

# DNB Working Paper

No. 534 / November 2016

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EUROSYSTEEM

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\* Views expressed are those of the author and do not necessarily reflect official positions of De Nederlandsche Bank.

Working Paper No. 534

November 2016

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# Aggregate liquidity and banking sector fragility\*

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21 November 2016

## Abstract

As compared to non-banks, banks adopt relatively fragile balance sheet structures characterized by leverage, maturity mismatch, and asset diversification. This paper offers a new potential explanation for this observation, within a model where banks face lower aggregate (funding) liquidity risk than non-banks. This single difference between both provides banks with an incentive to adopt fragile balance sheets, even in the absence of tax distortions, moral hazard, or a special role for banks as liquidity providers. The model implies that banks engage in pro-cyclical risk-taking, are vulnerable to contagion, and will resist regulatory equity and liquidity requirements, while non-banks do not.

**Keywords:** banks, balance sheet fragility, aggregate liquidity, bank equity and liquidity requirements, financial stability.

**JEL classifications:** E50, G01, G21, G28.

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\* Part of this paper was written while the author was visiting the Federal Reserve Board of Governors in Washington, DC. Previous versions benefited from helpful comments by and discussions with Itai Agur, Jan Marc Berk, Kees Bouwman, Francesca Carapella, Paul Cavelaars, Diederik Dicou, Xavier Freixas, Gabriele Galati, Jakob de Haan, Skander van den Heuvel, Neeltje van Horen, Jan Jacobs, Diana Hancock, Michal Kowalik, Luc Laeven, Ralf Meisenzahl, Jochen Mierau, Mikhail Oet, Carol Osler, Joost Passenier, Enrico Perotti, Francesco Piersante, Sebastiaan Pool, Rodney Ramcharan, Stéphane Verani, Razvan Vlahu, Frans de Weert, and Sweder van Wijnbergen, as well as from comments received during various conference and workshop presentations. Any remaining errors are those of the author. The views expressed do not necessarily reflect those of De Nederlandsche Bank.

# 1 Introduction

As compared to non-banks, banks adopt balance sheet structures characterized by a relatively high leverage, maturity mismatch, and asset diversification. The global financial crisis has illustrated that these balance sheet characteristics may negatively affect the stability of individual banks, as well as that of the banking sector as a whole.<sup>1</sup> Since then, the academic literature has with renewed interest tried to understand why banks adopt such fragile balance sheet structures to begin with.<sup>2</sup> On the one hand, in line with Modigliani and Miller's (1958) classic irrelevance result, studies such as Admati, DeMarzo, Hellwig, Pfleiderer (2010) argue that banks' choice of balance sheet structure is driven by tax distortions and moral hazard from government safety nets. This view implies that social welfare can be improved by raising bank equity and liquidity requirements to reduce the fragility of banks' balance sheets. On the other hand, contributions such as DeAngelo and Stulz (2015) and Hanson, Shleifer, Stein and Vishny (2015) view banks' balance sheet structures as intrinsic to banks' role of producing safe, money-like claims that are useful for transaction purposes. In this case, requiring banks to change their balance sheet structure may have welfare costs as well. The present paper contributes to this literature by offering a new potential explanation for banks' choice to adopt a fragile balance sheet structure, which comes with different policy implications.

To analyze banks' and non-banks' choice of balance sheet structure, the paper presents a simple model where profit (value) maximizing firms choose their optimal amount of leverage, maturity mismatch, and asset diversification. The model abstains from frictions that may apply to banks and non-banks equally, such as the debt tax shield put forward by Modigliani and Miller (1963) and the agency problems going back to Jensen and Meckling (1976) and

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<sup>1</sup>Non-banks typically finance themselves with equity and long-term debt, and concentrate their assets by operating within a specific economic sector (e.g., food production, car manufacturing, or health care provision). Banks, by contrast, typically finance themselves with little equity and large amounts of short-term debt, and diversify their assets by lending to a broader range of economic sectors. The higher leverage and maturity mismatch associated with this balance sheet structure renders banks vulnerable to runs, as established by Bryant (1980) and Diamond and Dybvig (1983). Shaffer (1994), Wagner (2010) and Ibragimov, Jaffee and Walden (2011) show that asset diversification causes banks' asset portfolios to more closely resemble the 'market portfolio', thereby increasing the correlation between different banks' asset returns and raising the risk of joint bank defaults.

<sup>2</sup>Strictly speaking, asset diversification increases the fragility of the banking sector as a whole more than the fragility of individual banks. For lack of a better word, however, a balance sheet characterized by leverage, maturity mismatch and diversification will be referred to as 'fragile' throughout the analysis.

Myers and Majluf (1984). Moreover, there are no frictions in the model that limit firms' access to equity funding, so that equity earns a higher expected return than short-term debt only because of its longer maturity (the difference between both expected returns is determined by the term-premium, which reflects the market price of liquidity risk).<sup>3</sup> One implication of this setup is that all firms are indifferent about which balance sheet structure to adopt if they have the same exposure to aggregate liquidity risk, which is defined as the risk that an outflow of short-term funds from one firm coincides with an outflow of short-term funds from other firms.<sup>4</sup> However, the main insight from the paper is that this indifference result disappears once firms do not have the same exposure to aggregate liquidity risk. In that case, firms facing lower aggregate liquidity risk choose balance sheet structures with leverage, maturity mismatch, and asset diversification, while firms facing higher aggregate liquidity risk choose balance sheet structures without leverage, maturity mismatch, or asset diversification. Firms with an intermediate exposure to aggregate liquidity risk remain indifferent about their balance sheet structure.

The intuition behind the main result is as follows. If firms are identical except for the aggregate liquidity risk on their short-term debt, funding their entire balance sheet with equity is equally profitable for all of them. Funding with short-term debt, by contrast, is more profitable for firms with lower aggregate liquidity risk than for firms with higher aggregate liquidity risk. The reason is that firms with higher aggregate liquidity risk would need to hold higher cash reserves, which reduces profits as cash yields a return of zero. As firms with

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<sup>3</sup>In practice, several additional reasons exist why equity earns a higher expected return than short-term debt. For instance, unless all risk is diversifiable, the return required by equity holders includes a higher risk-premium since equity constitutes a subordinated claim on the value of the firm. In addition, debt holders require a lower return when their claims are explicitly or implicitly insured, such as through a deposit insurance scheme as in Furlong and Keeley (1987), through too-big-to-fail policies as reviewed by Morrison (2012), or through too-many-to-fail guarantees as in Acharya and Yorulmazer (2007) and Farhi and Tirole (2012). Debt holders can also require a lower return when debt markets are segmented as in Allen, Carletti and Marquez (2015)), or when debt claims yield 'money-services' as in Gorton and Pennacchi (1990), Gorton and Winton (2014), DeAngelo and Stulz (2015), and Hanson, Shleifer, Stein and Vishny (2015). Moreover, short-term debt can also be attractive funding source because it disciplines bank management, see Calomiris and Kahn (1991) and Diamond and Rajan (2001). The model omits these considerations to highlight that they do not drive the results.

