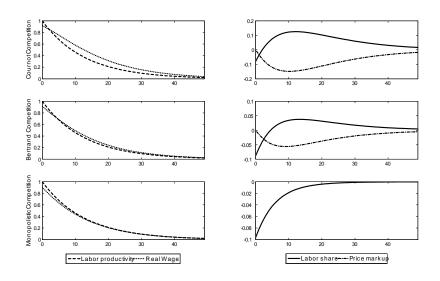


Figure 2: Impulse response functions to a technology shock. Top panel: Cournot competition; middle panel: Bertrand competition; bottom panel: monopolistic competition.

bor share is countercyclical due to Nash Bargaining. However the labor share overshoots its long run level after about five quarters, it peaks at about the fifth year at a level larger than its long-run value and seven years after the shock has hit the economy is still halfway toward its average. The key lies in the countercyclical and inertial response of the price markup. To see this, consider the log-deviations of the real wage and labor hours from their steady state. These are respectively

$$\hat{w}_t = \Upsilon_1 \left( \hat{A}_t - \hat{\mu}_t \right) - \Upsilon_2 \hat{h}_t + \Upsilon_3 E_t \hat{\Theta}_{t+1}$$
(25)

and

$$\hat{h}_t = \varphi \left( \hat{A}_t - \hat{\mu}_t - \hat{c}_t \right), \qquad (26)$$

where  $\Upsilon_1 = \frac{1}{\mu w} \left( \frac{\eta + \varphi}{1 + \varphi} \right)$ ,  $\Upsilon_2 = 1 - \Upsilon_1$ ,  $\Upsilon_3 = \frac{\eta \kappa \theta}{w}$  and  $\hat{\Theta}_{t+1} = \hat{\Lambda}_{t,t+1} + \hat{\theta}_{t+1}$ . Under all plausible parametrization, we find that  $\Upsilon_1$  is lower than one. As a result, only a fraction  $\Upsilon_1 < 1$  of the impact increase in productivity  $\hat{A}_t$  goes to workers. Further, equation (26) shows that labor hours increase with productivity and contribute to dampen the positive effect of productivity on real wages. Hence, the impact increase in real wages is lower than that of labor productivity and the labor share is countercyclical. In a model with endogenous market structures these are just partial effects. Technology shocks create expectations of future profits which lead to the entry of new firms. Stronger competition leads

to lower price markups. Given that entry is subject to a one period timeto-build lag, the total number of firms,  $N_t$ , does not change on impact, but builds up gradually. As shown in Figure 2, in the Cournot and in the Bertrand model this translates into an initially muted response of the markup. As entry increases the number of firms, however, the price markup starts declining. In particular it finds its negative peak after few periods and then gradually reverts to its long run value.<sup>10</sup> Equation (25) shows that a persistently lower markup acts as a shifter of the standard marginal product of labor schedule pushing the real wage above the average productivity of labor for several periods. Since  $\hat{ls}_t = \hat{w}_t - \hat{A}_t$ , this explains the overshooting of the labor share. Thus, we can state that the dynamic response of the markup to technology shocks is fundamental for the overshooting.<sup>11</sup>

In the Cournot model the initial drop of the labor share as well as the timing and amplitude of the overshooting are very close to their data counterpart (see Figure 1). In the Bertrand model, the magnitude of the overshooting is lower than in the data. The reason is the stronger markup variation under Cournot, which is reflected in a larger wedge between the real wage and average labor productivity.

#### 5.1 The role of real wage rigidity

Aggregate wages are characterized by an high degree of persistence, so that sudden and large shifts in the aggregate wage level are not observed. The existence of real wage rigidities has been pointed to by many authors as a feature needed to account for a number of labor market facts (see, e.g., Hall 2005).

Real wage rigidity leads to a slow adjustment of wages to labor market conditions. In particular, in response to a productivity shock it leads to a smoother and more inertial dynamics of the real wage than the average labor productivity. As emphasized above, this is the key feature a model should satisfy to address the overshooting of the labor share in response to a technology shock. For this reason we study the effect of real wage rigidity on the dynamics of the labor share. Following Hall (2005), we model real wage rigidity in the form of a backward looking

<sup>&</sup>lt;sup>10</sup>Notice that the shape of the response of the price markup to a technology shock is consistent with the evidence in Rotember and Woodford (1999) and the VAR evidence in Colciago and Etro (2010).

