# Funding Shocks and Credit Quality

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**Abstract**

Some credit booms, though by no means all, result in financial crises. While risk-taking incentives seem a plausible cause, market participants do not appear to anticipate increasing risk. We show how credit expansions driven by credit supply shocks may be misunderstood as productivity driven, due to the opacity of bank balance sheets. Large funding shocks may induce some intermediaries to scale up speculative lending, distorting price signals. Other banks and firms may misjudge actual profitability, reinforcing the credit expansion. Similarly, at times of low saving supply credit may be inefficiently low, and speculative assets underpriced.

## 1. Introduction

In the classic view of business cycle the volume of credit is driven by real shocks, and financial intermediation affects the cycle only via contractual frictions. However, recent evidence points to an independent role of credit supply in explaining output fluctuations (Krishnamurthy and Vissing-Jorgensen, 2012a; Mian et al., 2017a). Credit booms with weak productivity are more likely to end in financial crises (Gorton and Ordoñez, 2016; Mian et al., 2017b), suggesting instability may stem from excess lending relative to productive demand.

While a diffused view holds that the recent credit crisis was caused by deliberate risk taking, excess credit may also have resulted from economic agents overlooking the build up of risk. In the credit boom of the early 2000s, financial prices such as bank equity returns and credit spreads suggest that investors did not anticipate rising losses (Baron and Xiong, 2017; Krishnamurthy and Muir, 2016). Studies on historical credit booms confirm that risk accumulation is often not recognized, so financial instability comes as a surprise (Gennaioli et al., 2015; Reinhart and Rogoff, 2009; Richter et al., 2017). A natural question is why most investors and banks do not recognize increasing risk.

While behavioral drivers likely play a role, even rational agents may be unable to correctly assess the state of the economy. This paper offers a rational explanation for episodes of excess credit, in which deliberate risk taking by some agents may be amplified via an imperfect inference of underlying fundamentals by other players. It thus reconciles the risk shifting view with the evidence of market participants underestimating the underlying risk during booms.

In the model strong funding supply can induce banks to engage in speculative lending that distorts asset prices. Because bank balance sheets are opaque other agents are unable to disentangle demand and supply effects and may thus misjudge high prices as reflecting strong economic fundamentals. As a result, they may overestimate the quality of investment opportunities, amplifying the credit expansion. Likewise, when funding supply is weak, agents may underestimate the quality of opportunities and hence underinvest.

We study the investment choices of two types of agents: large intermediaries informed about aggregate productivity, and small uninformed firms. We think of the former as global banks, which have access to a broader set of information and international capital markets. Global banks receive insured debt funding and privately choose the levels of productive lending and speculative investment in a speculative asset. An aggregate shock determines the level of available bank funding. Uninformed firms cannot observe the realization of this shock nor the level of aggregate productivity. However, as the publicly observed price of the speculative asset depends on the productivity, it conveys some information to the less informed agents.

In equilibrium each bank chooses between a strategy that ensures solvency in all states, or a risk shifting strategy. The latter entails a higher exposure to the speculative asset relative to an optimal choice of an unlevered investor and may lead to bank default. The solvent strategy involves less risky investment, so that losses never compromise debt repayment.

Under low funding supply the speculative asset is under-priced, it reflects the available cash in the market and the relative marginal returns of the two investment opportunities. Under moderate funding supply all global banks follow the solvent strategy and their exposure to the risky asset is justified by its fundamental value. Consequently, the price of the asset reflects the aggregate productivity. Under large funding supply, banks are able to reach high leverage, which encourages risk shifting. The resulting speculative investment results in the asset being overpriced.

The same asset price may result from speculation investment in a low productivity and high funding state and from a solvent equilibrium in a high productivity and moderate funding state. Since the firms cannot observe the funding shock, their inference is distorted. They may be unable to determine whether the observed price indicates a good boom, characterized by high productivity and solvent investment by global banks or a bad boom, with a low productivity and excessive risk taking driven by an abundant supply of bank funding. If the expected payoff of the speculative investment is sufficiently sensitive to changes in aggregate productivity, firms underestimate productivity when funding supply is moderate and overestimate it when supply is abundant.

If uninformed agents are unlevered, the inference problem leads to a symmetric amplification: they over-invest when funding supply is high, and under-invest when it is low. In contrast, if the less informed agents are also levered, high uncertainty regarding the true productivity may alter their incentives. We study this possibility by extending our setting to include local banks, which similarly to the uninformed firms are unable to observe the aggregate productivity and funding flows towards global banks. The distorted inference may induce them to invest excessively, as they do not internalize the losses in case of default. The result is a form of uninformed risk shifting where lending decisions are ex-post optimal if the true productivity is high, but lead to default if high asset prices are caused by supply-driven speculation. Their risk choice represents an induced rather than fully deliberate form of risk taking.

The problem of local bank's gives raise to potential inefficiencies induced by the confused inference. We discuss the scope for a regulator to restore the efficient investment by local banks using a reserve requirement. The optimal requirement is positive requirement only if the observed prices are such that local banks are induced to risk-shift. However, the local regulator may be unable to restore the efficient solution if the uncertainty about the true productivity is too high. The key trade off for the regulator is between allowing for excessive lending by local banks and inducing an inefficient contraction.

The inference problem faced by investors in the model is analogous to the identification problem faced by an empirical researcher seeking to disentangle supply and demand effects. Quite unfairly to empirical researchers, rational agents are assumed to know all probability distributions and most parameters. In an extension we allow the uninformed agents to also observe the total investment by banks. We show that once agents observe both asset prices and credit volume they can precisely infer the underlying credit demand and supply. In reality the opacity of bank assets, imprecise measurement and gradual learning is likely to allow confusion to persist even when both price and volume signals are observed. We show that if uninformed agents face additional uncertainty regarding banks' risk shifting incentives (for instance, if they cannot observe bank's funding and it's exposure to legacy assets), the imperfect inference may persist even when agents observe both credit quantity and speculative price.