<sup>4</sup>More precisely, a firm's aggregate liquidity risk in the model is defined as the correlation between the liquidity shocks to its short-term debt and the liquidity shocks experienced by other firms. The concept of liquidity risk in the model thus refers to the funding liquidity risk of firms rather than to the market liquidity risk of assets. Market liquidity risk arises if asset prices can deviate from their fundamental values, for instance after fire sales such as in Shleifer and Vishny (1992) and in Brunnermeier and Pedersen (2009).

intermediate aggregate liquidity risk are indifferent between both balance sheet structures, those firms with relatively high aggregate liquidity risk prefer the typical non-bank's balance sheet and fund their long-term assets with equity. By contrast, those firms with relatively low aggregate liquidity risk prefer to fund with short-term debt while holding cash reserves. In addition, as these firms' higher leverage makes them vulnerable to asset losses, they diversify their assets to reduce solvency risk. What results is a combination of leverage, maturity mismatch, and asset diversification – the typical bank's balance sheet.

The mechanism established by the model provides a rationale for the differences between banks' and non-banks' balance sheet structures, since aggregate liquidity risk in the banking sector is lower than aggregate liquidity risk in the non-banking sector. This difference reflects how central banks implement their monetary policies, which typically involves determining a target for the short-term interest rate and then using open market operations to supply the banking sector with all liquidity demanded at this rate. In particular, if the banking sector is confronted with an aggregate outflow of short-term funds, the central bank provides additional liquidity to the sector so as to keep the short-term interest rate from rising above the monetary policy target. While in normal times the need for such additional liquidity is limited, the aftermath of the Lehman Brothers bankruptcy illustrated that if needed, central banks are willing to provide much larger amounts to maintain interest rates at their target values.<sup>5</sup> As a result, if a bank issues any short-term debt it faces less aggregate liquidity risk than a non-bank would. The main insight from the model is that this single difference provides banks with an incentive to adopt balance sheet structures characterized by leverage, maturity mismatch, and asset diversification, while non-banks do not.

One implication from the analysis is that banks may not be as prone to moral hazard as their fragile balance sheet structures suggest. In fact, it implies that banks would adopt such balance sheet structures even if tax incentives and government safety nets were absent and

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<sup>5</sup>This liquidity provision to the banking sector as a whole is part of standard monetary policy implementation, see Federal Reserve Board (2005). It thereby differs from central banks' lending of last resort policies, which involve lending to individual banks that are solvent but illiquid (Goodhart, 1999). Goodfriend and King (1988) argue that lending of last resort policies cause moral hazard, and recommend that central banks dispense with them altogether so as to provide liquidity through open market operations only. See Allen, Carletti and Gale (2009) and Freixas, Martin and Skeie (2011) for recent analyses of central bank lending and interbank market liquidity.

even if banks would aim to stay liquid and solvent under all circumstances (both are the case in the model). Hence, policy efforts to reduce moral hazard in the banking sector may not lead banks to adopt balance sheet structures that are markedly less fragile than their current ones. At the same time, the analysis implies that these balance sheet structures should not in themselves be taken as evidence that banks are ‘special’ compared to non-banks, other than for their access to central bank liquidity provision. Perhaps surprisingly, if central banks would provide liquidity to non-banks as well, all firms in the model may become indifferent about their balance sheet structure as they now have the same exposure to aggregate liquidity risk. Alternatively, and in line with recent bank regulation reforms, the fragility of banks’ balance sheets can be reduced by raising equity and liquidity requirements. A general equilibrium version of the model, for instance augmented with negative externalities from bank failures, may provide a useful framework to study the welfare effects of such regulations.

The remainder of this paper is organized as follows. Section 2 introduces a model to analyze banks’ choice of balance sheet structures. The first part of the section focuses on the case of a single bank, the second part introduces an additional bank to distinguish between aggregate and idiosyncratic liquidity risk, and the last part discusses the key role of aggregate liquidity risk. The subsequent section discusses the implications of the analysis, while the last section concludes. Finally, the appendix offers some further mathematical derivations.

## 2 The model

Consider a setup with three time periods  $t = 0, 1, 2$  in which a single risk-neutral bank maximizes its profits. At time  $t = 0$ , the bank makes an amount of long-term loans equal to  $L$  and decides to hold an amount of cash equal to  $C$ . At this time it also chooses how to fund these assets, using (uninsured) short-term deposits  $D$  and equity  $E$ . The bank’s balance sheet identity at  $t = 0$  therefore reads  $L + C \equiv D + E$ . The following discussion introduces these items in more detail and then defines the bank’s profit function.

Starting with the asset side, long-term loans  $L$  do not pay a return at  $t = 1$ , and can be of type  $x$  or type  $y$  with  $L \equiv L_x + L_y$ . At  $t = 2$ , the percentage return on a type  $x$  loan

equals  $\alpha_x r_b$ , where  $r_b$  is the bank lending rate (to be determined later) and  $\alpha_x \in (0, 2)$  is a random asset return shock with mean equal to one. Likewise, the return on a type  $y$  loan equals  $\alpha_y r_b$ , with  $\alpha_y - 1 \equiv 1 - \alpha_x$ . As a result of this specification the shocks are each other's mirror image, so that an above average return on one type of loan coincides with a below average return on the other. At  $t = 2$ , the total value of loans on the bank's balance sheet is  $L_x(1 + \alpha_x r_b) + L_y(1 + \alpha_y r_b)$ , which in expectation equals  $L(1 + r_b)$ . If the bank at  $t = 0$  makes both types of loans in equal amounts, the asset return shocks cancel each other out and the value of loans at  $t = 2$  equals  $L(1 + r_b)$  with certainty. In addition to making long-term loans, the bank can hold cash reserves  $C$ , which always maintain their value but yield a return of zero.

Turning to the bank's liability side, short-term deposits  $D$  pay the short-term interest rate  $r_s$  at  $t = 1$  as well as at  $t = 2$ . Having paid interest, the deposits at  $t = 1$  are hit by a random liquidity shock  $\lambda \in (0, 2)$  with mean equal to one, so that  $\lambda D(1 + r_s)$  deposits remain. A value of  $\lambda$  higher than one thus implies an inflow of deposits, while a value lower than one implies an outflow. If outflows at  $t = 1$  exceed the bank's cash reserves, the bank becomes illiquid.<sup>6</sup> Furthermore, if the value of deposits at  $t = 2$  exceeds the value of assets, the bank becomes insolvent. In addition to using deposits, the bank can fund its assets with equity  $E$ , which pays a residual return at  $t = 2$  if depositors have been fully repaid.

Focusing on the expected returns on liabilities, the short-term interest rate  $r_s \geq 0$  required by depositors at  $t = 1$  and at  $t = 2$  is exogenous to the bank (e.g., set by the central bank) and compounds to  $\bar{r}_s \equiv (1 + r_s)^2 - 1$  over both periods. This interest rate does not contain a risk-premium as lending risk is fully diversifiable (i.e., idiosyncratic) and therefore is not priced. For the same reason, equity holders do not earn a risk-premium either, but they do require to earn a term-premium as they provide funds over a longer (i.e., two-period) horizon. The long-term return required by equity holders at  $t = 2$  therefore equals  $r \equiv \bar{r}_s + \Delta r$ , where the term-premium  $\Delta r > 0$  reflects the (exogenous) compensation demanded by investors for holding a long-term security instead of investing in a series of short-term ones.<sup>7</sup> In this setup,

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<sup>6</sup>At  $t = 1$ , any inflows of deposits are invested in short-term loans  $S$  that at  $t = 2$  pay the short-term interest rate  $r_s$ . Later on, these loans have a role in interbank liquidity insurance, but for now they merely provide a source of income.

