Technological Change and the Evolution of Finance

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Abstract

We explore how asset prices evolve under technological progress. Technological change leads to human capital substituting physical capital and manual labor. As a result, firms have less tangible collateral to pledge for external finance, while the creation of intangible capital by skilled labor requires inside equity. The resulting lower demand for external finance by firms leads to excess savings over investment and to a decline in interest rates.

Tangible assets such as houses can also serve as collateral, so over time excess savings are redirected to mortgage credit. Under fixed land supply, house prices rise in real terms. In combination with growing wage inequality, this leads to high leverage for low-skill workers, increasing default rates and foreclosures. Restraining mortgage borrowing contains leverage and thus house price appreciation. Its general equilibrium effect is to lower interest rates, which spurs more investment and higher wages.

Keywords. Skill-biased technological change, mortgage credit, human capital, housing, inequality

JEL classifications. D33, E22, E44, R21

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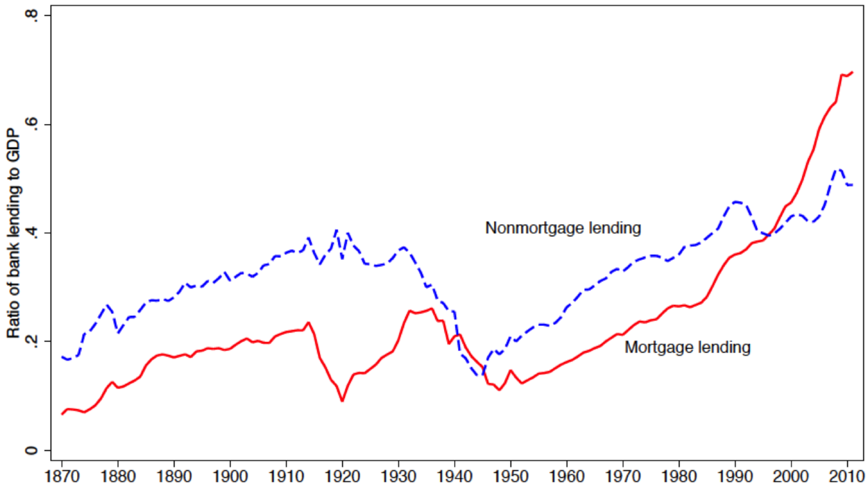

Figure 1: The Great Mortgaging. Source: Jorda et al. (2014)

1. Introduction

Interest rates have fallen in the last decades, while house prices have appreciated over time. At the same time, real estate credit has grown dramatically, recently surpassing business credit volume (Jorda et al., 2014, see figure 1). This credit expansion, dubbed the Great Mortgaging, funded overwhelmingly house purchases rather than new construction. In desirable locations, limited land and building restrictions limit the supply of housing. We suggest these trends may be interpreted in the light of the broad process of technological change towards a knowledge based economy. Information technology increases the productivity of intangible capital, substituting for physical capital and labor.

Progress in information technology since the 1980s has boosted the productivity of high-skill workers while largely substituting low-skill labor, increasing wage inequality (e.g. see Katz and Murphy, 1992; Autor et al., 1998, 2008; Acemoglu and Autor, 2011). At the same time technological change induces firms to use more intangible capital.[1] Corrado and Hulten (2010a) document how US corporations have been investing more in intangibles over time (see

[1] Falato et al. (2013) define intangibles as knowledge capital, organizational capability and computerized information and software, while Lev (2000) classifies intangibles as innovation-related, human and organizational capital.

Table A1.1 Nonfarm Business Fixed Investment rates[1]

	1948-2007 (1)	1948-1972 (2)	1973-1994 (3)	1995-2007 (4)
1. Tangible	11.4	11.2	12.3	10.4
1a. ICT equipment	1.3	.6	1.6	2.0
1b. Non-ICT equipment	5.9	5.9	6.2	5.4
1c. Nonresidential structures[2]	3.2	3.2	3.5	2.6
1d. Residential capital	1.1	1.3	1.1	.8
2. Intangible	8.6	5.9	9.2	12.8
2a. Computerized information[3]	.6	.1	.6	1.6
2b. Innovative property	3.2	1.9	3.7	4.8
(1) R&D (NSF/BEA)	1.4	.9	1.6	2.1
(2) Other R&D, etc.[4]	1.2	.5	1.3	2.2
(3) Mineral exploration	.6	.5	.7	.5
2c. Economic competencies	4.8	3.9	4.9	6.4
(1) Brand equity	1.6	1.6	1.5	1.8
(2) Firm-specific resources	3.2	2.3	3.4	4.6

Table 1: Evolution of intangible investment, from Corrado and Hulten (2010a)

table 1). We highlight here that this has implications for financial intermediation. Intangible capital cannot be easily pledged as collateral to investors, as it depreciates faster and is less appropriable by outsiders.[2] As a result, less external finance is used to fund production. Recent evidence relates the rising use of intangibles to falling corporate leverage (Bates et al., 2009; Falato et al., 2013).[3]. A falling business credit demand may also explain the steady decline of interest rates since the 1980s.

We introduce technological progress in an overlapping generations setup where housing has both consumption and storage value. Households save to consume in old age, by buying a house when young and reselling it when old, or by investing in financial claims backed by

[2]While public knowledge is by nature non excludable, the return to worker skills cannot be promised for moral hazard reasons. The only category of intangible capital that may be appropriated are patents and exploitation rights.

[3]The trend is related to a decline in the relative price of certain investment goods, highlighted by Eichengreen (2015) as the key cause of the growing excess of saving over investment.

collateral (either physical capital or houses). Buying a house offers some utility value[4] plus any price appreciation. Mortgage credit enables poorer households to buy a house, competing for net savings with business credit.

Production requires four inputs: physical capital, complementary with manual labor, and intangible capital, complementary with high-skill labor. Physical capital is produced by firms, while intangible capital is produced by some skilled workers with entrepreneurial talent (akin to the innovative producers of intermediate goods in endogenous growth models (Romer, 1986)).

Technological change alters the relative productivity of factors, leading to rising wage inequality over time. The shift from physical to intangible capital reduces credit demand of traditional firms, while intangible capital accumulation need to be supported by inside equity. As a result a lower fraction of production is financed externally, reducing the supply of savings vehicles. Over time, excess savings over physical investment are redirected to mortgage credit, secured by houses. The combination of lower rates and more credit boost house prices, as land is in fixed supply.[5]

The result of technological change is a financial system that overwhelmingly finances the purchase of real estate. Since land absorbs savings that could otherwise be invested productively we study policy options that limit mortgage credit, such as a limit to loan-to-value (LTV) ratios. Constraining mortgage credit has benefits and costs. On the downside, as low skill workers become credit constrained, the stock of housing is allocated less efficiently. On the upside, released savings are redirected towards corporate credit. This reduces the borrowing cost for firms and therefore leads to more physical investment, with a positive effect on wages. As a result, long-run consumption is maximized. Interestingly, the general equilibrium effect tends to benefit most those for whom the borrowing constraint becomes binding, namely young low-skill workers.

Trading off these costs and benefits we show that a limit to mortgage credit can improve

[4]This inherent consumption value means that houses are not a pure bubble as fiat money in Samuelson (1958)

[5]As housing has direct consumption utility, it maintains its value even with technological progress.

the utility of all agents in steady state. However, this is not a strict Pareto improvement, as the current generation of home owners loses out because houses drop in price. The problem is that as the borrowing limit is introduced the current generation of home owners loses out from dropping land prices. It is impossible to compensate this generation because the benefits of higher productive investment only accrue in the future.[6] Thus, even though in the long run society may be better off it is impossible to dynamically make every generation better off.

A stronger welfare result obtains when introducing a financial stability dimension to the model. We show that increasing leverage results in higher default rates. Since mortgage defaults arguably have significant welfare cost, an LTV limit improves financial stability. If the deadweight losses from default are sufficiently large we obtain dynamic inefficiency, and show that it is possible to implement an inter-generational transfer scheme that makes each and generation better off such that a borrowing limit can yield a strict Pareto improvement.

Mortgage subsidies, commonly practiced in many countries, lead to quite a different result. Since the subsidy is paid by all taxpayers, it is a transfer from rich lenders to poor borrowers and may therefore reduce defaults. However, as it encourages more mortgage credit it increases house prices and household leverage. The general equilibrium effect is exactly the opposite of an LTV limit, namely a reduction in physical investment and labor wages. Overall, poorer workers may therefore well be worse off despite receiving the subsidy. The difference between the limit and the subsidy is driven by the fact that the supply of land is fixed, so reducing demand is more effective at controlling prices than subsidizing transactions.

Finally, we study three extensions to the model. While in the main text we assume that the return to intangibles are not appropriable at all, in a first extension we analyze the case when returns are partially appropriable. Then intangible investments can support external financing in the form of outside equity. In line with the growth of US equity markets technological

[6]A standard result in the OLG literature is that welfare improvements are only possible in a dynamically inefficient economy. At present, LTV limits are rare, perhaps because current home owners would always resist them.

progress increases share prices as interest rates fall and intangible capital becomes more productive.

Second we analyze the role of government debt as an additional savings vehicle. As public debt diverts net savings away from housing and investment it results in lower wages and house prices. A potential benefit in the long-run equilibrium is that lower house prices result in less mortgage defaults.

In contrast, capital inflows from abroad add to net savings, increasing house prices and investment. This result is in line with the global savings glut view expressed by Bernanke (2005), that capital inflows in particular from Asian economies have been contributing to the growth in US house prices.

The model results need to be qualified by its simplifying assumptions. In the model agents can only consume when they are old, thus the overall supply of savings is not affected by interest rates.[7] The supply of skill is exogenous, ensuring a steady increase in skill rents. Endogenizing education would dampen the effect on relative wages and inequality, but not the overall trend towards less credit demand by firms. We assume that land supply is fixed, as it is the case for an urban location where housing supply is constrained. Higher prices may lead to more dense housing, but there are countervailing factors such as population growth and urban zoning limits.[8] While a more extensive model would incorporate a richer set of effects, our stylized approach enables to combine first order effects of technological change on labor and funding, and their implications on credit allocation, house prices and financial stability.

