DNB Working Paper

No 841/September 2025

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Stefan Wöhrmüller

DeNederlandscheBank

EUROSYSTEEM

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* Views expressed are those of the author and do not necessarily reflect official positions of De Nederlandsche Bank.

Working Paper No. 841

September 2025

De Nederlandsche Bank NV
P.O. Box 98
1000 AB AMSTERDAM
The Netherlands

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Stefan Wöhrmüller †

This draft: May 2025
First draft: February 2024

Abstract

How does uninsurable idiosyncratic risk shape the optimal design of carbon taxation? To answer this question, I augment a heterogeneous-agent incomplete-markets model with a climate externality on total factor productivity and dirty energy demand of households and firms. A government sets a carbon tax on energy and redistributes its revenue via lump-sum transfers. When labor tax instruments are held fixed, I find that the optimal carbon tax rises with the level of uninsurable idiosyncratic risk. In contrast, when labor taxes can adjust, the carbon tax remains relatively stable across different economic environments. Overall, welfare gains are primarily driven by improved insurance provision.

Keywords: heterogeneous agents, precautionary savings, carbon taxation
JEL codes: D31, D52, E21, H21, Q50

*I am indebted to Christian Stoltenberg for his continuous guidance and support during my PhD. I am very grateful to Dennis Bonam, Thomas Douenne, Sergio de Ferra, Kees Haasnoot, Timo Haber, Alejandro Hirmas, Albert Jan Hummel, Bas Jacobs, Eva Janssens, Franc Klaassen, Leonardo Melosi, Kurt Mitman, Marcelo Pedroni, Rick van der Ploeg, Elisabeth Pröhl, Federica Romei, Timo Schenk, Randolph Sloof, Andra Smadu, and Konstantin Sommer for many helpful comments and conversations. All errors are my own.

†De Nederlandsche Bank and University of Amsterdam: s.h.p.wohrmuller@dnb.nl.

1 Introduction

How does uninsurable idiosyncratic earnings risk influence the design of optimal carbon taxation in a heterogeneous-agent economy? To what extent do insurance and redistribution motives contribute to the welfare-maximizing level of the carbon tax? In this paper, I argue that, depending on the available tax instruments, an increase in the uninsurable idiosyncratic risk that households face gives rise to a higher optimal carbon tax in general equilibrium.

I begin by considering a standard heterogeneous-agent incomplete-markets economy á la Aiyagari (1994) and enrich it along four dimensions. First, households supply labor and have Stone-Geary preferences over a clean and an energy-intensive (dirty) consumption good, where the dirty energy good is subject to a subsistence level. The subsistence level implies that carbon taxation would be regressive per se, as poorer households spend a larger fraction of their income on dirty goods. Second, the supply side features an energy producer and a final goods producer that uses capital, labor, and energy for production. Third, I introduce a climate externality. Energy production is pollution-intensive and increases the stock of carbon in the atmosphere, which in turn decreases total factor productivity of the final good firm. Fourth, the government has access to non-individualized lump-sum transfers, capital taxes, labor income taxes, and carbon taxes.

In this model environment, households have a precautionary saving motive, because there is only one saving instrument available to self-insure against the idiosyncratic risk and borrowing is limited by an exogenous constraint. It is well-known that this precautionary saving behavior then implies a concave consumption function in current income and wealth (Carroll and Kimball, 1996; Jensen, 2018). Building on this insight, I show that this result carries over to static Stone-Geary preferences, which otherwise imply a linear expenditure system in a model environment without idiosyncratic risk. Hence, consumption functions with respect to the clean and dirty good are also concave and thus imply decreasing marginal propensities to consume with respect to both goods. Put differently, idiosyncratic risk and precautionary savings give rise to non-linear Engel curves.[1] As a result, the optimal carbon tax set by the government might also take into account distributional and insurance concerns in addition to the climate externality (Jacobs and van der Ploeg, 2019).

To quantitatively study this interaction between carbon taxation and uninsurable idiosyncratic risk and precautionary savings, I calibrate and estimate the economic part of the climate-economy to match features of the U.S. economy. Specifically, I use data from the Panel Study of Income Dynamics to estimate parameters on the household side of my economy and model a Pareto tail in labor productivity to capture the high level of wealth inequality in the economy. The calibration of the climate block is taken from the literature and represents climate impacts on a global scale. I then use my estimated model as a laboratory and let the government choose (combinations of) tax instruments to maximize welfare along a transition under a utilitarian welfare criterion when

[1] The term Engel curve is used with varying meanings in the literature. Here, I use it to refer to the mapping from current (after-tax) income to current consumption.

recycling revenue lump-sum back to households.[2] Due to the concavity of the utility function, the government exhibits an implicit preference for both redistribution and insurance.

In my scenarios, I allow the government to adjust the average income tax, τ_0, tax progressivity, τ_1, and the carbon tax, τ_d, all in various combinations, to maximize welfare along the transition. My findings with respect to the magnitude of the tax are as follows. When all three instruments are adjusted, I find that the carbon tax increases only slightly from its benchmark and income taxes rise significantly to finance higher transfers. The latter result is in line with Ferriere, Grübener, Navarro, Vardishvili and Irvine (2023). Similar overall welfare gains are achieved when only τ_0 and τ_d are adjusted, suggesting that the tax progressivity τ_1 plays a limited role in driving welfare outcomes. Indeed, in all scenarios, the adjustment of the average income tax accounts for the largest share of overall welfare gains. However, when the carbon tax is the only instrument used, it is set at a level nearly five times higher than under a coordinated tax reform suggesting that the carbon tax picks up existing inefficiencies in the economy in addition to the climate externality.

To better understand the sources and drivers of welfare gains, I use the decomposition of Bhandari, Evans, Golosov and Sargent (2023) to separate welfare gains into three components: (i) efficiency gains, (ii) redistribution gains, and (iii) insurance gains. I find that when all three instruments are adjusted simultaneously, the efficiency component of welfare is negative, which is partially offset by gains from insurance. Gains from redistribution are small and slightly negative. A similar pattern holds when only τ_0 and τ_d are adjusted, with nearly identical overall effects, again implying that tax progressivity has only minor effects. In contrast, when only the carbon tax is adjusted, the efficiency effect is positive for clean consumption and leisure but sharply negative for dirty consumption, suggesting a substitution away from this good. Even though this scenario also shows the largest redistribution gains, and while efficiency losses dominate in most cases, the insurance channel plays a substantial compensating role. As such, this paper is complementary and builds on the results from two-agent models as in Känzig (2023), which focus on the role of redistributive effects and where insurance concerns are absent by definition.

Insurance concerns arise from the presence of uninsurable idiosyncratic risk. Hence, to better understand what components give rise to a welfare-maximizing non-zero carbon tax in this economy and to study the relation between welfare-maximizing carbon taxation and this type of risk, I conduct several exercises that change various features of the model. In particular, I repeat the main quantitative exercise above when (i) increasing the degree of idiosyncratic risk, (ii) set the stock of government bonds to zero, (iii) do not model a Pareto tail in labor productivity, (iv) ignore climate damages, and (v) ignore the subsistence level of dirty goods consumption. I re-calibrate all model versions to the same initial steady state and I focus on two cases: adjusting both the average labor income tax and the carbon tax, and only the latter.

In the first case, I find that the carbon tax remains relatively stable across most scenarios. An exception is the exercise with no climate damages, when the planner sets the carbon tax at the lower

[2] An alternative would be to optimize over time-varying tax instruments, as in Douenne, Dyrda, Hummel and Pedroni (2024), and to solve the full Ramsey plan. However, given the high computational demands of this approach, I instead fix a transition path for the tax instruments and optimize over the terminal value.

bound of zero. However, the labor income tax exhibits more variation across the scenarios. Overall, this finding is consistent of the Tinbergen rule: one instrument for one objective. While the carbon tax remains largely unchanged, except when its primary purpose is eliminated, labor income taxes, in combination with transfers, are actively adjusted to provide redistribution and insurance.

Different from the first case, where the average income tax absorbed much of the adjustment, in the second case the carbon tax varies much more across scenarios. This reflects that in the presence of uninsurable idiosyncratic income risk, the carbon tax must now also bear the burden of providing redistribution and insurance. Indeed, I find that, compared to the benchmark, the carbon tax is higher with a higher degree of idiosyncratic risk and in the economy with zero bonds. The opposite holds true for the economy without the Pareto tail in labor productivity, as this modelling choice implies a reduction in idiosyncratic risk. Moreover, the carbon tax is positive in the scenario where climate damages are absent. Finally, across all scenarios, the contribution from insurance remains large and positive, underscoring its role as the primary driver of welfare gains.

In all scenarios where the average labor income tax increases, economic activity declines significantly. This raises the question of whether and to what extent the lower carbon tax in the combined scenario, where both labor and carbon taxes are adjusted, compared to the τ_d-only scenario, is partly driven by a weaker economy with lower emissions. Hence, in a final exercise, to fully isolate the role of idiosyncratic income risk and precautionary savings in shaping welfare-maximizing carbon taxation, I also consider a representative-agent version of the model. I only change the household block in the model and again re-calibrate the economy to the same initial steady state. In this setting, agents are always on their Euler equation and have no motive for precautionary savings. In the representative-agent economy, when adjusting τ_d only, the planner sets the carbon tax approximately 20% lower than in the heterogeneous-agent case, emphasizing again the role of idiosyncratic risk in raising the value of carbon taxation.

Related literature and contribution My paper contributes to several strands of the literature overarching optimal fiscal policy, consumption dynamics, and environmental economics.

My key contribution to this literature is the analysis of welfare-maximizing carbon taxation in a dynamic general equilibrium economy with a climate externality on production and where households face uninsurable idiosyncratic risk. This risk induces precautionary savings behavior, leading concave Engel curves with respect to current income and consumption. Notably, this shape is in line with empirical research that finds a concave relationship between the consumption of emissions and current after-tax income (Levinson and O'Brien, 2019; Sager, 2019; Wöhrmüller, 2024). Moreover, to assess the welfare gains associated with the tax reforms studied in this paper, I apply the welfare decomposition of Bhandari et al. (2023); a novel application in a climate-economy context.

The quantitative model combines a heterogeneous-agent incomplete-market economy in the spirit of Bewley (1986); Huggett (1993); Aiyagari (1994) with a climate sector, which yields an endogenous distribution over income and wealth, and heterogeneity in marginal propensities to consume. Hence, my model allows to study the interaction of climate policies and economic inequalities in a unified framework. Thereby, I connect two lines of research.

The first line is a rapidly growing literature which analyzes optimal carbon taxation in quantitative macroeconomic models. Building on the seminal work by Nordhaus (1992, 1993), who developed the first integrated assessment model (IAM) to analyze climate damages within a centralized economic framework, several papers moved to decentralized market structure. For instance, Golosov, Hassler, Krusell and Tsyvinski (2014) derive a formula for the optimal carbon tax in a dynamic stochastic general-equilibrium model with an externality and resource scarcity. Building on their quantitative work, Barrage (2020) quantifies optimal carbon taxation in a model with tax distortions, and in turn, Douenne, Hummel and Pedroni (2023) quantify the additional impact of inequality. Bourany (2025) studies optimal carbon taxation in a multi-country IAM. He shows analytically that the optimal uniform carbon tax follows the first-best Pigouvian rule, if cross-country transfers are possible. However, in the second-best where such transfers are absent, the carbon tax also reflects redistributive effects and distortions. My paper similarly highlights the importance of the instruments available to the planner.[3]

The second line investigates the impact of idiosyncratic uncertainty and borrowing constraints on individual consumption demand. In particular, in the presence of idiosyncratic risk both prudence in preferences as well as borrowing constraints give rise to a precautionary saving motive which renders the consumption function concave in current income and wealth (Leland, 1968; Sandmo, 1970; Zeldes, 1989*b*,*a*; Kimball, 1990*a*,*b*; Carroll and Kimball, 1996; Huggett and Ospina, 2001; Carroll, Holm and Kimball, 2021).[4] I build on this literature and show that this concavity is also present in a two-goods framework with quasi-homothetic preferences.

Turning to papers that connect these two lines of research, Bosetti and Maffezzoli (2014) is a very early example that introduces studies aggregate and distributional consequences of carbon taxation in a heterogeneous-agent model. Related, Benmir and Roman (2022) study the 2050 net-zero emissions target for the U.S. in a HANK model with a climate block. Moreover, in recent years and since this paper was first circulated, there are two contributions that study optimal climate policies in models with incomplete markets. Douenne et al. (2024) solve the full Ramsey problem of determining the optimal paths of carbon and labor income taxes in a climate-economy model, building on computational methods from Dyrda and Pedroni (2023). Belfiori, Carroll and Hur (2025) also consider a climate-economy with two goods to evaluate uniform and heterogeneous carbon taxes, but they do not model climate damages on production or a two-layer energy production structure. I discuss analytical results on optimal carbon taxation presented in these two papers below.

[3] Other quantitative examples study the optimal environmental policy in response to business cycles (Heutel, 2012) or nominal frictions and uncertainty (Annicchiarico and Di Dio, 2015), or in an overlapping generations framework (Kotlikoff, Kubler, Polbin and Scheidegger, 2021*a*; Kotlikoff, Kubler, Polbin, Sachs and Scheidegger, 2021*b*).

[4] Lugilde, Bande and Riveiro (2019) survey the empirical literature on precautionary savings. They conclude that papers which "test the effect of uncertainty about future income on consumption/saving decisions, especially [those] using micro data, tend to provide robust and convincing results as regards the existence of a precautionary motive for saving" (p.507). Examples of micro-panel studies in different countries include Carroll and Samwick (1997, 1998); Guariglia and Rossi (2002); Guariglia (2003); Lugilde, Bande and Riveiro (2018).

Conceptually, my exercise builds on the theoretical literature on optimal carbon taxation. In particular, Jacobs and van der Ploeg (2019) show that the optimal carbon tax should be equal to the marginal external damage of pollution if Engel curves are linear and the social planner has access to a non-individual lump-sum transfer and linear income taxes.[5] In other words, the optimal carbon tax follows the Pigouvian rule (Pigou, 1920).[6] Intuitively, any demand change induced by the carbon tax can be undone by changing the lump-sum transfer and the income tax. The main difference in this paper is that I consider a quantitative model with idiosyncratic risk, CRRA utility, and borrowing constraints in which, as explained above, non-linear Engel curves are microfounded.[7]

Recent studies further extend these theoretical analyses under deterministic environments with tax distortions (Barrage, 2020) and inequality (Douenne et al., 2023). Moreover, both Douenne et al. (2024) and Belfiori et al. (2025) study optimal carbon taxes in incomplete-market economies and, remarkably, derive analytical results. To achieve this, however, they have to assume that other tax instruments are optimized, or they have to introduce individualized lump-sum transfers in an essentially constrained efficient framework.

Compared to this theoretical literature, I do not derive analytical results for the optimal carbon tax, as closed-form solutions are not tractable within the class of models considered here, and I do not consider individualized lump-sum transfers. Instead, I conduct counterfactual analyses to disentangle the main forces behind my results, as is common in this literature (see e.g. Conesa, Kitao and Krueger, 2009; Guerrieri and Lorenzoni, 2017; Dyrda and Pedroni, 2023).

Lastly, my paper builds on the literature which studies how to optimally recycle carbon tax revenue (Fried, Novan and Peterman, 2018, 2024; Goulder, Hafstead, Kim and Long, 2019; Ascari, Colciago, Haber and Wöhrmüller, 2025). This paper, on the other hand, examines the welfare-maximizing level of the carbon tax. In other words, instead of fixing the carbon tax and adjusting the level of transfers, I adjust the carbon tax while redistributing the revenue lump-sum to households.