<sup>&</sup>lt;sup>11</sup>We consider alternative values of  $\eta$  and  $\varphi$  and we find that they do not alter qualitatively the overshooting result. This holds also in the case with fixed individual hours, that is with  $\varphi = 0$ .

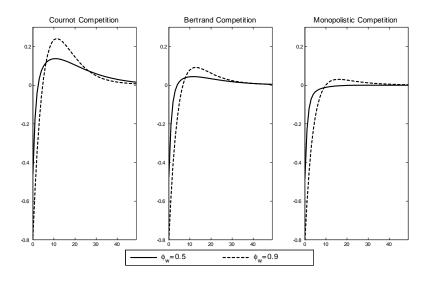


Figure 3: Labor share response to a technology shock under alternative degrees of real wage rigidity. Left panel: Cournot competition; middle panel: Bertrand competition; right panel: monopolistic competition.

social norm:<sup>12</sup>

$$w_t = \phi_w w_{t-1} + (1 - \phi_w) w_t^{nash}$$
(27)

where  $\phi_w$  is an index reflecting the degree of real wage rigidity and  $w_t^{nash}$  is the wage obtained under Nash Bargaining, i.e. that in equation (18). Notice that  $\phi_w = 1$  implies a fixed real wage, while  $\phi_w = 0$  corresponds to the case of Nash bargaining analyzed earlier. As observed by Blanchard and Galì (2007), equation (27), even though admittedly ad-hoc, is a parsimonious way of introducing a slow adjustment of real wages to labor market conditions.<sup>13</sup>

Figure 3 displays the response of the labor share to a one percent increase in technology in the Bertrand and the Cournot models as well as in the model with monopolistic competition. Since there is no evidence on the degree of real wage rigidities, we consider two alternative values of the parameter  $\phi_w$ . Dashed lines refer to the case  $\phi_w = 0.5$ , the midpoint of the admissible range. Solid lines depict the extreme case where  $\phi_w = 0.9$ .<sup>14</sup>

<sup>&</sup>lt;sup>12</sup>Blanchard and Galì (2007), Christoffel and Linzert (2010), Ascari and Rossi (2011) and Faia and Rossi (2012) take a similar approach.

<sup>&</sup>lt;sup>13</sup>The authors consider alternative formalizations, explicitly derived from staggering of real wage decisions. Although the algebra is more involved, the basic conclusions are the same as those obtained with the ad-hoc formulation.

 $<sup>^{14}\</sup>mathrm{A}$  value of  $\phi_w=0.9$  implies a real wage adjustment of about 6 quarters.

In the model with constant price markups the labor share overshoots its long run level just in the case of extreme real wage rigidity. Nevertheless the overshooting is negligible. This confirms that countercyclical price markups are key for the overshooting of the labor share.

Augmenting the Cournot and Bertrand competitive frameworks with a limited degree of real wage rigidity, does not alter the previous findings substantially, nevertheless it improves the matching of the amplitude of the overshooting from a quantitative point of view. Our view is that real wage rigidity does not seem to play a crucial role for the dynamics of the labor share of income.

#### 6 Conclusion

Recent U.S. evidence suggests that the response of labor share to a productivity shock is characterized by *countercyclicality* and *overshoot-ing*. To account for these empirical findings a model should satisfy two desiderata. The first one is that the impact increase in the real wage must be lower than that of average labor productivity. The second one is the presence of a persistent wedge between average labor productivity and real wages such that the response of the latter is smoother and more inertial with respect to that of the former.

We propose a DMP model characterized by firms' entry and oligopolistic competition between producers that addresses this evidence. Nash bargaining delivers the countercyclicality of the labor share. The countercyclicality of price markup originating from strategic interactions in the goods market acts as a shifter of the standard marginal product of labor and allows the labor share of income to overshoot.

While real wage rigidity helps accounting for a number of labor market facts, such as the variability of unemployment in response to a technology shock and the slow response of real wages to labor market conditions, it does not seem to play a crucial role for the dynamics of the labor share of income.