1.1. Related literature

Our approach relates to several established empirical trends. First, the secular rise of mortgage credit and house prices depicted in figure 1 (Turner, 2014b,a; Reinhart and Rogoff, 2013;

[7] We take here some comfort from the observed "savings glut" discussed by Bernanke (2005).

[8] A recent survey in the Economist (2015) magazine stressed the explosive increase in urban real estate prices due to strict zoning rules.

Schularick and Taylor, 2012; Jorda et al., 2014). In line with the model Mian and Sufi (2009) show how the growth in mortgage credit before the crisis was concentrated among low-income households and how an increasing supply of local credit drove up house prices.

Second, the increasing use of intangible inputs (Corrado and Hulten, 2010b), and its relation to lower corporate leverage (Bates et al., 2009). Different studies have shown that the fall in net leverage can be explained by the increasing use of intangibles (Falato et al., 2013; Giambona et al., 2014), R&D activity (Bates et al., 2009), and is driven by innovative technology-based firms (Hyytinen and Pajarinen, 2005; Hogan and Hutson, 2005).

Third, the rise in wage inequality, driven by the rising demand for educated workers (Katz and Murphy, 1992; Autor et al., 2008; Acemoglu and Autor, 2011) who benefit from computerization, substituting more simple labor engaged in routine tasks (Autor et al., 1998; Akerman et al., 2015).

We adopt the OLG approach developed around Samuelson (1958), Diamond (1965) and Tirole (1985) to describe intertemporal saving behavior. Here land is a durable good that can fulfill a storage of value role (as fiat money in Samuelson (1958)), while contributing to agents' utility as a consumption good.

The two papers most related to ours are Thwaites (2014) and Giglio and Severo (2012). Using a similar OLG model Thwaites (2014) explains low interest rates and high household debt by the fall in the relative price of investment goods. The channel is that when a given level of capital investments requires less upfront investment slack savings are redirected to housing. Relative to Thwaites (2014) the exogenous driver in our model is technological change that affects the relative productivity of intangible vs tangible capital, and additionally has implications for wage inequality.

Giglio and Severo (2012) study rational bubbles in the context of an OLG model with tangible and intangible investments. As in our model, intangible capital cannot support outside finance. Therefore, a technological shift from physical to intangible capital results in excess savings and low interest rates that create an environment for rational asset bubbles. In contrast to Giglio and Severo (2012) land is not a pure bubble in our model. Additionally, by highlighting the impact of technological change on wage inequality we are able to study the evolution of mortgage relative to corporate credit.

As most likely cause of excess savings, Eichengreen (2015) sees the fall in the relative price of certain investment goods. Our general equilibrium approach endogenizes these trends, though in a restated version emphasizing the drop in productivity of tangible capital rather than its cost, which in our model is endogenous.

Our analysis of mortgage defaults complements the political economy narrative of Rajan (2010). In Rajan (2010) the rise of mortgage credit is explained by politicians reacting to increasing inequality, catering low income constituencies. Our dynamic model with endogenous inequality enables to assess policy choices aimed at broadening house ownership, and clearly indicate that restraining leverage is better than subsidizing mortgage credit. The intuition is that because of land fixed supply, subsidizing demand is not efficient, as it only increases price competition and leverage.

In our welfare results we obtain a strict Pareto-improvement only if mortgage defaults have significant welfare cost. This notion is supported by several empirical studies. Jordà et al. (2015) find that the growth of mortgage lending has led to property bubbles and financial instability. Property price busts tend to follow credit booms, are long lasting and result in large output losses (for example, see IMF, 2003, 2009; Claessens et al., 2009). The trigger of the recent crisis was default among highly levered, low income households.

The rest of the paper is organized as follows. Section 2 derives the basic model and defines the equilibrium. Section 3 develops our main narrative, connecting technological change to mortgage finance. Section 4 introduces default to the model and analyzes different policy measure. Finally, section 6 concludes.

2. The model

This section describes the baseline model and equilibrium. We start by outlining the model environment.

Time Overlapping generations live for two periods. Generation t is young at time t and old at $t+1$. Time starts at $t=0$ and goes on to infinity. There is an initial generation $t=-1$.

Goods There are two consumption goods, corn and land.[9] There is a fixed amount of land $\bar{L}$ that is infinitely durable and does not depreciate. The relative price of land in terms of corn at period t is denoted by p_t.

Agents Each generation consists of a unit mass of households indexed by i. Household i of generation t has utility function $U(c_{t+1}^i, L_t^i) = c_{t+1}^i + v(L_t^i)$, where c_{t+1}^i denotes i's consumption of corn and L_t^i its land holdings at the end of period t. The function $v(L)$ with $v'(L) > 0$, $v''(L) < 0$ captures the utility households achieve from living in their house. A fraction ϕ of households is born high skill ($i = h$) and the rest is low-skill ($i = l$). Each low-skill worker is endowed with $\tilde{l}$ units of manual labor, while a high skill worker has $\tilde{h}$ units of high-skill labor, which we will refer to as human capital. Thus, the distribution of skills is given exogenously. Both types of labor endowments are supplied inelastically to firms and the initial old generation is endowed with all the land $\bar{L}$.

Representative firm There is an infinitely lived representative firm set up at the initial period with a mandate for value maximization. It has access to a nested CES production technology that takes as inputs physical capital K_t, manual labor l_t, intangible capital H_t and high-skill labor h_t:

$$Y_t = \left[\eta_t (H_t^{\alpha} h_t^{1-\alpha})^{\rho} + (1-\eta_t)(K_t^{\alpha} l_t^{1-\alpha})^{\rho}\right]^{\frac{1}{\rho}}. \tag{1}$$

Technological evolution is driven by changes in η_t, associated with the relative productivity of intangible capital. The firm can invest $I_{K,t}$ units of corn at t to install $K_{t+1} = I_{K,t}$ units of physical capital that can be used in production at $t+1$, producing $K_{t+1} + Y_{t+1}$ units of corn. In contrast, intangible capital is created by some high skill workers with entrepreneurial

[9] We do not distinguish between houses and the underlying land and will use the two terms interchangeably.

talent using their human capital. Both types of capital fully depreciate after production, and the firm starts with an initial stock (K_0, I_0).

Innovative entrepreneurs A fraction ε of high-skill agents have some entrepreneurial talent to create intangible capital. When supplying their labor endowment to firms they simultaneously create $H^i_{t+1} = \beta h^i_t$ of intangible capital for firms.

We follow Hart and Moore (1994) in assuming that intangible capital is inherently linked to the human capital of its creator. Therefore, intangibles are only productive if entrepreneurs do not withdraw their human capital at $t + 1$. Crucially, human capital is not alienable and therefore no contract can be written that commits the human capital to stay inside the firm. Finally, we assume that when withdrawing their human capital from a firm, entrepreneurs can employ a fraction γ of the intangibles they created in another firm. This gives them a credible threat that makes intangibles inappropriable for outside investors. For the main part of the text we focus on the case $\gamma = 1$, s.t. the returns to intangibles are fully captured by entrepreneurs. In section 5 we analyze the case $\gamma < 1$, where some returns can be captured by outside equity.

The creation of intangible capital in our model can be interpreted in two ways. One can either think of some key employees with entrepreneurial talent, creating intangibles inside existing firms. In an alternative interpretation entrepreneurs start a new innovative firm that produces intangible capital outside existing firms. We leave it to the reader to pick the favorite interpretation and refer to the agents creating intangible capital simply as (innovative) entrepreneurs.

Financial contracts Borrowing is only feasible if backed by land or by physical capital.[10] Lending may take place as mortgages backed by land and corporate debt backed by physical capital. There is no credit risk, so both types of debt earn the same market rate r_t.

[10] We ignore here that land may be a superior form of collateral than productive capital. In general, capital is movable while land is not. Moreover, land may be more redeployable.

2.1. Households

Households supply their labor endowment inelastically to the representative firm, generating income $y_t^i \in \{w_t\tilde{l}, q_t\tilde{h}\}$ when young, where w_t denote wages for manual workers and q_t wages of high-skill workers. Households without entrepreneurial talent have no income when old, $y_{t+1}^i = 0$, while high-skill workers with entrepreneurial talent may generate some additional income from the intangible capital they create, $y_{t+1}^i = R_{t+1}H_{t+1}$, where R_t denotes the return on intangible capital. Households only enjoy consumption at $t+1$, so they save all income for retirement. They can buy a house and sell it to the next generation when they are old, earning some utility plus a possible price appreciation. They can also invest in the financial market, lending to firms or to households via mortgages. Let s_t^i denote net financial savings. We will refer to households with $s_t^i \geq 0$ as lenders and those with $s_t^i < 0$ as borrowers.

The maximization problem of a household is given in (2), taking prices p_t and the market rate r_t as given:

$$\begin{aligned} \max_{c_{t+1}^i, L_t^i, s_t^i} \; & U(c_{t+1}^i, L_t^i) = c_{t+1}^i + v(L_t^i) \\ s.t. \quad & s_t^i \leq y_t^i - p_t L_t^i \\ & c_{t+1}^i \leq y_{t+1}^i + p_{t+1}L_t^i + (1+r_{t+1})s_t^i \\ & s_t^i \geq -\bar{s} \\ & c_{t+1}^i, L_t^i \geq 0 \end{aligned} \tag{2}$$

The first two budget constraints for young and old respectively are binding, else households could increase consumption and thus utility at no cost. The third constraint limits an agent's mortgage borrowing to $\bar{s}$. For now we assume that it is not binding, and that borrowers can fully collateralize the future value of their house, i.e. $\bar{s} = \frac{p_{t+1}L_t^i}{(1+r_{t+1})}$.[11] In section 4 we relax this assumption and solve the model with a general $\bar{s}$ at the corner solution where the borrowing constraint is binding.

