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Sebastiaan Pool

DeNederlandscheBank
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De Nederlandsche Bank NV
P.O. Box 98
1000 AB AMSTERDAM
The Netherlands

Credit Defaults, Bank Lending and the Real Economy*

Sebastiaan Pool[a]

[a]*De Nederlandsche Bank and University of Groningen, the Netherlands*

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Abstract

This paper examines how the materialization of credit defaults affects the real economy. I estimate a DSGE model including banks, firms and financial frictions using euro area data. The estimation results show that a positive credit default shock, which is identified as an unanticipated increase in credit default losses, complicates monetary policy because output falls while inflation goes up. The monetary authority must choose between stabilizing output and inflation and is therefore less effective. Inflation increases slightly because firms experience besides a demand contraction also a cost-push effect when banks increase the lending rate. Countercyclical capital buffers can in this case complement conventional monetary policy but there is a trade-off: they effectively attenuate macroeconomic fluctuations, but increase the persistence of the slump as banks rebuild their capital more slowly. A bank recapitalization overcomes this trade-off and significantly reduces macroeconomic fluctuations.

Keywords: Banking, Credit risk, Credit defaults, Countercyclical Capital Buffer, Bayesian Estimation.
JEL classifications: E44, E51, E52.

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1 Introduction

The global financial crisis demonstrated the importance of banks in determining macroeconomic fluctuations. Particularly, the materialization of unexpected credit defaults had severe consequences for the real economy because, in response to the losses, banks ceased credit supply which increased firms' cost of lending. The impact was especially large because banks had high leverage – assets-to-capital ratios – which compelled an excessively large reduction in credit supply. Even though almost a decade has passed since the start of the financial crisis, non-performing loans are still a major issue in the euro area. Whether conventional monetary policy is sufficiently adequate to attenuate financial fluctuations caused by defaulting loans and whether macro-prudential measures can complement conventional monetary policy is, however, largely unknown.

This paper examines the effects of an unanticipated increase in credit default losses, a *credit default shock*, on macroeconomic fluctuations. Banks provision for anticipated credit default losses and target a maximum attainable leverage ratio. As more risk corresponds to potentially higher returns with limited liability for the equity holders, banks that maximize profits maximize bank leverage conditional on regulatory and bankruptcy constraints by extending the balance sheet (Shin, 2010). Accordingly, a deterioration of financial conditions, i.e, an unexpected increase in credit default losses, could trigger a bank balance sheet recession and severely impact economic activity. To quantify the results, I estimate the model for the euro area where firms are largely bank financed. Thereafter, I use the model to perform a counterfactual analysis to analyze whether alternative policy instruments can complement conventional monetary policy measures.

A vast literature has studied the interactions between the financial sector, the Central Bank and the real economy. Presumable the most salient line of literature concerns work of, among others, Bernanke and Gertler (1989); Kiyotaki and Moore (1997); Bernanke et al. (1999); Curdia and Woodford (2009, 2010); Del Negro et al. (2010), and Christiano et al. (2014). These models show the importance of financial frictions caused by asymmetric information because they increase the persistence of an adverse shock and amplify the impact of the shock on the real economy. However, in these models the financial sector is often a veil because firms borrow essentially directly from households. These models are therefore not suitable to analyze the importance of the balance sheet of banks or the relation between bank and firm leverage.

Other studies, such as Freixas and Rochet (1997); Goodfriend and McCallum (2007); Christiano et al. (2010); Gerali et al. (2010); Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), model the impact of bank balance sheets on the real economy more explicitly. However, in these models there is only a limited role for credit risk or endogenous credit defaults. Goodhart et al. (2006, 2012) examine the consequences of endogenous defaults, but focus mostly on the (shadow) banking sector and less so on the real side of the economy. Borio and Zhu (2012) therefore argue that these models fail to endogenize the disruptive consequences of defaults on the real economy. Benes and Kumhof (2015) calibrate the impact of countercylical capital buffers on welfare when banks manage credit risk. Their approach is closely related to the approach presented in this paper, but has a pure theoretical focus. In contrast, the model in this paper is estimated to identify the impact of a credit default shock on the real economy.

Credit defaults force banks to alter their behavior via at least two channels. First, if banks anticipate more credit defaults in the future because of deteriorating economic conditions, they raise the lending rate. The increase in the lending rate decreases credit supply to the real sector and tightens the balance sheet of borrowers. The literature regarding credit defaults focuses mainly on this *credit risk channel*. Banks manage credit risk through diversification and securitization. However, if realized credit defaults are different from anticipated credit defaults, a second effect materializes as banks incur unanticipated losses. These losses wear down their banking capital. As a result bank leverage rises contemporaneously via a denominator effect which raises banks' marginal costs and induces an increase in the lending rate.

The latter increase in the lending rate is suboptimal from a finance perspective as the decision to finance new projects should be independent from costs made on previous projects, i.e., losses on previous projects should be treated as a sunk cost. Nevertheless, if leverage ratios become too high, financial market discipline or regulation forces banks to deleverage.[1] Banks can deleverage by accumulating more capital and by decreasing loans and deposits thereby shrinking their balance sheet.[2] Since rebuilding capital from retained earnings is a slow process, bank capital is fixed in the short run (Adrian and Shin, 2010). Therefore a deterioration of bank capital could restrain both liquidity creation (Diamond and Rajan, 2009; Van den Heuvel, 2008) and credit supply (Woodford, 2010). Consequently, unanticipated credit default losses force banks via a *balance sheet channel* to decrease credit supply in order to deleverage.

In this study I examine how unanticipated credit defaults affect credit risk, interest rates, bank lending and inflation. In the model the inability of banks to perfectly assess the borrower's creditworthiness generates a financial friction similar to Bernanke et al. (1999). Although diversification in theory mitigates the unanticipated consequences of credit defaults, the sub-prime crisis exemplified the risks of diversification when the recent past is not representative for the future. The credit default shock is interpreted as a realization of credit default losses higher than anticipated levels. In the model, a realization of credit losses higher than anticipated ex-ante deteriorates banking capital. This modeling assumption is in line with existing evidence in the empirical literature on loan loss provisioning (Laeven and Majnoni, 2003; Bikker and Metzemakers, 2005; Pool et al., 2015), i.e., credit default losses impact bank capital directly because banks do not provision extra in good times to build a buffer for bad times.

The theoretical model is estimated for the euro area with Dynare using a Bayesian estimation approach for the period 2000Q1-2014Q2.[3] Apart from macroeconomic data (output, consumption, investment, inflation, hours worked, wages, outstanding corporate loans, and outstanding deposits), I also use financial data: policy rate, interest rate on loans, interest rate on deposits and credit spreads. The credit spread data is constructed by Gilchrist and Mojon (2014). As only short time series are available for the euro area, the parameters are divided into two groups. The first group contains conventional parameters which are calibrated based on the literature. The second group of parameters

[1] Specifically, regulation prescribes banks to lower their value at risk (VaR). Banks could, when it concerns bank capital requirements, deleverage by lowering the risk level of their assets. Below I consider only one investment asset, loans to firms, which have the same risk level ex-ante and a risk weight of 100%.

[2] Bank can also issue equity to raise capital; however, during a downturn or in financial distress, issuing equity is expensive as it might signal insolvency.

[3] See Adjemian et al. (2011) for the appropriate documentation.

are estimated. The prior specification for the second group is taken from the literature or set according to historical sample averages.

The estimation results show that conventional monetary policy effectively attenuates macroeconomic fluctuations after an adverse technology shock and a contractionary monetary policy shock. A credit default shock complicates conventional monetary policy. Banks become constrained by their leverage ratio and raise lending rates. Credit supply declines and economy wide demand falls. Firms experience, besides a demand contraction, a cost-push effect because their funding costs increase. The increase in costs forces firms to raise prices when their net worth is insufficient to incur losses. The net effect is a small increase in the inflation rate. The monetary authority must choose between stabilizing output and inflation and conventional monetary policy is therefore not effective in attenuating macroeconomic fluctuations.

As conventional monetary policy is less effective after a credit default shock, I introduce two alternative policy instruments that address the bank capital constraint more directly: a countercyclical capital buffer and an endogenous recapitalization. Countercyclical capital buffers as prescribed by the Basel committee can be effective in mitigating the effects of a credit default shock. However, there is a trade-off when the countercyclical capital buffer is activated because banks rebuild their capital more slowly. The countercyclical capital buffer smooths the effects of a credit default shock over time; as a consequence, the downturn is more persistent. In contrast, an endogenous recapitalization financed by lump-sum taxation effectively overcomes this trade-off problem as rebuilding bank capital is no longer the bank's choice. An endogenously recapitalization is therefore an effective instrument to mitigate macroeconomic fluctuations after a credit default shock.

The results suggest that it is important to identify a shock and tailor the policy response. When the economy is for example hit by a technology shock, conventional monetary policy is effective. However, when the banking sector is hit by a shock that deteriorates bank capital, macro-prudential policy measures like countercyclical capital buffers or a bank recapitalization might be more effective and can complement conventional monetary policy measures. While there is a trade-off between the depth and the length of the crisis when the countercyclical capital buffers is activated, one might argue that also an endogenous recapitalization raises moral hazard issues not explicitly modeled in this paper. It is important to note, however, that the model is no able to distinguish between a taxed-financed recapitalization, a bail-in or a mandatory equity issuance as the consumer is the taxpayer, the bank equity holder and the depositor. A mandatory equity issuance therefore seems the preferred option to formalize the endogenous recapitalization without the corresponding moral hazard problems.

The remainder of this paper is structured as follows. Section 2 describes the model. Section 3 discusses the data, the calibration and the Bayesian methodology. Section 4 presents the estimation results. Section 5 concludes.

2 The model

The real side representation of the model follows closely Smets and Wouters (2003) and Christiano et al. (2005). The banking sector is embedded following Gerali et al. (2010)

and credit risk is modeled according to the financial accelerator mechanism described in Bernanke and Gertler (1989) and formalized in Bernanke et al. (1999). The model contains two types of agents, households denoted by the superscript H and entrepreneurs denoted by E who operate the firms. Households and entrepreneurs interact via a banking sector which competes in a monopolistic competitive environment because they offer heterogeneous financial products.

Households represent the labor force in the economy. They maximize utility over an infinite life span. Households differ from one another in that they supply a differentiated type of labor. As a consequence, each household has some monopoly power over the supply of its labor which generates wage rigidity. Entrepreneurs operate the intermediate firms. They maximize utility and supply their labor inelastically. Entrepreneurs accumulate capital and hire labor from households to produce a differentiated intermediate product. Entrepreneurs borrow to buy capital which they use to produce the intermediate good. Borrowing and saving in this economy is intermediated by a banking sector. The banks face the risk that the intermediate firms default on their loan repayment. If an intermediate firm defaults, the bank realizes bankruptcy costs which deteriorates the bank balance sheet and lowers credit supply to the real economy.

2.1 The real side: households, entrepreneurs and firms

Households

The representative household maximizes its expected utility by choosing consumption, leisure and deposits.[4] The inter-temporal utility function is separable in consumption and leisure:

$$\max_{C_t^H(i), 1-L_t^H(i), D_t(i)} \mathrm{E}_t \sum_{t=0}^{\infty} (\beta^H)^t U(C_t^H(i), L_t^H(i)), \tag{1}$$

where

$$U(C_t^H(i), L_t^H(i)) \equiv \eta_t^c \left(\frac{1}{1-\sigma_c} (C_t^H(i) - hC_{t-1}^H(i))^{1-\sigma_c} - \frac{\eta_t^l}{1+\sigma_h} (L_t^H(i))^{1+\sigma_h} \right), \tag{2}$$

where $C_t^H(i)$ denotes consumption of agent i at time t, $L_t^H(i)$ denotes hours worked, $D_t(i)$ denotes deposits, β^H is the household discount factor, E_0 is an expectation operator, σ_c is the coefficient of relative risk aversion (inverse of the inter-temporal elasticity of substitution), σ_h represents the inverse of the elasticity of work effort with respect to the real wage, h denotes the habit parameter, η_t^c and η_t^l represent a preference shock and a labor supply shock, respectively. Both shocks follow a stochastic process of the form $\eta_t^c = \rho^c \eta_{t-1}^c + \varepsilon_t^c$ and $\eta_t^l = \rho^l \eta_{t-1}^l + \varepsilon_t^l$, $0 < \rho^b, \rho^l < 1$, where ε_t^c and ε_t^l are error terms i.i.d.$\sim$ (μ_c, σ_c) and i.i.d.$\sim$ (μ_l, σ_l). The representative household maximizes expected utility subject to the budget constraint:

$$C_t^H(i) + D_t(i) = w_t^H L_t^H(i) + \frac{1 + r_{t-1}^d}{\pi_t} D_{t-1}(i) + RP_t(i) + BP_t(i) - T_t, \tag{3}$$

[4] Here I consider a cashless limit economy, similar to e.g. Smets and Wouters (2007) and Gerali et al. (2010). Consequently, the role of liquidity cannot be analyzed as in e.g. Christiano et al. (2005), Goodfriend and McCallum (2007) and Mierau and Mink (2016).

where w_t^H denotes the real household wage, $\pi_t \equiv P_t/P_{t-1}$ is the inflation rate, where P_t denotes the price level, r_t^d is the deposit rate such that $(1 + r_{t-1}^d)D_{t-1}(i)/\pi_t$ denotes real interest income on last period's deposits, $RP_t(i)$ and $BP_t(i)$ denote real profits from the intermediate firms and real profits from the banking sector that households' receive in a lump-sum fashion (so households are the true owners of the firms and the banks) and T_t is a lump-sum tax levied by the monetary authority for the recapitalization of the banking sector.

I follow the mainstream approach and assume that actuarially fair priced state-contingent securities exist that insure each household against idiosyncratic variations in labor and dividend income. Consequently, as in the Arrow–Debreu model, individual household income will correspond to aggregate household income.

Maximizing (1) with respect to consumption and deposits subject to the budget constraint (3) yields the standard consumption Euler equation:

$$\eta_t^c(C_t^H(i) - hC_{t-1}^H(i))^{-\sigma_c} = \frac{1 + r_t^d}{1 + \rho^H} \mathrm{E}_t \left\{ \frac{\eta_{t+1}^c(C_{t+1}^H(i) - hC_t^H(i))^{-\sigma_c}}{\pi_{t+1}} \right\}. \tag{4}$$

Labor supply

Households are wage setters in the labor market. I adopt a commonly used wage-adjustment formulation which is a variant of Calvo (1983) pricing. Following Smets and Wouters (2003), I assume that wages can only change after receiving a random wage signal. The probability of receiving the signal is equal to $1 - \xi^w$. If a household receives a wage signal, it sets a new wage denoted by $\tilde{W}_t$. In addition, if no wage signal has been received, wages are indexed partially. If households cannot re-optimize, the wage rate is indexed according the following indexation equation:

$$W_t(i) = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma^w} W_{t-1}(i), \tag{5}$$

where γ_w is the degree of wage indexation. Households maximize utility (1) subject to the budget constraint (3) and demand for labor which is assumed to be presented by:

$$L_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}} L_t. \tag{6}$$

Aggregate labor demand and aggregate labor supply are given by the following Dixit and Stiglitz (1977) type aggregation functions:

$$L_t = \left(\int_0^1 (L_t(i))^{\frac{1}{1+\lambda_t^w}} di\right)^{1+\lambda_t^w}, \tag{7}$$

$$W_t = \left(\int_0^1 (W_t(i))^{-\frac{1}{\lambda_t^w}} di\right)^{-\lambda_t^w}, \tag{8}$$

where $\lambda_t^w = \lambda^w + \eta_t^w$ determines the wage mark-up, η_t^w is a wage cost-push shock following a stochastic progress: $\eta_t^w = \rho^w \eta_{t-1}^w + \varepsilon_t^w$, $0 < \rho^w < 1$, where ε_t^w is an error term i.i.d.$\sim$

(μ_w, σ_w). The maximization problems results in a wage mark-up equation:

$$\frac{\tilde{W}_t(i)}{P_t} \mathrm{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} (\xi^w)^{\tau} \left(\frac{\left(\frac{P_t}{P_{t-1}}\right)^{\gamma^w}}{\frac{P_{t+\tau}}{P_{t+\tau-1}}} \right) \frac{L^H_{t+\tau}(i)(C^H_{t+\tau}(i) - hC^H_{t+\tau-1}(i))^{-\sigma_c}}{1+\lambda^w_t} =$$
$$- \mathrm{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} (\xi^w)^{\tau} L^H_{t+\tau}(i)(L^H_t(i))^{\sigma_h}. \tag{9}$$

Equations (9) and (5) determine the law of motion for the wage process, i.e., the so called New Keynesian wage curve. As the probability of a wage signal is equal to $1 - \xi^w$ and taking (8) into account, the aggregate wage rate develops according to:

$$W_t^{-\frac{1}{\lambda^w_t}} = \xi^w \left(\left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma^w} W_{t-1} \right)^{-\frac{1}{\lambda^w_t}} + (1 - \xi^w) \tilde{W}_t^{-\frac{1}{\lambda^w_t}}. \tag{10}$$

Entrepreneurs

Entrepreneurs operate the intermediate firms. They invest in real physical capital K_t at time t which has a nominal price q_t and use capital together with labor to produce. The capital accumulation identity is denoted as follows:

$$K_t(j) \equiv (1 - \delta^k) K_{t-1}(j) + \left[1 - \phi \left(\frac{\eta^i_t I_t(j)}{I_{t-1}(j)} \right) \right] I_t(j) \tag{11}$$

where $I_t(j)$ denotes investment by entrepreneur j, and $\phi(\cdot)$ captures capital adjustment costs, where $\phi(\cdot)' > 0$, $\phi(\cdot)'' < 0$. Following Christiano et al. (2005), I assume that $\phi(\cdot) = 0$ and $\phi'(\cdot) = 0$ in steady state, so the adjustment costs will only depend on the second-order derivative $\phi''(\cdot)$. Here η^i_t is an investment shock which follows a stochastic process of the form $\eta^i_t = \rho^i \eta^i_{t-1} + \varepsilon^i_t$ where ε^i_t is an error term i.i.d.$\sim (\mu_i, \sigma_i)$.

Return to capital is subject to idiosyncratic risk. Ex-post gross return on capital is given by $\omega_t(j) r^k_t$ where $\omega_t(j)$ is an idiosyncratic disturbance term realized by entrepreneur j and r^k_t is the aggregate gross return on capital averaged over all firms. Similar to Bernanke et al. (1999), I assume that $\omega_t(j)$ is independently and identically distributed across entrepreneurs and time and follows a log-normal distribution with density and cumulative distributions functions denoted by $f(\omega_t(j))$ and $F(\omega_t(j))$, respectively.