The next section discusses the related evidence and theoretical contributions. Section 3 introduces the baseline model and offers the main results. Section 4 discusses the investment problem by uninformed local banks and the optimal policy response of a local regulator. Section 5 introduces the additional signal and additional source of balance sheet opacity. Section 6 concludes

## 2. Related Literature

Recent evidence on credit booms suggests that abundant bank funding is more likely to lead to a crisis. High credit growth is one of the best predictors of bank distress (Borio, 2014; Reinhart and Rogoff, 2009; Schularick and Taylor, 2012). The quality of credit tends to deteriorate during booms with weak productivity growth, suggesting endogenous build-up of risk in the expansion phase (Bolton et al., 2016; Dell'Ariccia et al., 2012; Greenwood and Hanson, 2013). Credit booms characterized by surging house prices and loan-to-deposit ratios suggest an imbalance between new investment and funding volumes, and are more likely to end in crises (Richter et al., 2017). In contrast, credit expansions accompanied by high and sustained productivity growth are unlikely to be followed by a crisis (Gorton and Ordoñez, 2016).

While credit supply may be boosted by demographic trends, safety demand and capital inflows, it is also enhanced by financial liberalization and bank deregulation. These trends were believed to attenuate financial frictions, enhancing access to finance and boosting growth. Yet evidence on the recent credit expansion suggests it was driven by abundant bank funding rather than a relaxation of borrowers' collateral constraints (Justiniano et al., 2015).

US evidence suggests that local credit supply shocks boosted house prices rather than local productivity (Mian and Sufi, 2009; Mian et al., 2017b), and led to a sharper cycle correction in the bust (Di Maggio and Kermani, 2015). The result is an amplified business cycle, with higher growth in output, employment and house prices in good times and sharper declines in recessions. The model offers a rationalization of recent evidence on the quality of credit boom, in particular on the combination of rapid credit expansion with weak productivity growth at times of high asset values.

A prominent view why rising risk may not be properly anticipated is that a collective misjudgement reflects diffused irrationality. While models with overconfident agents can produce rising leverage and asset prices under marked differences in opinions (Geanakoplos, 2010), there is little evidence of wide skepticism over risk during the recent boom, even among bankers (Baron and Xiong, 2017). The nonfictional book "The Big Short" (Lewis, 2011) describes how hardly any investor realized even late in the boom how risky lending had become. In fact, many of the few who did had autistic traits that may have made them less prone to social herding.

Behavioral rules of posterior belief formation such as representativeness heuristics Gennaioli et al. (2015) do explain how market participants may underestimate the chance of a crisis in an economic expansion. Without denying any role for limited rationality, we seek to offer a rational benchmark on why public inference may be confused.

Our work relates to the literature on overconfidence arising via rational learning (Pástor and Veronesi, 2003; Biais et al., 2015). Since market participants who observe the improving economic conditions during the boom phase may be unable to recognize the growing risk, misinterpreting high asset prices as reflecting buoyant productivity. Our insight is that opacity of bank balance sheets (perhaps justified by optimal risk sharing (Dang et al., 2017)) may be an important factor obstructing the interpretation of price and quantity signals by market participants. A related effect is suggested by Thakor (2016), who shows that uncertainty over banker quality may result in under- or overestimation of risk, and by Lee (2016) where investors are unable to recognize the quality of bank lending by observing bank funding demand alone.

Abundant credit supply may result from capital inflows driven by safety needs (Caballero and Krishnamurthy, 2008), demographic trends or secular stagnation (Eichengreen, 2015; Doettling et al., 2017). A relaxed regulatory stance may also contribute, though this view raises the question why regulators become more confident about risk.

Our work is related to the literature suggesting a link between low interest rates and risk-taking (Dell'Ariccia et al., 2017; Jiménez et al., 2014). Recent work explains possible channels when rates are exogenously low (Acharya and Plantin, 2017; Dell'Ariccia et al., 2014; Martinez-Miera and Repullo, 2017). Our setup suggests low rates may reflect abundant funding supply, which affects risk incentives directly.

The analysis adopts a partial equilibrium setting to illustrate a possible channel for distorted inference from financial prices. A simple framing enables to interpret some empirical finding on credit cycles. We abstract from bank funding costs, and focus on the volume of available funds. We assume suppliers of bank funding are insured or able to avoid losses (running is safe if there is adequate insured deposit funding). Savings appear fairly inelastic to interest rates, and a segmented demand for safe assets may eliminate an important price signal (Krishnamurthy and Vissing-Jorgensen, 2012b; Gorton et al., 2012).

A more complete model may enable to study welfare issues related to long term speculative assets that reflect pure rents, such as land or real estate (net of their productive component). When funding supply and profitable demand are not too far the asset is fairly priced, revealing the true value of productivity and contributing to efficient lending. When risk shifting triggered by excess or scarce credit supply causes mispricing, it distorts both incentives and rational inference on productive investment.

## 3. The Model

There are two dates ($t = 0, 1$) and two types of active agents: a unit mass of large (global), informed banks and a unit mass of uninformed firms (in the next Section we study the case when uninformed agents are local banks). At $t = 0$ there is uncertainty about productivity and savings supply.