As the budget constraints are binding, households choose the optimal amount of housing

[11]This also implies that here the borrowing constraint is equivalent to the non-negativity constraint $c_{t+1}^i \geq 0$

demand by the first order condition

$$\frac{(p_{t+1} - p_t) + v'(L_t^i)}{p_t} = r_{t+1}.$$

This has a simple interpretation. Households allocate their savings by weighing the potential price appreciation plus the utility from their house (LHS) against the mortgage interest (RHS, for a borrower), or alternatively, against the opportunity cost of investing in the financial markets (RHS, for a lender). The optimal demand for land follows readily:

$$L_t^* = v'^{-1}\left((1 + r_t)p_t - p_{t+1}\right), \tag{3}$$

which is independent of the worker's type. Thus, low-skill and high-skill workers have the same demand for housing.[12] The role of mortgage credit is to allocate land efficiently to equalize across agents the marginal utility from housing. Financial savings now follow as a residual $s_t^i = y_t^i - p_t L_t^*$. Households with $y_t^i \geq p_t L_t^*$ have an income high enough to buy their house and invest the remainder in the financial market. In contrast, those with $y_t^i < p_t L_t^*$ take out a mortgage to buy L_t^*.

As land has both a consumption and a storage use, mortgage credit serves to separate these two uses: while high income agents need a lot of storage, it would be inefficient if they held a lot of land directly. Instead agents with low incomes sell claims on their houses, providing the necessary storage through the financial market while still enjoying the consumption value of the house.

2.2. The firm's optimization

The representative firm maximizes profits employing the four input factors physical capital K_t, intangible capital H_t, low-skill labor l_t and high-skill labor h_t.

Under perfect competition, workers are compensated according to their marginal productivity $w_t = Y_{l,t}$ and $q_t = Y_{h,t}$. Since physical capital is fully eligible as collateral, the firm can always finance any tangible investments externally. As long as the return on physical capital

[12] This is particular to the case where the borrowing constraint remains slack

exceeds the interest rate paid on business loans, the firm scales up tangible investment until:

$$r_t = Y_{K,t}. \tag{4}$$

Consequently, the firm refinances investment each period, so the total demand for corporate debt is $d_t = I_{K,t} = K_{t+1}$.

Finally, because entrepreneurs have the credible threat to withdraw their human capital and moving the intangibles they created to another firm, the value created by intangibles is fully captured by its creator. Thus the return to intangibles is equal to its marginal productivity, $R_t = Y_{H,t}$.

While intangibles are an investment in that they create *future* returns, these returns are not appropriable. As a consequence they can only be captured by inside equity. This funding structure implies that as intangible capital plays an increasing role in production, overall a smaller fraction of final output is financed externally and relatively less savings are intermediated towards physical capital via corporate debt. As we will show, the excess savings are then channeled to housing, the only other tangible savings vehicle.

2.3. Equilibrium

Market clearing in the land market requires that $\int_0^1 L_t^i di = \bar{L}$, while in the financial market it requires $\int_0^1 s_t^i di = d_t$. Using (3) leads to the land market equilibrium:

$$p_t = \frac{v'(\bar{L}) + p_{t+1}}{1 + r_t}. \tag{5}$$

Plugging this price in (3) it follows that $L_t^i = L_t^* = \bar{L}$ for all i. As demand for housing does not depend on wealth, all agents consume the same amount. Similarly the financial market clearing condition can be written as

$$(1 - \alpha)Y_t = p_t\bar{L} + K_{t+1}, \tag{6}$$

where the LHS is the savings supply (the total income earned by workers saved for retirement), while the RHS are tangible assets that can carry savings over time. Intuitively, savings are intermediated to either land, using mortgages, or to tangible investments, using corporate debt.

The core of the model is described by (1), (4), (5) and (6).

3. Technological change

In this section we study how technological change affects the evolution of interest rates, asset prices and mortgage finance. In line with the literature, we view technological change as knowledge-driven innovation that disproportionately benefits skilled agents. More precisely, in the model we define technological progress as follows:

Definition 1. *Technological progress is the non-stochastic evolution of knowledge* η_t *in a parameter range where it increases steady-state output, i.e.*

$$\frac{\partial Y}{\partial \eta} > 0 \tag{H1}$$

We first derive some comparative statics on η in steady state. Then we calibrate the model to match the evolution of intangible-tangible investment ratios from the 1980s to the 2000s, as documented by Corrado et al. (2009) and simulate the transition path generated by the model.

For the sake of tractability, for our analytical results we focus on the Cobb-Douglas case where $\rho \to 0$ in (1). Additionally, we make one more parameter assumption:[13]

Assumption 1. *High-skill labor is relatively scarce:*

$$\frac{\phi(1-\varepsilon)}{(1-\phi)} < \frac{\eta_t}{1-\eta_t}$$

This assumption ensures that at any point in time high-skill workers have a higher income than low-skill workers. While highly plausible, it may become less obvious in a setup with endogenous education choice.

[13]Earlier we restricted attention to the case where the borrowing constraint in (2) is not binding. We will relax this assumption in the next section.

3.1. Steady state comparative statics

We now derive some comparative static results on the effect of changing η on the steady state equilibrium. First we show that technological progress reduces steady state interest rates and drives up house prices, and then analyze under what conditions it changes the composition of credit towards mortgage finance.

First, note that in the Cobb-Douglas formulation wage inequality is given by a simple expression

$$\frac{q}{w} = \frac{\eta}{1-\eta}\frac{l}{h} \tag{7}$$

where $l \equiv (1-\phi)\tilde{l}$ and $h = \phi\tilde{h}$ are the aggregate supply of low and high skill labor respectively. Because of its direct effect on the relative productivity of factors in (1) technological change directly increases wage inequality - a result well known in the literature. A second and more novel result is given in the following proposition:

Proposition 1. *Technological progress reduces steady-state interest rates:* $\frac{dr}{d\eta} < 0$

A proof can be found in appendix A. As intangibles are not appropriable by outside investors, a shift from physical capital to intangibles results in a falling demand for corporate debt. The freed savings then put downward pressure on interest rates.

Low rates have an impact on asset prices. Evaluating (5) in steady state the land price takes the following simple form:

$$p = \frac{v'(\bar{L})}{r}. \tag{8}$$

The numerator reflects the consumption value of land, which is independent of income and therefore of η. The denominator reflects its storage value. As higher η leads to falling interest rates, the value of land as storage increases. Put differently, it has to be higher to absorb slack savings. Thus, p is decreasing in r.

To understand this effect it is instructive to look at the financial market clearing condition (6), here evaluated in steady state:

$$(1-\alpha)Y = p\bar{L} + K$$

Clearly, as the relative use of physical capital $\frac{K}{Y}$ decreases, savings have to be absorbed by land implying that $\frac{p}{Y}$ has to rise.

Technological progress affects the composition of credit, resulting in a shift from financing production to financing land purchases. The following result formalizes this notion:

Proposition 2. *If and only if*

$$\frac{dY/d\eta}{Y} \leq \frac{1}{1-\eta}\left[1 + \frac{(1-\alpha)^2 r}{\alpha(1-\phi)(p\bar{L}/Y)}\right] \tag{H2}$$

technological progress increases steady-state mortgage credit to GDP: $\frac{d(m/Y)}{d\eta} > 0$

To gain some intuition first note that for the financial market to clear high-sill workers must have positive net financial savings, $s_t^h \geq 0$. Thus, an increase in the aggregate demand for mortgage credit must be coming from low-skill workers, who are affected by technological progress in the following way. On the one hand, low rates, high house prices and rising income inequality boost their demand for mortgage credit. On the other hand, to the extent that progress increases overall output it benefits all factors, including low-skill workers. Under condition (H2) growth is not too strong and the first effect dominates, resulting in rising leverage.

To summarize, technological progress results in output growth, falling interest rates, increasing house prices and income inequality. As long as these effects are not dominated by overall growth, mortgage credit to GDP increases as a result. Thus, our model suggests that the fall of interest rates and rise of mortgage credit since the 1980s may be interpreted in the light of technological change. The next section illustrates this result in a calibrated example.

3.2. Simulated time path of technological change

We turn now to assess how well this setup explains the evolution of the main endogenous variables by simulating its dynamic version. We calibrate the start and end point of our simulation by assuming that an initial steady state with a low η was in place in advanced economies before the IT revolution started in the 1980s, using contemporary measures of the

Period	1950 - 59	1960 - 69	1970 - 79	1980 - 89	1990 - 99	2000 - 03
Intangible/tangible ratio	0.54	0.62	0.60	0.82	1.10	1.36

Table 2: Intangible-tangible investment ratios, annual averages (Corrado et al. (2009))

scale of intangible capital. The end point is based on the assumption of a final steady state with a high η, based on the most recent measures of intangible investment. We then simulate a transition time path from a low-η to a high-η steady state to dynamically illustrate the results from the comparative statics.

Corrado et al. (2009) measure intangible investment ratios for a growth accounting description for the US economy. The authors offer a series of estimates for the intangible-tangible investment ratio, replicated in table 2. The relative investment in intangibles appears to have grown over time from roughly 0.6 before the 1980s to 1.36 in the early 2000s. We simulate a transition path from a low-η to a high-η steady state, calibrated on these ratios and on the relevant US data in the early 1980s.

Since our model is deliberately kept simple we do not make quantitative statements. Rather, we illustrate the qualitative mechanism of the model in a reasonable parameter range. Details on the calibration can be found in appendix C.

The simulated time path is presented in figure 2. In the simulation η_t linearly rises over 10 periods, after which the system reaches a new steady state in a few periods. While technological change is associated with GDP growth (upper left panel), it does not benefit all factors equally, so wage inequality rises (upper right panel). Over time investment in physical capital and interest rates decrease (middle left panel), while a lower share of production is funded externally (middle right panel). To absorb slack savings, house prices rise. Finally, in this calibration condition (H2) in proposition 2 holds, and leverage as well as mortgage credit as a percentage of GDP rise, while the share of business credit falls (lower panels).