#### Appendix

Let us provide some terminology before starting the analysis. The term new entrants refers to the firms which enter the market at time t. The value of these firms is denoted by  $V_t^e$ . The term new producers refers to firms which entered the market in t-1 and at time t produce for the first time (these firms are a fraction  $(1 - \delta)$  of time t-1 new entrants). The term incumbent firms refer to firms which entered the market in period t-2 or earlier. Notice that new producers and incumbent firms have the same value, which we denote with  $V_t$ . This is so since new producers close their size gap with incumbent firms in their first period of activity. For this reason after their first period of activity new producers are indistinguishable from firms that entered in t-2 or earlier.

#### Households

We assume that households invest in both incumbent firms and new entrants. Bonds and stocks are denominated in terms of the final good. The budget constraint expressed in nominal terms is

$$P_{t}B_{t+1} + P_{t}C_{t} + P_{t} \int_{0}^{1} V_{jt}N_{jt}s_{jt+1}dj + P_{t} \int_{0}^{1} V_{jt}^{e}N_{jt}^{e}s_{jt+1}^{e}dj$$
  
=  $W_{t}L_{t}h_{t} + (1 - L_{t}) P_{t}b + (1 + r_{t})P_{t}B_{t} + (1 - \delta) P_{t} \int_{0}^{1} \left[\pi_{jt}(\varepsilon, N_{jt}) + V_{jt}\right] N_{jt-1}s_{jt}dj +$   
+  $(1 - \delta) P_{t} \int_{0}^{1} \left[\pi_{jt}^{new}(\varepsilon, N_{jt}) + V_{jt}\right] N_{jt-1}^{e}s_{jt}^{e}dj - P_{t}T_{t}$  (28)

where  $B_t$  is net bond holdings with interest rate  $r_t$ ,  $V_{jt}$  is the value of an incumbent firm in sector j and  $V_{jt}^e$  is the value of a new entrant in the same sector. The variables  $N_{jt}$  and  $N_{it}^e$  represent the number of active firms in sector j and the new entrants in this sector at the end of the period, respectively. The variable  $s_{it}$  represents the share of the portfolio of incumbent firms belonging to sector j that is owned by the household, while  $s_{jt}^{e}$  is the share of portfolio of new entrants held by the household. The term  $(1-\delta) P_t \int_0^1 [\pi_{jt}(\varepsilon, N_{jt}) + V_{jt}] N_{jt-1} s_{jt}$  represents the sum between the value of the portfolio of firms which entered the market in period t-2 or earlier held by the household and the profits distributed by these firms. Notice the number of these firms is equal to  $(1 - \delta) N_{jt-1}$  in each sector. The term  $(1-\delta) P_t \int_0^1 \left[ \pi_{jt}^{new}(\varepsilon, N_{jt}) + V_{jt} \right] N_{jt-1}^e s_{jt}^e$  denotes the sum be-tween the value of the portfolio of new producers, where  $(1-\delta) N_{jt-1}^e$  is the number of firms which produce for the first time at time t. In the budget constraint we have imposed the symmetry in the value of new firms and incumbent firms. Finally  $P_t T_t$  represent nominal lump sum taxes imposed to finance unemployment benefits. The household recognizes that employment is determined by the flows of its members into and out of employment according to

$$L_t = (1 - \delta) \varrho L_{t-1} + z_t u_t \tag{29}$$

Equations (28) and (29) represent the constraint to the utility maximization problem. We denote with  $\xi_t$  the Lagrangian multiplier of the first constraint, while  $\Gamma_t$  is the one of the second constraint.

The intertemporal optimality conditions with respect to  $s_{jt+1}$ ,  $s_{jt+1}^e$  for each sector, and with respect to  $B_{t+1}$  are, respectively

$$P_t V_{jt} = \beta E_t (1 - \delta) \frac{\xi_{t+1}}{\xi_t} P_{t+1} [\pi_{jt+1}(\varepsilon, N_{jt+1}) + V_{jt+1}]$$
(30)

$$P_t V_{jt}^e = \beta E_t \left( 1 - \delta \right) \frac{\xi_{t+1}}{\xi_t} P_{t+1} \left[ \pi_{jt+1}^{new}(\varepsilon, N_{jt+1}) + V_{jt+1} \right]$$
(31)

$$P_t \xi_t = \beta E_t (1 + r_{t+1}) P_{t+1} \xi_{t+1}$$
(32)

