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The natural rate of interest from a monetary and financial perspective

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The natural rate of interest from a monetary and financial perspective

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Table of contents

Introduction and summary	7
1 Definition and model estimations of the natural rate of interest	9
1.1 r^* as real long-term rate of interest	9
1.2 r^* as real short-term rate of interest	10
1.3 Model-based approaches	11
2 Determinants of r^* in literature	13
2.1 Mechanisms underlying r^*	13
2.2 Global savings surplus	14
2.3 Secular stagnation versus financial cycle	15
2.4 Risk premium	16
2.5 Monetary policy	17
2.6 Hysteresis effects	19
3 Trends and cycles in the natural rate of interest	21
3.1 Historical interest rate series	21
3.2 Model estimations	23
4 Influence of monetary policy	45
4.1 Model-based approach	45
4.2 Outcomes	48
References	54
Annex 1 The natural rate of interest in a general equilibrium model	60
Annex 2 Time series models used	63

Introduction and summary

7

The natural rate of interest (r^*) is an important monetary policy variable in economic literature. It serves as a benchmark for the policy rate in an equilibrium. It also plays a role in the ongoing debate about unconventional monetary policy, for instance in the development of opinions on the lower bound of the policy rate and on the current low market interest rates. To illustrate: the 'secular stagnation' hypothesis posits that the low real market interest rates are an expression of a negative value of r^* . This hypothesis argues that this has consequences for monetary policy, which – according to the predominant theory – stimulates the economy by lowering the policy rate (adjusted for inflation) to below r^* . When r^* is negative, however, this is not possible because of the lower bound set for the policy rate. This, it is argued, impedes the ability of monetary policy to stimulate the economy.

One complication in analyses concerning r^* is that it is a theoretical concept, which means there are several definitions in existence. The natural rate of interest cannot be directly observed, which is why empirical research uses approximations of r^* . This study contributes to this body of research by using time series methodology to shed light on monetary and financial factors that influence r^* , such as the financial cycle, monetary policy and risk premium (the fee for investing in relatively high-risk bonds). What distinguishes this study is its use of long historical time series for several countries.

The main empirical finding in this study is that estimations of r^* and its drivers are beset with great uncertainty. This is evident from our own estimations using time series methods, but also from models used in other studies. Bearing in mind this uncertainty, this study shows that the value of r^* has fallen over recent decades, but that this downward trend is less marked when viewed over a period of 200 years. A second finding is that

real interest rates have been in a downward phase of a medium-term cycle since the 1980s, possibly reflecting the influence of financial factors on r^* , such as deleveraging. These factors play a role in the 'financial cycle' hypothesis, which also posits the long-lasting accommodative monetary policy as a reason for low real interest rates. That said, the precise effect of monetary policy on r^* – and this is our third finding – is difficult to determine in an empirical analysis and varies from country to country.

The main policy conclusion is that the uncertainty regarding the level of r^* constrains its practical usefulness as a benchmark for monetary policy. Not only are model estimations of r^* beset with great uncertainty, there is no uniform definition of the natural rate of interest either. Specifically for the euro area, the value of r^* as a benchmark for monetary policy is limited by the fact that approximations of its level, in the form of real long-term yields on government bonds issued in euro-area countries, are influenced by the risk premium on relatively high-risk government bonds. Partly because of this, there is no uniform benchmark for measuring the natural rate of interest in the different countries in the euro area.

Chapter 1 offers a conceptual framework for r^* and describes a number of models that are commonly used to estimate it. Chapter 2 discusses the factors that help to determine r^* , as found in literature. Chapter 3 analyses the trend-based and cyclical components of r^* . The final chapter discusses the effect of monetary policy on r^* .

1 Definition and model estimations of the natural rate of interest

9

There are several concepts and definitions of the natural rate of interest (r^*).¹ This chapter summarises them and demonstrates that the concepts of r^* found in the literature depend on such aspects as the model framework and policy application used. There is a clear differentiation between the definition of r^* as real long-term interest rate where there is equilibrium on the capital markets, or as the real short-term interest rate consistent with equilibrium in the economy.

1.1 r^* as real long-term rate of interest

Swedish economist Knut Wicksell was one of the originators of the theory of r^* . According to Wicksell (1898), r^* is the interest rate at which the (global) demand for and supply of capital are in balance. From this perspective, r^* can also be interpreted as the equilibrium interest rate that corresponds to the marginal product of capital. In practice, the supply of and demand for capital find each other on markets for long-term finance, so that r^* can be approximated using real long-term interest rates. If the actual real interest rate, i.e. the market interest rate, is below the natural rate, demand for capital will exceed its supply. Put differently, investors will want to borrow more money than savers are willing to save. Banks, which serve as intermediaries between investors and savers, can accommodate excess demand by investors by increasing the supply of credit. This then leads to a rise in demand for goods, which – when there is a finite supply of goods – pushes up prices. Only when the real market interest rate once again corresponds to the natural rate, and excess demand for credit is lifted will price stability be achieved.

1 r^* is sometimes also referred as the *equilibrium interest rate* or *neutral rate of interest*. In this paper, we use the term *natural rate of interest*.

1.2 r^* as real short-term rate of interest

Using the natural rate of interest as the real short-term interest rate plays a prominent role in modern, New Keynesian macroeconomic models. According to Woodford (2003), the natural rate of interest in these models is the rate at which the economy is in equilibrium while prices are fully flexible. The natural rate of interest is not necessarily constant in this equilibrium, but can fluctuate under the influence of all kind of shocks, such as aggregated demand and productivity shocks, or changes in the preferences of households. If the economy is not in balance, for instance because prices are not able to adjust freely, the true real market interest rate can deviate from the natural rate; in the spirit of Wicksell, this will lead to inflationary or deflationary pressure.

This approach, in which the definition of r^* is based on the real short-term interest rate, allows the concept to be applied to monetary policy. By looking at the difference between the true real short-term market rate and the natural rate, or the 'interest rate gap', the central bank can make a judgement on its monetary stance, i.e. the degree to which it eases or tightens monetary policy. If the policy rate is above (below) the natural rate, monetary policy is too restrictive (accommodative) and the central bank can alleviate excessive pressure on prices by bringing its policy rate more in line with the natural rate of interest. We need to remember that such 'corrections' are not always possible if the natural rate of interest is negative, inflation is low and the (nominal) policy rate is tied to a floor (the 'lower bound'). It is also important to be able to make a good estimation of the natural rate of interest, but the fact that this rate is not observable makes this more difficult in practice. Models are generally used to nonetheless gain an understanding of the natural rate and the factors that potentially influence it.

1.3 Model-based approaches

Several models have been developed recently to estimate the natural rate of interest. They can be divided into three groups: (i) time series models; (ii) semi-structural models; and (iii) general equilibrium models.² This section summarises these models and shows that the type of model used determines the estimations and explanatory factors of r^* .

1.3.1 Time series models

In time series models, r^* is seen as the long-term trend in real interest rates. This trend is 'filtered' out of the data, as it were (Del Negro et al., 2017; Johanssen et al., 2016), with non-observable variables (such as r^*) being estimated using observable variables. In the models developed by Del Negro et al. and Johanssen et al., for example, the inflation rate and the business cycle are used as observable variables. Harvey (1990) and Durbin and Koopman (2012) also impose some economic structure using long-term relationships between the trend variables. This technique is also used in the time series model presented in Chapter 3. An alternative time series model is the Vector Autoregression (VAR) model developed by Lubik and Matthes (2015), using time-variable parameters. The outcomes of time series models are sensitive to assumptions that are imposed on the estimation process. Both short-term and long-term real interest rates are used as approximations for r^* in time series models. The next chapter uses a time series model to estimate the natural rate of interest in six developed economies and the euro area.

1.3.2 Semi-structural models

Semi-structural models are theoretically based but flexible enough to fit the data. The most popular semi-structural model for estimating r^* is the

² These models are referred to in literature as DSGE models, where DSGE stands for Dynamic Stochastic General Equilibrium.

one developed by Laubach and Williams (2003, henceforth LW) and the alternative developed by Holston, Laubach and Williams (2017, henceforth HLW). This model draws a correlation between the natural (short-term) rate of interest and the potential economic activity. The non-observable natural rate of interest is then filtered out of these data. As well as r^* , the model also estimates the potential output and trend growth. The research by LW shows that estimations of the natural rate of interest are highly inaccurate and can vary widely depending on the model specification applied (see Box 1 in Chapter 3).

1.3.3 General equilibrium models

In general equilibrium models, economic agents take optimum decisions (about aspects such as consumption and investments) based on rational expectations about the present and future state of the economy. In these models, all markets are interconnected, making it possible to analyse the effects of market-specific shocks in a macroeconomic perspective. It is also possible to estimate such models and to filter out non-observable variables, such as the natural rate of interest and its drivers, from the data.

Recent examples of studies analysing the trend in the natural rate of interest using an estimated general equilibrium model include Del Negro et al. (2017) for the United States and Gerali and Neri (2017) for the United States and the euro area. In these models, the natural rate of interest is influenced by both aggregated demand shocks and shocks in the risk premium, i.e. the difference between the return on high-risk and risk-free assets. The drawback is that general equilibrium models assume the risk premium shocks to be exogenous, which limits a sound explanation of r^* within models of this type (see Section 2.4 in Chapter 2).

2 Determinants of r^* in literature

Developments in the natural rate of interest can be attributed to various factors. This chapter provides an overview of the main determinants of the fall in r^* over recent decades, as presented in the literature.

13

2.1 Mechanisms underlying r^*

The specification for r^* can be derived analytically in general equilibrium models, providing an understanding of the factors that influence the natural rate. This analytical specification depends on the model used. In a standard New Keynesian general equilibrium model, for example, r^* depends on the intertemporal substitution elasticity of households, which determines how strongly households react to a change in real interest rates (see Box in Annex 1). In this model, r^* is also determined by shocks influencing household savings decisions, such as aggregated demand shocks and changes in productivity growth (Woodford, 2003; Galí, 2008). An increase in aggregated demand (e.g. due to a rise in public expenditure) knocks the savings market out of balance, causing r^* to rise until the equilibrium has been restored. It is worth noting in this context that a demand shock driven by government spending can produce short-term 'crowding out' effects and an increase in debt sustainability risks, potentially leading to an increase in private savings. Over the long term, a projected acceleration in productivity growth can result in a higher natural rate of interest because households are willing to consume more and save less in response to a higher anticipated income. If real interest rates do not rise in tandem with r^* , the result will be excess demand and concomitant upward pressure on prices. The analysis in Annex 1 demonstrates how demand and productivity shocks influence r^* in an economy in equilibrium with flexible prices. If capital is added to the standard model, r^* will also be a function of the capital stock, because this influences the marginal product of capital (i.e. the return). If international trade is also added, r^* is further influenced by international demand shocks (Clarida et al., 2001; Galí and Monacelli, 2008). Van Wijnbergen (2018)

shows, for example, that in open economies r^* determines the equilibrium in the international capital market. Country-specific factors, such as fiscal policy, can lead to different levels of r^* in different countries. Frictions in the economy can also affect r^* ; frictions can impede the allocation of investments through their influence on savings and investment decisions by households and businesses.