<sup>7</sup>This compensation depends, amongst other things, on the market price of bearing undiversifiable liquidity



equity thus comes with a higher expected return than deposits not because it is riskier, as in most models of banking, but because it has a longer maturity.<sup>8</sup>

The above yields the following expected profit function (see the mathematical appendix for the derivations underlying the model):

$$\Pi = L(r_b - \bar{r}_s) - C\bar{r}_s - E\Delta r. \quad (1)$$

The bank is assumed to maximize these profits under the two constraints that it stays liquid and solvent under all possible realizations of the liquidity and asset return shocks (e.g., to avoid an infinitely large bankruptcy cost). These constraints simplify the exposition of the results and strengthen them at the same time, as they highlight that banks will choose a fragile balance sheet structure even if they are fully committed to avoiding liquidity and solvency risk. Of course, this commitment is weaker in practice, if only because of tax distortions and moral hazard created by deposit insurance and government safety nets. The next section formalizes both constraints and then solves for the optimal balance sheet structure.

## 2.1 Constraints for a single bank

The bank structures its balance sheet at  $t = 0$  to maximize its expected profits in (1). A first constraint it faces in doing so is that the balance sheet identity holds, which reads  $L + C \equiv E + D$ . Second, the bank wants to remain liquid even if  $\lambda = 0$  and all deposits are withdrawn. The appendix shows that the corresponding liquidity constraint reads:

$$C \geq D(1 + r_s), \quad (2)$$

which indicates that the bank needs to hold sufficient cash reserves to pay out all its depositors their notional claim plus interest at  $t = 1$ . Third, the bank wants to remain solvent even if the asset return shocks  $\alpha_x$  and  $\alpha_y$  are at their lowest possible value. As the appendix shows,

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risk. Adrian, Crump and Moench (2013) estimate that over the past fifty years, the 10-year term-premium fluctuated around 1.75 percent, as illustrated in Figure 2.

<sup>8</sup>The model omits long-term debt as a potential funding source, as banks in the model would never prefer funding with long-term debt over funding with equity. The reason is that both equity and long-term debt have the same expected return  $r$ , while only equity has the added benefit of reducing solvency risk.

if the bank specializes in a single type of loan, the corresponding solvency constraint reads:

$$E \geq D\bar{r}_s. \tag{3.a}$$

This constraint indicates that if the lending portfolio yields a return of zero, so that loans preserve their notional value but do not yield interest, the bank needs to have sufficient equity to pay its depositors the compound short-term interest rate. If the bank diversifies by making both types of loans in equal amounts, the solvency constraint reads:

$$E \geq D\bar{r}_s - Lr_b. \tag{3.b}$$

Comparing both solvency constraints shows that diversifying between both loan types reduces the amount of equity that the bank needs to hold in order to eliminate its solvency risk. The intuition behind this result is that the diversified loan portfolio yields a percentage return of  $r_b$  with certainty, while the percentage return on a portfolio with just one type of loan is risky, and can turn out as low as zero. Diversifying assets or holding equity thus provide two alternative approaches for the bank to reduce its solvency risk.

### 2.1.1 Optimal balance sheet structure

Solving the model (see the appendix) shows that the bank at  $t = 0$  chooses  $C = D = 0$ . The intuition behind this result is that funding with deposits does not maximize profits, because deposits earn a positive return  $r_s$  but have to be invested in cash against a return of zero. Hence, the bank chooses  $L = E$ , and is indifferent about specializing or diversifying its loan portfolio as it would fully finance itself with equity in both cases. Furthermore, with  $r_b = r$ , the bank lending rate is equal to the long-term interest rate  $r$ , and as  $\Pi = 0$ , expected profits are equal to zero. While this balance sheet structure is quite uncharacteristic for the typical bank, it changes below where we introduce an additional bank to distinguish between aggregate and idiosyncratic liquidity risk and to allow for interbank liquidity insurance.

## 2.2 Constraints for multiple banks

Consider a second bank  $j$  which is ex ante identical to the original bank  $i$ . The only difference between both banks involves bank  $j$ 's liquidity shock  $\lambda_j$ , which is defined as  $1 - \lambda_j \equiv \lambda_i - 1 + 2\rho(1 - \lambda_i)$ . As a result of this specification, the amount of aggregate (i.e., systematic) liquidity risk can conveniently be defined as the value of  $\rho \in (0, 1)$ , which reflects the correlation between the liquidity shocks of both banks. If  $\rho = 0$ , there is no aggregate liquidity risk as an outflow at one bank coincides with an equally large inflow at the other. By contrast, if  $\rho = 1$ , aggregate liquidity risk is at its maximum as an outflow of deposits at one bank coincides with an equally large outflow at the other. Section 2.3 discusses the origins of such aggregate liquidity risk in a bit more detail, although for the present analysis one may remain agnostic about these as well.

By itself, the presence of bank  $j$  does not affect the optimization problem of bank  $i$ . However, this changes when a bank with a liquidity deficit at  $t = 1$  can obtain a short-term loan  $S$  against interest rate  $r_s$  from a bank with a liquidity surplus. This loan was already introduced in the above as a source of income (see fn. 6), but from now on also enables banks to provide each other with liquidity insurance (as first studied by Bhattacharya and Gale (1987)).<sup>9</sup> In particular, at  $t = 1$ , the bank with a deposit inflow provides a loan to the bank with a deposit outflow, so that both of them remain liquid as long as the banking sector is liquid in the aggregate. The appendix shows that this modification changes the liquidity constraint into:

$$C \geq \rho D(1 + r_s). \tag{4}$$

For  $\rho = 1$ , this liquidity constraint is the same as in (2), so that the profit maximization problem remains unchanged. By contrast, for  $\rho = 0$ , all liquidity shocks are idiosyncratic, so that the constraint simplifies to the redundant  $C \geq 0$ .

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<sup>9</sup>In practice, frictions in the market for interbank loans may hamper banks from completely insuring each other's idiosyncratic liquidity risks. Acharya, Gromb and Yorulmazer (2012) show that when the interbank lending market suffers from imperfect competition, strategic behaviour can cause a bank with a liquidity surplus to refuse making a loan to a bank with a liquidity deficit, in an attempt to eliminate a competitor. The authors show that the central bank can ameliorate these effects by acting as lender of last resort and offering emergency liquidity assistance to individual banks.

### 2.2.1 Optimal balance sheet structure

Solving the model (see the appendix) shows that the optimal balance sheet structure at  $t = 0$  depends crucially on the correlation  $\rho$  between banks' liquidity shocks, i.e., on the aggregate liquidity risk. In particular, letting  $\rho^*$  denote a threshold value for aggregate liquidity risk, to be defined in (5) below, three cases can be distinguished. First, if aggregate liquidity risk exceeds the threshold value, i.e.,  $\rho > \rho^*$ , banks as before choose  $L = E$  and  $C = D = 0$ . Again, they are indifferent between specializing and diversifying, set  $r_b = r$ , and make expected profits equal to  $\Pi = 0$ . The intuition behind this result is that if banks would fund themselves with deposits, they would have to hold an unprofitably high amount of cash reserves as a buffer against the relatively high level of aggregate liquidity risk. Therefore, they fund themselves with equity instead.