The paper is organized as follows. Section 2 describes the quantitative model. Section 3 presents the data and outlines the estimation strategy. Section 4 briefly discusses the main quantitative exercise and presents the main results. Section 5 concludes.

2 An Economy with Idiosyncratic Risk, Two Goods, and a Climate Externality

This first two parts of this section describe the economic model used in the quantitative analyses to study the interaction between uninsurable idiosyncratic risk and optimal carbon taxes. Households face uninsurable idiosyncratic productivity risk and borrowing constraints, supply labor and consume clean and dirty goods. The structure of production and the climate sector largely follows

[5] This result is reminiscent of earlier studies by Angus Deaton (Deaton, 1979, 1981) in which he demonstrates that uniform commodity taxation is desirable under linear Engel curves and separability in consumption and leisure.

[6] This refers to Proposition 2 in Jacobs and van der Ploeg (2019).

[7] Jacobs and van der Ploeg (2019) is a specific application of a more general result that the optimal carbon tax equals the Pigouvian rate adjusted by the marginal cost of public funds (Sandmo, 1975; Bovenberg and de Mooij, 1994), which equals one under the optimal tax system (Jacobs and de Mooij, 2015; Jacobs, 2018).

Barrage (2020) and Golosov et al. (2014), respectively. In the last part, I discuss household consumption and saving decisions in more detail. In particular, I discuss the emergence of non-linear Engel curves over consumption in this framework, even under quasi-homothetic preferences, which provides the theoretical rationale for carbon taxes to take into account distributional concerns (Jacobs and van der Ploeg, 2019).

2.1 Setup

Households Time is discrete, $t \in \{0, 1, \dots\}$, and there is no aggregate risk. The time period in the model is five years. The economy is populated by a continuum of infinitely-lived households of measure one. Households' preferences are represented by the utility function

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \frac{\left(c_{it}^{\eta} (d_{it} - \underline{d})^{1-\eta}\right)^{1-\gamma}}{1-\gamma} + \chi \frac{(1-n_{it})^{1-\epsilon}}{1-\epsilon} \tag{1}$$

where c_{it} denotes the consumption flow of the clean good, d_{it} denotes the consumption flow of the dirty good, and n_{it} denotes labor supply of household i at time t. The time endowment of each household is normalized to 1. The future is discounted with factor β.

The first part of the preferences in Equation (1) nests a Stone-Geary utility in a CRRA specification. In particular, γ denotes the coefficient of relative risk aversion and $\underline{d}$ is the subsistence level for the dirty consumption goods. It is important to note that the elasticity of substitution between the clean and the dirty good is decreasing in the subsistence level (Baumgärtner, Drupp and Quaas, 2017).[8] η and $(1-\eta)$ are expenditure shares based on total income net subsistence consumption, as will become clear below. Regarding the second part, χ denotes the disutility of labor supply, and ϵ is related to the Frisch elasticity of labor supply, which is equal to $\frac{1}{\epsilon}\frac{1-n_{it}}{n_{it}}$ under these preferences.

Households are subject to idiosyncratic productivity risk captured by a first-order Markov chain $\theta \in \Theta$ with $|\Theta| = \mathfrak{S} < \infty$ and transition matrix $\underset{\mathfrak{S}\times\mathfrak{S}}{\Gamma}$. An agents' pre-tax income is then determined by her productivity, the equilibrium wage per unit of productivity, w_t, and the amount of labor supply: $y_{it}^{pre} = w_t \theta_{it} n_{it}$. Pre-tax income is transformed into net (or after-tax) income using a net income function $\mathcal{T}_t(y) = y - T_t^y(y)$, with the tax function denoted by $T_t^y(\cdot)$. Moreover, households have access to a one-period risk-free bond, a_{it}, to partially insure their consumption stream against idiosyncratic risk. Capital income is taxed at rate τ^k and borrowing is restricted by an ad-hoc constraint $\underline{a}$. Lastly, the government pays lump-sum transfers g_t to the household.

Hence, the household budget constraint at time t is

$$c_{it} + p_{d,t} d_{it} + a_{it+1} = \mathcal{T}_t(y_{it}^{pre}) + (1 + r_t(1-\tau^k))a_{it} + g_t,$$

where $p_{d,t}$ denotes the price of the dirty good and r_t is the equilibrium interest rate.

[8] Under no subsistence consumption this elasticity is one (usual Cobb-Douglas case).

Recursive problem A household is characterized by the pair $(a_{it} = a, \theta_{it} = \theta)$, the household state, and solves the following optimization problem (for notational brevity, I omit that decision rules are functions of the state):

$$
\begin{aligned}
V_t(a,\theta) = \max_{c,d,n,a'} \; & u(c,d,n) + \beta \mathbb{E}_\theta V_{t+1}(a',\theta') \\
& \text{subject to} \\
& c + p_{d,t} d + a' \le (1 + r_t(1-\tau^k))a + \underbrace{w_t\theta n - T_t^y(w_t\theta n)}_{\mathcal{T}_t(w_t\theta n)} + g_t \\
& a' \ge \underline{a}
\end{aligned} \tag{2}
$$

Production I model two production sectors (Barrage, 2020; Douenne et al., 2023).

Final good sector In the final goods sector, indexed by 1, the final good Y_t is produced using a neoclassical aggregate production function

$$
Y_t = (1 - \mathfrak{D}(S_t))\tilde{Z}\tilde{F}_1(K_{1,t}, L_{1,t}, E_t^p) = F_1(K_{1,t}, L_{1,t}, E_t^p; \tilde{Z}, S_t) \tag{3}
$$

with $K_{1,t}$ units of capital, $L_{1,t}$ efficiency units of labor, E_t^p units of energy as inputs. The final good can either be consumed or invested. Moreover, $\mathfrak{D}(S_t)$ represents climate damages to output as a function of the stock of atmospheric carbon S_t with $\mathfrak{D}'(S_t) > 0$. This modelling approach of climate damages follows the seminal work by Nordhaus (1991) and the more recent environmental macroeconomic literature such that total factor productivity (TFP) is denoted by $Z_t = (1 - \mathfrak{D}(S_t))\tilde{Z}$, and TFP net of climate damages is denoted by $\tilde{Z}$.

Energy sector In the energy sector, indexed by 2, energy E_t is produced using a neoclassical aggregate production function

$$
E_t = F_2(K_{2,t}, L_{2,t}) \tag{4}
$$

with $K_{2,t}$ units of capital and $L_{2,t}$ efficiency units of labor. Following Barrage (2020), producers can provide a share μ_t from clean energy production, such that only $E_t^m = (1-\mu_t)E_t$ contributes to the stock of emissions. This clean technology is available at a cost of $\Psi(\mu_t)$ per unit of energy. Dirty energy is taxed at excise rate $\tau_{d,t}$ such that energy firm profits are given by $\Pi_t^E = p_{d,t}E_t - \tau_{d,t}E_t^m - (r_t + \delta)K_{2,t} - w_t L_{2,t} - \Psi(\mu_t)E_t$.[9]

Note that even though households are not taxed directly, the current assumptions on the production function of the energy producer and market structure imply that the pass-through rate of carbon taxes to prices is high, even in general equilibrium. See Appendix A for more details and an example.

Energy is either consumed by households (dirty good) or used in production of the final good

[9] The availability of this clean technology also implies that "dirty" energy could become clean if abatement was large enough. Since I do not achieve full abatement in any of my scenarios and to make the distinction about which good is responsible for the externality as clear as possible, I keep the label clean and dirty good throughout the paper.

such that market clearing on the energy market requires $E_t = D_t + E_t^p$, where D_t denotes aggregate dirty good consumption by households. Lastly, capital and labor are fully mobile across sectors, having in mind a time period of 5 years, such that market clearing implies:

$$K_t = K_{1,t} + K_{2,t} \tag{5}$$

$$L_t = L_{1,t} + L_{2,t} \tag{6}$$

Government The government levies labor taxes on pre-tax income y_t^{pre} using the non-linear labor tax function $T_t^y(y_t^{pre})$, a (constant) linear capital income tax τ^k as well as a carbon tax on energy production $\tau_{d,t}$. Moreover, it issues a constant stock of government debt B, and chooses lump-sum transfers g_t to balance its budget:

$$g_t = r_t B + \mathfrak{T}_t, \tag{7}$$

where $\mathfrak{T}_t$ denotes total tax revenue from labor, capital, and carbon taxes.

Climate sector In modelling the carbon cycle, I follow Golosov et al. (2014).

Carbon cycle The current level of atmospheric carbon concentration, S_t, depends on current and past emissions. In my case, emissions are related to the amount of energy produced net of the abated share:

$$S_t = \sum_{\tau=0}^{\infty} (1 - \Phi_\tau) \left[(1 - \mu_{t-\tau}) E_{t-\tau}\right] = \sum_{\tau=0}^{\infty} (1 - \Phi_\tau) E_{t-\tau}^m$$

where $1 - \Phi_\tau = \varphi_L + (1 - \varphi_L)\varphi_0(1 - \varphi)^\tau$. The three terms in $1 - \Phi_\tau$ have the following interpretation: φ_L is the share of carbon emitted which stays in the atmosphere forever; a share of $1 - \varphi_0$ of the remaining $1 - \varphi_L$ exits the atmosphere immediately; and a remaining share $(1 - \varphi_L)\varphi_0$ that decays at geometric rate φ. To write it recursively, I follow Känzig (2023) and set and $\varphi_L = 0$ to write:

$$S_t = (1 - \varphi) S_{t-1} + \varphi_0 E_t^m. \tag{8}$$

2.2 Equilibrium

Let $A \equiv [\underline{a}, \overline{a}]$ be the set of possible values for a_{it}. Define the state space by $\mathbb{S} \equiv A \times \Theta$ and let the σ-algebra $\Sigma_\mathbb{S}$ be defined as $B_A \otimes P(\Theta)$, where B_A is the Borel σ-algebra on A and $P(\Theta)$ is the power set of Θ. Finally, let $\mathcal{S} = (\mathcal{A} \times \Theta)$ denote a typical subset of $\Sigma_\mathbb{S}$. I define a recursive competitive equilibrium as follows.

Definition 1 (Recursive competitive equilibrium)**.** Given a sequence of government policies $\{\tau_{d,t}, T_t^y\}$, constant capital taxes τ^k, constant net TFP $\tilde{Z}$, and constant supply of government bonds, B, a recursive competitive equilibrium is a sequence of aggregate quantities $\{Y_t, K_t, K_{1,t}, K_{2,t}, L_t, L_{1,t}, L_{2,t}, \mu_t, E_t, E_t^p, S_t\}$, probability measures $\{\Xi_t\}$, each defined over the measurable space $(\mathbb{S}, \Sigma_\mathbb{S})$, decision rules

$\{\mathbf{c}_t(a,\theta), \mathbf{d}_t(a,\theta), \mathbf{n}_t(a,\theta), \mathbf{a}_{t+1}(a,\theta)\}$, prices $\{r_t, w_t, p_{d,t}\}$, and transfers $\{g_t\}$ such that, for all t: (i) given government policies and prices, the decision rules solve the optimization problem Equation (2), (ii) the final goods firm chooses capital $K_{1,t}$, labor in efficiency units $L_{1,t}$, and energy E_t^p to maximize profits, (iii) the energy producer chooses capital $K_{2,t}$, labor in efficiency units $L_{2,t}$, and abatement μ_t to maximize profits, (iv) the government budget constraint

$$g_t + rB = \int_{(A\times\Theta)} T^y(w_t\theta\mathbf{n}_t(a,\theta))d\Xi + \tau^k r_t(B+K_t) + \tau_{d,t}(1-\mu_t)E_t$$

holds, (v) the asset market clears

$$\int_{(A\times\Theta)} \mathbf{a}_{t+1}(a,\theta)d\Xi_t = B + K_{t+1}$$

(vi) factor markets and the energy market clear

$$K_{1,t} + K_{2,t} = K_t, \quad L_{1,t} + L_{2,t} = L_t, \quad \int_{(A\times\Theta)} \mathbf{d}_t(a,\theta)d\Xi_t + E_t^p = E_t,$$

(vii) the goods market clears[10]

$$\int_{(A\times\Theta)} \mathbf{c}_t(a,\theta)d\Xi_t + I_t + \Psi(\mu_t)E_t = Y_t,$$

(viii) the probability measure Ξ satisfies for all $\mathcal{S} \in \Sigma_S$

$$\Xi_{t+1}(\mathcal{S}) = \int_{(A\times\Theta)} Q_t((a,\theta),\mathcal{S})d\Xi_t,$$

where Q_t is the associated Markov transition function induced by Γ and $\mathbf{a}_{t+1}$, and (ix) the stock of emissions evolves according to $S_t = (1-\varphi)S_{t-1} + \varphi_0 E_t^m$.

2.3 Quasi-homothetic preferences and concave consumption functions

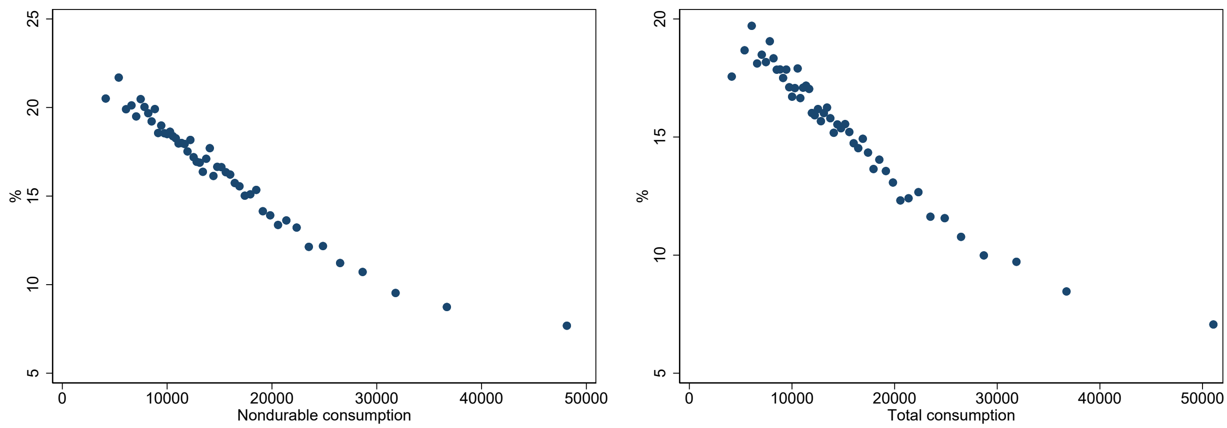
Having presented the full model, I now focus on the household block of the model. The analysis takes as its theoretical starting point the results of Jacobs and van der Ploeg (2019). They show, in a static setting where expenditure equals income, that under linear Engel curves, pollution taxes should be set at the Pigouvian rate, correcting only the externality. This subsection shows, however, that the dynamic household setting outlined above implies *concave* consumption functions for clean and dirty goods over income and wealth. This concavity is due to the uninsurable idiosyncratic risk and precautionary saving behavior. Based on this deviation from linearity, I argue that there is a rationale for the carbon tax to take distributional aspects into account.

Concave consumption functions and Stone-Geary preferences I start by arguing that this concavity is present even for static Stone-Geary preferences as in Equation (1), which imply a linear

[10] This market clearing condition is actually redundant by Walras's law.

Figure 1: Energy expenditure relative to...

(a) ... nondurable consumption expenditure (b) ... total consumption expenditure



Note. This figure shows energy expenditure relative to consumption expenditure with and without durables for households in consumption expenditure 100 bins. Consumption includes expenses for food, gasoline, rent, utilities, communication, transportation, education, childcare, medical needs, vacations, clothing, and recreational activities. Durable components are car repair expenses, down-, loan-, and lease-payments for vehicle loans as well as other expenditure regarding vehicles. All variables have been adjusted using the OECD equivalence scale and are expressed in 2010-$.