2.2 Global savings surplus

Despite the great uncertainty of estimates of the natural rate of interest, most studies suggest a persistent global downward trend in r^* since the 1980s. Rachel and Smith (2017) show that the 'global' natural rate has fallen over recent decades due to a decline in (projected) global growth.³ According to the authors, much of this decline can be explained by a shift in global savings and investments. An increase in global savings is caused by demographic factors (Gottfries and Teulings, 2015). To illustrate: the global population of working age (20–64 years) has grown in recent years relative to the number of people who do not work, due to a decline in population growth. Since working people have the highest savings, this has led to an increase in total savings. A reduction in population growth can also reduce the return on capital due to the declining number of new workers, thus depressing investment demand. These shifts in savings and investment needs at global level lead to a downward adjustment in the global natural rate of interest. Financial innovations provide another explanation of the increase in global savings. These innovations have increased the availability of finance in recent decades and have therefore lowered interest rates. New financial instruments, such as complex derivatives and securitisations, have contributed to this. The development of these instruments has been

3 In Rachel and Smith (2017), the 'global natural interest rate' refers to average real long-term rates in the G7 countries, excluding Italy.

fostered by technological developments combined with liberalisation and deregulation of financial markets (ECB, 2002).

2.3 Secular stagnation versus financial cycle

In response to the observed fall in r^* at global level, a debate has arisen among policymakers on the question of whether the low natural rate of interest (r^*) is correlated with secular stagnation or stems from the financial cycle.⁴ Both views adopt a long-term perspective and assume that, following a crisis shock, an economy may not automatically return towards equilibrium. The secular stagnation hypothesis goes further by assuming that this equilibrium has fallen to a lower level. It explains the low natural rate of interest by a structural demand shortfall, for which various reasons are put forward. The fall in r^* may be due to a decline in population growth, a reduction in investment demand, an increase in income inequality and a drop in the price of investment goods, which together lead to a savings surplus (see e.g. Summers, 2014). According to the secular stagnation hypothesis, low real interest rates are an equilibrium phenomenon. If r^* has a negative value and inflation is close to zero, conventional monetary policy is not able to stimulate the economy, because the lower bound in the policy rate prevents it from being cut to below the negative natural rate. In this situation, fiscal policy can help to stimulate investment demand, for example through tax cuts and higher public spending.

In the financial cycle hypothesis – introduced by the Bank for International Settlements (BIS) – low interest rates are associated with financial booms

⁴ See e.g. Geneva Report on “Low for Long? Causes and Consequences of Persistently Low Interest Rates”, Bean et al. eds., 2015. Borio, C. (2017a), Secular stagnation or financial cycle drag?, Keynote speech, National Association for Business Economics, 33rd Economic Policy Conference, 5-7 March 2017, Washington DC.

and busts, in combination with an inadequate response from policymakers. Several countries had built up large financial imbalances before the crisis, and when these collapsed, this led to a deep worldwide recession in 2008-2009. Central banks responded to this by cutting interest rates. The expansionary monetary policy was sustained for a considerable time, because deleveraging by households and businesses blunted the impact of policy. Based on the financial cycle hypothesis, persistently low policy rates contributed to a reduction in r^* , because accommodative monetary conditions encouraged a misallocation of production factors and thus depressed production growth (see Section 2.6). The low natural rate of interest is thus associated with the course of the financial cycle and is not an equilibrium phenomenon, as posited in the secular stagnation hypothesis (Borio et al., 2017). Nonetheless, both hypotheses stress the downside risks for potential growth and are therefore not mutually exclusive.

2.4 Risk premium

The risk premium plays a role in the r^* dynamic in both the secular stagnation and the financial cycle hypothesis. This has been demonstrated empirically in such models as estimated general equilibrium models. Del Negro et al. (2017) and Gerali and Neri (2017) show, for example, that r^* has declined since the 1990s in both the US and the euro area as a result of a rise in the risk premium. However, as these models treat the risk premium as an exogenous factor, it is difficult to ascertain the reliability and interpretation of the outcomes. It is possible that the risk premium is affected by other factors that are responsible for the reduction in the natural rate of interest, but not explicitly factored into the model, or that an increase in the risk premium is the result of a fall in r^* rather than the other way around. In a theoretical model, Caballero and Farhi (2014) show that a positive risk premium is the result of a shortage of risk-free assets and that this pushes r^* below the lower bound of the policy rate, the reason

being that r^* is the interest rate on risk-free assets and the shortage of these assets means investors are willing to accept a lower interest return on them. As policy rates cannot be reduced below the lower bound, this leads to excess demand for risk-free assets and a demand shortfall for goods, or in other words a deficient aggregated demand, as posited in the secular stagnation hypothesis (in the model of Caballero and Farhi, demand is determined by the equilibrium on asset markets).

2.5 Monetary policy

Monetary policy is not regarded in the literature as a driver of the natural rate of interest (r^*). It is assumed that policy rates follow r^* and that wages and prices are able to adapt flexibly over the long term to economic developments such as rising spending. As a result, monetary policy has no real-economic effects. This mechanism forms the basis for the neoclassical concept that money is neutral, a concept that also underlies most New Keynesian models (see Chapter 1).

The theory that monetary policy has no real impact over the long term means that monetary policy has only a temporary, cyclical effect. Despite this, monetary policy has been accommodative for much longer than usual since 2007 due, in part, to the unconventional measures. As a result, the accommodative monetary policy has acquired a more permanent character⁵ and it is more likely to have real-economic effects. Long lasting accommodative monetary policy can for example encourage investments in less profitable sectors; this can lead to misallocation of production factors

5 "...over periods as long as a decade or more, money is not neutral, at least for practical policy purposes", Borio (2017b).

in the economy, undermining the growth potential.⁶ Misallocation can also occur if monetary policy influences the pricing of risks, for instance by encouraging risk-taking on financial markets, potentially leading to long-term disruption of the pricing mechanism for capital allocation.

The influence of monetary policy on the financial cycle can also give rise to real-economic effects (Borio, 2017a). Accommodative monetary policy can exacerbate a financial boom and accumulation of debt; this in turn can lead to a financial crisis with problems in the banking sector and forced deleveraging. In these circumstances, capital allocation via the banking sector can be disrupted, undermining the economic growth potential. If monetary policy responds asymmetrically to booms and busts, by not constraining rising asset prices but easing policy if they fall, interest rates can decline over a long period. This mechanism fits in with the financial cycle hypothesis.

Expansionary monetary policy also influences the scarcity of risk-free assets ('safe assets') through large-scale asset-purchasing programmes, also referred to as quantitative easing (QE), in which the central bank buys bonds in exchange for central bank reserves. As QE reduces the supply of government bonds and boosts demand for assets by increasing the liquidity on financial markets, it can lead to excess demand for safe assets. Only banks can hold central bank reserves, which means non-banks need alternative safe assets. This increases the demand for and hence the scarcity of safe assets. That scarcity obstructs efficient resource allocation and is

6 Several empirical studies, focusing on the supply side of the economy, have found that long-term low interest rates lead to misallocation of production factors. See Caballero et al. (2008) for Japan; Barnett et al. (2014) for the UK; Cetto et al. (2016), Gopinath et al. (2017) for Southern Europe and Hoerberichts and Van den End (2018) for multiple countries.

accompanied by a risk premium on other assets, both exerting downward pressure on r^* (Caballero and Farhi, 2014).

19

2.6 Hysteresis effects

Finally, pessimistic expectations about the economy over a long period of expansionary monetary policy can push the economy into a deflationary spiral (Schmitt-Grohé and Uribe, 2010). Economic agents can read the persistent low interest rates as a negative signal about the economy, which may lead to lower inflation expectations. This in turn can disrupt the equilibrium, with persistent deflation and a shrinking economy. Temporary shocks can also have long-lasting effects on the economy in such a scenario, a phenomenon referred to as 'hysteresis'. To illustrate: if pessimism in the short term leads to lower investment growth, this can have a negative knock-on effect on productivity growth and so on the long-term potential growth of the economy. Hysteresis effects can also depress productivity growth by disrupting the allocation of labour and capital. This can manifest itself through the labour market if it becomes less dynamic due to a recession and through banks if they channel capital to businesses less efficiently due to their weak balance sheet position following a recession. Research has shown that banks with a weak balance sheet tend to fund relatively more unproductive 'zombie companies', with a potential negative impact on productivity growth (Andrews and Petroulakis, 2017).

Blanchard and Summers (2017) conclude that monetary policy needs to be relaxed aggressively at the start of a recession in order to avoid hysteresis effects. In its commitment to price stability, the central bank must also take into account fluctuations in the output gap, the reason being that hysteresis pushes up equilibrium unemployment, and unemployment figures therefore give misleading information about underutilisation in the economy and inflationary pressure. According to Blanchard (2017), this

means that the trade-off between inflation and output is more favourable than the unemployment figures suggest, creating scope for using extended monetary stimulation to restore the labour supply and reverse hysteresis effects without this necessarily leading to very high inflation. In other words, low inflation expectations mean that monetary policy has (positive) real-economic effects over the long term. Blanchard does not mention the fact that – due to low interest rates and risk premiums – misallocation can also occur during this phase, with concomitant negative real-economic effects. He only associates misallocation with hysteresis effects on a downward trend. This makes it a partial analysis and means it cannot be assumed that long-term monetary stimulation offers a solution for hysteresis effects.

3 Trends and cycles in the natural rate of interest

This chapter uses advanced time series methods to analyse long-term historical interest rate series. The outcomes of this analysis are used to estimate trends and cycles in the natural rate of interest (r^*) for six countries and for the euro area. The assumption is that r^* can best be described using trends and long-term cycles in the real bond yield.

21

3.1 Historical interest rate series

We analyse the trend in short-term and long-term interest rates using historical long-term series based on quarterly and annual data for six countries (the US, Japan, the Netherlands, Germany, Italy and Spain) and for the euro area.⁷ Most annual series start in 1800 and quarterly series are available from 1900.⁸ Using long data series allows us to include multiple financial cycles – which, as Drehmann et al. (2012) show, are by their nature long-lasting developments – and monetary regimes. Other time series generally use series spanning 50 or 60 years, and often only for the US (e.g. Johansson and Mertens, 2016).