In conclusion, our model shows how the growth of mortgage credit is driven by increasing income inequality, low interest rates and higher house prices, all independently induced by

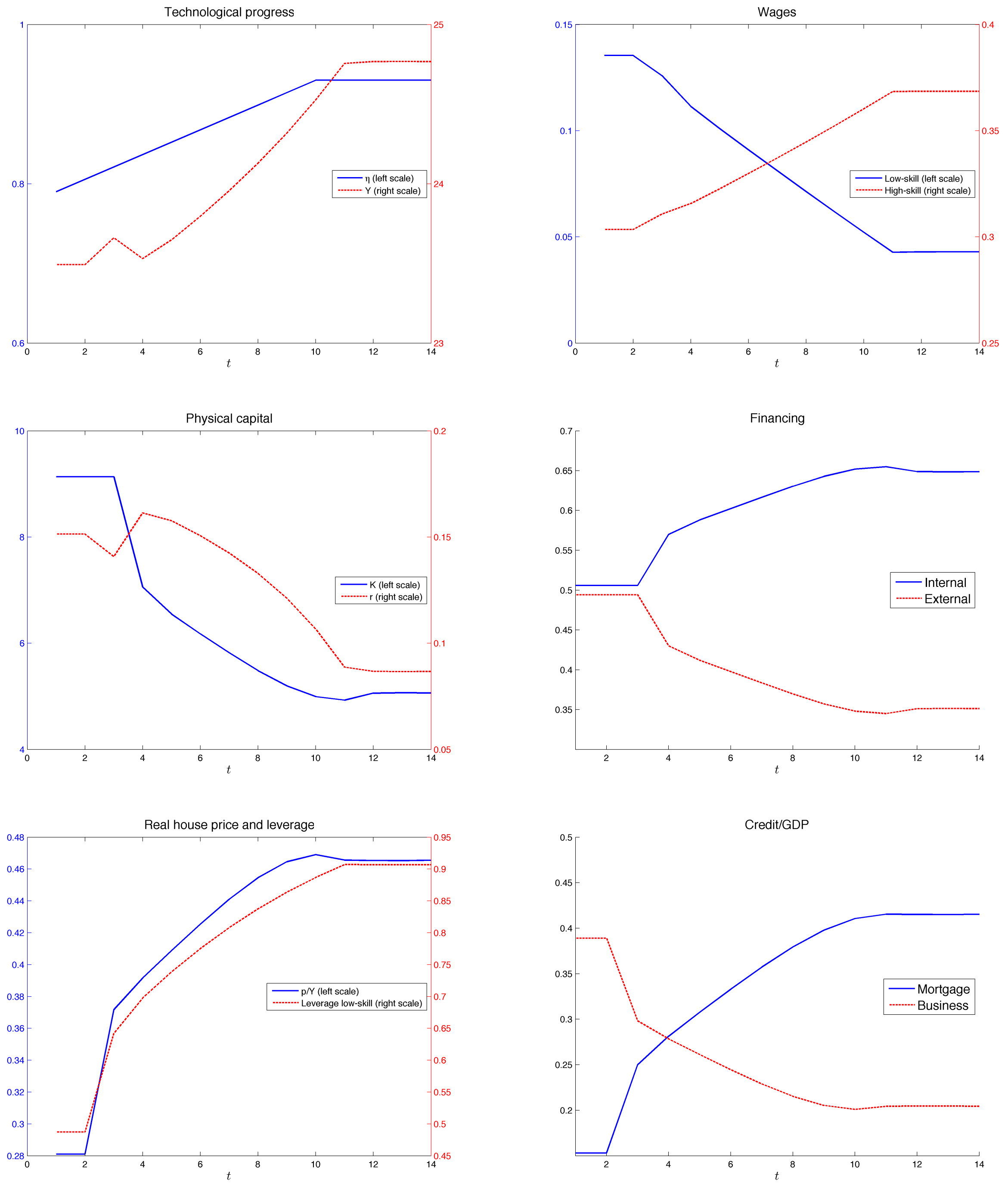

Figure 2: Simulated time path of technological change

technological change.

3.3. Default

The growing role of mortgage credit has consequences for financial stability, as the experience from the recent crisis has demonstrated. Poorer households received abundant credit in the boom to purchase houses, but could not keep up payments on their high debt burden. As a consequence, much policy debate has centered on how to control credit risk in housing and mortgage markets.

This section introduces some uncertainty in house prices that may produce default. In the context of our dynamic model, we show that under technological change mortgage defaults become more frequent, as low-skill workers increase their leverage.

Suppose that after yielding utility to their owner, but before it is sold to the next generation, agent i's house receives a temporary shock ξ_t^i with zero mean that alters the value of i's house. In appendix D we solve the model with default and elaborate on one interpretation of the shock being a "bad weather shock" that damages houses. In this interpretation houses hit by a shock need repair and therefore trade at a discount, $p_t^i = p_t(1-\xi_t^i)$, defining a threshold

$$\hat{\xi}_t^i = 1 - \frac{p_{t-1}}{p_t} LTV_{t-1}^i$$

such that for realizations of $\xi_t^i > \hat{\xi}_t^i$ household i defaults on her mortgage, and where $LTV_{t-1}^i \equiv \frac{(1+r_t)(-s_{t-1}^i)}{p_{t-1}L_{t-1}^i}$ is defined as the loan-to-value (LTV) ratio of a home buyer.

As we show in the appendix, the advantage of introducing default in this simple way is that it leaves the equilibrium allocation unchanged. As a result, with default in the model we immediately have the following corollary to proposition 2:

Corollary 1. *Define $\chi_t \equiv 1 - G(\hat{\xi}_t^l)$ as the aggregate default of low-skill workers. Technological progress that results in rising mortgage credit relative to GDP (i.e. satisfies (H2)) also produces increasing steady-state default among low-skill workers ($\frac{d\chi_t}{d\eta} \geq 0$)*

As technological progress increases income inequality and house prices, low-skill workers end up with higher LTV-ratios, pushing down the threshold $\hat{\xi}^l$ such that for a given distribution of shocks default occurs more frequently.

Note that in our current formulation default is just an ex-post transfer. Lenders charge borrowers a risky interest rate compensating them for the possibility of default, thus receive the risk-less rate in expectations. In the following section we show in our welfare analysis that a deadweight cost to default that is not fully internalized in prices may motivate a limit to mortgage credit.

4. Public policy

The textbook role of financial intermediation is to take resources from savers and channel them to businesses for productive purposes. However, in advanced economies the role of credit has shifted from financing productive investments towards financing real estate, i.e. the purchase of existing assets (see figure 1 from the introduction). As the scarce asset land absorbs rents an increasing fraction of wealth is concentrated in housing (Rognlie, 2015). Beyond that, mortgage defaults arguably have significant negative welfare effects.[14]

In this section we analyze policy options in the mortgage market. The basic policies that may be adopted are either ratio based (loan-to-value (LTV) limits on mortgages) or price based (fiscal subsidies or other financial benefits of mortgage borrowing). Our first result is that a limit to real estate credit may be motivated by its positive general equilibrium effects. When less savings are intermediated through house purchases, the released supply of savings is channeled towards corporate debt, resulting in higher capital investment, and ultimately higher wages. In other words, a policy to limit mortgage credit partially counters the effect of technological change by indirectly subsidizing corporate borrowing and thus encouraging traditional investment in tangible assets.

While limiting mortgage credit may maximize the utility of households in steady state,

[14]Several studies have found that property price busts are long lasting and result in large output losses (for example, see IMF, 2003, 2009; Claessens et al., 2009). Mian et al. (2013) and Mian and Sufi (2014b) show that the 2007 housing downturn was responsible for the drop in demand and high unemployment.

one old generation loses out as the borrowing limit results in a drop of land prices. When a borrowing limit is introduced the value of land drops. In our dynamically efficient OLG economy it is impossible to compensate the old generation that loses out from falling house prices. Thus, the long run benefits rely on a one-off redistribution from the old to the young generation.

Consequently, a limit to real estate credit can only generate a strict welfare improvement if mortgage defaults impose a deadweight cost that is not fully internalized. We show that in this case it is possible to implement an inter-generational transfer scheme (e.g. in form of a pension) that compensates the old generation losing out from falling house prices.

4.1. Limiting mortgage credit

Consider the policy option to directly limit the quantity of mortgage credit, such as a limit to loan-to-value (LTV) ratios. In the context of our model we analyze this policy by choosing $\bar{s}$, the maximum allowed size of a mortgage in the households problem (2). Throughout this section we make the functional form assumption $v(L) = \ln(L)$ to obtain closed form solutions.

The borrowing constraint becomes binding for low-skill workers when $p_t\bar{L} - w_t\tilde{l} > \bar{s}$. This forces them to reduce their demand for land to $L_t^l = \frac{w_t\tilde{l}+\bar{s}}{p_t} < \bar{L}$. In equilibrium, high-skill agents will acquire the rest, $L_t^h > \bar{L}$. Imposing land market clearing now results in the following land market clearing price evolution, analogous to (5):

$$p_{t+1} = (1 + r_{t+1})p_t - \frac{\phi p_t}{p_t\bar{L} - (1-\phi)(w_t\tilde{l} + \bar{s})}$$

In a steady state, this results in the expression

$$p = \frac{\phi + (1-\phi)r(w\tilde{l} + \bar{s})}{r\bar{L}},$$

which is easily seen to be increasing in $\bar{s}$. Similarly, L^l increases in $\bar{s}$ and L^h decreases in $\bar{s}$. Thus, for a tighter borrowing limit (lower $\bar{s}$) the allocation of land becomes more inefficient, and this is reflected in lower land prices.