Engel curves in settings with no uninsurable idiosyncratic risk and are often used in the environmental literature. The reason for choosing Stone-Geary type preferences is that they replicate the empirical fact of declining expenditure shares of carbon-intensive goods (dirty goods) such as energy.

Indeed, this declining relationship between energy expenditure share and total expenditure is also visible in the PSID data that I later use for estimation. As Figure 1 shows, the expenditure share of US households on energy - defined as the sum of home fuel, heating, and electricity expenditure as a share of two different consumption measures in the PSID - decreases from 20% at the lower end of the expenditure distribution to around 8% at the upper end. Under Cobb-Dogulas utility, the expenditure share would be constant and independent of the expenditure level. However, the introduction of a subsistence level, $\underline{d}$, generates this pattern as households first have to cover the subsistence level before equating the (price-weighted) marginal utilities of the two goods.

Static household problem To further understand the role of the subsistence level, and to facilitate the discussion below, it is instructive to separate the household problem into a dynamic and a static one. In the dynamic problem, the household chooses how much to save for the next period, a_{it+1}, and how much to spend on consumption. Denote this latter total expenditure by e_{it}. In the static problem, the household allocates total expenditure between the clean and the dirty good, respectively. Formally, the household solves the following simple problem, in which e_{it} is predetermined:

$$\max_{c_{it}\geq 0, d_{it}\geq \underline{d}} c_{it}^{\eta}(d_{it}-\underline{d})^{1-\eta} \quad \text{subject to:} \quad c_{it}+p_d d_{it}=e_{it}$$

The solution to this problem is

$$
\begin{aligned}
c_{it} &= \eta\left(e_{it} - p_d \underline{d}\right), \\
d_{it} &= (1-\eta)\frac{e_{it}}{p_d} + \eta \underline{d}.
\end{aligned} \tag{9}
$$

Hence, in this simple setting, decision rules for clean and dirty consumption are linear in total expenditure. The subsistence level is merely a shifter of the expenditure expansion paths. This is an important feature of these particular preferences. In fact, the system of demand equations implied by them are referred to as the Linear Expenditure System (Stone, 1954).

Dynamic household problem Importantly, the main point of this subsection is then the following: Engel curves are *not* linear under the dynamic model described in Section 2.1, which features uninsurable risk and precautionary savings, even with Stone-Geary preferences (nested in a CRRA specification). The key to this observation lies in the concavity of the consumption function in heterogeneous-agent incomplete-markets models (Zeldes, 1989*b*; Carroll and Kimball, 1996), which is inextricably linked to the saving behavior of households (Huggett, 2004; Jensen, 2018). Due to uncertain future income or productivity, households accumulate precautionary savings and especially so when asset and/or income levels are low. Intuitively, the precautionary desire for households to self-insure against possible future negative income realizations increases with lower resources.[11] Hence, poorer households with a relatively stronger precautionary motive have lower consumption and higher marginal propensities to consume (Jappelli and Pistaferri, 2017). In other words, the dirty good Engel curve is non-linear.[12] Note that this concavity is in line with empirical literature that studies the shape of Environmental Engel Curves, that is, the relation between consumption of emissions and current after-tax income (Levinson and O'Brien, 2019; Sager, 2019; Wöhrmüller, 2024).

Figure 2 illustrates these points and highlights the distinction between the static and dynamic framework. Panel 2a shows consumption functions for two productivity types as a function of cash on hand; both are concave and more so for lower levels of assets. Panel 2b shows expenditure on the dirty good as a function of total expenditure. We see that this relation is linear, relating to the static subproblem of the household (Equation (9)). Finally, Panel 2c replicates the decreasing expenditure shares.

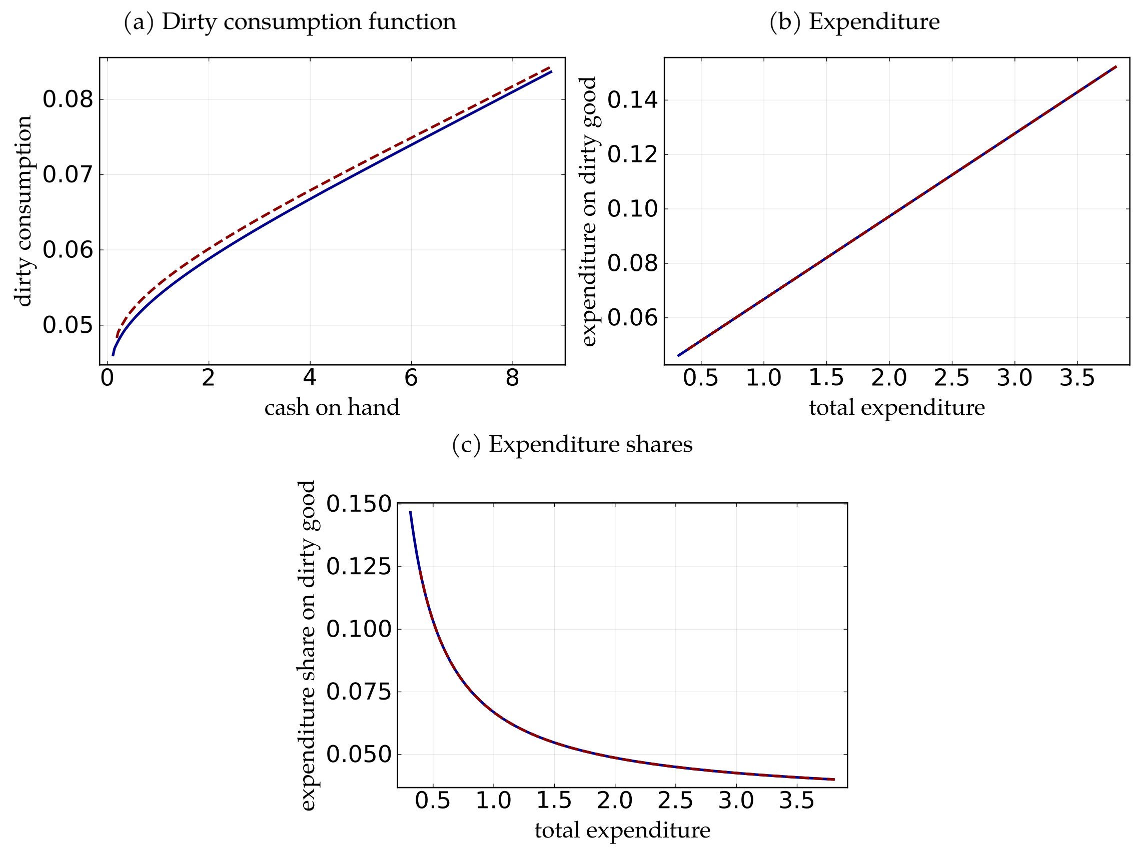
Overall, in the static stage, there is a linear mapping from total expenditures to expenditures for the dirty good. In the dynamic stage, however, there is a concave mapping from income or assets to total expenditure. Both taken together imply a concave mapping from income or assets to dirty goods expenditure or consumption.

The following proposition formalizes this discussion:

[11] Note that the precautionary motive to save is distinct from the smoothing motive, which describes households' desire to save (dissave) when current productivity is high (low) relative to its expected level.

[12] Of course, this result also holds for the clean consumption good. In fact, one can show that the curvature of the two consumption functions are linearly related. See Appendix B.1.

Figure 2: Decision rules, expenditure, and MPCs



Note. The top left panel shows the standard concave (dirty) consumption function in incomplete-markets models due to precautionary savings from the dynamic problem. The top right panel shows the linear relationship between total expenditure and dirty goods expenditure arising from the static problem. The bottom panel shows the decreasing expenditure share on dirty goods induced by the subsistence level on dirty goods consumption.

Proposition 1 (Non-linear Engel curves)**.** *Under (quasi-)homothetic preferences, inelastic labor supply, and for any labor-productivity Markov chain which induces non-negative consumption decisions, both the clean and dirty consumption good exhibits concave Engel curves w.r.t. to income and wealth:*

$$\mathbf{c}_{aa}(a,\theta) < 0,\ \mathbf{c}_{\theta\theta}(a,\theta) < 0 \quad \textit{and} \quad \mathbf{d}_{aa}(a,\theta) < 0,\ \mathbf{d}_{\theta\theta}(a,\theta) < 0$$

Proof. The proof of this proposition is a straightforward application of Theorem 1 in Carroll and Kimball (1996) for a finite horizon or Theorem 4 in Jensen (2018) for an extension to an infinite horizon and borrowing constraints. The proofs of these theorems go through without any modification, but the period utility function is replaced by the indirect utility of the static subproblem, and households choose total expenditure and savings instead of consumption and savings. Intuitively, it relies on the fact that the composition of a linear (decision rule in the static problem) and a concave function (decision rule in the dynamic problem) yields a concave function. □

The takeaway of Proposition 1 and the preceding discussion is that, based on the theoretical results in Jacobs and van der Ploeg (2019), the optimal carbon tax should take into account distributional concerns in quantitative heterogeneous-agent incomplete-market models with precautionary savings.

3 Bringing the model to the data

The discussion in the last part of the previous section suggests that the presence of idiosyncratic risk and precautionary savings matters - in theory - qualitatively for optimal carbon taxation. In light of this, the rest of this paper asks whether these features are also quantitatively important? Hence, I calibrate the model from Section 2.1 and choose functional forms and parameter values. The latter are chosen in two steps. First, I set values according to the literature or to match aggregate data targets from the US economy and the global climate. Second, I estimate the remaining set of structural parameters, which only belong to the household problem, from US microdata and using simulated method of moments in general equilibrium. Table 1 summarizes the parameter values.

Preferences I choose a standard value for relative risk aversion, $\gamma = 2$, and set ϵ to target an average Frisch elasticity of labor supply of unity. The discount factor β is set to achieve asset market clearing given an annual interest rate of 3%, since the estimation of the model is done in general equilibrium. As described later, the preference parameters η and $\underline{d}$ are estimated on microdata and using simulated method of moments (SMM).

Labor productivity process For the labor productivity process, I follow the interpretation of Hubmer, Krusell and Smith, Jr. (2021) and model the process in two steps. First, I model the idiosyncratic productivity process as the sum of a persistent and a transitory shock:

$$\log(\theta_{it}) = \kappa_{it} + \psi_{it}$$
$$\kappa_{it} = \rho\kappa_{it-1} + \varepsilon^{\kappa}_{it}.$$

In particular, the persistence process κ is modelled as an AR(1) with persistence ρ and variance of its innovation of $\sigma^2_{\varepsilon^\kappa}$; the transitory shocks ψ are independently and identically distributed with zero mean and variance σ^2_ψ.

Second, I then assume that productivity levels, in the top 10% are spread out according to a scaled Pareto distribution:

$$\theta = \begin{cases} \exp(\kappa + \psi) & \text{if } F_\theta(\theta) \leq 0.9 \\ F^{-1}_{Pareto(\omega)}\left(\dfrac{F_\theta(\theta) - 0.9}{1 - 0.9}\right) & \text{if } F_\theta(\theta) > 0.9 \end{cases} \tag{10}$$

where F_θ is the cdf of θ and $F^{-1}_{Pareto(\omega)}$ is the inverse cdf of a Pareto distribution with lower bound $F_\theta^{-1}(0.9)$ and tail coefficient ω.

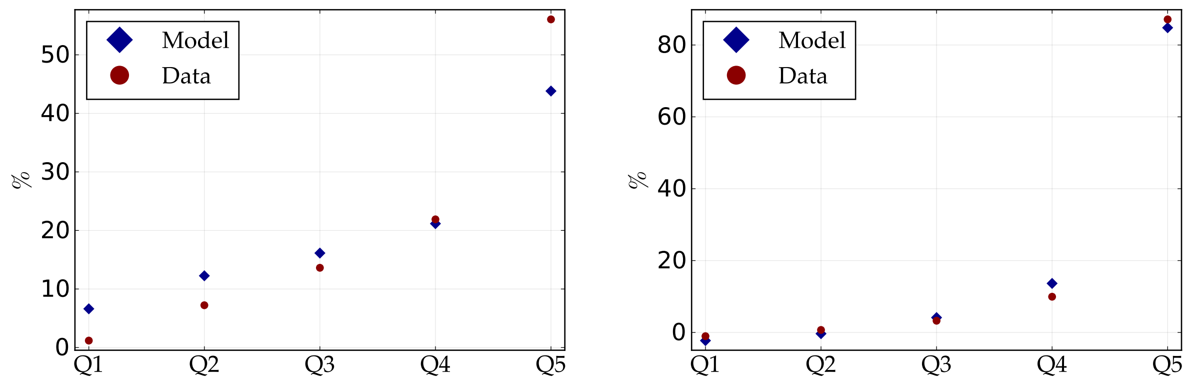
I determine the (annual) variances of the parameters using pre-tax log hourly wage residuals estimated from annual PSID data between 1967-1996, and translate them into the 5-year period unit of the model. Finally, I set the Pareto shape parameter $\omega = 1.6$ as in Aoki and Nirei (2017) and

Ferriere et al. (2023). In Appendix D.2, I describe the estimation procedure in detail.

Figure 3 shows untargeted labor income and wealth shares by quintile from the Survey of Consumer Finances (SCF) in the data and the model counterparts implied by this calibration strategy. The model slightly undershoots labor income shares for the bottom three quintiles, aligns well for Q4, and overshoots at the top. However, the wealth shares across quintiles match the data.

Figure 3: Untargeted quintile shares

(a) Labor income shares by labor income quintile (b) Wealth shares by wealth quintile



Note. This figure shows untargeted labor income shares and wealth shares by their respective quintiles in the initial steady state. The data quintiles are constructed using SCF data spanning waves 2006 to 2018.

Final goods production The technology $\tilde{F}_1(K_{1,t}, L_{2,t}, E_t^p)$ is assumed to be of the constant elasticity of substitution (CES) form

$$\left[(1-s)(K_{1,t}^{\alpha} L_{1,t}^{1-\alpha})^{\frac{\lambda-1}{\lambda}} + s(E_t^p)^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}} \tag{11}$$

with λ as the elasticity of substitution between the capital-labor bundle and energy, and s a share parameter.[13] In equilibrium, the factors of production are rented at rates $r_t + \delta$, w_t, and $p_{d,t}$.

I fix the gross capital share in production α at 0.36 based on standard estimates from the literature (Rognlie, 2016) and the elasticity of substitution between the capital-labor composite and energy λ at 0.547 as found in van der Werf (2008). I follow Straub (2019) and set δ to match a capital-to-output ratio of 3.05. The implied wealth to output ratio is 3.8, close to the most recent estimate of 4 in Piketty and Zucman (2014, Figure IV) for the US. The share parameter s is set to match an energy share of production of five percent as in Fried et al. (2018).

In the initial steady state, I normalize output to unity using the technology parameter Z. Moreover, recall that Z is a product of net of climate damages and damages: $Z = \tilde{Z}(1 - \mathfrak{D}(S))$. Hence, during estimation, I ignore the stock of carbon in the atmosphere in the economy, for I could always update $\tilde{Z}$ to cancel out any resulting damages.

[13] van der Werf (2008) writes that "the (KL)E nesting structure, that is a nesting structure in which capital and labor are combined first, fits the data best, but we generally cannot reject that the production function has all inputs in one CES function". Another recent example where this particular nesting structure is used is Hassler, Krusell and Olovsson (2021).