The optimal choice of consumption requires

$$\frac{1}{P_t C_t} = \xi_t \tag{33}$$

Notice that  $\Gamma_t$  has the meaning of the marginal value to the household of having a member employed rather than unemployed. The latter affects bargaining over the real wage and individual hours and it is given by

$$\Gamma_t = \frac{1}{C_t} \left( w_t h_t - b \right) - \chi \frac{h_t^{1+1/\varphi}}{1 + 1/\varphi} + \beta E_t \left[ (1 - \delta) \rho - z_{t+1} \right] \Gamma_{t+1}$$
(34)

where  $w_t = \frac{W_t}{P_t}$  is the real wage.

#### **Profit Maximization Problem**

Consider Bertrand competition. We initially consider the problem of an incumbent firm. Substituting the direct demand for the individual good into period t real profits, we obtain

$$\pi_{t} = \frac{p_{t}(i)^{1-\varepsilon}}{\left[\sum_{i=1}^{N_{t}} p_{t}(i)^{-(\varepsilon-1)}\right]} \frac{EXP_{t}}{P_{t}} - w_{t}(i) n_{t}(i) h_{t}(i) - \kappa v_{t}(i) \qquad (35)$$

The profit maximization problem of an incumbent firm reads as

$$\max_{\{p_t(i), n_t(i), v_t(i)\}_t^\infty} \pi_t + E_t \sum_{s=t+1}^\infty \Lambda_{t,s} \pi_s$$
(36)

subject to

$$A_t n_t(i) h_t(i) = \frac{p_t(i)^{-\varepsilon} E X P_t}{\left[\sum_{i=1}^{N_t} p_t(i)^{(1-\varepsilon)}\right]}$$
(37)

and

$$n_{t}(i) = \rho n_{t-1}(i) + v_{t}(i) q_{t}$$
(38)

Lagrangian multipliers on constraints (37), and (38) are respectively  $mc_t(i)$  and  $\phi_t(i)$ . Setting up the Lagrangian  $\mathbb{L}$ , the FOCs with respect to  $n_t(i)$ ,  $v_t(i)$  and  $p_t(i)$  are, respectively

$$\frac{\partial \mathbb{L}}{\partial n_t(i)} = 0: w_t(i) h_t(i) + \phi_t(i) - mc_t(i) A_t h_t(i) = \varrho E_t \Lambda_{t,t+1} \phi_{t+1}(i)$$
(39)

$$\frac{\partial \mathbb{L}}{\partial v_t(i)} = 0 : \kappa = \phi_t(i) q_t \tag{40}$$

and

$$\frac{\partial \mathbb{L}}{\partial p_{t}(i)} = 0: \frac{(1-\varepsilon)\left[\sum_{i=1}^{N_{t}} p_{t}(i)^{(1-\varepsilon)}\right] - (1-\varepsilon)p_{t}(i)^{1-\varepsilon}}{\left[\sum_{i=1}^{N_{t}} p_{t}(i)^{1-\varepsilon}\right]^{2}} p_{t}(i)^{-\varepsilon}\frac{EXP_{t}}{P_{t}} + \frac{\left[\sum_{i=1}^{N_{t}} p_{t}(i)^{1-\varepsilon}\right]^{2}}{mc_{t}(i)} \frac{\varepsilon p_{t}(i)^{-1}\left[\sum_{i=1}^{N_{t}} p_{t}(i)^{(1-\varepsilon)}\right] + (1-\varepsilon)p_{t}(i)^{-\varepsilon}}{\left[\sum_{i=1}^{N_{t}} p_{t}(i)^{1-\varepsilon}\right]^{2}} p_{t}(i)^{-\varepsilon}EXP_{t}}$$
$$= 0$$
(41)