Limiting mortgage borrowing has benefits and costs. On the downside, whenever $\bar{s}$ is binding, land can no longer be allocated efficiently (that is, uniformly). On the upside, there is a positive general equilibrium effect on output. As credit for land purchase is constrained, freed savings can be intermediated to production, resulting in higher investment and wages in the long run:

Lemma 1. *A binding limit to mortgage credit*

1. *Increases steady-state output* ($\frac{dY}{d\bar{s}} \leq 0$) *and reduces steady-state defaults* ($\frac{d\chi}{d\eta} \leq 0$)

2. *May increase the utility of all agents in steady state* $\int_0^1 \mathbb{E}U^i di$

The first result is immediate, as a binding credit limit lowers leverage and house prices and therefore reduces defaults and maximizes long-run capital investment and thus output.[15] To see the second point, suppose a social planner concerned with the long-run utility of all agents would set $\bar{s}$ to maximize $\int_0^1 \mathbb{E}U^i di$, trading off costs and benefits. Using market clearing conditions it can be shown that $\int_0^1 \mathbb{E}U^i di = \int_0^1 v(L^i)di + Y$ and the first oder condition to this problem would be

$$\int_0^1 v'(L^i)\frac{dL^i}{d\bar{s}}di = -\frac{dY}{d\bar{s}}. \tag{9}$$

The LHS is the cost of tightening the borrowing constraint (lower $\bar{s}$), namely a less efficient land allocation. Here $\int_0^1 v'(L^i)\frac{dL^i}{d\bar{s}}di > 0$, as the role of mortgage credit is to allocate land to equalize the marginal utility from land across agents. The RHS is the general equilibrium benefit of limiting mortgage credit. Because higher capital investments increase consumption $\frac{dY}{d\bar{s}} < 0$.

Trading off cost and benefit, a limit to mortgage borrowing can generally improve the welfare of households living in steady state. Interestingly, low-skill workers tend to benefit most from the LTV ceiling although they are precisely the agents for whom the constraint becomes binding. The reason is that low-skill workers benefit most from the general equilibrium effects of higher investments in physical capital. First, a higher steady state K implies lower interest rates. This is good for low-skill workers because they are borrowers. Second, to the extent

[15] Note that this general equilibrium effect is beneficial because we do not allow for the possibility of over-accumulation of capital in our model. In the case of over-accumulation lower house prices would take the economy further away from the Golden Rule level of capital.

that physical capital is more complementary with low-skill than with high-skill labor wages of low-skill workers are boosted disproportionately.

4.2. Deadweight losses from mortgage defaults

While an LTV limit may benefit society in the long run a strict welfare improvement is not possible, as our model economy is dynamically efficient.[16] This implies that the moment the borrowing limit is introduced, the old generation of home owners will always lose out as their houses drop in value. Thus, although it is possible to make agents living in steady state better off, dynamically it is impossible to make each and every generation better off.

However, note that the results of lemma 1 solely rely on a redistribution of savings from a fixed supply asset (land) to production (capital), while defaults constitute a pure ex-post transfer. However, as experienced in the recent crisis, mortgage defaults may have significant negative welfare effects. Deadweight losses associated with default may be an additional motivation to curb real estate credit.

We can introduce this notion to our model in a stylized way by assuming that every agent of generation $t-1$ incurs a utility loss $C(\chi_t)$ from aggregate mortgage defaults, with $C'(\chi_t) > 0$.[17] In the following proposition we show that with deadweight losses from default it can be possible to implement an inter-generational transfer scheme that compensates the old generation losing out, such that each and every generation can benefit from a limit to mortgage credit:

Proposition 3. *If mortgage defaults produce deadweight losses that are not fully internalized,*

[16] This is a standard result from the OLG literature, see Tirole (1985). An economy may be dynamically inefficient when there is over-investment, as savings are very abundant and the interest rate is below the rate of population growth. In such a case, introducing a pure asset bubble consumes savings and increases interest rates, reducing excess investment.

[17] Arguably, this is an extreme case where atomistic agents do not internalize the deadweight losses associated with default at all. Our result would also go through if agents would partially internalize the losses, but it is easiest to show the result in a formulation where agents do not adjust their decisions at all, leaving the equilibrium allocation unaltered otherwise.

a limit to mortgage credit in combination with an inter-generational transfer scheme can yield a strict welfare improvement.

As one old generation loses out from dropping house prices, without default costs a strict welfare improvement is not possible. Essentially, the problem is that the old generation that is worse off dies before the benefits of reduced mortgage credit are realized. However, if mortgage default is associated with sufficiently large deadweight losses it is possible to implement an inter-generational transfer scheme (e.g. in form of a pension) such that a borrowing limit makes each and every generation better off. In appendix E we show under what conditions such a transfer scheme indeed exists.

To gain intuition how the transfer scheme works, suppose up to time $T-1$ the economy is in a steady state (p, K, Y), and from time T onwards a borrowing limit $\bar{s}$ is introduced (unanticipated). As land prices drop from p to p_T generation $T-1$ needs to be compensated by a transfer $x_T = (p_{T-1} - p_T)\bar{L}$ from young generation T to not be worse off. Note that the transfer also ensures that defaults do not increase despite falling land prices. Due to the transfer, initially the total resources available for capital investment do not increase, $K_{T+1} = (1-\alpha)Y_T - p_T - x_T = (1-\alpha)Y - p = K$. Thus, in the short run there is no benefit in terms of higher capital investment.

However, generation T still benefits to the extent that lower land prices improve financial stability, $C(\chi_T) < C(\chi_{T-1})$. Thus, at $T+1$ a smaller transfer of $x_{T+1} < x_T$ suffices for generation T to not be worse off relative to the initial steady state. With this transfer, at $T+1$ more resources are available for capital investments, such that $K_{T+2} > K_{T+1}$, and thus $Y_{T+2} > Y$. As a result, for subsequent generations there is a positive feedback between higher capital investments and lower necessary transfers, such that at some point no more transfers are necessary.

In appendix E we show that a sufficient condition for such a transfer scheme to exist is that the reduction in deadweight losses outweighs the utility loss coming from a less efficient land allocation. Thus, with costly default we obtain dynamic inefficiency even though there is no over-accumulation of capital in our model.

Numerical example Our dynamic result is illustrated in a simulation without default costs, where borrowing is gradually limited over 10 periods in figure 3.[18] The upper left panel plots the demand for mortgage credit by low-skill workers along with the borrowing limit. It can be seen that the constraint actually only becomes binding around period 7, where the two lines meet.

Mortgage credit already starts falling before because house prices drop (top right panel) in anticipation that the constraint will be binding in the future. The bottom left panel illustrates the positive general equilibrium effects in form of higher wages. Wages of low-skill workers increase roughly from 0.043 to 0.047, or 10%, while those of high-skill workers rise more moderately by a bit less than 1%. Finally, the bottom right panel displays the utilities of low-skill workers, along with the sum of utilities of all agents. While both are higher in the new steady state, the initial dip in the time path illustrates that the current generation is hit by the introduction of the borrowing ceiling as it results in an unanticipated drop in house prices.

The dynamic perspective on welfare highlights why it may politically actually be quite difficult to introduce limits to mortgage borrowing. The generation of current home owners will always resist such policies, even though in the long run society may be better off.[19] In contrast, with sufficiently costly default, an inter-generational transfer scheme that compensates the old generation can improve overall welfare.

4.3. Mortgage subsidies

Many countries implicitly or explicitly subsidize home mortgages. For example, in the US and in the Netherlands interest payments on mortgages are tax-deductible. The UK government

[18]The calibration is the same as in the high-η steady state in section 3

[19]This result mirrors the analysis of Diamond (1965) who shows that less government debt as an OLG bubble can increase steady state welfare, but that a Pareto-improvement is only possible by increasing government debt when the economy is dynamically inefficient.

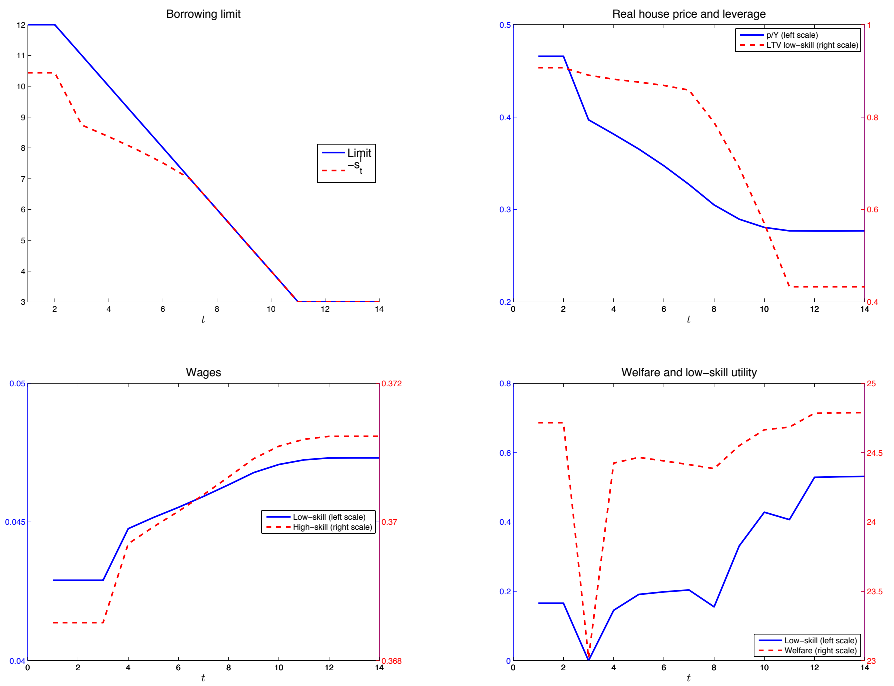

Figure 3: Simulated time path when gradually limiting mortgage credit

recently introduced the "Help for home buyers" program that grants an interest-free equity loan to home buyers. In the US, government sponsored agencies reduce the lending cost of banks by buying home mortgages. Finally, an OECD study Catte et al. (2004) highlights how in many countries the sale of residential property is exempt from capital gains taxes.

While different in their details, all these policies subsidize property and mortgage financing. We analyze them in a stylized way. Suppose the government pays a subsidy of $\tau_t < r_t$ on every mortgage, and raises lump-sum taxes T_t from the old generation to finance the subsidy. This redistribution favors borrowers while sharing costs with lenders. The subsidy reduces the interest paid on mortgages to $(r_t - \tau_t)$. The government budget constraint then requires $-\tau \int_D (s_t(i))di = T_t$, where D is the set of all households that are debtors in the mortgage market $(s_t(i) < 0)$.