**Assets**

There are two assets: productive investment (available to all agents) and a speculative asset in fixed supply (eg. land and real estate, available only to large banks). Each unit $x_i$ invested by an agent $i$ at $t = 0$ into the productive technology yields $f(x_i) = \alpha x_i^\gamma$ at $t = 1$, where productivity $\alpha$ is drawn from a uniform distribution $\alpha \sim U[\underline{\alpha}, \overline{\alpha}]$. Large banks receive a precise signal about the realized productivity $\alpha$ at $t = 0$, other agents know only the distribution. In order to be able to derive closed form solutions we focus on the case when $\gamma = \frac{1}{2}$ (in the appendix we discuss our results in the general formulation with $\gamma \in (0, 1)$)

The payoff of the speculative asset at $t = 1$ is subject to aggregate risk. It's realization is drawn from a binomial distribution at the final date:

$$y \rightarrow \begin{cases} Ry & \text{with prob. } q(\alpha) \\ 0 & \text{with prob. } 1 - q(\alpha) \end{cases}$$

An interpretation is that the end date return represents the long term value of the durable asset. From ex-ante perspective, the asset value is uncertain and correlated with current productivity (long term value depends on future prospects, which are more likely to be high when the current fundamentals are strong $q'(\alpha) > 0$). We assume that the upside payoff of the asset is not too high relative to the productivity: $R < (\frac{\alpha}{2})^2$.

Banks can purchase the asset from the hands of the initial asset owners, who are uninformed and derive utility only from $t = 0$ consumption, so that they accept any positive price. Thus, the asset price $p$ is determined by bank's demand at $t = 0$. Uninformed firms can observe the price and may use it to infer the realized productivity.

For simplicity we assume that only large banks are able to invest in the speculative asset [1]. We denote the amount of the asset purchased at $t = 0$ by bank $i$ as $y_i$. All agents in the economy have access to a private storage technology with return of $1$ on any stored unit $z_i$.

[1] Allowing uninformed players to invest in the speculative asset does not affect the main results. Due to asymmetric information about asset quality, smaller agents are unable to assess whether the asset is fairly priced. Since in equilibrium, the risky asset is overpriced with positive probability, the expected return of the asset is always lower than the return on storage for the uninformed agents.

**Funding supply**

Available funding $s$ represents aggregate savings by agents seeking a safe asset (for example: global savers). Large banks choose how much out of available funds to accept as deposits, with any residual placed in private storage by the savers. We assume that bank deposits are insured. The bank funding cost thus equals the unit return of storage[2]. The amount of funding available to global banks is subject to an aggregate shock. It's realization is drawn at the beginning of $t = 0$ and is private information of large banks:

$$s = \begin{cases} s^L & \text{with prob. } \rho \\ s^H & \text{with prob. } 1 - \rho \end{cases}$$

We assume that the lowest feasible funding supply is larger than then the supply of the asset $s^L > 1$. Uninformed firms are endowed with a fixed amount of own capital $c_j = c$. They seek to infer productivity from traded asset prices.

**Timing**

The timing is as follows:

- At $t = 0$:
  - Aggregate productivity $\alpha$ is realized and observed by large banks
  - Large banks receive funding supply $s$ and choose their portfolio
  - All funding not taken on by banks is stored by savers
  - Uninformed firms observe the asset price $p$, and choose their productive investment
- At $t = 1$:
  - The long-term state (ie. the speculative payoff) is realized
  - All assets pay off
  - Banks or deposit insurance pay back depositors

[2] An alternative would be to model a decision of risk-neutral uninformed savers to deposit funding at a bank. In such a setting, depositors charge a premium to compensate for the expected losses. As long as depositors are uninformed about the size of the aggregate savings, the only difference from the setting with the deposit insurance is that risk-shifting by banks emerges under a higher funding supply.

### 3.1. Investment by large banks

After observing aggregate productivity and their own funding, each large bank chooses an investment into productive lending ($x_i$) and the long term asset ($y_i$) to maximize expected profits subject to funding and no-short-selling constraints. Since the funding and the storage offer the same return, banks are indifferent between refusing some of the available funding and storing that amount. Thus, the level of $z_i$ and $s_i$ is indeterminate and we can only pin down the size of the balance sheet net of storage: $s_i = s - z_i = py_i + x_i$.

**Unlevered investment benchmark**

As a benchmark we first analyze the optimal investment by an unlevered informed investor with endowment of $s$. This case represents the frictionless subgame of informed investors: this is how the problem would look like if banks were faced with unlimited liability or if their funding was uninsured but instead provided by informed, risk-neutral savers requiring appropriate risk compensation. Since these unlevered investors use their endowment instead of deposits, their gross balance sheet size $s$ is fixed. Thus we can pin down their investment into productive technology $x_i$, speculative asset $y_i$ and storage $z_i$ by solving the following:

$$max_{x_i,y_i,z_i}\ \alpha\sqrt{x_i} + q(\alpha)Ry + z - s$$

subject to:

$$x_i + py_i + z_i = s \qquad \text{(budget constraint)}$$

$$x_i \geq 0,\ y_i \geq 0,\ z_i \geq 0 \qquad \text{(no-short-selling constraint)}$$

The optimal investment into productive technology by an unlevered agent is such that its marginal return is equal to the opportunity cost of capital. The relevant opportunity cost depends on the price of the speculative asset, taken as given by atomistic investors.

$$x_i^* = \begin{cases} \min[(\frac{\alpha p}{2q(\alpha)R})^2, s] & \text{if } p < q(\alpha)R \\ \min[(\frac{\alpha}{2})^2, s] & \text{if } p \geq q(\alpha)R \end{cases}$$

An unlevered informed investor has no incentives to take excessive risk, and evaluates investment opportunities solely on their expected returns. If the expected speculative return is higher than the return on storage $p < q(\alpha)R$, an unlevered investor allocates all of his endowment between the productive and speculative investment ($y_i^* = \frac{s-x_i^*}{p}$, $z_i^* = 0$). Otherwise (when $p > q(\alpha)R$), the informed investor chooses storage over the risky investment ($y_i^* = 0$, $z_i^* = \frac{s-x_i^*}{p}$).