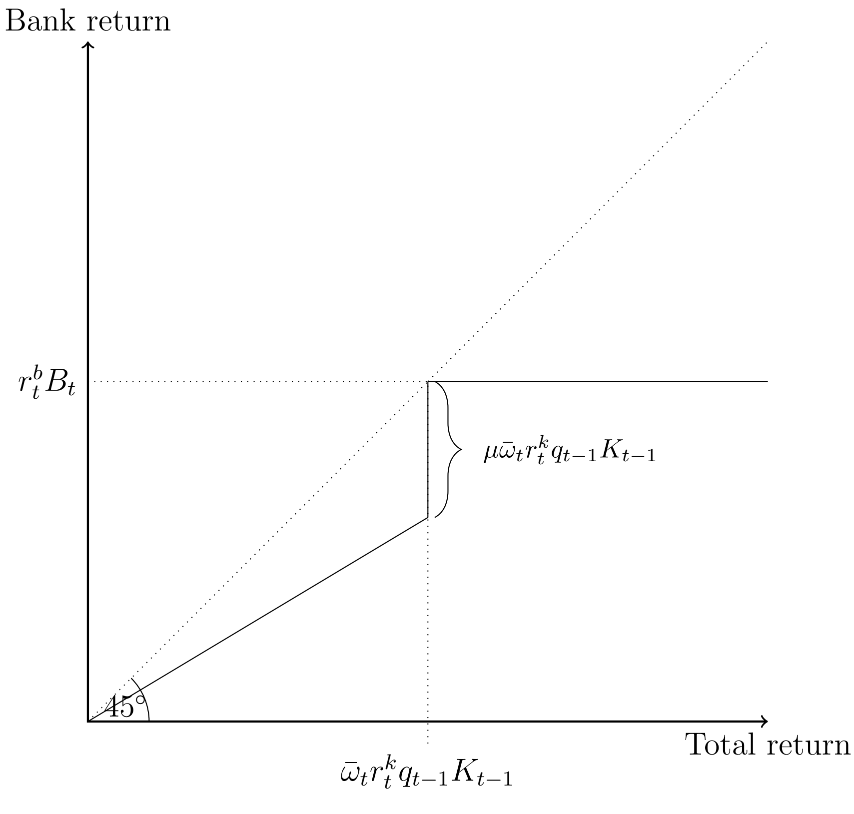
The entrepreneur finances the acquisition of capital with wealth and borrows the remaining funds from the banking sector pledging expected return to capital as collateral, i.e., $\mathrm{E}_t\{\omega_{t+1}(j) r^k_{t+1}\} q_t K_t(i)$. Households will not directly finance entrepreneurs because, by assumption, they do not have the means or skills to eliminate all idiosyncratic risk via diversification. If the realization of $\omega_t(j)$ is below a certain threshold $\bar{\omega}_t$, the firm defaults because realized return to capital is insufficient to repay the amount borrowed. The entrepreneur chooses physical capital, the amount of borrowing and a default threshold by maximizing the firm's expected return to capital given a gross non-default lending rate r^b_t. The threshold below which the entrepreneur defaults is set according:

$$r^b_t B_t(j) = \mathrm{E}_t\{\bar{\omega}_{t+1} r^k_{t+1}\} q_t K_t(i), \tag{12}$$

where $B_t(j)$ denotes the amount borrowed by the entrepreneur. Hence, the expected default threshold, $\mathrm{E}_t\{\bar{\omega}_{t+1}\} = \frac{r_t^b}{\mathrm{E}_t\{r_{t+1}^k\}} \frac{B_t(j)}{q_t K_t(j)}$ is determined by the relation between the gross non-default lending rate and the expected aggregate gross return to capital averaged over all firms $\frac{r_t^b}{r_{t+1}^k}$ and a measure related to entrepreneurial leverage, $\frac{B_t(j)}{q_t K_t(j)}$. In addition, entrepreneurial leverage (and so the default rate) is affected by the price of capital q_t. If the price of capital falls, $\bar{\omega}_{t+1}$ increases which entails that more firms strategically default on their loan repayment.

Figure 1 graphically describes the payoff structure of the loan contract. If realized return $\omega_t(j) > \bar{\omega}_t$, the entrepreneur repays the debt $r_t^b B_t(j)$ and is entitled to any remaining profits, the area to the right of $\bar{\omega}_t r_t^k q_{t-1} K_{t-1}$ on x-axis. If $\omega_t(j) < \bar{\omega}_t$, realized return is insufficient to repay the loan and the entrepreneur defaults without realizing any profits, the area to the left of $\bar{\omega}_t r_t^k q_{t-1} K_{t-1}$. The entrepreneur's stake in the project is therefore similar to common equity. If the entrepreneur defaults the banks can only recover a fraction $(1-\mu)$ of the gross return, where $0 < \mu < 1$ denotes the cost of bankruptcy. In case of default the banks claim the project's total return after paying bankruptcy costs and receive $(1-\mu)\omega_t(j) r_t^k q_{t-1} K_{t-1}(j)$ and lose the remaining part, see Figure 1. Thus, the entrepreneur uses the project's expected return to capital as collateral.

Figure 1: Payoff structure of the loan contract.



All entrepreneurs who realize an idiosyncratic disturbance term $\omega_t(i) < \bar{\omega}_t$ default. Using the cumulative distribution function and the law of large numbers, I can express the fraction of loans that default at period t as $F(\bar{\omega}_t) = \int_0^{\bar{\omega}_t} f(\omega_t) d\omega_t$. Using the same notation as in Bernanke et al. (1999) $\Gamma(\bar{\omega}_t)$ is the share of gross return to capital that goes to the bank:

$$\Gamma(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} \omega_t f(\omega_t) d\omega_t + \bar{\omega}_t \int_{\bar{\omega}_t}^{\infty} f(\omega_t) d\omega_t, \tag{13}$$

and $\mu G(\bar{\omega}_t)$ is the share of gross returns that is lost in the bankruptcy process:

$$\mu G(\bar{\omega}_t) \equiv \mu \int_0^{\bar{\omega}_t} \omega_t f(\omega_t) d\omega_t. \tag{14}$$

Hence, the net share of gross profits that the bank appropriates is equal to $\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)$. The financial intermediary has opportunity costs denoted by the riskless gross rate of return r_t which is the interest rate set by the Central Bank. However, different from the financial intermediaries in Bernanke et al. (1999), the banking sector might be balance sheet constrained. When the bank is balance sheet constrained, its appropriate opportunity costs are no longer the risk-free rate, but a higher lending interest rate, r_t^{wb}. The bank will therefore only agree with the loan contract if it receives on average at least this lending rate.

Entrepreneurs' optimization problem

Entrepreneurs only care for consumption because they supply their labor $L_t^E(j)$ inelastically to the firms for a wage w_t^E. They maximize the discounted sum of expected future utility, where the utility function takes a logarithmic form, by choosing consumption, capital, loans, the default threshold value, the capital utilization rate, and labor input $(C_t^E(j), K_t(j), B_t(j), \bar{\omega}_{t+1}(j), \psi(u_t(j))$ and $L_t(j))$:

$$\max_{C_t^E(j), K_t(j), B_t(j), \bar{\omega}_{t+1}(j), \psi(u_t(j)), L_t(j)} \mathrm{E}_t \sum_{t=0}^{\infty} (\beta^E)^t \left[\ln \left(C_t^E(j) - h C_{t-1}^E(j) \right) \right], \tag{15}$$

subject to their budget constraint:

$$\begin{aligned} &[1 - \Gamma(\bar{\omega}_t(j))](r_t^k u_t(j) - \psi(u_t(j))) q_t K_{t-1}(j) + B_t(j) + w_t^E L_t^E(j) = \\ &\qquad \frac{1 + r_{t-1}^b}{\pi_t} B_{t-1}(j) + I_t(j) + w_t L_t(j) + C_t^E(j), \end{aligned} \tag{16}$$

the capital accumulation identity (11) and the participation constraint of the banks:

$$[\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] r_t^k(j) q_t u_t(j) K_{t-1}(j) \geq \frac{r_{t-1}^{wb}}{\pi_t} B_{t-1}(j), \tag{17}$$

where w_t and $L_t(j)$ are composite factors consisting of the household wage and entrepreneurial wage and household labor and entrepreneurial labor respectively, δ^k is the depreciation rate of physical capital and $\psi(u_t(j))K_{t-1}(j)$ is the real cost of setting a capital utilization rate equal to u_t. Inequality (17) states that banks invest only if they expect that the project's return is higher than their opportunity costs of supplying credit denoted by r_t^{wb}. In the steady state this opportunity cost is equal to the risk-free interest rate r_t. Banks, however, might be constrained by the amount of capital they have. In that case, capital requirements limit the bank to supply more credit, raising the lending rate. As the right hand side of (17) increases the left hand side must increase as well.

That is, if credit conditions tighten firms need more capital, a higher rate of return, higher capital prices or a lower default probability to obtain a similar amount of funding.

Maximizing the entrepreneur's maximization problem (15) subject to the budget constraint (16), participation constraint (17) and the capital accumulation identify (11) with respect to consumption, loans and capital investment, the default threshold and the utilization rate and dropping the entrepreneurial indexation gives:

$$\frac{1}{C_t^E - hC_{t-1}^E} = \lambda_t^E, \tag{18}$$

$$q_t\left(1-\phi\left(\frac{\eta_t^i I_t}{I_{t-1}}\right)\right) =$$

$$q_t\phi'\left(\frac{\eta_t^i I_t}{I_{t-1}}\right)\frac{\eta_t^i I_t}{I_{t-1}} - \beta^E \,\mathrm{E}_t\left\{\frac{\lambda_{t+1}^E}{\lambda_t^E}\phi'\left(\frac{\eta_{t+1}^i I_{t+1}}{I_t}\right)\frac{\eta_{t+1}^i I_{t+1}}{I_t}\frac{I_{t+1}}{I_t} + 1\right\}, \tag{19}$$

$$\beta^E \,\mathrm{E}_t\left\{\frac{\lambda_{t+1}^E}{\lambda_t^E}\frac{1+r_t^b}{\pi_{t+1}}\right\} = 1, \tag{20}$$

$$q_t \,\mathrm{E}_t\left\{\frac{\lambda_t^E}{\lambda_{t+1}^E}\right\}\frac{1}{\beta^E} = \mathrm{E}_t\left\{q_{t+1}(1-\delta^k) + [1-\Gamma(\bar{\omega}_{t+1})](r_{t+1}^k q_{t+1} u_{t+1} - \psi(u_{t+1}))\right\}. \tag{21}$$

where λ_t^E is the entrepreneur's marginal utility of consumption. In addition, I used that $r_t^k = \psi'(u_t)$, i.e. firms increase capital utilization up to the point where the marginal benefit of an extra unit of capital r_t^k is equal to the marginal cost of utilizing an extra unit capital $\psi'(u_t)$. This ensures that the derivatives with respect to the utilization rate and the default threshold are equal to zero and drop out of the system.

Firms

Entrepreneurs own firms active in the intermediate goods sector and produce a unique variety of a wholesale good $Y_t(i)$ according to the following Cobb-Douglas production function:

$$Y_t(j) = \eta_t^a A_t[K_t(j)^\alpha L_t(j)^{1-\alpha}], \tag{22}$$

where A_t is a Hicks-neutral technology parameter and L_t is a composite of household labor L_t^H and entrepreneurial labor L_t^E, η_t^a is a technology shock and follows a stochastic progress of the form $\eta_t^a = \rho^a \eta_{t-1}^a + \varepsilon_t^a$ where ε_t^a is an error term i.i.d.$\sim (\mu_a, \sigma_a)$. The firm produces the wholesale good using capital, labor hired from the household sector at a real wage rate w_t^H and labor hired from the entrepreneur sector at a real wage w_t^E. Household labor and entrepreneurial labor are used according the following technology:

$$L_t(j) = L_t^H(j)^\Omega L_t^E(j)^{1-\Omega}, \tag{23}$$

where I assume that entrepreneurs supply labor in-elastically. Note that (23) implies $w_t = \left(\frac{w_t^H}{\Omega}\right)^\Omega \left(\frac{w_t^E}{1-\Omega}\right)^{1-\Omega}$. Similar to Bernanke et al. (1999) I set Ω low to ensure that entrepreneurial income has no effect on the results.

Entrepreneurs sell their intermediate product to retailers who transform the intermediate product in a homogeneous product by application of a CES production function.

The introduction of the retail sector is merely a mechanical device to keep the model analytically tractable. The CES production function is represented by:

$$Y_t = \left[\int_0^1 Y_t(i)^{1-1/\lambda^p} di \right]^{1/(1-1/\lambda^p)}. \tag{24}$$

They minimize costs, $\int_0^\infty P_t(i) Y_t(i)$, subject to the CES production function, (24). The solution to the retailers' optimization problem defines how the price and output of retailer i, $P_t(i)$ and $Y_t(i)$ respectively, relate to aggregate prices and aggregate output P_t and Y_t respectively. I assume that retailers compete in a perfectly competitive market which implies that prices can be rewritten as:

$$P_t = \left[\int_0^1 P_t^{1-\lambda^p} di \right]^{1/1-\lambda^p}. \tag{25}$$

Entrepreneurs maximize their profits by setting prices and compete in a monopolistic competitive market. Monopolistic competition is modeled by application of a Dixit and Stiglitz (1977) framework. I adopt Calvo (1983) pricing as entrepreneurs can only change their price after receiving a random price change signal. The exogenous probability of receiving the price signal is equal to $(1 - \xi^p)$. If the entrepreneur receives the price signal (s)he sets a new price denoted by $\tilde{P}_t$. If the entrepreneur does not receive the price signal I allow for partial indexation. Partial indexation is done in a way similar to the indexation of wages:

$$P_t(i) = \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma^p} P_{t-1}(i). \tag{26}$$

As a consequence, prices in the model are sticky. In Appendix 5 I solve the entrepreneur's optimization problem. Given (25) the law of motion of the price level is given by:

$$(P_t)^{1-\lambda_t^p} = \xi^p \left(\left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} P_{t-1}(i) \right)^{1-\lambda_t^p} + (1 - \xi^p)(\tilde{P}_t)^{1-\lambda_t^p}, \tag{27}$$

where $\lambda_t^p = \lambda^p + \eta_t^\pi$ denotes the price mark-up, η_t^π is a cost-push shock and follows a stochastic progress of the form $\eta_t^\pi = \rho^\pi \eta_{t-1}^\pi + \varepsilon_t^\pi$ where ε_t^π is an error term i.i.d.$\sim (\mu_\pi, \sigma_\pi)$.

Profits

As argued below (1), households are the true owners of firms. Entrepreneurs operate the firms, they supply their labor inelastically for which they receive a wage rate and the aggregate gross return to capital as long as they do not default. Households are entitled to any remaining profits. Households perfectly diversify between all the firms in the economy. The law of large numbers ensures that the representative households receives dividend payments equal to:

$$RP_t = [1 - F(\omega)][P_t Y_t - L_t W_t - r_t^K q_t K_{t-1}]. \tag{28}$$

Equation (28) states that remaining profits consist of total return minus labor and capital costs in case the firm does not default.

2.2 The financial side: banks

Banks act as intermediaries for all financial transactions between households and entrepreneurs. Households save deposits to smooth consumption over time and entrepreneurs borrow to finance production. Following Gerali et al. (2010) the banking sector is modeled as a collaboration between three branches, i.e., a bank holding company and two retail branches: a funding branch and a lending branch. This approach ensures tractability of the model. The two retail branches are responsible for the collection of deposits and the allocation of loans and set interest rates in a monopolistic competitive fashion. The bank holding company manages the capital position of the banking entity.

2.2.1 Loan and deposit demand

Banks and borrowers often engage in long-term relationships which are vulnerable to asymmetric information problems. Due to this market characterization switching banks is considered costly, because the lender has to allocate costs to screen potential new borrowers and the borrower has to signal creditworthiness to the new lender. The presence of switching costs due to asymmetric information is often mentioned as a reason for market power in the banking sector, see Diamond and Dybvig (1983), Greenbaum et al. (1989) and Sharpe (1990). [5]

Market power in the banking sector is modeled by application of a Dixit-Stiglitz framework. Each bank produces a unique variety of loans $B_t(\ell)$ and deposits $D_t(\ell)$. Accordingly, loan demand by entrepreneurs and deposit demand by households at bank ℓ are given by, see Appendix 5 for the derivations:

$$B_t(\ell) = \left(\frac{r_t^b(\ell)}{r_t^b}\right)^{\mu_t^b} B_t, \tag{29}$$

$$D_t(\ell) = \left(\frac{r_t^d(\ell)}{r_t^d}\right)^{\mu_t^d} D_t, \tag{30}$$

where $\mu_t^b = \mu^b + \eta_t^b$ and $\mu_t^d = \mu^d + \eta_t^d$ denote the elasticity of substitution for loans and deposits, respectively. Here η_t^b and η_t^d are a loan demand and deposit demand shock, respectively, following a stochastic process equal to $\eta_t^b = \rho^b \eta_{t-1}^b + \varepsilon_t^b$ and $\eta_t^d = \rho^d \eta_{t-1}^d + \varepsilon_t^d$ where ε_t^b and ε_t^d are error terms i.i.d.$\sim (\mu_b, \sigma_b)$ and i.i.d.$\sim (\mu_d, \sigma_d)$.

Bank holding company

The bank holding company operates under perfect competition and combines bank capital $K_t^b(\ell)$ and deposits $D_t(\ell)$ on the liability side and supplies loans $B_t(\ell)$ on the asset side to maximize its profits. Moreover, $V_t^r(\ell)$ denotes the realized (superscript r) level of losses from loan defaults.

[5] The market structure within the banking sector is also an often cited source of market power. Berger et al. (2004) link market concentration to market power and the interest rate setting behavior of banks. They find evidence that high market concentration in the banking sector increases market power of banks. Other studies report limited contestability and regulatory restrictions as a source of market power, e.g. Demirguc-Kunt et al. (2004). Several empirical papers confirm the presence of market power in the banking sector, see Berger et al. (2004) and Degryse and Ongena (2008) for a discussion.

The bank holding company expects each period that a number of entrepreneurs default on their loan repayment because they realize an idiosyncratic return that is too low to repay the loan. If an entrepreneur defaults, the bank only receives the residual claim net of monitoring costs, $(1-\mu)\omega_t(j)r_t^k q_{t-1}K_{t-1}(j)$, which happens if $\omega_t(j) < \bar{\omega}_t$. Aggregating over all entrepreneurs that default gives the aggregate return on loans that default: $(1-\mu)\int_0^{\bar{\omega}_t}\omega_t f(\omega_t)d\omega_t r_t^k q_{t-1}K_{t-1}$. The default probability is equal to $F(\bar{\omega}_t)$ and accordingly the realized amount of bank losses in period t is determined by:

$$V_t^r(\ell) = \left(F(\bar{\omega}_t)r_t^b(\ell)B_{t-1}(\ell) - (1-\mu)\int_0^{\bar{\omega}_t}\omega_t f(\omega_t)d\omega_t r_t^k q_{t-1}K_{t-1}\right)\eta_t^v. \tag{31}$$

where η_t^v denotes a loan default shock. The loan default shock follows a stochastic process $\eta_t^v = \rho^v\eta_{t-1}^v + \varepsilon_t^v$ where ε_t^v is an error term i.i.d.$\sim (\mu_v, \sigma_v)$. Intuitively η_t^v describes the deviation of realized from anticipated losses. Bank holding companies predict future losses based on historical information assuming a log-normal distribution for ω_t. Yet, during a financial crisis ($\eta_t^v > 1$) the log-normal distribution or the historical default losses, may not be representative for the actual default process. Consequently, actual losses might differ from expected losses.

Bank holding companies determine expected default losses in the next period, $\mathrm{E}_t\{V_{t+1}^r(\ell)\}$, and reserve the equivalent today denoted by $V_t^e(\ell)$ (superscript e). Hence, $V_t^e(\ell) = \mathrm{E}_t\{V_{t+1}^r(\ell)\}$ and the amount of funds reserved for future losses is based on today's information set. In contrast to Christiano et al. (2014), I do not allow the volatility of cross-sectional idiosyncratic uncertainty to fluctuate over time, but allow realized losses to differ from expected losses. A higher level of expected losses increases loan provisioning which lowers the amount of funds available for loans and raises lending rates.

Each bank holding company has a balance sheet constraint which is given by:

$$B_t(\ell) = D_t(\ell) + K_t^b(\ell). \tag{32}$$

Note that bank capital in (32) is not valued at the price of capital q_t as is done in Gerali et al. (2010). Bank capital, or bank equity, is, however, not the same as physical capital but a residual claim on the bank's future cash flow after having payed the debtors. In an upturn, capital prices increase because demand for capital is high. Consequently, bank equity might increase in value, not directly because capital prices increase, but because the value of the expected future cash flow increases. The increase in capital prices increases the value of the assets pledged as collateral by the entrepreneur and lowers the expected costs of default, see (31). Accordingly, bank profits and retained earnings increase which increases the expected future cash flow and therefore the value of bank capital.

As bank capital is a residual claim on the bank's future cash flow after having payed the debtors, i.e., $B_t(\ell) - D_t(\ell)$, it is possible to write bank capital as a law of motion process:

$$\begin{aligned} B_t(\ell) - D_t(\ell) &= (1-\delta^b)B_{t-1}(\ell) - D_{t-1}(\ell) + \omega_b J_t^b(\ell) + T_t, \\ K_t^b(\ell) &= K_{t-1}^b(\ell) - \delta^b B_{t-1}(\ell) + \omega_b J_t^b(\ell) + T_t, \end{aligned} \tag{33}$$

where we used (32) to get from the first to the second line, J_t^b denotes overall bank profits of the retail banks and bank holding company, $(1-\omega_b)$ denotes the dividend payout ratio of the bank, δ^b denotes resources used to manage the assets and T_t captures the effect of a bank recapitalization. The precise bank recapitalization rule is specified below. In this model there is by assumption no dividend payout ($\omega_b = 1$). All retained earnings therefore accumulate to bank capital and are reinvested the next period.

Moreover the bank holding company faces adjustment costs, denoted by the parameter κ_w, whenever the value of the capital-to-assets ratio K_t^b/B_t (the inverse of the bank's leverage ratio) deviates from the optimal capital-to-asset ratio ν_w. If the capital-to-asset ratio is below the target ratio, banks are too highly leveraged and might incur insolvency costs. Insolvency costs are not explicitly modeled because banks cannot default in this model. For this reason, I try to capture the insolvency costs via the parameter κ_w. Absence this constraint banks would have an incentive to increase leverage indefinitely.