For simplicity, suppose the subsidy can be targeted at low-skill workers only, such that rich borrowers (in our model, the entrepreneurs) do not receive the subsidy. Then the aggregate tax burden is $T_t = -(1-\phi)\tau_t s_t^l$, and every low skill household receives a subsidy of $-\tau_t s_t^l$.

This amounts to a net subsidy of $-\phi^l \tau_t s_t$ per household.

With the subsidy housing demand of low skill workers now is $L_t^p = v'^{-1}\left((1 + r_t - \tau_t)p_t - p_{t+1}\right)$, while the demand of high skill and rich is the same as before, since they receive no subsidy. Since L_t^p is increasing in τ_t the subsidy induces low-skill workers to consume more housing. We focus our analysis on how the subsidy changes the steady state allocation. The land price with a subsidy τ is

$$p = \frac{r - \phi\tau}{r(r-\tau)\bar{L}},$$

which is monotonically increasing in τ for $\tau < r$. For $\tau = 0$ the land price is $p = \frac{1}{r\bar{L}}$, as in the baseline model of section 2. Thus, the subsidy distorts the land price upwards and induces more savings to be intermediated towards housing. Therefore it has exactly the opposite general equilibrium effect of an LTV limit: business credit and investment are discouraged, and wages fall.

Analyzing steady state welfare the mortgage subsidy has two negative effects: first, it results in an inefficient land allocation as low-skill workers demand too much land. Second, there is an output loss because less credit is channeled to production. The effect of the subsidy on default and foreclosures is ambiguous. On the one hand transferring funds to borrowers makes it easier for them to repay their mortgage, so the threshold shock above which low-skill workers default increases to

$$\hat{\xi}^l = p - \frac{(1 + r - \tau)(-s^l)}{L^l}.$$

However, through the general equilibrium effects of the subsidy interest rates and house prices increase and wages decrease, so low-skill workers end up with higher leverage. Thus, in sum even defaults may increase and so all three welfare components - land allocation, output and foreclosures - may affect steady state welfare negatively.[20] These results are illustrated in a numerical example in figure 4, where we compare the steady states with different levels of τ. It can be seen in the upper panels that as investment in physical capital goes down, wages

[20] A strict welfare improvement by reducing mortgage subsidies is again not possible because this economy is dynamically efficient, as discussed in the context of the borrowing limit.

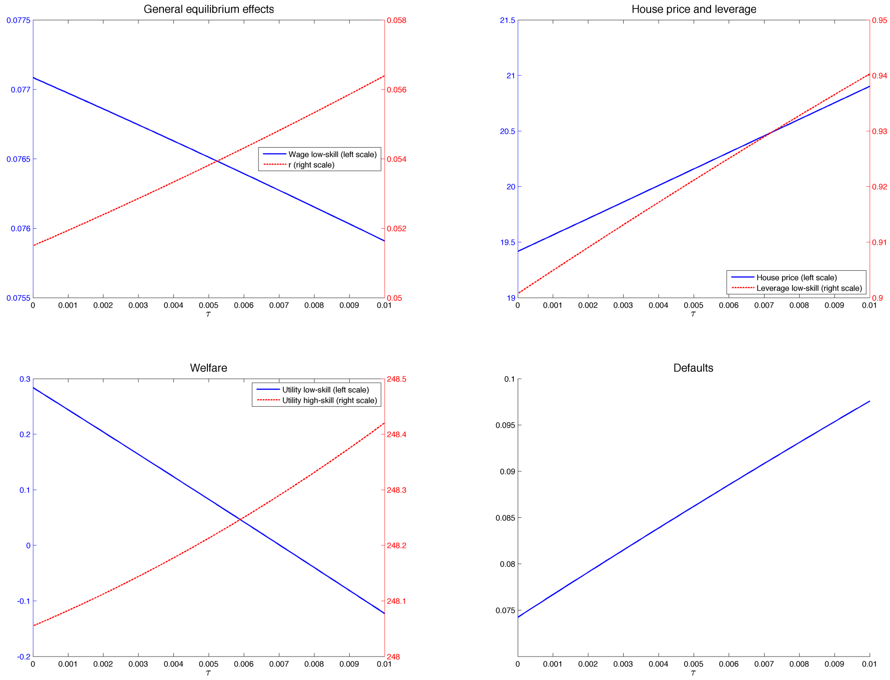

Figure 4: Steady states with different levels of the mortgage subsidy τ.

decrease and interest rates and house prices increase. In this example the leverage of low-skill workers increases too, resulting in more defaults in equilibrium (bottom right panel). Finally, the lower left panel illustrates the surprising result that due to its general equilibrium effects the receiver of the subsidy (i.e. low-skill workers) may actually be worse off in steady state. In this calibration it is actually high-skill workers who benefit from the subsidy as it results in higher interest rates.

To summarize the policy analysis we compare a mortgage borrowing limit to a subsidy along the three welfare components land allocation, output and foreclosures. First, both policies come with the welfare cost of distorting the land allocation. Second, a mortgage ceiling has positive general equilibrium effects on output, investment and wages, while a subsidy destroys output by diverting savings towards land. Third, a borrowing limit is always effective at containing defaults and foreclosures because it reduces leverage. In contrast, because a mortgage subsidy encourages higher leverage it may actually increase default rates despite transferring resources to borrowers.

We conclude that if the purpose of a mortgage subsidy is to support lower income households with buying a house this could more easily be achieved by direct transfers that do not distort the housing market.

5. Extensions

This section presents extensions to the basic model of section 2 that alter the relative supply of savings vehicles and demand for savings. First we consider the case $\rho < 1$, when intangibles are partially appropriable and support equity investments. We then add government debt and finally analyze the impact of capital inflows from abroad. Like in Diamond (1965), public debt is an additional savings vehicle, on net subtracting savings from mortgage and business finance. In contrast, capital inflows from abroad add savings to the economy. In line with Bernanke's (2005) global savings glut narrative they push up house prices and investment.

5.1. Outside equity

Suppose that $\rho < 1$, such that a fraction $(1-\rho)$ of the returns to intangibles are appropriable. Note that we still assume that debt finance must be backed by tangible collateral. As a result, even with $\rho < 1$ intangibles can still not support corporate debt. However, with $\rho < 1$ some of the returns may be promised to outside equity. We introduce this notion by assuming that there is one unit of shares and that the initial generation $t = -1$ is endowed with all shares.

We denote the share price at time t by f_t. Now, after production there is a return $(1 - \rho)Y_{H,t}H_t$ that can either be retained or paid out as dividend. Retained earnings would be used to support further capital investments and over time shift the financing of physical capital from debt to equity. Because equity holders apply the market discount rate r_t (i.e. they compare the return on equity to the alternative of investing in corporate or mortgage debt), they are indifferent between the two policies and for concreteness we assume that the returns are fully paid out as dividends. Then at any point in time the relation

$$f_t = \frac{f_{t+1} + (1-\rho)Y_{H,t}H_t}{1 + r_t}$$

has to hold, and in the long run the following steady state share price obtains:

$$f = \frac{(1-\rho)Y_H H}{r}.$$

Technological change boosts share prices through two channels. First, in the numerator the increasing productivity of intangible capital directly increases the value of shares. Second, lower interest rates imply higher asset prices in general.

The financial market clearing condition would now be

$$(1-\alpha)Y_t = p_t \bar{L} + f_t + K_{t+1},$$

and shares show up as an additional savings vehicle on the right hand side, as expected.

A caveat is that in our model shares are a pure store of value. This owes to the fact that intangibles are produced by human capital. To give outside equity a more productive role one could think of a version of the model where in addition to the human capital some other resources are needed to produce intangible capital. While the notion of equity as a pure store of value is arguably a bit peculiar, it is actually surprisingly close to developments in US equity markets since the 1980s. While the market value of equities has been growing strongly, net equity issuance of non-financial corporates has actually been negative.[21]

5.2. Government Debt

Another potential savings vehicle is government debt. Suppose the government raises an amount G_t by issuing one-period bonds, and repays principal and interest either by raising lump-sum taxes T_{t+1} or by issuing new bonds G_{t+1}. By a no-arbitrage argument government debt must yield the same return r_t as mortgages and business loans. The government budget constraint is thus

$$(1+r_t)G_{t-1} = G_t + T_t.$$

Since the public debt needs to be financed by net financial savings of households the market clearing condition now becomes $\int_0^1 s_t^i di = d_t + G_t$.

[21] see Federal Reserve Board Flow of Funds Accounts

Now, in a steady state the stock of government debt is at a constant level $G_t = G_{t+1} = G$. From the government budget constraint, for the stock of debt to be constant interest payments must be financed by taxes, $T = rG$.

To complete the analysis we have to take a stance on who pays the taxes, i.e. who effectively pays for the interest cost of the public debt. First note that agents buy bonds when they are young and sell them when they are old, thus the receiver of the interest payment is always the old generation. Thus, taxing the young generation effectively constitutes a transfer from young to old. In this case the cost of interest subtracts from net financial savings and market clearing can be written as

$$(1-\alpha)Y_t - r_t G_{t-1} = p_t \bar{L} + K_{t+1} + G_t.$$

In contrast, taxing the old means that the same generation that receives the interest payments has to pay for it through taxes, netting out the cost of interest in the aggregate. Market clearing then requires

$$(1-\alpha)Y_t = p_t \bar{L} + K_{t+1} + G_t.$$

Evaluating these expressions in steady state it is clear that in both cases the stock of public debt G is an additional savings vehicle that shows up on the right hand side. If taxes are paid by the young, additionally the cost of interest is subtracted from savings, such that on net a total of $(1+r)G$ of savings are diverted towards public debt. If taxes are paid by the old still a total G is saved through government debt.