**Bank's investment choice**

We next study the investment choice of a levered bank subject to limited liability and deposit insurance:

$$max_{x_i,y_i,s_i} q(\alpha)(\alpha\sqrt{x_i} + Ry_i - s_i) + (1-q(\alpha))\max[\alpha\sqrt{x_i} - s_i, 0]$$

subject to:

$$x_i + py_i = s_i \qquad \text{(budget constraint)}$$
$$s_i \leq s \qquad \text{(funding constraint)}$$
$$x_i \geq 0,\ y_i \geq 0 \qquad \text{(no-short-selling constraint)}$$

An optimal investment by a bank entails a choice between a solvent strategy and a risk-shifting strategy. A solvent strategy ensures solvency even in the bad state, but may involve some risky investment as long as its losses never compromise deposit repayment. A risk-shifting strategy is characterized by over-investment into speculative asset relative to the unlevered-investment benchmark, induced by the interaction of default risk and bank's limited liability. The optimal strategy choice is determined by the level of available funding supply.

**Lemma 1.** *There exists an individual risk-shifting threshold of funding supply $\hat{s}(\alpha, p)$, such that:*

- *When bank's funding supply is below the threshold $s < \hat{s}(\alpha, p)$, it invests according to a solvent strategy:*

$$x_s^* = \min[(\frac{\alpha p}{2q(\alpha)R})^2, s] \qquad y_s^* = \frac{s - x_s^*}{p} \qquad \textit{if } p < q(\alpha)R$$
$$x_s^* = \min[(\frac{\alpha}{2})^2, s] \qquad y_s^* \in [0, \frac{s - x_s^*}{p}] \qquad \textit{if } p = q(\alpha)R$$
$$x_s^* = \min[(\frac{\alpha}{2})^2, s] \qquad y_s^* = 0, \qquad \textit{if } p > q(\alpha)R$$

- *When bank's funding supply is above the threshold $s > \hat{s}(\alpha, p)$, it invests according to a risk-shifting strategy:*

$$x_r^* = (\frac{\alpha p}{2R})^2, \qquad y_r^* = \frac{s - x_r^*}{p} \qquad \forall p$$

*Proof.* In Appendix □

In a solvent strategy, bank's portfolio choice is similar to the choice by an unlevered agent. It will exclude any investment in a speculative asset with negative expected return ($p > q(\alpha)R$), as its' return does not compensate for bank's cost of funding. Productive investment is chosen such that its marginal return equals to the cost of funding, and the bank declines any more funds. When the speculative asset has a positive expected return ($p < q(\alpha)R$), the bank invests in the productive technology until its productivity matches the speculative return. All residual available funding is invested into the speculative asset. As long as available funding is below the risk-shifting threshold, the potential losses are not large enough to compromise the repayment of deposits.

In a risk-shifting strategy the bank defaults with probability $1 - q(\alpha)$, thus it maximizes profits conditional on the good state outcome. Its portfolio choice equalizes the marginal productive return with the opportunity cost: the speculative return in the good state ($\frac{R}{p}$). Higher opportunity cost implies that risk-shifting banks invest less into productive technology (compared to solvent banks or unlevered agents). They allocate all remaining funds to the speculative asset.

The risk-shifting strategy is preferred over the solvent investment only if the bank can achieve a sufficient scale of speculation. Thus, risk-shifting is optimal if and only if the supply of available funding exceeds the individual risk-shifting threshold $s > \hat{s}(\alpha, p)$. This is our basic insight: for a given price level banks choose a risk-shifting strategy after a large supply shock, since it enables a high scale of speculative investment. The strategic shift occurs as a large supply shock enables higher leverage, so the expected return on a larger risky portfolio exceeds the solvent strategy return on a smaller investment scale.

**Lemma 2.** *The individual risk-shifting threshold is given by:*

$$\hat{s}(\alpha, p) = \begin{cases} (\frac{\alpha}{2})^2 \frac{p}{q(\alpha)R}[1 + q(\alpha)] & \text{if } p < q(\alpha)R \\ (\frac{\alpha}{2})^2 \frac{1 - q(\alpha)\frac{p}{R}}{q(\alpha)(\frac{R}{p} - 1)} & \text{if } p \geq q(\alpha)R \end{cases}$$

*The threshold:*

- *increases in the price of the speculative asset* ($\frac{\partial \hat{s}}{\partial p} > 0$)
- *decreases in the productivity* ($\frac{\partial \hat{s}}{\partial \alpha} > 0$) *whenever:*
  - *If* $p < q(\alpha)R$, *then* $1 + q(\alpha) < \frac{\alpha}{2}q'(\alpha)$, *or*
  - *If* $p \geq q(\alpha)R$, *then* $q(\alpha)(1 - q(\alpha)\frac{p}{R}) < \frac{\alpha}{2}q'(\alpha)$

*Proof.* In Appendix □

The price of the speculative asset affects the individual risk shifting threshold directly through its effect on the return. Everything else equal, higher price leads to lower profits under the risk shifting strategy, moving the threshold upwards.

An increase in productivity implies higher return on productive investment and an increase in probability of high speculative payoff. If the latter effect is strong ($q'(\alpha)$ is sufficiently high), an increase in productivity disproportionately increases the profits from risk shifting, moving the risk shifting threshold downwards. If the expected payoff of the speculative asset is not too sensitive to the changes in productivity, the risk shifting threshold will increase as a result of a rise in productivity.

Since the individual risk-shifting threshold of funding is monotonically increasing in the level of price, we can reformulate the result of Lemma 2. At any given level of funding $s$, there exists a lower bound price $\hat{p}(\alpha, s) = \hat{s}^{-1}(\alpha, s)$, such that when the asset price is below the threshold the bank prefers the risk-shifting strategy. Whenever the price is above the threshold the solvent strategy is optimal. We refer to this price as the solvent investment lower bound: it is the lowest price at which an individual bank has no risk-shifting incentives.