By contrast, if  $\rho < \rho^*$ , the amount of aggregate liquidity risk in the banking sector is sufficiently low for banks to choose a different balance sheet structure. In particular, banks set  $E = 0$  and  $C = \rho D(1 + r_s)$ , so that  $L + C = D$ . In addition, they diversify their loan portfolio to eliminate asset risk. The corresponding bank lending rate equals  $r_b = \frac{\bar{r}_s}{1 - \rho(1 + r_s)}$ , with expected profits  $\Pi = 0$ . The intuition behind this result can most easily be illustrated for  $\rho = 0$ , in which case  $C = E = 0$ ,  $L = D$  and  $r_b = \bar{r}_s$ . Banks then hold no cash reserves as there is no aggregate liquidity risk, which implies that all liquidity shocks can be managed through the market for interbank loans. In addition, by diversifying their loan portfolios, banks remain solvent without holding equity. The bank lending rate then equals the interest rate on deposit funding.

The third case occurs if  $\rho = \rho^*$  and aggregate liquidity risk is equal to its threshold value, which renders banks indifferent between the above two balance sheet structures. If they choose  $E = 0$  while diversifying and setting  $C = \rho^* D(1 + r_s)$ , their expected profits are zero and, using (5) below, their lending rate equals  $r_b = \frac{\bar{r}_s}{1 - \rho^*(1 + r_s)} = r$ . This lending rate and profitability are the same as those obtained when banks choose  $L = E$  and  $C = D = 0$ , which renders banks indifferent between both balance sheet structures.

Completing the solution, the threshold value for aggregate liquidity risk is defined as:

$$\rho^* \equiv \frac{\Delta r}{(\Delta r + \bar{r}_s)(1 + r_s)}, \quad (5)$$

which is larger for higher values of the term-premium and for lower values of the (compound) short-term interest rate. The intuition behind this result is that a higher term-premium increases the expected return on equity relative to the expected return on deposits, while a lower short-term interest rate reduces the loss in income from holding cash instead of making loans. Both effects make funding with deposits and holding cash reserves more attractive, so that banks prefer doing so even for higher values of aggregate liquidity risk. The expression for the threshold value can also be rewritten as  $\Delta r \equiv (C^*/L^*)\bar{r}_s$ , which implies that a bank is indifferent about which balance sheet structure to adopt if the term-premium is equal to its cash-loan ratio multiplied by the compound short-term interest rate.

### 2.3 The key role of aggregate liquidity risk

The analysis so far was framed in terms of ‘banks’ but could just as well have been framed in terms of the more general ‘firms’, as this only requires to replace in the above the words ‘long-term loans’ and ‘short-term deposits’ by ‘long-term assets’ and ‘short-term debt’. Figure 1 then illustrates how differences in aggregate liquidity risk across firms lead them to adopt different balance sheet structures. The left part of the figure shows the optimal balance sheet associated with  $\rho = 0$ , which resembles the typical bank’s balance sheet characterized by leverage, maturity transformation, and asset diversification. The right part of the figure illustrates the optimal balance sheet structure for  $\rho = 1$ , which resembles the typical non-bank’s balance sheet financed with equity and focused on a specific asset. The middle part shows the balance sheet for  $\rho = \rho^*$ , where the firm is indifferent between adopting a bank balance sheet or a non-bank balance sheet (note that if it adopts a bank balance sheet, it holds higher cash reserves the further  $\rho$  is away from zero).

It is worth emphasizing that the middle case in the figure echoes the classic irrelevance result by Modigliani and Miller (1958), as this case involves firms being indifferent about

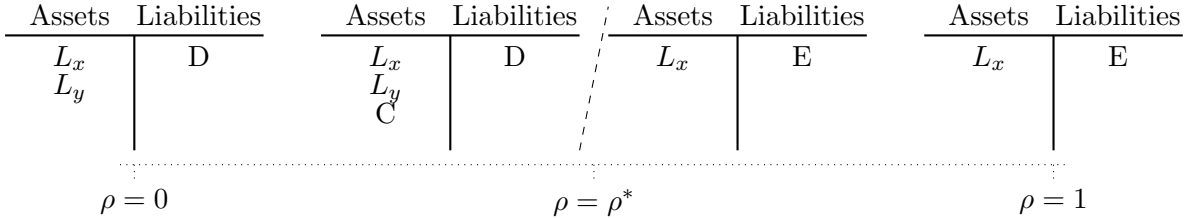


Figure 1: Aggregate liquidity risk and balance sheet structure

diversifying or specializing their asset portfolio and about funding with equity or with short-term debt. Hence, if one believes that the average firm in the economy is indifferent about which balance sheet structure to adopt, which may or may not be true in practice, the model can be made to accommodate this case by adding the assumption that the average firm's aggregate liquidity risk is equal to  $\rho^*$ . This assumption would imply that price formation in financial markets causes the term-premium to come about in such a way that fluctuations of the threshold  $\rho^*$  defined in (5) track the average firm's aggregate liquidity risk. Figure 2 shows that this threshold value attains a trough just before the start of the 2007 financial crisis, and peaks during the heat of it.

The key result from the analysis is that the model provides an understanding of the differences between banks' and non-banks' balance sheets if:

$$\rho^{\text{banks}} < \rho^* < \rho^{\text{non-banks}}, \quad (6)$$

which indicates that banks face lower aggregate liquidity risk than non-banks do. This aggregate liquidity risk originates from the fact that depositors can withdraw some or even all of their funds from banks and non-banks, and exchange these for claims on the central bank such as banknotes or reserve balances.<sup>10</sup> The reason that aggregate liquidity risk in practice is lower for banks than for non-banks, so that inequality (6) is met, is that central banks explicitly ensure the aggregate liquidity of the banking sector when they implement their monetary policy using open market operations. In particular, according to the U.S. Federal

<sup>10</sup>The origin of aggregate liquidity risk is most easily understood when thinking about the most extreme realization of an aggregate liquidity shock, which occurs if all depositors at banks and non-banks withdraw all their funds simultaneously. The only place where these depositors can then put their funds is in a physical storage of banknotes and coins or, if they have one, in their reserve account at the central bank.

