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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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The Effects of Fiscal Policy when Planning Horizons are Finite *

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Abstract

How important is the planning horizon of households for the effects of fiscal plans? We address this question through the lens of a New-Keynesian model where households are boundedly rational and plan over a finite number of periods. We show that the planning horizon affects the medium-run cumulative multipliers significantly. Government spending cumulative multipliers increase with the horizon while labor tax cumulative multipliers drop, in absolute terms. In light of the recent debt crisis in the Euro Area, we look at spending cuts and labor income tax hikes. In line with the empirical literature, we find in our benchmark calibration that spending cuts are less recessionary than tax hikes. Looking at the interaction between the planning horizon and the persistence of the fiscal plans, we find that spending cuts become less recessionary while tax hikes more recessionary when the persistence of fiscal plans rise, and that this effect is amplified by shorter planning horizons. Introducing wage stickiness, we show that for very sticky wages, tax hikes become less recessionary than spending cuts, but that intermediate levels of wage stickiness have the opposite effect on fiscal multipliers when planning horizons are finite.

Keywords: Fiscal policy, Finite planning horizons, Bounded rationality

JEL Classification: E60, E62, E63, H63

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1 Introduction

The effects of changes in fiscal policy on the economy have attracted much attention in the empirical literature to date. There are a number of contributions that have focused on the distinct effects of changes in government spending and/or tax revenues (Alesina and Perotti (1995), Alesina et al. (2019*a*), Blanchard and Perotti (2002), Mountford and Uhlig (2009) among others). Furthermore, there is a long-standing debate in the empirical literature about fiscal multipliers. The consensus, here, has become that, normally, government spending multipliers are smaller in absolute value than labor tax multipliers. At the same time, the theoretical literature has rested on models featuring various frictions in order to explain key empirical findings. The vast majority of the theoretical literature, though, has embarked on models where agents plan and form expectations over an infinite number of periods into the future. Moreover, the largest part of the literature assumes that agents have fully rational expectations even if they might have finite lifetimes à la Blanchard-Yaari. To that end, there has been little attention in the theoretical literature on the effects of forecast errors on multipliers, as documented empirically in Blanchard and Leigh (2013).

In this paper, instead, we develop a New-Keynesian model where agents have finite planning horizons and exhibit dynamic inconsistency in their consumption plans due to forecast errors that arise from the cognitive limitations of their planning horizon. We find that fiscal multipliers are considerably affected by the length of agent's planning horizons. Moreover, finite planning horizons and the induced forecast errors contribute to explaining the difference between government spending and labor tax multipliers. Finally, we find interesting interactions between planning horizons and other features of the model (e.g. persistence of fiscal plans, wage stickiness, intertemporal substitution) that affect the difference between these two fiscal multipliers.

We introduce finite planning horizons into a closed economy dynamic stochastic general equilibrium model with nominal rigidities, investment adjustment costs and labor unions. Embarking on Woodford (2018), we assume that households decide upon their consumption, capital accumulation and asset accumulation over a finite number of future periods. Decisions over a finite number of periods into the future, in our framework, lead to dynamic inconsistency in consumption plans. As regards firms, we consider two cases, namely one where they are also assumed to have finite planning horizons and another where they are rational and plan over the infinite future. We show that these two cases lead to very similar results, implying that the main findings of the paper are driven by finite planning horizons of households. Agents' finite horizons imply that the model has a non-recursive representation so that we have to embark on non-standard solution techniques.

In the literature, there have been other approaches as well which break down the Ricardian equivalence. Galí et al. (2007) assume that a fraction of households have *rule-of-thumb* behavior in a dynamic stochastic general equilibrium model. This type of households have zero net worth and decide their consumption upon their current disposable income. Hence the permanent income hypothesis does not hold for those households. The rest of the households are conventional infinite-horizon Ricardian consumers. In our model, there is only one type of household that maximizes over a finite number of periods into the future. Consumption decisions depend both on their current disposable income as well as on the stream of their wealth within their finite planning horizon. As such, our agents are non-Ricardian but not as mechanical as rule-of-thumb agents. In Galí et al. (2007) the effects of changes in government spending depend crucially on the fraction of non-Ricardian households and its interaction with price and wage stickiness. In our model instead, the effects of changes government spending depend entirely on the households' planning horizon and the resulting expectations, as well as on the interactions with the persistence in the model and in the fiscal interventions.

Blanchard (1985) examines the dynamic effects of government deficit finance when the lifetime of the representative household is finite.[1] In the Blanchard-Yaari approach, households can save, but discount the future heavily which makes them non-Ricardian and which causes the permanent income hypothesis not to hold. Moreover, in a Blanchard-Yaari structure, households' commit to a consumption plan which, abstracting from shocks, they will stick to. Our approach differs in that we allow for dynamic inconsistency in consumption

[1] This approach, known as the Blanchard-Yaari structure, has been used extensively in the literature in the analysis of monetary and fiscal policy interactions. See Devereux (2010), Mavromatis (2020), Richter (2015), Smets and Trabandt (2012) and the references therein.

plans.[2] Furthermore, in Blanchard-Yaari households have fully rational expectations.

Finite planning horizons in our model, instead, are associated with cognitive limitations and can result in suboptimality of decisions. The first one is that agents ignore what happens after their horizon, completely. The second is that, because of the cognitive limitations of their planning horizons, agents are incapable of forming fully rational expectations even for the periods that lie within their horizon. Consequently, their expectations are subject to errors. This is entirely due to their finite planning horizons and the fact that they can be wrong in their assessments of the true state of the economy at the end of their planning horizon.[3]

Our model is symmetric in the sign of fiscal changes (cuts or increases) and our main focus lies on fiscal multipliers. However, in light of the recent debt crisis in the Euro Area and the fiscal packages followed by some of its member states in order to guarantee fiscal solvency, we consider the effects of government spending cuts and labor income tax hikes as particular examples of fiscal policy changes. Following an approach similar to Alesina et al. (2017), we assume fiscal consolidations are persistent changes in government spending or taxes. Alesina et al. (2017) and Alesina et al. (2018) distinguish between anticipated and unanticipated fiscal plans. We restrict our focus on unanticipated changes in government spending or in taxes only. As regards spending-based consolidations we distinguish between changes in wasteful government spending and government lump-sum transfers and we look at them separately. Our finite planning horizon structure implies that Ricardian equivalence breaks down allowing thus for lump-sum transfers to have real effects.[4]

We show that planning horizons matter for the effects of fiscal plans, especially in the medium-run. In particular, we find that as the planning horizon falls, the medium-run cumulative labor tax multiplier becomes more negative. This implies that, following tax hikes,

[2]This feature is also absent in Galí et al. (2007). Our approach is closer to a myopic individual consumption decision which is time-inconsistent. That is, actual consumption at date t may be different from what the household had previously planned to consume at that date. This is because at any date t, the household plans its consumption accounting for elements not considered in its plans computed previously. For a more detailed analysis of those issues see Lovo and Polemarchakis (2010) and the references therein.

[3]Blanchard and Leigh (2013) highlight the impact of forecast errors on multipliers following fiscal consolidations and find that growth has been lower than expected in advanced economies. Our model captures these discrepancies whereas the models discussed above cannot.

[4]Given that transfers are lump-sum, changes in lump-sum taxes simply mirrors our results from transfers.

recessions become deeper in the medium-run as agents' planning horizon falls. This is explained by the response of investment for various horizons, which differs due to differences in expectations. As horizons shorten, investment drops even more after tax hikes. In fact, for infinite horizons, investment drops slightly for only a few quarters and then stabilizes until it reverts back to its pre-shock level. Finite planning horizons, on the other hand, can capture continued declines in investment that are closer to those documented empirically (see e.g. Alesina et al., 2017, Blanchard and Perotti, 2002 and Mountford and Uhlig, 2009).[5]

At the same time, medium-run cumulative spending multipliers decrease as planning horizons shorten. This result is stronger as the cumulative multiplier is calculated over more periods. The intuition for this result is the stronger investment crowding-in effect as the planning horizon falls. Lower real interest rates after government spending cuts imply an increase in investment for any planning horizon.[6] However, as real rates start to increase again, the total increase in investment is of limited magnitude under longer planning horizons. Due to the limitations of their planning horizons, agents with shorter planning horizons, on the other hand, overestimate the response of the real interest rate and increase investment more. As a result, the shorter the planning horizon the stronger the crowding-in effect in investment and the faster the induced recovery of the economy.

In line with the existing literature on the effects of spending cuts and tax increases (Alesina et al. (2015), Alesina et al. (2017), Guajardo et al. (2014) among others), we find that tax-based consolidations are more contractionary, both on impact and in the medium-run.[7] Importantly, we show that the differences in the performance of the two types of consolidation

[5]In their seminal paper Baxter and King (1993) introduce capital in a neoclassical model to analyze the effects of permanent changes in government purchases and find that the effects on investment and hence on output to be highly persistent.

[6]Alesina et al. (2019*a*) uncover some strong empirical regularities about the effects of austerity. One of those regularities is that investment responds positively to spending-based plans and negatively to tax-based plans.

[7]There has been much research over the effects of different types of fiscal consolidations (e.g. spending-based and tax-based). A large empirical literature provides evidence supporting the expansionary fiscal consolidations hypothesis (see Alesina and Perotti (1995), Perotti (1996), Alesina and Ardagna (1998, 2010), Ardagna (2004)). On the other hand, another strand of the empirical literature, using narrative data to identify consolidations, initially introduced by Romer and Romer (2010), finds that output drops following both types of consolidations and that recessions are deeper after tax hikes (Guajardo et al. (2014)). Along the same lines Alesina et al. (2015) find that spending-based consolidations are less costly, in terms of output losses, than tax-based ones. However, as Guajardo et al. (2014) argue the negative effects of consolidations on output may be understated due to the induced bias. This is the case with spending cuts in many instances, where the announced cuts were stronger than those actually implemented (Beetsma et al. (2016)).

depend on the planning horizon as well, and they widen as horizons shorten. Moreover, by changing one by one different parts of the calibration, we are able to obtain insight in what is making spending multipliers smaller in absolute value than tax based ones.

In our benchmark calibration, we assume a persistent autoregressive process for spending and labor income taxes similar to Alesina et al. (2017). In general, we find that spending-based consolidations become less recessionary the more persistent are the cuts in spending. Conversely, for labor tax hikes we find that when they are very persistent the tax hikes to deeper recessions, regardless of the planning horizon. Persistent increases in labor taxes make the static substitution effect between labor and leisure more permanent, increasing thus the tax multiplier (in absolute value).[8] These results are in line with evidence on the persistence of fiscal plans (e.g. Alesina et al. (2017), Alesina et al. (2019*b*) among others). As regards the interaction between planning horizons and persistence, we find that there are stronger planning horizon effects on medium-run cumulative multipliers when consolidations are very persistent. In particular, for shorter planning horizons, the gap between spending and tax-based consolidations widens *more* as their persistence increases.

Wage stickiness is an additional factor determining the performance of fiscal plans. We find that when wages are very sticky, tax-based consolidations result in milder recessions than spending cuts, independently of planning horizons. The intuition is that, in the absence of sticky wages, labor unions bargain higher wages to offset the drop in current disposable labor income. As a result, real wages increase. Higher wages lead to lower labor demand and a decline in firms' profits, causing a recession. When wages are very sticky, instead, this mechanism is very weak simply because labor unions reset wages infrequently. As a result, the recession is milder than under fully flexible wages. Tax multipliers are then lower which makes tax-based consolidations less costly in terms of output losses. For shorter planning horizons, very sticky wages additionally imply larger government spending multipliers, which further improves the relative performance of tax-based consolidations. For intermediate or low degrees of wage stickiness, on the other hand, spending multipliers remain considerably smaller, in

[8] In a model without labor unions, the static substitution between labor and leisure would be directly made by the household. In our model, this trade-off is made *indirectly* by labor unions who set wages in order to maximize the utility of households and thereby indirectly determine the amount of labor that will be supplied by households given the labor demand curve of firms.

absolute value, than labor tax multipliers. In fact, for shorter planning horizons the absolute difference between the two multipliers becomes even larger (spending multipliers decrease while tax multipliers become more negative) for intermediate values of wage stickiness.

Finally, a lower elasticity of intertemporal substitution increases government spending multipliers and decreases, in absolute value, labor tax multipliers. This is because agents with a lower elasticity of intertemporal substitution smooth consumption more and, therefore, respond less strongly to temporary changes in their expected future disposable income and in expected future real interest rates. We find, however, that medium-run cumulative government spending multipliers under shorter planning horizons remain lower in absolute value than cumulative labor tax multipliers, also for relatively low values of the elasticity of intertemporal substitution. This is not the case for larger planning horizons.

The paper is organized as follows. In the next section, we briefly discuss the literature dealing with the optimization problem of boundedly rational agents who form expectations over a finite number of periods, and explain how our approach differs from the existing literature. In Section 3, we present the New Keynesian model with finite planning horizons. Section 4 presents the relation between the planning horizon and fiscal multipliers as well as impulse responses to fiscal shocks. In Section 5, we investigate what features of the model make government spending multipliers smaller in absolute value than labor tax multipliers. Section 6 considers the case where firms have infinite planning horizons, and Section 7 concludes.

2 Literature on Bounded Rationality and Bounded Optimality

Our model is related to a large literature on bounded rationality and bounded optimality. First of all, under Euler equation learning (see Honkapohja et al. (2013)) agents form expectations only up to one period into the future. When these expectations are not fully rational (and hence do not implicitly take the infinite future into account through a recursive formulation), agents have a one-period-ahead planning horizon. Resting on this finite horizon learning approach, Evans and McGough (2015) build a framework that formulates the agents' optimization in a way that is consistent with their short-sightedness in forecasting.[9] In their approach, agents learn about the shadow price of their wealth in a recursive way. They show that such a problem can be cast in a dynamic programming setting, which is consistent with the time inconsistency of consumption plans.

Our approach differs from Evans and McGough (2015) in some crucial aspects. First, our agents are uncertain not only about the correct valuation of their end-of-horizon wealth but also about the state of the economy at the end of their planning horizon. This means that even if our agents might know the true valuation of their wealth, they can still be time-inconsistent in their consumption decisions if their beliefs about the state of the economy at the end of their horizon are wrong. As such, even though in a two-period setting our approach is similar to theirs, the implied expectations path in our case is different. Moreover, we apply our approach to cases where agents optimize and form expectations for more than two (but a finite number of) periods. Consequently, our approach incorporates important wealth effects that are absent in Evans and McGough (2015).

Branch et al. (2010) consider the case of finite horizons. In their model, it is however required that agents form expectations about their end of horizon wealth and optimize based on these expectations. In contrast, in our model, choosing optimal end-of-horizon bond holdings and capital is in every period part of the agents' optimization problem.

The analysis of the scenario where agents do not fully understand the world and especially events that are far into the future is explored delicately in Gabaix (2020). In that model, it is assumed that as agents simulate the future they tend to simplify their simulations by narrowing them down to a simple benchmark, namely the steady state of the economy. In that framework, agents solve infinite horizon problems but receive noise signals about the economy. Consequently, an innovation happening far into the future is shrunk by a factor less than unity relative to the rational response. In this way, agents are myopic with respect to deviations around the steady state, especially when they are far in the future.

Our approach is very similar to recent work by Woodford (2018). Woodford argues about the plausibility of designing the household maximization problem in a way that deviates from

[9] Essentially Evans and McGough (2015) highlight the difference between learning to forecast with learning to optimize and build an optimizing framework satisfying both kinds of learning.

the assumption that households plan over the infinite future and commit to their consumption and saving plans. Instead, in his and our approach households decide over a finite number of periods. Specifically, households decide upon their consumption path from period t up to a finite period $t+k$. When period $t+1$, arrives households decide upon their consumption path from period $t+1$ up to a finite period $t+k+1$ and so on. In other words, the households in period $t+1$ do not stick to the path that was decided in period t. Consequently, Woodford's and our approach account for the time inconsistency in consumption plans as well, in addition to the short-sightedness of the households.

Our approach differs though from Woodford's in what agents believe about the future beliefs of all agents in the economy. In order to keep the whole planning problem within the k period horizon, Woodford assumes that in period t, agents believe that all agents in period $t+1$ will only have a planning horizon of $k-1$ periods, that in period $t+2$ all agents will only have a planning horizon of $k-2$ periods, etc. We, on the other hand, assume that all agents know that all agents in future periods will have the same planning horizon of k periods (or T in the notation of our paper), but that they will think that from period $t+k+1$ onward the model will be in steady state. This alternative assumption allows us to make sure that in period t, agents do not have to sophisticatedly think about periods after $t+k$. We then assume that expectations are formed in a manner that is consistent with the cognitive limitations of the planning horizon, which is not done in Woodford (2018).

Finally, we consider a model with government debt and distortionary taxes and look at fiscal consolidations whereas Woodford (2018) focuses on the implications for the conduct of monetary policy.

3 The model

Below we outline the model. In Section 3.1 we discuss the optimization problem and first-order conditions of households, and in Section 3.2 we present the wage setting of labor unions. In Section 3.3 we discuss the firm problem, and the government sector, monetary policy rule and market clearing are presented in Section 3.4. In Appendix D the model is log-linearized and aggregated. Finally, we discuss how expectations are formed in Section 3.5 and present our calibration in Section 3.6.

3.1 Households

We assume that the economy is made up of a large number of infinitely lived identical households that plan over a finite number of periods into the future. Households want to maximize their discounted utility of consumption and leisure over their planning horizon (T periods), and they also value the state they expect to end up in at the end of these T periods (their state in period T+1). They are however not able to rationally induce (by solving the model forward until infinity), how exactly they should value their state in period T+1. Instead, households use a rule of thumb to evaluate the value of their state. Household i chooses consumption, C_t^i, investment, I_t^i, capital (K_t^i) and bond (B_t^i) holdings and capital utilization, Z_t^i, so as to maximize the following objective function:

$$\tilde{E}_t^i \sum_{s=t}^{t+T} \beta^{s-t} u(C_s^i, H_s) + \beta^{T+1} V^{i,t}\left(\frac{B_{t+T+1}^i}{P_{t+T}}, K_{t+T+1}^i\right), \tag{1}$$

subject to

$$P_s C_s^i + P_s I_s^i + \frac{B_{s+1}^i}{R_s} \leq \left(1-\tau_s^W\right) W_s H_s + B_s^i + R_s^k Z_s^i K_s^i - P_s a\left(Z_s^i\right) \bar{K}_s^i + P_s \Xi_s - T_s, \qquad s = t, t+1, ...t+T. \tag{2}$$

where B_s is a one period riskless bond issued by the government, yielding interest rate R_s. W_s is the nominal wage and τ_s^W is a distortionary labor income tax rate. H_t denotes hours worked, while R_s^K is the return on capital ownership.[10] $\alpha(Z_s)$ captures costs related to capital utilization and is a strictly convex function, so that $\alpha'() > 0$ and $\alpha''() > 0$. Ξ_s denotes real profits from firm ownership while T_s denotes lump-sum taxes. The value function $V^{i,t}(.)$ of the household at the end of her horizon T takes the following form:

$$V^i(B, K) = (1-\beta)^{-1} u\left((1-\beta)\left(\frac{1}{\beta}K + \frac{B}{\bar{\Pi}}\right) + \bar{X}\right), \tag{3}$$

[10] As will be discussed in the next section, wages are set by labor unions. As it is assumed that the continuum of different labor types and corresponding labor unions is distributed equally across households, the individual quantity of hours worked is common across households. We therefore denote H_t without an i superscript.

where $\bar{X}$ consists of steady state labor income, profits and taxes/transfers. In Appendix B we derive and discuss this value function in detail in the same spirit as in Woodford (2018). In particular, this value function is the solution of a simplified dynamic programming problem, where there are no shocks and where it is assumed that the variables outside the direct control of the household are at their steady state levels.