Energy production Energy is produced using a Cobb-Douglas technology in capital and labor:

$$E_t = K_{2,t}^{\alpha_E} L_{2,t}^{1-\alpha_E}. \tag{12}$$

I set $\alpha_E = 0.597$ following Barrage (2020). Moreover, the abatement cost function is

$$\Psi(\mu_t) = c_1 \mu_t^{c_2}. \tag{13}$$

I follow DICE and set c_2 to 2.6. Hence, the cost function is convex in μ_t, implying that marginal costs are increasing in abatement. To pin down c_1, I use initial steady-state values from Douenne et al. (2023), who largely follow DICE 2016 in their calibration. In particular, the backstop price describes the price of emissions at which there is full abatement, $\mu = 1$, which implies for marginal abatement costs: $c_1 c_2 \mu^{c_2 - 1} E = c_1 c_2 E = P^{backstop} E$. The parameter c_1 is then chosen such that the backstop-price implied energy costs to GDP ratio in initial steady state, $\frac{P^{backstop} E}{Y}$, is equal to 0.27 as in Douenne et al. (2023).

Government I use the three parameter functional form by Gouveia and Strauss (1994) to model the labor income tax function:

$$T^y(y^{pre}; \tau_{0,t}, \tau_{1,t}, \tau_2) = \tau_{0,t} \left(y^{pre} - \left((y^{pre})^{-\tau_{1,t}} + \tau_2 \right)^{-1/\tau_{1,t}} \right). \tag{14}$$

Note that for this particular functional form of the tax function, both the limiting marginal and average tax rate are equal to τ_0. That is, $\lim_{y^{pre} \to \infty} \frac{T^y(y^{pre})}{y^{pre}} = \lim_{y^{pre} \to \infty} (T^y)'(y^{pre}) = \tau_0$.

In the initial steady state, I fix $\tau_0 = 0.264$ and $\tau_1 = 0.964$ based on estimates by Guner, Kaygusuz and Ventura (2014). τ_2 is determined in general equilibrium and is adjusted such that the government budget constraint is satisfied. I set the capital income tax τ^k to 0.36 as in Trabandt and Uhlig (2011). Lump-sum transfers g are set to 0.114 to match a transfer-to-GDP ratio of 11.4% (Dyrda and Pedroni, 2023).

Climate sector

Carbon cycle To calibrate φ and φ_0 I follow Golosov et al. (2014).[14] φ is set to capture the fact that excess carbon has a mean-lifetime of about 300 years (60 periods) such that $(1-\varphi)^{60} = 0.5$, while the calibration for φ_0 captures that half of the CO_2 emissions into the atmosphere are removed after 30 years (6 periods): $\varphi_0 = \frac{0.5}{(1-\varphi)^6}$

Damage function The functional form for the damage function is taken from Golosov et al. (2014):

$$1 - \mathfrak{D}(S) = e^{-\xi S_t}, \tag{15}$$

[14] Golosov and co-authors, in turn, cite Archer (2005) and the 2007 technical summary of the IPCC report (IPCC, 2007)

Table 1: Preset parameters for estimation

	Description	Value	Target/source
Preferences			
γ	Risk aversion	2.0	literature
ϵ	Curvature of utility from leisure	4.06	Average Frisch elasticity of unity
η	Relative weight on clean consumption	0.9354	PSID
β	Discount factor (annual)	0.9632	$r = 0.03$
Productivities (annual)			
ρ	Productivity shock persistence	0.9577	PSID
$\sigma^2_{\epsilon^\kappa}$	Variance of innovations to persistent shock	0.0203	PSID
σ^2_ψ	Variance of transitory shocks	0.0556	PSID
σ^2_ν	Variance of measurement error	0.02	French (2004, p.608, Table 5)
ω	Pareto tail parameter	0.16	Aoki and Nirei (2017)
Production			
Final goods production			
λ	Substitution elasticity	0.547	van der Werf (2008, p.2972, Table 3)
α	Capital share	0.36	literature
δ	Depreciation (annual)	0.115	annual $K/Y = 3.05$ (FRED)
X	Net total factor productivity	2.9017	Normalize output to unity
s	Share parameter	0.0054	Energy share in production of 5%
Energy production			
α_2	Capital share	0.597	Barrage (2020)
Abatement			
c_1	Scale abatement cost function	1.242	Backstop price to GDP (see text)
c_2	Exponent abatement cost function	2.6	DICE 2016
Government			
τ_0	Average labor income tax	0.264	Guner et al. (2014, p.573,Table 10)
τ_1	Progressivity of labor income tax	0.964	Guner et al. (2014, p.573,Table 10)
τ_2	Scaling parameter	1.2038	Government budget constraint
τ^k	Capital income tax	0.36	Trabandt and Uhlig (2011, p.311,Table 1)
B/Y	Public debt (annual) / GDP	0.146	FRED
g/Y	Transfers / GDP	0.114	Dyrda and Pedroni (2023)
Climate sector			
Damages			
ξ	Damage parameter	0.0032	GDP loss of 2.5% when doubling steady state emissions
Carbon cycle			
φ	Emissions decay parameter	0.0115	Golosov et al. (2014)
φ_0	Emissions share parameter	0.5359	Golosov et al. (2014)

Note. This table shows preset and calibrated parameters of the quantitative model which is used to estimate the remaining parameters via indirect inference. FRED datasources can be found in Appendix D.1.

where ξ governs the strength of output damages of a marginal increase in atmospheric carbon.[15] As in Heutel (2012) and Känzig (2023), I set the parameter ξ such that a doubling of the initial steady state carbon emissions, without carbon taxes, would imply an output loss of 2.5% of GDP. This damage loss thus represents the business-as-usual scenario.[16]

Parameters pinned down by microdata and SMM The remaining structural parameters are (i) the utility elasticity η, (ii) the subsistence level $\underline{d}$, (iii) the disutility of labor χ, and (iv) the borrowing limit $\underline{a}$. I will estimate η directly from microdata and pin down $\underline{d}$, χ, and $\underline{a}$ via SMM. In the following two subsections, I will describe the microdata and targets I use.

3.1 Data

I use data from the Panel Study of Income Dynamics between 2005-2018 to compute the micro moments which I target in estimation. The PSID is a widely used longitudinal survey containing information on household demographics, income, and wealth. In the waves of 1999 and 2005, respectively, the PSID extended its collection of consumption expenditure data. It now captures over 70 percent of all consumption items available in the Consumer Expenditure Survey (CEX) and around 70 percent of aggregate consumption in the national income and product accounts (NIPA) (Blundell, Pistaferri and Saporta-Eksten, 2016). The PSID was attested to be a high quality dataset in terms of general low sample attrition rates and high response rates (Andreski, Li, Samancioglu and Schoeni, 2014).

Variables The following variables are all on the household level. For instance, income refers to income from both the head and the spouse in the household, if present. Moreover, all monetary variables in the analysis have been adjusted using the OECD equivalence scale and are expressed in 2010-$.

Income *Labor income* refers to all income from wages, salaries, commissions, bonuses, overtime and the labor part of business income *Total income* in addition includes transfers such as as well as social security income. Both income variables are net of taxes, which were computed using NBER's Taxsim program.

Consumption *Nondurable consumption* includes expenses for food, gasoline, rent, utilities, communication, transportation, education, childcare, medical needs, vacations, clothing, and recreational activities. *Total consumption* also includes durable components such as car repair expenses, down-, loan-, and lease-payments for vehicle loans as well as other expenditure regarding vehicles. Moreover, I define *energy expenditure* as expenditure on gasoline, electricity, and heating. All three categories are greenhouse gas intensive goods and are thus used as a data counterpart for the dirty consumption good in the model.

[15] As Golosov et al. (2014) explain, Equation (15) is an approximation that conflates the *concave* relationship between CO_2 concentrations and temperature, and a *convex* relationship between temperature and damages. In particular, it implies constant marginal damages - measured as a share of GDP: $\frac{\partial Y / \partial S}{Y} = -\xi$

[16] Recent empirical evidence by Bilal and Känzig (2024) suggests that the macroeconomic damages from climate change could be considerable larger.

Wealth My wealth variable refers to financial wealth net of liabilities. In particular, I include the value of one's real estate assets net of remaining mortgages, checking and saving accounts, stocks, bonds, business assets, IRAs or other annuities, and cars. I subtract liabilities such as credit card debt, student debt, outstanding medical bills, legal debt, loans obtained from relatives, and business debt.

Sample My baseline sample includes all PSID waves from 2005-2019, and consists of households where the head is between 25 and 60 years. I exclude observations for which information on consumption, income, wealth, education, household size, or region is missing. Furthermore, I remove observations with labor income below half the state minimum wage as well as top and bottom 1% of the remaining observations on consumption, income, and wealth. This leaves me with a sample of 21,750 households, around 2700 observations per year.

3.2 Data targets

Based on the PSID sample and variables as just described, I first present the regression to estimate η. Thereafter, I present the moments I use to estimate the remaining three structural parameters.

Dirty good allocation rule In the data I only observe expenditures, that is, the product of price and quantity. Hence, the data counterpart to Equation (9) is

$$p_d d_{it} = \alpha_i + \delta_1 e_{it} + \mathbf{X}_{it}'\omega + \varepsilon_{it}, \tag{16}$$

where now e_{it} denotes *observed* total expenditure, $p_d d_{it}$ *observed* expenditure on dirty goods of household i at time t. $\mathbf{X}$ is a vector of controls including a constant, household-size dummies, household head's five-year age bracket, region, household and year dummies (Pedroni, Singh and Stoltenberg, 2022; Straub, 2019). Total expenditure will be instrumented by total post-tax income as in Blundell, Chen and Kristensen (2007). The coefficient of interest is δ_1 which informs on the magnitude of η.

Table 2 shows estimation results for the baseline specification (1) and various robustness exercises (2)-(4) of Equation (16). Column (2) omits the region dummies in the control vector. Column (3) additionally controls for liquid assets. Column (4) uses an alternative measure for consumption as the (instrumented) regressor. In all specifications, the first stage F-statistic is well above 10.[17]

To interpret the coefficients, let us look at column (1). The coefficient on total expenditure is equal to 0.0646. According to Equation (9), this coefficient identifies $1-\eta$, which suggests an η of 0.9354. This value is remarkably similar to Fried et al. (2018) who find 0.931 using a different calibration strategy.

Data targets to identify $\underline{d}$, χ, and $\underline{a}$

[17] This rule of thumb is valid as I only consider one endogenous regressor and one instrument (Stock and Yogo, 2005).

Table 2: Dirty good regression

	dirty goods expenditure			
	(1)	(2)	(3)	(4)
Total expenditure	0.0646	0.0643	0.0623	0.0486
	(0.0152)	(0.0152)	(0.0156)	(0.0113)
Observations	22033	22033	22033	22027
R-squared	0.3059	0.2996	0.3077	0.3100

Note. This table shows second stage (IV) coefficients δ_1 and δ_0 of Equation (16) for different specifications. Column (1) is the baseline case as specified in the text. Column (2) omits the region dummies in the control vector. Column (3) additionally controls for liquid assets. Column (4) uses an alternative measure for consumption as the (instrumented) regressor. Standard errors are corrected for heteroskedasticity and clustered at the household level.

Table 3: SMM estimation results

Parameter	Estimate	Standard Error	Target	Data Value	Model Value
χ	0.6661	(0.0106)	$\int \mathbf{n}(a,\theta)d\Xi$	0.3627	0.3627
$\underline{d}$	0.0389	(0.0004)	$\int \frac{p_d d_i}{e_i} d\Xi$	0.164	0.164
$\underline{a}$	-0.0946	(0.0108)	$\frac{a}{y}\Big\vert_{10}$	-0.2527	-0.2527

Note. Asymptotic standard errors in parentheses are computed using a non-parametric panel bootstrap with 200 repetitions; see Appendix E for details.

Average expenditure shares on energy As shown in Figure 1, the expenditure shares on energy are decreasing with higher total consumption expenditures. To pin down $\underline{d}$, I thus target the average expenditure shares on energy with respect to total nondurable consumption expenditures in my PSID data.

Hours worked The cross-sectional average of weekly working hours of household heads in my sample is 40.61. Given that a full week has 168 hours and assuming 8 hours per day for sleep and other personal care leaves 112 hours per week as time endowment (Guerrieri and Lorenzoni, 2017). Hence, average hours worked as a share of the total time endowment gives 36.3% which is targeted in estimation.

Wealth-to-income ratio To consider the distribution of endogenous variables in my model, I follow Stoltenberg and Uhlendorff (2023) and target the 10$^{\text{th}}$ percentile of the wealth-to-income distribution. A 10$^{\text{th}}$ quantile regression on a constant yields $\hat{\beta}_{10} = -0.253$. The value is precisely estimated.

3.3 SMM results

Table 3 shows the estimation results. I construct asymptotic standard errors using a non-parametric panel bootstrap with 200 repetitions (Appendix E).

Regarding the subsistence level, I estimate $\underline{d} = 0.0389$. The parameter is precisely estimated. Average hours worked, as a fraction of agents' time endowment, is pinned down well by the disutility of labor χ, which is estimated to equal 0.6661. Again, this parameter is estimated precisely. The borrowing limit is $\underline{a} = -0.0946$, which amounts to 15% of average gross income that can be borrowed every period. The model matches the targeted moment of the wealth-to-income distribution well.

Identification As common in these type of models, I have no proof of global identification of my parameters. For this reason, I assess how different model-implied moments are affected when I change two of the three parameters and fix the remaining one at its best fit value. Figure E.12 in Appendix E shows that every parameter is well identified.

4 Quantitative exercise

Having brought the model to the data, I now use the estimated economy as laboratory to answer the main question of this paper. To what extent do idiosyncratic risk and precautionary savings matter for the welfare-maximizing carbon tax? To this end, I first specify a social welfare function as the objective for the government (social planner). Next, I look for the welfare-maximizing carbon tax in general equilibrium, when the lump-sum transfer is adjusted to clear the government budget constraint. I consider tax reforms that are introduced gradually along a linear path over five periods (25 years), that is, reaching their terminal values in the fifth period and staying constant thereafter. Hence, to compute the welfare implications, I do take into account the transitional dynamics.[18]

Social welfare function I assume that the social planner is utilitarian and maximizes social welfare defined over households' value functions along the transition:

$$SW = \int \underbrace{\mathbb{E} \sum_{t=0}^{\infty} \tilde{\beta}^t u(c_t, d_t, n_t)}_{\tilde{V}_{t,(\tau_d,\tau_0,\tau_1)}} \, d\Xi_0, \tag{17}$$

where the welfare paths are weighted by the initial steady state distribution, and $\tilde{\beta}$ denotes a social discount factor with $\tilde{\beta} > \beta$. I set $\tilde{\beta} = 0.95$.

The planner chooses tax instruments τ_d, τ_0, and τ_1, or various combinations thereof, to maximize Equation (17) while setting g to balance the government budget. Hence, the subscript (τ_d, τ_0, τ_1) stresses that the value functions along the transition are associated with the tax scenario in which all taxes parameters are adjusted. In this specification, every household gets the same welfare weight. However, due to concavity in the utility function, the government has an implicit preference for redistribution, as the marginal utility of poorer households is higher than that of the rich.

[18] The (negative) social welfare function is minimized using a multistart global optimization algorithm ("Tik-Tak"). For local minimization routines in this algorithm, I use the derivative-free BOBYQA routine from starting points that are determined using the Sobol sequence and a pre-testing phase (Guvenen, 2011; Arnoud, Guvenen and Kleineberg, 2019).