The (three-monthly) money market interest rate is used for short-term interest rates and the 10-year rate on government bonds for long-term rates. As r^* is a real variable, the nominal interest rate is deflated with the expected inflation rate to determine a real interest rate. An autoregressive (AR) model forecast is used to approximate expected inflation, as is common

7 A weighted average of the five largest euro area countries was used to cover the period for which no data is available for the euro area as a whole. There are a number of gaps in the data series during the years of the two world wars.

8 Sources: Central banks, national statistics offices, IMF, Global Financial Data, Jorda-Schularick-Taylor macroeconomic database, Lawrence H. Officer, Measuring Worth, 2017.

in the literature (Binder, 2016).⁹ A further explanation of the model and the deflation of the nominal interest rate is provided in Annex 2.

Viewed over a long period, nominal long-term interest rates have shown a downward trend since 1800. This is apparent from statistical tests, which demonstrate that the series are non-stationary (see Table 3.1). Following a temporary peak in the 1970s and 1980s, the downward long-term trend has accelerated over recent decades (Figure 3.1; short-term rates show a similar pattern). This acceleration is partly related to the downward trend in inflation and policy rates in the countries studied.

Although nominal interest rates are currently at historically low levels, real long-term rates show a different pattern (Figure 3.1; short-term rates show a similar picture). Real rates have sometimes recorded sharply negative values in the past – especially during periods of hyperinflation caused by wars or crises – but in historical perspective have not been exceptionally low over the recent period. Jordà et al. (2017) reach the same conclusion based on an international study of long-term returns. The reason that real interest rates are not exceptionally low lies in the fact that the very low nominal interest rates are accompanied by an equally low inflation rate.

Unlike nominal rates, real interest rates generally do not display a long-term downward trend measured over a long historical period (statistical tests show that real interest rates are nearly always stationary, see Table 3.1)¹⁰. It can be deduced from this that the level of and downward trends in

9 We have also experimented with a mixed backward and forward expectations model, but this technique is unsuitable for a long series because it is based on inflation forecasts from surveys that are not available prior to 1989.

10 The trend is analysed using a different method in Section 3.2.

nominal rates are driven mainly by inflation. During the decades following the Second World War, real interest rates rose in most countries. Population growth also accelerated during that period, and this is one of the drivers of r^* . We have seen a clear downward trend in real interest rates in recent decades. This turning point coincides with a levelling off of global population growth and an increased savings surplus, driven partly by demographic factors and financial innovations. The liberalisation of financial markets for instance, in combination with new financial products, has increased the availability of finance since the 1980s (see Section 2.2).

Real interest rates are also less uniform across countries than nominal rates (Table 3.2). The correlation coefficients of nominal long-term rates are above 0.5 for all countries, and in most cases higher than 0.8. The correlation is lower for real rates, but is still positive in all cases. This illustrates the global dimension of movements in long-term rates. Interest rate parity offers an analytical framework for explaining differences in real interest rates between countries. It assumes that, over the long term, investors cannot achieve surplus returns on investments in a higher-interest-rate country because the real exchange rate for that country is likely to depreciate. The spreads in real interest rates are eliminated by capital flows. The finding that real interest rates are not perfectly correlated could be explained by restrictions on capital flows. Another explanation that poses an obstacle to real interest rate parity is that government bonds issued by the various countries are not perfect substitutes.

3.2 Model estimations

In this section, we estimate the natural rate of interest using time series models. These models are sensitive to the model specification and the priors imposed on the estimation process. As a result, conclusions about r^* based on this mere time series analysis should be treated with caution.

Our estimates are based on a Multivariate Unobserved Component (MUC) model, as used by Harvey (1990) and Durbin and Koopman (2012), among others. The main feature of models of this type is that they facilitate decomposition of a time series into a trend, cycle, season and irregular components. The statistical calculations are based on a Kalman filter and related methods. These not only produce point estimates of the components, but interval estimates as well. Annex 2 describes the model, in which r^* is regarded as a long-term concept of real long-term interest rates.¹¹ This concept assumes that r^* has both a trend component and a component that is associated with the real interest rate cycle.

3.2.1 Trends

The MUC model estimated using quarterly data demonstrates that r^* has shown a downward trend in all countries since the 1980s or 1990s (Figure 3.2). This was a period of a rising savings surplus worldwide and a global decline in population growth, two factors that shape the natural rate of interest (see Section 2.2). The estimations using annual data confirm the downward trend over recent decades, but also show that the decline in the trend component in real long-term rates has been stronger and more persistent at certain points in history (Figure 3.3¹²). The different outcomes for r^* estimated using quarterly data (Figure 3.2) and annual data (Figure 3.3) illustrate the sensitivity of r^* to the model approach used and data availability. To illustrate: the estimations using annual data are based on a different specification of the MUC model (see Annex 2) and cover a longer period than the quarterly estimations. The latter means that r^* estimated using annual

¹¹ An analysis based on real short-term rates generates comparable results.

¹² The euro area is not included because the data series are too short for a decomposition analysis. The annual data sometimes contains gaps (for Germany and Japan), which means it is not possible to estimate two cycles for all countries.

data shows fewer fluctuations because the trend component is smoothed out with more historical information.

3.2.2 Cycles

In the MUC model estimated using annual data, a (medium-term) cycle can also be distinguished for each time series in addition to the trend component (see Annex 2). The estimated length of this cycle is between nine and 17 years, which corresponds to the length of a financial cycle as estimated in the literature, where medium-term financial cycles are of comparable length.¹³ The outcomes of the MUC model show that real interest rates have been in a downward phase of a cycle in all countries since the 1980s. As a result, the positive contribution of the cycle to long-term interest rates over the past two decades has broadly reversed into a negative contribution (see dark purple bars in Figure 3.4). As mentioned, the medium-term cycle in real interest rates is comparable in length to the financial cycle. Combined with the assumption that the cyclical interest rate component in the annual data contains information about long-term developments in real interest rates (and consequently about r^*), these outcomes suggest that financial factors, such as deleveraging and changing risk preferences, have probably contributed to the recent fall in r^* . This supports the financial cycle hypothesis.¹⁴

3.2.3 Recent outcomes

Recent outcomes of the MUC model in Figure 3.2 show that r^* is positive in the US, the euro area, the Netherlands, Italy and Spain and stands at roughly

¹³ See e.g. Galati et al. (2016), Rünstler and Vlekke (2016), and De Winter et al. (2017).

¹⁴ The financial cycle hypothesis is further supported by the outcomes of empirical research on benchmarks of the financial cycle and their interaction with real variables. See e.g. Galati et al. (2016), De Winter et al. (2017) and ECB (2018).

0% in Japan and Germany. Only the Italian natural rate of interest shows a different trend, but due to the very wide confidence intervals, those findings are highly uncertain. They are moreover heavily influenced by the model used. The outcomes for r^* based on estimations using the semi-structural HLW model are different, for instance (see Box 1). Nonetheless, the common denominator is still the wide confidence intervals around r^* . This aligns with Weber et al. (2007), who have concluded that the great uncertainty surrounding attempts to determine r^* limits its value as a benchmark for monetary policy. More generally, policymakers ought to be cautious in basing their policy on non-observable variables such as r^* (Tarullo, 2017).

Another observation from the MUC model outcomes is that, in Germany and the Netherlands, r^* has fallen considerably more sharply in recent years than in Italy, Spain and the euro area average. One explanation for this is the relatively high risk premium on government bonds of Italy and Spain, both euro area countries with a higher risk profile. As described in Chapter 2, the risk premium is a determinant of r^* . The premium on bonds issued by countries with a higher risk profile means the natural rate of interest of countries with a lower risk profile, such as Germany and the Netherlands, is relatively low. Given the relative safety of bonds issued by those countries, investors are prepared to accept a lower interest return. These may be institutional investors with a preference for safe bonds because of their investment mandate.

Box 1. Estimations of r^* using the Holston, Laubach and Williams model

r^* was estimated for the euro area using the semi-structural model developed by Holston, Laubach and Williams (2017) as described in Chapter 1. This showed a clear downward trend in r^* (Figure A, light blue line, HLW r^*). Two frequently cited criticisms of the HLW model are the relatively unrealistic course of the output gap and the calculation of expected inflation (moving average of inflation rates in the preceding year). DNB is developing an alternative model that addresses both these points (Hindrayanto and Li, forthcoming). This model uses the relationship between the trend in output and unemployment (Okun's law) to determine the trend growth in output. Expected inflation is also calculated simultaneously within the model, on the assumption that this corresponds to the non-observable stable component of actual inflation (estimated local trend of the observed inflation rate). Estimating the trend growth in output and expected inflation within the model produces a more plausible output gap trend and shows a different development of r^* (grey line, r^* Alt in Figure A). The alternative model estimates r^* in the euro area in 2017 at between -1.4% and +1.5%. The dotted lines in the figure show the confidence interval of ± 1 times the standard deviation; in other words, there is a 70% probability that the actual r^* lies within this interval. Consequently, the message from the alternative model is the same as that from the original HLW model, i.e. that the estimated r^* is surrounded by very great uncertainty.

Figure A Estimations of r^* for the euro area using the HLW model

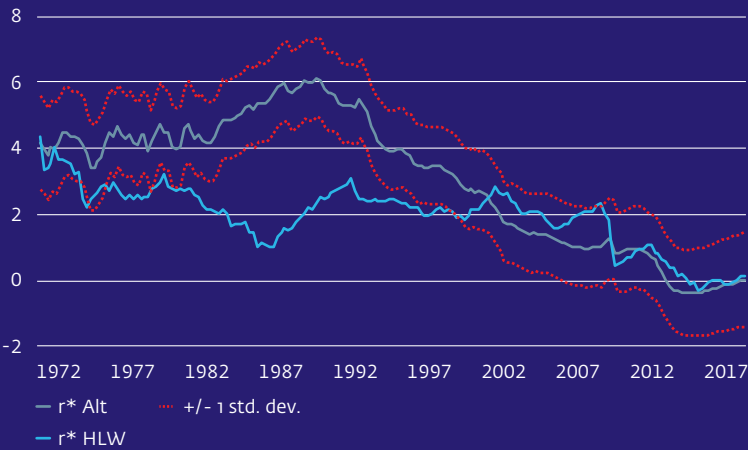
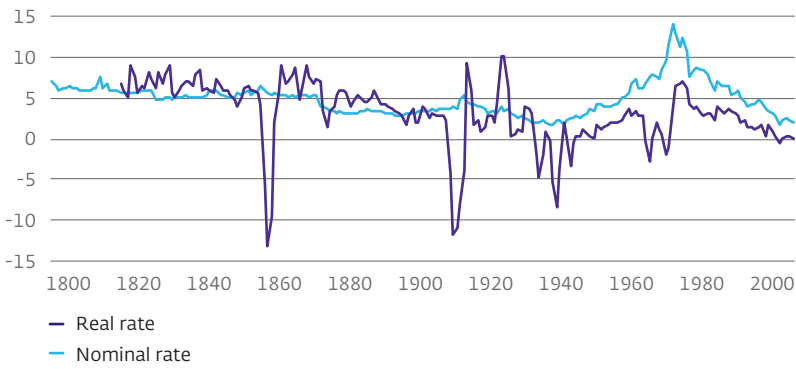