The real budget constraint reads as

$$C_s^i+I_s^i+\frac{B_{s+1}^i}{R_sP_s} \leq \left(1-\tau_s^W\right) w_sH_s+\frac{B_s^i}{P_s}+\frac{R_s^kZ_s^iK_s^i}{P_s}-a\left(Z_s^i\right)\bar{K}_s^i+\Xi_s-\frac{T_s}{P_s}, \qquad s=t,t+1,...t+T, \tag{4}$$

where w_s is the real wage rate. The capital accumulation equation is

$$K_{s+1}^i = (1-\delta)\,K_s^i + \left[1-S\left(\frac{I_s^i}{I_{s-1}^i}\right)\right] I_s^i, \qquad s=t,t+1,...t+T. \tag{5}$$

The function $S\,(.)$ in the capital accumulation equation (5) is the adjustment cost function, with $S\,(\theta)=0$, $S'\,(\theta)=0$, $S''\,(.)>0$. The amount of effective capital that households can rent to the firm is:

$$\tilde{K}_t^i = Z_s^iK_s^i. \tag{6}$$

The income from renting capital services to firms is $R_s^kZ_s^iK_s^i$, while the cost of capital utilization is $P_sa\left(Z_s^i\right)\bar{K}_s^i$.

In line with CEE and SGU we set

$$S\left(\frac{I_s^i}{I_{s-1}^i}\right) = \frac{\kappa}{2}\left(\frac{I_s^i}{I_{s-1}^i}-1\right)^2, \tag{7}$$

and

$$a\left(Z_s^i\right) = \gamma_1\left(Z_s^i-1\right)+\frac{\gamma_2}{2}\left(Z_s^i-1\right)^2. \tag{8}$$

The composite consumption good, C_s^i, is a Dixit-Stiglitz aggregate,

$$C_s^i = \left[\int_0^1 \left(C_s^i(j)\right)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}, \tag{9}$$

of the household's expenditure $C_s^i(j)$ on each of the continuum of differentiated varieties j, where $\theta > 1$ is the intertemporal elasticity of substitution across varieties.

Households are identical in our model which means that they have the same utility function and face the same constraints. The first order conditions of the maximization problem are:

$$C_t^i: \ \varpi_s^i = u_C(C_s^i, H_s), \qquad s = t, t+1, ...t+T, \tag{10}$$

$$B_{t+1}^i: \ \varpi_s^i = \beta E_s^i \left[\frac{\varpi_{s+1}^i R_s}{\Pi_{s+1}} \right], \qquad s = t, t+1, ...t+T-1, \tag{11}$$

$$K_{t+1}^i: \ \gamma_s^i = \beta E_s^i \left[\varpi_{s+1}^i \left(\frac{R_{s+1}^K}{P_{s+1}} Z_{s+1}^i - \alpha(Z_{s+1}^i) \right) + (1-\delta)\, \gamma_{s+1}^i \right], \qquad s = t, t+1, ...t+T-1, \tag{12}$$

$$\begin{aligned} I_s^i: \ \varpi_s^i = {} & \gamma_s^i \left[1 - S\left(\frac{I_s^i}{I_{s-1}^i} \right) - S'\left(\frac{I_s^i}{I_{s-1}^i} \right) \frac{I_s^i}{I_{s-1}^i} \right] \\ & + \beta E_s \left[\gamma_{s+1}^i S'\left(\frac{I_{s+1}^i}{I_s^i} \right) \left(\frac{I_{s+1}^i}{I_s^i} \right)^2 \right], \qquad s = t, t+1, ...t+T-1, \end{aligned} \tag{13}$$

$$Z_s^i: \ \frac{R_s^K}{P_s} = \alpha'\left(Z_s^i\right), \qquad s = t, t+1, ...t+T. \tag{14}$$

where ϖ_t^i and γ_t^i are the lagrange multipliers associated with the budget constraint and the capital accumulation constraint respectively. $\Pi_s = P_s/P_{s-1}$ is gross inflation in any period s. In the F.O.C. above, the optimal decisions capital utilization, Z_t^i, concerns the entire planning horizon of households. The optimal decisions instead for bond holdings, B_t^i, capital, K_t^i, and investment, I_t^i, derived above concern the optimal path up to period $T-1$. That is, one period before the end of its horizon. This is because these three are state variables and determine the end-of-horizon continuation value of households' utility. For B_{t+T}^i, K_{t+T}^i and I_{t+T}^i, different first-order conditions follow from the above optimization problem. These conditions determine the optimal trade-off between consumption, saving and investing at the very end of their horizon. These conditions capture the fact that households care about how much wealth they will have after the end of the planning horizon and need thus to determine the optimal trade-off. The three conditions read as follows:

$$B^i_{t+T+1}:\ \frac{\varpi^i_{t+T}}{R_{t+T}} = \frac{\beta}{\bar{\Pi}} E^i_t \left[u'\left((1-\beta)\left(\frac{1}{\beta} K^i_{t+T+1} + \frac{B^i_{t+T+1}}{P_{t+T}\bar{\Pi}} \right) + \bar{X} \right) \right], \tag{15}$$

$$K^i_{t+T+1}:\ \gamma^i_{t+T} = E^i_t u'\left((1-\beta)\left(\frac{1}{\beta} K^i_{t+T+1} + \frac{B^i_{t+T+1}}{P_{t+T}\bar{\Pi}} \right) + \bar{X} \right), \tag{16}$$

$$I^i_{t+T}:\ \varpi^i_{t+T} = E^i_t \gamma^i_{t+T} \left[1 - S\left(\frac{I^i_{t+T}}{I^i_{t+T-1}} \right) - S'\left(\frac{I^i_{t+T}}{I^i_{t+T-1}} \right) \frac{I^i_{t+T}}{I^i_{t+T-1}} \right]. \tag{17}$$

Our utility function takes the functional form $u(C^i_s, H_s) = \frac{(C^i_s)^{1-\sigma}}{1-\sigma} - \frac{(H^i_s)^{1+\eta}}{1+\eta}$, so that households have CRRA preferences where σ is the inverse of the elasticity of intertemporal substitution and η is the inverse of the Frisch elasticity of labor supply. Defining Tobin's $Q^i_t = \frac{\gamma^i_t}{\varpi^i_t}$ and using the first order condition with respect to investment (13) we receive the following:

$$\begin{aligned} 1 = Q^i_s &\left(1 - S\left(\frac{I^i_s}{I^i_{s-1}} \right) - S'\left(\frac{I^i_s}{I^i_{s-1}} \right) \frac{I^i_s}{I^i_{s-1}} \right) \\ &+ \beta \frac{\varpi^i_{s+1}}{\varpi^i_s} \left[Q^i_{s+1} S'\left(\frac{I^i_{s+1}}{I^i_s} \right) \left(\frac{I^i_{s+1}}{I^i_s} \right)^2 \right], \qquad s = t, t+1, ...t+T-1. \end{aligned} \tag{18}$$

Using the first order condition with respect to capital, K^i_s, we end up to the following expression for the law of motion of Tobin's Q^i_s

$$Q^i_s = \beta \frac{\varpi^i_{s+1}}{\varpi^i_s} \left[r^k_{s+1} Z^i_{s+1} - a\left(Z^i_{s+1} \right) + Q^i_{s+1} (1-\delta) \right], s = t, t+1, ...t+T-1 \tag{19}$$

$$\text{with} \quad r^k_{s+1} = \frac{R^k_{s+1}}{P_{s+1}}. \tag{20}$$

Tobin's Q in the terminal period T is specified as $Q^i_{t+T} = \frac{\gamma^i_{t+T}}{\varpi^i_{t+T}}$.

Finally, using Equation (8) for $\alpha(Z)$, the first order condition (14) with respect to capital utilization, Z_s, reads as follows:

$$\frac{R_s^k}{P_s} = a'(Z_s) = \gamma_1 + \gamma_2 \left(Z_s^i - 1\right), \tag{21}$$

$$\hat{r}_s^k = \frac{\gamma_2}{\gamma_1} \hat{Z}_s^i.$$

3.2 Wage setting

In the monopolistic competitive labor market, labor unions act as wage setters. We assume a continuum of labor types j which are distributed uniformly across households. Each labor union sets the wage of a given labor type j. This means that there is a continuum of labor unions which coincides with the labor types. Households always offer the demanded quantity of labor of type j at the current wage W_{t+s}^j of the labor union of type j (as in Galí et al., 2007).

We assume that labor unions set wages in a flexible manner at date t.[11] Unions set the nominal wage by maximizing the expected discounted sum of the utility of the households subject to the households' budget constraint and the demand for labor of type j.

$$h_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} H_t, \tag{22}$$

where θ_w is the intratemporal elasticity of substitution between labor types and W_t is the nominal wage aggregated over all labor types. H_t is the aggregate demand for labor given by

$$H_s = \left[\int_0^1 \left(h_s^j\right)^{\frac{\theta_w - 1}{\theta_w}} dj\right]^{\frac{\theta_w}{\theta_w - 1}}. \tag{23}$$

Since the union sets the wage at every date, the maximization problem of the union becomes static. At a generic time t, it reads as follows:

$$\max_{W_t^j} \left[\frac{\left(C_{t+s}^j\right)^{1-\sigma}}{1-\sigma} - \frac{\left(h_{t+s}^j\right)^{1+\eta}}{1+\eta}\right] \tag{24}$$

subject to (22) and (4). The first order condition is:

[11] In Section 5.2 and Appendix C.1 we also consider the case where labor unions set the wages infrequently assuming Calvo-type wage stickiness.

$$W_t^j\left(1-\tau_t^w\right)=\frac{\theta_w}{\theta_w-1}\frac{\left(h_t^j\right)^{\eta}}{\left(C_t^j\right)^{-\sigma}}. \tag{25}$$

From the equation above, it becomes clear that the labor union sets the wage with a mark-up above the marginal rate of substitution between consumption and labor.

3.3 Firms

There is a continuum of monopolistically competitive firms producing the final differentiated goods. Each firm produces its good using all differentiated labor types. Each firm then combines its share of the aggregate labor input, H_t, with the capital, $\tilde{K}_t$ that it rents from households, using a Cobb-Douglas technology:

$$Y_t(j)=A\tilde{K}_t(j)^{\zeta}H_t(j)^{1-\zeta}, \tag{26}$$

where A is aggregate productivity, which is assumed to be constant. Firm's j nominal profit is given by:

$$P_t\Xi_t(j)=P_t(j)Y_t(j)-W_tH_t(j)-R_t^k\tilde{K}_t(j) \tag{27}$$

Cost minimization implies that the marginal cost is equal to:

$$MC_t=\zeta^{-\zeta}\left(1-\zeta\right)^{-(1-\zeta)}W_t^{1-\zeta}R_t^{k\,\zeta} \tag{28}$$

or in real terms:

$$mc_t=\zeta^{-\zeta}\left(1-\zeta\right)^{-(1-\zeta)}w_t^{1-\zeta}r_t^{k\,\zeta} \tag{29}$$

Since all firms face the same marginal cost, we have dropped the j indicator.

Cost minimization also implies:

$$r_t^k=mc_tA\zeta\left(\frac{\tilde{K}_t(j)}{H_t(j)}\right)^{\zeta-1} \tag{30}$$

$$w_t = mc_t A(1-\zeta)\left(\frac{\tilde{K}_t(j)}{H_t(j)}\right)^{\zeta} \tag{31}$$

Firms are run by households, and hence we assume they also have finite planning horizons. That is, they will form expectations about their marginal costs and the demand for their product for T periods ahead only. In Section 6 we check robustness to this assumption by considering the case where firms have an infinite planning horizon.

We assume that in each period a fraction $(1-\omega)$ of firms can change their price. The problem of firm j that can reset its price is then to maximize the discounted value of its nominal profits for the next T periods.

$$\tilde{E}_t^j\left(\sum_{s=0}^{T}\omega^s Q_{t,t+s}^j\left[p_t(j)Y_{t+s}(j) - P_{t+s}mc_{t+s}Y_{t+s}(j)\right] + \omega^{T+1}\beta^{T+1}\frac{P_t}{\left(C_t^j\right)^{-\sigma}}\tilde{V}(\frac{p_t(j)}{P_{t+T}})\right), \tag{32}$$

where

$$Q_{t,t+s}^j = \beta^s\left(\frac{C_{t+s}^j}{C_t^j}\right)^{-\sigma}\frac{P_t}{P_{t+s}}. \tag{33}$$

is the stochastic discount factor of the household that runs firm j. Moreover, as in Woodford (2018), $\tilde{V}()$ describes the continuation value of real profits in utility terms as a function of the relative price. As in case of the household, this value function is obtained from the assumption that all variables other than the price of the firm (such as output, wages and the aggregate price level) are in steady state. Assuming zero steady state inflation, this value function therefore satisfies

$$\tilde{V}(r) = \max_c\{\left(\bar{C}\right)^{-\sigma}\left(r^{1-\theta}\bar{Y} - r^{-\theta}\bar{Y}\bar{mc}\right) + \omega\beta\tilde{V}(r) + (1-\omega)\beta\tilde{V}^{opt}\}, \tag{34}$$

where $\tilde{V}^{opt}$ is next periods value for a firm that can re-optimize next period. Since $\tilde{V}^{opt}$ does not influence the current decision problem of the firm (since it is independent of r), we ignore it and let the functional form of $\tilde{V}(r)$ be

$$\tilde{V}(r) = \frac{1}{1-\omega\beta}\left(\bar{C}\right)^{-\sigma}\left(r^{1-\theta}\bar{Y} - r^{-\theta}\bar{Y}\bar{mc}\right). \tag{35}$$

Using the demand for good j, and multiplying by $\frac{(C_t^j)^{-\sigma}}{P_t}$, the firm's profit maximization problem writes as follows

$$\max \tilde{E}_t^j \Bigg(\sum_{s=0}^{T} \omega^s \beta^s \left(C_{t+s}^j\right)^{-\sigma} \left[\left(\frac{p_t(j)}{P_{t+s}}\right)^{1-\theta} Y_{t+s} - mc_{t+s} \left(\frac{p_t(j)}{P_{t+s}}\right)^{-\theta} Y_{t+s} \right] \tag{36}$$
$$\frac{(\omega\beta)^{T+1}}{1-\omega\beta} (\bar{C})^{-\sigma} \left[\left(\frac{p_t(j)}{P_{t+T}}\right)^{1-\theta} \bar{Y} - \bar{mc} \left(\frac{p_t(j)}{P_{t+T}}\right)^{-\theta} \bar{Y} \right] \Bigg).$$

The first order condition for $p_t(j)$ is

$$\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \frac{\left(C_{t+s}^j\right)^{-\sigma}}{P_{t+s}} Y_{t+s} \left[(1-\theta) \left(\frac{p_t^*(j)}{P_{t+s}}\right)^{-\theta} + \theta mc_{t+s} \left(\frac{p_t^*(j)}{P_{t+s}}\right)^{-1-\theta} \right] \tag{37}$$
$$+ \frac{(\omega\beta)^{T+1}}{1-\omega\beta} \frac{\left(\bar{C}\right)^{-\sigma}}{P_{t+T}} \bar{Y} \left[(1-\theta) \left(\frac{p_t^*(j)}{P_{t+T}}\right)^{-\theta} + \theta \bar{mc} \left(\frac{p_t^*(j)}{P_{t+T}}\right)^{-1-\theta} \right] = 0,$$

where $p_t^*(j)$ is the optimal price for firm j if it can re-optimize in period t. Multiplying by $\frac{p_t^*(j)^{1+\theta}}{1-\theta}$, this can be written as

$$\frac{p_t^*(j)}{P_t} \left[\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \left(C_{t+s}^j\right)^{-\sigma} \left(\frac{P_{t+s}}{P_t}\right)^{\theta-1} Y_{t+s} + \frac{(\omega\beta)^{T+1}}{1-\omega\beta} \bar{Y} (\bar{C})^{-\sigma} \left(\frac{P_{t+T}}{P_t}\right)^{\theta-1} \right] \tag{38}$$
$$= \frac{\theta}{\theta-1} \left[\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \left(C_{t+s}^j\right)^{-\sigma} \left(\frac{P_{t+s}}{P_t}\right)^{\theta} Y_{t+s} mc_{t+s} + \frac{(\omega\beta)^{T+1}}{1-\omega\beta} \bar{Y} (\bar{C})^{-\sigma} \bar{mc} \left(\frac{P_{t+T}}{P_t}\right)^{\theta} \right]$$

Finally, the aggregate price level evolves as

$$P_t = [\omega P_{t-1}^{1-\theta} + (1-\omega) \int_0^1 p_t^*(j)^{1-\theta} dj]^{\frac{1}{1-\theta}}. \tag{39}$$

3.4 Completing the model

In this section, we close the model by characterizing the budget constraint of the government, the interest rate rule of the central bank and the market clearing condition. The government issues bonds and levies lump-sum taxes, T_t, and labor income taxes, τ_t^W, to finance its (wasteful) spending, G_t, as well as Transfers to households, $Trans_t$. Its budget constraint is given by

$$\frac{B_{t+1}}{R_t} = P_t G_t - \tau_t^W W_t H_t - T_t + B_t, \tag{40}$$

with $H_t = \int H_t^i di$ and $B_t = \int B_t^i di$ aggregate labor and aggregate bond holdings respectively. Dividing by $\bar{Y} P_t$ gives

$$\frac{b_{t+1}}{R_t} = g_t - \tau_t^W w_t \frac{H_t}{\bar{Y}} - \tau_t + \frac{b_t}{\Pi_t}, \tag{41}$$

where $b_t = \frac{B_t}{P_{t-1}\bar{Y}}$ and $g_t = \frac{G_t}{\bar{Y}}$ are the ratios of debt to steady state GDP and government expenditure to steady state GDP, respectively. Lump-sum taxes as a fraction of steady-state nominal output at current prices, $\tau_t \equiv \frac{T_t}{P_t\bar{Y}}$, are adjusted to stabilize public debt. Additionally, there may be transfers which are equivalent to negative lump-sum taxes. The evolution of lump-sum taxes hence is given by

$$\tau_t = \phi_{B_y}\left(b_t - \bar{b}\right) - \tau_t^r, \tag{42}$$

where $\tau_t^r \equiv \frac{Trans_t}{P_t\bar{Y}}$. g_t, τ_t^w and τ_t^r are exogenously set by the government. In particular, these are the three fiscal instruments of which we study multipliers in the next section. Here, these variables will be assumed to follow AR(1) processes.

The central bank sets the policy rate following a Taylor rule targeting inflation and output:

$$R_t = \left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_Y} \tag{43}$$

where π^* denotes the inflation target which we assume to coincide with steady state inflation. $\bar{Y}_t$ is steady state output.[12]

Finally, market clearing is given by:

$$Y_t = C_t + I_t + \bar{Y} g_t + \alpha\left(Z_t\right) K_t \tag{44}$$

[12]We derive the steady state of the model in appendix A.

3.5 Expectations

We assume that agents form expectations in a forward-looking manner that is as close to rational expectations as the cognitive limitations of their planning horizon allow. Forming fully rational expectations would, however, require agents to anticipate what the future paths of all variables will be until infinity. This is not consistent with the assumption of finite planning horizons.

We, therefore, assume that agents rationally use the model equations within their horizon to form model consistent expectations, but that they are not able to form expectations for variables outside their horizon in a sophisticated manner. Agents thus know the structural form of the minimum state variable solution of the model but are not always fully correct in computing the parameters of this solution, as we describe below.