### 3.2. Equilibrium investment and prices

The individual decision rule of the banks derived in the previous section allows us to determine the equilibrium investment and prices as a function of productivity and funding supply.

**Proposition 1.** *There is an equilibrium risk shifting threshold of funding supply given by:*

$$\hat{s}^*(\alpha) = \hat{s}(\alpha, q(\alpha)R)$$

*When funding supply is funding is below the threshold all banks invest according to the solvent strategy. If the funding is above the threshold, a positive fraction of banks engages in risk-shifting.*

*Proof.* In the Appendix □

To understand this result let's assume that at a given $s$ and $\alpha$ all banks invest according to the solvent strategy. In this case the price is given by $p_s^* = min[s - (\frac{\alpha p}{2q(\alpha)R})^2, q(\alpha)R]$. If the resulting price is higher than the solvent investment lower bound $p_s^* > \hat{p}(\alpha, s)$, banks have no risk-shifting incentives. Thus, all invest according to the solvent strategy. However, if the solvent price is below the lower bound $p_s^* < \hat{p}(\alpha, s)$, risk-shifting is a dominating strategy. Thus, banks prefer switching from the solvent to the risk-shifting strategy up until the speculative demand drives asset price to the solvent investment lower bound: $p^* = \hat{p}(\alpha, s)$. The graph below plots the price for some fixed productivity $\alpha$ as a function of funding supply under all solvent $p_s^*(s)$ and all risk-shifting investment $p_r^*(s)$ together with the solvent investment lower bound on price $\hat{p}(s)$.

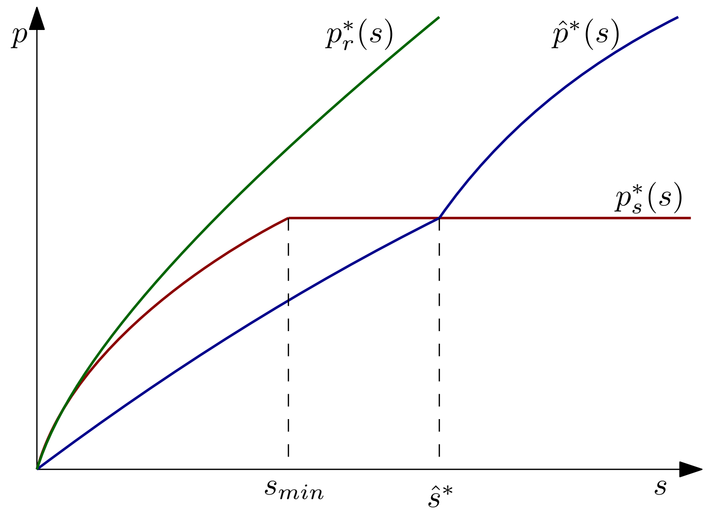


Figure 1: Prices as a function of funding supply if all banks invest solvently $p_s^*(s)$, red curve, and if all banks risk-shift $p_r^*(s)$, green curve. The blue curve corresponds to the solvent investment lower bound.

Initially the solvent investment lower bound price increases in the funding supply at a slower pace then the price under "all solvent" investment. Because banks investing solvently demand the asset only until $p = q(\alpha)R$, the price under "all solvent" investment remains fixed as the funding supply increases above the minimum unconstrained level ($s > (\frac{\alpha}{2})^2 + q(\alpha)R = s_{min}(\alpha)$). The solvent investment lower bound continues to increase in the funding supply. Thus, once the supply exceeds the individual risk shifting threshold at the highest feasible asset price under the "all solvent" investment ($p = q(\alpha)R$), banks prefer to engage in risk-shifting.

**Lemma 3.** *For a given level of productivity*

- *if funding supply is low ($s < s_{min}(\alpha)$): all banks invest solvently $x^* = (\frac{\alpha p^*}{2q(\alpha)R})^2$, $y^* = s - (\frac{\alpha p^*}{2q(\alpha)R})^2$ and the asset is underpriced: $y^* = p^* < q(\alpha)R$*
- *if funding supply is moderate ($s_{min}(\alpha) < s < \hat{s}^*(\alpha)$) all banks invest solvently: $x^* = (\frac{\alpha}{2})^2$, $y^* = q(\alpha)R$ and the asset is fairly priced $y^* = p^* = q(\alpha)R$*
- *if funding supply is high ($s > \hat{s}$): fraction $\psi^*(\alpha, s) = \hat{s}(\alpha)/[s - (\frac{\alpha p^*}{2R})^2]$ of banks is risk-shifting: $x_r^* = (\frac{\alpha p^*}{2R})^2$, $y_r^* = s - (\frac{\alpha p^*}{2R})^2$, the remaining banks invest solvently: $x_s^* = (\frac{\alpha}{2})^2$, $y_s^* = 0$ and the asset is overpriced: $p^* = \hat{p}(\alpha, s) > q(\alpha)R$*

*Proof.* In the Appendix □

When the funding supply is low, none of the banks is willing to risk shift and the demand for the speculative asset is insufficient to drive the asset price up to its fundamental value. If the funding is above the minimum-unconstrained level but below the equilibrium risk shifting threshold the solvent investors demand the speculative asset only until its' price is equal to the expected payoff. Thus, any funding above the minimum unconstrained level $s_{min}(\alpha)$ will be rejected by banks (or equivalently: invested in storage). When funding supply is higher than the risk shifting threshold evaluated at the fair price of the asset, the high leverage makes excessive speculation attractive. A fraction of banks engages in risk-shifting, thus bringing the asset price to the solvent investment lower bound. At this price level banks are indifferent between risk shifting and investing solvently. The resulting asset price is higher than its' expected value- thus the asset is unattractive for banks pursuing a solvent strategy. Our setting gives rise to three distinct states of the global economy: a stagnation, a good boom and a bad boom. In stagnation, available funding is insufficient for the banks to utilize all of the available investment opportunities. In the good boom, the funding is sufficient for all of the good opportunities to be exploited but not too high, so that bank's leverage remains limited and their incentives are sound. In the bad boom, the funding supply is excessive relative to the quality of the investment opportunities. This gives rise to risk shifting incentives as global banks invest in an asset with a negative expected return, hoping to benefit from its' upside. Their exorbitant demand for the speculative investment comes at a cost an inefficiently low productive lending. Thus, in the bad boom the total investment is excessive and the allocation of the funds to the available opportunities is misguided. The bad boom is associated with a positive probability of bank default.