Figure 3.1 Nominal and real long-term interest rates (annual data)

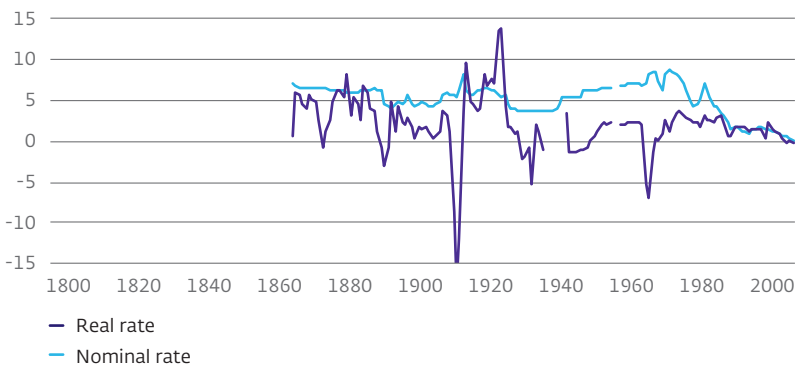
US

percentage



Japan

percentage



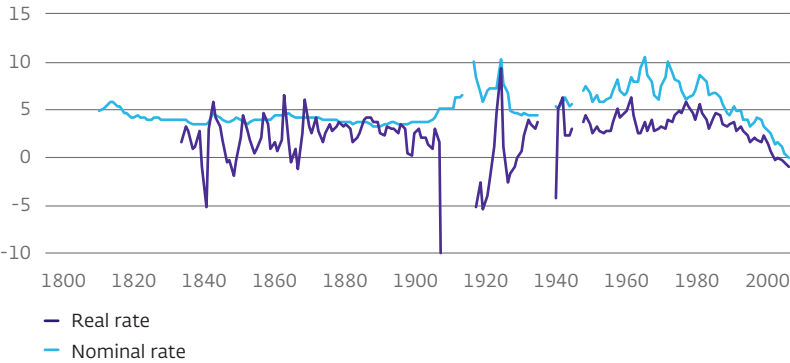
Euro area

percentage



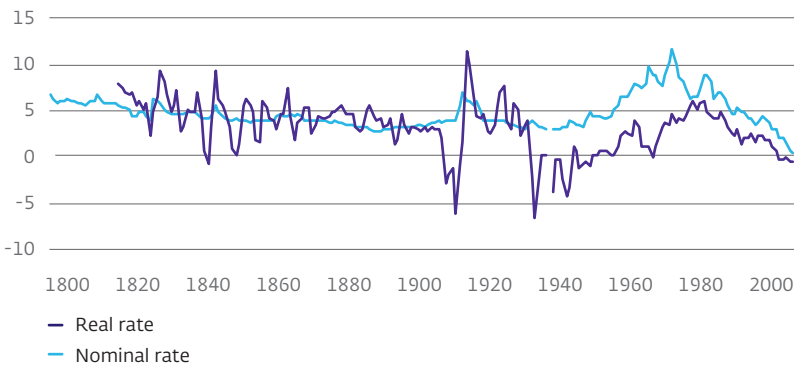
Germany

percentage



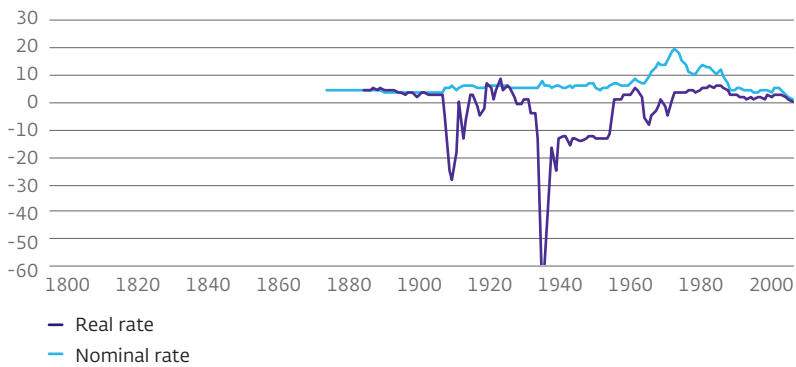
Netherlands

percentage



Italy

percentage



Spain

percentage

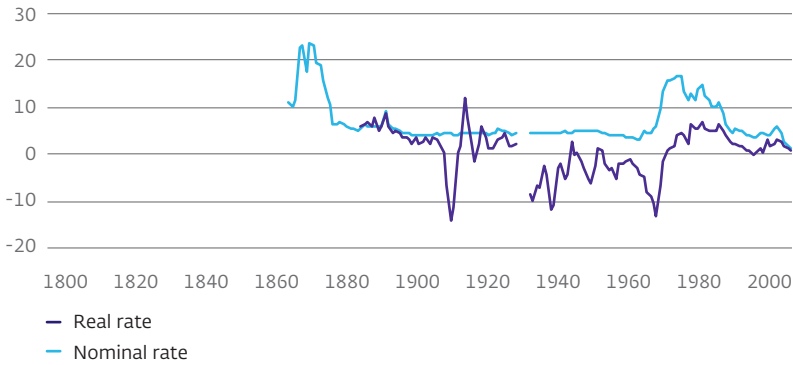


Table 3.1 Statistical properties of long-term interest rates

(annual data, period 1800-2016, where available)

Nominal long-term interest rates							
	US	JP	EA	NL	DE	IT	SP
min	1.7	-0.1	0.9	0.3	0.1	1.5	1.4
max	13.9	8.9	14.6	11.6	10.4	20.2	23.7
trend ¹	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)

Real long-term interest rates							
	US	JP	EA	NL	DE	IT	SP
min	-13.3	-17.6	-0.1	-6.7	-43.1	-71.7	-14.2
max	10.2	13.8	6.1	11.3	9.2	8.9	12.2
trend ¹	I(0)	I(1)	I(1)	I(0)	I(0)	I(0)	I(0)

¹ Stationarity based on Dicky Fuller unit root test, with 1 lag.
I(1): trend, I(0): stationary, at 5% confidence interval.

34

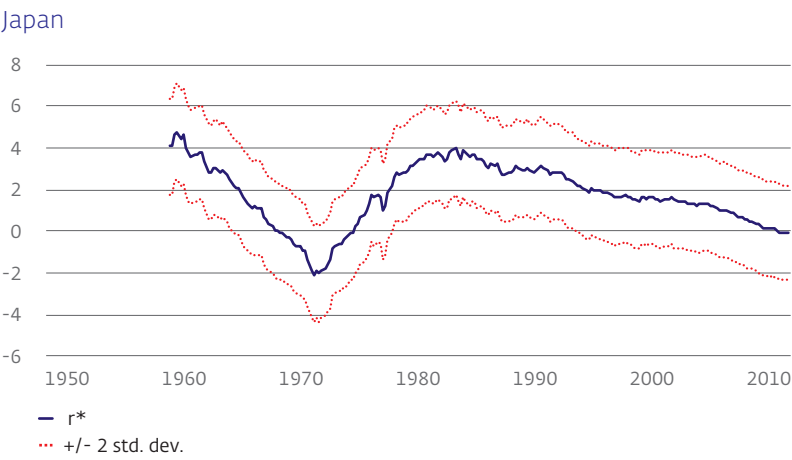
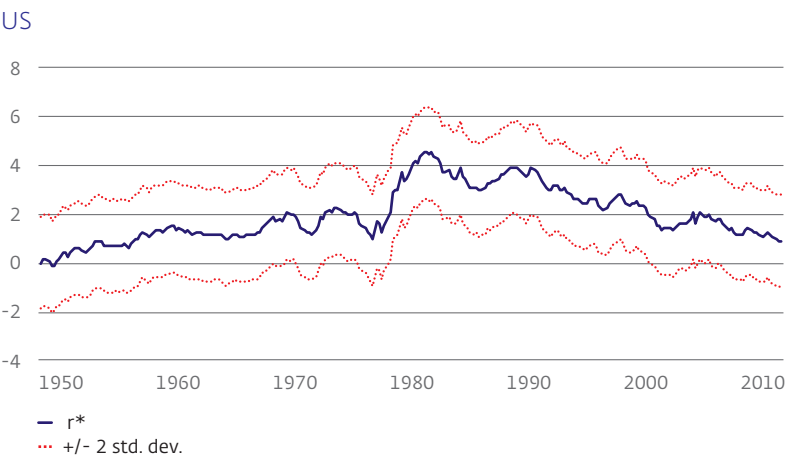
Table 3.2 Correlation coefficients of long-term interest rates

(annual data, period 1800-2016, where available)

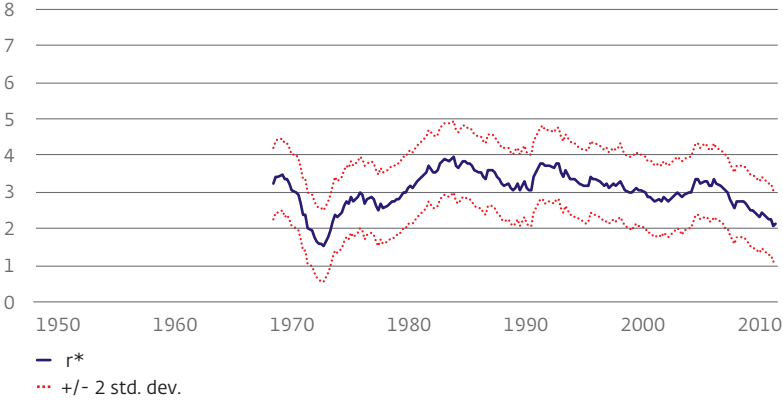
Nominal long-term interest rates						
	US	JP	EA	NL	DE	IT
JP	0.84					
EA	0.95	0.92				
NL	0.89	0.94	0.95			
DE	0.84	0.91	0.91	0.98		
IT	0.92	0.86	0.97	0.88	0.80	
SP	0.84	0.60	0.83	0.67	0.60	0.87