Agents start with forming expectations about the final period of their horizon. To do this, they take the model equations of period $t+T$. However, the IS curve, Phillips curves and investment equation contain finite sums up to T periods ahead (see the equations in Appendix D.5). Therefore, when these equations are forwarded T periods, finite sums with expectations about period $t+T+1$ up to period $t+T+T$ appear. That is, in the model equations that agents are considering, expectations of variables outside their planning horizon show up. Since agents are not able to solve, in a sophisticated manner, what will happen after their planning horizon, they cannot come up with a rational value for these expectations. Instead, they assume that in the periods after their horizon, the model will have converged to its steady state.

With this assumption, agents are able to solve for period $t+T$ variables in terms of the state variable in period $t+T$. They then move to the model equations of period $t+T-1$. Here, they plug in the solution of period $t+T$ variables as expectations about period $t+T$ and again assume steady state levels for expectations of variables outside their horizon. They can then solve for period $t+T-1$ variables in terms of state variable of period $t+T-1$. This process goes on until they have solved for all expectations within their horizon in terms of the observed state variables of period t. Expectations of variables for the periods within the horizon can then be obtained from these policy functions by plugging in the values of the current state variables.

To what extent expectations formed in this manner deviate from fully rational expectations depends on the persistence of the model, the shocks hitting the economy and, mainly, on agents' planning horizon. First of all, if the planning horizon is large enough relative to the persistence in the economy, the assumption that the model will have returned to steady state after the horizon is rational. In that case, the above algorithm converges to the fully rational expectation solution.

If, on the other hand, the model is so persistent relative to the planning horizon that shocks hitting the economy will have effects that last longer than the planning horizon, then the expectations of agents with finite planning horizons will be biased. This is because, when forming expectations about variables in periods towards the end of their horizon, agents ignore the effect that expectations about periods after the end of their horizon will have on the realizations of these variables.

The impact of this bias on the dynamics in the economy crucially depends on the planning horizon. When agents have longer planning horizons, their decisions will only marginally be affected by biases in their expectations, even if the economy is very persistent. This is because the cognitive limitations of their planning horizon will bias only expectations about periods that are close to the end of their horizon. Their expectations about all other periods will be close to fully rational.

In case of shorter planning horizons, this is different. Here, a considerable bias in the expectations about all periods within their horizon can arise when the economy is very persistent. This can potentially lead agents to make considerably different decisions about, e.g, investment than fully rational agents.

3.6 Calibration

In this section, we discuss the calibration of the deep parameters of the model. Our intension is to assign values to parameters which are as close as possible to the existing theoretical or empirical literature on DSGE models. One period corresponds to a quarter. Regarding the finite planning horizons in our model, we focus on analyzing the effects of changing this parameter and do not consider a benchmark horizon. In illustrations and exercises where we change other parameters, we always show two cases: a horizon of $T = 20$ which is representative for shorter horizons, and a horizon of $T = 100$ which is representative for longer (and infinite) planning horizons. The rest of the model parameters are specified in table 1 below.

Table 1: Parameter values

Parameter	Description	Value
β	Subjective disc. factor	0.99
σ	Inv. intertemp. substitution	2
η	Inv. Firsch elasticity	1.5
$\frac{\theta}{\theta-1}$	Price mark-up	2
ω	Calvo - price	0.66
ζ	Share of capital	0.2
γ_1	Capital utilization	0.035
γ_2	Capital utilization	0.05
δ	Capital depreciation	0.025
κ	Capital adjustment cost	17
$\frac{\theta_w}{\theta_w-1}$	Wage mark-up	1.3
ϕ_π	Taylor Rule - inflation	1.5
ϕ_Y	Taylor Rule - output	0.05
ϕ_{B_y}	Tax Rule - debt	0.06
ρ_g	AR(1) coef. gov't spending	0.97
ρ_τ	AR(1) coef. labor income tax	0.97
$\bar{\tau}$	Steady state tax rate	0.4
$\bar{g}$	Steady state gov't spending	0.15

4 Fiscal multipliers under finite planning horizons

In this section, we study the role of planning horizons for fiscal multipliers. It turns out that especially medium-run cumulative multipliers are affected by agents' planning horizon considerably. We first present, in Section 4.1, fiscal multipliers for government expenditures, labor taxes and transfers for different planning horizons. Next, we investigate where the differences in multipliers stem from by studying impulse response functions in Section 4.2.

4.1 Multipliers

Following Mountford and Uhlig (2009) and Bi et al. (2013), we calculate present value multipliers as follows

$$\Gamma_{t+k}^{y} = \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1} \right) \left(Y_{t+j}^{s} - Y_{t+j}^{ns} \right) / \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1} \right) \left(x_{t+j}^{s} - x_{t+j}^{ns} \right), \tag{45}$$

where r_t is the gross interest rate, and x denotes the fiscal instrument considered: $x_t = G_t$ for the government expenditure multiplier, $x_t = \tau_t^w \bar{w} \bar{H}$ for the labor tax multiplier, and $x_t = \frac{Trans_t}{P_t}$ for the transfer multiplier.[13] Y_t^s and x_t^s indicate values taken when there is a fiscal shock, and Y_t^{ns} and x_t^{ns} indicate values that would have occurred in the absence of a fiscal shock.

Figure 1 presents fiscal multipliers that we find under our benchmark calibration for different planning horizons. Government expenditure multipliers, labor tax multipliers and transfer multipliers are depicted in respectively panels (a), (b) and (c). The dashed blue curves present impact multipliers. In all figures, the effect of planning horizons on impact multipliers is negligible, the only exception being very short (less than 15 quarters) planning horizons.

This is different for the other two curves which represent cumulative present value multipliers in the medium run. The dotted orange curves depict cumulative multipliers after 12 quarters, whereas the solid green curves depict cumulative multipliers after 20 quarters. In all graphs, there is a clear effect of the planning horizon on the medium-run cumulative multipliers.

In particular, in panel (a) of Figure 1 it can be seen that once agents have a planning horizon that is shorter than approximately 50 quarters, the medium-run cumulative effect

[13] We multiply taxes by $\bar{w}\bar{H}$ to get a change in tax income due to a changed tax rate, rather than the change in the tax rate itself. This facilitates compatibility with changes in government spending and transfers. Using the definitions of (log)-linearized variables, we can calculate the labor tax multipliers as $\Gamma_{t+k}^{y} = \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1} \right) \left(\hat{Y}_{t+j}^{s} - \hat{Y}_{t+j}^{ns} \right) / \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1} \right) \left(\bar{w}(\tilde{\tau}_{t+j}^{w,s} - \tilde{\tau}_{t+j}^{w,ns}) \right)$, the government spending multiplier as $\Gamma_{t+k}^{y} = \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1} \right) \left(\hat{Y}_{t+j}^{s} - \hat{Y}_{t+j}^{ns} \right) / \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1} \right) \left(\tilde{g}_{t+j}^{s} - \tilde{g}_{t+j}^{ns} \right)$, and the transfer multipliers as $\Gamma_{t+k}^{y} = \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1} \right) \left(\hat{Y}_{t+j}^{s} - \hat{Y}_{t+j}^{ns} \right) / \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1} \right) \left(\tilde{\tau}_{t+j}^{r,s} - \tilde{\tau}_{t+j}^{r,ns} \right)$. In computing the multipliers, we use the realized real interest rates under the transition path with the fiscal shock.

of a change in government spending on output starts becoming smaller. For a planning horizon of 20 periods, the 20-quarter cumulative government spending multipliers are more than 0.1 lower than for large planning horizons. Planning horizons of this kind of magnitude can definitely be considered relevant, as many households report relatively low values when they are asked about their financial planning horizon. Hong and Hanna (2014), for example, report a median reported planning horizon of "the next few years" in the Survey of Consumer Finances (SCF).

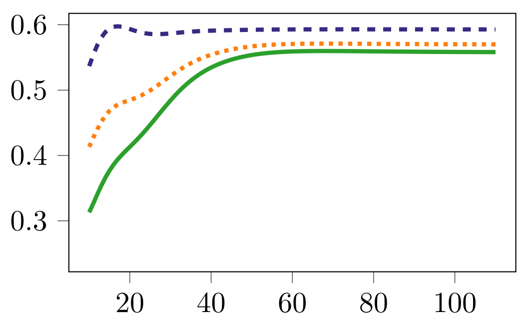
A similar pattern can be seen in panel (b) for labor-tax multipliers. However, here cumulative multipliers become more negative for shorter planning horizons, meaning that they become larger in absolute value. Shorter planning horizons hence lead to a smaller medium-run cumulative effect of government spending changes on output but to a larger medium-run cumulative effect of labor tax changes on output.

Further, comparing the multipliers in panel (a) and panel (b), it can be observed that both impact multipliers and cumulative multipliers are lower in absolute value for government spending than for labor taxes. Moreover, the difference between the cumulative multipliers of the two instruments becomes larger for shorter planning horizons. We will investigate this further in Section 5.

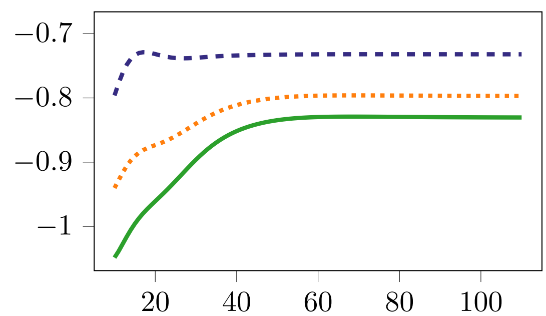
Finally, turning to panel (c) of Figure 1, a clear deviation of Ricardian equivalence can be observed for shorter planning horizons. For planning horizons of 50 and larger, the effect of a change on transfers is practically zero, both in the short- and in the medium-run. However, for shorter planning horizons, the medium-run transfer multiplier becomes negative. Although the multiplier is never more negative than -0.075, the deviation from Ricardian equivalence for shorter planning horizons can be considered economically significant.

4.2 Impulse responses

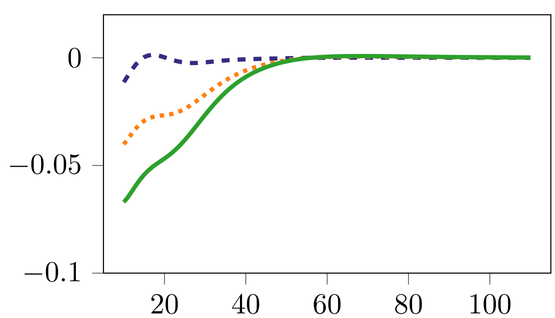
To get more intuition about the mechanisms that lead to different multipliers for different planning horizons, we now present impulse responses to a government spending shock, a labor tax shock and a transfer shock. This is done in Figures 2, 3 and 4, respectively. In particular, Figure 2 presents the case of a 1% cut in government spending, while Figure 4 presents the



(a) Government spending multipliers



(b) Labor tax multipliers



(c) Transfer multipliers

Figure 1: Fiscal multipliers (vertical axis) for government spending, labor taxes and transfers for different planning horizons (horizontal axis). Dashed blue are impact multipliers and dotted orange and solid green cumulative multipliers after respectively 12 and 20 quarters.

case of a 1% cut in transfers. In Figure 3, we present an increase in labor taxes that is scaled by $\frac{1}{\bar{w}}$ so that all fiscal shocks are comparable in terms of their (direct) effect on the government budget constraint. In each figure, the red dashed curves depict responses for a planning horizon of $T = 100$, while the solid blue curves depict the case of a planning horizon of $T = 20$.

First focusing on the longer planning horizon case (red dashed curves), it can be seen in Figure 2 that the persistent cut in government spending leads to a persistent fall in output and to a reduction in the debt level. At the same time, there is an increase in consumption and a fall in inflation and the nominal interest rate. Even though capital utilization falls as a consequence of the recession, lower real interest rates lead to a slight increase in investment and capital.

In Figure 3, the persistently higher labor income taxes reduce the stock of debt, but lead households to cut consumption, and firms to increase prices. Given an active monetary policy, the central bank raises the interest rate to combat the inflationary pressures. The resulting increase in the real interest rate and in capital utilization lead households to cut investment, so that the capital stock goes down. These eventualities lead to a persistent fall in output.

Finally, in Figure 4, it can be seen that no variable other than the stock of government debt visibly responds to a persistent decrease in transfers when planning horizons are long enough (red dashed lines). This reflects that Ricardian equivalence holds in our model when agents have very long or infinite planning horizons. In the current model specification, a planning horizon of 100 periods, hence, seems to be enough for agents to be almost fully Ricardian.

Next, we turn to the comparison between impulse responses of long planning horizons and those of shorter (20 quarters) planning horizons. The differences that arise here, are mainly driven by different expectations of agents with different planning horizons. As discussed in Section 3.5, a finite planning horizon not only limits an agent's ability to make optimal decisions but also her ability to form rational expectations. This is because forming fully rational expectations requires taking account of the future evolution of all variables up to infinity.

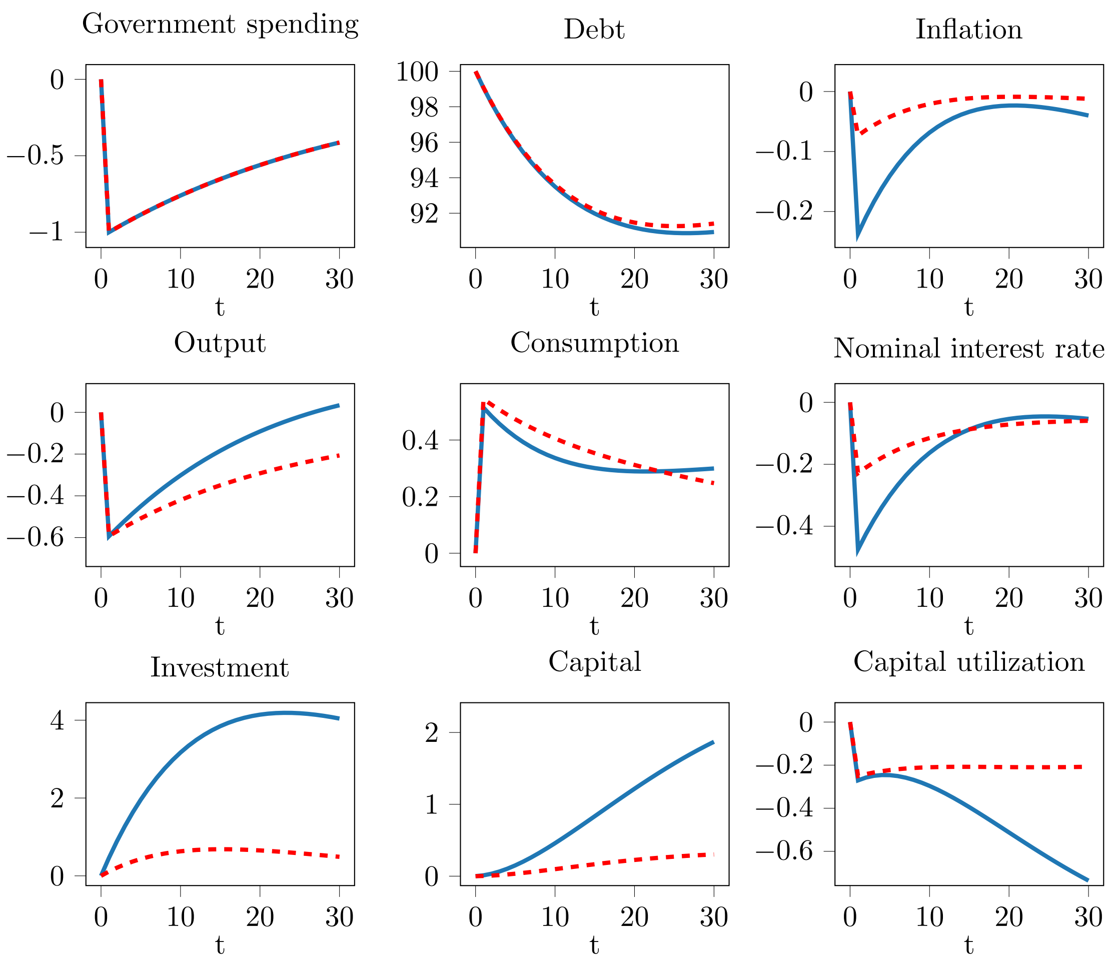


Figure 2: Impulse responses after a persistent cut in government spending. The solid blue curves depict the case of a shorter planning horizon ($T = 20$) and the dashed red curves the case of a long planning horizon ($T = 100$).

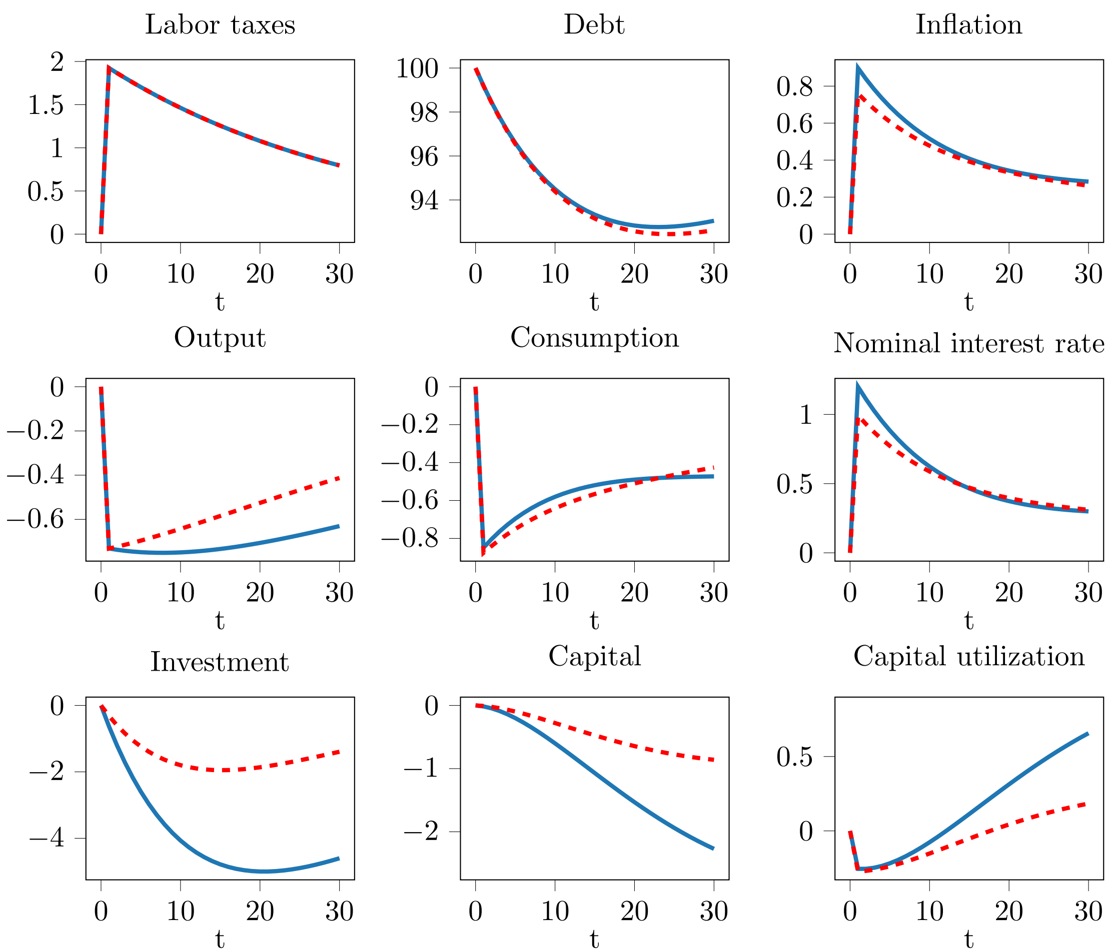


Figure 3: Impulse responses after a persistent increase in labor taxes. The solid blue curves depict the case of a shorter planning horizon ($T = 20$) and the dashed red curves the case of a long planning horizon ($T = 100$).