Notice that we assume that firms take individual wages as given when choosing employment. Also notice that since there is a continuum of sectors, the individual firm takes the aggregate price level as given. The second condition shows that  $\phi_t(i)$ , the surplus created by a match, is identical across incumbent firms. Before providing an explicit formula for the individual price level and the price markup, we turn to the profit maximization problem of a first period producer which sets the price for the first time. The relevant difference with respect to the previous case is represented by the form of constraint (38) which reads as  $v_t(i) q_t = n_t(i)$ , since producers in their first period of activity have no initial workforce. However, FOCs with respect to  $p_t(i), n_t(i)$  and  $v_t(i)$  are identical to those reported above. Since the surplus  $\phi_t$  created by a match is identical across all producers , they will face the same wage bargaining problem, thus will face the same wage,  $w_t(i) = w_t$ , the same marginal cost,  $mc_t(i) = mc_t$ , and will demand the same amount of hours,  $h_t(i) = h_t$ . As a result the third condition can be written as

$$(1-\varepsilon) N_t P_t^{1-\varepsilon} - (1-\varepsilon) p_t(i)^{1-\varepsilon} = M C_t \left[ (\varepsilon - 1) p_t(i)^{-\varepsilon} - \varepsilon p_t(i)^{-1} N_t P_t^{1-\varepsilon} \right]$$

$$(42)$$

where  $MC_t (= P_t m c_t)$  is the nominal marginal cost, which shows that  $p_t (i)$  does not depend on any firm specific variable. In other words all firms which

are active at time t, no matter the period of entry, choose the same price. Since firms face the same demand function and adopt the same technology, it follows that  $y_t(i) = y_t$  and  $n_t(i) = n_t$ . We are now ready to provide an expression for the common price chosen by firms. Given that firms choose the same price level, it follows that  $p(i) = p_t = P_t$ . Imposing symmetry and rearranging, condition (14) can be rewritten as

$$p_t = \mu_t M C_t \tag{43}$$

where

$$\mu_t = \frac{\varepsilon \left(N_t - 1\right) + 1}{\left(\varepsilon - 1\right) \left(N_t - 1\right)} \tag{44}$$

Further, notice that, after imposing symmetry, by combining equation (39) and (40) we get the JCC reported in the main text. Under Cournot competition profit maximization must take the inverse demand function as a constraint. The latter is

$$p_t(i) = \frac{y_t(i)^{-\frac{1}{\varepsilon}}}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}} EXP_t$$

which implies that period profits can be written as

$$\pi_{t} = \frac{y_{t}(i)^{1-\frac{1}{\varepsilon}}}{\sum_{j=1}^{N_{t}} y_{t}(j)^{\frac{\varepsilon-1}{\varepsilon}}} \frac{EXP_{t}}{P_{t}} - w_{t}(i) n_{t}(i) h_{t}(i) - kv_{t}(i)$$

Setting up a Lagrangian function as in the proof of Proposition 1 and differencing with respect to  $y_t(i)$ ,  $n_t(i)$ ,  $v_t(i)$ , it can be easily verified that the FOCs with respect to  $n_t(i)$ ,  $v_t(i)$  are unchanged with respect to the Bertrand case.

#### Wage setting

The real wage and hours worked are set to maximize the product

$$\left(\phi_t\right)^{1-\eta} \left(\Gamma_t C_t\right)^\eta \tag{45}$$

where the term in the first bracket,  $\phi_t$ , is the value to the firm of having an additional worker, i.e.,

$$\phi_t = \frac{\rho_t}{\mu_t} A_t h_t - w_t h_t + \varrho E_t \Lambda_{t,t+1} \phi_{t+1}$$
(46)

the second term,  $\Gamma_t$ , is the household's surplus expressed in units of consumption,

$$\Gamma_t = \frac{1}{C_t} w_t h_t - \chi \frac{h_t^{1+1/\varphi}}{1+1/\varphi} - \frac{b}{C_t} + \beta E_t \left[ (1-\delta) \,\rho - z_{t+1} \right] \Gamma_{t+1} \tag{47}$$

The FOC with respect to the wage is

$$(1-\eta)\left(\phi_{t}\right)^{-\eta}\left(\Gamma_{t}C_{t}\right)^{\eta}\frac{d\phi}{d_{w}}+\eta\left(\Gamma_{t}C_{t}\right)^{\eta-1}\left(\phi_{t}\right)^{1-\eta}\frac{d\Gamma_{t}}{d_{w}}C_{t}=0$$
(48)