Real long-term interest rates						
	US	JP	EA	NL	DE	IT
JP	0.67					
EA	0.79	0.69				
NL	0.63	0.57	0.91			
DE	0.62	0.37	0.83	0.84		
IT	0.67	0.79	0.73	0.55	0.32	
SP	0.47	0.59	0.54	0.49	0.16	0.76

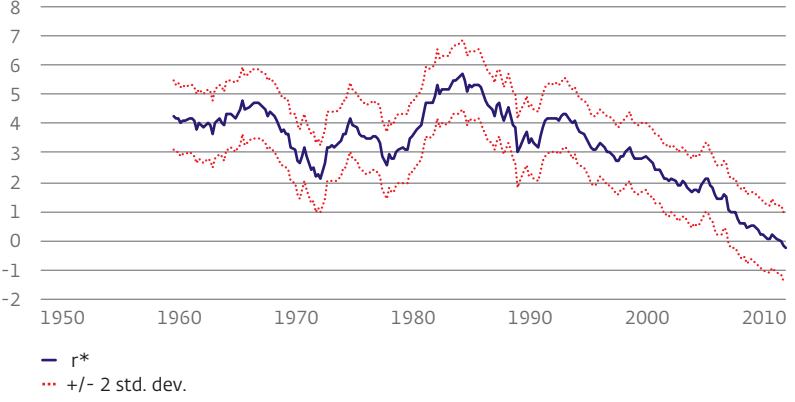
Figure 3.2 Estimates of r^* using MUC model (percentage, quarterly data), where r^* is the trend component of the real long-term interest rate



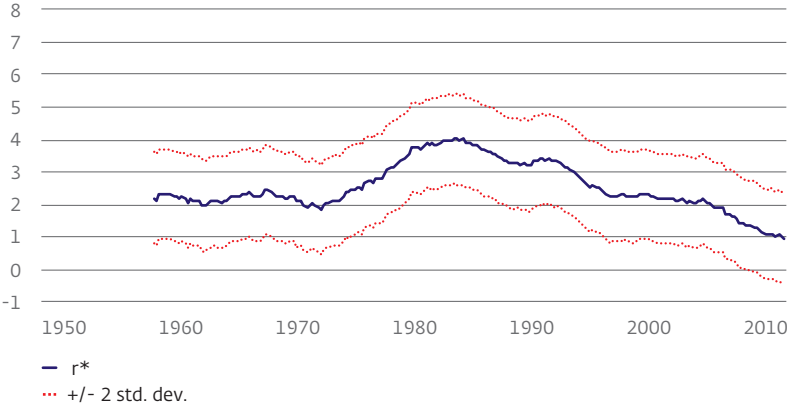
Euro area



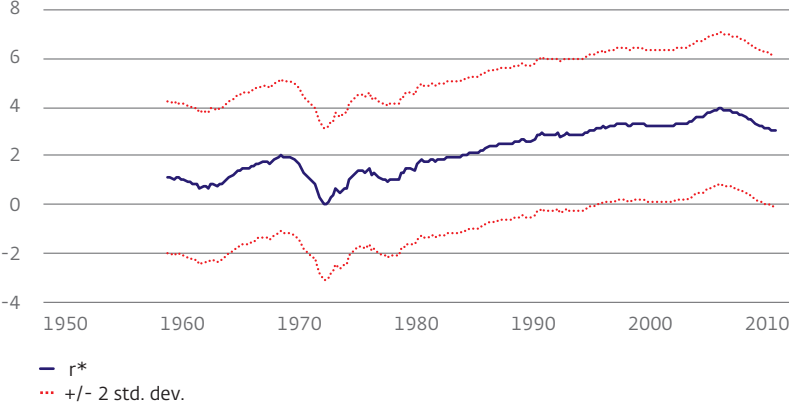
Germany



Netherlands



Italy



Spain

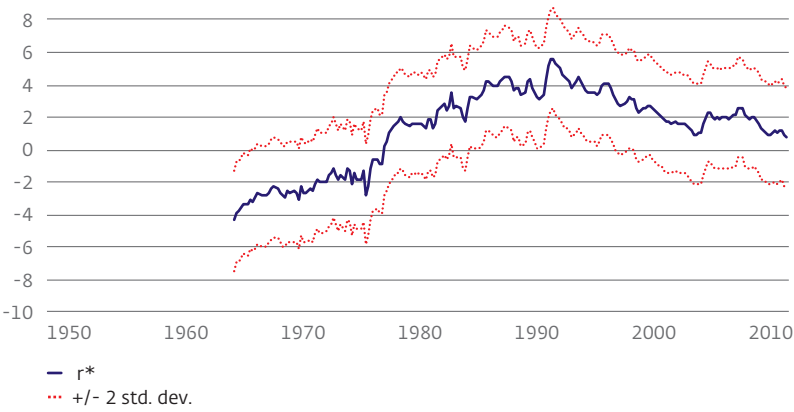
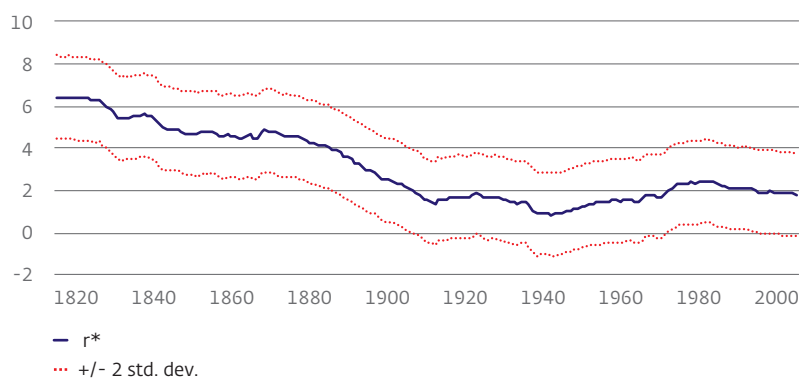


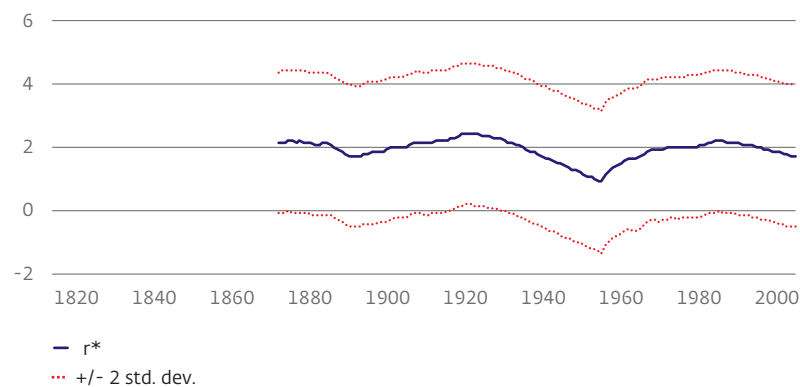
Figure 3.3 Estimates of r^* using MUC model (percentage, annual data), where r^* is the trend component of the real long-term interest rate

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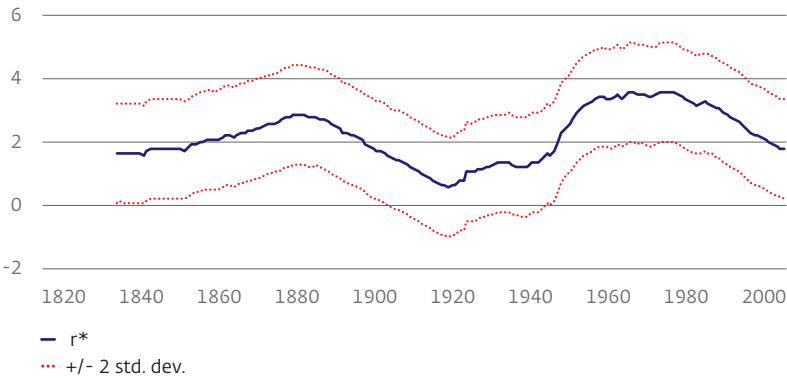
US