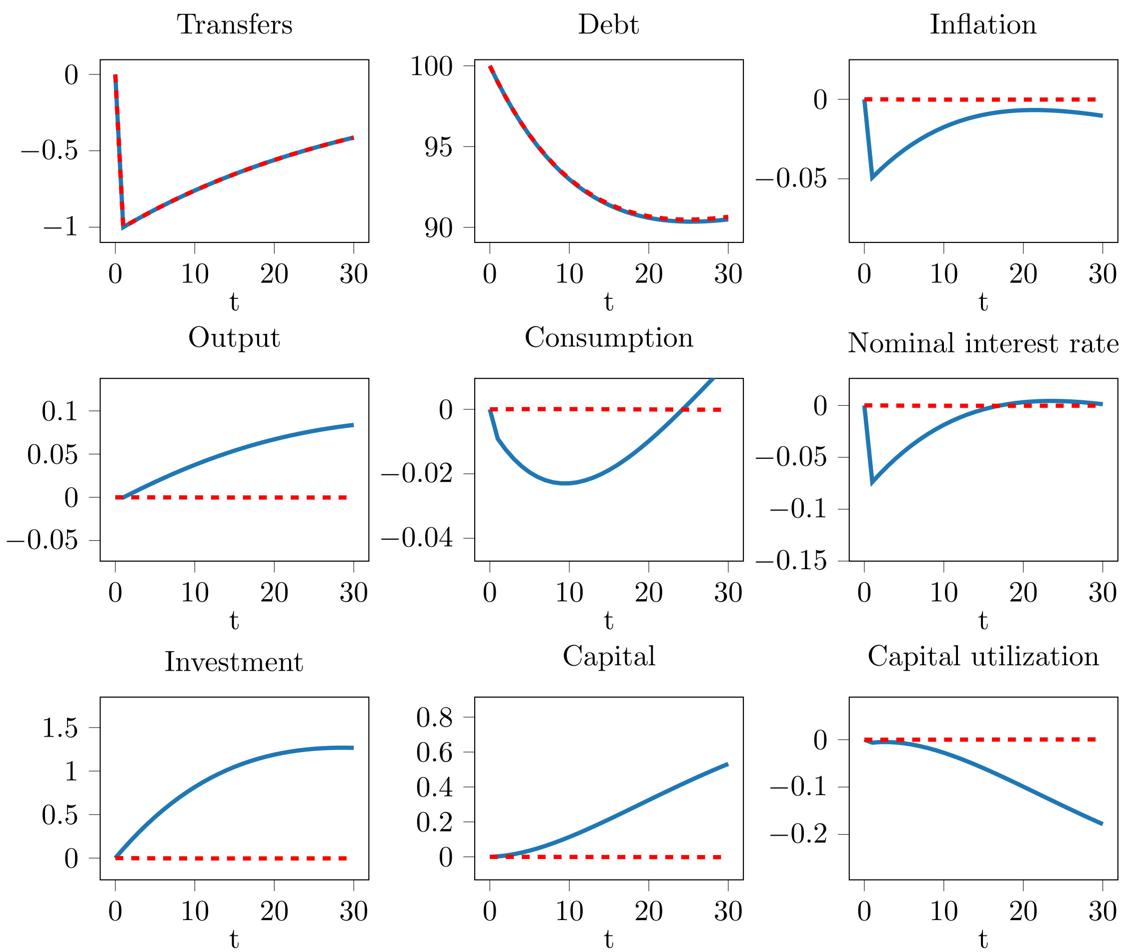


Figure 4: Impulse responses after a persistent cut in transfers. The solid blue curves depict the case of a shorter planning horizon ($T = 20$) and the dashed red curves the case of a long planning horizon ($T = 100$).

Agents with a planning horizon of $T = 100$ use the model equations of 100 different quarters to form expectations about all variables in all these periods. Using so many model equations and sophisticatedly thinking about all these 100 quarters moves their expectations very close to the fully rational expectations benchmark.

Agents with a planning horizon of $T = 20$, on the other hand, have the cognitive ability to sophisticatedly think about what will happen for the next 20 quarters only. Therefore, when forming expectations about what happens 20 quarters from now, they ignore any effects on period 20 variables that arise due to expectations formed in period 20 about later periods. This is because they lack the cognitive ability to form accurate expectations about what will happen in these periods and hence on what expectations formed in period 20 about periods 21 to 40 should be. Since the model is highly persistent, all variables will actually deviate from steady state in periods 21 through 40 significantly. Not being able to take this into account biases the expectations of agents with short planning horizons.

In particular, it turns out that households with a planning horizon of 20 periods overestimate the effect that fiscal shocks will have on inflation and the nominal interest rate. Therefore, for the case of a negative shock to government spending, they expect a more negative path of the real interest than what the fully rational expectations path would be.

As can be seen in the solid blue curve in the bottom left panel of Figure 3, this leads agents to invest more in response to a negative shock to government spending than in the case of a large planning horizon. Higher investment accumulates over time and leads to a considerably less negative path of output compared to the case of long planning horizons.[14] The persistent increase in investment further leads to a larger accumulation of capital and a fall in capital utilization.

For the case of labor tax increases, an overestimation of the responses of inflation and the nominal interest rate implies that agents expect a path of the real interest rate that is too high. This leads them to lower investment more than agents with long planning horizons, which causes a deeper recession. At the same time, capital falls more, and capital utilization

[14]Note that, at the same time, deviations from model consistent expectations about other variables lead to lower consumption than in the case of infinite planning horizons. This effect is, however, smaller and the final effect of the planning horizon on the government expenditure multiplier is dominated by the investment response.

increases more when agents have shorter planning horizons.

Following spending cuts, the demand shift due to the spending cuts starts to be dominated by the impact of forecast errors when planning horizons become shorter. In particular, the adverse effect on demand is alleviated due to the fact that households expect even lower real rates when they have shorter planning horizons compared to the case with long horizons, as discussed above.[15] This causes households to raise investment more, thus alleviating the adverse effects on demand and lowering the spending multiplier. Similarly, when the government raises labor income taxes, households with short horizons anticipate higher real rates than under longer planning horizons. This causes a larger drop in investment and in aggregate demand. This is the reason for the deeper recessions after tax hikes when horizons become shorter.

Let us now consider the response of agents with a planning horizon of $T = 20$ to a transfer shock. The fact that this shock leads the debt level to still considerably be different from its steady state level in the periods after the agent's horizon prevents agents from forming fully rational expectations. As a result, they do not expect all variables other than debt and transfer to remain at zero but, instead, expect a negative path of inflation and the nominal interest rate. This leads them to increase investment in response to a cut in transfers, which results in an increase in output. This implies a negative transfer multiplier for shorter planning horizons.

One might think that our results and the mechanisms behind these are similar to those in the Blanchard-Yaari framework where households have finite lifetimes. However, the intuition behind the results obtained above is different than that in the Blanchard-Yaari setup in various respects. In the latter setup, households commit to a specific path of consumption and do not make expectational errors (i.e. they have fully rational expectations). Obviously, finite lifetimes imply that households are non-Ricardian which means that bond holdings become net wealth. At the same time, finite lifetimes create a stronger investment crowding-in effect than infinite lifetimes. As a result, investment increases more than under infinite lifetimes. Therefore, the recession is milder as lifetimes become shorter. Similar reasoning applies in the

[15] Importantly, this effect is stronger the more persistent the spending cuts are. This will be discussed in detail in Section 5.1.

case of labor income tax hikes or after increases in transfers. In our setup, instead, the driving force behind the different investment responses and multipliers are the cognitive limitations of households that result in expectational errors.

Another crucial difference between our setup and Blanchard-Yaari is that the interaction of the persistence of the fiscal plan with the planning horizon matters for the comparison between spending cuts and tax hikes, as we show below. This is not true in the Blanchard-Yaari setup. Moreover, the notion of the horizon of the household in our setup is completely different from that in the Blanchard-Yaari setup. In the latter, the horizon has the meaning of the expected lifetime or the effective planning horizon corresponding to the average working life, as in Trabandt and Smets (2012) and Mavromatis (2020). Importantly, in this setup and in the absence of bequest motives, households do not care about their end of lifetime wealth and they do not need to form expectations about what happens after that. In our framework, instead, they care about what happens after the end of their planning horizon as they will be alive. They form thus expectations about what they believe the state of the economy to be after the end of their horizon. The shorter the horizons the more wrong these expectations are and hence the larger their forecast errors. This feature is absent in Blanchard-Yaari. As we show in our analysis in the following section, the planning horizons, their associated forecast errors and their interactions with various parameters affect the effects of fiscal policy to a great extent.

The mechanism behind our results is also quite different from that in models with rule-of-thumb consumers. As these consumers consume only their disposable income, do not accumulate wealth and do not form any expectations, any mechanism that goes through expectations is absent for them. This in contrast to our model, where cognitive limitations and the resulting expectations are the main drivers of results. Moreover, models with rule-of-thumb consumers exhibit a dampened reaction of investment to fiscal changes rather than an amplified reaction (which is what we find). This is because rule-of-thumb consumers do not make any investment, independent of what happens to fiscal policy and other variables.

5 Comparing government expenditure and labor tax multipliers

In Section 4, we found that under the benchmark calibration, government spending multipliers are lower in absolute value than labor tax multipliers. Moreover, for medium-run cumulative multipliers, this difference is larger than for short-run multipliers. Finally, we found that the differences in medium-run multipliers are amplified when agents have shorter planning horizons. This implies that labor tax increases, as a means to consolidate government debt, are considerably more harmful to output than government spending cuts.

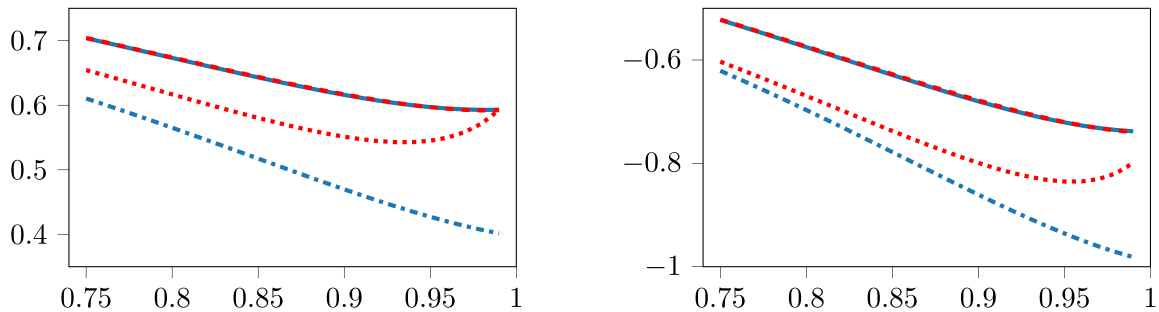
In this section, we investigate why changes in government spending impact output less than changes in labor taxes and what role is played here by agents' planning horizon. We do so by studying, one by one, different parameterizations and features of the model that could potentially reverse this ranking of government spending and labor tax multipliers.

We start, in Section 5.1, with studying the role of the persistence of the fiscal shocks. In Section 5.2, we consider what happens if we introduce sticky wages while in Section 5.3, we vary the inter-temporal substitution of households.

5.1 Persistence of fiscal shocks

Figure 5 presents multipliers for different values of the persistence parameter in the fiscal shocks. Panels (a) and (b) correspond to government spending and labor tax multipliers, respectively. On the horizontal axis in each panel, we vary the persistence from 0.75 to 0.99. Impact multipliers (dashed) and 20-quarter cumulative multipliers (dotted) for the case of $T = 100$ are depicted in red. Impact and 20-quarter multipliers for the case of $T = 20$ are depicted in solid and dash-dotted blue, respectively.

In the figure, it can be seen that the impact multipliers for $T = 20$ and $T = 100$ lie on top of each other and are downward sloping. For both planning horizons, the downward slope of impact multipliers in both panels implies that more persistent fiscal shocks lead to smaller impact multipliers for government spending but to larger impact multipliers, in absolute value, for labor tax cuts.



(a) Government spending multipliers

(b) Labor tax multipliers

Figure 5: Fiscal multipliers (vertical axis) for government spending and labor taxes for different values of the persistence of the fiscal shocks (horizontal axis). The solid blue and dashed red curves correspond to the impact multipliers for respectively $T = 20$ and $T = 100$. The dash-dotted blue and dotted red curves depict the respective medium-run cumulative multipliers (after 20 quarters).

We further find in Figure 5 that if the persistence of the fiscal shocks is lower than 0.85, impact multipliers of labor taxes become smaller in absolute value than those of government expenditures. It hence follows that the finding that government expenditure multipliers are smaller in absolute value than those of labor taxes is for a large part driven by the persistence of fiscal shocks.

The cumulative multiplier curves are also generally downward sloping in both panels. However, the blue and red curves now do not lie on top of each other. After a government spending shock, the cumulative multiplier is smaller for short planning horizons compared to longer planning horizons. At the same time, shorter planning horizons lead to more negative labor tax multipliers. Moreover, the gaps between the cumulative multipliers for the different horizons widen as the persistence of the fiscal shock increases.

Alesina et al. (2017) and Alesina et al. (2018) show that spending cuts are less recessionary the more long-lived they are, while tax hikes are more recessionary the longer lasting are the increases in taxes. In fact, when spending cuts are very persistent, agents anticipate a very persistent drop in interest rates. Investment then increases more, the more persistent are those cuts. As a result, the adverse demand effect is alleviated, and the output contracts less. At the same time, more persistent spending cuts lead to a larger increase in wages and lower labor demand because households are expected to pay less lump sum taxes in the future.[16]

[16]In our model, this increase in wages arises because of labor unions that are optimizing the expected utility

Persistent tax hikes, instead, lead to a more persistent substitution effect between labor and leisure. In our model, this substitution is made *indirectly* by labor unions who set wages in order to maximize the utility of households. In particular, persistently higher labor taxes will induce labor unions to set higher wages, so that, given the labor demand curve of firms, households will have to supply less labor.[17] Moreover, the persistent increase in prices due to persistently higher marginal costs creates expectations of a long-lived rise in the real interest rate. As a result, the more persistent the tax hikes are, the deeper is the induced recession.

For the case of a planning horizon of $T = 100$ (dotted red), there is however a reversal around 0.97. When the persistence becomes larger than that, the 20-quarter cumulative multiplier becomes somewhat smaller again for government expenditure shocks and less negative for labor tax shocks. This result reflects a more general property of infinite horizon DSGE models. Alesina et al. (2017), for example, also find such a reversal in their model when adopting the Christiano et al. (2011) calibration. Interestingly, our results show that when horizons become shorter (i.e. T=20) the reversal disappears and multipliers change monotonically with the persistence of fiscal plans. In fact, for a positive labor tax shock with persistence of 0.99, we find that, for shorter planning horizons, output is not only very slow to recover (as in Figure 3) but that it actually keeps decreasing further for a number of periods. This leads to very negative cumulative multipliers.[18]

Moreover, the slopes of the dash-dotted blue curves are somewhat steeper than those of the dotted red curves, also for lower persistence levels. This implies that, generally, for shorter planning horizons, more persistence in fiscal shocks leads to a larger change in medium-run multipliers than for longer horizons. Hence the difference in multipliers between shorter and longer planning horizons becomes larger as the persistence of shocks increases.

The above implies that, for shorter planning horizons, persistent tax-based consolidation packages are more recessionary in the medium run than spending based ones, especially for very high persistence levels. This difference between the two fiscal instruments then is

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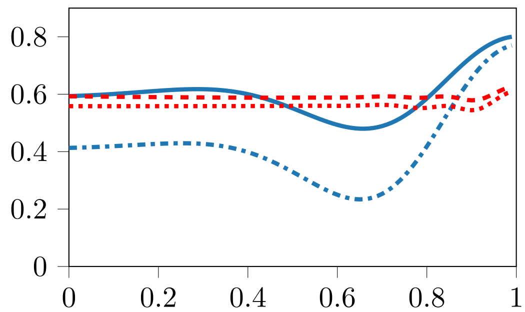
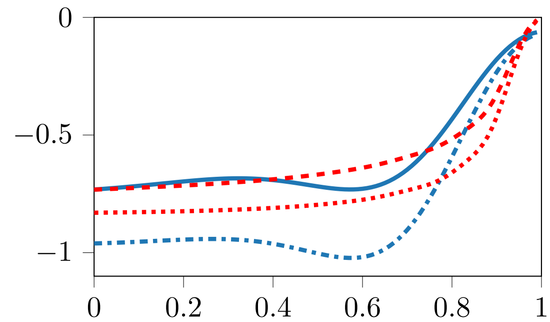
of households. In a model without labor unions, an equivalent increase in wages would arise due to a lower labor supply decision of the households themselves.

[17]In a model without labor unions, the same substitution from labor to leisure would be obtained *directly*, through the labor supply decision of households.

[18]Using a New-Keynesian DSGE model, Alesina et al. (2017) show that output contracts for more than three years after labor income tax hikes. Mountford and Uhlig (2009) find similar results.



(a) Government spending multipliers



(b) Labor tax multipliers

Figure 6: Fiscal multipliers (vertical axis) for government spending and labor taxes for different levels of wage stickiness (horizontal axis). The solid blue and dashed red curves correspond to the impact multipliers for respectively $T = 20$ and $T = 100$. The dash-dotted blue and dotted red curves depict the respective medium-run cumulative multipliers (after 20 quarters).

considerably larger than under longer planning horizons.

5.2 Wage stickiness

So far, we have assumed that wages were flexible. Now, we consider how multipliers change when wages are sticky. In particular, we assume Calvo wage stickiness where labor unions have a probability ω_w of not being able to reset the wage at any given time t. Details of this model specification can be found in Appendix C.1.

In Figure 6, we present impact and 20-quarter cumulative multipliers for different degrees of wage stickiness (along the horizontal axis). Again, impact multipliers and 20-quarter cumulative multipliers for the case of $T = 100$ are depicted in respectively dashed and dotted red, whereas impact and 20-quarter multipliers for the case of $T = 20$ are depicted in solid and dash-dotted blue.

It can immediately be seen in Figure 6 that for low levels of wages stickiness (below 0.4) none of the multipliers are significantly affected compared to the case of flexible wages. Moreover, government spending multipliers for the case of $T = 100$ (red curves in the left panel) are also insensitive to larger degrees of wage stickiness.

Labor tax multipliers for the case of $T = 100$, on the other hand, considerably change when wages become very sticky. Both red curves in the right panel of Figure 6 go up considerably, especially for values of wage-stickiness above 0.8, implying lower multipliers in absolute value.

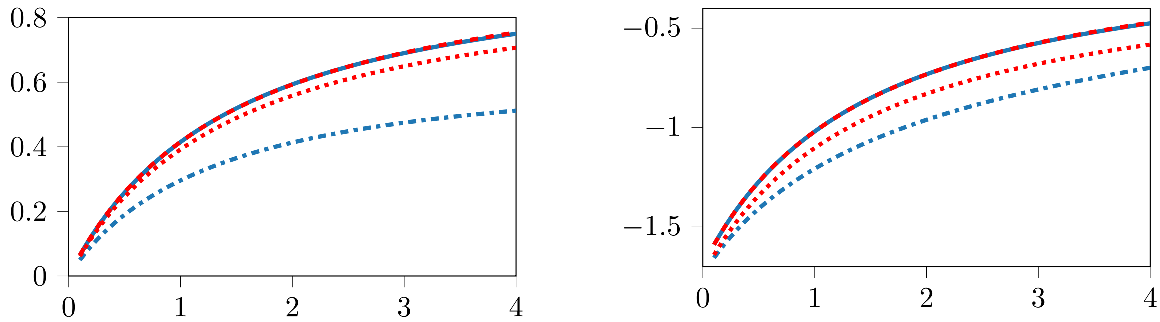
In fact, as wage stickiness goes to 1, both the impact multiplier and the 20-quarter cumulative multiplier go to 0. When wages are (somewhat) flexible, (some) labor unions bargain higher wages when labor taxes increase in order to offset the drop in households' labor income. The resulting higher real wages lead to lower labor demand and a decline in firms' profits, causing a recession. Therefore, the more unions can re-optimize, the larger the recession and the larger the multiplier. When, on the other hand, wagers are not at all allowed to change, firms choose the same labor demand and same production, independent of the labor tax rate. In this case, no recession arises and the multiplier is 0.