#### Comparative statics

The relationship between equilibrium prices and funding supply is reflected in Figure **??**. In the graph, the equilibrium price of the speculative asset is given by the maximum of the all solvent price and the solvent investment lower bound: $p^* = \max[p_s^*(s), \hat{p}^*(s)]$. We never observe all banks risk-shifting in the equilibrium, because the resulting price would be too high for the risk-shifting to remain optimal for all: $p_r^*(s) > \hat{p}^*(s)$.

**Corollary 1.** *For a given level of productivity, the equilibrium price is a non-decreasing function of the funding supply. The equilibrium price is strictly increasing in the funding supply for low* $s < s_{min}(\alpha)$ *and high funding supply* $s > \hat{s}(\alpha)$.

The relationship between price and productivity is less straightforward. It depends on the parameter assumptions regarding the sensitivity of expected asset payoff to the aggregate productivity.

**Corollary 2.** *The equilibrium price increases in productivity:*

- *when funding supply is low (*$s < s_{min}(\alpha)$*) if and only if the expected payoff of the asset increases steeply in productivity* $\frac{\partial p^*}{\partial \alpha} > 0 \iff q'(\alpha)\frac{\alpha}{q(\alpha)} > 1$
- *whenever the funding supply is moderate (*$s_{min}(\alpha) < s < \hat{s}^*(\alpha)$*)*
- *when funding supply is high (*$s > \hat{s}$*) if and only if the expected payoff of the asset increases steeply in productivity:* $\frac{\partial p^*}{\partial \alpha} > 0 \iff q'(\alpha)\alpha > 2q(\alpha)(1 - q(\alpha)\frac{p^*}{R})$.

The panel below illustrates the relationship between price and productivity for the four combinations of parameters.

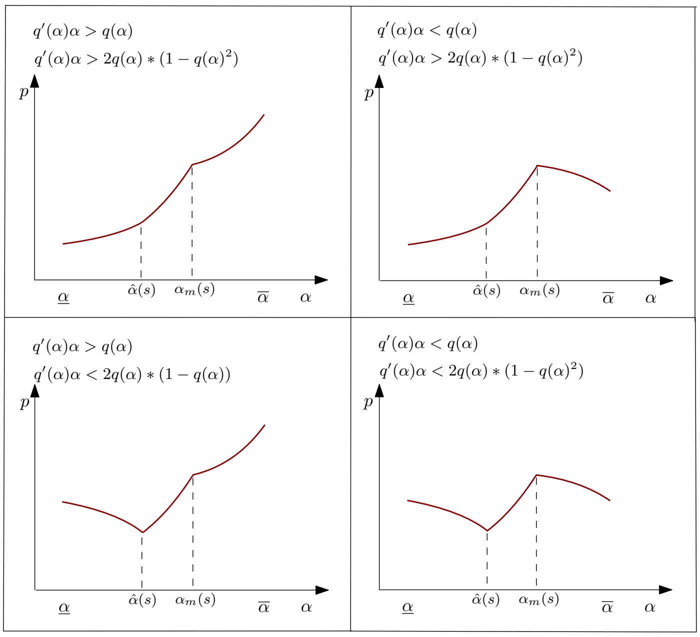


Figure 2: Prices as a function of productivity for different combinations of parameters. A risk-shifting equilibrium emerges when $\alpha < \hat{\alpha}$, all banks invest solvently when $\alpha \geq \hat{\alpha}$, investment by the banks is constrained if $\alpha > \alpha_m$, where $s_{min}(\alpha_m) = s$.

When bank funding supply is low so that banks have insufficient funding to drive the asset price to its fair value, the demand for the speculative asset is given by the residual funding after the the productive investment is set. The optimal level of productive investment is pinned down by the marginal productivity of technology relative to the return of the asset. Higher productivity increases the return on both- the productive and speculative investment. If the probability of a high asset payoff increases in productivity sufficiently fast, an increase in productivity decreases the optimal productive investment for any price level. This generates an upward shift in the demand for the speculative asset, resulting in a higher equilibrium price.

When bank funding supply is moderate, all banks invest solvently and the asset demand is sufficient for it to be fairly priced. In this range of funding supply the expected value of the speculative asset, and thus it's price, increases in productivity.

When bank funding is high, so that a fraction of banks' is risk-shifting, the equilibrium price of the asset ensures that banks are indifferent between the risk-shifting and the solvent strategy. Thus, the equilibrium price increases in productivity as long as the solvent investment lower bound price $\hat{p}(\alpha, s)$ increases in productivity. This is the case if and only if the risk-shifting incentives increase with productivity. So we need, that the increase in productivity needs to affect the speculative payoff more strongly than the productive return.

### 3.3. Inference and investment by uninformed firms

The relationship between the productivity and the speculative asset demand varies changes with the level of the funding supply. Since the uninformed agents do not observe the level of funding supply, they may be unable to recognize whether a given price follows from a stagnation, a good or a bad boom. Thus, precise inference of the productivity from asset prices may be impossible.