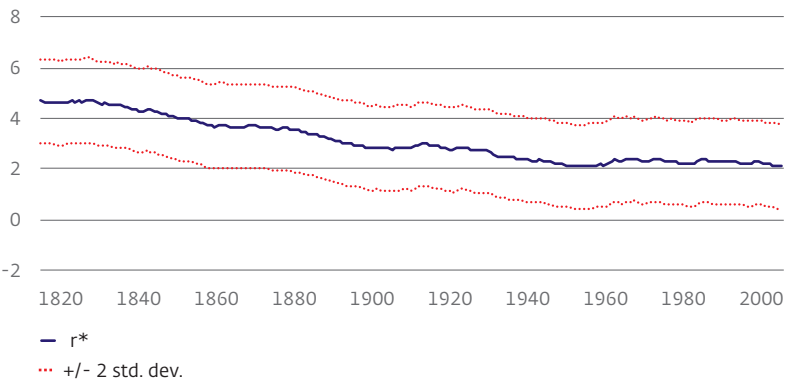
Japan



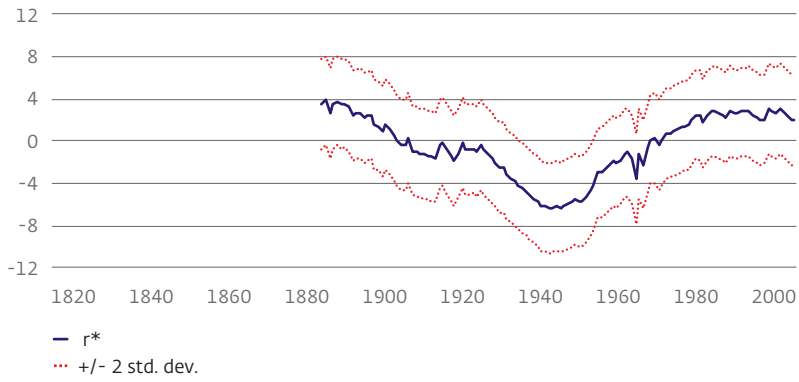
Germany



Netherlands



Italy



Spain

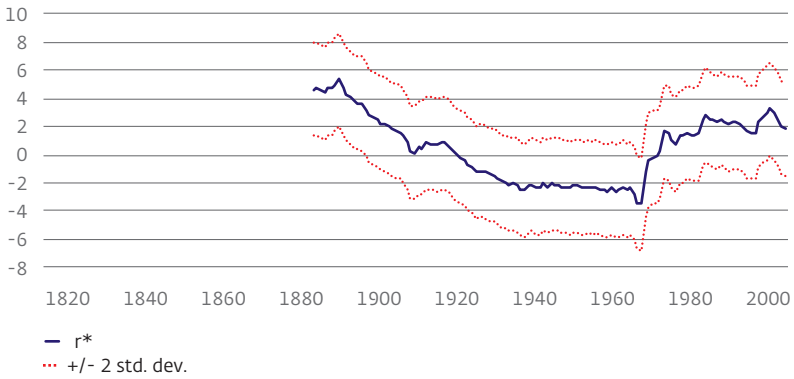
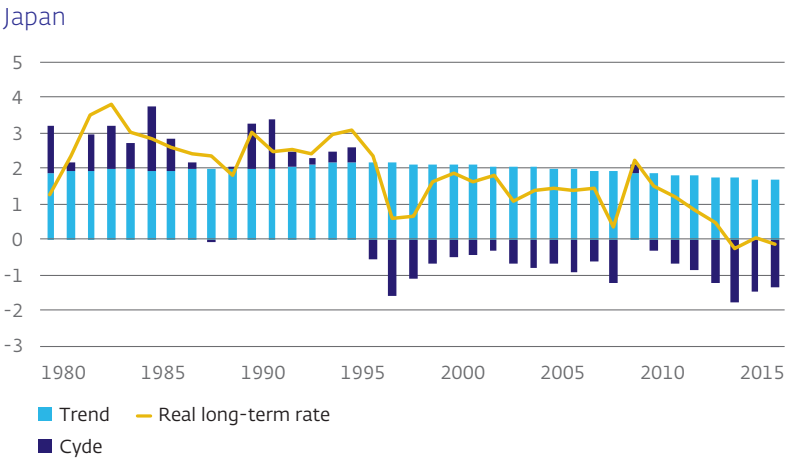
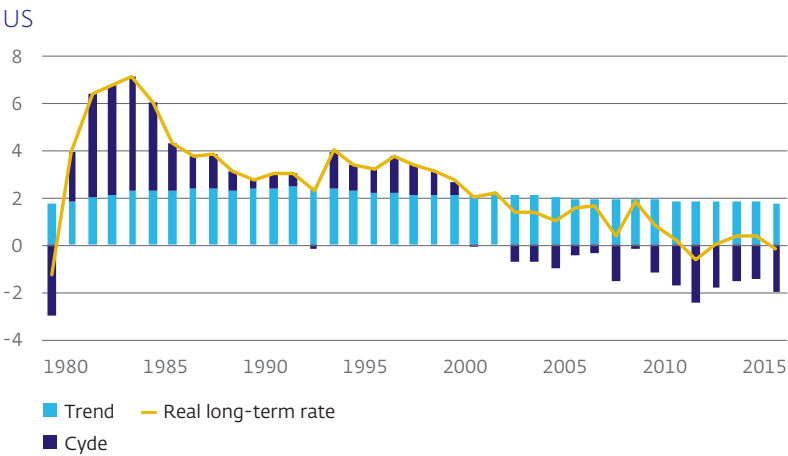
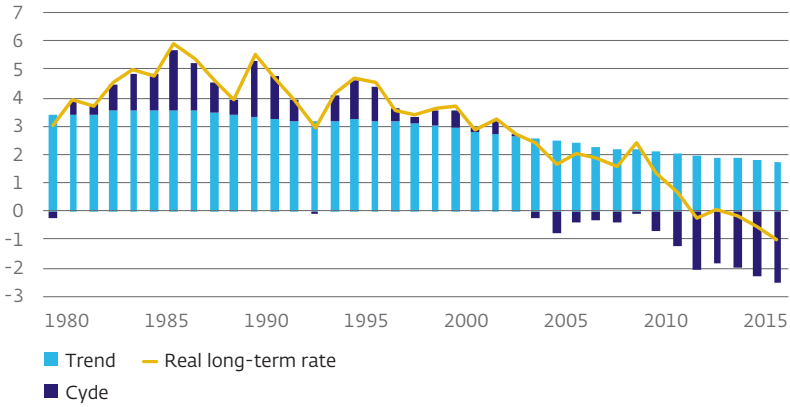


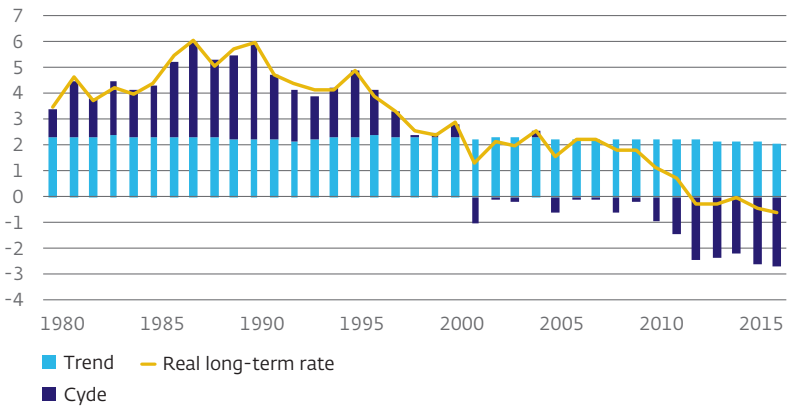
Figure 3.4 Real long-term interest rate, together with trend and cycle as estimated using MUC model (percentage, annual data)



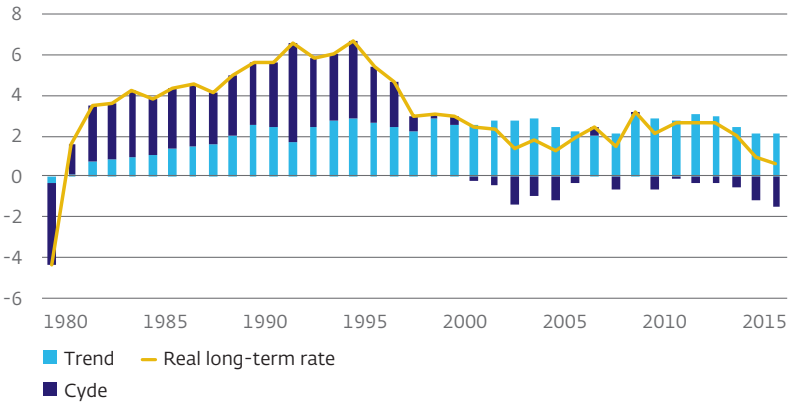
Germany



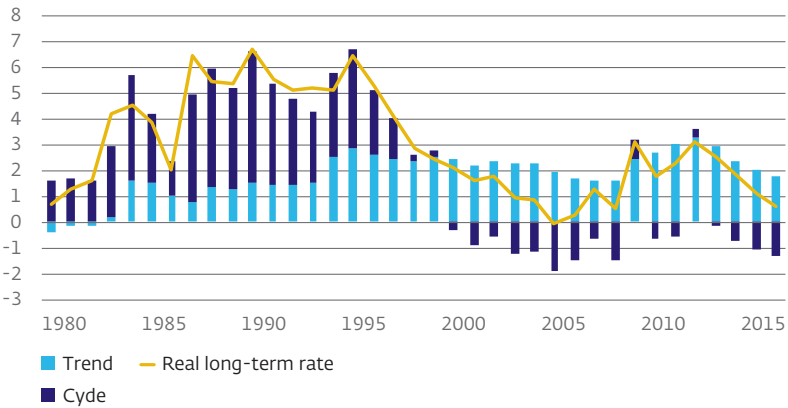
Netherlands



Italy



Spain



4 Influence of monetary policy

This chapter presents estimates of the influence of monetary policy on r^* in countries that are representative for the three major currency areas: Germany, the US and Japan. The estimates are based on a Structural Vector Autoregressive (SVAR) model, another frequently used time series model.

45

4.1 Model-based approach

The model takes the potential economic growth as an approximation for r^* rather than the values of r^* as estimated in Chapter 3, the reason being that the estimates stem from a model and are therefore sensitive to model assumptions and less suitable for use as variables in the SVAR model. Estimations of the potential growth are also uncertain, but they are based on observable survey information. Moreover, potential economic growth is also regarded as a determinant of r^* in the literature (HLW, 2017). The underlying mechanism is that higher potential growth is accompanied by a higher expected return on capital, which is a proxy for r^* (a high expected return stimulates demand for capital, leading to higher interest rates; see Chapter 2). The potential growth is approximated by the expected real GDP growth over the long term, based on the assumption that the economy will be in equilibrium in the long term and that there is no difference between actual and potential growth.¹⁵ This proxy for r^* is included in the SVAR model along with five other variables and a number of control variables:

$$\begin{bmatrix} Y_t \\ X_t \end{bmatrix} = C_0 + A(L) \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \varepsilon_t$$

¹⁵ The expected real GDP growth in the long term follows from expectations for the coming five to ten years according to the Consensus Economics survey, for which data is available at six-monthly intervals for the period 1990–2016.

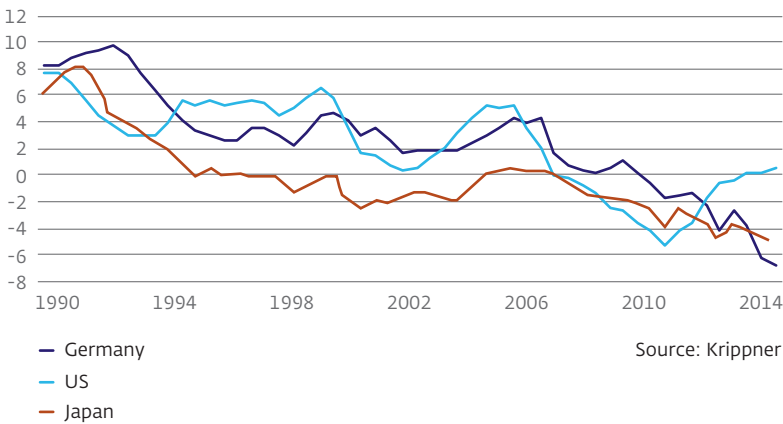
In addition to r^* , vector Y also includes actual real GDP growth (gdp) and inflation (p), the risk premium (rp ; the difference between the interest rate on corporate and government bonds), movements in real effective exchange rates (e) and the monetary policy variable (mp ; i.e. the shadow interest rate). The shadow interest rate measures both conventional and unconventional monetary policy and can turn sharply negative, unlike the money market rate, which has a floor. At the effective floor, the (negative) shadow interest rate measures the effect of the expansionary balance sheet policy. The shadow interest rate (shown in Figure 4.1¹⁶) was calculated by Krippner (2013) by deducting the value of the option of retaining cash from the short-term market rate. This option value increases at an interest rate of zero percent. X_t is a vector with exogenous control variables, i.e. labour productivity growth and population growth, which according to the literature are explanatory factors for r^* (see Chapter 2). Vector C_0 is the vector with constant terms and ε is the vector with residuals.¹⁷

The SVAR model is estimated and the influence of monetary policy subsequently determined with a historical decomposition of the shock effects on r^* . This allows the effect of the monetary policy variable (the shadow interest rate) on r^* to be calculated. According to the theoretical literature, monetary policy has no influence on the real economy in the steady state, although it can influence r^* in the short term – in the shift towards equilibrium – because of short-term price rigidity. The short-term effect can operate both directly and indirectly, via the other model variables.