Next, turning to the case of $T = 20$ it can be seen that the effect of wage stickiness on the labor tax multipliers is similar to the case of $T = 100$. However, for government spending multipliers a different pattern arises. For larger values of ω_w, these multipliers first become smaller and then considerably larger for values of ω_w above 0.9. This implies that for shorter planning horizons, tax-based consolidations outperform spending-based consolidations to an even larger extent when wages are extremely sticky compared to the case of larger planning horizons. However, for wage stickiness around $\omega_w = 0.7$, the opposite case arises. Here, shorter planning horizons lead spending-based consolidations to outperform tax-based consolidation by more than would be the case under larger planning horizons.

5.3 Intertemporal substitution

Finally, we present, in Figure 7, multipliers for different values of σ, the inverse of the elasticity of intertemporal substitution. Impact and 20-quarter multipliers for $T = 20$ and $T = 100$ are presented by the same line styles and colors as in the previous two subsections.

In Figure 7, it can be seen that larger values of σ (lower elasticity of intertemporal substitution) lead to larger government expenditure multipliers and to less negative labor tax multipliers. The reason for this is that agents with a lower elasticity of intertemporal substitution (higher σ) smooth consumption more and, therefore, respond less strongly to temporary changes in their expected future disposable income and in expected future real interest rates. Hence, agents with a low elasticity of inter-temporal substitution increase their consumption by less in response to the anticipated low future real interest rates implied by spending cuts.



(a) Government spending multipliers

(b) Labor tax multipliers

Figure 7: Fiscal multipliers (vertical axis) for government spending and labor taxes for different values of the inverse of the elasticity of intertemporal substitution (horizontal axis). The solid blue and dashed red curves correspond to the impact multipliers for respectively $T = 20$ and $T = 100$. The dash-dotted blue and dotted red curves depict the respective medium-run cumulative multipliers (after 20 quarters).

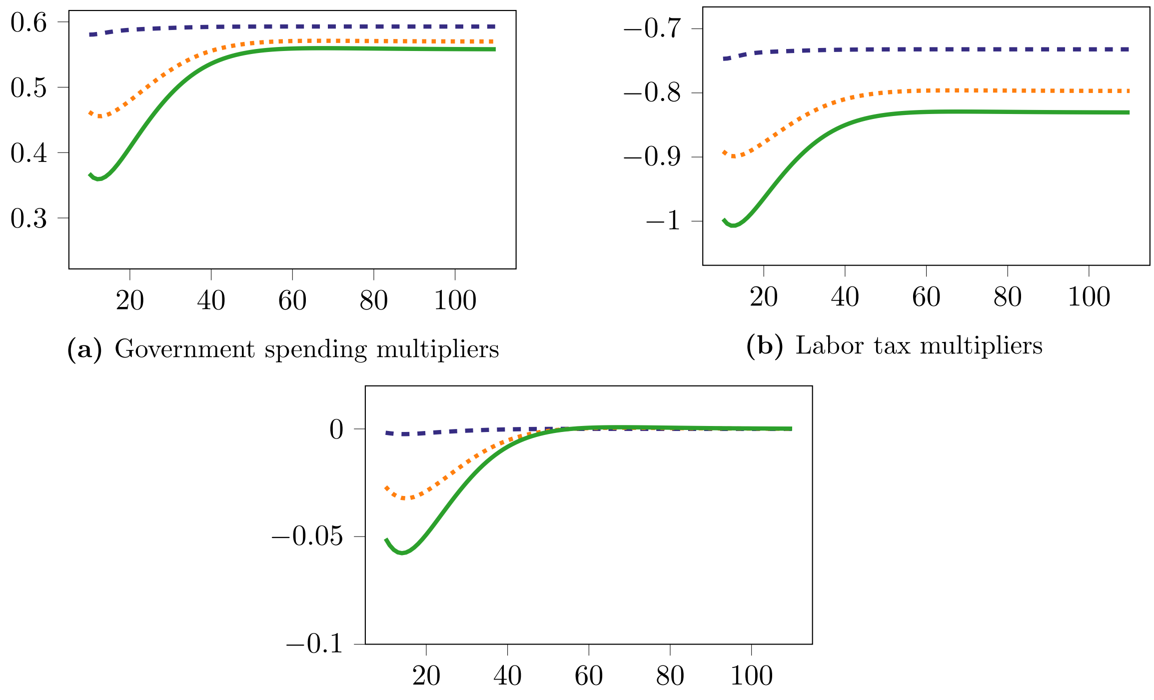
Similarly, agents with a lower elasticity of intertemporal substitution (higher σ) reduce their current consumption less when they anticipate a temporary lower income due to tax increases.

As a result, the recession after a spending cut becomes deeper and the recession after a tax increase becomes smaller when σ is larger. For very large values of σ (around 4) impact multipliers of government spending become larger in absolute value than those of labor taxes. This holds for both short and long planning horizons.

Further, for $T = 100$, the same conclusion can be drawn for 20-quarter cumulative multipliers. For shorter horizons, on the other hand, we find that σ should be even larger for tax-based consolidations to become less harmful than spending-based consolidations in the medium run. This is because under shorter planning horizons the investment response of households is amplified, leading to lower expenditure and more negative labor tax multipliers, as was discussed in Section 4.2.

6 Robustness: Firms with infinite planning horizons

As discussed in Section 4, the finding that multipliers are different for shorter horizons is mainly driven by the expectations of households and their resulting investment decisions. The fact that firms also have finite planning horizons in our model plays little to no role for the results of the paper.



(a) Government spending multipliers

(b) Labor tax multipliers

(c) Transfer multipliers

Figure 8: Fiscal multipliers (vertical axis) for the case of fully rational firms (with an infinite planning horizon) for government spending, labor taxes and transfers for different planning horizons (horizontal axis). Dashed blue are impact multipliers and dotted orange and solid green cumulative multipliers after respectively 12 and 20 quarters.

In this section, we show this with a robustness exercise where we let firms have an infinite planning horizon and fully rational expectations. Pricing decisions are then made based on the discounted sum of all expected future marginal costs until infinity.

Households, on the other hand, are still assumed to have finite planning horizons and form expectations accordingly. Since they do not have the cognitive ability to anticipate how fully rational firms with infinite planning horizons will set their prices in future periods, households still need to rely on the same expectation algorithm outlined in section 3.5. Households thereby implicitly assume that firms make their decisions and form expectations with the same planning horizon and cognitive limitations as the households themselves. Given the limitations of their planning horizon, these are the most sophisticated higher-order beliefs about the expectations and future decisions of firms that households are able to form.

Comparing Figure 8 with Figure 1 it, first of all, can be seen that the dashed purple impact multiplier curves are even flatter when firms have infinite planning horizons. Apparently, the difference in impact multipliers for very short horizons in Figure 1 are driven by an interaction between boundedly rational firms and boundedly rational households. When firms have infinite planning horizons and fully rational expectations, this impact effect largely disappears.

This finding also somewhat alters the shape of the cumulative multipliers after 12 and 20 quarters (dotted orange and solid green respectively). However, the main results of the paper still hold when firms have infinite planning horizons: there is a clear relation between the planning of households and the cumulative multipliers. If anything, for planning horizons between 15 and 50 quarters, this relation becomes even cleaner and smoother. Moreover, the exact sizes of the multipliers for different planning horizons are also very similar in Figures 1 and 8.

Finally, we find that all exercises of Section 5 lead to qualitatively the same and quantitatively very similar results when firms have infinite planning horizons as when they do not.[19]

7 Conclusion

In this paper, we analyze the effects of finite planning horizons on fiscal multipliers in a closed economy dynamic stochastic general equilibrium model with nominal rigidities, investment adjustment costs and labor unions. In particular, agents are infinitely lived but optimize over a finite number of periods. This is because they do not have the cognitive ability to form expectations and make rational inferences over an infinite horizon. Because of these cognitive limitations, agents also make expectational errors which are the main driver of differences in fiscal multipliers. Moreover, we find that all our main results are driven by the finite horizons of households, as we show that similar results are obtained whether or not firms also have finite planning horizons.

We show that especially medium-run fiscal multipliers are considerably affected by the planning horizon. Medium-run government spending multipliers become smaller for shorter planning horizons, whereas labor tax multipliers become more negative (i.e. larger in absolute

[19]Results are available on request.

value). Finite planning horizons can hence contribute to explaining why, in the empirical literature, spending-based consolidations are found to be considerably less recessionary than consolidations based on labor tax increases.

We also find a breaking of Ricardian equivalence for shorter planning horizons, which is most clearly reflected in negative transfer multipliers. The main driver of this Ricardian equivalence are expectational errors due to the cognitive limitations of finite planning horizons. This mechanism is, therefore, quite different than in models with a Blanchard-Yaari setup or with Rule of thumb consumers.

We provide further insight into why government spending multipliers are smaller in absolute value than labor tax multipliers by varying, one by one, different features of the model that might reverse this finding. In particular, we find that the persistence of fiscal plans plays an important role, and that the effects of this persistence on the difference between government spending and labor tax multipliers are amplified by finite planning horizons. Additionally, we find that very large degrees of wage stickiness make government spending multipliers larger and labor tax multipliers smaller in absolute value. However, for intermediate values of wage-stickiness, the difference between the two multipliers actually becomes larger under finite planning horizons. Finally, a smaller elasticity of intertemporal substitution in households' preferences leads to a relative advantage for tax-based consolidations. Under finite planning horizons, medium-run government spending multipliers remain, however, lower in absolute value than labor tax multipliers for all values of this parameter that we consider.

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Appendices

A Steady state

In this section, we derive the steady state around which the model is log-linearized, where gross inflation equals 1.

Evaluating (38) at the zero inflation steady state gives

$$\bar{mc} = \frac{\theta - 1}{\theta} = \zeta^{-\zeta} (1-\zeta)^{-(1-\zeta)} \bar{w}^{1-\zeta} \left(r^K\right)^{\zeta}. \tag{46}$$

From the first order conditions of the households it follows that in this steady state we must have

$$\bar{R} = \frac{\bar{\Pi}}{\beta}, \tag{47}$$

$$r^K = \frac{1-(1-\delta)\beta}{\beta \bar{Z}}, \tag{48}$$

$$\bar{C}^{-\sigma} = \bar{\gamma}, \tag{49}$$

$$\bar{r}^K = \gamma_1 + \gamma_2 \left(\bar{Z} - 1\right). \tag{50}$$

Normalizing Z to one yields:

$$\bar{r}^K = \gamma_1. \tag{51}$$

Having set $Z = 1$ simplifies the resource constraint at the steady state which reads as:

$$\bar{Y} = \bar{C} + \bar{I} + \bar{Y}\bar{g}. \tag{52}$$

Moreover, from the wage setting equation of labor unions, it follows that

$$\bar{C}^{-\sigma}\bar{w}\left(1 - \bar{\tau}^{w}\right) = \bar{H}^{\eta}\frac{\theta_w}{\theta_w - 1}. \tag{53}$$

Furthermore, normalizing technology A, to 1, it follows from (26) that

$$\bar{Y} = \bar{K}^{\zeta}\bar{H}^{1-\zeta}. \tag{54}$$

Using the law of motion of capital, we can express steady state investment as a function of steady state capital:

$$\bar{I} = \delta\bar{K}. \tag{55}$$

So that the resource constraint writes as:

$$\bar{Y} = \frac{1}{1 - \bar{g}}\left(\bar{C} + \delta\bar{K}\right). \tag{56}$$

Using the steady state marginal cost (46) and (51), we derive the following relation for the real wage at the steady state

$$\bar{w} = \left(\frac{\theta - 1}{\theta}\right)^{\left(\frac{1}{1-\zeta}\right)}\zeta^{\frac{\zeta}{1-\zeta}}\left(1 - \zeta\right)\gamma_1^{\frac{-\zeta}{1-\zeta}}. \tag{57}$$

From the firm's cost minimization problem we may write the steady state real wage and rental rate of capital as follows:

$$\begin{aligned} \bar{r}^{K} &= \bar{mc}\zeta\left(\frac{\bar{H}}{\bar{K}}\right)^{1-\zeta} \\ \bar{w} &= \bar{mc}\left(1 - \zeta\right)\left(\frac{\bar{K}}{\bar{H}}\right)^{\zeta}. \end{aligned} \tag{58}$$

Combining the two equations above and (51) we receive:

$$\bar{H} = \gamma_1 \frac{1}{\bar{w}} \frac{1-\zeta}{\zeta} \bar{K}. \tag{59}$$

Substituting (59) in the production function (54), we get:

$$\bar{Y} = \left(\frac{\gamma_1}{\bar{w}} \frac{1-\zeta}{\zeta} \right)^{1-\zeta} \bar{K}. \tag{60}$$

Combining (56) and (60), we receive:

$$\left(\frac{\gamma_1}{\bar{w}} \frac{1-\zeta}{\zeta} \right)^{1-\zeta} \bar{K} = \frac{1}{1-\bar{g}} \left(\bar{C} + \delta \bar{K} \right). \tag{61}$$

Solving for $\bar{C}$, we get:

$$\bar{C} = (1-\bar{g}) \left(\left(\frac{\gamma_1}{\bar{w}} \frac{1-\zeta}{\zeta} \right)^{1-\zeta} - \frac{\delta}{1-\bar{g}} \right) \bar{K}. \tag{62}$$

Plugging (62) into (53) we get:

$$\left[(1-\bar{g}) \left(\left(\frac{\gamma_1}{\bar{w}} \frac{1-\zeta}{\zeta} \right)^{1-\zeta} - \frac{\delta}{1-\bar{g}} \right) \bar{K} \right]^{-\sigma} (1-\bar{\tau}^w)\, \bar{w} = \bar{H}^{\eta} \frac{\theta_w}{\theta_w - 1}. \tag{63}$$

Exploiting relation (59), we can expand further:

$$\left[(1-\bar{g}) \left(\left(\frac{\gamma_1}{\bar{w}} \frac{1-\zeta}{\zeta} \right)^{1-\zeta} - \frac{\delta}{1-\bar{g}} \right) \bar{K} \right]^{-\sigma} (1-\bar{\tau}^w)\, \bar{w} = \left[\gamma_1 \frac{1}{\bar{w}} \frac{1-\zeta}{\zeta} \bar{K} \right]^{\eta} \frac{\theta_w}{\theta_w - 1}. \tag{64}$$

Gathering terms and solving for $\bar{K}$, we end up to the following expression:

$$\bar{K} = \left\{ \left[(1-\bar{g}) \left(\left(\frac{\gamma_1}{\bar{w}} \frac{1-\zeta}{\zeta} \right)^{1-\zeta} - \frac{\delta}{1-\bar{g}} \right) \right]^{-\sigma} (1-\bar{\tau}^w)\, \bar{w} \left[\frac{\gamma_1}{\bar{w}} \frac{1-\zeta}{\zeta} \right]^{-\eta} \frac{\theta_w - 1}{\theta_w} \right\}^{\frac{1}{\eta+\sigma}}, \tag{65}$$

where the equilibrium steady state real wage is given by (57). Using (59) and (65) the equilibrium value of labor reads as:

$$\bar{H} = \frac{\gamma_1}{\bar{w}}\frac{1-\zeta}{\zeta}\left\{\left[(1-\bar{g})\left(\left(\frac{\gamma_1}{\bar{w}}\frac{1-\zeta}{\zeta}\right)^{1-\zeta} - \frac{\delta}{1-\bar{g}}\right)\right]^{-\sigma}(1-\bar{\tau}^w)\,\bar{w}\left[\frac{\gamma_1}{\bar{w}}\frac{1-\zeta}{\zeta}\right]^{-\eta}\frac{\theta_w - 1}{\theta_w}\right\}^{\frac{1}{\eta+\sigma}}. \tag{66}$$

Since we have expressed private consumption, $\bar{C}$ and investment $\bar{I}$ as a function of $\bar{K}$, we are able to pin them down by plugging (65) in (55) and (62), respectively. We can pin down output by plugging (65) into (60).

Then we turn to the government budget constraint. In steady state, it reduces to

$$\bar{b} = \frac{\bar{\Pi}\left(\bar{\tau}^W\bar{w}\frac{\bar{H}}{\bar{Y}} - \bar{g}\right)}{1-\beta}, \tag{67}$$

where we assume that steady state lump sum taxes and transfers are 0, and where we used (47) to substitute for the gross interest rate. Having pinned down $\bar{w}$, $\bar{H}$ and $\bar{Y}$ above, we are able to also pin the steady state debt ratio down. Clearly, the model features a unique steady state.