If the capital-to-asset ratio is above the target ratio, banks are not maximizing their profits as a lower ratio (higher bank leverage) would result in higher returns. In this case, the bank's portfolio is not on the Markowitz frontier because banks could increase the expected return without increasing volatility. In a Modigliani-Miller world a lower leverage ratio corresponds to a lower risk level such that investors in the bank would require a lower return. In this model banks cannot default. Hence, a lower leverage ratio does not correspond to a lower risk level. Banks have therefore an incentive to maximize their leverage ratio.

Shin (2010) shows that even if banks face bankruptcy risk, leverage targeting is optimal as long as the maximum leverage ratio required by the market absent regulation is higher than the maximum leverage ratio required by the Central Bank. In the real world banks have implicit government guarantees if they are perceived "too big to fail" and depositors are protected via deposit insurance systems. As the market is aware of these implicit guarantees, leverage ratios required by the market often do not bind because of regulatory constraints.

The model does not allow banks to sell loans to other market participants or to issue new shares thereby lowering leverage contemporaneously. This reflect the situation in the wake of the financial crisis, i.e, a systemic crisis during which all banks try to sell their assets simultaneously. Liquidity markets dried up completely and prevented banks to sell their loan portfolio, usually in the form of asset backed securities, to other market participants. In addition, issuing equity was considered too expensive by the current shareholders and only done by a few banks.

The bank holding company maximizes the discounted sum of the expected future cash flows by choosing the appropriate loan and deposit levels subject to the balance sheet constraint:

$$\begin{aligned}\max_{B_t(\ell),D_t(\ell)} \mathrm{E}_t \sum_{t=0}^{\infty} \Lambda_t^p \Bigg[& (1+r_t^{wb}(\ell))B_t(\ell) - B_{t+1}(\ell) + D_{t+1}(\ell) - (1+r_t^{wd}(\ell))D_t(\ell) + \\ & \Delta K_{t+1}^b(\ell) - \frac{\kappa_w}{2}\left(\frac{K_t^b(\ell)}{B_t(\ell)} - \nu_w\right)^2 K_t^b(\ell)\Bigg], \\ \text{subject to} \quad & B_t(\ell) = D_t(\ell) + K_t^b(\ell), \end{aligned} \tag{34}$$

where Λ_t^p is the stochastic discount rate of the bank holding company, r_t^{wd} and r_t^{wb} denote the deposit rate and the loan rate charged by the bank holding company to the corresponding retail branches, respectively, and ΔK_{t+1}^b denotes the bank's gains or losses on banking capital. As the model is estimated in linear form, the leverage adjustment costs are completely specified by the parameter κ_w. The quadratic term is postulated for mathematical convenience. Using the balance sheet constraint at time t and $t+1$ in the objective function, (34) can be rewritten as:

$$\max_{B_t(\ell),D_t(\ell)} \left\{ r_t^{wb}(\ell)B_t(\ell) - r_t^{wd}(\ell)D_t(\ell) - \frac{\kappa_w}{2}\left(\frac{K_t^b(\ell)}{B_t(\ell)} - \nu_w\right)^2 K_t^b(\ell) \right\}. \tag{35}$$

The FOCs link the bank holding rates on loans and on deposits to the degree of leverage B_t/K_t^b. As banks are always solvent, they are never financing constrained and can always borrow from the Central Bank at rate r_t. However, they cannot use these funds to increase their loan portfolio unlimitedly as they are constrained by their leverage ratio. Arbitrage opportunities ensure that $r_t^{wd} = r_t$. Using these results the FOCs can be rewritten as:

$$s_t(\ell) \equiv r_t^{wb}(\ell) - r_t = -\kappa_w \left(\frac{K_t^b(\ell)}{B_t(\ell)} - \nu_w\right)\left(\frac{K_t^b(\ell)}{B_t(\ell)}\right)^2 \eta_t^s, \tag{36}$$

where η_t^s denotes a credit spread shock which follows a stochastic process $\eta_t^s = \rho^s\eta_{t-1}^s + \varepsilon_t^s$ where ε_t^s is an error term i.i.d.$\sim (\mu_s, \sigma_s)$. Equation (36) links the rate of the bank holding company to the Central Bank interest rate and to bank leverage. The difference between the bank holding rate and the risk-free rate, s_t, is determined by bank leverage. When expected defaults increase, expected profits and future bank capital decline. Banks decrease credit supply to ensure that in expectation credit remains constant and the spread is equal to zero. However, when the realization of defaults turns out to be higher than anticipated, profits decline which increases leverage contemporaneously. As a consequence, the rate set by the bank holding company increases and s_t can be interpreted as a credit spread that increases (decreases) due to unanticipated firm defaults (survivals), because absent unexpected defaults it would be constant and equal to zero.

In the seminal contribution of Bernanke et al. (1999) an increase in $\bar{\omega}_t$ is accounted for via an increase of the credit spread. All losses that materialize precipitate on the real side of the economy and are accounted for in the goods market equilibrium. While an amplification of the downturn can be expected, leverage or balance sheet constraints do not play a significant role. In this paper, an increase in the default thresholds not only increases credit spreads, but also actual defaults which deteriorates banking capital of leveraged banks.

Retail branches

The retail branches are monopolistic competitors on both the loan and deposit markets, i.e., both the lending branch as well as the funding branch produce a differentiated product.

Lending branch. The lending branch maximizes its profits by lending to entrepreneurs while financing these lending activities by borrowing from the bank holding company at rate r_t^{wb}. The lending branch maximizes its profits by choosing the appropriate lending rate r_t^b facing quadratic interest rate adjustment costs denoted by the parameter κ_b:

$$\max_{r_t^b(\ell)} \sum_{t=0}^{\infty} \Lambda_t^p \left[(r_t^b(\ell) - r_t^{wb}(\ell))B_t(\ell) - \frac{\kappa_b}{2} \left(\frac{r_t^b(\ell)}{r_{t-1}^b(\ell)} - 1 \right)^2 r_t^b B_t \right], \tag{37}$$

subject to the loan demand schedule (29). The interest rate adjustment costs are introduced to mimic the empirical evidence of a sluggish lending rate rate pass-through (see, for example, Sørensen and Werner (2006)). The solution to the lending branch optimization problem, dropping the bank indexation parameter ℓ which imposes a symmetric equilibrium, yields:

$$\mu^b - \mu^b \frac{r_t^{wb}}{r_t^b} - \kappa_b \left[\left(1 - \frac{r_t^b}{r_{t-1}^b} \right) \frac{r_t^b}{r_{t-1}^b} + \beta_t^H \frac{\lambda_{t+1}^p}{\lambda_t^p} \mathrm{E}_t \left\{ \left(\frac{r_{t+1}^b}{r_t^b} - 1 \right) \left(\frac{r_{t+1}^b}{r_t^b} \right)^2 \frac{B_{t+1}}{B_t} \right\} \right] = 1, \tag{38}$$

where λ_t^p t is the multiplier on the patient household budget constraint (3).

Funding branch Similarly to the lending branch, the funding branch of bank j collects deposits $D_t(j)$ from households and passes these to the bank holding company which compensates them at rate $r_t = r_t^{wd}$ (the interest rate set by the central bank). In addition, the funding branch faces quadratic interest rate adjustment costs which are denoted by the parameter κ_d. The funding branch maximization problem becomes:

$$\max_{r_t^d(\ell)} \sum_{t=0}^{\infty} \Lambda_t^p \left[(r_t - r_t^d(\ell))D_t(\ell) - \frac{\kappa_d}{2} \left(\frac{r_t^d(\ell)}{r_{t-1}^d(\ell)} - 1 \right)^2 r_t^d D_t \right], \tag{39}$$

subject to deposit demand (30). Interest rate adjustment costs are introduced also for the funding branch to mimic the empirical evidence of a sluggish deposit rate pass-through (Sørensen and Werner, 2006). The solution to the funding branch optimization problem, dropping the bank indexation parameter ℓ, yields:

$$\mu^d - \mu^d \frac{r_t}{r_t^d} + \kappa_d \left[\left(1 - \frac{r_t^d}{r_{t-1}^d} \right) \frac{r_t^d}{r_{t-1}^d} + \beta_t^H \frac{\lambda_{t+1}^p}{\lambda_t^p} \mathrm{E}_t \left\{ \left(\frac{r_{t+1}^d}{r_t^d} - 1 \right) \left(\frac{r_{t+1}^d}{r_t^d} \right)^2 \frac{D_{t+1}}{D_t} \right\} \right] = 1. \tag{40}$$

Bank profits. Combined real profits of the bank holding company, loan, and funding branches are equal to:

$$J_t^b = r_t^b B_t - r_t^d D_t - \frac{\kappa_w}{2} \left(\frac{q_t K_t^b}{B_t} - \nu_w \right)^2 q_t K_t^b - V_{t+1}^E + V_t^E - V_t^r - C_t^b, \tag{41}$$

where C_t^b are the adjustment costs for changing the interest rates at the retail level. Hence, bank profits are determined by interest income on loans minus interest expenses on deposits, deviations from the optimal capital-to-asset ratio, anticipated default losses, unanticipated default losses, and adjustment costs for changing interest rates.

Aggregation and equilibrium

The good market is in equilibrium if production equals consumption and the resources absorbed in the production of capital:

$$Y_t = C_t + I_t + \psi(u_t)K_{t-1}, \tag{42}$$

where $C_t \equiv C_t^H + C_t^E$. The rental market for capital is in equilibrium when the demand for capital by entrepreneurs equals supply by capital producers: $K_t = \int_0^1 K_t(i)di$. The labor market is in equilibrium when labor demand by entrepreneurs equals labor supply of households and entrepreneurs: $L_t = \int_0^1 L_t(i)di$. Finally, a conventional Taylor rule is postulated to close the model:

$$(1 + r_t) = (1 + r)^{(1-\delta_r)}(1 + r_{t-1})^{\delta_r} \left(\frac{\pi_t}{\pi}\right)^{\delta_\pi(1-\delta_r)} \left(\frac{y_t}{y_{t-1}}\right)^{\delta_y(1-\delta_r)} \varepsilon_t^m, \tag{43}$$

where ε_t^m is a monetary policy shock which follows an AR(1) process $\varepsilon_t^m = \rho^m \varepsilon_{t-1}^m + \eta_t^m$ and η_t^m is *i.i.d.* $\sim (\mu_m, \sigma_m)$ distributed. Equation (43) assumes that the Central Bank has two objectives, a stable inflation rate around the steady state inflation rate π and a stable growth path described by the deviation of y_t from y_{t-1}. Moreover, I assume interest rate smoothing by the Central Bank and include the lagged policy rate.

Countercyclical buffer and endogenous recapitalization

Since the global financial crisis, monetary authorities have, besides the conventional policy rate adjustments, a wide range of instruments at their disposal. Amongst these instruments is the countercyclical capital buffer which suggests that authorities could tighten capital constraints during a boom and ease capital constraints during a downturn. If bank capital constraints are binding they might induce pro-cyclical bank behavior; countercyclical capital buffers are suggested to alleviate this pro-cyclical nature.

Countercyclical capital buffers are only advantageous when required bank leverage after activating the countercyclical capital buffer is below the leverage ratio required by the market. Specifically, in a downturn the Central Bank may decide that capital buffers are allowed to fall below the regulatory capital requirement, i.e. leverage may increase. Yet, if the market requires a lower leverage ratio for solvency reasons, an increase in the bank's funding costs will force the bank to decrease leverage to a level required by the market, see for example Clerc et al. (2015). In this case the capital requirement set by the Central Bank is not binding and has no effect on credit supply.

For this reason, we might postulate that the Central Bank introduces a steady state leverage ratio widely below the market requirement in a financial crisis. So widely that even after activating the countercyclical capital buffer, the leverage ratio requirement of the Central Bank is still below the market requirement. In practice, leverage requirements are set at a relatively low level to ensure that countercyclical capital buffers bind even if financial conditions deteriorate.

Another way to operationalize the countercyclical capital buffer is to offer simultaneously a government guarantee to the banking sector. In this case, required leverage after activating the countercyclical capital buffer binds because the leverage ratio required by the market increases. If the government guarantee is credible, bank leverage is allowed to increase and the implicit guarantee ensures that market requirements increase accordingly. The model presented here is consistent with both representation.

In this respect it is important to distinguish between a government guarantee and a recapitalization as the former does not need funding if credible but the latter must be funded by means of e.g. taxation. To compare both policy instruments, both the countercyclical capital buffer as well as the endogenous recapitalization are specified according the same policy rule.

Following Pariès et al. (2011) I focus on the joint determination of a monetary policy rule and a macroprudential policy rule. Moreover, I abstract from welfare calculations and specify an ad hoc macroprudential policy rule to analyze the consequences for macroeconomic fluctuations. First, I introduce a policy rule that is contingent on the ratio of credit over GDP, M_t^y, which is the most commonly used "trigger variable" that activates countercyclical capital buffers. In practice countercyclical capital buffers and bank recapitalizations have a binary structure, i.e., they are either off and no capital surcharge (undercharge) is required (allowed), or they are activated and banks are required (allowed) to hold (release), say, an extra percent of capital relative to their assets. Here an endogenous rule is introduced which is not binary but continuous:

$$M_t^y = \left(\frac{B_t}{Y_t} - \frac{B^*}{Y^*} \right)^{\varrho_y}, \tag{44}$$

where $\frac{B^*}{Y^*}$ is the steady state credit to GDP ratio and ϱ_y is a policy parameter and denotes the degree to which the countercyclical capital buffer or recapitalization is affected by changes in the credit to GDP ratio. So, M_t^y is activated ($M_t^y \neq 0$) when credit over GDP differs from the steady state level.

Rational expectation models are, however, notorious for not having a role for inflationary bubbles. As credit is used to acquire capital for production, credit and GDP grow more or less in accordance. Also in reality it is hard to distinguish between a bubble and an increase in credit supported by fundamentals. My prior is therefore that the aforementioned policy rule will not affect macroeconomic fluctuations much. It is therefore informative to examine a more direct and therefore more effective instrument: a policy rule that is contingent on aggregate bank leverage:

$$M_t^{k^b} = \left(\frac{K_t^b}{B_t} - \frac{K^{b*}}{B^*} \right)^{\varrho_{k^b}}, \tag{45}$$

where $\frac{K^{b*}}{B^*} = \nu_w$ is the inverse of steady state bank leverage, and ϱ_{k^b} is a policy parameter and denotes the degree to which the countercyclical capital buffer or recapitalization is affected by changes in the leverage ratio. So, $M_t^{k^b}$ is activated ($M_t^{k^b} \neq 0$) when bank leverage differs from the steady state bank leverage ratio.

Formally the Central Bank alleviates the bank leverage constraint in the following way when the countercyclical capital buffer is applied:

$$s_t \equiv r_t^{wb} - r_t = -\kappa_w \left(\frac{K_t^b}{B_t} - (\nu_w + M_t) \right) \left(\frac{K_t^b}{B_t} \right)^2 \eta_t^s, \tag{46}$$

where M_t can be either M_t^y or $M_t^{k^b}$ depending on which activation rule the Central Bank adheres to. If the government decides on a recapitalization rather than activating the countercyclical capital buffer, the tax T_t is set equal to the endogenous buffer M_t^y or $M_t^{k^b}$. In this case the government directly injects the bank with additional capital.

A final remark, I assume that banks can decide on credit supply and interest rates unaware of the presence of a countercyclical capital buffer or a recapitalization, i.e., no moral hazard issues are introduced in the model. The rationale for this assumption is that each individual bank has only marginal influence on the aggregate development of the credit-to-GDP ratio or the aggregate leverage ratio.

Reduced form representation

For the empirical analysis in Section 3 I linearize the model around the non-stochastic steady state, see Appendix A.3 for the details. Throughout this paper a hat denotes a log-linearized variable. It is convenient to summarize the model using matrix notation. The reduced form of the model can be represented as:

$$\mathrm{E}_t\{\mathbf{Z}_{t+1}\} = \mathbf{\Gamma}_0^{-1}\mathbf{\Gamma}_1\mathbf{Z}_t + \mathbf{\Gamma}_0^{-1}\mathbf{\Gamma}_2\mathbf{Z}_{t-1} + \mathbf{\Gamma}_0^{-1}\mathbf{\Upsilon}_0\,\mathrm{E}_t\{\boldsymbol{\eta}_{t+1}\} + \mathbf{\Gamma}_0^{-1}\mathbf{\Upsilon}_1\boldsymbol{\eta}_t. \tag{47}$$

where I multiplied both sides by $\mathbf{\Gamma}_0^{-1}$, $\mathbf{\Gamma}_0$ is a coefficient matrix specifying the contemporaneous response of each variable at time $t+1$ to all variables at time $t+1$, $\mathbf{Z}_t = \left[\hat{y}_t, \hat{c}_t, \hat{i}_t, \hat{w}_t, \hat{\pi}_t, \hat{b}_t, \hat{d}_t, \hat{r}_t, \hat{r}_t^b, \hat{r}_t^d, \hat{s}_t,\right]'$ is a vector of observed variables, $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$ are respectively the coefficient matrices specifying the response of each variable at time $t+1$ to the time t and $t-1$ variables, $\mathbf{\Upsilon}_0$ and $\mathbf{\Upsilon}_1$ are coefficient matrices specifying the response of each variable at time $t+1$ to the vectors $\boldsymbol{\eta}_{t+1}$ and $\boldsymbol{\eta}_t = \left[\hat{\eta}_t^a, \hat{\eta}_t^v, \hat{\eta}_t^m, \hat{\eta}_t^l, \hat{\eta}_t^i, \hat{\eta}_t^\pi, \hat{\eta}_t^d, \hat{\eta}_t^b, \hat{\eta}_t^c, \hat{\eta}_t^i, \hat{\eta}_t^q\right]'$, respectively. Each $\hat{\eta}_t^\iota$ where $\iota \in \{a, v, m, l, i, \pi, d, b, c, i, q\}$ follows an autoregressive stationary process described by $\hat{\eta}_t^\iota = \rho^\iota\hat{\eta}_{t-1}^\iota + \hat{\varepsilon}_t^\iota$, all shocks $\hat{\varepsilon}_t^\iota$ are i.i.d.$\sim (\mu_\iota, \sigma_\iota^2)$. Whereas exact identification in the empirical part requires the number of shocks to be equal to the number of observable variables, I am mostly interested in the response to a positive default shock $\hat{\varepsilon}_t^v$, an adverse technology shock $\hat{\varepsilon}_t^a$ and a contractionary monetary policy shock $\hat{\varepsilon}_t^m$.

3 Methodology

The set of parameters is divided into two partitions. Partition one contains parameters that control the steady state. This set of parameters entails conventional parameters, e.g. the share of capital in the production function and the physical capital depreciation rate. These parameters are taken from the literature. The other parameter values in this set entail steady state averages which are calibrated to reproduce steady state averages of the data set. Partition two contains parameters that are estimated using a Bayesian estimation procedure.

3.1 Calibrated parameters

The conventional calibrated parameters, categorized in partition one and presented in the first column of Table 1, are taken from the literature. First, $\beta^E = 0.975$ and $\beta^H = 0.994$ which are also used by Gerali et al. (2010). The share of capital in the production function $\alpha = 0.3$; the depreciation rate of physical capital $\delta^k = 0.025$; the habit parameter $h = 0.7$;

the price mark-up $\lambda_p = 5/4$ the wage mark-up $\lambda_w = 3/2$; the coefficient of relative risk aversion $\sigma_c = 1$; the inverse of the elasticity of work effort $\sigma_l = 2$; and the probability of a price and wage update $\varepsilon_p = \varepsilon_w = 0.75$ (see Smets and Wouters (2003) and Christiano et al. (2005)). I set the share of inelastic entrepreneurial labor in production $\Omega = 0.01$ which is similar to Bernanke et al. (1999). The inverse of the elasticity of capital utilization $\Psi \equiv \psi^{'}/\psi^{''} = 0.25$. Less conventional, but nonetheless calibrated parameters, are the optimal capital-to-loan ratio of the bank, $1/\nu_w$, which is set equal to 8%, the capital requirement imposed by Basel III on corporate loans[6].