Table 4: Welfare-maximizing carbon taxes and consumption equivalent variations

Tax Parameter	Benchmark	τ_d, τ_0 & τ_1	τ_d & τ_0	τ_d	τ_0 & τ_1
τ_d	0.0000	0.0183	0.0183	0.0930	0.0000
τ_0	0.2640	0.6135	0.6131	0.2640	0.6143
τ_1	0.9640	0.9669	0.9640	0.9640	0.9662
CEV	–	2.750	2.750	0.046	2.748

Note. This table shows the welfare-maximizing tax rates in the different scenarios. The column names describe which tax rates are adjusted, relative to the benchmark, in the respective scenario. The last row gives the consumption equivalent variation as derived in Appendix C.3.1.

Finally, I motivate the choice of a social discount factor larger than the private one with two points. First, from a normative perspective, governments often place greater weight on the welfare of future generations than private agents do, reflecting societal concerns for intergenerational equity. Second, I want to avoid the theoretical possibility that a Ramsey steady state may not exist, or that potential near-immiseration is optimal, as in Auclert, Cai, Rognlie and Straub (2024), since my analysis also considers perfect foresight transitions while adjusting labor income taxes.

4.1 Welfare-maximizing carbon taxes

Table 4 shows the welfare-maximizing carbon taxes in the new steady state for different combinations of tax instruments that are adjusted. Overall, I consider four main scenarios. The second column shows the benchmark values as calibrated in the initial steady state. The third column shows the combination of taxes which maximizes welfare along the transition if τ_d, τ_0 and τ_1 get adjusted. Similar for the remaining columns. Transitional dynamics of different aggregate variables and parameters are depicted in Appendix C.1.

There are three important takeaways. First, when only τ_d is optimized, while keeping the labor tax parameters at their benchmark values, the welfare-maximizing value of the carbon tax increases substantially to 0.0930, over five times higher than when all tax parameters are jointly optimized. The welfare gain is small, however, with a CEV of 0.046. Second, if adjusted, the average labor income tax more than doubles, suggesting that the initial steady state is characterized by a tax system that is too regressive. This result is in line with Ferriere et al. (2023), who also find a stark increase in the average and marginal tax rates.[19] Third, adjusting the tax progressivity parameter has only minimal impact. Indeed, the results highlight that substantial welfare gains are primarily associated with changes to τ_0.

Before discussing these results further, I want to dig deeper into where the welfare gains are coming from. Hence, the following section presents a welfare decomposition for each of these scenarios.

[19] In Appendix B.2, I compare my average and marginal income tax rate under different scenarios to those implied by Ferriere et al. (2023).

Table 5: Welfare Decomposition: Efficiency, Redistribution, Insurance

	Clean consumption	Dirty consumption	Leisure	Overall
τ_d, τ_0 & τ_1				
Efficiency	-2.684	-0.254	1.929	-1.009
Redistribution	-0.052	-0.004	0.0	-0.055
Insurance	1.669	0.115	0.28	2.064
τ_d & τ_0				
Efficiency	-2.694	-0.25	1.939	-1.005
Redistribution	-0.068	-0.005	-0.002	-0.074
Insurance	1.681	0.116	0.282	2.079
τ_d				
Efficiency	1.513	-3.619	1.405	-0.701
Redistribution	0.165	0.011	0.073	0.249
Insurance	1.077	0.074	0.301	1.452
τ_0 & τ_1				
Efficiency	-2.706	-0.239	1.905	-1.04
Redistribution	-0.068	-0.005	-0.002	-0.075
Insurance	1.709	0.118	0.289	2.115

Note. This table shows the welfare decomposition á la Bhandari et al. (2023) for clean consumption, dirty consumption, and leisure along the transition in the different scenarios. The decomposition decomposes welfare gains into Efficiency (E), Redistribution (R), and Insurance (I) components for each good. Every row sums up to the "Overall" column. Within each scenario, all three components in the "Overall" column sum up to 1. The mathematical description behind the decomposition can be found in Appendix C.3.2.

4.2 Sources of welfare gains along the transition

I follow Bhandari et al. (2023) and decompose the welfare gains along the transition in three components. First, an efficiency component (E) that captures changes in total resources. Second, a redistribution component (R) that captures changes in consumption shares that households expect to receive. Third, an insurance component (I) that captures changes in households' consumption risk. A bit more precise, a household's choice of a particular good k satisfies the identity $c_{k,i} = \mathbb{E}c_{k,i} \frac{\mathbb{E}_i c_{k,i}}{\mathbb{E}c_{k,i}} \frac{c_{i,k}}{\mathbb{E}_i c_{k,i}} \equiv C \times w_{i,k} \times (1 + \epsilon_{k,i})$, where C denotes the aggregate quantity, w denotes the share that the household expects to receive of that aggregate, and ϵ captures the uncertainty that the household faces. Hence, the three terms capture the three components. Appendix C.3.2 describes the implementation in detail.

In Table 5, I present this decomposition along the transition for clean consumption, dirty consumption, and leisure ($l = 1 - n$). There are two points to keep in mind when reading this table. First, for each component, the effects are decomposed into their contribution from the respective goods. That is, the first three columns sum up to the last column, labeled "Overall". Second, for each scenario, the overall welfare gain is decomposed into the three components. This means that, within each scenario, the "Overall" columns sums up to 1.

My results are as follows. First, across all scenarios where the insurance channel is the main driver of overall welfare gains. With respect to the scenarios involving labor income taxes, this is in line with Ferriere et al. (2023). However, this results also holds for the scenario in which only the carbon tax is adjusted. Second, the redistribution effects are generally small and negative in most cases. An exception here is the third scenario. Adjusting carbon taxes only yields a positive redistributive contribution. Of course, this is not due to the carbon tax alone, but rather in combination with the lump-sum transfer. Together, these two findings underscore a central message of this paper: while the redistributive effects of carbon taxation have been emphasized in the literature (Känzig, 2023), the welfare gains from improved insurance represent an important and previously underappreciated channel. Importantly, this insurance channel is absent by construction in representative-agent or two-agent models and can only be analyzed in a model with idiosyncratic risk and precautionary savings.

Lastly, the efficiency component is always negative, but its decomposition differs across scenarios. Leisure does increase and dirty consumption does decrease in every scenario. However, the contribution of clean consumption is positive in the third scenario when only carbon taxes are adjusted. It is also negative otherwise. A higher carbon tax increases the price of the dirty good and hence, households substitute away from dirty to clean. Moreover, since the elasticity of substitution between the two goods is increasing in cash-on-hand due to the subsistence level (Baumgärtner et al., 2017), richer households decrease their dirty goods consumption by more and the dirty goods distribution gets compressed from above.

4.3 Sensitivity analysis

In this section, I analyze how the welfare-maximizing taxes and the contributions of the welfare components change if I change several details about the heterogeneous-agent model. Thereby, I focus on the second and third scenario and only adjust τ_0 & τ_d and τ_d, respectively. Finally, I also repeat the third scenario in a representative-agent economy. In all robustness exercises, I re-calibrate the model to the same initial steady state.

Different model specifications I consider five different model specifications. First, increasing the degree of idiosyncratic risk. Here, I increase the variance of the persistent idiosyncratic shock to raise the standard deviation of the (log) productivity process by 0.04, similar to the exercise in Auclert and Rognlie (2020). Second, I set the stock of government bonds to zero, i.e. $B = 0$, to limit the supply of assets for self-insurance. Third, I do not model a Pareto tail in labor productivity, i.e. I do not adjust the idiosyncratic process with Equation (10). Fourth, I ignore climate damages by setting $\xi = 0$. Finally, I ignore the subsistence level of dirty goods consumption by setting $\underline{d} = 0$.

Table 6 shows the results for all of these specifications if both the average labor income tax parameter and the carbon tax get adjusted. Overall, the table shows that when average labor income taxes can be adjusted, the welfare-maximizing carbon tax varies only slightly. The carbon tax varies between 0.02 in the specification with higher idiosyncratic risk and no subsistence level and 0.013 when there is no Pareto tail, compared to a benchmark level of 0.018. An exception is the specifi-

Table 6: Taxes and welfare decomposition under several specifications for the τ_0 & τ_d scenario

Model	Carbon Tax (τ_d)	Avg. Tax (τ_0)	Contribution E	Contribution R	Contribution I
Benchmark	0.018	0.613	-1.005	-0.074	2.079
Higher income risk	0.02	0.65	-0.893	-0.034	1.927
Zero bonds	0.018	0.68	-0.669	0.081	1.588
No pareto tail	0.013	0.509	-1.388	0.09	2.298
No damages	0.0	0.613	-1.019	-0.057	2.076
No subsistence level	0.02	0.566	-1.291	-0.211	2.502

Note. This table shows the welfare-maximizing tax rates as well as the welfare composition as in Bhandari et al. (2023) if both labor income taxes and carbon taxes can be adjusted for different model specification.

Table 7: Taxes and welfare decomposition under several specifications for the τ_d scenario

Model	Carbon Tax (τ_d)	Avg. Tax (τ_0)	Contribution E	Contribution R	Contribution I
Benchmark	0.093	0.264	-0.701	0.249	1.452
Higher income risk	0.115	0.264	-0.679	0.237	1.442
Zero bonds	0.103	0.264	-0.644	0.223	1.421
No pareto tail	0.051	0.264	-0.738	0.232	1.507
No damages	0.058	0.264	-1.835	0.414	2.421
No subsistence level	0.159	0.264	-1.099	0.338	1.761

Note. This table shows the welfare-maximizing tax rates as well as the welfare composition as in Bhandari et al. (2023) if only carbon taxes can be adjusted for different model specification.

cation without climate damages when the carbon tax drops to zero, the lower bound considered in the optimization exercise. These findings reflect the logic of the Tinbergen rule: one instrument per objective. The carbon tax remains relatively stable across specifications, except when climate damages are removed and its core purpose is eliminated. Instead, labor income taxes and transfers are actively adjusted to fulfill the distinct objectives of redistribution and insurance. Indeed, the insurance component of welfare remains the dominant one in all specifications.

In contrast, Table 7 shows the results for all of these specifications if only the carbon tax gets adjusted. The average labor income tax parameter is kept fixed at its benchmark value. In this case, the welfare-maximizing carbon tax varies much more across the different specifications, reflecting its expanded role in internalizing climate damages as well as redistribution and insurance motives in an economy with uninsurable idiosyncratic risk. This is evident in the higher carbon taxes observed under greater idiosyncratic risk (0.115) and in the zero-bonds economy (0.103). In the specification without the Pareto tail, where inequality at the top is reduced, the carbon tax is significantly lower (0.051), consistent with weaker redistributive and insurance motives. Moreover, the carbon tax remains positive even when climate damages are eliminated. Finally, eliminating the regressive impact of the carbon tax by removing the subsistence level of consumption increases the carbon tax to 0.159. Again, the ordering of welfare components remains unchanged compared to the benchmark, with the insurance component being the largest one.

Representative-agent model Finally, we see in Appendix C.1 that when average labor income taxes get increased to their welfare-maximizing level, economic activity is declining strongly. Output drops by about 12% and energy production by about 7%. This gives rise to the question whether and to what extent this slower economic activity leads to a lower welfare-maximizing carbon tax.

Hence, to fully remove redistributive and insurance motives of the planner, I optimize over the carbon tax when replacing the household block in the model with a representative agent (RA). In this model, there is no precautionary savings motive and agents are always on the Euler equation. I find that when adjusting τ_d only, the planner sets the carbon tax approximately 20% lower than in the heterogeneous-agent case. As a result, transfers, abatement, and the price of energy are lower in the RA economy, as can be seen from the transitional dynamics in Appendix C.2. This finding underscores again how uninsurable idiosyncratic income risk increase the value of the welfare-maximizing carbon tax, not only as a corrective tool for environmental externalities but also as a mechanism for redistribution and insurance in the absence of more flexible fiscal instruments.

5 Conclusion

In this paper, I studied the welfare-maximizing carbon tax in a climate-economy model with idiosyncratic risk and borrowing constraints in general equilibrium. I first calibrated and estimated the model on U.S. household panel data. In a next step, I used the model as a laboratory and optimized over the carbon tax and labor income tax instruments in general equilibrium taking transitional dynamics into account.

When labor tax instruments are held fixed, I find that the optimal carbon tax rises with the level of uninsurable idiosyncratic risk. In contrast, when labor taxes are allowed to adjust, the carbon tax remains relatively stable across different economic environments. Overall, welfare gains are primarily driven by improved insurance provision.

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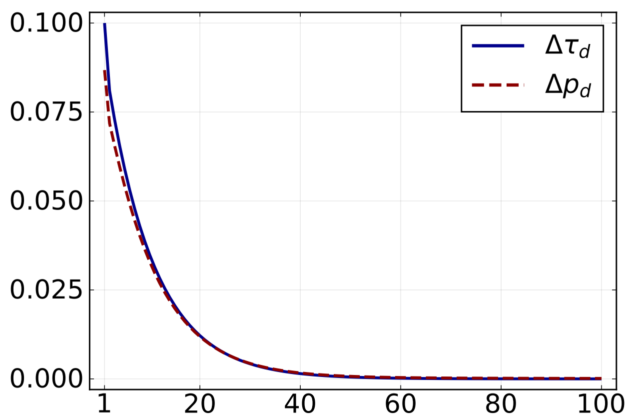
Appendix

A Carbon tax pass-through

The carbon tax in the model is placed on the energy firm. However, the model specifications imply a large pass-through of carbon taxes to energy prices such that households (and the final good firm) are significantly affected by carbon tax increases. In particular, under Cobb-Douglas preferences and perfect competition, marginal costs of the energy producer not a function of energy produced, making the supply of energy perfectly elastic. As a result, carbon taxes are largely passed through to energy prices. Solely general equilibrium considerations alleviate this effect.

Figure A.4 shows an example impulse response, where I increase the carbon tax by 0.1 and let it revert to the initial steady state. In this particular example, the pass-through rate, defined as the change in prices divided by the change in carbon taxes, is about 85%.

Figure A.4: Pass-through of carbon taxes to prices in general equilibrium



Note. This figure shows an example of the pass-through of the carbon tax to the price of energy in general equilibrium, when the carbon tax shock is transitory.

B Additional figures - Steady state

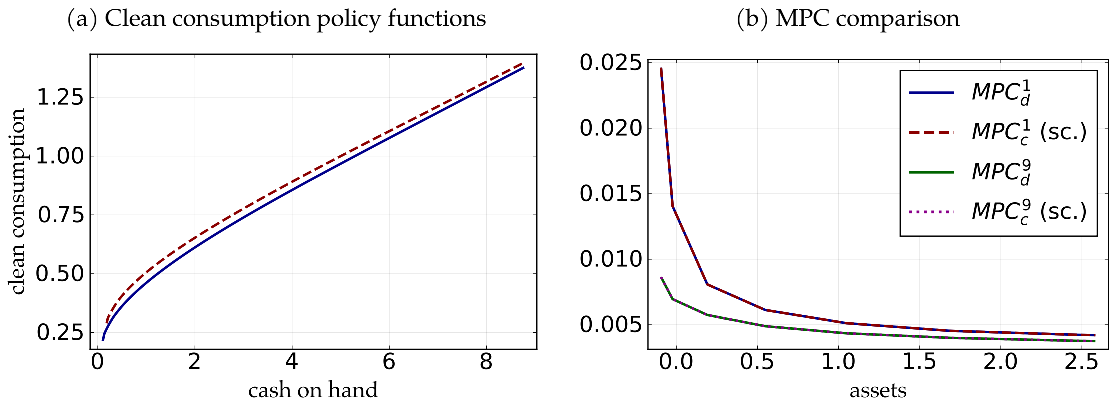
B.1 Policy functions and MPCs

I want to stress that the curvature in the consumption functions stems entirely from the dynamic stage of the budgeting problem and the precautionary savings behavior of agents and not due to non-linear allocation of expenditure in the static sub-problem. For this reason, I refer to the Stone-Geary preferences as quasi-homothetic, as opposed to non-homothetic preferences.