B Derivation of Household's Value Function

Households solve a maximization problem that includes utility within the planning horizon as well as a valuation of the state at the end of the horizon. This valuation can be seen as an approximation of the utility that households are expected to obtain after the end of their planning horizon, given their accumulated wealth at the end of their horizon. In particular, in the spirit of Woodford (2018), we assume that this approximation is obtained by using a value function which is consistent with a future stationary equilibrium where there are no shocks or other frictions and all variables other than then the household's own consumption, bond capital holdings and investment/capital decision are at the steady state.[20] Here we also assume that capital utilization is fixed at $Z^i = 1$, and that there are no investment adjustment costs: $S(.) = 0$. The F.O.C. w.r.t. capital and investment, (14), at the stationary equilibrium

[20]Given that we also assume a non-stochastic steady state as well as no growth in total factor productivity in the long-run, all variables do not grow at a stationary equilibrium, like the steady state.

then implies:

$$\frac{R^k}{P} = \gamma_1. \tag{68}$$

Our considerations regarding households' assumptions about what happens after the end of their horizon imply that the maximization problem can be cast as follows:

$$V(B,K) = \max_C \{u(C) + \beta V(B', K')\} \quad s.t.\ K' + \frac{\beta}{\bar{\Pi}} B' = \bar{X} + (1 - \delta + \gamma_1) K + \frac{B}{\bar{\Pi}} - C, \tag{69}$$

where the last term of the RHS of the budget constraint, $\bar{X}$, consists of labor income, profits and taxes/transfers which the household considers to be at the stationary equilibrium. To derive the value function, $V(.)$, we implement value function iteration. We start with:

$$V^{i,0}(B,K) = 0. \tag{70}$$

Here, the optimal consumption at the initial period (period zero) is determined by the household's budget constraint, assuming $K' = B' = 0$:

$$C = (1 - \delta + \gamma_1) K + \frac{B}{\bar{\Pi}} + \bar{X}. \tag{71}$$

At the steady, we have that $(1 - \delta + \gamma_1) = \frac{1}{\beta}$. Therefore, optimal consumption at iteration 0 reads also as follows:

$$C = \frac{1}{\beta} K + \frac{B}{\bar{\Pi}} + \bar{X}. \tag{72}$$

The above expression produces an optimized value:

$$V^{i,1}(B,K) = u\left(\frac{1}{\beta} K + \frac{B}{\bar{\Pi}} + \bar{X}\right), \tag{73}$$

or

$$V^{i,1}(B,K) = u\left(\frac{1}{\beta}\left(K + \frac{\beta B}{\bar{\Pi}}\right) + \bar{X}\right). \tag{74}$$

which is the updated value function in the household's new maximization problem in iteration

1. Using thus this value function, the household's maximization problem reads as follows:

$$V(B,K) = \max_{C,K',B'} \{u(C) + \beta u\left(\frac{1}{\beta}\left(K' + \frac{\beta B'}{\bar{\Pi}}\right) + \bar{X}\right)\}$$
$$s.t.\ K' + \frac{\beta}{\bar{\Pi}}B' = \bar{X} + \frac{1}{\beta}K + \frac{B}{\bar{\Pi}} - C. \tag{75}$$

Plugging the budget constraint into the value function, the problem writes as follows:

$$V(B,K) = \max_{C} \{u(C) + \beta u\left(\frac{1}{\beta}\left(\bar{X} + \frac{1}{\beta}K + \frac{B}{\bar{\Pi}} - C\right) + \bar{X}\right)\}. \tag{76}$$

From the F.O.C. of the maximization problem above and by monotonicity of the utility function we get that:

$$C = \frac{1}{1+\beta}\left(\frac{1}{\beta}K + \frac{B}{\bar{\Pi}}\right) + \bar{X}, \tag{77}$$

and the optimal decision for B' and K' is thus:

$$K' + \frac{\beta B'}{\bar{\Pi}} = \frac{\beta}{1+\beta}\left(\frac{1}{\beta}K + \frac{B}{\bar{\Pi}}\right). \tag{78}$$

Therefore, the value function for optimization problem for iteration 2 is computed as follows:

$$V^{i,2}(B,K) = u(C) + \beta u\left(\frac{1}{\beta}\left(K' + \frac{\beta B'}{\bar{\Pi}}\right) + \bar{X}\right). \tag{79}$$

Substituting for the optimal consumption and optimal choice for the assets from (77) and (78), we get

$$V^{i,2}(B,K) = u\left(\frac{1}{1+\beta}\left(\frac{1}{\beta}K + \frac{B}{\bar{\Pi}}\right) + \bar{X}\right) + \beta u\left(\frac{1}{\beta(1+\beta)}\left(K + \frac{\beta B}{\bar{\Pi}}\right) + \bar{X}\right)$$
$$= (1+\beta)\, u\left(\frac{1}{1+\beta}\left(\frac{1}{\beta}K + \frac{B}{\bar{\Pi}}\right) + \bar{X}\right). \tag{80}$$

Continuing in a similar vein up to infinity we get that the limiting function for the optimal consumption decision is given by:

$$C = (1-\beta)\left(\frac{1}{\beta}K + \frac{B}{\bar{\Pi}}\right) + \bar{X}, \tag{81}$$

and the optimal decision for B' and K' is thus:

$$K' + \frac{\beta B'}{\bar{\Pi}} = (1-\beta)\beta\left(\frac{1}{\beta}K + \frac{B}{\bar{\Pi}}\right), \tag{82}$$

while the value function is given by:

$$V^i(B,K) = (1-\beta)^{-1} u\left((1-\beta)\left(\frac{1}{\beta}K + \frac{B}{\bar{\Pi}}\right) + \bar{X}\right). \tag{83}$$

Similar toWoodford (2018), this problem describes the optimal intertemporal consumption decision of the household assuming that all variables that the household is not directly choosing are in steady state. Under this assumption households make fully optimal decisions in steady state. Moreover, when they have more wealth at the end of the horizon, they realize how this will allow them to consume more after their horizon and hence obtain more utility. However, they are not sophisticated enough to plan how e.g. wages and profits would change after their horizon if they would have different consumption levels. The value function hence captures partly how future utility depends on end of horizon wealth, but in a boundedly rational matter that only approximates the complete true value function.

C Sticky wages

C.1 Dynamic Wage setting

In Section 5.2, we assume Calvo wage stickiness with a probability ω_w of not being able to reset the wage at any give time t. Unions then still set the nominal wage by maximizing the expected discounted sum of the utility of households subject to the households' budget constraint and the demand for labor,

$$h^j_{t+s} = \left(\frac{W^j_t}{W_{t+s}}\right)^{-\theta_w} H_{t+s}, \tag{84}$$

where H_{t+s} is defined as in (23).

When choosing the optimal nominal wage in period t, $\tilde{W}_t$, re-optimizing unions will anticipate that in the future it might not be able to re-optimize. Unions have finite planning horizons which we assume to be equal to that of households. Therefore, unions evaluates all possible situations that may arise within their planning horizon, T, using a value function in order to estimate the continuation value of the maximization problem. Hence, the maximization problem of labor unions that can re-optimize in period t receives the following form:

$$\max_{\tilde{W}_t} E_t \sum_{s=0}^{T} \omega_w^s \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \frac{\left(h_{t+s}^j\right)^{1+\eta}}{1+\eta} \right] + (\beta\omega_w)^{T+1} \upsilon \left(\frac{\tilde{W}_t}{W_{t+T}}, \frac{\tilde{W}_t}{P_{t+T}} \right). \tag{85}$$

In the maximization problem (85) above, h_{t+s}^j is given by (84). $\upsilon()$ represents the union's value function which we derive in Appendix C.2 and which receives the following form:

$$\upsilon_w^j(r,w) = \frac{1}{1-\omega_w\beta} \left[u\left((1-\bar{\tau}^w)\, w\,(r)^{-\theta_w} \bar{H} + \bar{Z} \right) - f\left((r)^{-\theta_w} \bar{H} \right) \right], \tag{86}$$

where $\bar{Z}$ captures wealth from profits, capital and bond holdings net of lump-sum taxes which the union assumes to be at the stationary equilibrium beyond its planning horizon. Given this maximization problem, the first order condition is

$$\begin{aligned} & E_t \sum_{s=0}^{T} (\omega_w\beta)^s \varpi_{t+s} h_{t+s}^j \left[\tilde{W}_t - \frac{\theta_w}{\theta_w - 1} mrs_{t+s} \right] \\ & - \frac{(\beta\omega_w)^{T+1}}{1-\omega_w\beta} \left(\bar{W}\bar{H} + \bar{Z} \right)^{-\sigma} \left(\tilde{W}_t \right)^{1-\theta_w} \left(\frac{1}{W_{t+T}} \right)^{-\theta_w} \bar{H} \\ & - \frac{(\beta\omega_w)^{T+1}}{1-\omega_w\beta} \frac{\theta_w}{\theta_w - 1} \left(\tilde{W}_t \right)^{-\theta_w(1+\eta)} \left(\left(\frac{1}{W_{t+T}} \right)^{-\theta_w} \bar{H} \right)^{1+\eta} = 0, \end{aligned} \tag{87}$$

which can be written as

$$E_t \sum_{s=0}^{T} (\omega_w \beta)^s \varpi_{t+s} \left(\frac{\tilde{W}_t}{W_{t+s}} \right)^{-\theta_w} H_{t+s} \left[\left(1-\tau_{t+s}^{w}\right) \frac{\tilde{W}_t}{P_{t+s}} - \frac{\theta_w}{\theta_w - 1} \left(\frac{\tilde{W}_t}{W_{t+s}} \right)^{-\theta_w \eta} \frac{H_{t+s}^{\eta}}{\varpi_{t+s}} \right]$$

$$+ \frac{(\beta\omega_w)^{T+1}}{1-\omega_w\beta} \bar{\varpi} \left(\frac{\tilde{W}_t}{W_{t+T}} \right)^{-\theta_w} \bar{H} \tag{88}$$

$$\left[\frac{\left(\left(1-\bar{\tau}^{w}\right) \frac{\tilde{W}_t}{P_{t+T}} \left(\frac{\tilde{W}_t}{W_{t+T}} \right)^{-\theta_w} \bar{H} + \bar{Z} \right)^{-\sigma}}{\bar{\varpi}} \left(1-\bar{\tau}^{w}\right) \frac{\tilde{W}_t}{P_{t+T}} - \frac{\theta_w}{\theta_w - 1} \left(\frac{\tilde{W}_t}{W_{t+T}} \right)^{-\theta_w \eta} \frac{\bar{H}^{\eta}}{\bar{\varpi}} \right] = 0.$$

C.2 Derivation of the Value Function of the Union

As described in the main text, we consider a continuum of unions, each of which represents a set of households specialized in given type of labor service. Unions set the wages on behalf of the households they represent. To that end, they seek to maximize the utility of the household. In deriving its value function, the union takes into account the fact that it sets the wage infrequently. Moreover, analogue to Appendix B the labor unions assumes that all variables that are not in their direct control are at the stationary equilibrium in the periods that the concern the value function. Following these assumptions the budget constraint of the household from the union perspective reads as follows:

$$C = \left(1-\tau^{w}\right) \tilde{w}^{1-\theta_w} \left(\frac{1}{\bar{W}} \right)^{-\theta_w} \bar{H} + \bar{Z}, \tag{89}$$

where we have used the demand for labor, (84), and where $\bar{Z} = \frac{1-\beta}{\beta}\bar{K} + (1-\beta)\frac{\bar{B}}{\bar{\Pi}} - \bar{T} + \bar{\Xi}$. The value function of the union thus receives the following form:

$$\begin{aligned} \upsilon_w^j\left(\tilde{w}\right) =& u(\left(1-\tau^{w}\right) \tilde{w} \left(\frac{\tilde{w}}{\bar{W}} \right)^{-\theta_w} \bar{H} + \bar{Z}) - f\left(\left(\frac{\tilde{w}}{\bar{W}} \right)^{-\theta_w} \bar{H} \right) + \omega_w \beta \upsilon_w^j\left(\tilde{w}\right) \\ &+ \left(1-\omega_w\right) \beta \upsilon_w^{opt}, \end{aligned} \tag{90}$$

where, in the equation above, $u(.)$ represents utility from consumption satisfying $u'(.) >$

$0, u''(.) < 0$ and $f(.)$ represents disutility from labor supply satisfying $f'(.), f''(.) > 0$. In the equation above, we have substituted individual labor in the disutility term by the demand for labor condition (84) evaluated at the steady state.

Analogue to the firm case in Equation (34), υ_w^{opt} represents the value in case the union can re-optimize in the next period. Again, this term is independent of current decisions and can be ignored in the derivation of the value function that we need. We therefore take

$$\upsilon_w^j(\tilde{w}, r) = \frac{1}{1 - \omega_w \beta} \left[u\left((1 - \tau^w) \tilde{w} r^{-\theta_w} \bar{H} + \bar{Z} \right) - f\left(r^{-\theta_w} \bar{H} \right) \right], \tag{91}$$

where $r = \frac{\tilde{w}}{\bar{W}}$.

D Log-linear model

D.1 Optimal consumption decision

The log-linearized optimality conditions of the household (including budget constraints) are given by

$$\hat{C}_s^i = \hat{C}_{s+1}^i - \frac{1}{\sigma}(E_t^i \hat{R}_s - E_t^i \pi_{s+1}), \qquad s = t, t+1, ...t+T-1, \tag{92}$$

$$\hat{I}_s^i = \frac{1}{1+\beta} \hat{I}_{s-1}^i + \frac{1}{\kappa(1+\beta)} \hat{Q}_s^i + \frac{\beta}{(1+\beta)} \hat{I}_{s+1}^i, \qquad s = t, t+1, ...t+T-1, \tag{93}$$

$$\hat{Q}_s^i = \beta(1-\delta) \hat{Q}_{s+1}^i + \hat{\varpi}_{s+1}^i - \hat{\varpi}_s^i + \beta r^k \hat{r}_{s+1}^k, \qquad s = t, t+1, ...t+T-1, \tag{94}$$

$$\tilde{b}_{t+T+1}^i = -\frac{\bar{K}}{\beta \bar{Y}} \hat{K}_{t+T+1} + \frac{\bar{C}}{\Upsilon \bar{Y}} \hat{C}_{t+T}^i + \frac{\bar{C}}{\sigma \Upsilon \bar{Y}} \hat{R}_{t+T}, \tag{95}$$

$$\hat{K}_{t+T+1}^i = -\frac{\beta \bar{\gamma}^{-\frac{1}{\sigma}}}{\sigma \Upsilon \bar{K}} \hat{\gamma}_{t+T}^i - \frac{\beta \bar{Y}}{\bar{K}} \tilde{b}_{t+T+1}^i, \tag{96}$$

$$\hat{I}_{t+T}^i = \hat{I}_{t+T-1}^i - \frac{1}{\kappa} \left(\sigma \hat{C}_{t+T}^i + \hat{\gamma}_{t+T}^i \right), \tag{97}$$

$$\tilde{b}^i_{s+1} = \frac{\bar{w}}{\beta}((1-\bar{\tau}^W)(E^i_t\hat{w}_s + E^i_t\hat{H}_s) - E^i_t\tilde{\tau}^W_s) + \frac{1}{\beta}\tilde{b}^i_s + \bar{b}(\hat{R}_s - \frac{1}{\beta}E^i_t\hat{\pi}_s) + \frac{\bar{\Xi}}{\bar{Y}\beta}E^i_t\hat{\Xi}^i_s$$
$$+ \frac{\gamma_1\bar{K}}{\beta\bar{Y}}\left(\hat{r}^k_t + \hat{K}^i_t\right) - \frac{\bar{C}}{\beta\bar{Y}}\hat{C}^i_s - \frac{\bar{I}}{\beta\bar{Y}}\hat{I}^i_s - \frac{1}{\beta}\tilde{\tau}_s, \qquad s = t, t+1, ...t+T. \tag{98}$$

Plugging the capital accumulation equation (5) in its log-linear form to substitute out for investment, $\hat{I}_t$, we receive the following expression for the household budget constraint:

$$\tilde{b}^i_{s+1} = \frac{\bar{w}}{\beta}((1-\bar{\tau}^W)(E^i_t\hat{w}_s + E^i_t\hat{H}_s) - E^i_t\tilde{\tau}^W_s) + \frac{1}{\beta}\tilde{b}^i_s + \bar{b}(\hat{R}_s - \frac{1}{\beta}E^i_t\hat{\pi}_s) + \frac{\bar{\Xi}}{\bar{Y}\beta}E^i_t\hat{\Xi}_s$$
$$+ \frac{\gamma_1\bar{K}}{\beta\bar{Y}}\left(\hat{r}^k_s + \hat{K}^i_s\right) - \frac{\bar{C}}{\beta\bar{Y}}\hat{C}^i_s - \frac{\bar{K}}{\beta\bar{Y}}\left(\hat{K}^i_{s+1} - (1-\delta)\hat{K}^i_s\right) - \frac{1}{\beta}\tilde{\tau}_s. \qquad s = t, t+1, ...t+T, \tag{99}$$

Taking the $\hat{K}_{t+1}$ term to the LHS we receive:

$$\tilde{b}^i_{s+1} + \frac{\bar{K}}{\beta\bar{Y}}\hat{K}^i_{s+1} = \frac{\bar{w}}{\beta}((1-\bar{\tau}^W)(E^i_t\hat{w}_s + E^i_t\hat{H}_s) - E^i_t\tilde{\tau}^W_s) + \frac{1}{\beta}\tilde{b}^i_s + \bar{b}(\hat{R}_s - \frac{1}{\beta}E^i_t\hat{\pi}_s) + \frac{\bar{\Xi}}{\bar{Y}\beta}E^i_t\hat{\Xi}_s$$
$$+ \frac{\gamma_1\bar{K}}{\beta\bar{Y}}\left(\hat{r}^k_s + \hat{K}^i_s\right) - \frac{\bar{C}}{\beta\bar{Y}}\hat{C}^i_s + \frac{\bar{K}}{\beta\bar{Y}}(1-\delta)\hat{K}^i_s - \frac{1}{\beta}\tilde{\tau}_s, \qquad s = t, t+1, ...t+T. \tag{100}$$

Gathering the $\hat{K}^i_s$ terms and using the fact $1-\delta+\gamma_1 = 1/\beta$, we receive the following:

$$\tilde{b}^i_{s+1} + \frac{\bar{K}}{\beta\bar{Y}}\hat{K}^i_{s+1} = \frac{\bar{w}}{\beta}((1-\bar{\tau}^W)(E^i_t\hat{w}_s + E^i_t\hat{H}_s) - E^i_t\tilde{\tau}^W_s) + \bar{b}(\hat{R}_s - \frac{1}{\beta}E^i_t\hat{\pi}_s) + \frac{\bar{\Xi}}{\bar{Y}\beta}E^i_t\hat{\Xi}_s$$
$$+ \frac{\gamma_1\bar{K}}{\beta\bar{Y}}\hat{r}^k_s - \frac{\bar{C}}{\beta\bar{Y}}\hat{C}^i_s + \frac{1}{\beta}\left(\tilde{b}^i_s + \frac{\bar{K}}{\beta\bar{Y}}\hat{K}^i_s\right) - \frac{1}{\beta}\tilde{\tau}_s, \qquad s = t, t+1, ...t+T. \tag{101}$$

Iterating the log-linearized budget constraints from period $t+T$ backward gives:

$$\beta^{T+1}\left(\tilde{b}^i_{t+T+1} + \frac{\bar{K}}{\beta\bar{Y}}\hat{K}^i_{t+T+1}\right) = \left(\tilde{b}^i_t + \frac{\bar{K}}{\beta\bar{Y}}\hat{K}^i_t\right) - \frac{\bar{C}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(\hat{C}^i_{t+s}) + \frac{\bar{\Xi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(E^i_t\hat{\Xi}_{t+s})$$
$$+\,\bar{b}\sum_{s=0}^{T}\beta^s\left(\beta E^i_t\hat{R}_{t+s} - E^i_t\hat{\pi}_{t+s}\right) + \bar{w}\sum_{s=0}^{T}\beta^s\left(\left(1-\bar{\tau}^W\right)\left(E^i_t\hat{w}_{t+s} + E^i_t\hat{H}_{t+s}\right) - E^i_t\tilde{\tau}^W_{t+s}\right)$$
$$+\,\frac{\gamma_1\bar{K}}{\bar{Y}}\sum_{s=0}^{T}\beta^s E^i_t\hat{r}^k_{t+s} - \sum_{s=0}^{T}\beta^s E^i_t\tilde{\tau}_{t+s}. \tag{102}$$

Using (95) we receive

$$\beta^{T+1}\frac{\bar{C}}{\Upsilon\bar{Y}}\hat{C}^i_{t+T} + \beta^{T+1}\frac{\bar{C}}{\sigma\Upsilon\bar{Y}}E^i_t\hat{R}_{t+T}$$
$$= \left(\tilde{b}^i_t + \frac{\bar{K}}{\beta\bar{Y}}\hat{K}^i_t\right) - \frac{\bar{C}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(\hat{C}^i_{t+s}) + \frac{\bar{\Xi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(E^i_t\hat{\Xi}_{t+s})$$
$$+\,\bar{b}\sum_{s=0}^{T}\beta^s\left(\beta E^i_t\hat{R}_{t+s} - E^i_t\hat{\pi}_{t+s}\right) + \bar{w}\sum_{s=0}^{T}\beta^s\left(\left(1-\bar{\tau}^W\right)\left(E^i_t\hat{w}_{t+s} + E^i_t\hat{H}_{t+s}\right) - E^i_t\tilde{\tau}^W_{t+s}\right)$$
$$+\,\frac{\gamma_1\bar{K}}{\bar{Y}}\sum_{s=0}^{T}\beta^s\hat{r}^k_{t+s} - \sum_{s=0}^{T}\beta^s E^i_t\tilde{\tau}_{t+s}. \tag{103}$$