The probability of default is set $F(\bar{\omega}) = 3\%$. Bernanke et al. (1999) base this value loosely on the United States historical average. I assume a similar value for the Euro Area but experimented with higher and lower values. These experiments show that the model outcome is rather insensitive to the default threshold value because it is not the steady state value that is important but changes in the default probability and the mismatch between what is expected and what is realized. $F(\bar{\omega}) = 3\%$ pins down the entrepreneurial physical capital-to-loan ratio (K/B) at 0.4, which is close to the historical average capital to loan ratio of entrepreneurs. Assuming a log normal distribution $\sim \mathcal{N}(0,1)$ determines $f(\bar{\omega}) \approx 0.446$ and $\bar{\omega} \approx 0.152$. Optimization and linearization of the system results in first and second order derivatives w.r.t. $\Gamma(\bar{\omega})$ and $\mu G(\bar{\omega})$. Nevertheless, note that $\Gamma(\bar{\omega}_t) = F(\bar{\omega})\frac{1}{2}\bar{\omega}^2 + \bar{\omega}[1 - F(\bar{\omega})]$, $\mu G(\bar{\omega}_t) = \mu F(\bar{\omega})\frac{1}{2}\bar{\omega}^2$, $\Gamma^{'}(\bar{\omega}) = 1 - F(\bar{\omega})$, $\Gamma^{''}(\bar{\omega}) = -f(\bar{\omega})$, $G^{'}(\bar{\omega}) = \mu\bar{\omega}f(\bar{\omega})$ and $G^{''}(\bar{\omega}) = \mu(f(\bar{\omega}) + \bar{\omega}f^{'}(\bar{\omega}))$. Hence, deciding on the steady state default probability determines all steady state values w.r.t. the default threshold.

The parameters representing steady state averages are calibrated based on their historical averages. The fractions $\frac{C}{Y}$ and $\frac{I}{Y}$ are calculated by means of the data series and incorporate government consumption and government investment; $\frac{wL}{D}$ is the labor income to savings ratio and is set equal to 0.23, the historical average of the data series. Following Gerali et al. (2010) the share of entrepreneurs in the economy $\frac{C^E}{C} = 0.2$. The fractions concerning banking profit, $\frac{r^bB}{J^b}$, $\frac{r^dD}{J^b}$, and $\frac{V}{J^b}$ can easily be constructed from the values of the structural parameters described below.

3.2 Data

The model is estimated for the euro area for the period 2000:Q1-2014:Q2. The dataset contains real economic variables: output, consumption, investment, hours of work, wages, outstanding loans to firms and outstanding deposits; and price and interest variables: inflation, nominal policy rate, nominal interest rate on loans, nominal interest rate on deposits and credit spreads. The real economic variables are de-trended and expressed as log-deviations from their trend. The trend value of the variables is constructed using the HP-filter and a smoothing parameter equal to 1600. The prices and interest rate variables are expressed as absolute deviations from the sample mean. Figure 2 shows the resulting time series.

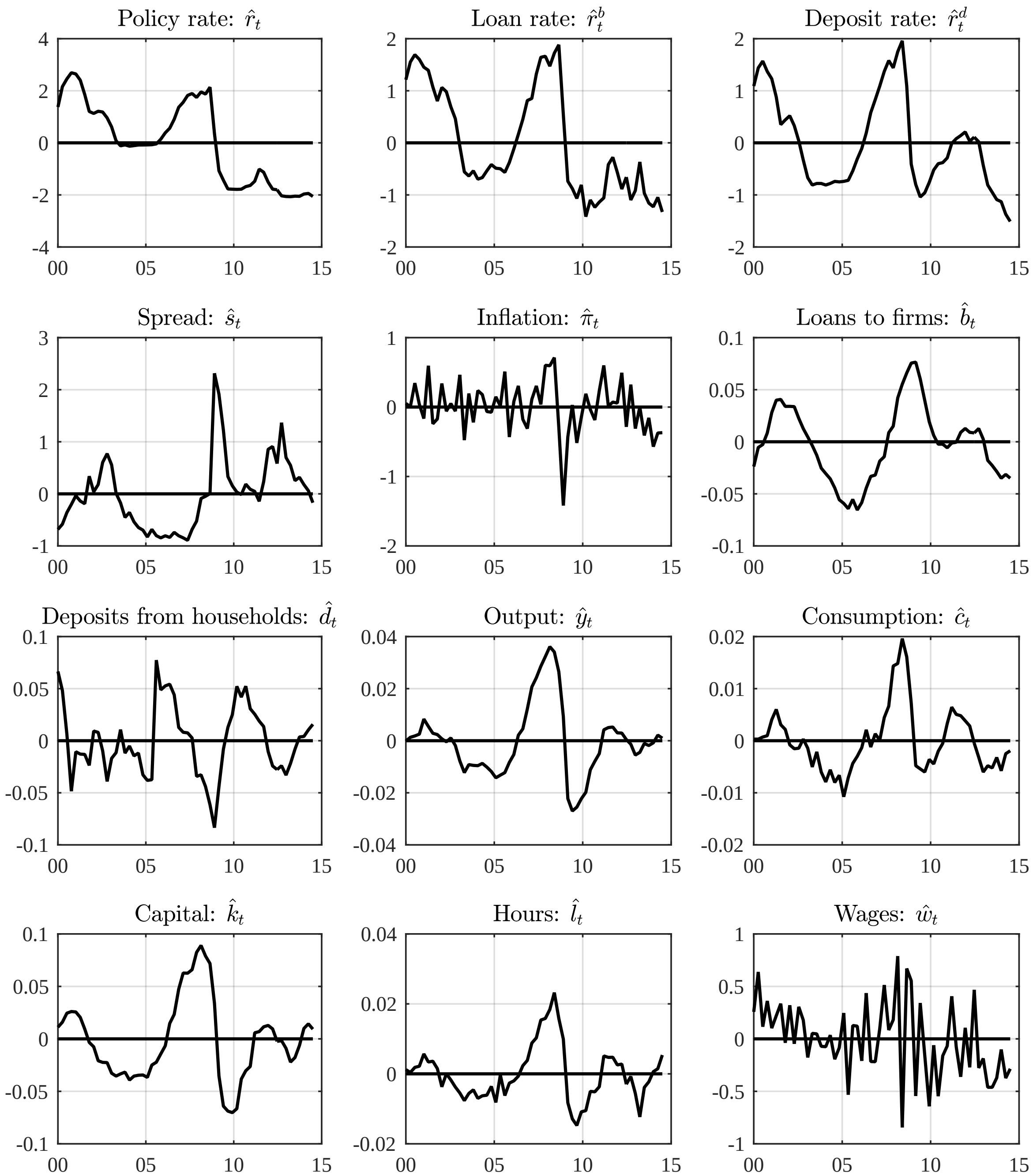
The difference between the lending rate set by bank holding company r_t^w and the policy rate r_t is interpreted as a credit spread. I use credit spread data of European firms constructed by Gilchrist and Mojon (2014) who use individual firm level securities data

[6]Basel III denotes capital requirement based on the risk characteristics of the asset. The model simplifies this characteristic and only considers one asset, i.e. loans to firms, which are homogeneous in risk and have a 100% risk weight.

Table 1: Calibrated parameters

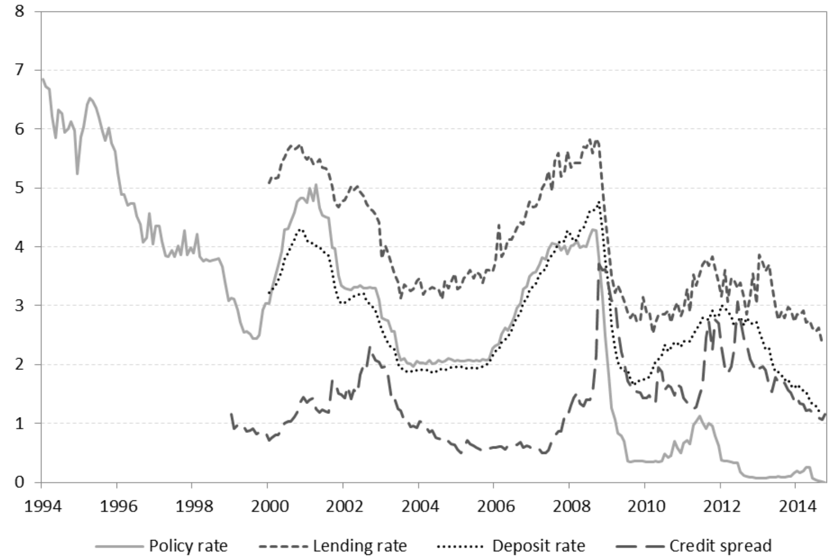
Parameters	Description	Value
β^E	Discount factor entrepreneurs	0.975
β^H	Discount factor households	0.994
α	Share of capital in the production function	0.300
h	The household habit parameter	0.700
σ_c	Relative risk aversion households	1.000
σ_l	Inverse of the elasticity of work effort	2.000
δ^k	Depreciation rate of physical capital	0.025
Ω	Share of households in composite labor factor	0.990
ν_w	Optimal leverage ratio banks	0.080
ε_p	Probability of a price update	0.750
ε_w	Probability of a wage update	0.750
λ_w	Wage mark-up	1.500
λ_p	Price mark	1.250
$\Psi = \frac{\psi'(1)}{\psi''(1)}$	Capital utilization	0.250
$F(\bar{\omega})$	Probability of default	0.030
$\frac{C}{Y}$	Consumption to GDP ratio	0.780
$\frac{I}{Y}$	Investment to GDP ratio	0.220
$\frac{wL}{D}$	Households savings quote	0.230
$\frac{C^E}{B}$	Consumption borrowing ratio	0.230
$\frac{K}{B}$	Inverse of loan to value ratio	0.400

Figure 2: Time series plot of observable variables.



Notes: Interest rates, inflation and credit spreads are plotted on a quarterly basis and in absolute deviations from the sample mean. The real variables, loans to firms, deposits to households, consumption, output, capital and hours worked, are plotted on a quarterly basis and expressed as log deviations from the HP-filtered trend.

Figure 3: Time series plot of Euro Area policy rate, lending rate, deposit rate and credit spreads.



to construct security-specific credit spreads which are aggregated for the euro area to construct aggregated credit spread indicators. As the aggregate indicator is constructed from micro level data, the spreads are, according to the authors, more informative about aggregate credit spreads than aggregate approximations. The credit spread data is added to be able to identify the default shock.

All real variables presented in Figure 2 show more or less the same pattern. Output, consumption, capital, hours and wages peak before the global financial crisis at an unprecedented level after which they appear to go in free fall. Although they return swiftly to pre-crisis levels the resurrection might also be a characteristic of the HP-filter applied to the data series as the estimated trend is not insensitive to the financial crisis. Loans show also a sharp decline during this crisis and the series returns quickly to pre-crisis levels. Deposits show the inverse pattern. Before the global financial crisis they are at their lowest point after which they steeply rise. The policy rate, the loan rate and to a lesser extent the deposit rate appear to be de-trending over the sample. Credit spreads, in contrast, appear to be trending up; they hit a minimum just before the burst of the global financial crisis at about 5 percent point below their mean, while during the crisis they appear to peak. The inflation rate is relatively stable over time reflecting effective monetary policy over the sample period.

Figure 3 plots the Euro Area policy rate, lending rate, deposit rate and credit spreads in percentages. During tranquil times (2000:2008) the deposit rate closely follows the policy rate; however, during the Fall of 2008 the policy rate, in reaction to the deteriorating conditions in financial markets, decreases sharply by almost 4 percent while the deposit rate falls roughly to 2 percent. Hence, after 2008 these rates appear to be less connected. The lending rate shows a pattern roughly similar to the deposit rate. The movements in the lending rate correspond to the movements in the policy rate, yet with a liquidity and credit spread premium. Figure 3 also shows the credit spread premium which clearly shows a countercyclical pattern: the premium is low before the global financial crisis and peaks in the fall of 2008. Moreover, the difference between the lending rate and the credit spread corresponds more or less to the policy rate. This observation suggests that once the credit spread is subtracted from the policy rate the lending rate also moves close to zero.

3.3 Bayesian estimation

The second partition of parameters, containing the less conventional parameters, are estimated using the Bayesian estimation algorithm in Dynare. The Bayesian algorithm chooses the parameter values that minimize the difference between the theoretical moments and the empirical moments. The mean and posterior distributions of the parameters and the impulse response functions are constructed by application of the Metropolis-Hastings algorithm.

The estimated parameters are presented in Table 2. Bayesian estimation requires the modeler to specify the prior mean, the prior distributions and the prior standard deviation. I assume that all the structural parameters follow a gamma distribution. The standard deviations are set large to give the data the opportunity to determine the value of the parameter.

Table 2: Table - Estimated parameters

		Prior		Posterior			
Param.	Distr.	Mean	S.D.	Mean	10%	Median	90%
κ_b	Gamma	20.00	1.00	13.13	9.17	13.08	16.81
κ_d	Gamma	10.00	1.00	10.36	9.03	10.34	11.72
κ_w	Gamma	10.00	1.00	7.63	6.39	7.60	8.97
δ^b	Gamma	0.03	0.01	0.04	0.03	0.05	0.06
μ_b	Gamma	4.30	0.50	4.68	4.21	4.67	5.14
μ_d	Gamma	2.60	0.50	2.60	2.23	2.61	3.02
δ_y	Gamma	0.50	0.10	0.94	0.82	0.94	1.03
δ_π	Gamma	1.50	0.20	1.18	0.99	1.16	1.36
δ_r	Beta	0.75	0.10	0.66	0.61	0.66	0.71
γ_p	Gamma	0.75	0.10	0.59	0.49	0.59	0.71
γ_w	Gamma	0.75	0.10	0.48	0.40	0.49	0.56
φ	Gamma	0.25	0.02	0.18	0.16	0.18	0.20
μ	Gamma	0.50	0.05	0.53	0.48	0.53	0.59

The parameters representing the loan rate, the deposit rate and the capital ratio adjustments costs, κ_b, κ_d and κ_w respectively, are less conventional. I follow Gerali et al. (2010) by setting $\kappa_b = 10$, and $\kappa_d = 10$. These values ensure that the model approximates the observed interest rate pass-through documented by the European Central Bank (2009). Gerali et al. (2010) argue that κ_w is hard to determine, for this reason they set the standard deviation equal to 5 and decide on a prior value equal to 20 such that the data is able to determine this parameter. I follow a similar approach. Setting $\kappa_w = 20$ implies that for each percentage point deviation of the banks capital-to-asset ratio from the optimal level, the interest rate spread increase by 4.166 basis point. I experimented with different values and found that the data is sufficiently able to identify this parameter.

For μ_b and μ_d also no conventional values are available. I therefore calibrate these values to ensure that they mimic historical averages. In the Appendix Section A.3 I show that in the steady state $\mu_b = \frac{r^{b*}}{r^{b*}-r^*}$ and $\mu_d = \frac{r^{d*}}{r^{d*}-r^*}$. These historical averages are calculated by transforming the quarterly data series representing annual rates, r^y, into quarterly rates, r^q, i.e. $r^q = \exp(\ln(\frac{1+r^y}{4}))$. Successively, I use the historical average of these transformed loan, deposit and policy rates to calculate $\mu_d = 4.299$ and $\mu_b = 2.605$. In Gerali et al. (2010) μ_b is calculated to be negative which implies that the deposit rate is on average below the policy rate. Here, I find that the deposit rate is on average higher than the policy rate, see Figure 3.

I follow Smets and Wouters (2007) and set the partial indexation parameters γ_p and γ_w equal to 0.75. I set the costs of managing bank assets equal to $\delta^b = 0.01$; this value ensures a steady state loan-to-capital ratio of approximately 8%. The value of $\varphi = 0.25$ is controversial as there is no consensus in the literature about its value. Reasonable values, however, lie within the range $0.0 - 0.5$ (Bernanke et al., 1999). Finally, $\delta_r = 0.75$, $\delta_y = 0.5$ and $\delta_\pi = 1.5$ which correspond to the conventional Taylor rule values. I set $\mu = 0.5$ because the value of a credit default swap is calculated under the assumption of a loss given default of 50%.

The prior specification of the persistence parameters and the variance of the shocks is shown in Table 3. For the persistence parameters of the shocks ρ^ι, where ι indexes a particular shock, I choose a prior value of 0.75. A quarterly autoregressive parameter of 0.75 represents a rapid decay. After a year, only 23% of the initial increase remains. As conventional, I choose a beta distribution with a standard deviation equal to 0.1. Finally, the prior mean and variances of the shock terms are set equal to zero and infinity: $\mu_\iota = 0$ and $\sigma_\iota = \infty$ and follow by assumption an inverse gamma distribution.

4 Empirical results

Figure 4 shows the prior and posterior distributions of the estimated parameters and the resulting posterior mode and Table 2 shows the posterior summary statistics. The posterior parameter values are drawn using the Metropolis Hastings algorithm by running 5 chains of $100,000$ draws. The convergence properties of the model are assessed by means of the convergence statistics suggested by Brooks and Gelman (1998). Overall, the convergence statistics indicate that the convergence properties are satisfied.

The posterior distributions suggest that the data is informative about most estimated parameters as the posterior distribution is significantly different from the prior distribution. However, for κ_b and μ_d the posterior distribution is close to the prior distribution which might indicate that the data is uninformative about these parameters. To determine whether the data is uninformative, or whether the prior value is simply close to the value implied by the data, I experimented with different prior values. Changing the prior mean of κ_b shift the posterior mean a little, but the impulse response function show comparable results. Changing μ_d does not affect the posterior distribution much suggesting that the prior value is close to the value implied by the data. This results is not surprising as the prior value of μ_d is calibrated from the data.

Table 3: Estimated autocorrelation and standard deviation parameters

Parameters	Label	Distribution	Prior	S.D.	Mean	10%	Median	90%
ρ^a	Technology	Beta	0.75	0.10	0.43	0.30	0.44	0.57
ρ^v	Default	Beta	0.75	0.10	0.63	0.54	0.63	0.73
ρ^{rb}	Lending rate	Beta	0.75	0.10	0.67	0.61	0.67	0.73
ρ^{rd}	Deposit rate	Beta	0.75	0.10	0.57	0.50	0.57	0.63
ρ^q	Capital price	Beta	0.75	0.10	0.68	0.60	0.68	0.77
ρ^s	Spread	Beta	0.75	0.10	0.83	0.78	0.83	0.89
ρ^π	Cost push	Beta	0.75	0.10	0.34	0.19	0.33	0.44
ρ^l	Labor supply	Beta	0.75	0.10	0.99	0.99	0.99	1.00
ρ^i	Investment	Beta	0.75	0.10	0.68	0.60	0.68	0.78
ρ^c	Consumption	Beta	0.75	0.10	0.75	0.70	0.76	0.81
ρ^w	Wage	Beta	0.75	0.10	0.34	0.27	0.34	0.40
ρ^{rn}	Monetary Policy	Beta	0.75	0.10	0.45	0.37	0.46	0.54
σ_a	Technology	Inv. gam	0.01	∞	0.02	0.02	0.02	0.02
σ_v	Default	Inv. gam	0.01	∞	0.01	0.00	0.01	0.01
σ_{rb}	Lending rate	Inv. gam	0.01	∞	0.17	0.13	0.17	0.20
σ_{rd}	Deposit rate	Inv. gam	0.01	∞	0.15	0.13	0.15	0.18
σ_q	Capital price	Inv. gam	0.01	∞	8.69	5.70	8.53	11.64
σ_s	Spread	Inv. gam	0.01	∞	0.40	0.33	0.39	0.46
σ_π	Cost push	Inv. gam	0.01	∞	4.03	3.37	4.02	4.75
σ_l	Labor supply	Inv. gam	0.01	∞	0.09	0.05	0.09	0.13
σ_i	Investment	Inv. gam	0.01	∞	13.17	9.35	13.14	16.25
σ_c	Consumption	Inv. gam	0.01	∞	1.66	1.34	1.65	1.96
σ_w	Wage	Inv. gam	0.10	∞	39.24	35.32	39.31	43.14
σ_{rn}	Monetary Policy	Inv. gam	0.10	∞	0.45	0.36	0.44	0.53

The validity of the model is assessed by comparing the impulse response functions resulting from an adverse technology shock and a contractionary monetary policy shock with the impulse response functions generated by canonical DSGE models. Thereafter I interpret the impulse response functions following a default shock and conduct a number of counter factual policy experiments by introducing alternative monetary policy rules.