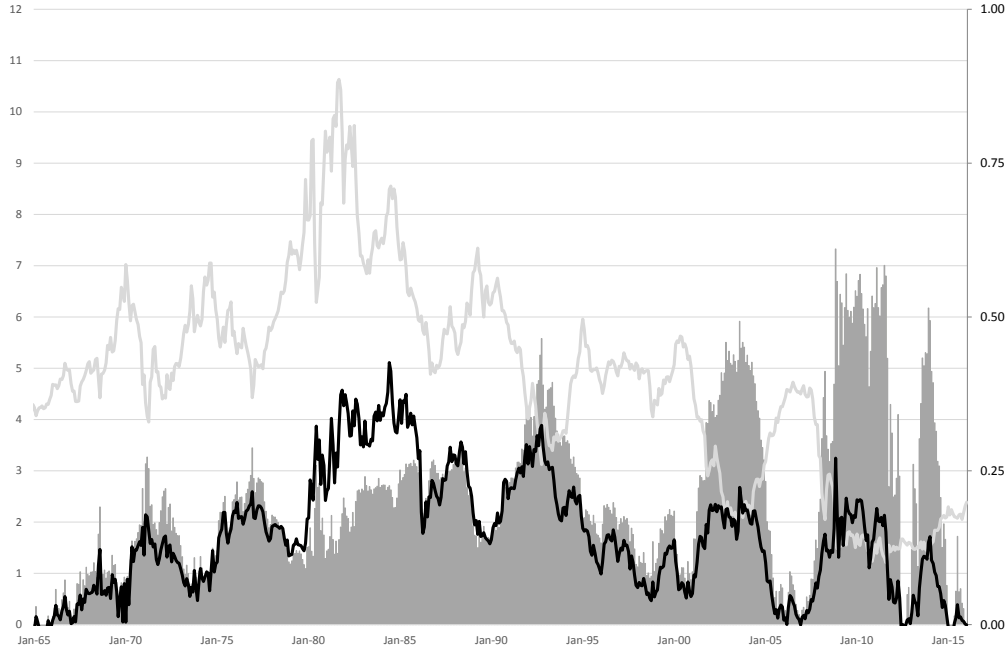


Figure 2: The threshold value for aggregate liquidity risk. Note: the graph shows the 10-year term-premium ( $\Delta r$ , black line, left axis) as well as the 10-year risk-neutral yield ( $\bar{r}_s$ , grey line, left axis) on U.S. treasuries, which add up to the 10-year U.S. interest rate. Both series are provided by the New York Federal Reserve, which estimates them using the methodology by Adrian, Crump and Moench (2013). The threshold value for aggregate liquidity risk ( $\rho^*$ , grey surface, right axis) is calculated from expression (5), measuring  $r_s$  as the federal funds rate.

Reserve System, “these operations, which are arranged nearly every business day, are designed to bring the supply of Federal Reserve balances in line with the demand for those balances at the FOMC’s [Federal Open Market Committee] target [short-term monetary policy interest] rate.” Moreover, “if, even after an open market operation is arranged, the supply of balances falls short of demand, then discount window lending provides a mechanism for expanding the supply of balances to contain pressures on the funds rate.” (Federal Reserve Board, 2005, p.35). Hence, after deciding upon the monetary policy target value for  $r_s$ , central banks implement this policy by supplying the banking sector with the full amount of liquidity demanded at this target rate, which considerably reduces, if not eliminates, the banking sector’s

aggregate liquidity risk. Benes and Kumhof (2012) summarize this notion by concluding that “at all times, when banks ask for reserves, the central bank obliges,” which if so would imply that  $\rho^{\text{banks}} = 0$ .

The strength of central banks’ commitment to ensure the aggregate liquidity of the banking sector was illustrated after the bankruptcy of Lehman Brothers in September 2008. This event shocked financial markets and triggered an outflow of short-term funds from the banking sector as a whole. As a result, liquidity constrained banks bid up the U.S. short-term interest rate above the Federal Reserve’s target value, thereby hampering the implementation of U.S. monetary policy. As Figure 3 shows, the Federal Reserve responded by rapidly expanding its lending to the banking sector, so that the short-term rate declined towards its policy target again. By targeting the short-term interest rate against which banks lend to each other, the Federal Reserve (and other central banks) thus effectively ensures the aggregate liquidity of the banking sector. As central banks do not lend to non-banks, aggregate liquidity risk in the banking sector is lower than in the non-banking sector.

The above reasoning explains why inequality (6) will hold in the current economic environment, but does not immediately explain the structure of banks’ balance sheets before central banks were established. For example, before the Federal Reserve System was founded in 1913, U.S. commercial banks also engaged in leverage and maturity transformation, albeit to a far lesser extent than currently is the case.<sup>11</sup> One possible reason why banks at the time faced lower aggregate liquidity risk than non-banks, is that banks promised their depositors to repay them in gold if they demanded so.<sup>12</sup> Non-banks did not make such a promise to their financiers, so that even if non-banks had enough gold reserves, they could still decide to repay their financiers with a bank deposit instead of in gold. As a result, even if banks and non-banks had exactly the same balance sheet structure, a deposit at a bank was a safer store of value than a deposit at a non-bank, so that banks faced lower aggregate liquidity risk

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<sup>11</sup>This difference with current banks’ balance sheet structures is described by Hanson, Shleifer, Stein and Vishny (2015), who show that in the decades before the Federal Reserve was founded, U.S. commercial banks financed as much as a quarter of their balance sheets with equity and held a roughly equal share of their assets in the form of cash. Their lending business focused on the provision of short-term loans to finance working capital, while the making of long-term loans was typically avoided.

<sup>12</sup>For example, during the decades before the Federal Reserve was established, U.S. banks issued national bank notes which were redeemable in gold or legal tender notes. See Elwell (2011) for a brief history of the Gold Standard in the United States.



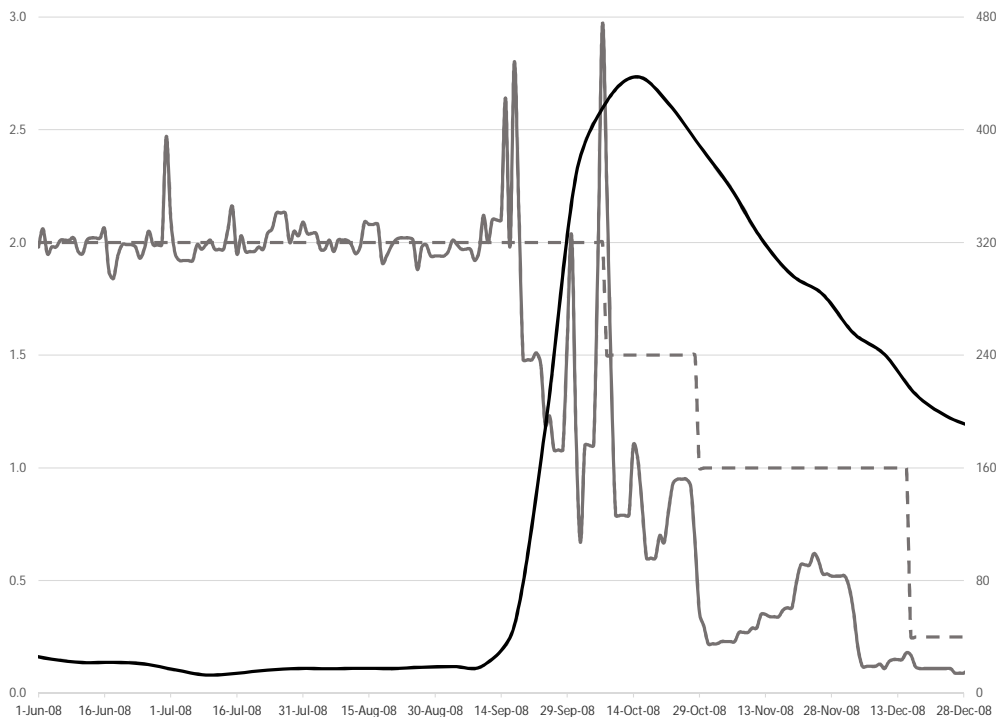


Figure 3: Federal Reserve lending around the bankruptcy of Lehman Brothers. Note: the dashed grey line shows the federal funds target short-term interest rate, the solid grey line shows the federal funds effective interest rate, and the black line shows the amount of lending by the Federal Reserve to the banking sector. All series were obtained from the St. Louis Federal Reserve Economic Database.

than non-banks. Still, banks faced frequent runs by depositors who were concerned about the adequacy of banks' gold reserves, so that aggregate liquidity risk at the time was much higher than it is today. The establishment of the Federal Reserve allowed banks to receive with liquidity support when such runs occurred, thereby cementing as well as expanding the difference between banks' and non-banks' aggregate liquidity risk. As the model shows, this difference in aggregate liquidity risk between banks and non-banks is sufficient for them to adopt different balance sheet structures, even if the average firm is indifferent about its choice.