Notice that  $\frac{d\Gamma_t}{dw_t}C_t = -\frac{d\phi_t}{dw_t} = h_t$ , thus (48) can be simplified as follows

$$\eta \phi_t = (1 - \eta) \, \Gamma_t C_t \tag{49}$$

Multiply both sides of equation (49) by  $\rho\beta(1-\delta)\frac{C_{t-1}}{C_t}$  yields

$$\eta \varrho \beta \left(1-\delta\right) \frac{C_{t-1}}{C_t} \phi_t = \left(1-\eta\right) \varrho \beta \left(1-\delta\right) C_{t-1} \Gamma_t, \tag{50}$$

leading one period and taking expectations as of time t leads to

$$\eta \varrho E_t \Lambda_{t,t+1} \phi_{t+1} = (1-\eta) \, \varrho \beta \, (1-\delta) \, C_t E_t \Gamma_{t+1}, \tag{51}$$

substituting for  $\phi_t$  and  $\Gamma_t C_t$  and simplifying

$$\eta \frac{\rho_t}{\mu_t} A_t h_t = w_t h_t - (1 - \eta) \left( \chi \frac{h_t^{1 + 1/\varphi} C_t}{1 + 1/\varphi} + b + \beta E_t z_{t+1} \Gamma_{t+1} C_t \right).$$
(52)

Multiplying both sides of (49) by  $z_t \frac{C_{t-1}}{C_t}$ , leading one period and taking expectation as of time t, we can rewrite

$$\eta z_{t+1} \frac{C_t}{C_{t+1}} \phi_{t+1} = (1 - \eta) \, z_{t+1} C_t \Gamma_{t+1}, \tag{53}$$

using the latter it follows that

$$(1-\eta)\,\beta C_t E_t z_{t+1} \Gamma_{t+1} = \eta \beta E_t \frac{C_t}{C_{t+1}} z_{t+1} \phi_{t+1} = \frac{\eta}{(1-\delta)} \Lambda_{t,t+1} z_{t+1} \phi_{t+1},$$
(54)

substituting into (52) delivers

$$\eta \frac{\rho_t}{\mu_t} A_t h_t = w_t h_t - (1 - \eta) \chi \frac{h_t^{1 + 1/\varphi} C_t}{1 + 1/\varphi} + (1 - \eta) b + \frac{\eta}{(1 - \delta)} \Lambda_{t, t+1} z_{t+1} \phi_{t+1},$$
(55)

finally, using  $\phi_t = \frac{\kappa}{q_t}$  and  $\frac{z_t}{q_t} = \theta_t$ , and rearranging, we get

$$w_t h_t = (1 - \eta) b + \eta A_t \frac{\rho_t}{\mu_t} h_t + (1 - \eta) \chi \frac{h_t^{1+1/\varphi}}{1 + 1/\varphi} C_t + \frac{\eta \kappa}{(1 - \delta)} E_t \Lambda_{t, t+1} \theta_{t+1},$$
(56)

which is the wage equation in the text. Similarly, the FOC for hours Nash bargaining is

$$(1-\eta)\left(\phi_{t}\right)^{-\eta}\left(\Gamma_{t}C_{t}\right)^{\eta}\frac{d\phi}{dh}+\eta\left(\Gamma_{t}C_{t}\right)^{\eta-1}\left(\phi_{t}\right)^{1-\eta}\frac{d\Gamma_{t}}{dh}C_{t}=0.$$
(57)

Considering that  $\frac{d\phi_t}{dh_t} = \frac{\rho_t}{\mu_t} A_t - w_t$ , and that  $\frac{d\Gamma_t}{dh_t} C_t = w_t - \chi h_t^{1/\varphi} C_t$ , equation (57) can be written as

$$(1-\eta)\Gamma_t C_t \left(\frac{\rho_t}{\mu_t} A_t - w_t\right) + \eta \phi_t \left(w_t - \chi h_t^{1/\varphi} C_t\right) = 0.$$
 (58)

Finally, using equation (49), equation (58) simplifies to

$$h_t = \left(\frac{1}{\chi} \frac{\rho_t}{\mu_t} \frac{A_t}{C_t}\right)^{\varphi} \tag{59}$$

which is the equation for hours worked in the text.

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