4.1 Technology and monetary policy shock

Figure 5 presents the impulse response functions after an adverse technology shock (dots) represented by a 0.5% decrease in the firm technology parameter and a monetary policy shock (dashes) represented by a 25 basis point increase in the policy rate. The estimated parameters are set at their posterior median.[7]

Technology shock

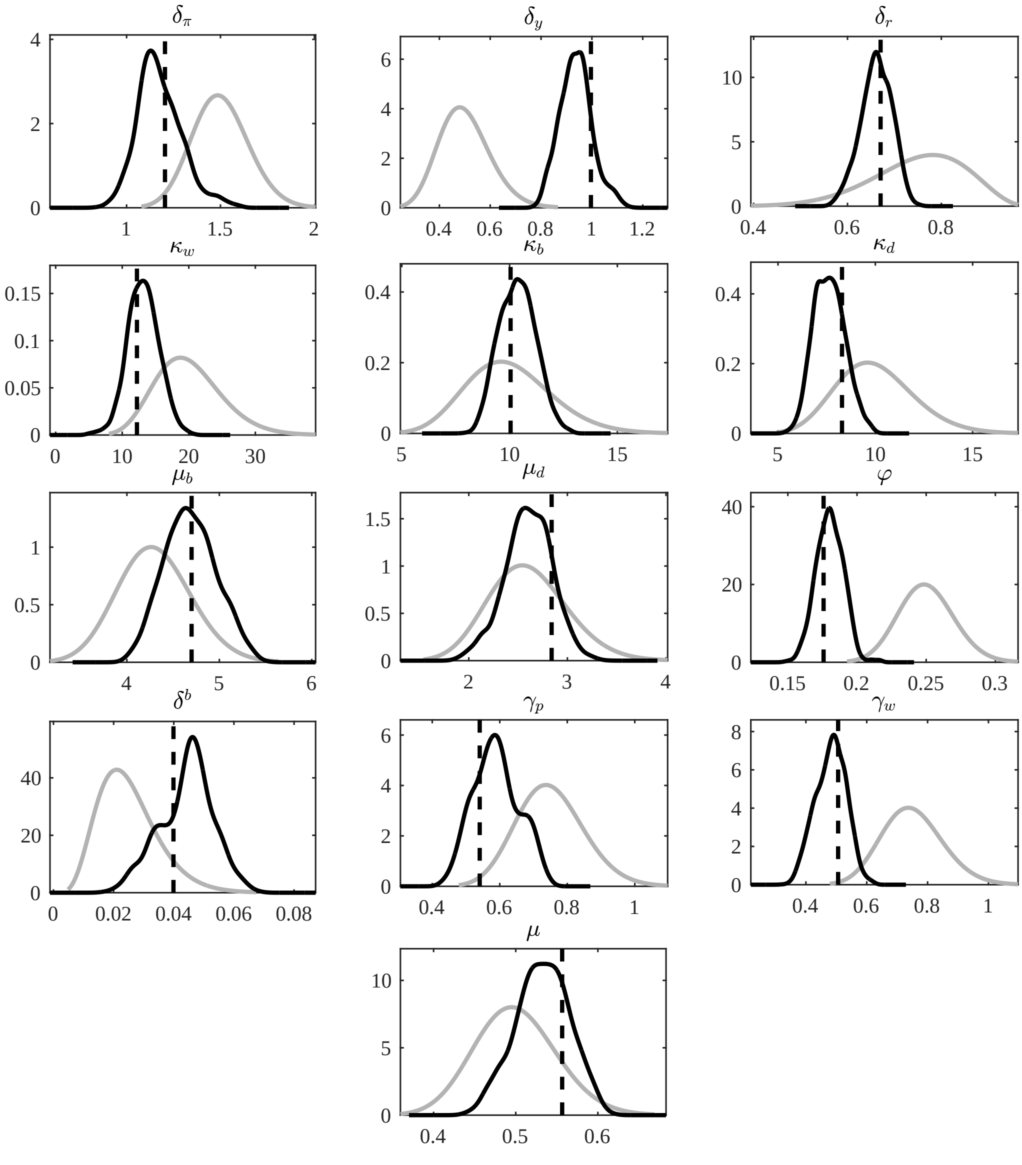
In general the results are comparable to the results presented in Smets and Wouters (2003), Christiano et al. (2005) and Gerali et al. (2010). The technology shock decreases the productivity of both capital and labor and pushes down the investment schedule and labor demand. Output decreases while inflation increases because the marginal cost of production increases. Firm balance sheets deteriorate because the real value of physical capital pledged to acquire credit decreases in value. However, as the real cost of borrowing declines on impact, entrepreneurs borrow more to finance their consumption expenditure. The Central Bank increases the policy rate in response to the inflationary pressure.

Some notable difference with the aforementioned literature arise due to the specifics of this model. The monetary transmission channel ensures that both the deposit rate and the lending rate follow the policy rate, but in real terms both interest rates decline on impact. The net effect on the banking sector is a small decline in banking capital. This result is partly in line with Gerali et al. (2010). In their model bank capital is countercyclical because the difference between the lending and deposit rate, i.e., the banks' profit margin, increases, but operating costs are not conditional on the size of the banks loan portfolio. In the model presented here, the banks' profit margin also increases, but not enough to offset the increase in operating costs because the bank manages a larger loan portfolio.

Default losses as well as credit spreads show a tenuous response. Default losses decrease on impact as the default threshold falls: less firms realize a productivity parameter below the default threshold. Credit spreads increase slightly on impact and return gradually to pre-shock levels which suggests that bank leverage increases. Nonetheless, the overall impact on the real economy is small and short-lived. These results suggest that banks can manage the changes in credit risk and leverage resulting from a technology shock without restricting economic activity too much.

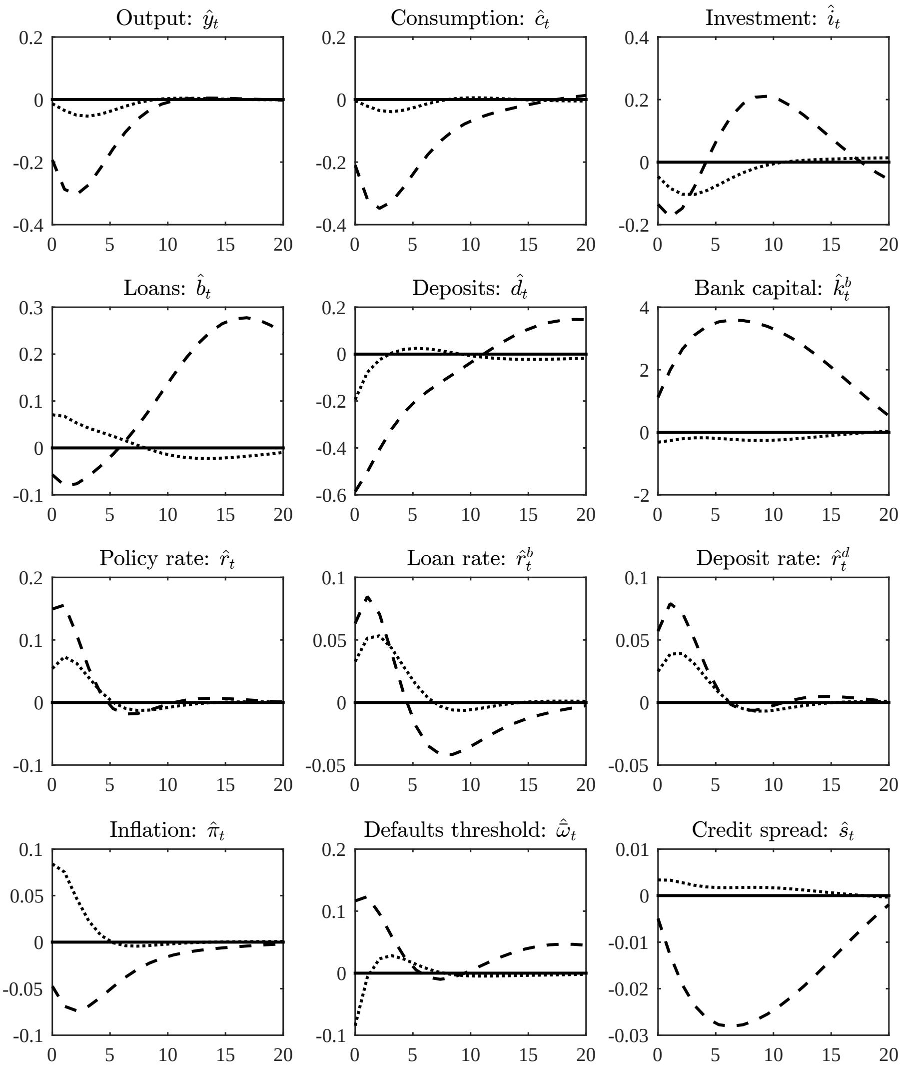
[7] I do not plot the confidence intervals in Figure 5 because they are tightly centered around the median response. These tight confidence intervals result as I estimate primarily parameters in the banking sector while all parameters in the real economy are calibrated. As the mechanisms operating on the real side of the economy dominate the feedback effects stemming from the banking side, the estimated parameter uncertainty that trickles through the monetary transmission mechanism is small. As a consequence, confidence intervals stemming from shock originated in the real sector are tight.

Figure 4: Priors and Posteriors



Grey curve: prior; black curve: posterior; dotted line: posterior mode.

Figure 5: Impulse response functions of a technology shock and monetary policy shock.



Notes: responses to a technology shock represented by dots and to a monetary policy shock represented by dashes. Prices and interest rates are shown as absolute deviations from their steady state, expressed in percentage points and real variables are percentage deviations from steady state.

Monetary policy shock

The impulse response functions after a contractionary monetary policy shock are similar to the responses presented by Gerali et al. (2010). On impact the policy rate increases. As banks face tighter funding conditions from the Central Bank (their opportunity costs increase), both the lending and deposit rate increase. Output declines because both consumption and investment decrease. Consumption declines because the real deposit rate increases and investment falls because the real lending rate increases and the balance sheet of the entrepreneur tightens. The increase in interest rates economy wide lowers demand and as a result both output and inflation fall.

The contractionary monetary policy shock increases bank capital because the bank's profits margin increases but also because deposits fall due to an income effect. In addition, the increase in the default threshold and credit defaults decreases bank capital, but not enough to offset the increase in bank profitability. As expected from the increase in bank capital, bank leverage decreases which is confirmed by the decrease in credit spreads. Overall, the effect on the real economy are relatively small and short-lived even though the effects on the banking sector are much more persistent compared to the technology shock.

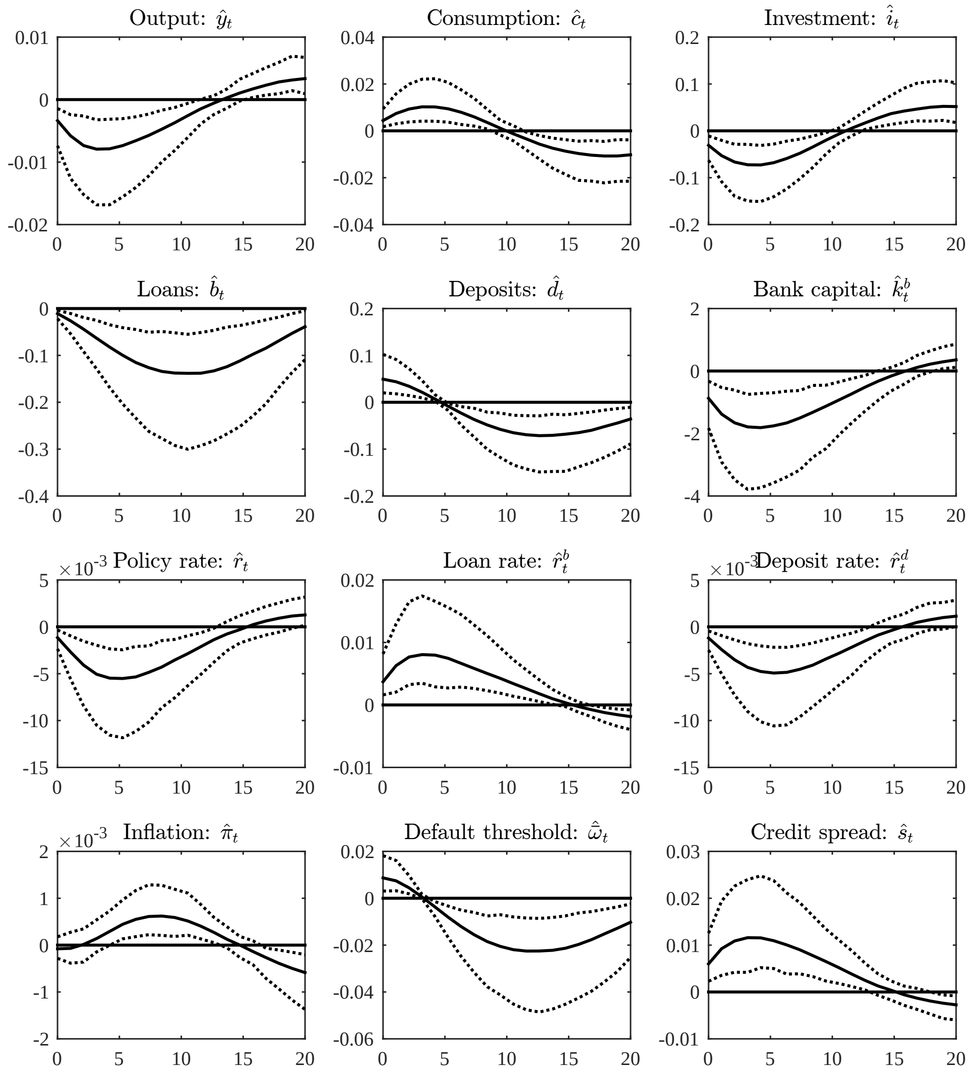
4.2 Credit default shock

This section describes the effects of a realization of credit defaults different from anticipated levels which is represented by a 1% increase in unexpected default losses. The parameters are set at the estimated posterior median and in addition I plot the 10% and 90% confidence intervals.

Figure 6 shows that, following a positive default shock, bank profits fall as the funds reserved for credit default losses appear insufficient to cover this period's losses. The unanticipated loss must be incurred on the profit and loss account. Even though default losses are only a small fraction of the bank's total operations, bank profits decrease sharply over time as the effect on bank leverage forces banks to lower their lending activities. In response to the increase in leverage they must decrease the amount of loans via an increase in the lending rate. The real side of the economy experiences a persistent decline in credit supply. Consequently, the amount of funds available for investment in the physical capital stock declines which explains the contemporaneous fall in investment. The fall in investment and physical capital lowers production and the economy moves into recession.

The default shock leads to an increase in the inflation rate because firms borrow to finance production and the cost of borrowing increases. The result is consistent with theoretical evidence presented by Christiano et al. (2005) and Gerali et al. (2010). Gilchrist et al. (2015) present empirical evidence for the presence of this "cost channel" in the U.S.: when firms have weak balance sheets, they may pass costs increases, i.e. higher lending rates, on to their customers in the form of higher prices. The Central Bank must consequently choose between inflationary pressure and output stabilization and as output falls more than inflation increases, the policy rate declines slightly. While the decrease in bank leverage affects the lending rate directly, it does not affect the deposit rate which follows the policy rate. As a result the lending rate increases while the deposit rate falls.

Figure 6: Impulse response functions of a default shock



Notes: prices and interest rates are shown as absolute deviations from steady state, expressed in percentage points and real variables are percentage deviations from steady state.

Comparing Figure 5 and Figure 6 suggests that the effects of a default shock are more persistent. Bank profitability is low because banks and entrepreneurs have not enough capital to increase investment. Importantly, conventional monetary policy, a decrease of the policy rate, is not effective to rebuild the bank balance sheets swiftly because the decline in the policy rate is restrained by inflationary pressure. Lowering the policy rate further would put additional upward pressure on the inflation rate and is therefore not consistent with the inflation objective.

4.3 Countercyclical capital buffer

As the Central Bank must choose between stabilizing inflation or output, the slight decrease in the policy rate is not enough to alleviate the bank capital constraints. As a consequence, the effects of a credit default shock on the real economy are very persistent. Moreover, the increase in the lending rate by banks after a credit default shock is suboptimal from a finance perspective as the decision to finance new projects should be independent from the costs made on previous projects, i.e., losses on previous projects should be treated as a *sunk cost*. Nevertheless, if leverage ratios become too high, financial market discipline or regulation forces banks to deleverage and restrict credit supply.

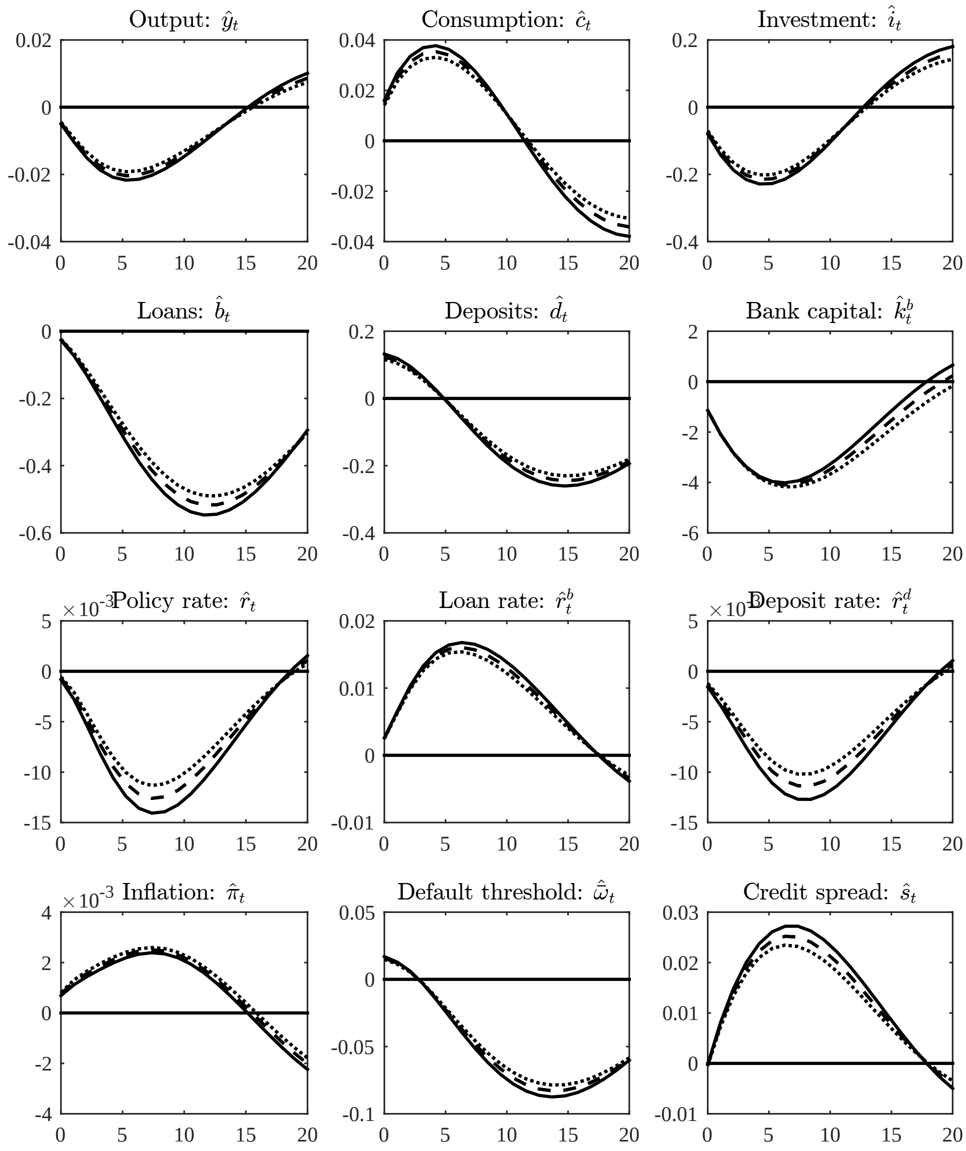
For these reasons, an alternative policy instrument is introduced that alleviates bank capital constraints more directly, i.e., a countercyclical capital buffer. Figure 7 shows the response to a default shock when the Central Bank implements the countercyclical capital buffer which is activated when the credit-to-GDP ratio differs from its steady state ratio specified by (44). I experiment with different values for the policy parameter ϱ_y. The solid line represents the benchmark model and sets $\varrho_y = 0$ (no countercyclical capital buffer), the dashed line represents $\varrho_y = 1$ and the dotted line represents $\varrho_y = 3$. If $\varrho_y = 1$ a 1% increase in the credit-to-GDP ratio corresponds to a 1% increase in bank leverage requirements.

The calibration results show that the countercyclical capital buffer attenuates the fluctuations caused by a credit default shock slightly. The buffer is activated and attenuates the business cycle, but the impact is minimal. Even if $\varrho_y = 3$ which implies that for a 1% decrease in the credit-to-GDP ratio from its steady state level the bank capital constraint is relaxed by 3%, the attenuation is minimal. The impact of the countercyclical capital buffer is minimal because the decline in credit is roughly similar to the decline in GDP. Consequently, the credit-to-GDP ratio does not change and the countercyclical capital buffer is not activated.