### 3 Discussion

The above analysis implies that considerable synergies exist between the leverage, maturity mismatch, and asset diversification that characterize the typical bank's balance sheet. By diversifying their assets, banks can increase their leverage without increasing their solvency risk. By increasing their leverage, in turn, banks can increase their maturity mismatch and fund themselves to a larger extent with short-term debt. As a result of these synergies, optimal balance sheet structures in the model either exhibit zero leverage, maturity mismatch, and asset diversification, or all of these three together. Notably, the model implies that these synergies are not bank-specific, but would apply to any firm that has a lower exposure to aggregate liquidity risk. Such differences in aggregate liquidity risk are the only deviation from Modigliani and Miller (1958) in the model, which echoes these authors' capital structure irrelevance result by admitting the case where the average firm is indifferent about its balance sheet structure (i.e, about adopting a bank or a non-bank one).

The analysis implies that changes of aggregate liquidity risk over time do not affect firms' choice between a bank and a non-bank balance sheet structure, as long as these fluctuations do not affect which firms' exposure is relatively high and which firms' exposure is relatively low. Still, such changes do affect the balance sheet structure of banks. That is, following a decline in aggregate liquidity risk, banks reduce their cash reserves and increase their holdings of risky long-term loans. At the same time, they lower their lending rates, which is likely to stimulate the demand for bank credit. Hence, if aggregate liquidity risk in the economy declines during economic upswings, the analysis implies that economic booms coincide with an increase in banks' balance sheet fragility and an increase in their credit supply. As the balance sheet structures of non-banks in the model remain unaffected by fluctuations in aggregate liquidity risk, the analysis highlights why bank behavior can be pro-cyclical even if non-bank behavior is not.

The use of the word 'risk-taking' in the above is a bit misleading, as one of the model's features is that banks choose a fragile balance sheet structure despite their aim to stay liquid and solvent under all circumstances. The analysis thereby illustrates that the fragility of

banks' balance sheets should not in itself be taken as evidence that bank managers, for instance, have an excessively high risk-appetite. Still, as in Gennaioli, Shleifer and Vishny (2013), an important source of instability in the banking sector can stem from a sudden increase in the correlation between the returns on different types of assets. Such an increase in correlations reduces the benefits from asset diversification, and can thereby lead to a shortfall of banks' equity buffers. As non-banks in the model do not rely on asset diversification to maintain solvent, the analysis highlights why even banks that are risk-averse may be vulnerable to a sudden increase in correlations, while non-banks are not. Moreover, this instability may spill over between banks as they are mutually exposed to each other through the short-term loans they use to manage their idiosyncratic liquidity risk. The analysis thus also implies that banks are vulnerable to contagion, as in Rochet and Tirole (1996) and Allen and Gale (2000), while non-banks are not.

To reduce the risk of bank instability, regulators require banks to meet minimum equity and liquidity requirements. The analysis implies that equity is an expensive funding source for banks irrespective of its riskiness, but because of its longer maturity. This longer maturity causes equity financiers to demand a term-premium in return for bearing liquidity risk (i.e., for not being able to withdraw their funds in the way that short-term financiers can). From a bank's perspective this term-premium is relatively high, as banks prefer to bear the liquidity risk themselves while relying on central bank liquidity provision to the sector as a whole. Following a similar logic, the model implies that liquidity requirements are expensive for banks too, as banks prefer to rely on central bank liquidity provision rather than to hold cash reserves. The analysis thus implies that equity and liquidity requirements are costly for banks, even though for non-banks they are not.<sup>13</sup>

Finally, the model draws attention to the financial stability implications of the way in which central banks implement their monetary policies. Typically, central banks' lending to the banking sector as a whole is considered to be fairly innocuous, especially when compared to the moral hazard from central banks' lending of last resort policies (which involve support-

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<sup>13</sup>This result does not imply that bank equity and liquidity requirements reduce social welfare. For instance, their benefits include a more stable provision of banking services to the economy and less reliance on government safety nets, both of which are not incorporated in the model.

ing individual banks that are illiquid but solvent, see Goodhart (1999)).<sup>14</sup> The key insight from the analysis, however, is that even central bank lending to the banking sector as a whole gives banks an incentive to adopt fragile balance sheet structures. This incentive could be interpreted as a previously unrecognized moral hazard effect from monetary policy implementation, but this interpretation would not be entirely accurate. After all, if central banks would expand their liquidity provision to non-banks as well so that the term-premium ultimately declines towards zero, all firms become indifferent about which balance sheet structure to adopt.<sup>15</sup> Moreover, while central banks only lend to banks, this policy also affects the balance sheet structures of non-banks, which respond by avoiding leverage, maturity mismatch, and asset diversification. Hence, rather than the existence of central bank liquidity provision in itself, the unequal availability thereof to banks and non-banks in the model is why both adopt different balance sheet structures.

## 4 Conclusion

As compared to non-banks, banks adopt relatively fragile balance sheet structures characterized by leverage, maturity mismatch, and asset diversification. This choice of balance sheet structure can be understood within a simple model where banks and non-banks maximize profits under the same constraints, except for the fact that banks have a lower exposure to aggregate liquidity risk. This lower exposure of banks reflects how central banks implement their monetary policy, which typically involves determining a target for the short-term interest rate and then using open market operations to supply the banking sector with all liquidity demanded at this rate. The resulting difference between banks' and non-banks' aggregate liquidity risk provides banks with an incentive to engage in leverage, maturity mismatch, and asset diversification, while non-banks do the opposite. The same mechanism implies that

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<sup>14</sup>The moral hazard from lending of last resort policies is sometimes considered to be limited as well, with for instance Freixas, Giannini, Hoggarth and Soussa (2000, p.73) concluding that "in principle, where liquidity support clearly can be separated from the provision of risk capital, the moral hazard created will be limited to possible mismanagement of liquidity risk."

<sup>15</sup>Firms in the model may already become indifferent about their balance sheet structure if the expansion of central bank liquidity provision renders each of them equally exposed to aggregate liquidity risk, irrespective of whether this exposure and the term-premium become equal to zero. However, only if this exposure and the term-premium equal zero does this indifference result emerge as the model's single solution, since the threshold in (5) then equals zero as well.

banks, but not non-banks, will engage in pro-cyclical risk-taking, are vulnerable to contagion, and will resist regulatory equity and liquidity requirements. These findings may benefit future economic research, as well as the policy debate on bank regulation and financial stability that was sparked by the recent financial crisis.