16 The shadow interest rate is available for 1995 and subsequent years; for earlier years, we estimate it using the nominal money market rate. To estimate the non-available shadow interest rate for Germany, we take that for the euro area. See also Pattipeilohy et al. (2017) for an explanation of the concept of shadow interest rate.

17 The model is estimated with one lag, based on tests for the number of lags.

Figure 4.1 Shadow interest rate (mp)



Identification of shocks is based on assumptions made in comparable studies (Gerlach and Smets, 1995; Bernanke and Blinder, 1992). It is assumed that shocks in the monetary variable, the exchange rate, real GDP growth and inflation have no long-term effect on r^* (comparable with the general assumption that demand shocks have no long-term effect on supply factors). In the evolution towards long-term equilibrium, however, these shocks can influence r^* . The risk premium does have a long-term effect on r^* , reflecting findings in the literature (see Chapter 2). It is also assumed that a monetary policy shock has no short-term (immediate) effect on real GDP growth and inflation, assuming lags in the transmission process. Monetary policy does by contrast have an immediate effect on the risk premium and exchange rate, but these variables do not influence monetary policy (the assumption for the exchange rate follows Peersman and Smets, 2003). Table 4.1 summarises the assumptions.

Table 4.1 Identification method using SVAR model

effect	shock					
	<i>e</i>	<i>rp</i>	<i>mp</i>	<i>gdp</i>	<i>p</i>	<i>r*</i>
<i>e</i>	-	-	-	-	-	-
<i>rp</i>	KT=0	-	-	-	-	-
<i>mp</i>	KT=0	KT=0	-	-	-	-
<i>gdp</i>	KT=0	KT=0	KT=0	-	-	-
<i>p</i>	KT=0	KT=0	KT=0	KT=0	-	-
<i>r*</i>	LT=0	KT=0	LT=0	LT=0	LT=0	-

4.2 Outcomes

The contribution made by structural shocks in the monetary policy variable (shadow interest rate) to *r** is shown in Figure 4.2. This demonstrates the difference between the observed real long-term interest rate and the baseline forecast from the SVAR model (forecast without shock effects). The difference is determined by cumulative effects of structural shocks in *r** and shocks in the other endogenous variables: the monetary variable (*mp*), the exchange rate (*e*), the risk premium (*rp*), real GDP growth (*gdp*) and inflation (*p*)

The contribution of monetary policy to *r** varies from country to country and from period to period. The negative contribution of monetary policy in Germany was visible in 2015, when the asset purchase programme (APP) began. Quantitative easing (QE) was introduced much earlier in the US, i.e. in 2008, and the shadow interest rate started to fall sharply from that year onwards. As a consequence, monetary policy had a downward influence on *r**. The shadow interest rate started to rise again in 2013, when the Fed began tapering its QE programme. This explains the positive contribution to *r** in recent years. Japan has had in place an accommodative monetary

policy since the 1990s, as a result of which the Japanese shadow interest rate has been systematically below that of Germany and the US (Figure 4.1). Monetary policy in Japan made a negative contribution to r^* principally between 1996 and 2006.

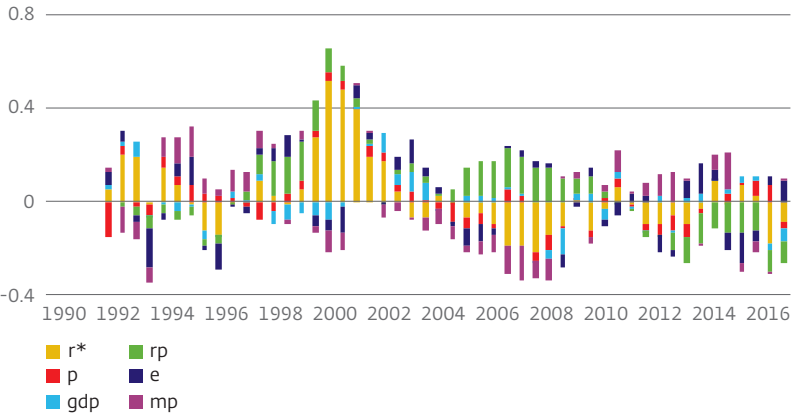
A key advantage of VAR models is that they are able to generate ‘impulse response’ functions, enabling the effect of an isolated shock in a variable on all other variables in the system to be calculated. Impulse response functions show that a monetary policy shock only has a statistically significant effect on r^* in Japan, where a monetary tightening (*mp*) leads to an increase in r^* (and a monetary easing to a reduction in r^*).

Looking at the influence of the other variables on r^* , we find that monetary policy has a relatively big influence on r^* in Japan (Table 4.2). Depending on the restrictions imposed in the model, the average contribution of monetary policy to r^* is between 25% and 32% in Japan, between 15% and 26% in the US and between 13% and 16% in Germany. These estimates are beset with the uncertainty inherent in the fact that the proxy used for r^* is an approximation of the natural rate of interest. Moreover, the impulse response functions show that the effect of monetary policy on r^* is only statistically significant in Japan.

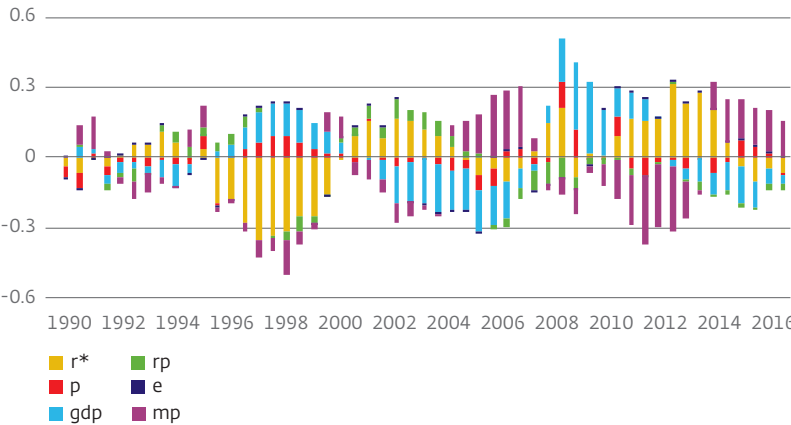
As regards the other determinants, r^* is relatively heavily influenced by shocks in the exchange rate (*e*) in Germany and Japan and by shocks in real GDP growth (*gdp*) in the US. The relatively large contribution of the risk premium to r^* in Germany is striking, being substantially greater than in Japan and the US. This is in line with Gerali and Neri (2017), who find that the fall in the natural rate of interest in the euro area in particular is driven by shocks in the risk premium, which reflect changes in such aspects as the preference for safe assets (see Chapter 2).

Figure 4.2 Historical decomposition of shocks on r^*
(based on short-term and long-term restrictions,
percentage points on y-axis¹⁾)

Germany

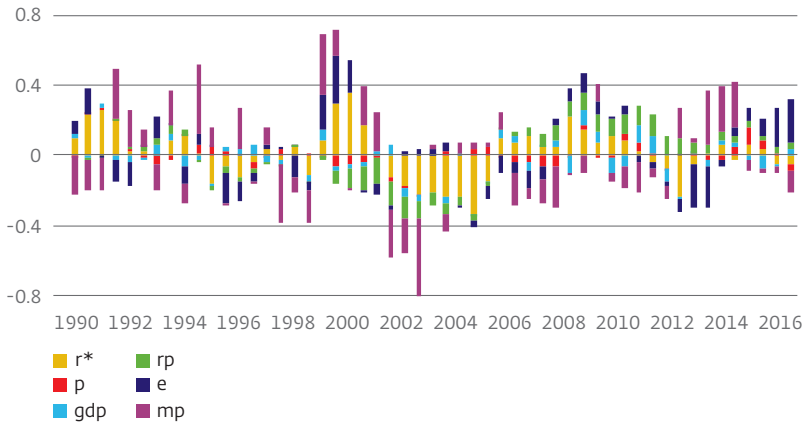


US



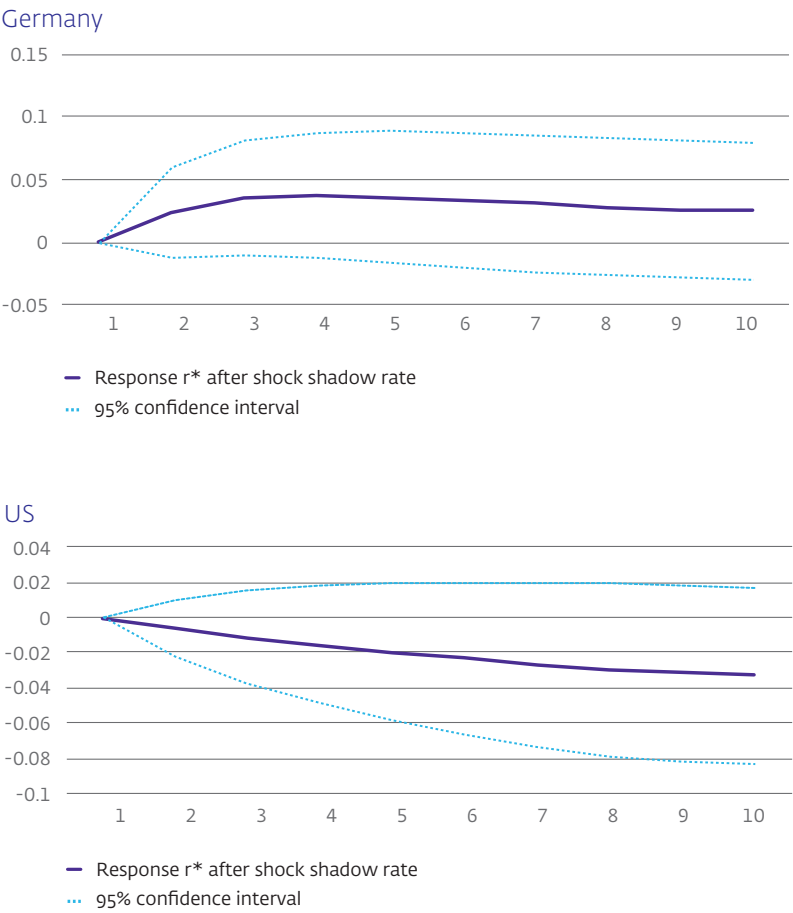
Japan

51



¹ The decomposition is determined by shocks in the natural rate of interest itself (r^*), the monetary variable (mp), the exchange rate (e), risk premium (rp), real GDP growth (gdp) and inflation (p).