Next, we use the Euler equation to substitute for future consumption. Iterating the Euler equation gives

$$\hat{C}^i_{t+s} = \hat{C}^i_t + \sum_{j=0}^{s-1}\frac{1}{\sigma}(E^i_t\hat{R}_{t+j} - E^i_t\pi_{t+j+1}), \qquad T-s\geq 1. \tag{104}$$

Substituting for future consumption in (103) gives:

$$\beta^{T+1}\frac{\bar{C}}{\Upsilon\bar{Y}}\left(\hat{C}_t^i+\sum_{j=0}^{T-1}\frac{1}{\sigma}(E_t^i\hat{R}_{t+j}-E_t^i\pi_{t+j+1})\right)+\beta^{T+1}\frac{\bar{C}}{\sigma\Upsilon\bar{Y}}\hat{R}_{t+T}$$
$$=\left(\tilde{b}_t^i+\frac{\bar{K}}{\beta\bar{Y}}\hat{K}_t^i\right)-\frac{\bar{C}}{\bar{Y}}\sum_{s=0}^{T}\beta^s\left(\hat{C}_t^i+\sum_{j=0}^{s-1}\frac{1}{\sigma}(E_t^i\hat{R}_{t+j}-E_t^i\pi_{t+j+1})\right)$$
$$+\frac{\bar{\Xi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(E_t^i\hat{\Xi}_{t+s})+\bar{b}\sum_{s=0}^{T}\beta^s\left(\beta E_t^i\hat{R}_{t+s}-E_t^i\hat{\pi}_{t+s}\right)$$
$$+\bar{w}\sum_{s=0}^{T}\beta^s\left(\left(1-\bar{\tau}^W\right)\left(E_t^i\hat{w}_{t+s}+E_t^i\hat{H}_{t+s}\right)-E_t^i\tilde{\tau}_{t+s}^W\right)$$
$$+\frac{\gamma_1\bar{K}}{\bar{Y}}\sum_{s=0}^{T}\beta^sE_t^i\hat{r}_{t+s}^k-\sum_{s=0}^{T}\beta^sE_t^i\tilde{\tau}_{t+s}. \tag{105}$$

Taking contemporaneous consumption to one side of the equation gives the current decision of consumer i

$$\frac{\bar{C}}{\bar{Y}}\left(\frac{\beta^{T+1}}{\Upsilon}+\frac{1-\beta^{T+1}}{1-\beta}\right)\hat{C}_t^i$$
$$=\left(\tilde{b}_t^i+\frac{\bar{K}}{\beta\bar{Y}}\hat{K}_t^i\right)-\frac{\bar{C}}{\bar{Y}}\sum_{s=1}^{T}\beta^s\left(\sum_{j=0}^{s-1}\frac{1}{\sigma}(E_t^i\hat{R}_{t+j}-E_t^i\pi_{t+j+1})\right)+\frac{\bar{\Xi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(E_t^i\hat{\Xi}_{t+s})$$
$$+\bar{b}\sum_{s=0}^{T}\beta^s\left(\beta E_t^i\hat{R}_{t+s}-E_t^i\hat{\pi}_{t+s}\right)-\beta^{T+1}\frac{\bar{C}}{\Upsilon\bar{Y}}\sum_{j=0}^{T-1}\frac{1}{\sigma}(E_t^i\hat{R}_{t+j}-E_t^i\pi_{t+j+1})$$
$$+\bar{w}\sum_{s=0}^{T}\beta^s\left(\left(1-\bar{\tau}^W\right)\left(E_t^i\hat{w}_{t+s}+E_t^i\hat{H}_{t+s}\right)-E_t^i\tilde{\tau}_{t+s}^W\right)$$
$$+\frac{\gamma_1\bar{K}}{\bar{Y}}\sum_{s=0}^{T}\beta^sE_t^i\hat{r}_{t+s}^k-\beta^{T+1}\frac{\bar{C}}{\sigma\Upsilon\bar{Y}}\hat{R}_{t+T}-\sum_{s=0}^{T}\beta^sE_t^i\tilde{\tau}_{t+s}. \tag{106}$$

Aggregating this equation over all households yields an expression for aggregate consumption as a function of aggregate expectations about aggregate variables, only.

$$
\begin{aligned}
&\frac{\bar{C}}{\bar{Y}}\left(\frac{\beta^{T+1}}{\Upsilon}+\frac{1-\beta^{T+1}}{1-\beta}\right)\hat{C}_t \\
&=\left(\tilde{b}_t^i+\frac{\bar{K}}{\beta\bar{Y}}\hat{K}_t^i\right)-\frac{\bar{C}}{\bar{Y}}\sum_{s=1}^{T}\beta^s\left(\sum_{j=0}^{s-1}\frac{1}{\sigma}(E_t\hat{R}_{t+j}-E_t\pi_{t+j+1})\right)+\frac{\bar{\Xi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(E_t\hat{\Xi}_{t+s}) \\
&+\bar{b}\sum_{s=0}^{T}\beta^s\left(\beta E_t\hat{R}_{t+s}-E_t\hat{\pi}_{t+s}\right)-\beta^{T+1}\frac{\bar{C}}{\Upsilon\bar{Y}}\sum_{j=0}^{T-1}\frac{1}{\sigma}(E_t\hat{R}_{t+j}-E_t\pi_{t+j+1}) \\
&+\bar{w}\sum_{s=0}^{T}\beta^s\left(\left(1-\bar{\tau}^W\right)\left(E_t\hat{w}_{t+s}+E_t\hat{H}_{t+s}\right)-E_t\tilde{\tau}_{t+s}^W\right) \\
&+\frac{\gamma_1\bar{K}}{\bar{Y}}\sum_{s=0}^{T}\beta^sE_t\hat{r}_{t+s}^k-\beta^{T+1}\frac{\bar{C}}{\sigma\Upsilon\bar{Y}}\hat{R}_{t+T}-\sum_{s=0}^{T}\beta^sE_t\tilde{\tau}_{t+s}.
\end{aligned} \tag{107}
$$

D.2 Investment decision

Iterating (94) forward, we obtain

$$
\begin{aligned}
\hat{Q}_t^i=\beta^T\left(1-\delta\right)^T\hat{Q}_{t+T}^i+\frac{1}{\beta\left(1-\delta\right)}\sum_{s=1}^{T}\beta^s\left(1-\delta\right)^s\hat{\varpi}_{t+s}^i-\sum_{s=0}^{T-1}\beta^s\left(1-\delta\right)^s\hat{\varpi}_{t+s}^i \\
+\frac{\beta\gamma_1}{\beta\left(1-\delta\right)}\sum_{s=1}^{T}\beta^s\left(1-\delta\right)^s\hat{r}_{t+s}^k.
\end{aligned} \tag{108}
$$

From (10) we have that:

$$
\hat{\varpi}_s=-\sigma\hat{C}_s, \qquad s=t,t+1,...t+T. \tag{109}
$$

Plugging (109) into (108) we receive:

$$
\begin{aligned}
\hat{Q}_t^i=\beta^T\left(1-\delta\right)^T\hat{Q}_{t+T}^i-\frac{\sigma}{\beta\left(1-\delta\right)}\sum_{s=1}^{T}\beta^s\left(1-\delta\right)^s\hat{C}_{t+s}^i+\sigma\sum_{s=0}^{T-1}\beta^s\left(1-\delta\right)^s\hat{C}_{t+s}^i \\
+\frac{\beta\gamma_1}{\beta\left(1-\delta\right)}\sum_{s=1}^{T}\beta^s\left(1-\delta\right)^s\hat{r}_{t+s}^k.
\end{aligned} \tag{110}
$$

Plugging (104) into (110) we end up to the following expression for $\hat{Q}_t^i$:

$$\hat{Q}_t^i = \beta^T (1-\delta)^T \hat{Q}_{t+T}^i - \frac{\sigma}{\beta(1-\delta)} \sum_{s=1}^{T} \beta^s (1-\delta)^s \left(\hat{C}_t^i + \sum_{j=0}^{s-1} \frac{1}{\sigma} (E_t^i \hat{R}_{t+j} - E_t^i \pi_{t+j+1}) \right)$$
$$+ \sigma \sum_{s=0}^{T-1} \beta^s (1-\delta)^s \left(\hat{C}_t^i + \sum_{j=0}^{s-1} \frac{1}{\sigma} (E_t^i \hat{R}_{t+j} - E_t^i \pi_{t+j+1}) \right) + \frac{\beta\gamma_1}{\beta(1-\delta)} \sum_{s=1}^{T} \beta^s (1-\delta)^s \hat{r}_{t+s}^k. \tag{111}$$

Gathering the consumption terms yields:

$$\hat{Q}_t^i = \beta^T (1-\delta)^T \hat{Q}_{t+T}^i - \beta(1-\delta) \sum_{s=1}^{T} \beta^s (1-\delta)^s \left(\sum_{j=0}^{s-1} (E_t^i \hat{R}_{t+j} - E_t^i \pi_{t+j+1}) \right)$$
$$+ \sum_{s=0}^{T-1} \beta^s (1-\delta)^s \left(\sum_{j=0}^{s-1} (E_t^i \hat{R}_{t+j} - E_t^i \pi_{t+j+1}) \right) + \frac{\beta\gamma_1}{\beta(1-\delta)} \sum_{s=1}^{T} \beta^s (1-\delta)^s \hat{r}_{t+s}^k. \tag{112}$$

From (95) and (96) and by combining (109), we receive the following law of motion for $\hat{Q}_t^i$:

$$\hat{Q}_t^i = -\beta^T (1-\delta)^T E_t^i \hat{R}_{t+T} - \beta(1-\delta) \sum_{s=1}^{T} \beta^s (1-\delta)^s \left(\sum_{j=0}^{s-1} (E_t^i \hat{R}_{t+j} - E_t^i \pi_{t+j+1}) \right)$$
$$+ \sum_{s=0}^{T-1} \beta^s (1-\delta)^s \left(\sum_{j=0}^{s-1} (E_t^i \hat{R}_{t+j} - E_t^i \pi_{t+j+1}) \right) + \frac{\beta\gamma_1}{\beta(1-\delta)} \sum_{s=1}^{T} \beta^s (1-\delta)^s \hat{r}_{t+s}^k. \tag{113}$$

Using (95) - (97) it follows that

$$\hat{I}_{t+T}^i = \left(\hat{E}_t \hat{I}_{t+T-1}^i + \frac{1}{\kappa} \hat{R}_{t+T} \right). \tag{114}$$

This can be used to solve (93) from period $t+T-1$ backward to

$$\hat{I}_t^i = \hat{I}_{t-1}^i + \frac{1}{\kappa} \sum_{s=0}^{T-1} \beta^s \hat{Q}_{t+s}^i + \beta^T \frac{1}{\kappa} \hat{R}_{t+T}. \tag{115}$$

Iterating (94) s times, in a similar way as the derivation of (113), we can obtain expressions for $\hat{Q}_{t+s}^i$ for any $s < T$:

$$\hat{Q}^i_{t+s} = -\beta^{T-s}\left(1-\delta\right)^{T-s} E^i_t \hat{R}_{t+T} - \beta\left(1-\delta\right)\sum_{l=1}^{T-s}\beta^l\left(1-\delta\right)^l\left(\sum_{j=0}^{l-1}(E^i_t\hat{R}_{t+s+j} - E^i_t\pi_{t+j+s+1})\right)$$
$$+\sum_{l=0}^{T-s-1}\beta^l\left(1-\delta\right)^l\left(\sum_{j=0}^{l-1}(E^i_t\hat{R}_{t+s+j} - E^i_t\pi_{t+j+s+1})\right) + \frac{\beta\gamma_1}{\beta\left(1-\delta\right)}\sum_{l=1}^{T-s}\beta^l\left(1-\delta\right)^l\hat{r}^k_{t+s+l}. \tag{116}$$

Plugging in Q^i_{t+s} in (115) gives

$$\hat{I}^i_t = \hat{I}^i_{t-1} + \frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\Bigg[-\beta^{T-s}\left(1-\delta\right)^{T-s} E^i_t\hat{R}_{t+T} - \beta\left(1-\delta\right)\sum_{l=1}^{T-s}\beta^l\left(1-\delta\right)^l\left(\sum_{j=0}^{l-1}(E^i_t\hat{R}_{t+s+j} - E^i_t\pi_{t+s+j+1})\right)$$
$$+\sum_{l=0}^{T-s-1}\beta^l\left(1-\delta\right)^l\left(\sum_{j=0}^{l-1}(E^i_t\hat{R}_{t+s+j} - E^i_t\pi_{t+s+j+1})\right) + \frac{\beta\gamma_1}{\beta\left(1-\delta\right)}\sum_{l=1}^{T-s}\beta^l\left(1-\delta\right)^l\hat{r}^k_{t+s+l}\Bigg] + \beta^T\frac{1}{\kappa}\hat{R}_{t+T}. \tag{117}$$

$$\hat{I}^i_t = \hat{I}^i_{t-1} - \frac{\beta^T}{\kappa}E^i_t\hat{R}_{t+T}\sum_{s=0}^{T-1}\left(1-\delta\right)^{T-s} + \beta^T\frac{1}{\kappa}\hat{R}_{t+T} + \frac{\beta\gamma_1}{\beta\left(1-\delta\right)}\frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\sum_{l=1}^{T-s}\beta^l\left(1-\delta\right)^l\hat{r}^k_{t+s+l}$$
$$-\beta\left(1-\delta\right)\frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\sum_{l=1}^{T-s}\beta^l\left(1-\delta\right)^l\left(\sum_{j=0}^{l-1}(E^i_t\hat{R}_{t+s+j} - E^i_t\pi_{t+s+j+1})\right)$$
$$+\frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\sum_{l=0}^{T-s-1}\beta^l\left(1-\delta\right)^l\left(\sum_{j=0}^{l-1}(E^i_t\hat{R}_{t+s+j} - E^i_t\pi_{t+s+j+1})\right). \tag{118}$$

Exploiting the definition for finite sums of fixed parameters, we may re-write equation

(118) above as follows:

$$\begin{aligned}\hat{I}_t^i =& \hat{I}_{t-1}^i + \left(1 - \frac{(1-\delta)(1-(1-\delta)^T)}{\delta}\right)\frac{\beta^T}{\kappa}E_t^i\hat{R}_{t+T} + \frac{\beta\gamma_1}{\beta(1-\delta)}\frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\sum_{l=1}^{T-s}\beta^l(1-\delta)^l\hat{r}_{t+s+l}^k \\ &-\beta(1-\delta)\frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\sum_{l=1}^{T-s}\beta^l(1-\delta)^l\left(\sum_{j=0}^{l-1}(E_t^i\hat{R}_{t+s+j} - E_t^i\pi_{t+s+j+1})\right) \\ &+\frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\sum_{l=0}^{T-s-1}\beta^l(1-\delta)^l\left(\sum_{j=0}^{l-1}(E_t^i\hat{R}_{t+s+j} - E_t^i\pi_{t+s+j+1})\right). \end{aligned} \tag{119}$$

Aggregating gives

$$\begin{aligned}\hat{I}_t =& \hat{I}_{t-1} + \left(1 - \frac{(1-\delta)(1-(1-\delta)^T)}{\delta}\right)\frac{\beta^T}{\kappa}E_t\hat{R}_{t+T} + \frac{\beta\gamma_1}{\beta(1-\delta)}\frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\sum_{l=1}^{T-s}\beta^l(1-\delta)^l\hat{r}_{t+s+l}^k \\ &-\beta(1-\delta)\frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\sum_{l=1}^{T-s}\beta^l(1-\delta)^l\left(\sum_{j=0}^{l-1}(E_t\hat{R}_{t+s+j} - E_t\pi_{t+s+j+1})\right) \\ &+\frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\sum_{l=0}^{T-s-1}\beta^l(1-\delta)^l\left(\sum_{j=0}^{l-1}(E_t\hat{R}_{t+s+j} - E_t\pi_{t+s+j+1})\right). \end{aligned} \tag{120}$$

D.3 Wage setting

When wages are flexible, they are simply set according to the aggregated loglinearized version of (25):

$$\hat{w}_{t+s} = \eta\hat{H}_{t+s} + \sigma\hat{C}_{t+s} + \frac{\tilde{\tau}_{t+s}^w}{1-\bar{\tau}^W} \tag{121}$$

If, instead, we have sticky wages, we log-linearize the FOC (88) around the zero steady state wage inflation to receive:

$$\left(\frac{1+\theta_w\eta}{1-\omega_w\beta}-\frac{(\omega_w\beta)^{T+1}}{1-\omega_w\beta}\frac{(1-\bar{\tau}^W)\bar{w}\bar{H}}{\bar{C}}(\theta_w-1)\right)\hat{\tilde{w}}_t= \tag{122}$$

$$+\sum_{s=0}^{T}(\omega_w\beta)^s\left(\eta\hat{H}_{t+s}+\sigma\hat{C}_{t+s}+\frac{\tilde{\tau}^w_{t+s}}{1-\bar{\tau}^W}-\hat{w}_{t+s}\right)+(1+\theta_w\eta)\sum_{s=1}^{T}(\omega_w\beta)^s\sum_{\tau=1}^{s}\pi^w_{t+\tau}$$

$$-\frac{(\omega_w\beta)^{T+1}}{1-\omega_w\beta}\left(1+\frac{(1-\bar{\tau}^W)\bar{w}\bar{H}}{\bar{C}}\right)\hat{w}_{t+T}+\frac{(\omega_w\beta)^{T+1}}{1-\omega_w\beta}\left(1+\theta_w\eta-\frac{(1-\bar{\tau}^W)\bar{w}\bar{H}}{\bar{C}}(\theta_w-1)\right)\sum_{\tau=1}^{T}\pi^w_{t+\tau},$$

where where $\hat{\tilde{w}}_t=\hat{\tilde{W}}_t-\hat{W}_t$ and $\pi^w_t=\hat{W}_t-\hat{W}_{t-1}$. Given the assumed wage setting structure, the evolution of the aggregate log-linearized nominal wage index is given by:

$$\hat{W}_t=\omega_w\hat{W}_{t-1}+(1-\omega_w)\,\hat{\tilde{W}}_t, \tag{123}$$

which after adding and subtracting $(1-\omega_w)\,\hat{W}_t$ and solving for $\hat{\tilde{W}}_t-\hat{W}_t$ yields the following expression:

$$\hat{\tilde{w}}_t=\frac{\omega_w}{1-\omega_w}\pi^w_t, \tag{124}$$

Plugging (124) into (122) and writing the double sum as a geometric series, we can obtain

$$\pi^w_t=\varphi_w\sum_{s=0}^{T}(\omega_w\beta)^s\left(\eta\hat{H}_{t+s}+\sigma\hat{C}_{t+s}+\frac{\tilde{\tau}^w_{t+s}}{1-\bar{\tau}^W}-\hat{w}_{t+s}\right)-\varphi_{wt}\hat{w}_{t+T}+\varphi_{pw}\sum_{s=1}^{T}(\omega_w\beta)^s\,\pi^w_{t+s}, \tag{125}$$

with

$$\varphi_w=\frac{(1-\omega_w)(1-\omega_w\beta)}{\omega_w}\frac{\bar{C}}{\bar{C}(1+\theta_w\eta)-(\omega_w\beta)^{T+1}(1-\bar{\tau}^W)\bar{w}\bar{H}(\theta_w-1)}, \tag{126}$$

$$\varphi_{wt}=\frac{(1-\omega_w)\,(\omega_w\beta)^{T+1}}{\omega_w}\frac{\bar{C}+(1-\bar{\tau}^W)\bar{w}\bar{H}}{\bar{C}(1+\theta_w\eta)-(\omega_w\beta)^{T+1}(1-\bar{\tau}^W)\bar{w}\bar{H}(\theta_w-1)}, \tag{127}$$

$$\varphi_{pw}=\frac{(1-\omega_w)}{\omega_w}. \tag{128}$$

D.4 linearized firm equations

Log linearizing (38) gives

$$\begin{aligned}&\hat{p}_t^*(j) - \hat{p}_t \\ &= (1-\omega\beta)\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \left(\hat{mc}_{t+s} + \hat{p}_{t+s} - \hat{p}_t\right) + (\omega\beta)^{T+1} \tilde{E}_t^j \left(\hat{p}_{t+T} - \hat{p}_t\right),\end{aligned}$$

which can be written in terms of inflation expectations as

$$\hat{p}_t^*(j) - \hat{p}_t = (1-\omega\beta)\left[\hat{mc}_t + \tilde{E}_t^j \sum_{s=1}^{T} \omega^s \beta^s \left(\hat{mc}_{t+s} + \sum_{\tau=1}^{s} \pi_{t+\tau}\right)\right] + (\omega\beta)^{T+1} \tilde{E}_t^j \sum_{\tau=1}^{T} \pi_{t+\tau}. \quad (129)$$