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## Mathematical Appendix

### Expected profits

Expected profits  $\Pi$  are equal to the expected income on assets minus the expected expenditure on liabilities. The expected income on loans equals  $Lr_b$  since  $\mathbb{E}(\alpha_x) = \mathbb{E}(\alpha_y) = 1$ . Since  $\mathbb{E}(\lambda) = 1$ , the expected amount of deposit inflows is zero so that the bank in expectation does not make any short-term loans  $t = 1$ . Expected expenditure, in turn, are equal to expenditures on equity  $Er$  plus the expected interest expense on deposits at  $t = 1$  and the expected interest expense on deposits at  $t = 2$ . Since  $\mathbb{E}(\lambda) = 1$  this interest expense equals  $Dr_s + D(1 + r_s)r_s = D\bar{r}_s$ . Hence, using  $L + C \equiv E + D$  and  $r \equiv \bar{r}_s + \Delta r$ , expected profits equal  $\Pi = Lr_b - Er - D\bar{r}_s = L(r_b - \bar{r}_s) - C\bar{r}_s - E\Delta r$ . This expression is equal to equation (1) in the main text.

### The liquidity constraint for a single bank

The liquidity constraint ensures that the bank remains liquid under all circumstances. To achieve this, the amount of cash available at  $t = 1$  has to be at least as large as the maximum amount of deposits that can be withdrawn at this time. This maximum outflow occurs if  $\lambda = 0$ , in which case the bank has to pay out all depositors their notional amount plus interest  $D(1 + r_s)$ . Hence, the corresponding liquidity constraint reads  $C \geq D(1 + r_s)$ , which is equal to expression (2) in the main text.

### The solvency constraint for a single bank

The solvency constraint ensures that the bank remains solvent under all circumstances. To achieve this, the income on assets plus the amount of available equity reserves always has to be at least as large as the interest expense on deposits. This implies that  $(L_x\alpha_x + L_y\alpha_y)r_b + Sr_s + E \geq Dr_s + \lambda D(1 + r_s)$ , where the last term reflects the interest payment to depositors at  $t = 2$ . As short-term loans have to be funded with deposit inflows at  $t = 1$ , the amount of short-term loans equals  $S = \max(0, \lambda - 1)D(1 + r_s)$ . Moving all items with the exception of  $E$  to the right-hand side shows that the constraint is the tightest if  $\lambda \geq 1$ . After substituting this result, if the bank specializes in  $L_x$ , the right-hand side side attains its highest possible value for  $\alpha_x = 0$ . This yields the solvency constraint  $E \geq D\bar{r}_s$ , which is equal to expression (3.a) in the main text. Alternatively, if the bank diversifies so that  $L_x\alpha_x + L_y\alpha_y = L$  with certainty, the solvency constraint equals  $E \geq D\bar{r}_s - Lr_b$ . This is expression (3.b) in the main text.

### The solution for a single bank

The Lagrangian and Kuhn-Tucker conditions for the bank's profit maximization problem are:

$$\begin{aligned}
\mathcal{L} &= L(r_b - \bar{r}_s) - C\bar{r}_s - E\Delta r + \mu_1(D + E - L - C) + \mu_2(C - D(1 + r_s)) + \mu_3(E - D\bar{r}_s), \\
\frac{\partial \mathcal{L}}{\partial L} &= r_b - \bar{r}_s - \mu_1 \leq 0, \\
\frac{\partial \mathcal{L}}{\partial C} &= -\bar{r}_s - \mu_1 + \mu_2 \leq 0, \\
\frac{\partial \mathcal{L}}{\partial D} &= \mu_1 - \mu_2(1 + r_s) - \mu_3\bar{r}_s \leq 0, \\
\frac{\partial \mathcal{L}}{\partial E} &= -\Delta r + \mu_1 + \mu_3 \leq 0, \\
\frac{\partial \mathcal{L}}{\partial \mu_1} &= D + E - L - C = 0, \\
\frac{\partial \mathcal{L}}{\partial \mu_2} &= C - D(1 + r_s) \geq 0, \\
\frac{\partial \mathcal{L}}{\partial \mu_3} &= E - D\bar{r}_s \geq 0,
\end{aligned}$$

with inequalities  $L, C, D, E, \mu_1, \mu_2, \mu_3 \geq 0$  and equalities  $\frac{\partial \mathcal{L}}{\partial L}L = \frac{\partial \mathcal{L}}{\partial C}C = \frac{\partial \mathcal{L}}{\partial D}D = \frac{\partial \mathcal{L}}{\partial E}E = \frac{\partial \mathcal{L}}{\partial \mu_1}\mu_1 = \frac{\partial \mathcal{L}}{\partial \mu_2}\mu_2 = \frac{\partial \mathcal{L}}{\partial \mu_3}\mu_3 = 0$ . The included solvency constraint corresponds to the case where the bank specialises, while the case of diversification is discussed below. To start with, it needs to hold that  $L > 0$ , since otherwise the bank has no source of income and cannot pay the expected returns on deposits and/or equity. For the potential case where  $E = 0$ , the inequality  $\frac{\partial \mathcal{L}}{\partial \mu_3} \geq 0$  implies that  $D = 0$ , which would invalidate the balance sheet constraint since  $D = E = 0$  is inconsistent with  $L > 0$ . Hence, it follows that  $E > 0$ . For the potential case that  $C > 0$ , it follows from  $\frac{\partial \mathcal{L}}{\partial C} = 0$  that  $\mu_2 = \bar{r}_s + \mu_1$ , which is larger than zero. Using this result in  $\frac{\partial \mathcal{L}}{\partial \mu_2}\mu_2 = 0$  then requires that  $D > 0$  as well. This cannot be a solution, however, as for  $D > 0$  the equality  $\frac{\partial \mathcal{L}}{\partial D}D = 0$  only holds if  $\frac{\partial \mathcal{L}}{\partial D} = 0$ , which is infeasible since  $\mu_2 = \bar{r}_s + \mu_1$  and  $\mu_1, \mu_3 \geq 0$ . Hence, it follows that  $C = 0$ , which in combination with  $\frac{\partial \mathcal{L}}{\partial \mu_2} \geq 0$  implies that also  $D = 0$ . The optimal balance sheet configuration is therefore given by  $L = E$  and  $C = D = 0$ . This is the balance sheet structure that is presented in the main text. Furthermore, using  $\frac{\partial \mathcal{L}}{\partial \mu_3}\mu_3 = 0$  yields  $\mu_3 = 0$ , which can be used in  $\frac{\partial \mathcal{L}}{\partial E}E = 0$  to obtain  $\mu_1 = \Delta r$ . Using this result in  $\frac{\partial \mathcal{L}}{\partial L}L = 0$  yields  $r_b = \bar{r}_s + \Delta r \equiv r$ , which is the lending rate stated in the main text. Substituting these results in the expected profit function yields  $\Pi = 0$  so that excess profits are zero. Note that because  $\mu_3 = 0$ , relaxing the solvency constraint by diversifying does not affect the solution. The main text therefore focuses on the solution where the bank specializes, with  $L = E$  and  $C = D = 0$  and  $r_b = r$ .