To see this, define the marginal propensity to spend out of current wealth of an agent in state (a, θ) as

$$\mathbf{MPE}(a, \theta) \equiv \frac{\partial}{\partial a}\mathbf{e}(a, \theta). \tag{B.18}$$

Figure B.5: Other steady state figures



Note. This figure shows the policy function of the clean consumption good as a function of cash-on-hand and the marginal propensities to consume both goods as a function of asset holdings.

Since agents allocate their expenditure linearly over consumption of either good, one can show that the marginal propensities to consume the clean good and the marginal propensity to consume the dirty good are linearly related up to scale. Using Equation (9)

$$\mathbf{MPC}_c(a,\theta) = \eta \frac{\partial}{\partial a}\mathbf{e}(a,\theta)$$
$$\mathbf{MPC}_d(a,\theta) = \frac{1-\eta}{p_d}\frac{\partial}{\partial a}\mathbf{e}(a,\theta)$$

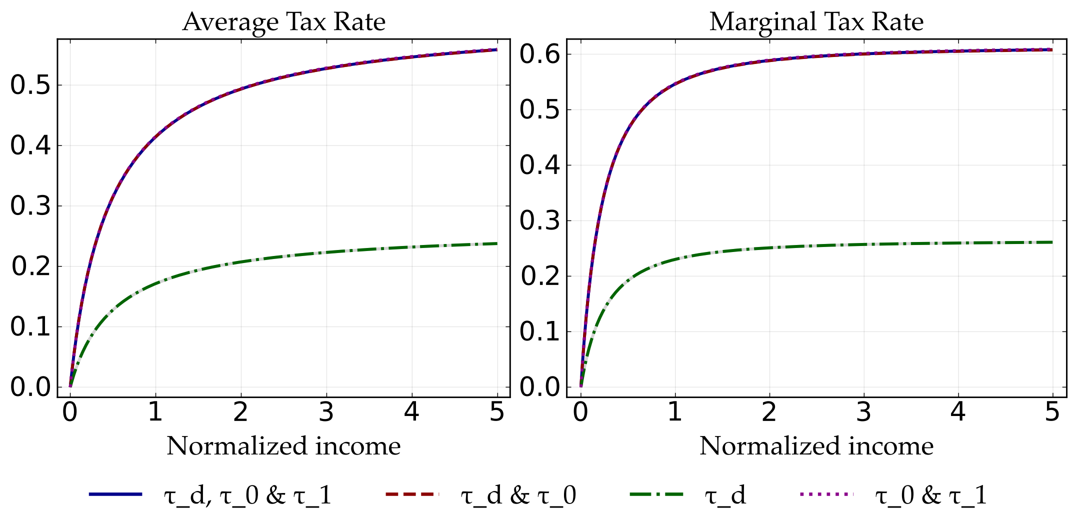
such that $\frac{\mathbf{MPC}_c(a,\theta)}{\mathbf{MPC}_d(a,\theta)} = \frac{\eta p_d}{1-\eta}$ gives the scaling factor. Panel B.5a shows that curvature is also present in the clean consumption function and Panel B.5b relates the marginal propensities to consume the clean and the dirty good for two labor productivity types using the scaling factor. We see that, conditional on a labor productivity type, once the scaling factor is applied to the MPC of the dirty good, it exactly overlaps with the MPC of the clean good.

B.2 Tax rates

This section presents the average and marginal tax rates associated with the terminal steady states of the economy under the different tax policy scenarios as well as a comparison with Ferriere et al. (2023).

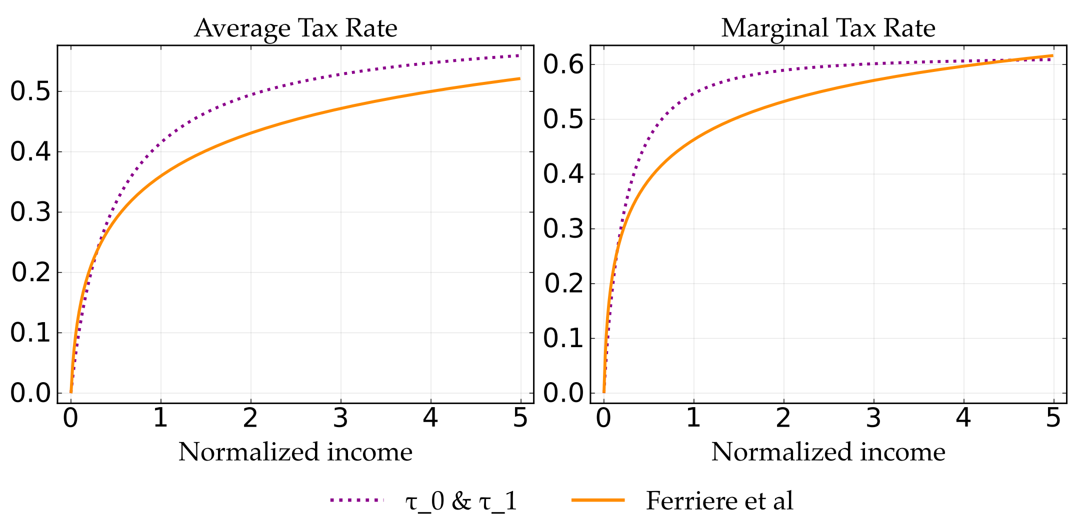
Figure B.6 shows the tax rates in the terminal steady states of my four benchmark scenarios. Figure B.7 shows the tax rates in the terminal steady states of the scenario when only the labor income tax parameters are adjusted.

Figure B.6: Implied tax rates



Note. This figure shows the average and marginal tax rates as a function of normalized income for the welfare-maximizing tax rates of the four scenarios in the benchmark of the main text. Income is normalized using average income.

Figure B.7: Comparison with Ferriere et al. (2023)

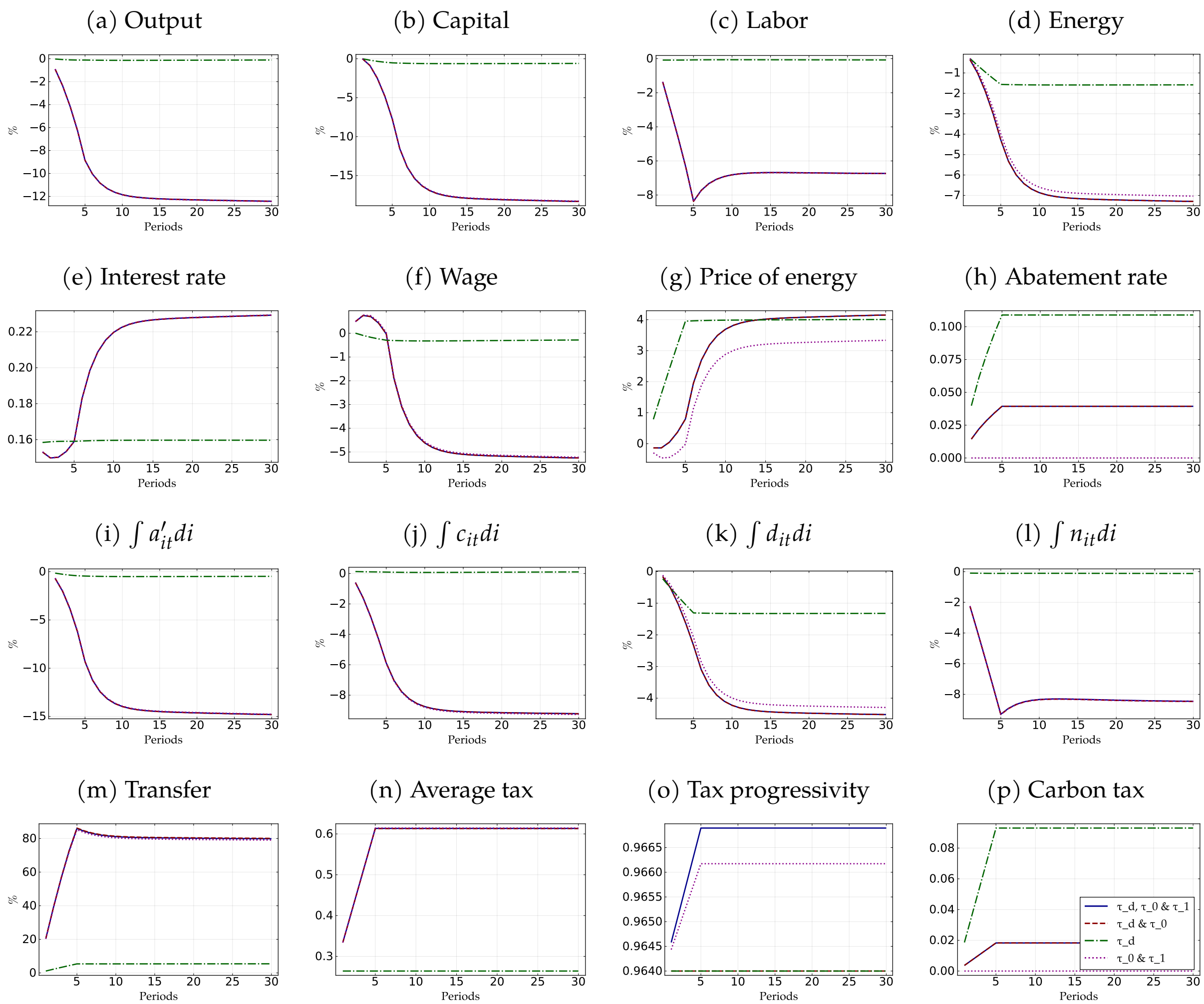


Note. This figure shows the average and marginal tax rates as a function of normalized income for the welfare-maximizing tax rates of the fourth scenario, when only labor income taxes are adjusted, and the tax rates implied by the benchmark results of Ferriere et al. (2023).

C Additional figures - Transitional dynamics

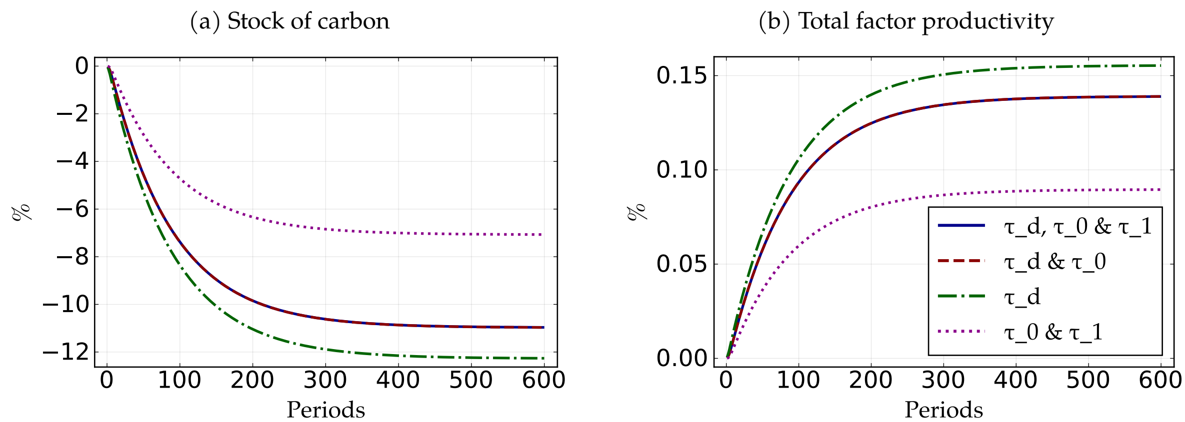
C.1 Short-run consequences of implementing the new baseline

Figure C.8: Transition dynamics



Note. This figure shows transitional dynamics of aggregates and parameters for the first 30 periods implied by the tax changes of all four scenarios. All numbers are in percentage deviation of the initial steady state, except the interest rate, the abatement rate, and the tax parameters.

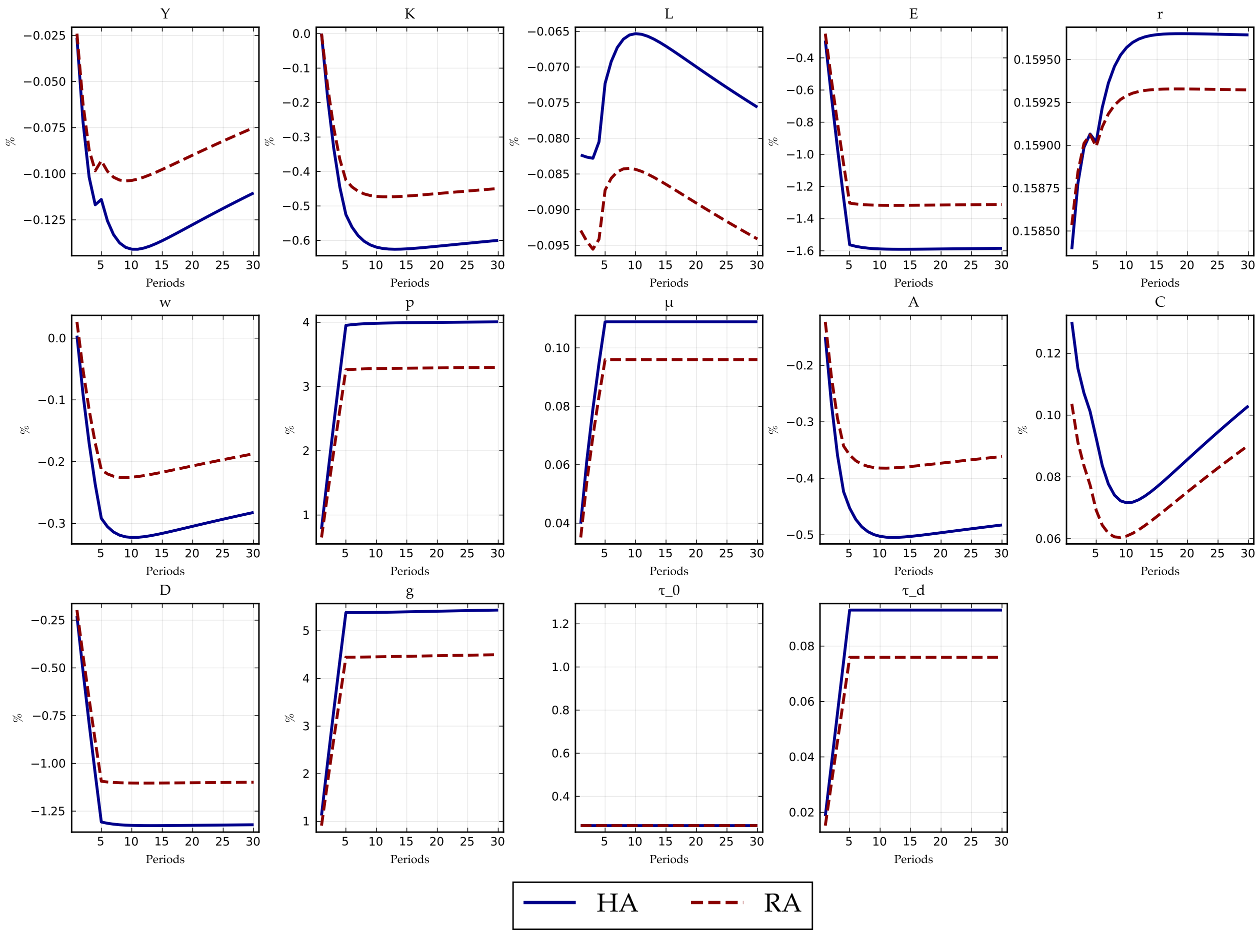
Figure C.9: Transition dynamics of slow moving variables



Note. This figure shows transitional dynamics of slow moving variables, the stock of carbon and TFP, over the full transition horizon implied by the tax changes of all four scenarios. All numbers are in percentage deviation of the initial steady state.