As a result, government debt diverts savings away from both land and physical capital. Interest rates rise, and house prices, investment and wages fall. The benefit of transporting some savings via relatively save government debt is that lower house prices imply lower leverage, which in turn improves financial stability.

5.3. Capital inflows from abroad

Bernanke (2005) warned of a global savings glut, pointing out that especially Asian economies were accumulating more and more safe Dollar-denominated assets. It is now often argued that the global savings glut contributed to the US housing bubbles of the 2000s (e.g. Mian and Sufi, 2014a). In this extension we analyze the effect of capital inflows on local house prices, in the context of our model.

Suppose foreigners want to invest an exogenous amount x_t in domestic financial markets. Since they live abroad foreigners do not gain any utility from living in domestic houses. Therefore holding land directly yields no return other than a potential price appreciation, and foreigners only invest in financial assets, which yield a return of r_t. With an amount x_t of capital inflows financial market clearing becomes

$$(1-\alpha)Y_t + x_t = p_t\bar{L} + K_{t+1}.$$

Because foreigners invest in domestic financial assets there is a higher supply of savings that need to be invested in either mortgages or business loans. Interest rates fall and house prices, investment in physical capital and wages rise.

This extension shows how a global savings glut can result in increasing house prices. One reading of the US housing bubble is that an increased demand for Dollar-denominated stores of value from abroad was essentially put into US land.

Interestingly, the analysis also shows how persistent current account surpluses can dampen house price appreciation. If capital actually flows abroad x_t is negative. This has exactly the opposite effect of a capital inflow, pushing up interest rates and dampening property prices. This may be one explanation for why the housing market in Germany has been relatively stable until before the crisis (Agnello and Schuknecht, 2011). Only since in recent years capital has started flowing back from the European periphery Germany has been experiencing considerable house price growth.[22]

6. Conclusion

This paper offers a stylized dynamic model where production, factor compensation and financing are driven by technological change, to study its effect on the price and the funding

[22]According to Bundesbank data German real estate prices have risen by 24% from 2010-2014.

of real estate, a durable asset in fixed supply. Next to established results on the evolution of relative wages, the framework explains concomitant trends in credit allocation and house prices. Our intent is not to offer a complete macroeconomic calibration of the effects of technological progress. We deliberately seek a simple framework to gain some clear insight on the dynamics of mortgage funding and house prices, reconciling several empirical trends. The critical channel is a long term decline in interest rates driven by an increasing use of intangible inputs.

Eichengreen (2015) discusses four factors that may contribute to an excess of saving over productive investment, and lead to a persistent period of below-potential growth (secular stagnation). He examines the rise in saving in emerging markets, the decline in population growth, a drop in investment opportunities, and finally concludes that a fall in the relative price of investment goods (see, e.g. Thwaites, 2014) appears to be the most likely explanation. Our analysis is related to this conclusion. In a world of incomplete contracting, the new technologies boost intangible vs tangible capital, which reduce corporate demand for credit (or more generally, less external financing). Indeed, corporate demand for credit has been falling for years, while net equity issues have been negative. This induces private savings to fund purchases of other durable assets such as housing. Critically, excess savings arise because savers cannot co-invest in the private development of intangible capital. In reality, innovative entrepreneurs are largely self financed, in part because they need modest upfront investment. There is much evidence that they receive external finance only at a fairly late stage, once their product is sufficiently tangible. Anecdotal evidence also suggests that value created by innovative firms largely accrues to founders and employees.

Rognlie (2015) shows that surging house prices are largely driving responsible for growing returns on capital. Technological change can help to explain this development. Starting with an unequal distribution of land, with bequests wealth inequality would rise over time, as under technological progress the value of land as store of value increases over time.

Arguably, we still miss a clear framework on the nature of intangible capital and its limited pledgeability. Future research should explore whether this arises from its intellectual nature.

Intellectual labor is hard to monitor, so it must be promised large rents to ensure performance. Another issue advanced by the endogenous growth literature is that knowledge, once shared, is non excludable, so it is hard for outsiders to benefit from it. Outside investors then need to rely on tangible assets.

In conclusion, more insight is needed on the evolution of investment funding for an economy driven by technological progress, and specifically knowledge.

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A. Proof of proposition 1

We need to show that in a steady state under technological progress ($\frac{dY}{d\eta} \geq 0$) interest rates fall in η, i.e. $\frac{dr}{d\eta} \leq 0$.

The proof uses two equilibrium conditions: the condition that the interest rate equals the marginal product of capital (A.1) and the market clearing condition (A.2), both evaluated in steady state:

$$r = \alpha(1-\eta)\frac{Y}{K} \tag{A.1}$$

$$(1-\alpha)Y = \frac{v'(\bar{L})\bar{L}}{r} + K \tag{A.2}$$

Differentiating (A.1) w.r.t. η gives:

$$\frac{dr}{d\eta} = \alpha\frac{Y}{K}\left[-1 + \frac{(1-\eta)}{Y}\frac{dY}{d\eta} - \frac{(1-\eta)}{K}\frac{dK}{d\eta}\right] \tag{A.1'}$$

Similarly, rearranging (A.2) for r and taking a derivative w.r.t. η yields:

$$\frac{dr}{d\eta} = \frac{r^2}{v'(\bar{L})\bar{L}}\left[\frac{dK}{d\eta} - (1-\alpha)\frac{dY}{d\eta}\right] \tag{A.2'}$$

From (A.2') it is easy to see that $\frac{dr}{d\eta} \leq 0$ if

$$\frac{dK}{d\eta} \leq (1-\alpha)\frac{dY}{d\eta}. \tag{A.3}$$

In what follows we show that condition (A.3) indeed holds.

As a first step, setting (A.1') = (A.2') and solving for $\frac{dK}{d\eta}$ results in

$$\frac{dK}{d\eta} = \frac{\left[\frac{r(1-\alpha)}{v'(\bar{L})\bar{L}} + \frac{1}{Y}\right]\frac{dY}{d\eta} - \frac{1}{1-\eta}}{\frac{r}{v'(\bar{L})\bar{L}} + \frac{1}{K}},$$

Plugging $\frac{dK}{d\eta}$ into (A.3) and rearranging, condition (A.3) can be rewritten as

$$\frac{dY}{d\eta}\left[\frac{1}{Y} - \frac{1-\alpha}{K}\right] \leq \frac{1}{1-\eta}$$

Again using (A.2) this can be rewritten as

$$-\frac{v'(\bar{L})\bar{L}}{r}\frac{dY}{d\eta} \leq \frac{1}{1-\eta}$$

Under technological progress $\frac{dY}{d\eta} \geq 0$, so the LHS is always negative, while the RHS is always positive. Thus condition (A.3) holds and we can conclude that indeed $\frac{dr}{d\eta} \leq 0$. □

B. Proof of proposition 2

We need to show that in a steady state under technological progress ($\frac{dY}{d\eta} \geq 0$), if and only if (H2) holds.

As a first step, note that for the financial market to clear mortgage credit cannot be demanded by high-skill workers. Using this result, aggregate mortgage credit m_t can be written as follows:

$$m_t = (1-\phi)\max\left\{0, p_t\bar{L} - w_t\tilde{l}\right\} \tag{B.1}$$

The first case in the max-function refers to the case where low-skill workers can still afford their house out of pocket and thus do not take out any mortgage, while in the second case they take out a mortgage. Focusing on the second case, evaluating (B.1) in steady state and dividing by Y mortgage credit to GDP can be written as

$$\begin{aligned}\frac{m}{Y} &= (1-\phi)\frac{p\bar{L}}{Y} - \frac{wl}{Y}\\ &= (1-\phi)\frac{v'(\bar{L})\bar{L}}{rY} - (1-\alpha)(1-\eta)\end{aligned} \tag{B.2}$$

Now, combining (A.1) and (A.2) (rY) can be written as

$$rY = \frac{v'(\bar{L})\bar{L}}{(1-\alpha)} + \frac{\alpha(1-\eta)}{(1-\alpha)}Y \tag{B.3}$$

Using (B.3) to eliminate r in (B.2) and taking a derivative w.r.t. η yields the result desired result that $\frac{d(m/Y)}{d\eta} \geq 0$ if and only if (H2) holds. □

C. Calibration

For the calibration we use the functional form $v(L) = \ln(L)$. The parameters of the model are $\alpha, \bar{L}, \phi, \rho, \varepsilon, \tilde{h}$ and $\tilde{l}$. We calibrate all parameters to fit US data from the early 1980s. Then we linearly change η_t over time, from a starting point $\underline{\eta}$ to an endpoint value $\bar{\eta}$ to match the tangible-intangible investment ratios reported by Corrado et al. (2009). Arguably, to get a more realistic time path one would need to also change other parameters over time. However, since the goal of the simulation is to illustrate the qualitative mechanism of the model we deliberately only change one parameter over time to get a clear picture of the impact of η_t in the model.

First we normalize $\bar{L} = 1$ and set $\alpha = 0.33$, a standard value in the literature. For ϕ we use the percent of the population with a Bachelor degree or higher in 1980, reported to be 17% in the Digest of Education Statistics 2013. To calibrate ρ we use the elasticity of substitution between high-skill and low-skill workers. In the SBTC literature this elasticity is measured to be between 1.4 and 2 (Acemoglu and Autor, 2011), so we set ρ to get an elasticity in the center of this range at 1.7. Finally we set $\varepsilon = 0.18$ to match the fraction of self-employed in the US in 1980 of roughly 3%.

Parameter	**Calibration method**
$\alpha = 0.33$	Income share capital
$\phi = 0.17$	Fraction of population with Bachelor degree or higher
$\rho = 0.7/1.7$	Elasticity of substitution between high-skill and low-skill = 1.7
$\varepsilon = 0.18$	Target fraction of population self-employed
$\bar{L} = 1$	Normalization
$\tilde{l} = 25, \tilde{h} = 305$	Target steady state interest rate and wage gap
$\underline{\eta} = 0.79, \bar{\eta} = 0.93$	Target steady state tangible-intangible ratios

This leaves us with two parameters that have no direct representation in the data, namely $\tilde{h}$ and $\tilde{l}$. Additionally, we still need to set the starting- and end values for η_t. To calibrate $\tilde{h}$, $\tilde{l}$ and $\underline{\eta}$ we target the wage gap $\frac{q}{w}$, the level of the interest rate r and the value of intangible-tangible investment ratios $\frac{RH}{(1+r)K} = 0.6$. The wage gap is taken from Autor (2014) who reports a college-high school wage gap of roughly 50% in 1980. For the interest rate we look at the yield on 10-year US treasury bonds, ranging just below 15% in 1980, and the intangible-tangible investment ratio is taken from Corrado et al. (2009).