**Proposition 2.** *There exists a set of parameters such that the uninformed infer two or more productivity values when the equilibrium price is in the "confused inference range": $p^* \in (\underline{p}, \overline{p})$ and make a correct inference of one productivity value when the equilibrium price is outside of this range.*

*Proof.* In the Appendix □

To illustrate the possibility of confused inference in the simplest setting that is relevant for our inquiry into good and bad booms, let us choose the parameters which ensure that bank investment is never constrained and the bad boom may occur if the supply of funding is high: $\hat{s} > s^L > s_{min}(\overline{\alpha})$ and $s^H = \hat{s}^*(\hat{\alpha})$ for some $\hat{\alpha} > \underline{\alpha}$. Furthermore, we assume that the probability of high payoff of the asset increases steeply in productivity, so that the equilibrium price increases in productivity for all values of $\alpha$ and $s$[3]. The graph below plots the equilibrium price as a function of productivity for the two feasible levels of available funding.

[3] These assumptions are not necessary for the confusion to arise. We can immediately observe from Figure **??**, that a confusion may arise also if prices are decreasing in the productivity in some range

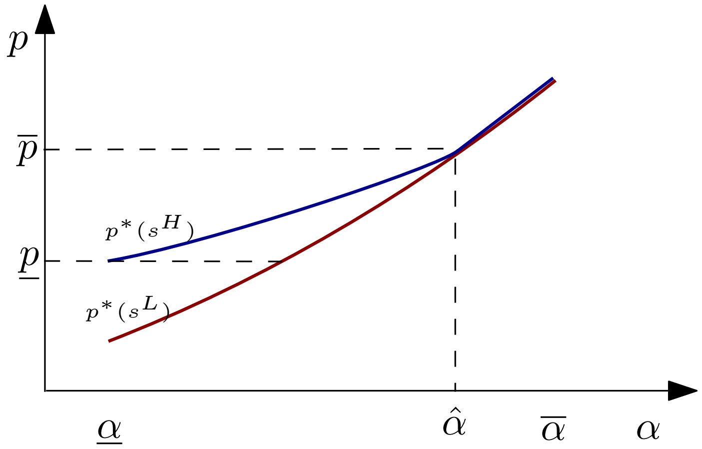


Figure 3: Prices as a function of productivity for the high and low supply realization if $s^L > s_{min}(\alpha)$, $p^{*\prime}(\alpha) > 0$ and $s^H = s^*(\hat{\alpha})$. Blue curve corresponds to the high supply shock, red curve corresponds to the low supply shock.

Under our assumptions on the model parameters whenever a low funding supply is realized all banks invest solvently and the equilibrium price is equal to the expected payoff of the asset. As the high funding supply is equal to the equilibrium risk-shifting threshold evaluated at $s_H = \hat{s}^*(\hat{\alpha})$, the risk-shifting equilibrium arises whenever the productivity is below $\hat{\alpha}$. Thus, can define the minimum price under the high funding shock as: $p_{min}(s_H) = \hat{p}(\underline{\alpha}, s_H)$. When the observed price is below that level, it can only emerge from a solvent equilibrium. This allows the uninformed agents to infer the productivity using $p^* = q(\alpha)R$.

If the productivity is above $\hat{\alpha}$ all banks invest solvently even under the high funding shock and the equilibrium price is given by $p = q(\alpha)R$. Such prices also allow a perfect inference using the solvent equilibrium price formula.

Whenever the observed price is between the minimum feasible price under risk-shifting and the fair price evaluated at $\hat{\alpha}$, uninformed agents are unable to determine whether the price results from a good boom (an all-solvent equilibrium under the low funding supply) or a bad boom (a risk-shifting equilibrium under the high funding). In this case they will form two estimates of the productivity: $\tilde{\alpha}(p, s^H) = \hat{p}^{-1}(s^H, p)$ and $\tilde{\alpha}(p, s^L) = q^{-1}(\frac{p}{R})$.

$$\tilde{\alpha} = \begin{cases} \tilde{\alpha}(p, s^H) \text{ with prob. } \rho \\ \tilde{\alpha}(p, s^L) \text{ with prob. } 1-\rho \end{cases} \tag{1}$$

**Lemma 4.** *If the parameters are such that the equilibrium price is increasing in the productivity, then whenever the asset price is in the confused inference range $p^* \in (\underline{p}, \overline{p})$, the expected value of the productivity according to the uninformed agents is:*

- *Higher than the actual productivity if the funding supply is low*
- *Lower than the actual productivity if the funding supply is high*

*Proof.* In the Appendix □

For a fixed level of productivity the price under the high funding supply is weakly higher than the price under the low funding supply. If the asset price increases in productivity, then whenever the distribution of the funding shock is such that two different levels of productivity can result in the same price, the productivity level that justifies the price conditional on high funding supply realization is lower than the productivity level under the low funding supply ($\tilde{\alpha}(p, s^H) < \tilde{\alpha}(p, s^L)$.

**Investment by uninformed firms**

Let $A$ denote the set of productivity estimates $\tilde{\alpha}_e$ inferred by the uninformed. They maximize the profits, subject to the budget constraint.

$$\max_{x_j} \sum_{\tilde{\alpha} \in A} P(\tilde{\alpha})[\tilde{\alpha}\sqrt{x_j} - x_j]$$

$$st.\ x_j \leq c$$

As far as firm's capital endowment is sufficient, it's optimal investment choice reflects the expected productivity of the technology. Let us focus on the parameters such that the local firm endowment is sufficient for any $\alpha < \overline{\alpha}$. In this case $x_j^* = (\frac{E(\alpha)}{2})^2$. Thus, under the conditions of Lemma 4 firms over-invest from ex-post perspective when the funding supply is high. They under-invest from the ex-post perspective when the funding supply is low.