It might therefore be more advantageous to specify a countercyclical capital buffer that is contingent on bank leverage. Figure 8 shows the responses to a default shock when the Central Bank activates the countercyclical capital buffer when bank leverage differs from steady state bank leverage specified by (45). The solid line represents the benchmark model and sets $\varrho_{k^b} = 0$, the bar striped line represents $\varrho_{k^b} = 1/3$ and the dotted line represents $\varrho_{k^b} = 2/3$. Note that $\varrho_{k^b} = 1$ is trivial as it dissolves the entire feedback effect between bank leverage and the lending rate.

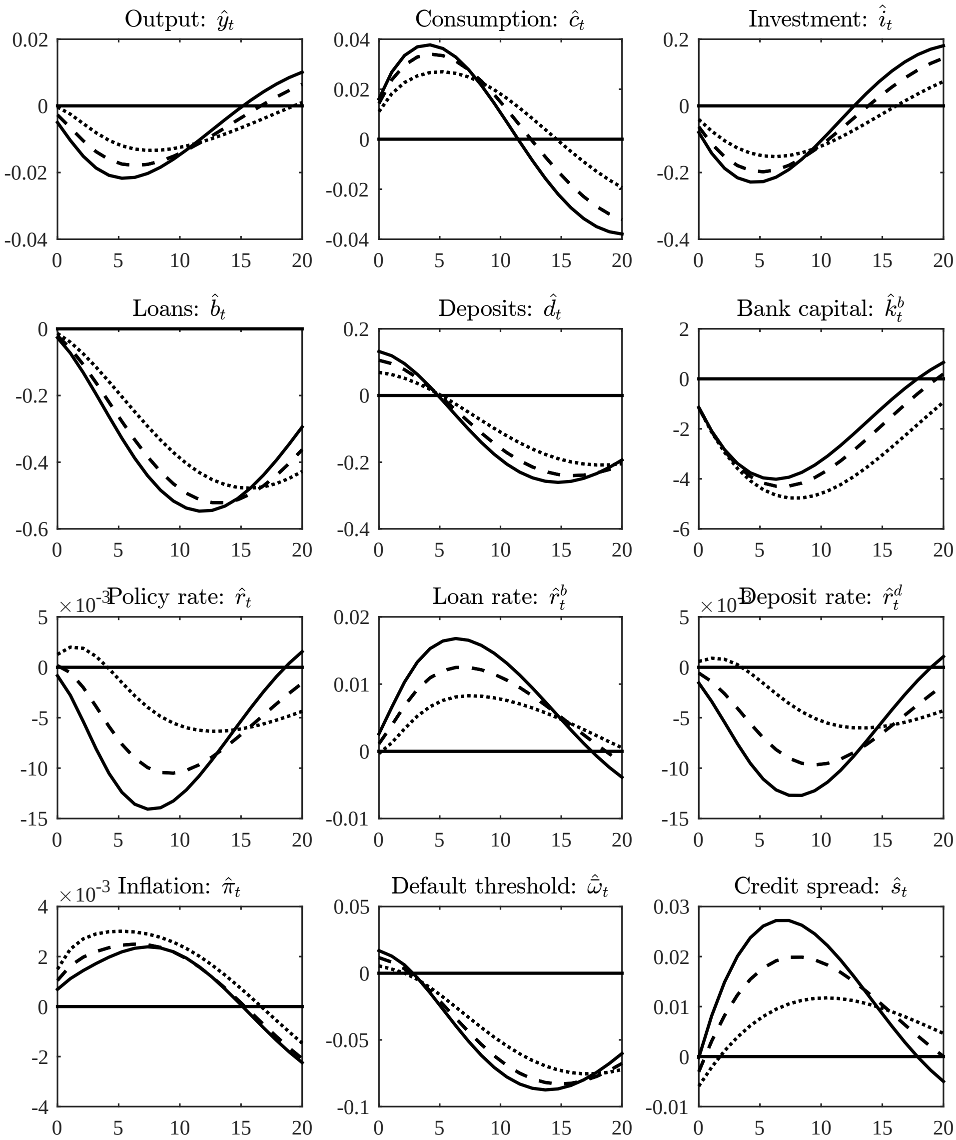
The calibration shows that the countercyclical capital buffer is much more stabilizing when it is activated when bank leverage rises. The endogenous activation of the buffer significantly attenuates macroeconomic fluctuations. However, not without costs as the macroeconomic fluctuations show more persistence. When the countercyclical capital

Figure 7: Impulse response functions of a default shock and a countercyclical capital buffer based on credit-to-GDP.



Notes: prices and interest rates are shown as absolute deviations from steady state, expressed in percentage points and real variables are percentage deviations from steady state. The solid line represents the benchmark model and sets $\varrho_y = 0$ (no countercyclical capital buffer), the bar stripes $(---)$ represents $\varrho_y = 1$ and the dotted line $(\cdots)$ represents $\varrho_y = 3$.

Figure 8: Impulse response functions of a default shock and a countercyclical capital buffer based on steady state bank leverage.



Notes: prices and interest rates are shown as absolute deviations from steady state, expressed in percentage points and real variables are percentage deviations from steady state. The solid line represents the benchmark model and sets $\varrho_{k^b} = 0$ (no countercyclical capital buffer), the bar striped line $(- - -)$ represents $\varrho_{k^b} = 1/3$ and the dotted line $(\cdots)$ represents $\varrho_{k^b} = 2/3$.

buffer is activated, banks have less incentive to rebuild bank capital as they are allowed to adjust more slowly. Specifically, the deterioration of bank capital is amplified and lasts longer and as a consequence it takes longer for credit supply to recover to pre-shock levels. Recently, Shin (2014) showed that since the global financial crisis banks have been very slow in rebuilding bank capital and prefer dividend payouts over retained earning.

4.4 Endogenous recapitalization

A potential solution to overcome the increase in persistence resulting from the countercyclical capital buffer is to specify an endogenous recapitalization. The countercyclical capital buffer analyzed in the previous section allows banks to operate at a lower leverage ratio. As a result, the bank no longer accrues bankruptcy costs, but has less incentives to rebuild its bank capital. A endogenous recapitalization could mitigate this incentive problem as restoring bank capital is no longer the bank's choice. An endogenous recapitalization could be specified as a mandatory equity issuance, a bail-in or a government facilitated tax-financed recapitalization. For modeling convenience I focus on the latter specification, but as the consumers is the tax-payer, bank shareholder and depositor, the model is not able to distinguish between these three options.

Figure 9 and Figure 10 show the effects of an endogenous bank recapitalization which is financed by lump-sum taxation, taxed from the patient consumer. The recapitalization and corresponding tax level are specified by exactly the same processes as the countercyclical capital buffers in the previous section. The results show that an endogenous recapitalization is very effective in attenuating the impact of a credit default shock on the real economy. Both the amplification as well as the persistence of the macroeconomic fluctuations after a credit default shock decrease significantly. The recapitalization contingent on bank leverage shows the strongest attenuation of macroeconomic fluctuations.

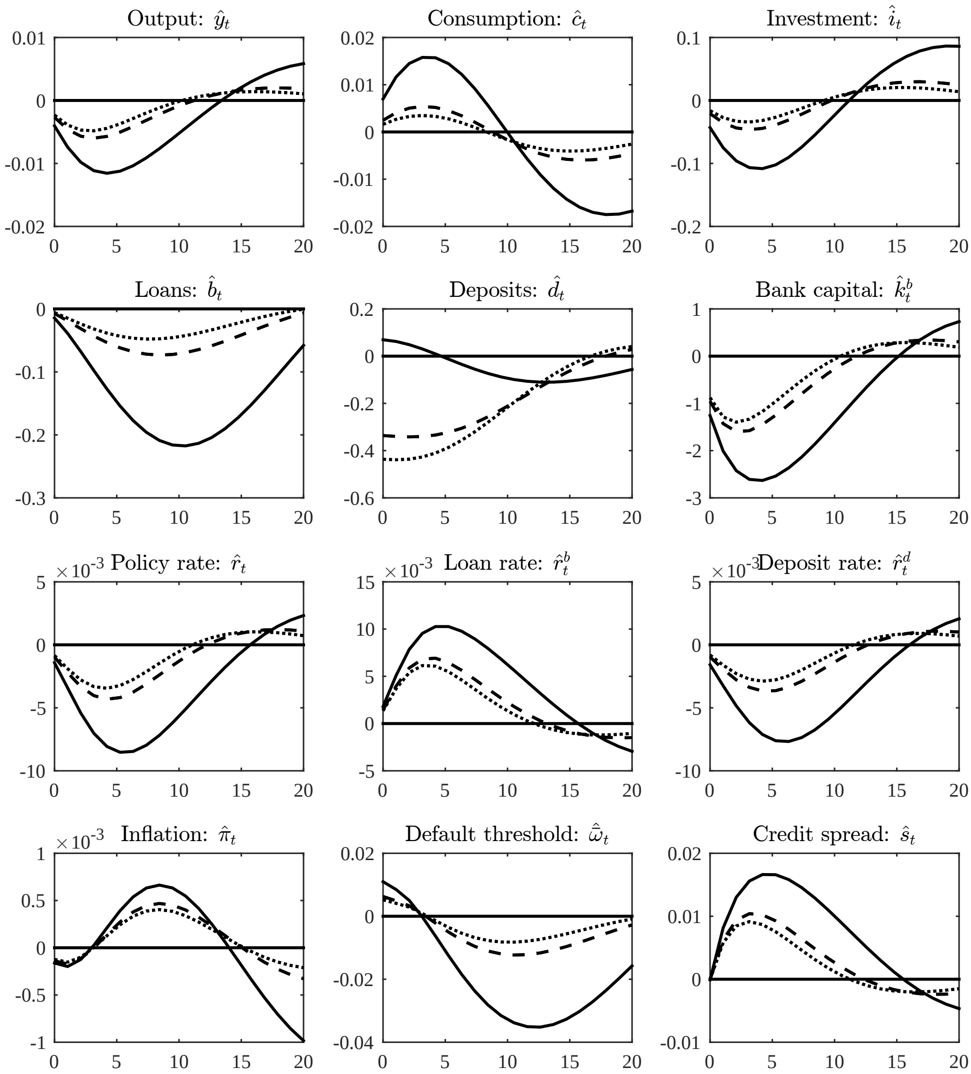
The bank recapitalization is effective because deposits are effectively converted in bank equity. For this reason, the model is not able to distinguish between a taxed-financed recapitalization, a bail-in or a mandatory equity issuance because all options would yield qualitatively the same results. The recapitalization ensures that bank capital deteriorates less severe. Consequently, banks only moderately restrict credit supply and investment. As the fall in output is small, the monetary authority does not need to intervene as strong as before. The real deposit rate continues to fall, but less than before which ensures that consumption rises less.

5 Conclusion and discussion

This study analyzed the effects of unanticipated credit defaults on the banking sector and the real economy. The model is estimated for the euro area to identify the effects of a credit default shock. In the model banks anticipate that some loans will default. However, if the materialization of credit defaults losses is higher than anticipated, a credit default shock, bank capital deteriorates to cover the losses. As a consequence, the bank balance sheet tightens, credit supply falls and lending rates rise.

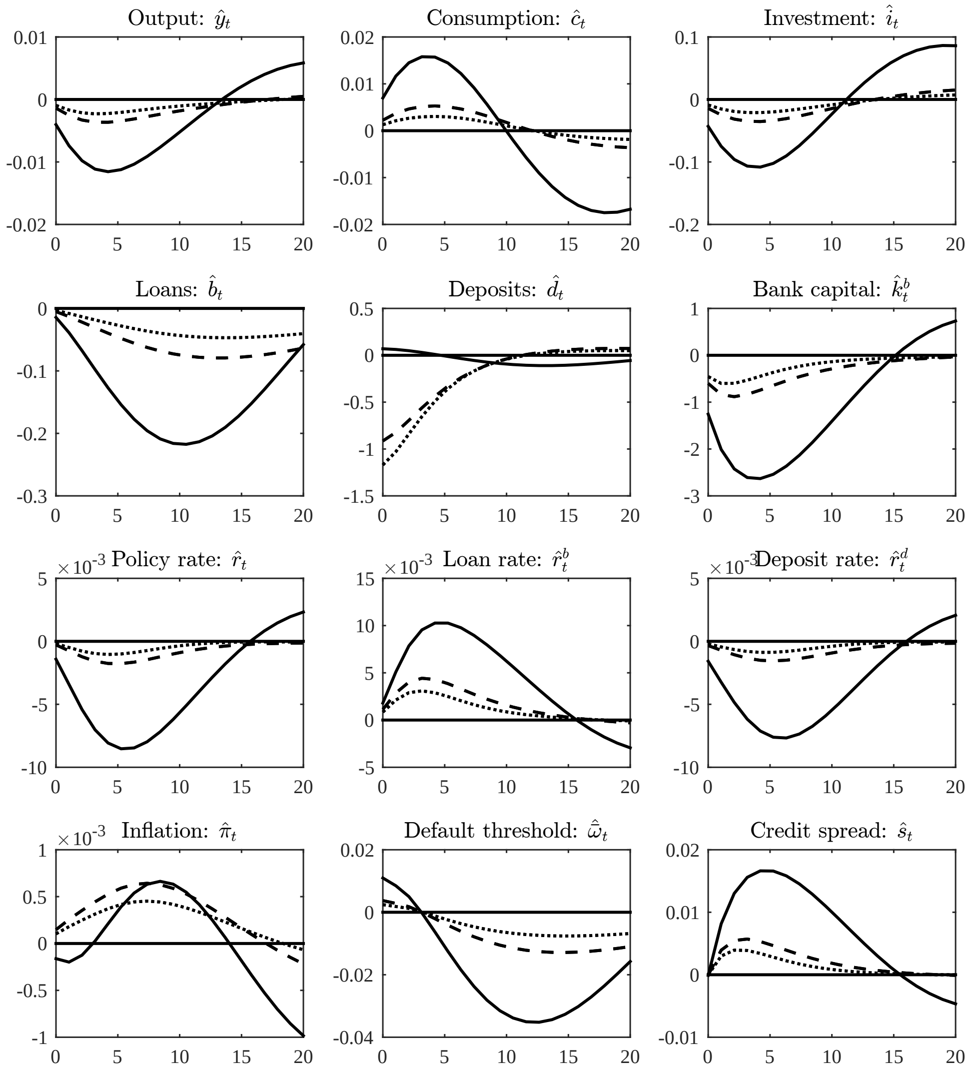
The results showed that conventional monetary policy effectively attenuates macroeconomic fluctuations after conventional shocks like an adverse technology shock and a

Figure 9: Impulse response functions of a default shock and a countercyclical capital buffer based on credit-to-GDP.



Notes: prices and interest rates are shown as absolute deviations from steady state, expressed in percentage points and real variables are percentage deviations from steady state. The solid line represents the benchmark model and sets $\varrho_y = 0$ (no countercyclical capital buffer), the bar stripes $(---)$ represents $\varrho_y = 1$ and the dotted line $(\cdots)$ represents $\varrho_y = 3$.

Figure 10: Impulse response functions of a default shock and a countercyclical capital buffer based on steady state bank leverage.



Notes: prices and interest rates are shown as absolute deviations from steady state, expressed in percentage points and real variables are percentage deviations from steady state. The solid line represents the benchmark model and sets $\varrho_{k^b} = 0$ (no countercyclical capital buffer), the bar striped line $(---)$ represents $\varrho_{k^b} = 1/3$ and the dotted line $(\cdots)$ represents $\varrho_{k^b} = 2/3$.

contractionary monetary policy shock. The effects of these shocks are relatively moderate because conventional monetary policy is accommodating and effective. These result contrast with the macroeconomic consequences of a credit default shock. The results suggested that a credit default shock initiates a persistent slump because conventional monetary policy is not effective in facilitating a quick recovery of the bank balance sheet. When the economic is hit by a credit default shock, banks increase lending rates and firms increase prices because they experience a cost-push effect when their lending costs increase. At the same time, the decline in credit supply lowers output via investment. The fall of output while inflation increases complicates conventional monetary policy as the monetary authority must choose between two contrasting objectives.

As conventional monetary policy is less effective after a credit default shock, I adapted the framework to allow for two alternative policy instruments that address the bank capital constraint more directly: a countercyclical capital buffer and an endogenous recapitalization. Countercyclical capital buffers attenuate macroeconomic fluctuations after a credit default shock. However, there is a trade-off between the length and the depth of the cycle because after activating the countercyclical capital buffer banks rebuild their capital more slowly. Consequently, the cycle is less deep but the recovery is more sluggish. In contrast, an endogenous recapitalization financed by lump-sum taxation effectively solves this trade-off problem as rebuilding bank capital is no longer the bank's choice. An endogenous recapitalization is therefore an effective instrument to reduce the macroeconomic fluctuations resulting from a credit default shock without increasing the persistence of the downturn.

The results indicate that conventional monetary policy effectively stabilizes the economy when the economy is hit by a conventional shock, for example a technology shock. However, when bank capital is hit directly by for example a credit default shock, macroprudential policy measures that directly alleviate bank capital constraints are effective and can complement conventional monetary policy. While depth and length of the recession are important considerations when the countercyclical capital buffer is implemented, one could argue that moral hazard issues should be considered when banks are recapitalized. Albeit a justified concern when banks are recapitalized with taxpayers' money, the model is not able to distinguish between a taxed-financed recapitalization, a bail-in or a mandatory equity issuance as consumers are the tax-payers, the bank shareholders and the depositors. As a mandatory equity issuance is less vulnerable for moral hazard issues, this seems the preferred option to formalize the endogenous recapitalization in practice.