C.2 Representative-agent

Figure C.10: Transition dynamics in the heterogeneous-agent and the representative-agent economy when carbon taxes get adjusted



Note. This figure shows transitional dynamics of aggregates and parameters for the first 30 periods of the HA economy and the RA economy for the scenario when only the carbon tax gets adjusted. All numbers are in percentage deviation of the initial steady state, except the interest rate, the abatement rate, and the tax parameters.

C.3 Welfare details

C.3.1 Consumption (composition) equivalent welfare

The CEV is the change in the consumption composite that makes an agent indifferent from switching from the pre-tax stationary equilibrium consumption-labor allocation to the one obtained under the optimal carbon tax, including the transition. In particular, the CEV solves

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \tilde{u}(\tilde{c}_*, n_*) = \mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \tilde{u}((1+CEV)\tilde{c}_0, n_0),$$

where the $\tilde{c}$ refers to the consumption composite $\tilde{c} = c^{\eta}(d - \underline{d})^{1-\eta}$. Denote the infinitely discounted sum of utilities as W such that the left-hand side is $W(\tilde{c}_*, n_*)$. Moreover, note that due to separability of the preferences, we can write $W(\tilde{c}_*, n_*) = W(\tilde{c}_*) + W(n_*)$. Then, the CEV is given by

$$CEV = \left(\frac{W(\tilde{c}_*, n_*) - W(n_0)}{W(\tilde{c}_0)} \right)^{\frac{1}{1-\gamma}} - 1.$$

C.3.2 Welfare decomposition

To understand where the welfare gains are coming from along the transition towards the new steady state, I compute the welfare decomposition as in Bhandari et al. (2023) for a multi-good economy. Compared to their paper, I use the same greek variables but with a tilde ($\sim$) above it. For ease of exposition, I will replicate their formulas here.

A households choice of a particular good k satisfies the identity $c_{k,i} = \mathbb{E}c_{k,i} \frac{\mathbb{E}_i c_{k,i}}{\mathbb{E}c_{k,i}} \frac{c_{i,k}}{\mathbb{E}_i c_{k,i}} \equiv C \times w_{i,k} \times (1 + \epsilon_{k,i})$, where C denotes the aggregate quantity, w denotes the share that the household expects to receive of that aggregate, and ϵ captures the uncertainty that the household faces.

Define for two different policies $j \in \{A, B\}$ for each good k

$$C_k^Z \equiv \sqrt{C_k^A C_k^B}, \quad w_{i,k}^Z \equiv \sqrt{w_{i,k}^A w_{i,k}^B}, \quad c_{i,k}^Z \equiv \sqrt{C_k^Z w_{i,k}^Z}.$$

Define quasi-weights as $\tilde{\phi}_{k,i} \equiv \tilde{\alpha}_i U_k(c_{k,i}^z) c_{k,i}^z$ and coefficients of relative risk aversion $\tilde{\gamma}_{km,i} \equiv -\frac{U_{km,i} c_{m,i}^z}{U_{k,i}}$. Moreover, define

$$\tilde{\Gamma}_k \equiv \ln C_k^B - \ln C_k^A, \quad \tilde{\Delta}_{k,i} \equiv \ln w_{k,i}^B - \ln w_{k,i}^A, \quad \tilde{\Lambda}_{km,i} \equiv -\frac{1}{2} \left[cov_i(\ln c_{k,i}^B, \ln c_{m,i}^B) - cov_i(\ln c_{k,i}^A, \ln c_{m,i}^A) \right].$$

Bhandari et al. (2023) then show that welfare differences between two policies can be approximated and decomposed as

$$\mathcal{W}^B - \mathcal{W}^A \approx \mathbb{E} \sum_k \tilde{\phi}_{k,i} (\tilde{\Gamma}_k + \tilde{\Delta}_{k,i} + \sum_m \tilde{\gamma}_{km,i} \tilde{\Lambda}_{km,i}), \tag{C.19}$$

where the first term represents aggregate efficiency, the second term represents redistribution, and the last term represents insurance.

Using these definitions, we can compute all terms within the current setup. Note that since utility is separable across time, different goods k also refer to goods in different time periods. However, within a period, we do have to take into account interactions between clean and dirty consumption. Finally, in my setup the formulas simplify considerably if we treat leisure l, instead of labor supply, as the third good.

Compute the cross-elasticities:

$$\begin{aligned}
\tilde{\gamma}_{cc} &= 1 - \eta(1-\gamma) \\
\tilde{\gamma}_{dd} &= (1 - (1-\gamma)(1-\eta)) \\
\tilde{\gamma}_{dc} &= -\eta(1-\gamma) \\
\tilde{\gamma}_{cd} &= -(1-\gamma)(1-\eta)
\end{aligned}$$

Compute the quasi-weights as:[20]

$$\begin{aligned}
\tilde{\phi}_{c,t}(a_0,\theta_0) &= \tilde{\beta}^{t-1} \cdot \tilde{\alpha} \cdot \eta \cdot (c_t^z(a_0,\theta_0))^{\eta\cdot(1-\gamma)-1} \cdot (d_t^z(a_0,\theta_0))^{(1.0-\gamma)(1-\eta)} \cdot (c_t^z(a_0,\theta_0)) \\
\tilde{\phi}_{d,t}(a_0,\theta_0) &= \tilde{\beta}^{t-1} \cdot \tilde{\alpha} \cdot (1-\eta) \cdot (c_t^z(a_0,\theta_0))^{\eta\cdot(1-\gamma)} \cdot (d_t^z(a_0,\theta_0))^{(1.0-\gamma)(1-\eta)-1} \cdot (d_t^z(a_0,\theta_0)) \\
\tilde{\phi}_{l,t}(a_0,\theta_0) &= \tilde{\beta}^{t-1} \cdot \tilde{\alpha} \cdot \chi \cdot (l_t^z(a_0,\theta_0))^{1-\epsilon}
\end{aligned}$$

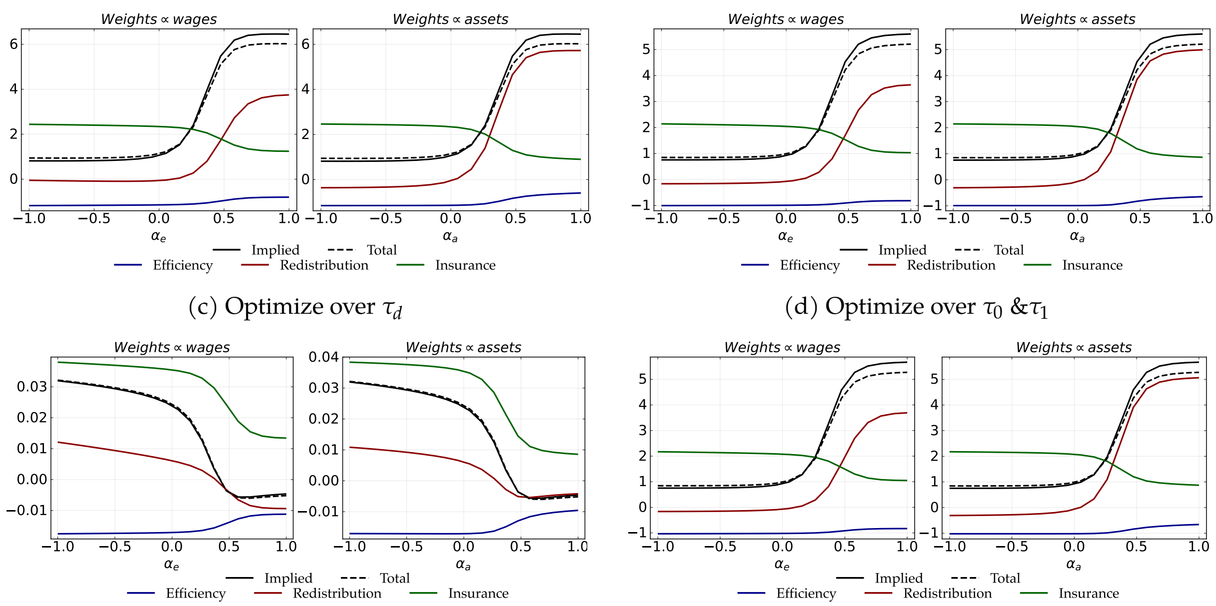
Compute the different components:

$$\begin{aligned}
E_c &= \sum_t \int \left(\tilde{\phi}_{c,t}\tilde{\Gamma}_{c,t}\right) d\Xi_0, \quad R_c = \sum_t \int \left(\tilde{\phi}_{c,t}\tilde{\Delta}_{c,t}\right) d\Xi_0, \quad I_c = \sum_t \int \left(\tilde{\phi}_{c,t}\left(\tilde{\gamma}_{cc,t}\tilde{\Lambda}_{cc,t} + \tilde{\gamma}_{cd,t}\tilde{\Lambda}_{cd,t}\right)\right) d\Xi_0, \\
E_d &= \sum_t \int \left(\tilde{\phi}_{d,t}\tilde{\Gamma}_{d,t}\right) d\Xi_0, \quad R_d = \sum_t \int \left(\tilde{\phi}_{d,t}\tilde{\Delta}_{d,t}\right) d\Xi_0, \quad I_d = \sum_t \int \left(\tilde{\phi}_{d,t}\left(\tilde{\gamma}_{dc,t}\tilde{\Lambda}_{dc,t} + \tilde{\gamma}_{dd,t}\tilde{\Lambda}_{dd,t}\right)\right) d\Xi_0, \\
E_l &= \sum_t \int \left(\tilde{\phi}_{l,t}\tilde{\Gamma}_{l,t}\right) d\Xi_0, \quad R_l = \sum_t \int \left(\tilde{\phi}_{l,t}\tilde{\Delta}_{l,t}\right) d\Xi_0, \quad I_l = \sum_t \int \left(\tilde{\phi}_{l,t}\epsilon\tilde{\Lambda}_{l,t}\right) d\Xi_0
\end{aligned}$$

To check the quality of my approximation, I reproduce Figure 2 in Bhandari et al. (2023). The result can be seen in Figure C.11 and plots the welfare gain and its components for different Pareto weights for all four scenarios considered in the baseline of the paper. The solid black line is the sum of the three components, that is, the right-hand side of Equation (C.19), whereas the dotted black line is the left-hand side of Equation (C.19). While it is interesting by itself of how the different components change if more/less weight is given to income or asset poor agents, the figure also shows that the approximation error of the decomposition, for different Pareto weights, is small. This is evident from the small difference between the actual and implied welfare gains.

[20] Note that $\tilde{\beta}$ denotes the discount factor of the social planner here.

Figure C.11: Welfare decompositions for different pareto weights



(a) Optimize over τ_d, τ_0 & τ_1

(b) Optimize over τ_d & τ_0

(c) Optimize over τ_d

(d) Optimize over τ_0 &τ_1

Note. This figure is based on Figure 2 in Bhandari et al. (2023) and plots the welfare gain and its components for different Pareto weights for all four scenarios considered in the baseline of the paper.

D Calibration - Details

D.1 Macroeconomic variables

Capital-output-ratio (K/Y) Current-Cost Net Stock of Fixed Assets (2014) in current dollars divided by Gross Domestic Product (2014) in current dollars; both series from the FRED database with series tags K1TTOTL1ES000 and GDPA, respectively.

Bond-output-ratio (B/Y) Federal Debt Held by the Public (2014) in current dollars divided by Gross Domestic Product (2014) in current dollars; both series from the FRED database with series tags FYGFDPUN and GDPA, respectively.

D.2 Estimation of the productivity process

In the following, I explain how I estimate the labor productivity process which I use in my quantitative model. [21] For my sample and data definitions I follow Heathcote, Perri and Violante (2010*a*) and Straub (2019).

I start from the core PSID surveys from 1967-1996, more precisely the SRC sample of Heathcote et al. (2010*a*), since these surveys are available at annual frequency. I focus on household heads as unit of observation and measure productivity as (log) hourly pre-tax wages, which I define as

[21] The exposition of the estimation follows the one in Straub (2019) who uses a similar strategy to estimate a process for log income and from whose description I learned a lot.

pre-tax labor income divided by annual hours of work. In terms of sample selection, I follow the steps by Heathcote et al. (2010*a*) precisely to construct their Sample C. In particular, I exclude

- observations with missing or miscoded household information
- observations with positive labor income but zero working hours
- observations with a wage smaller than half the minimum wage
- observations younger than 25 and older than 60
- observations with fewer than 260 annual working hours.

Finally, I only use observations from years after 1981 to exclude observations with income top-coding (Straub, 2019). Overall, this leaves me with 5429 distinct and 40660 total observations.

As my productivity process, I take the following standard persistent-transitory specification for log wages at year τ[22]:

$$\log \hat{\theta}_{i\tau} = f(\mathbf{X}_{i\tau}, \beta_\tau) + \kappa_{i\tau} + \psi_{i\tau} + \nu_{i\tau}$$
$$\kappa_{i\tau} = \rho\kappa_{i\tau-1} + \varepsilon^{\kappa}_{i\tau},$$

where $f(\mathbf{X}_{i\tau}, \beta_\tau)$ denotes a set of individual-specific controls, $\kappa_{i\tau}$ is an AR(1) process with persistence ρ and innovation-variance $\sigma^2_{\varepsilon^\kappa}$, $\psi_{i\tau}$ is a transitory component with variance σ^2_ψ, and $\nu_{i\tau}$ is measurement error.

At the outset, I follow the literature and set the variance of the measurement error term to 0.02 (French, 2004; Heathcote, Storesletten and Violante, 2010*b*; Straub, 2019), since measurement error cannot be identified separately from the transitory shock.

The estimation of the income process parameters $(\rho, \sigma^2_{\varepsilon^\kappa}, \sigma^2_\psi)$ then proceeds along the following two steps. First, on the sample described above, I residualize log wages separately for each year using

$$\log \theta_{i\tau} = \log \hat{\theta}_{i\tau} - f(\mathbf{X}_{i\tau}, \hat{\beta}_\tau),$$

where the controls $\mathbf{X}_{i\tau}$ are a cubic polynomial in age and education dummies. Before proceeding to the next step, I only keep observations which are present at least 8 times. This leaves 30434 residuals.

In the second step, I compute empirical variances and covariances from these residuals and stack them in the vector $\vec{\mathfrak{M}}$. The theoretical variances and covariances, on the other hand, can be computed using

$$var(\log \theta_{i\tau}) = \frac{\sigma^2_{\varepsilon^\kappa}}{1-\rho^2} + \sigma^2_\psi + \sigma^2_\nu$$
$$cov(\log \theta_{i\tau}, \log \theta_{i\tau-h}) = \rho^h \frac{\sigma^2_{\varepsilon^\kappa}}{1-\rho^2}.$$

[22] To avoid notational clutter, I denote yearly time-steps by τ, compared with a model period (5 years) denoted by t

Table D.8: Estimated parameters - Productivity process

ρ	$\sigma^2_{\varepsilon^\kappa}$	σ^2_{ψ}
0.9577	0.0203	0.0556
(0.0037)	(0.0018)	(0.0034)

Note. This table shows the estimated parameters of the productivity process. Standard errors are bootstrapped using a non-parametric block bootstrap at the household level with 500 iterations.

I denote the stacked theoretical (co)variances by $\vec{\mathfrak{m}}(\rho, \sigma^2_{\varepsilon^\kappa}, \sigma^2_{\psi})$. This formulation stresses that $\vec{\mathfrak{m}}$ is a function of the parameters that we seek to estimate.