Next, (39) can be log-linearized to

$$\hat{p}_t = \omega \hat{p}_{t-1} + (1-\omega) \int_0^1 \hat{p}_t^*(j) dj,$$

from which it follows that

$$\pi_t = \frac{1-\omega}{\omega} \left(\int_0^1 \hat{p}_t^*(j) dj - \hat{p}_t \right). \quad (130)$$

Aggregating (129) and plugging this in in the above expression gives

$$\pi_t = \frac{(1-\omega)(1-\omega\beta)}{\omega} \left(\hat{mc}_t + \sum_{s=1}^{T} \omega^s \beta^s \hat{E}_t \hat{mc}_{t+s} + \sum_{s=1}^{T} \omega^s \beta^s \sum_{\tau=1}^{s} \hat{E}_t \pi_{t+\tau} + \frac{(\omega\beta)^{T+1}}{1-\omega\beta} \sum_{\tau=1}^{T} \hat{E}_t \pi_{t+\tau} \right). \quad (131)$$

Writing the Double sum as a geometric series and combing with the final term we can rewrite this as

$$\pi_t = \kappa_{mc} \sum_{s=0}^{T} \omega^s \beta^s \hat{E}_t \hat{mc}_{t+s} + \frac{(1-\omega)}{\omega} \sum_{s=1}^{T} \omega^s \beta^s \hat{E}_t \pi_{t+s}, \quad (132)$$

$$\kappa_{mc} = \frac{(1-\omega)(1-\omega\beta)}{\omega}. \quad (133)$$

D.5 Final model

The log-linearized version of the resource constraint (44) reads as follows:

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}}\hat{C}_t + \frac{\bar{I}}{\bar{Y}}\hat{I}_t + \tilde{g}_t + \frac{\gamma_1 \bar{K}}{\bar{Y}}\hat{Z}_t. \tag{134}$$

Plugging (134) into (106) to substitute out for consumption yields:

$$\begin{aligned}
&\frac{\bar{C}}{\bar{Y}}\left(\frac{\beta^{T+1}}{\Upsilon} + \frac{1-\beta^{T+1}}{1-\beta}\right)\left(\frac{\bar{Y}}{\bar{C}}\hat{Y}_t - \frac{\bar{I}}{\bar{C}}\hat{I}_t - \frac{\bar{Y}}{\bar{C}}\tilde{g}_t - \frac{\gamma_1\bar{K}}{\bar{C}}\hat{Z}_t\right)\\
&= \left(\tilde{b}_t^i + \frac{\bar{K}}{\beta\bar{Y}}\hat{K}_t^i\right) - \frac{\bar{C}}{\bar{Y}}\sum_{s=1}^{T}\beta^s\left(\sum_{j=0}^{s-1}\frac{1}{\sigma}(E_t\hat{R}_{t+j} - E_t\pi_{t+j+1})\right) + \frac{\bar{\Xi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(E_t\hat{\Xi}_{t+s})\\
&+\bar{b}\sum_{s=0}^{T}\beta^s\left(\beta E_t\hat{R}_{t+s} - E_t\hat{\pi}_{t+s}\right) - \beta^{T+1}\frac{\bar{C}}{\Upsilon\bar{Y}}\sum_{j=0}^{T-1}\frac{1}{\sigma}(E_t\hat{R}_{t+j} - E_t\pi_{t+j+1})\\
&+\bar{w}\sum_{s=0}^{T}\beta^s\left(\left(1-\bar{\tau}^W\right)\left(E_t\hat{w}_{t+s} + E_t\hat{H}_{t+s}\right) - E_t\tilde{\tau}_{t+s}^W\right)\\
&+\frac{\gamma_1\bar{K}}{\bar{Y}}\sum_{s=0}^{T}\beta^s E_t\hat{r}_{t+s}^k - \beta^{T+1}\frac{\bar{C}}{\sigma\Upsilon\bar{Y}}\hat{R}_{t+T} - \sum_{s=0}^{T}\beta^s E_t\tilde{\tau}_{t+s}.
\end{aligned} \tag{135}$$

Rearranging terms and solving for output, $\hat{Y}_t$:

$$\begin{aligned}
\hat{Y}_t =&\\
&\frac{1}{\rho}\left(\tilde{b}_t^i + \frac{\bar{K}}{\beta\bar{Y}}\hat{K}_t^i\right) - \frac{\bar{C}}{\bar{Y}\rho}\sum_{s=1}^{T}\beta^s\left(\sum_{j=0}^{s-1}\frac{1}{\sigma}(E_t\hat{R}_{t+j} - E_t\pi_{t+j+1})\right) + \frac{1}{\rho}\frac{\bar{\Xi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(E_t\hat{\Xi}_{t+s})\\
&+\frac{1}{\rho}\bar{b}\sum_{s=0}^{T}\beta^s\left(\beta E_t\hat{R}_{t+s} - E_t\hat{\pi}_{t+s}\right) - \frac{1}{\rho}\beta^{T+1}\frac{\bar{C}}{\Upsilon\bar{Y}}\sum_{j=0}^{T-1}\frac{1}{\sigma}(E_t\hat{R}_{t+j} - E_t\pi_{t+j+1})\\
&+\frac{\bar{w}}{\rho}\sum_{s=0}^{T}\beta^s\left(\left(1-\bar{\tau}^W\right)(E_t\hat{w}_{t+s} + E_t\hat{H}_{t+s}) - E_t\tilde{\tau}_{t+s}^W\right)\\
&+\frac{1}{\rho}\frac{\gamma_1\bar{K}}{\bar{Y}}\sum_{s=0}^{T}\beta^s\hat{r}_{t+s}^k - \beta^{T+1}\frac{\bar{C}}{\sigma\Upsilon\bar{Y}}\hat{R}_{t+T} - \frac{1}{\rho}\sum_{s=0}^{T}\beta^s E_t\tilde{\tau}_{t+s} + \frac{\bar{I}}{\bar{Y}}\hat{I}_t + \tilde{g}_t + \frac{\gamma_1\bar{K}}{\bar{Y}}\hat{Z}_t,
\end{aligned} \tag{136}$$

where

$$\rho = \left(\frac{\beta^{T+1}}{\Upsilon} + \frac{1-\beta^{T+1}}{1-\beta} \right). \tag{137}$$

Next, log-linearizing profits of firm j to

$$\hat{\Xi}_t(j) = \frac{1}{1-\bar{mc}}(\hat{p}_t(j) - \hat{p}_t) + \hat{Y}_t(j) - \frac{\bar{mc}}{1-\bar{mc}}\hat{mc}_t. \tag{138}$$

Aggregate profits can be written as

$$\hat{\Xi}_t = \hat{Y}_t - (\theta - 1)\hat{mc}_t, \tag{139}$$

where we used that $\bar{mc} = \frac{\theta-1}{\theta}$.

Next, we write marginal costs as

$$\hat{mc}_t = (1-\zeta)\hat{w}_t + \zeta \hat{r}_t^k, \tag{140}$$

and use

$$\hat{r}_s^k = \frac{\gamma_2}{\gamma_1}\hat{Z}_s. \tag{141}$$

Finally, the loglinearized production function can be solved for aggregate labor as

$$\hat{H}_t = \frac{1}{1-\zeta}\hat{Y}_t - \frac{\zeta}{1-\zeta}\hat{K}_t - \frac{\zeta}{1-\zeta}\hat{Z}_t. \tag{142}$$

We now assume that agents know, or have learned about the above relations between aggregate variables (which hold in every period). Therefore, expectations about profits, the price of capital and aggregate can be substituted for. This gives

$$
\begin{aligned}
(1-\nu_y)\,\hat{Y}_t =& \frac{1}{\rho}\left(\tilde{b}_t + \frac{\bar{K}}{\beta\bar{Y}}\hat{K}_t\right) + \frac{\bar{I}}{\bar{Y}}\hat{I}_t + \tilde{g}_t + \frac{\gamma_1\bar{K}}{\bar{Y}}\hat{Z}_t - \frac{1}{\rho}\sum_{s=0}^{T}\beta^s E_t\tilde{\tau}_{t+s} + \nu_y\sum_{s=1}^{T}\beta^s(\hat{E}_t\hat{Y}_{t+s}) \\
& - \nu_k\sum_{s=0}^{T}\beta^s(\hat{E}_t\hat{K}_{t+s}) + \nu_z\sum_{s=0}^{T}\beta^s(\hat{E}_t\hat{Z}_{t+s}) - \nu_\tau\sum_{s=0}^{T}\beta^s\left(E_t\tilde{\tau}^W_{t+s}\right) + \nu_w\sum_{s=0}^{T}\beta^s\left(E_t\hat{w}_{t+s}\right) \\
& - \nu_r\sum_{s=1}^{T}\beta^s\sum_{j=1}^{s}(\hat{E}_tR_{t+j-1} - \hat{E}_t\pi_{t+j}) + \frac{1}{\rho}\bar{b}\sum_{s=0}^{T}\beta^s\left(\beta E_t\hat{R}_{t+s} - E_t\hat{\pi}_{t+s}\right) \\
& - \frac{1}{\rho}\beta^{T+1}\frac{\bar{C}}{\Upsilon\bar{Y}}\sum_{j=0}^{T-1}\frac{1}{\sigma}(E_t\hat{R}_{t+j} - E_t\pi_{t+j+1}) - \beta^{T+1}\frac{\bar{C}}{\rho\sigma\Upsilon\bar{Y}}\hat{R}_{t+T},
\end{aligned}
\tag{143}
$$

with

$$\nu_y = \frac{1}{\theta\rho} + \left(1-\bar{\tau}^W\right)\frac{\bar{w}}{\rho}\frac{1}{1-\zeta}, \tag{144}$$

$$\nu_z = \frac{\gamma_2}{\rho\gamma_1}\left(\frac{\gamma_1\bar{K}}{\bar{Y}} - \frac{\theta-1}{\theta}\zeta\right) - \left(1-\bar{\tau}^W\right)\frac{\bar{w}}{\rho}\frac{\zeta}{1-\zeta}, \tag{145}$$

$$\nu_k = \left(1-\bar{\tau}^W\right)\frac{\bar{w}}{\rho}\frac{\zeta}{1-\zeta}, \tag{146}$$

$$\nu_\tau = \frac{\bar{w}}{\rho}, \tag{147}$$

$$\nu_w = \left(1-\bar{\tau}^W\right)\frac{\bar{w}}{\rho} - \frac{\theta-1}{\theta\rho}(1-\zeta), \tag{148}$$

$$\nu_r = \frac{\bar{C}}{\bar{Y}\sigma\rho}. \tag{149}$$

Next, using (141) in (120) gives

$$
\begin{aligned}
\hat{I}_t =& \hat{I}_{t-1} + \left(1 - \frac{(1-\delta)(1-(1-\delta)^T)}{\delta}\right)\frac{\beta^T}{\kappa}E_t\hat{R}_{t+T} + \frac{\gamma_2}{(1-\delta)}\frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\sum_{l=1}^{T-s}\beta^l\left(1-\delta\right)^l\hat{Z}_{t+s+l} \\
& - \beta\left(1-\delta\right)\frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\sum_{l=1}^{T-s}\beta^l\left(1-\delta\right)^l\left(\sum_{j=0}^{l-1}(E_t\hat{R}_{t+s+j} - E_t\pi_{t+s+j+1})\right) \\
& + \frac{1}{\kappa}\sum_{s=0}^{T-1}\beta^s\sum_{l=0}^{T-s-1}\beta^l\left(1-\delta\right)^l\left(\sum_{j=0}^{l-1}(E_t\hat{R}_{t+s+j} - E_t\pi_{t+s+j+1})\right).
\end{aligned}
\tag{150}
$$

Using (134) and the production function we can write

$$\eta\hat{H}_t + \sigma\hat{C}_t = (\frac{\eta}{(1-\zeta)} + \frac{\sigma\bar{Y}}{\bar{C}})\hat{Y}_t - \frac{\eta\zeta}{(1-\zeta)}\hat{\hat{K}}_t - \frac{\sigma\bar{Y}}{\bar{C}}\tilde{g}_t - \frac{\sigma\bar{I}}{\bar{C}}\hat{I}_t - \frac{\sigma\gamma_1\bar{K}}{\bar{C}}\hat{Z}_t \tag{151}$$
$$= (\frac{\eta}{(1-\zeta)} + \frac{\sigma\bar{Y}}{\bar{C}})\hat{Y}_t - \frac{\eta\zeta}{(1-\zeta)}\hat{K}_t - \frac{\sigma\bar{Y}}{\bar{C}}\tilde{g}_t - \frac{\sigma\bar{I}}{\bar{C}}\hat{I}_t - \left(\frac{\sigma\gamma_1\bar{K}}{\bar{C}} + \frac{\eta\zeta}{(1-\zeta)}\right)\hat{Z}_t.$$

When wages are flexible, we can plug this into (121) to obtain

$$\pi_t^w = \varphi_y^f \sum_{s=0}^{T} (\omega_w\beta)^s \hat{Y}_{t+s} - \varphi_k^f \sum_{s=0}^{T} (\omega_w\beta)^s \hat{K}_{t+s} - \varphi_g^f \sum_{s=0}^{T} (\omega_w\beta)^s \tilde{g}_{t+s} + \varphi_\tau^f \sum_{s=0}^{T} (\omega_w\beta)^s \tilde{\tau}_{t+s}^W \tag{152}$$
$$- \varphi_i^f \sum_{s=0}^{T} (\omega_w\beta)^s \hat{I}_{t+s} - \varphi_z^f \sum_{s=0}^{T} (\omega_w\beta)^s \hat{Z}_{t+s}$$

Moreover, using (134) for the case of sticky wages, the wage Phillips curve, (125), becomes

$$\pi_t^w = \varphi_y \sum_{s=0}^{T} (\omega_w\beta)^s \hat{Y}_{t+s} - \varphi_k \sum_{s=0}^{T} (\omega_w\beta)^s \hat{K}_{t+s} - \varphi_g \sum_{s=0}^{T} (\omega_w\beta)^s \tilde{g}_{t+s} + \varphi_\tau \sum_{s=0}^{T} (\omega_w\beta)^s \tilde{\tau}_{t+s}^W \tag{153}$$
$$- \varphi_i \sum_{s=0}^{T} (\omega_w\beta)^s \hat{I}_{t+s} - \varphi_z \sum_{s=0}^{T} (\omega_w\beta)^s \hat{Z}_{t+s} + \varphi_{pw} \sum_{s=1}^{T} (\omega_w\beta)^s \pi_{t+s}^w - \varphi_w \sum_{s=0}^{T} (\omega_w\beta)^s \hat{w}_{t+s} - \varphi_{wt}\hat{w}_{t+T},$$

with

$$\varphi_y = \varphi_w(\frac{\eta}{(1-\zeta)} + \frac{\sigma\bar{Y}}{\bar{C}}), \tag{154}$$

$$\varphi_k = \varphi_w\frac{\eta\zeta}{(1-\zeta)}, \tag{155}$$

$$\varphi_g = \varphi_w\frac{\sigma\bar{Y}}{\bar{C}}, \tag{156}$$

$$\varphi_\tau = \varphi_w\frac{1}{1-\bar{\tau}^W}, \tag{157}$$

$$\varphi_i = \varphi_w \frac{\sigma \bar{I}}{\bar{C}}, \tag{158}$$

$$\varphi_z = \varphi_w \left(\frac{\sigma \gamma_1 \bar{K}}{\bar{C}} + \frac{\eta \zeta}{(1-\zeta)} \right). \tag{159}$$

The flexible coefficients with superscript f in Equation (153) are obtained by replacing φ_w with 1 in the above 6 definitions.

When wages are sticky, the evolution of the aggregate *real* wage, can then be calculated as

$$\hat{w}_t = \hat{w}_{t-1} + \pi_t^w - \pi_t. \tag{160}$$

Further, plugging in marginal costs in (132) gives

$$\pi_t = \kappa_w \sum_{s=0}^{T} \omega^s \beta^s \hat{E}_t \hat{w}_{t+s} + \kappa_z \sum_{s=0}^{T} \omega^s \beta^s \hat{E}_t \hat{Z}_{t+s} + \kappa_\pi \sum_{s=1}^{T} \omega^s \beta^s \hat{E}_t \pi_{t+s},$$

$$\kappa_w = \kappa_{mc}(1-\zeta), \tag{161}$$

$$\kappa_z = \kappa_{mc} \zeta \frac{\gamma_2}{\gamma_1}, \tag{162}$$

$$\kappa_\pi = \frac{(1-\omega)}{\omega}. \tag{163}$$

The capital accumulation equation can be log-linearized to

$$\hat{K}_{t+1} = (1-\delta) \hat{K}_t + \frac{\bar{I}}{\bar{K}} \hat{I}_t. \tag{164}$$

Log-linearizing (30) gives

$$\hat{r}_t^k = \hat{mc}_t + (\zeta - 1)(\hat{\hat{K}}_t - \hat{H}_t). \tag{165}$$

Combining this with (140), we can obtain an equation for capital utilization.

$$\left(\frac{\gamma_2}{\gamma_1} + \frac{1}{1-\zeta} \right) Z_t = \frac{1}{1-\zeta} \hat{Y}_t - \frac{1}{1-\zeta} \hat{K}_t + \hat{w}_t. \tag{166}$$

To complete the model we log-linearize the government budget constraint, and substitute for Labor using the production function (41), to

$$\tilde{b}_{t+1} = \frac{1}{\beta}\tilde{g}_t - \frac{\bar{w}}{\beta}(\bar{\tau}^W(\hat{w}_t + \hat{H}_t) + \tilde{\tau}_t^W) + \frac{1}{\beta}\tilde{b}_t + \bar{b}(\hat{R}_t - \frac{1}{\beta}\hat{\pi}_t) - \frac{1}{\beta}\tilde{\tau}_t, \tag{167}$$

$$\begin{aligned}\tilde{b}_{t+1} =& \frac{1}{\beta}\tilde{g}_t - \frac{1}{\beta}\tilde{\tau}_t - \frac{\bar{w}}{\beta}\bar{\tau}^W\hat{w}_t - \frac{\bar{w}}{\beta}\bar{\tau}^W\frac{1}{(1-\zeta)}\hat{Y}_t + \frac{\bar{w}}{\beta}\bar{\tau}^W\frac{\zeta}{(1-\zeta)}\hat{K}_t \\ &+ \frac{\bar{w}}{\beta}\bar{\tau}^W\frac{\zeta}{(1-\zeta)}\hat{Z}_t - \frac{\bar{w}}{\beta}\tilde{\tau}_t^W + \frac{1}{\beta}\tilde{b}_t + \bar{b}(\hat{R}_t - \frac{1}{\beta}\hat{\pi}_t).\end{aligned} \tag{168}$$

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