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Appendix A: Model solution

A.1 Household maximization problem

The households maximize their utility subject to the budget constraint by choosing $C_t^H(i)$, $(1 - L_t^H(i))$ and $D_t(i)$:

$$\mathcal{L}_t \equiv \mathrm{E}_t \sum_{t=0}^{\infty} (\beta^H)^t \Bigg\{ \eta_t^c \left(\frac{1}{1-\sigma_c} (C_t^H(i) - hC_{t-1}^H(i))^{1-\sigma_c} - \frac{\eta_t^l}{1+\sigma_h} (L_t^H(i))^{1+\sigma_h} \right) +$$
$$\lambda_t^{h,1}(i) \left(W_t(i) L_t^H(i) + \frac{1+r_{t-1}^d}{\pi_t} D_{t-1}(i) + RP_t(i) - C_t^H(i) - D_t(i) \right) \Bigg\}$$
$$\lambda_t^{h,2}(i) \left(L_t^H(i) - \left(\frac{W_t(i)}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_w}} L_t^H \right). \tag{A.1}$$

Notice $(\beta^H)^t \equiv \frac{1}{1+\rho^H}$, ρ^H is the household patience parameter. The FOCs are:

$$\frac{\partial \mathcal{L}_t}{\partial C_t^H(i)} = (\beta^H)^t \left[\eta_t^c (C_t^H(i) - hC_{t-1}^H(i))^{-\sigma_c} - \lambda_t^{h,1}(i) \right] = 0, \tag{A.2}$$

$$\frac{\partial \mathcal{L}_t}{\partial W_t(i)} = -\frac{1+\lambda_w}{\lambda_w} \left(\frac{W_t(i)}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_w}-1} L_t^H + \lambda_t^{h,1}(i) L_t^H(i), \tag{A.3}$$

$$\frac{\partial \mathcal{L}_t}{\partial D_t(i)} = (\beta^H)^t \left[-\lambda_t^{h,1}(i) + \mathrm{E}_t \left\{ \frac{\lambda_{t+1}^{h,1}(i)}{1+\rho^H} \frac{1+r_t^d}{\pi_{t+1}} \right\} \right] = 0, \tag{A.4}$$

$$\frac{\partial \mathcal{L}_t}{\partial \lambda_t^{h,1}(i)} = W_t(i) L_t^H(i) + \frac{1+r_{t-1}^d}{\pi_t} D_{t-1}(i) + RP_t(i) - C_t^H(i) - D_t(i) = 0, \tag{A.5}$$

$$\frac{\partial \mathcal{L}_t}{\partial \lambda_t^{h,2}(i)} = L_t(i) - \left(\frac{W_t(i)}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_w}} L_t = 0. \tag{A.6}$$

Iterating (A.2) forward in time for one period and substituting the result and (A.2) in (A.4) gives the consumption Euler Equation for households:

$$\eta_t^c (C_t^H(i) - hC_{t-1}^H(i))^{-\sigma_c} = \frac{1+r_t^d}{1+\rho^H} \mathrm{E}_t \left\{ \frac{\eta_{t+1}^c (C_{t+1}^H(i) - hC_t^H(i))^{-\sigma_c}}{\pi_{t+1}} \right\}. \tag{A.7}$$

In the logarithmic case, $\sigma_c = 1$, (A.8) becomes:

$$C_t^H(i) = \left(\frac{1+r_t^d}{(1+r_t^d)\eta_t^c + (1+\rho^H) h \, \mathrm{E}_t \left\{ \eta_{t+1}^c \pi_{t+1} \right\}} \right) \left(\eta_t^c h C_{t-1}^H(i) + \frac{1+\rho^H}{1+r_t^d} \mathrm{E}_t \left\{ \eta_{t+1}^c C_{t+1}^H(i) \pi_{t+1} \right\} \right). \tag{A.8}$$

Substituting (A.2) in (A.3) and noticing (7) and (8) gives:

$$\frac{\tilde{w}_t(i)}{P_t} \mathrm{E}_t \sum_{\tau=0}^{\infty} \beta^\tau (\xi^w)^\tau \left(\frac{\left(\frac{P_t}{P_{t-1}} \right)^{\gamma_w}}{\frac{P_{t+\tau}}{P_{t+\tau-1}}} \right) \frac{L_{t+\tau}^H(i) (C_{t+\tau}^H(i) - hC_{t+\tau-1}^H(i))^{-\sigma_c}}{1+\lambda_w} =$$
$$- \mathrm{E}_t \sum_{\tau=0}^{\infty} \beta^\tau (\xi^w)^\tau L_{t+\tau}^H(i) (L_t^H(i))^{\sigma_l} \tag{A.9}$$

which can be used to define the aggregate law of motion for the wage rate:

$$W_t^{-\frac{1}{\lambda_w}} = (\xi^w)^\tau \left(\left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} W_{t-1} \right)^{-\frac{1}{\lambda_w}} + (1 - (\xi^w)^\tau) \tilde{w}_t^{-\frac{1}{\lambda_w}} \tag{A.10}$$

Aggregating (A.4) gives the deposit demand Equation:

$$D_t = w_t L_t^H + \frac{1 + r_{t-1}^d}{\pi_t} D_{t-1} + RP_t - C_t^H. \tag{A.11}$$

A.2 Entrepreneur maximization problem

Entrepreneurs maximize their utility subject to the budget constraint and the participation constraint of the banks by choosing $C_t^E(j), K_t(j), I_t(j), B_t(j)$, and $\bar{\omega}_{t+1}$:

$$\begin{aligned}
\mathcal{L}_t \equiv \mathrm{E}_t \sum_{t=0}^{\infty} (\beta^E)^t \Bigg\{ & \left[\ln \left(C_t^E(j) - h C_{t-1}^E(j) \right) \right] + \\
& \lambda_t^1(j) \Bigg([1 - \Gamma(\bar{\omega}_t(j))] (r_t^k u_t(j) - \psi(u_t(j))) q_t K_{t-1}(j) + B_t(j) + w_t^E L_t^E(j) - \\
& \frac{1 + r_{t-1}^b}{\pi_t} B_{t-1}(j) - I_t(j) - w_t L_t(j) - C_t^E(j) \Bigg) + \\
& \lambda_t^2(j) \Bigg([\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] r_{t+1}^k u_t q_t K_t(j) - \frac{r_t^{wb}}{\pi_{t+1}} B_t(j) \Bigg), \\
& \lambda_t^3(j) \Bigg((1 - \delta^k) K_{t-1}(j) + \left[1 - \phi \left(\frac{\eta_t^i I_t(j)}{I_{t-1}(j)} \right) \right] I_t(j) - K_t(j) \Bigg) \Bigg\}.
\end{aligned} \tag{A.12}$$

where $w_t = \left(\frac{w_t^H}{\Omega}\right)^{\Omega}\left(\frac{w_t^E}{1-\Omega}\right)^{1-\Omega}$. The FOCs, using the bank participation constraint with equality[8], denote:

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial C_t^E(j)} =& \frac{1}{C_t^E(j) - hC_{t-1}^E(j)} - \lambda_t^1(j) = 0, && \text{(A.13)}\\
\frac{\partial \mathcal{L}_t}{\partial (K_t(j))} =& \lambda_t^1(j)\left([1-\Gamma(\bar{\omega}_{t+1})](r_{t+1}^k u_t - \psi(u_t))q_t - q_t\right) + \frac{\lambda_{t+1}^1(j)}{1+\rho^E}\left(q_{t+1}(1-\delta^k)\right)\\
&+ \lambda_t^2(j)\left([\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})]r_{t+1}^k u_t q_t\right) = 0,\\
\frac{\partial \mathcal{L}_t}{\partial B_t(j)} =& \frac{\lambda_{t+1}^1(j)}{1+\rho^E}\frac{1+r_t^b}{\pi_{t+1}} + \lambda_t^2(j)\frac{r_t^{wb}}{\pi_{t+1}} = \lambda_t^1(j), && \text{(A.14)}\\
\frac{\partial \mathcal{L}_t}{\partial \bar{\omega}_{t+1}} =& \lambda_t^1(j)\Gamma'(\bar{\omega}_{t+1})(r_{t+1}^k u_t - \psi(u_t)) = \lambda_t^2(j)[\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1})](r_{t+1}^k u_t), && \text{(A.15)}\\
\frac{\partial \mathcal{L}_t}{\partial I_t} =& \lambda_t^1(j)\left(q_t\left[1-\phi\left(\frac{\eta_t^i I_t(j)}{I_{t-1}(j)}\right) - \phi'\left(\frac{\eta_t^i I_t(j)}{I_{t-1}(j)}\right)\frac{\eta_t^i I_t(j)}{I_{t-1}(j)}\right] - 1\right),\\
&\lambda_{t+1}^1(j) q_{t+1}\phi'\left(\frac{\eta_t^i I_t(j)}{I_{t-1}(j)}\right)\left(\frac{\eta_t^i I_t(j)}{I_{t-1}(j)}\right)\left(\frac{I_t(j)}{I_{t-1}(j)}\right) = 0, && \text{(A.16)}\\
\frac{\partial \mathcal{L}_t}{\partial u_t} =& \lambda_t^1(j)[1-\Gamma(\bar{\omega}_{t+1})](r_{t+1}^k - \psi'(u_t)) + \lambda_t^2(j)[\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})]r_{t+1}^k = 0 && \text{(A.17)}
\end{aligned}$$

and the two constraints (16) and (17) which are not presented for conciseness. The maximization problem with respect to consumption determines aggregate consumption:

$$\frac{1}{C_t^E - hC_{t-1}^E} = \lambda_t^1, \tag{A.18}$$

aggregate demand for loans

$$\beta^E \frac{\lambda_{t+1}^1}{\lambda_t^1}\frac{1+r_t^b}{\pi_{t+1}} + \frac{\lambda_t^2}{\lambda_t^1}\frac{r_t^{wb}}{\pi_{t+1}} = 1, \tag{A.19}$$

aggregate demand for capital:

$$\begin{aligned}
q_t =& \frac{\lambda_t^2}{\lambda_t^1}[\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})]r_{t+1}^k u_t, +\\
&\beta^E \frac{\lambda_{t+1}^1}{\lambda_t^1}\left(q_{t+1}(1-\delta^k) + [1-\Gamma(\bar{\omega}_{t+1})](r_{t+1}^k u_t - \psi(u_t))\right), && \text{(A.20)}
\end{aligned}$$

aggregate demand for investment:

$$\begin{aligned}
&q_t\phi'\left(\frac{\eta_t^i I_t}{I_{t-1}}\right)\frac{\eta_t^i I_t}{I_{t-1}} - \beta^E\frac{\lambda_{t+1}^1}{\lambda_t^1}\phi'\left(\frac{\eta_{t+1}^i I_{t+1}}{I_t}\right)\frac{\eta_{t+1}^i I_{t+1}}{I_t}\frac{I_{t+1}}{I_t} + 1 =\\
&q_t\left(1-\phi\left(\frac{\eta_t^i I_t}{I_{t-1}}\right)\right), && \text{(A.21)}
\end{aligned}$$

[8] I assume free entry and exit in the banking sector such that competition drives down the profit margin.

the optimal utilization level:

$$\frac{\lambda_t^2}{\lambda_t^1} = -\frac{[1-\Gamma(\bar{\omega}_t)](r_t^k - \psi'(u_t))}{[\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)]r_t^k} = 0, \tag{A.22}$$

and the optimal default threshold:

$$\frac{\Gamma'(\bar{\omega}_t)(r_t^k u_t - \psi(u_t))}{[\Gamma'(\bar{\omega}_t) - \mu G'(\bar{\omega}_t)]r_t^k u_t} = \frac{\lambda_t^2}{\lambda_t^1} = 0. \tag{A.23}$$

Note that, as in Smets and Wouters (2003), I assume that $r_t^k = \psi'(u_t)$ and so $\frac{\lambda_t^2}{\lambda_t^1} = 0$. Using this simplification gives the following results for the aggregate loan demand:

$$\beta^E \frac{\lambda_{t+1}^1}{\lambda_t^1} \frac{1+r_t^b}{\pi_{t+1}} = 1, \tag{A.24}$$

and aggregate demand for capital:

$$q_t = \beta^E \frac{\lambda_{t+1}^1}{\lambda_t^1} \left(q_{t+1}(1-\delta^k) + [1-\Gamma(\bar{\omega}_{t+1})](r_{t+1}^k u_{t+1} q_{t+1} - \psi(u_{t+1})) \right) \tag{A.25}$$

A.2.1 Retailers

Retailers buy the intermediate products produced in the intermediate good sector and transform it into a homogeneous good Y_t using a CES production function, see Dixit and Stiglitz (1977):

$$Y_t = \left[\int_0^1 Y_t(i)^{1-1/\lambda_p} di \right]^{1/(1-1/\lambda_p)}, \tag{A.26}$$

where $Y_t(i)$ is an unique input variety produced by entrepreneur i and λ_p is the elasticity of substitution in production. The retailer minimizes costs $\int_0^\infty P_t(i)Y_t(i)$ subject to the CES production function, Equation (A.26). Hence, the retailer optimization problem becomes:

$$\mathcal{L}_t \equiv \int_0^1 P_t(i)Y_t(i)di + \lambda_t \left[Y_t - \left[\int_0^1 Y_t(i)^{1-1/\lambda_p} di \right]^{1/(1-1/\lambda_p)} \right]. \tag{A.27}$$

The solution defines unit costs P_t and demand for $Y_t(i)$:

$$P_t \equiv \left[\int_0^1 P_t^{1-\lambda_p} di \right]^{1/1-\lambda_p}, \tag{A.28}$$

$$Y_t(i) = Y_t \left[\frac{P_t(i)}{P_t} \right]^{-\lambda_p}. \tag{A.29}$$

A.2.2 Intermediate good sector

Entrepreneurs own firms therefore the maximization problem of firm i is part of the entrepreneurs decision problem. From Equation (22) I derive the intermediate firm optimization problem. This is an intermediate step to determine the optimal capital-labor mix:

$$\begin{aligned} &\min_{K_t(i), L_t^H(i)} \quad r_t^k q_t K_t(i) + W_t L_t^H(i), \\ &\text{subject to} \quad Y_t(i) = A_t[K_t(i)^\alpha L_t^H(i)^{1-\alpha}] + q_t[(1-\delta^k)K_{t-1} - K_t]. \end{aligned} \tag{A.30}$$

From the Lagrangian I derive the following FOCs:

$$E_t r_{t+1}^k = E_t \left[\frac{\lambda_t \alpha \frac{Y_{t+1}(i)}{K_{t+1}(i)} + (1-\delta^k) q_{t+1}}{q_t} \right] \tag{A.31}$$

$$W_t = \lambda_t (1-\alpha) \frac{Y_t(i)}{L_t(i)}, \tag{A.32}$$

where λ_t are the Lagrangian multipliers for the production constraint. Equations (A.31) and (A.32) determine capital demand and labor demand respectively. Using Equations (A.31), (A.32) and Equation (22) I can write λ_t as:

$$MC_t \equiv \lambda_t = \frac{1}{A_t} \left(\frac{r_t^k q_{t-1} - (1-\delta^k) q_t}{\alpha} \right)^\alpha \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha}, \tag{A.33}$$

λ_t may be interperted as the marginal costs MC_t of producing one extra unit of the intermediate good $Y_t(i)$. I assume that entrepreneurs maximize firms value by setting prices. First I define nominal profits $NP_t(i)$ as:

$$NP_t(i) \equiv P_t(i) Y_t(i) - TC_t(i), \tag{A.34}$$

where $TC_t(i)$ denotes total costs:

$$TC_t(i) \equiv r_t^k K_t(i) + W_t L_t^H(i) = \lambda_t Y_t(i). \tag{A.35}$$

Substituting the optimal capital and labor mix, Equations (A.31) and (A.32), the total costs function, Equation (A.35), and marginal costs, Equation (A.33), in the nominal profits function, Equation (A.34), I obtain

$$NP_t(i) = P_t \left(\frac{Y_t(i)}{Y_t} \right)^{-1/\lambda_p} Y_t(i) - MC_t Y_t(i), \tag{A.36}$$

where I used the Dixit-Stiglitz demand function for output variety $Y_t(i)$.

Firms maximize expected firm value by setting prices $P_t(i)$. I define the real profit function $RP_t(i)$ as:

$$RP_t(i) \equiv \frac{NP_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t} - mc_t \right] Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\lambda_p}, \tag{A.37}$$

where $mc_t \equiv MC_t/P_t$. Firms maximize their real profits $RP_t(i)$ by choosing the price level $P_t(i)$.

Using Calvo Pricing (Calvo, 1983), and denoting the probability that a firm is able to change its price by the probability $(1-\eta)$ I obtain the expected value of the firm that has just received a "green light", i.e., the firm is allowed to change it price in period t, and has set a new price $P_t^n(i)$:

$$\begin{aligned}\max_{P_t^n(i)} \mathrm{E}_t \Bigg[\left[\frac{P_t^n(i)}{P_t} - mc_t\right] Y_t \left(\frac{P_t^n(i)}{P_t}\right)^{-\lambda_p} + \frac{\eta}{r_{t+1}^k}\left[\frac{P_t^n(i)}{P_{t+1}} - mc_{t+1}\right] Y_{t+1}\left(\frac{P_t^n(i)}{P_{t+1}}\right)^{-\lambda_p} \\ \frac{\eta^2}{r_{t+1}^k r_{t+2}^k}\left[\frac{P_t^n(i)}{P_{t+2}} - mc_{t+2}\right] Y_{t+2}\left(\frac{P_t^n(i)}{P_{t+2}}\right)^{-\lambda_p} + \ldots\Bigg], \\ = \mathrm{E}_t \sum_{\tau=0}^{\infty} \eta^\tau S_{t+\tau}\left[\frac{P_t^n(i)}{P_{t+\tau}} - mc_{t+\tau}\right] Y_{t+\tau}\left(\frac{P_t^n(i)}{P_{t+\tau}}\right)^{-\lambda_p},\end{aligned} \tag{A.38}$$

where $S_{t+\tau}$ is the discount factor. Entrepreneurs use the cost of capital to discount, i.e., $S_{t+\tau} \equiv 1/(1+r_t^k), \forall \tau$. Maximizing Equation (A.38) w.r.t. the new price $P_t^n(i)$ and rewriting gives:

$$P_t^n(i) = P_t^n = \frac{\lambda_p}{\lambda_p - 1} \mathrm{E}_t \frac{\sum_{\tau=0}^{\infty} \eta^\tau S_{t+\tau} P_{t+\tau}^{\lambda_p} Y_{t+\tau} mc_{t+\tau}}{\sum_{\tau=0}^{\infty} \eta^\tau S_{t+\tau} P_{t+\tau}^{\lambda_p - 1} Y_{t+\tau}}. \tag{A.39}$$

Log-linearizing Equation (A.39) gives Equation (A.65) in the main text.

To derive the relationship between the aggregate price level and the new price level set s periods ago I rewrite the aggregate price level as:

$$P_t^{1-\lambda_p} = \int_0^1 P_t(i)^{1-\lambda_p} di. \tag{A.40}$$

Subsequently, using the law of large numbers we know that $(1-\eta)\eta^s$ is the fraction of firms that has changed its price s periods ago. Hence, Equation (A.40) can be rewritten as:

$$P_t^{1-\lambda_p} = (1-\eta)(P_t^n)^{1-\lambda_p} + (1-\eta)\left[\eta(P_{t-1}^n)^{1-\lambda_p} + \eta^2(P_{t-2}^n)^{1-\lambda_p} + \ldots\right]. \tag{A.41}$$

Using Equation (A.40) for $P_{t-1}^{1-\lambda_p}$ I can rewrite Equation (A.40) as:

$$P_t^{1-\lambda_p} = (1-\alpha_p)(P_t^n)^{1-\lambda_p} + \alpha_p P_{t-1}^{1-\lambda_p}. \tag{A.42}$$

Moreover, the link between aggregate production and the aggregate factor inputs L_t and K_t can be defined as follows:

$$Y_t^a \equiv \int_0^1 Y_t di = A_t K_t^\alpha L_t^{1-\alpha}, \tag{A.43}$$

where Y_t^a is an alternative output measure. Defining an alternative price index P_t^a:

$$(P_t^a)^{-\lambda_p} \equiv \left[\int_0^1 P_t^{-\lambda_p} di\right]^{-1/\lambda_p}, \tag{A.44}$$

Using the alternative price index I can link aggregate output Y_t to the aggregate factor inputs:

$$Y_t = \left(\frac{P_t^a}{P_t}\right)^{\lambda_p} Y_t^a, \tag{A.45}$$

In equilibrium $P_t^a = P_t$ and $Y_t^a = Y_t$.

A.2.3 Loan and deposit demand

The demand functions for loans is derived in a similar fashion as demand for product $Y_t(i)$. Entrepreneurs minimize their loan payments subject to the aggregation technology, see again Dixit and Stiglitz (1977):

$$\mathcal{L}_t \equiv \int_0^1 r_t^b(j)B_t(j)dj + \lambda_t \left[B_t - \left[\int_0^1 B_t(j)^{1-1/\mu_b}dj\right]^{1/(1-1/\mu_b)}\right]. \tag{A.46}$$

Solving this problem by aggregating all FOCs across all entrepreneurs gives loan demand at bank j:

$$B_t(j) = \left(\frac{r_t^b(j)}{r_t^b}\right)^{\mu_b} B_t. \tag{A.47}$$

In a similar fashion the deposit demand function is derived. Households maximize interest revenue from deposits subject to the aggregation technology:

$$\mathcal{L}_t \equiv \int_0^1 r_t^d(j)D_t(j)dj + \lambda_t \left[B_t - \left[\int_0^1 D_t(j)^{1-1/\mu_d}dj\right]^{1/(1-1/\mu_d)}\right]. \tag{A.48}$$

Solving this problem by aggregating all FOCs across all household gives deposit demand at bank j:

$$D_t(j) = \left(\frac{r_t^d(j)}{r_t^d}\right)^{\mu_d} D_t. \tag{A.49}$$

A.2.4 Retail Branch

The retail branch of bank j consist out of 2 parts, a lending branch and a funding branch. Both maximize their profits subject to the demand schedules by choosing the appropriate interest rates. Substituting demand for loans, Equation (29), in Equation (37) gives the following maximization problem:

$$\max_{r_t^b(j)} \sum_{t=0}^{\infty}([1-F(\bar{\omega}_{t+1})]r_t^b(i) - r_t^{wb})\left(\frac{r_t^b(j)}{r_t^b}\right)^{\mu^b} B_t - \frac{\kappa_b}{2}\left(\frac{r_t^b(j)}{r_{t-1}^b(j)} - 1\right)^2 r_t^b B_t, \tag{A.50}$$

The solution to the problem is:

$$\begin{aligned}&[1-F(\bar{\omega}_{t+1})]\left(\frac{r_t^b(j)}{r_t^b}\right)^{\mu^b} B_t + ([1-F(\bar{\omega}_{t+1})]r_t^b(j) - r_t^{wb})\frac{\mu^b B_t}{r_t^b}\left(\frac{r_t^b(j)}{r_t^b}\right)^{\mu^b-1} - \\ &\frac{\kappa_b r_t^b B_t}{r_{t-1}^b(j)}\left(\frac{r_t^b(j)}{r_{t-1}^b(j)} - 1\right) + \beta_t^b \,\mathrm{E}_t\left\{\kappa_b\left(\frac{r_{t+1}^b(j)}{r_t^b(j)} - 1\right) r_{t+1}^b B_{t+1}\frac{r_{t+1}^b(j)}{(r_t^b(j))^2}\right\} = 0.\end{aligned} \tag{A.51}$$

Rewriting Equation (A.51) gives:

$$\left(\frac{r_t^b(j)}{r_t^b}\right)^{\mu^b} B_t\left([1-F(\bar{\omega}_{t+1})](1+\mu^b)-\mu^b\frac{r_t^{wb}}{r_t^b(j)}\right)+$$
$$\kappa_b\left[\left(\frac{1}{r_{t-1}^b(j)}-\frac{r_t^b(j)}{(r_{t-1}^b(j))^2}\right)r_t^b B_t+\beta_t^b\,\mathrm{E}_t\left\{\left(\frac{r_{t+1}^b(j)}{r_t^b(j)}-1\right)\left(\frac{r_{t+1}^b(j)}{r_t^b(j)}\right)^2 B_{t+1}\right\}\right]$$
$$=0. \tag{A.52}$$

Imposing a symmetric equilibrium $r_t^b(j)\equiv r_t^b$, $\forall j$:

$$[1-F(\bar{\omega}_{t+1})](1-\mu^b)+\mu^b\frac{r_t^{wb}}{r_t^b}+$$
$$\kappa_b\left[\left(1-\frac{r_t^b}{r_{t-1}^b}\right)\frac{r_t^b}{r_{t-1}^b}+\beta_t^b\,\mathrm{E}_t\left\{\left(\frac{r_{t+1}^b}{r_t^b}-1\right)\left(\frac{r_{t+1}^b}{r_t^b}\right)^2\frac{B_{t+1}}{B_t}\right\}\right]=0. \tag{A.53}$$

Notice that if Equation (A.53) is log-linearized we obtain Equation (A.67).