#### The liquidity constraint for multiple banks

The liquidity constraint for bank  $i$  can be augmented to take into account that banks with a liquidity surplus can make a short-term loan  $S$  to banks with a liquidity shortage at  $t = 1$ . In particular, the constraint that needs to be met for bank  $i$  to always be liquid then becomes  $C_i + S \geq (1 - \lambda_i)D_i(1 + r_s)$ . This expression states that bank  $i$ 's cash reserves plus the interbank loan obtained from  $j$  together need to cover  $i$ 's deposit outflows. In addition, bank  $j$  wants to remain liquid with certainty too, which requires that  $C_j \geq (1 - \lambda_j)D_j(1 + r_s) + S$ ,

which states that bank  $j$ 's cash reserves need to be sufficiently large to cover its outflow of deposits plus the interbank loan it makes to bank  $i$ . Adding up both expressions yields the amount of cash reserves required for the banking sector as a whole to remain liquid  $C_i + C_j \geq (1 - \lambda_i)D_i(1 + r_s) + (1 - \lambda_j)D_j(1 + r_s)$ . This expression can be simplified by substituting  $1 - \lambda_j \equiv \lambda_i - 1 + 2\rho(1 - \lambda_i)$  and using  $C_i = C_j = C$  and  $D_i = D_j = D$  as banks are ex ante identical. Evaluating this constraint at  $\lambda_i = 0$  then yields the requirement that ensures that the banking sector is always liquid in the aggregate:  $C \geq \rho D(1 + r_s)$ . This liquidity constraint is equal to expression (4) in the main text.

#### The solution for multiple banks

The Lagrangian and Kuhn-Tucker conditions for the bank's profit maximization problem are:

$$\begin{aligned}
\mathcal{L} &= L(r_b - \bar{r}_s) - C\bar{r}_s - E\Delta r + \mu_1(D + E - L - C) + \mu_2(C - \rho D(1 + r_s)) + \mu_3(E - D\bar{r}_s + Lr_b), \\
\frac{\partial \mathcal{L}}{\partial L} &= r_b - \bar{r}_s - \mu_1 + \mu_3 r_b \leq 0, \\
\frac{\partial \mathcal{L}}{\partial C} &= -\bar{r}_s - \mu_1 + \mu_2 \leq 0, \\
\frac{\partial \mathcal{L}}{\partial D} &= \mu_1 - \mu_2 \rho(1 + r_s) - \mu_3 \bar{r}_s \leq 0, \\
\frac{\partial \mathcal{L}}{\partial E} &= -\Delta r + \mu_1 + \mu_3 \leq 0, \\
\frac{\partial \mathcal{L}}{\partial \mu_1} &= D + E - L - C = 0, \\
\frac{\partial \mathcal{L}}{\partial \mu_2} &= C - \rho D(1 + r_s) \geq 0, \\
\frac{\partial \mathcal{L}}{\partial \mu_3} &= E - D\bar{r}_s + Lr_b \geq 0,
\end{aligned}$$

with inequalities  $L, C, D, E, \mu_1, \mu_2, \mu_3 \geq 0$  and equalities  $\frac{\partial \mathcal{L}}{\partial L} L = \frac{\partial \mathcal{L}}{\partial C} C = \frac{\partial \mathcal{L}}{\partial D} D = \frac{\partial \mathcal{L}}{\partial E} E = \frac{\partial \mathcal{L}}{\partial \mu_1} \mu_1 = \frac{\partial \mathcal{L}}{\partial \mu_2} \mu_2 = \frac{\partial \mathcal{L}}{\partial \mu_3} \mu_3 = 0$ . The included solvency constraint corresponds to the case where the bank diversifies, while the case of specialization is discussed below. To start with, note that  $\frac{\partial \mathcal{L}}{\partial D}$  is strictly smaller than zero if  $\rho > \rho^*$  with  $\rho^* \equiv \frac{\mu_1 - \mu_3 \bar{r}_s}{\mu_2(1 + r_s)}$ , i.e. if the fraction of aggregate liquidity risk exceeds a threshold value  $\rho^*$ . Focusing on the case  $\rho > \rho^*$ , it follows from  $\frac{\partial \mathcal{L}}{\partial D} D = 0$  that  $D = 0$ , which since  $L > 0$  implies that  $E > 0$ . Using  $D = 0$  in  $\frac{\partial \mathcal{L}}{\partial \mu_3} \mu_3 = 0$  then implies that  $\mu_3 = 0$ , which can be substituted in  $\frac{\partial \mathcal{L}}{\partial E} = 0$  to yield  $\mu_1 = \Delta r$ . Subsequently, through  $\frac{\partial \mathcal{L}}{\partial L} = 0$  it follows that  $r_b = \bar{r}_s + \Delta r = r$ . Furthermore,  $C = 0$ , since otherwise  $\frac{\partial \mathcal{L}}{\partial C} = 0$ , but this yields  $\mu_2 > 0$  and therefore  $\frac{\partial \mathcal{L}}{\partial \mu_2} = 0$ , which given  $D = 0$  yields  $C = 0$  after all. Note that because  $D = 0$ , the solvency constraint does not bind so that there is no need for the bank to diversify. The main text therefore focuses on the solution where the bank specializes,  $L = E$ ,  $C = D = 0$ , and  $r_b = r$ . Substituting these results in the expected profit function yields  $\Pi = 0$ , so that excess profits are zero.

The alternative case  $\rho < \rho^*$  implies that aggregate liquidity risk is below the threshold value. Focusing on the solution where  $D > 0$  implies through  $\frac{\partial \mathcal{L}}{\partial \mu_2} \geq 0$  that  $C > 0$  as long as  $\rho > 0$ . Using  $\frac{\partial \mathcal{L}}{\partial C} = 0$  then yields

$\mu_2 = \mu_1 + \bar{r}_s > 0$ , which implies that  $\frac{\partial \mathcal{L}}{\partial \mu_2} = 0$  so that  $C = \rho D(1 + r_s)$ . Exploring the case where  $E = 0$  and  $\mu_3 = 0$  then implies through  $\frac{\partial \mathcal{L}}{\partial L} = 0$  that  $\mu_1 = r_b - \bar{r}_s$ , which through  $\mu_2 = \mu_1 + \bar{r}_s > 0$  implies that  $\mu_2 = r_b$ . Substituting these results in  $\frac{\partial \mathcal{L}}{\partial D} = 0$  then yields  $\mu_2 = r_b = \frac{\bar{r}_s}{1 - \rho(1 + r_s)}$ , which in addition implies that  $\mu_1 = \frac{\rho(1 + r_s)\bar{r}_s}{1 - \rho(1 + r_s)}$ . It can easily be verified that for these values  $\frac{\partial \mathcal{L}}{\partial D} = 0$  for all  $\rho < \rho^*$ , which confirms that  $D > 0$  is a solution. This is the solution presented in the main text.

Finally, if  $\rho = \rho^*$ , both of the above balance sheet structures are a solution so that banks are indifferent about which balance sheet structure to adopt. Since  $\rho^* \equiv \frac{\mu_1 - \mu_3 \bar{r}_s}{\mu_2(1 + r_s)}$  can under both solutions be written as  $\rho^* = \frac{r_b - \bar{r}_s}{r_b(1 + r_s)}$ , which follows because  $\mu_2$  in the first solution can take any non-negative value, the threshold  $\rho^*$  lies above zero and below one. This implies that some values in  $\rho \in (0, 1)$  will be lower than the threshold while others will be higher. At the threshold, both balance sheet structures yield  $r_b = r$  so that  $\rho^* = \frac{\Delta r}{(\Delta r + \bar{r}_s)(1 + r_s)}$ . This is the threshold value presented in the main text.

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