Figure 4.3 Impulse response r^* after 1 standard deviation shock in shadow interest rate (Cholesky decomposition, percentage points on y-axis)



Japan

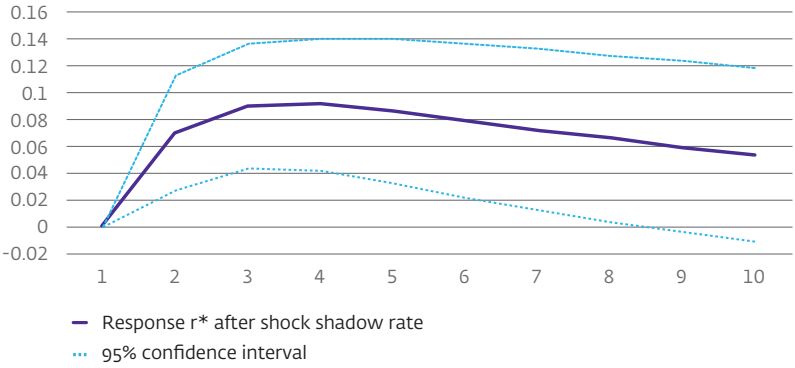


Table 4.2 Effect of shocks in shadow interest rate on r^* (sample period: 1990-2016)

Cumulative absolute effect of structural shock in mp as percentage of cumulative absolute effect of all shocks on r^* (excluding the effect of shocks to r^* itself)

Germany		US		Japan	
<i>LT & ST restrictions</i>	<i>ST restrictions</i>	<i>LT & ST restrictions</i>	<i>ST restrictions</i>	<i>LT & ST restrictions</i>	<i>ST restrictions</i>
16%	13%	26%	15%	32%	25%

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54

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Annex 1 The natural rate of interest in a general equilibrium model

60

This box shows how the natural rate of interest can be derived analytically from a simple general equilibrium model. The model is made up of households, companies and a central bank. The optimum distribution between consumption and saving is determined by the following demand function:

$$c_t = c_{t+1}^e - \sigma r_t + \sigma v_t \quad (1)$$

where c_t stands for consumption in period t , c_{t+1}^e expected consumption in the following period ($t + 1$), r_t the real interest rate, and v_t a demand shock. The higher the interest rate, the greater the propensity to save and therefore the lower the level of consumption. A demand shock leads to an increase in consumption. The parameter $\sigma > 0$ reflects the extent to which households are willing to trade off present and future consumer spending against each other (the higher the value of σ , the less pronounced the need for such a trade-off). Companies produce the supply of consumer goods, y_t , based on the following production function:

$$y_t = p_t + a_t \quad (2)$$

where p_t is a production shock and a_t the number of workers hired by companies. The optimum demand for labour is determined by equating the real marginal costs, mc_t , to the wage base, w_t , adjusted for productivity:

$$mc_t = w_t - p_t \quad (3)$$

The optimum labour supply is determined using the following equation:

$$a_t = \varphi w_t - \varphi \sigma c_t \quad (4)$$

The higher the wage, the greater the willingness to work and the higher the labour supply. The last term in equation (4) shows the income effect: the more affluent someone is, the fewer hours they wish to work. The parameter $\varphi > 0$ measures the labour elasticity, conditional on the income effect. Finally, the demand for and supply of consumer goods must correspond:

$$y_t = c_t \quad (5)$$

We now have all the ingredients needed to derive r^* . Remember that, in this type of model, r^* corresponds to the interest rate that occurs in an equilibrium with flexible prices, in which marginal costs are constant. By way of illustration, we assume that the following applies in this equilibrium: $mc_t = 0$. If we now combine equations (2) through (4), we find the specification for potential output, y_t^* :

$$y_t^* = \frac{1 + \varphi}{1 + \varphi\sigma} p_t$$

An increase in productivity leads to an increase in potential output. Using this outcome and equation (5), we can rewrite equation (1):

$$x_t = x_{t+1}^e - \sigma(r_t - r_t^*)$$

where $x_t = y_t - y_t^*$ is the output gap, x_{t+1}^e the expected output gap, and r_t^* the natural rate of interest:

$$r_t^* = \frac{1 + \varphi}{\sigma(1 + \varphi\sigma)} (p_{t+1}^e - p_t) + v_t \quad (6)$$

with p_{t+1}^e being the expected productivity in the subsequent period.

An increase in expected productivity growth, i.e. a rise in $p_{t+1}^e - p_t$, means that households will earn more in the future and thus be able to consume more. As households seek to 'smooth' their consumption pattern over time as far as possible, their consumption today will rise too. This causes savings to fall and r_t^* rises to bring the savings market back into balance. The higher the value of σ , the less need there is for intertemporal smoothing of consumption and the weaker the effect of the expected increase in productivity growth on r_t^* . A positive demand shock, i.e. an increase in v_t , leads to a direct increase in consumption, causing r_t^* to rise once again in order to bring savings supply and demand into balance.

Annex 2 Time series models used

63

Autoregressive (AR) model for expected inflation

As r^* is a real variable, the nominal interest rate is deflated with the expected inflation rate to determine a real interest rate (r_t), which is constructed as:

$$r_t = \left(\left(\frac{1 + i_t}{1 + \pi_{t+1}^e} \right) - 1 \right) \times 100$$

where π_{t+1}^e is the expected inflation rate for the coming year and i_t is the nominal interest rate. The prediction by an autoregressive (AR) model is used as a proxy for the expected inflation rate; expected inflation is estimated in this model with one lag, which applies consistently for a shifting constituent period. This allows the parameters of the model to vary over time. The entire sample period runs from $t = 1, \dots, T$. If n is the length of the constituent period, we take $n=20$ for the annual data and $n=40$ for the quarterly data. As usual in a rolling regression, the most recent n observations are used for the model estimation at time t (no future observations are used). This is how $T - n + 1$ rolling model estimations are made.

Both the short-term and long-term interest rates are made real with the one-year estimated inflation expectation, as in the equation above.¹⁸ Expected inflation for the series based on annual data is the inflation forecast one year ahead; for the quarterly series, expected inflation is the average of the predicted inflation rate for the ensuing one to four quarters inclusive. Although these are imperfect approximations of the actual inflation expectations, the model predictions are close to the actual

¹⁸ As an alternative, the real long-term interest rate was also constructed on the basis of inflation predictions for five years ahead, but these proved to be too unstable to allow a real interest rate to be constructed.

inflation rate, which is plausible if it is assumed that expectations of future inflation are based on today's inflation rate.

Multivariate Unobserved Component (MUC) model for r^*

The MUC model is used to estimate r^* with three variables: the real long-term interest rate, the real short-term interest rate and the real GDP of the country concerned. The nominal interest rates are deflated with inflation expectations (as simulated in the AR model above). The model assumes that the observable variables (y_{it}) can be split into a trend (μ_{it}), a cyclical (ψ_{it}) and an error component (ε_{it}). The proxy for r^* is the trend component of the real long-term interest rate.

In mathematical terms, the model is as follows, for $t = 1, \dots, T$,

$$\begin{aligned}
 y_{it} &= \mu_{it} + \psi_{it}^S + \psi_{it}^L + \varepsilon_{it}, \quad \varepsilon_{it} \sim NID(0, \sigma_{\varepsilon,i}^2), \quad i = 1, 2, 3, \\
 \mu_{it} &= \mu_{i,t-1} + \eta_{it}, \quad \eta_{it} \sim NID(0, \sigma_{\eta,i}^2), \quad i = 1, 2, \\
 \mu_{it} &= \mu_{i,t-1} + \beta_{it}, \quad \beta_{it} = \beta_{i,t-1} + \zeta_{it}, \quad \zeta_{it} \sim NID(0, \sigma_{\zeta,i}^2), \quad i = 3, \\
 \begin{bmatrix} \psi_{it}^S \\ \psi_{it}^{S*} \end{bmatrix} &= \begin{bmatrix} \cos \lambda_S & \sin \lambda_S \\ -\sin \lambda_S & \cos \lambda_S \end{bmatrix} \otimes I_2 \begin{bmatrix} \psi_{t-1}^S \\ \psi_{t-1}^{S*} \end{bmatrix} + \begin{bmatrix} \kappa_t^S \\ \kappa_t^{S*} \end{bmatrix}, \quad \begin{bmatrix} \kappa_t^S \\ \kappa_t^{S*} \end{bmatrix} \sim NID \left(0, \begin{pmatrix} \Sigma_{\kappa,S} & 0 \\ 0 & \Sigma_{\kappa,S} \end{pmatrix} \right), \\
 \begin{bmatrix} \psi_{it}^L \\ \psi_{it}^{L*} \end{bmatrix} &= \begin{bmatrix} \cos \lambda_L & \sin \lambda_L \\ -\sin \lambda_L & \cos \lambda_L \end{bmatrix} \otimes I_2 \begin{bmatrix} \psi_{t-1}^L \\ \psi_{t-1}^{L*} \end{bmatrix} + \begin{bmatrix} \kappa_t^L \\ \kappa_t^{L*} \end{bmatrix}, \quad \begin{bmatrix} \kappa_t^L \\ \kappa_t^{L*} \end{bmatrix} \sim NID \left(0, \begin{pmatrix} \Sigma_{\kappa,L} & 0 \\ 0 & \Sigma_{\kappa,L} \end{pmatrix} \right),
 \end{aligned}$$

where index $i=1$ represents the real long-term interest rate, $i=2$ the real short-term interest rate, and $i=3$ the logarithm of real GDP. The last variable was included in order to determine the cyclical component in the data. Variable β_t is the slope of the trend and ζ_t its error term. Vector $\psi_t = (\psi_{1t}, \psi_{2t}, \psi_{3t})'$ is the change in the set of stochastic cycles for the three observed variables, and vector ψ_t^*

a construct variable whereby the individual cycle is rewritten as a stationary ARMA(2,1) process. The model estimated using quarterly data allows for a short (ψ_t^S) and a long cycle (ψ_t^L) for each of the three variables. In the model estimated using annual data, one (medium-term) cycle is distinguished for each time series. This cycle lies between the two cycles distinguished in the quarterly data. The estimated length of the cycle in the annual data lies between nine and 17 years. Vector $\kappa_t = (\kappa_{1t}, \kappa_{2t}, \kappa_{3t})'$ is the set of error terms.

In addition to the restriction in the cyclical component (which imposes the requirement that the cycles of the three variables have the same length and persistence), a restriction is also imposed on the trend component in the model, i.e. that both interest rate variables share a common trend, which implies that they are co-integrated. This restriction follows the expectation hypothesis in which the long-term interest rate is a function of the short-term interest rate, so that both variables share the same underlying trend.

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