In a similar fashion the funding branch maximizes its profits subject to the deposit demand schedule. Substituting demand for deposits, Equation (30), in Equation (39) gives the following maximization problem:

$$\max_{r_t^d(j)}\mathrm{E}_0\sum_{t=0}^{\infty}\Lambda_{0,t}^p\left[(r_t-r_t^d(j))\left(\frac{r_t^d(j)}{r_t^d}\right)^{\mu^d}D_t-\frac{\kappa_d}{2}\left(\frac{r_t^d(j)}{r_{t-1}^d(j)}-1\right)^2 r_t^d D_t\right]. \tag{A.54}$$

The solution, after rewriting in a similar fashion as in Equation (A.52), is:

$$\left(\frac{r_t^d(j)}{r_t^d}\right)^{\mu^d} D_t\left(-1+\mu^d-\mu^d\frac{r_t}{r_t^d(j)}\right)+\kappa_d\left[\left(\frac{1}{r_{t-1}^d(j)}-\frac{r_t^d(j)}{(r_{t-1}^d(j))^2}\right)r_t^d D_t\right.$$
$$\left.+\beta_t^H\,\mathrm{E}_t\left\{\left(\frac{r_{t+1}^d(j)}{r_t^d(j)}-1\right)\left(\frac{r_{t+1}^d(j)}{r_t^d(j)}\right)^2 D_{t+1}\right\}\right]=0. \tag{A.55}$$

Imposing again a symmetric equilibrium $r_t^d(j)\equiv r_t^d$, $\forall j$:

$$-1+\mu^d-\mu^d\frac{r_t}{r_t^d}+\kappa_d\left[\left(1-\frac{r_t^d}{r_{t-1}^d}\right)\frac{r_t^d}{r_{t-1}^d}+\beta_t^H\,\mathrm{E}_t\left\{\left(\frac{r_{t+1}^d}{r_t^d}-1\right)\left(\frac{r_{t+1}^d}{r_t^d}\right)^2\frac{D_{t+1}}{D_t}\right\}\right]=0. \tag{A.56}$$

Notice that if Equation (A.56) is log-linearized we obtain Equation (A.68).

A.3 The log-linear model

For the empirical analysis I linearize the model around the non-stochastic steady state. A hat denotes a log-linearized variable. The household consumption Euler (4) is linearized and represented by:

$$\hat{c}_t^H=\frac{h}{1+h}\hat{c}_{t-1}^H+\frac{1}{1+h}\mathrm{E}_t\{\hat{c}_{t+1}^H\}-\frac{1-h}{1+h}\left(\hat{r}_t^d-\mathrm{E}_t\{\hat{\pi}_{t+1}\}\right)+\frac{1-h}{1+h}(\hat{\eta}_t^c-\mathrm{E}_t\{\hat{\eta}_{t+1}^c\}). \tag{A.57}$$

This is the conventional forward-looking consumption equation with external habit formation, i.e., consumption today depends on past consumption and expected consumption.

Linearizing the wage setting equation (10) gives:

$$\begin{aligned}\hat{w}_t =& \frac{\beta^H}{1+\beta^H}\,\mathrm{E}_t\{\hat{w}_{t+1}\} + \frac{1}{1+\beta^H}\hat{w}_{t-1} + \frac{\beta^H}{1+\beta^H}\,\mathrm{E}_t\{\hat{\pi}_{t+1}\} - \frac{1+\beta^H\gamma_w}{1+\beta^H}\hat{\pi}_t + \frac{\gamma_w}{1+\beta^H}\hat{\pi}_{t-1} \\ &- \frac{1}{1+\beta^H}\frac{(1-\beta^H\varepsilon_w)(1-\varepsilon_w)}{\left(1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w}\right)\varepsilon_w} \times \left(\hat{w}_t - \sigma_l \hat{l}^H_t - \frac{\sigma_c}{1-h}(\hat{c}^E_t - \hat{c}^E_{t-1}) - \eta^l_t\right). \end{aligned} \tag{A.58}$$

The wage rate depends via partial indexation positively on the past wage rate and positively on the expected future wage rate via Calvo pricing; consequently, wages also depend on past, current and future inflation. Additionally, wages also depend positively on household consumption and labor demand via the household and firm optimization problem.

The entrepreneur consumption Euler (18) is given by:

$$\hat{c}^E_t = \frac{1}{1+h}\,\mathrm{E}_t\{\hat{c}^E_{t+1}\} + \frac{h}{1+h}\hat{c}^E_{t-1} - \frac{1-h}{1+h}(\hat{r}^b_t - \mathrm{E}_t\{\hat{\pi}_{t+1}\}) + \frac{1-h}{1+h}(\hat{\eta}^c_t - \mathrm{E}_t\{\hat{\eta}^c_{t+1}\}). \tag{A.59}$$

Consumption of the entrepreneur evolves in an equivalent fashion as consumption of the households; it only depends on past and and expected consumption. Entrepreneurial consumption depends, in contrast to households consumption, on the real lending rate rather than the real deposit rate. Moreover, the probability of default does not affect the consumption decision of the entrepreneur.

The investment equation (19), is linearized and represented by:

$$\hat{i}_t = \frac{1}{1+\beta^E}\hat{i}_{t-1} + \frac{\beta^E}{1+\beta^E}\,\mathrm{E}_t\{\hat{i}_{t+1}\} + \frac{1}{1+\beta^E}\frac{1}{\varphi}\hat{q}_t + \frac{1}{1+\beta^E}\hat{\eta}^i_t - \frac{\beta^E}{1+\beta^E}\,\mathrm{E}_t\{\hat{\eta}^i_{t+1}\}, \tag{A.60}$$

where $\varphi = \phi''$. Investment depends on past and expected investment via the capital adjustment costs function. Investment also depends on the capital price. Although, investment does not depend directly on the probability of default, it does via the price of capital. The capital pricing equation (21) is linearized and represented by:

$$\hat{q}_t = -(\hat{r}^b_t - \hat{\pi}_{t+1}) + \hat{q}_{t+1} + \frac{[1-\Gamma(\bar{\omega^*})]r^{k*}}{1-\delta^k+[1-\Gamma(\bar{\omega^*})]r^{k*}}\hat{r}^k_{t+1} - \frac{r^{k*}\Gamma'(\bar{\omega^*})\bar{\omega^*}}{1-\delta^k+[1-\Gamma(\bar{\omega^*})]r^{k*}}\hat{\bar{\omega}}_{t+1}, \tag{A.61}$$

where I used the steady state condition $\beta^E = (1-\delta^k+[1-\Gamma(\bar{\omega}^*)]r^{k*})^{-1}$. β^E is higher than the discount rate used in Smets and Wouters (2003) because firms need to pay banks a mark-up for the possibility of bankruptcy. As such their expected return is not r^k but $[1-\Gamma(\bar{\omega})]r^k$. So, correcting for the probability of default makes entrepreneurs more impatient. Moreover, the capital price not only depends on the real lending rate, the expected capital price and the return on capital, but also on the shift in the default threshold. If the default threshold increases, it is more difficult to borrow. Demand for capital will therefore fall and capital prices follow.

Linearizing the bank participation (17 threshold gives:

$$\frac{[\Gamma'(\bar{\omega})-\mu G'(\bar{\omega})]}{[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}\,\mathrm{E}_t\{\hat{\bar{\omega}}_{t+1}\}+\mathrm{E}_t\{\hat{r}^k_{t+1}\}+\hat{q}_t+\hat{k}_t=\hat{b}_t+\hat{r}^{wb}_t-\hat{\pi}_{t+1}, \tag{A.62}$$

where $\hat{\bar{\omega}}_{t+1}$ is a threshold below which entrepreneurs default on their loan repayment because the return on the investment project is insufficient to cover the loan.

The model of the production side of the economy is standard. The Cobb-Douglas production function is represented by:

$$\hat{y}=\hat{a}_t+\alpha\hat{k}_{t-1}+(1-\alpha)\hat{l}_t. \tag{A.63}$$

Labor demand is given by:

$$\hat{l}_t=-\hat{w}_t+\left(1+\frac{\psi'(1)}{\psi''(1)}\right)\hat{r}^k_t+\hat{k}_{t-1}, \tag{A.64}$$

and the inflation rate is given by:

$$\begin{aligned}\hat{\pi}_t=&\frac{\beta^E}{1+\beta^E\gamma_p}\,\mathrm{E}_t\{\hat{\pi}_{t+1}\}+\frac{\gamma_p}{1+\beta^E\gamma_p}\hat{\pi}_{t-1}+\\&\frac{1}{1+\beta^E\gamma_p}\frac{(1-\beta^E\varepsilon_p)(1-\varepsilon_p)}{\varepsilon_p}(\alpha\hat{r}^k_t+(1-\alpha)\hat{w}_t-\hat{\eta}^a_t+\hat{\eta}^p_t).\end{aligned} \tag{A.65}$$

The financial side of the economy is represented by the lending rate set by bank holding company:

$$\hat{r}^{wb}_t-\hat{r}_t=-\frac{\kappa_w\nu_w^3}{r}(\hat{k}^b_t+\hat{q}_t-\hat{b}_t). \tag{A.66}$$

Hence, the spread between the lending rate set by the bank holding company and the risk-free rate $\hat{r}^{wb}_t-\hat{r}_t$ can be either positive or negative depending on leverage. If, for example, leverage is low such that the capital-to-asset ratio is above the optimal capital-to-asset ratio ν_w, there is not enough banking capital $\hat{k}^b_t+\hat{q}_t$ to cover the outstanding loans $\hat{b}_t$. The interest spread will have to rise to decrease the amount of outstanding loans.

Log-linearizing the loan rate setting equation gives:

$$\hat{r}^b_t=\zeta^b_1\hat{r}^b_{t-1}+\zeta^b_2\,\mathrm{E}_t\{\hat{r}^b_{t+1}\}+\zeta^b_3\hat{r}^{wb}_t, \tag{A.67}$$

where $\zeta^b_1\equiv\frac{\kappa_b}{\mu_b-1+(1+\beta^H)\kappa_b}, \zeta^b_2\equiv\frac{\beta^H\kappa_b}{\mu_b-1+(1+\beta^H)\kappa_b}, \zeta^b_3\equiv\frac{\mu_b-1}{\mu_b-1+(1+\beta^H)\kappa_b}$. Equation (A.67) states that the loan rate depends on the loan rate in the previous period, the expected loan rate in the next period and the borrowing costs charged by the bank holding company. If the loan rate is perfectly flexible ($\kappa_b=0$), the maximization problem simplifies to $r^b_t=\frac{\mu_b}{\mu_b-1}r^{wb}_t$ and $\hat{r}^b_t=\hat{r}^{wb}_t$. Hence each branch simply sets the loan rate as a mark-up over its marginal costs. Note that default losses do not affect the optimization problem of the retail branch, but they do affect the rate set by the bank holding company, r^{wb}_t, bank holding companies charge their retail branches. Hence, an increase in default losses does affect r^b_t via an increase in r^{wb}_t.

The solution to the funding branch optimization problem is:

$$\hat{r}_t^d = \zeta_1^d \hat{r}_{t-1}^d + \zeta_2^d \,\mathrm{E}_t\{\hat{r}_{t+1}^d\} + \zeta_3^d \hat{r}_t, \tag{A.68}$$

where $\zeta_1^d \equiv \frac{\kappa_d}{\mu_d+1+(1+\beta^H)\kappa_d}$, $\zeta_2^d \equiv \frac{\beta^H\kappa_d}{\mu_d+1+(1+\beta^H)\kappa_d}$, and $\zeta_3^d \equiv \frac{\mu_d+1}{\mu_d+1+(1+\beta^H)\kappa_d}$. Similar to the lending branch, the deposit rate depends on the deposit rate in the previous period, the deposit rate in the coming period, and the lending rate offered by the bank holding company. If the deposit rate is perfectly flexible ($\kappa_d = 0$) the maximization problem simplifies to $r_t^d = \frac{\mu_d}{\mu_d - 1} r_t$ and $\hat{r}_t^d = \hat{r}_t$.

The Central Bank stabilizes the economy via a simple Taylor rule:

$$\hat{r}_t = \delta_r \hat{r}_{t-1} + (1-\delta_r)\delta_\pi(\hat{\pi}_t + \mathrm{E}_t\{\hat{\pi}_{t+1}\}) + (1-\delta_y)\delta_r(\hat{y}_t - \hat{y}_{t-1}) + \eta_t^m \tag{A.69}$$

All that rests is the evolution of the state variables and the goods market equilibrium that closes the model. The capital accumulation identities are:

$$\hat{k}_t = \delta^k \hat{i}_t + (1-\delta^k)\hat{k}_{t-1}, \tag{A.70}$$

$$\hat{k}_t^b = \hat{k}_{t-1}^b - \frac{\delta^b B}{K^b}\hat{b}_{t-1} + \frac{J^b}{K^b}\hat{j}_t^b + \frac{T^*}{K^b}\hat{t}_t, \tag{A.71}$$

where in steady state $\frac{\delta^b B}{K^b} = \frac{J^b}{K^b} = \frac{\delta^b}{\nu_w}$ and I set $\frac{T^*}{K^b} = \frac{\delta^b}{\nu_w}$, i.e, the bank recapitalization is added to bank profits.

As the changes in deposits and bonds are important for leverage in the banking sector, these variables are modeled explicitly via the budget identities of the households and the entrepreneurs:

$$\hat{d}_t = (1+r^d)\hat{d}_{t-1} + \hat{r}_{t-1}^d - \hat{\pi}_t + \frac{w^* l^{H*}}{D^*}(\hat{w}_t + \hat{L}_t^H - \hat{t}_t) + \frac{RP^*}{D^*}\hat{rp}_t - \frac{C^{H*}}{D^*}\hat{c}_t^H, \tag{A.72}$$

$$\hat{b}_t = -\frac{r^{k*}K^*}{B^*}(-\Gamma(\bar{\omega})\hat{\bar{\omega}}_{t+1} + (1-\Gamma(\bar{\omega})(\hat{r}_{t+1}^k + \hat{q}_t + \hat{K}_t)) - \frac{w^E L^E}{B}(\hat{w}_t^{E*} + \hat{l}_t^{E*}) +$$

$$(1+r^b)\hat{b}_{t-1} + \hat{r}_{t-1}^b + \hat{\pi}_t + \frac{w^* L^*}{B^*}(\hat{w}_t + \hat{l}_t) + \frac{I^*}{B^*}\hat{i}_t + \frac{C^{E*}}{B^*}\hat{c}_t^E \tag{A.73}$$

where lower case letters with a star ($*$) denote steady state values. As T^* is in steady state equal to zero, I simply subtract the lump-sum tax from labor income. Bank profits and anticipated losses are represented by:

$$\hat{j}_t^b = \frac{r^{b*}b^*}{j^{b*}}(\hat{b}_t + \hat{r}_t^b) - \frac{r^{d*}d^*}{j^{b*}}(\hat{d}_t + \hat{r}_t^d) - \frac{v^*}{j^{b*}}(\hat{v}_t + \mathrm{E}_t\{\hat{v}_{t+1}\} - \mathrm{E}_{t-1}\{\hat{v}_t\}), \tag{A.74}$$

$$\hat{v}_t = \xi^{v*}\hat{b}_t + (1-\xi^{v*})(\hat{r}_{t+1}^k + \hat{q}_t + \hat{k}_t) - \xi^{v*}\left(\frac{f(\bar{\omega}^*)\omega^*}{2} + F(\bar{\omega}^*)\right)\hat{\bar{\omega}}_t + \eta_t^v, \tag{A.75}$$

where $\xi^{v*} \equiv \frac{F(\bar{\omega}^*)b^*}{F(\bar{\omega}^*)b^* - (1-\mu)\int_0^{\bar{\omega}^*}\omega^* f(\omega^*)d\omega^* r^{k*}k^*} = \frac{F(\bar{\omega}^*)b^*}{V^*} \approx \frac{1}{\mu}$. The model is closed by the goods market equilibrium condition:

$$\hat{y}_t = \left(1 - \delta^k \frac{k^*}{y^*}\right)\hat{c}_t + \delta^k \frac{k^*}{y^*}\hat{i}_t. \tag{A.76}$$

Linearization of the countercyclical capital buffers yields:

$$\hat{m}_t^y = \varrho_y \left(\hat{b}_t - \hat{y}_t\right), \tag{A.77}$$

$$\hat{m}_t^{k^b} = \varrho_{k^b} \left(\hat{k}_t^b - \hat{b}_t\right), \tag{A.78}$$

and once activated the lending rate set by the bank holding company is determined by the whole sale bank as:

$$\hat{r}_t^{wb} - \hat{r}_t = -\frac{\kappa_w \nu_w^3}{r}(\hat{q}_t + \hat{k}_t^b - \hat{b}_t - \hat{m}_t^y), \tag{A.79}$$

or

$$\hat{r}_t^{wb} - \hat{r}_t = -\frac{\kappa_w \nu_w^3}{r}(\hat{q} + \hat{k}_t^b - \hat{b}_t - \hat{m}_t^{k^b}), \tag{A.80}$$

depending on which activation rule the Central Bank applies.

B Appendix: variables

Consumption: Final consumption aggregates - Current prices seasonally adjusted and adjusted data by working day in millions of euros. Source: Eurostat.

Output: GDP and main components - Current prices seasonally adjusted and adjusted data by working day in millions of euros. Source: Eurostat.

Investment: Gross fixed capital formation - Current prices seasonally adjusted and adjusted data by working day in millions of euros. Source: Eurostat.

Wages: Gross wages and salaries seasonally adjusted and adjusted data by working day in millions of euros. Source: Eurostat.

Inflation: HICP (2005=100) monthly data. Eurostat.

Nominal policy rate: Money market interest rates - monthly data (day-to-day) Eurostat.

Outstanding loans to firms: Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Loans, Total maturity, All currencies combined - Euro area (changing composition) counterpart, Non-Financial corporations (S.11) sector, denominated in Euro, data Neither seasonally nor working day adjusted. ECB.

Outstanding deposits to households: Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Overnight deposits, Total maturity, All currencies combined - Euro area (changing composition) counterpart, Households and non-profit institutions serving households (S.14 &

S.15) sector, denominated in Euro, data Neither seasonally nor working day adjusted. ECB.

Nominal interest rate on loans: Euro area (changing composition), Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks) reporting sector - Loans other than revolving loans and overdrafts, convenience and extended credit card debt [A20-A2Z], Over 1 and up to 5 years initial rate fixation, Over 1 and up to 5 years, Over EUR 1 million amount, New business coverage, Non-Financial corporations (S.11) sector, Euro. ECB.

Nominal interest on deposits: Euro area (changing composition), Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks) reporting sector - Deposits with agreed maturity, Up to two years original maturity, Up to two years, New business coverage, Households and non-profit institutions serving households (S.14 and S.15) sector, Euro.

Credit risk: the credit risk indicator is constructed by Gilchrist and Mojon (2014).

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De Nederlandsche Bank N.V.
Postbus 98, 1000 AB Amsterdam
020 524 91 11
dnb.nl