Lastly, with the initial steady state calibrated we choose the endpoint $\bar{\eta}$ to match the Corrado et al. (2009) tangible-intangible investment ratio in the early 2000s of $\frac{RH}{(1+r)K} = 1.36$.

D. The model with default

We now introduce some idiosyncratic risk to allow for the possibility of default. Suppose that after yielding utility to their owner, but before it is sold to the next generation, agent i's house receives a temporary "bad weather shock" ξ_t^i, with a CDF $G(\xi)$ with support $[-1,1]$ and zero mean.

The weather shock is drawn every period, and its effects are thus temporary. Realizations of $\xi_t < 0$ mean the house stands in a neighborhood that temporarily experiences particularly good weather, yielding their owner some additional utility $-p_t\xi_t$ per unit of land. In contrast, realizations of $\xi_t > 0$ are bad weather shocks that damage the house. A damaged house will not yield any utility to the next owner unless it is repaired at cost $p_t\xi_t$ per unit of land. As the cost has to be ultimately borne by the seller, a damaged house trades at a discount such that $p_t^i = p_t(1-\xi_t^i)$.

As a result of the shocks, households with very larger shocks default. In particular, default occurs if $-(1+r_{t+1})s_t^i \geq p_{t+1}^i L_t^i$, defining a threshold

$$\begin{aligned}\hat{\xi}_t^i &= 1 - \frac{(1+r_t)(-s_{t-1}^i)}{p_t L_{t-1}^i} \\ &= 1 - \frac{p_{t-1}}{p_t} LTV_{t-1}^i\end{aligned}$$

such that for realizations of $\xi_t^i > \hat{\xi}_t^i$ a household defaults on her mortgage, and where $LTV_{t-1}^i \equiv \frac{(1+r_t)(-s_{t-1}^i)}{p_{t-1}L_{t-1}^i}$ is defined as the loan-to-value ratio of a home buyer. Note that default can only occur if $\hat{\xi}_t^i < 1$, i.e. if $\hat{\xi}_t^i$ is within the support of ξ_t^i. As a result, whenever i is a borrower ($s_{t-1}^i \leq 0$), she may default.

To compensate lenders for the possibility of default, agent i has to pay a risky interest rate. We assume that savers pool their lending through an intermediary that just breaks even, and pays lenders the riskless rate r_t. Therefore individual lenders always receive r_t, while borrowers pay it in expectation. The household maximization is analogous to (2). In particular, denoting the risky rate by rr_t^i the maximization problem of household i in the model with default becomes

$$\max_{c^i_{t+1},L^i_t,s^i_t} \mathbb{E}_t U(c^i_{t+1}, L^i_t) = \mathbb{E}_t c^i_{t+1} + v(L^i_t)$$

$$s.t. \quad s^i_t \leq y^i_t - p_t L^i_t$$

$$c^i_{t+1} \leq \max\left\{y^i_{t+1} + p_{t+1}(1 + \xi^i_{t+1} L^i_t + (1 + rr^i_{t+1})s^i_t, 0\right\}$$

$$s^i_t \geq -\bar{s}$$

$$c^i_{t+1}, L^i_t \geq 0$$

where the max-function in the $t+1$ budget constraint reflects that households are protected by limited liability. The probability of default is $[1 - G(\hat{\xi}^i_t)]$, so that expected consumption at $t+1$ can be written as

$$\mathbb{E}_t c_{t+1} = G(\hat{\xi}^i_{t+1}) \left\{p_{t+1}(1 + \mathbb{E}_t[\xi^i_{t+1}|\xi^i_{t+1} \leq \hat{\xi}^i_{t+1}])L^i_t + (1 + rr^i_{t+1})s^i_t\right\}$$

Now, the break even condition for the intermediary on borrower i is

$$-(1 + r_{t+1})s^i_t = -G(\hat{\xi}^i_{t+1})(1 + rr^i_t)s^i_t + (1 - G(\hat{\xi}^i_t))p_{t+1}(1 + \mathbb{E}_t[\xi^i_{t+1}|\xi^i_{t+1} > \hat{\xi}^i_{t+1}])L^i_t.$$

Plugging this condition into $\mathbb{E}_t c_{t+1}$ the objective function can be written as follows

$$\mathbb{E}_t U(c^i_{t+1}, L^i_t) = y^i_{t+1} + p_{t+1}L^i_t + (1 + r_{t+1})(y^i_t - p_t L^i_t) + v(L^i_t)$$

The household problem boils down to choosing L^i_t to maximize $\mathbb{E}_t U(c^i_{t+1}, L^i_t)$. Differentiating w.r.t L^i_t results in the first order condition and thus demand for land (3). It follows that the allocation in the model with default is equivalent to the model without default. However, now households with $\xi^i_t > \hat{\xi}^i_t$ default.

D.1. Technological Change and the evolution of defaults (proof of corollary 1)

TO DO

E. Proof of proposition 3

PRELIMINARY

In this appendix we want to proof that if default creates a deadweight loss that is not internalized a limit to mortgage borrowing can improve welfare.

To that end, suppose every agent of generation $t-1$ incurs a utility loss $C(\chi_t)$ from aggregate mortgage defaults, with $C'(\chi_t) > 0$. Further suppose that up to time $T-1$ the economy is in a steady state (p, K, Y) without borrowing limit and from time T onwards a limit $\bar{s}$ that maximizes long-run welfare $\int_0^1 \mathbb{E}U^i di$ is introduced (unanticipated).

In lemma 2 we will show that if the deadweight costs associated with default are large enough we can construct an inter-generational transfer scheme such that each and every generation is better off with the policy.

Let $V_t(\bar{s}) \equiv \int_0^1 v'(L_t^i)di$ be the aggregate utility from land, and $C_t(\bar{s}) \equiv C(\chi_{t+1})$ the default costs incurred by generation t, given a borrowing limit $\bar{s}$. Similarly, let $V_t \equiv \int_0^1 v'(\bar{L})di$ and C_t be the respective values of these functions absent a borrowing limit.

Lemma 2. *If for every* $t \geq 0$

$$C - C_{T+t}(\bar{s}) \geq V - V_{T+t}(\bar{s}) \tag{W1}$$

there exists an inter-generational transfer scheme $\{x_{T+t}\}_{t=0}^{\infty}$ *with non-negative transfers* $x_t \geq 0$ *from young generation* t *to old generation* $t-1$ *that satisfies for all* $t \geq 0$

$$\begin{aligned} x_{T+t+1} \geq\ & x_{T+t} + [p_{T+t} - p_{T+t+1}]\bar{L} - [\alpha Y_{T+t} + (1-\alpha)Y_{T+t-1} - Y] \ldots \\ & - [(C - C_{T+t}(\bar{s})) - (V - V_{T+t}(\bar{s}))], \end{aligned} \tag{X1}$$

$$x_{T+t+1} \leq x_{T+t} + (1-\alpha)[Y_{T+t+1} - Y_{T+t}] - [p_{T+t+1} - p_{T+t}]\bar{L}, \tag{X2}$$

As a result the borrowing limit in combination with the transfer scheme generates a strict Pareto-improvement.

To proof that a borrowing limit yields a strict welfare improvement we have to show that all agents of each generation are better off relative to the initial steady state. Note that it suffices to show that each generation as a whole is better off, since intra-generational transfers can always be used to split the surplus generated. More specifically, letting $U_t = \int_0^1 U_t^i di$ it suffices to show that $U_{T+t} - U \geq 0 \forall t \geq 0$. What is more complicated is to make each and every generation better off, since each generation can only be compensated while it is alive.

First, using market clearing conditions U_t can be written as

$$U_t = (p_{t+1} - p_t)\bar{L} + x_{t+1} - x_t + \alpha Y_{t+1} + (1-\alpha)Y_t + V_t(\bar{s}) - C_t(\bar{s}),$$

and thus

$$U_t - U = (p_{t+1} - p_t)\bar{L} + x_{t+1} - x_t + (\alpha Y_{t+1} + (1-\alpha)Y_t - Y) + [(C - C_t(\bar{s})) - (V - V_t(\bar{s}))].$$

After rearranging it can be seen that $U_{T+t} - U \geq 0$ if (X1) holds. Thus, under any transfer scheme that satisfies (X1) all generations are better off relative to the initial steady state.

However, we still need to verify that the transfer scheme is viable. To check this it suffices to show that investments in physical capital grows over time, because then there is a positive feedback between higher investment at time t, increasing output and thus reducing necessary transfers at $t+1$, again allowing higher investments. If capital grows over time this positive feedback has to result in fading transfers eventually. Suppose not, then output and necessary transfers would grow together. Inspecting (X1), under (W1) this could only be if land prices kept falling. But falling land prices are imply rising interest rates, contradicting increasing investments in physical capital.

Thus, it suffices to show that physical capital grows over time, which holds true under condition (X2). To see this note that

$$K_{t+1} = (1-\alpha)Y_t - p_t\bar{L} - x_t,$$

and therefore, after rearranging it can be seen that $K_{T+t+2} - K_{T+t+1} \geq 0$ if (X2) holds.

Finally, combining (X1) and (X2) it can be seen that the two can simultaneously be satisfied if and only if

$$Y_{T+t+1} - Y \geq -[(C - C_t(\bar{s})) - (V - V_t(\bar{s}))].$$

for all $t \geq 0$. Since capital grows over time the LHS is always positive, while under (W1) the RHS is always negative. Hence the condition holds, completing the proof. □