Lastly, I apply a minimum distance estimation (MDE) to minimize the weighted distance, $\mathfrak{U}(\rho, \sigma^2_{\varepsilon^\kappa}, \sigma^2_{\psi}) = \vec{\mathfrak{M}} - \vec{\mathfrak{m}}$, between theoretical and empirical moments/covariances:

$$\min_{\rho, \sigma^2_{\varepsilon^\kappa}, \sigma^2_{\psi}} \mathfrak{U}(\rho, \sigma^2_{\varepsilon^\kappa}, \sigma^2_{\psi})' \, \mathfrak{W} \, \mathfrak{U}(\rho, \sigma^2_{\varepsilon^\kappa}, \sigma^2_{\psi})$$

As is standard in this procedure, I use the identity matrix as weighting matrix $\mathfrak{W}$ which was shown to be more robust to small sample bias (Altonji and Segal, 1996).

Table D.8 shows the result of the MDE. Standard errors are obtained by using a non-parametric block bootstrap (Cameron and Trivdei, 2005, p.362/p.377).

5-year time period To translate these values that were estimated on annual data to their 5-year model counterparts, I proceed in two steps: First, I iterate the persistent component backward such that

$$\kappa_{i\tau} = \rho^5 \kappa_{i\tau-5} + \underbrace{\sum_{s=0}^{4} \rho^s \varepsilon^\kappa_{i\tau-s}}_{\tilde{\varepsilon}^\kappa_{i\tau}} . \tag{D.20}$$

I can compute the variance of $\tilde{\varepsilon}^\kappa_{i\tau}$ given the annual estimate:

$$\sigma^2_{\tilde{\varepsilon}^\kappa} = var(\tilde{\varepsilon}^\kappa_{i\tau}) = \sum_{s=0}^{4} \rho^{2s} var(\varepsilon^\kappa_{i\tau-s}) = \sum_{s=0}^{4} \rho^{2s} \sigma^2_{\varepsilon^\kappa} \tag{D.21}$$

This gives $\tilde{\rho} = \rho^5 = 0.8057$ and $\sigma^2_{\tilde{\varepsilon}^\kappa} = 0.0869$. Second, I set $\sigma^2_{\tilde{\psi}} = \sigma^2_{\psi}$.

E Estimation - Details

E.1 Standard errors

Gourieroux, Monfort and Renault (1993) show that under no observable exogenous variables that enter the moments, the asymptotic variance-covariance matrix is

$$\mathbb{COV} = \left(1 + \frac{1}{B}\right) \left[\frac{\partial \mathcal{M}^*}{\partial \Theta}^\top W \; \frac{\partial \mathcal{M}^*}{\partial \Theta} \right]^{-1} \frac{\partial \mathcal{M}^*}{\partial \Theta}^\top W \, cov(\mathcal{M}^B) \, W \; \frac{\partial \mathcal{M}^*}{\partial \Theta} \left[\frac{\partial \mathcal{M}^*}{\partial \Theta}^\top W \; \frac{\partial \mathcal{M}^*}{\partial \Theta} \right]^{-1}$$

where B denotes the number of bootstrap repetitions, $\mathcal{M}^* = \mathcal{M}(\Theta^*)$ and $cov(\mathcal{M}^B)$ is the covariance matrix of the bootstrapped moments.

In particular, I have 5771 unique households in my sample. I draw $B = 200$ random samples, with replacement, of these households to construct $cov(\mathcal{M}^B)$ from the data. The draws are panel draws (block draws), that is, when I draw a specific household, I keep all household-year observations.

The gradient $\frac{\partial \mathcal{M}^*}{\partial \Theta}$ is a Jacobian, where the elements give partial derivatives from the structural parameters to the model moments:

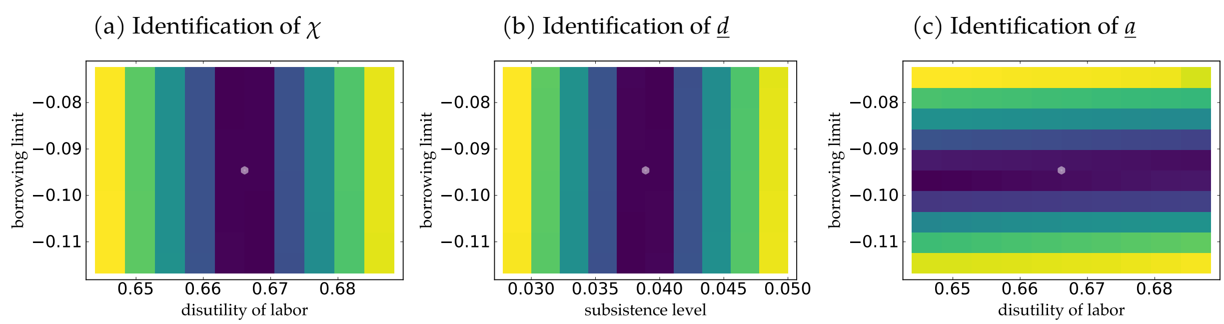
$$\frac{\partial \mathcal{M}^*}{\partial \Theta} = \begin{bmatrix} \frac{\partial M_1^*}{\partial \chi} & \frac{\partial M_1^*}{\partial \underline{d}} & \frac{\partial M_1^*}{\partial \underline{a}} \\ \frac{\partial M_2^*}{\partial \chi} & \frac{\partial M_2^*}{\partial \underline{d}} & \frac{\partial M_2^*}{\partial \underline{a}} \\ \frac{\partial M_3^*}{\partial \chi} & \frac{\partial M_3^*}{\partial \underline{d}} & \frac{\partial M_3^*}{\partial \underline{a}} \end{bmatrix}$$

The partial derivatives are approximated with a numerical two-sided difference. Finally, the weighting matrix W is an identity matrix. The standard errors are then on the diagonal of $\mathbb{COV}$.

E.2 Identification plots

In this section, I show that the parameters I estimate using SMM are identified. Figure E.12 depicts how different model-implied moments are affected when I change two of the three parameters and fix the remaining one at its best fit value. Lighter regions indicate a larger absolute deviation from the data moment. We see that in all three plots, only one of the parameters on the axis identifies the moment. From left to right: Conditional on the borrowing limit, the disutility of labor parameter χ identifies average hours worked; conditional on the borrowing limit, $\underline{d}$ identifies the average expenditure share; conditional on the disutility of labor, $\underline{a}$ identifies the 10th percentile of the wealth-to-income ratio.

Figure E.12: Illustrating the identification of χ, $\underline{d}$, and $\underline{a}$



Note. This figure shows absolute deviation of targeted moments implied by the structural model from their data counterparts when parameters on the respective axis are varying. Other parameters are fixed at their calibrated and estimated value. Lighter regions indicate higher deviation.

F Computational appendix

Discretization, Pareto tail, and asset grid I discretize the labor productivity process using Rouwenhorst (1995)'s method. I choose 7 gridpoints for the persistent part and 3 gridpoints for the transitory one. Denote the resulting cumulative distribution function over productivity states by F_θ, the gridpoints by θ_j, and the invariant distribution over these gridpoints by π. Denote by $\theta_{0.9}$ for which $F_\theta(\theta_{0.9}) \approx 0.9$, that is the productivity value on the discretized grid that determines the cutoff for the top 10%. I then adjust the grid values for all j gridpoints larger than $\theta_{0.9}$ by

$$\theta_j^{pareto} = \frac{\theta_{0.9}}{\left(1 - \frac{F_\theta(\theta_j) - F_\theta(\theta_{0.9})}{1 - F_\theta(\theta_{0.9})}\right)^{\frac{1}{\omega}}}.$$

Note that $F_\theta(\theta_{max}) = 1 - \pi_{max} < 1$ such that the denominator is well-defined for the largest gridpoint. I normalize aver labor productivity to 1.

To construct the asset grid, I use a quadratic transformation between $\underline{a}$ and $\bar{a}$. I use 150 asset grid points and make sure to set the upper bound large enough such that it never becomes binding.

F.1 Stationary steady state

My methods to compute the stationary steady state are relatively standard. To solve the household problem I use a version of the endogenous grid method (EGM) by Carroll (2006) that accommodates the non-linear tax function. I explain it in detail in Appendix F.3. To obtain the stationary distribution, I rely on the histogram method by Young (2010). I solve for general equilibrium using a Broyden root-finding algorithm. In the initial steady state, take r and g as given. Then:

1. Guess β, D, L, τ_2, χ, $\underline{d}$, $\underline{\mathrm{a}}$
2. Set the asset grid based on $\underline{a}$.

3. Solve for $w, K_1, K_2, L_1, L_2, p_d, \mu, E, E^p, Y, \delta, s, \tilde{Z}$ using the six first order conditions of the firms, labor market clearing between sectors, energy market clearing, energy firm technology, final firm technology, and three aggregate data targets with a non-linear solver.

4. Given prices solve the household problem using the EGM.

5. Given policies and the law of motion of the exogenous state, solve for the stationary distribution using the histogram method.

6. Compute aggregate quantities.

7. Check if the asset market clears (tolerance 10^{-6}), if the household choices of L and D are in line with the initial guesses, the government budget clears, the three data targets are satisfied. If not, update guesses and go back to Step 2.

The procedure for the terminal steady state is similar, only that $\chi, \underline{d}, \underline{a}, \delta, s, \tilde{Z}$ are not updated anymore and the data targets are thus not considered. τ_2 is replaced with g to get government budget clearing and r, instead of β, is updated to deliver asset market clearing. Moreover, in the terminal steady state, the damage function and climate cycles are enforced, since Z is now explicitly a function of S.

F.2 Transitional dynamics

To solve the transitional dynamics between two steady states, I rely on the sequence-space Jacobian method by Auclert, Bardóczy, Rognlie and Straub (2021). Denote the new terminal steady state by subscripts ss.

My vector of unknown paths are capital demand, aggregate dirty good consumption, labor demand, transfers, and the stock of atmospheric carbon: $\boldsymbol{U} = \{\boldsymbol{K}, \boldsymbol{D}, \boldsymbol{L}, \boldsymbol{g}, \boldsymbol{S}\}$. The five corresponding targets are then

$$H(\boldsymbol{U}, \boldsymbol{\tau}) = \begin{pmatrix} \text{Asset market clearing} \\ \text{Dirty goods market clearing} \\ \text{Labor market clearing} \\ \text{Government budget constraint} \\ \text{Carbon cycle} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

where $\boldsymbol{\tau}$ denotes the exogenous paths of the tax variables. I then compute the general equilibrium Jacobian $H_{\boldsymbol{U}}$ based on forward accumulation along a directed acyclical graph of the model. With $H_{\boldsymbol{U}}$ in hand, I compute the nonlinear perfect foresight transition as in Auclert et al. (2021). Starting from iteration $j = 0$, I guess a path for $\boldsymbol{U}^0 = \boldsymbol{U}_{ss}$ and update according to

$$\boldsymbol{U}^{j+1} = \boldsymbol{U}^j - [H_{\boldsymbol{U}}(\boldsymbol{U}_{ss}, \boldsymbol{\tau}_{ss})]^{-1} H(\boldsymbol{U}^j, \boldsymbol{\tau})$$

Note that I use the GE Jacobian around the new terminal steady state for updating.

Representative-agent model The RA version of the model is solved similarly, however, with different targets. Asset market clearing and labor market clearing are replaced by the Euler equation and goods market clearing, respectively.

F.3 Computing the household's optimal decision rules and invariant distribution

I use a variant of the endogenous gridpoint method (EGM) to solve the household's decision problem. Compared to the basic version developed by Carroll (2006), my version accommodates two goods and endogenous labor supply with non-linear taxation.

Grids I represent asset positions by discrete points on an exponentially-spaced grid $\mathcal{A} \subset [\underline{a}, \overline{a}]$, where $\overline{a}$ is chosen large enough such that the upper bound is never binding. I discretize the productivity Markov process with a finite-state Markov chain using Rouwenhorst (1995)'s method. The inputs for this method, such as the persistence parameter ρ, are obtained in Appendix D.2.

Endogenous gridpoint method

Step 1 I start with a guess of the clean consumption policy function defined on the *future* asset and productivity grid, $\mathbf{c}(a', \theta')$. Using the intra-temporal first-order condition between clean and dirty consumption, I can express the dirty consumption policy function $\mathbf{d}(a', \theta')$ as a function of $\mathbf{c}(a', \theta')$:

$$\mathbf{d}(a', \theta') = \frac{1-\eta}{p_d \eta} \mathbf{c}(a', \theta') + \underline{d}. \tag{F.22}$$

Step 2 For each pair (a', θ) where the household is not constrained and the Euler equation (EE) holds with equality, I can solve *analytically* for the value $\mathbf{c}(a', \theta)$.[23] This follows from the inversion of the Euler equation and $\mathbf{c}(a', \theta)$ represents the value of consumption today, which is consistent with having a' assets tomorrow if the productivity shock today is θ:

$$u_c(\mathbf{c}(a', \theta)) = \beta(1+\tilde{r})\mathbb{E}_\theta \left[u_c(\mathbf{c}(a', \theta')) \right] \tag{F.23}$$

Note that I write u_c explicitly as a function of $\mathbf{c}$ only, as the utility is separable in consumption and labor, and $\mathbf{d}$ is implied by Equation (F.22).

Step 3 With $\mathbf{c}(a', \theta)$ in hand, I can solve for $\mathbf{n}(a', \theta)$ using the intra-temporal FOC between clean consumption and labor. In the following, I assume an interior solution:

$$-u_n(\mathbf{n}(a', \theta)) = u_c(\mathbf{c}(a', \theta))\mathcal{T}_n, \tag{F.24}$$

where $\mathcal{T}_n$ denotes $\frac{\partial \mathcal{T}}{\partial n}$. Under linearity of $\mathcal{T}$, Equation (F.24) can also be solved analytically for $\mathbf{n}(a', \theta)$. Otherwise, a root-finding step has to be implemented at every point in the state space. In

[23] Of course, this step depends on the invertability of the utility function. Other functional forms for the consumption composite might not make this feasible.

the benchmark case, I use a version of Brent's method, modified to take into account the corner solution if $\mathbf{n}(a', \theta) = 0$. Moreover, since the root-finding step to find optimal labor supply as a function of assets, given a labor productivity type, is computationally expensive, I only apply this step for a subset of asset grid points. Having the pairs $(\mathbf{n}, a'_{coarse})$ on the coarse grid, I then linearly interpolate the labor supply policy function on the full grid.

Step 4 I can then invert the budget constraint to solve for the value of assets today, $a^*(a', \theta)$, which are consistent with the future assets (on grid) and the choices made above.

$$a^* = \frac{1}{1+\tilde{r}}\bigg(\mathbf{c}(a', \theta) + (p_d + \tau_d)\mathbf{d}(a', \theta) + a' - \mathcal{T}\big(w\theta\mathbf{n}(a', \theta)\big)\bigg), \tag{F.25}$$

implying $\tilde{c}(a^*, \theta) = \mathbf{c}(a', \theta)$. Note that these a^* are not on the grid (whence the name) and change each iteration. To obtain a new guess for the clean consumption policy function which is defined on the grid, I linearly interpolate on $(a^*, \tilde{c}(a^*, \theta))$ and apply this mapping to the exogenous grid a'. Use the new guess as a starting point in Step 2 above.

I repeat the above iteration procedure until convergence between two successive clean consumption policy functions is achieved: $||\mathbf{c}^{n+1} - \mathbf{c}^n|| < 10^{-8}$, where $|| \cdot ||$ denotes the supnorm and n is the iteration counter.

Density discretization With the policy functions in hand, I discretize the invariant density and iterate on it using Young (2010)'s lottery method (tolerance 10^{-10}).

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De Nederlandsche Bank N.V.
Postbus 98, 1000 AB Amsterdam
020 524 91 11
dnb.nl