## 4. Uninformed local banks

In this section we study the implications of the imperfect inference from prices on the investment decision by levered agents. We modify the baseline set up by assuming that the uninformed agents are small, local banks with access to a fixed deposit base $s_k = s_u$ (here too we assume it is sufficient to exclude corner solutions). We maintain the assumption on the information structure. The interpretation is that local banks do not have the ability to observe the aggregate productivity nor the funding flows towards the big, global banks. Local banks may still posses superior information about the quality of local projects, but in as far as the aggregate productivity affects their returns, the inference problem remains relevant. One could simply introduce superior local information by assuming that local banks receive idiosyncratic signals on a component $\lambda_k$, that affect the total productivity of his local investment opportunities. Since, this additional insights are not interesting for current study we keep the simplest version of the model, without local information.

We assume that the deposits of local banks are insured and thus their cost of funding is equal to 1, the return of storage[4]. Finally, local banks, like firms in the baseline model, are assumed to not have access to the market for the speculative asset. For ease of notation we focus on the illustrative case introduced in the previous section, where the funding supply shocks are such that the global economy is either in an unconstrained solvent equilibrium or in risk-shifting equilibrium.

The local bank maximizes profits subject to the funding constraint.

$$\max_{x_k} \sum_{\tilde{\alpha} \in A} P(\tilde{\alpha}) \max[\tilde{\alpha}\sqrt{x_j} - x_k, 0]$$

$$st.\ x_k < s_u$$

Under the assumptions on the shock structure the inferred productivity values are either correctly inferred as $\tilde{\alpha} = \alpha = q^{-1}(\frac{p}{R})$, in which case the problem is trivial, or given by (1). If prices are such that the inference is confused, the optimal investment by the local banks can differ from the choice of local firms due to the limited liability.

**Lemma 5.** *Consider the case when confused inference yields two productivity estimates: $\tilde{\alpha}_L < \tilde{\alpha}_H$. If the dispersion of the productivity estimates is not too high ($2\tilde{\alpha}_L > E(\alpha)$), local uninformed banks invest like firms: $x_k^* = (\frac{E(\alpha)}{2})^2 = x_j^*$. Otherwise, they risk-shift on the productive technology: $x_k^* = (\frac{\tilde{\alpha}_H}{2})^2 > x_j^*$.*

*Proof.* In the Appendix. □

[4] Our results maintain if we drop the deposit insurance and assume that local banks raise deposits before the global asset prices are realized, for example at the beginning of $t = 0$

High uncertainty about the productivity of the technology may result in risk-shifting by local banks. If the investment based on the expected productivity would result in losses at low productivity values ($2\tilde{\alpha}_L > E(\alpha)$), it is optimal for banks to bet on the high productivity estimates. As a result, local banks invest more than the uninformed firms. If the true productivity is low, this results in larger losses than those faced by firms and a default by local banks. If the true productivity is high, banks' investment is closer to the ex-post optimal level than that of the firm's. Thus local banks engage in a form of risk-shifting induced by the confused inference

If the conditions of Lemma 4 are satisfied, so that the equilibrium prices increase in productivity in the risk-shifting range of funding supply, the low productivity estimate corresponds to the high funding state. In this setting, risk shifting by local banks results in their default whenever the global economy is the bad boom. Thus, uncertainty among local banks may further amplify the "cycle". At $t = 0$, the supply driven over-investment by global banks may come hand in hand with confusion driven over-investment by local banks. As a consequence at $t = 1$, we will observe default of some global and all local banks.

## 4.1. Local regulator

The possibility of risk-shifting by local banks implies a scope for regulation by local authorities. In this section we discuss the optimal choice of the local regulator, who observes the global prices like other uninformed agents and may impose requirements on local banks accordingly.

We assume that the regulator acts like a local social planner, aiming to maximize the total output. As means of macro-prudential policy the regulator may impose a proportional tax on the investment by local banks: $\tau x_k$ and redistribute the proceeds to all banks at the final date. The policy operates in via two channels. On one hand, the tax increases the cost of investment allowing the regulator to curb potential over-investment. On the other hand, by redistributing the proceeds at date $t = 1$ the policy maker improves bank's incentives. Thus, this reduced form definition of the policy tool could represent a reduced form capital or reserve requirement.

In the presence of the macro-prudential policy, the problem of the local bank can be expressed as:

$$\max_{x_k} \sum_{\tilde{\alpha} \in A} P(\tilde{\alpha}) \max[\tilde{\alpha}\sqrt{x_k} - (1+\tau)x_k + \tau x, 0]$$

$$st.\ x_k < s_u$$

$$x = \int_k x_k$$

In the presence of the requirement the optimal investment of the local bank is given by $x_k^* = (\frac{E(\alpha)}{2(1+\tau)})^2$ if the productivity estimates are not too dispersed $2\tilde{\alpha}_L > (1+\tau)E(\alpha)$. Otherwise, the investment into productive technology is $x_k^* = (\frac{\tilde{\alpha}_H}{2(1+\tau)})^2$.

The problem of the regulator can be expressed as:

$$\max_{\tau} \sum_{\tilde{\alpha} \in A} P(\tilde{\alpha}) \max[\tilde{\alpha}\sqrt{x} - (1+\tau)x + \tau x, 0]$$

Where: $x = \begin{cases} (\frac{E(\alpha)}{2(1+\tau)})^2 \text{ if } 2\tilde{\alpha}_L(1+\tau) > E(\alpha) \\ (\frac{\tilde{\alpha}_H}{2(1+\tau)})^2 \text{ if } 2\tilde{\alpha}_L(1+\tau) \leq E(\alpha) \end{cases}$.

It is straightforward to see that if the productivity estimates are not too dispersed, so that local banks have no risk-shifting incentives and the private solution is efficient, the optimal tax set by the regulator is equal to zero. If the local banks have risk-shifting incentives the regulator prefers to set macro-prudential tax to either limit the over-investment or eliminate the risk-shifting incentives.

**Lemma 6.** *If the productivity estimates are such that the local banks have risk-shifting incentives, the regulator sets a positive macro-prudential tax $\tau^* > 0$. The resulting investment may remain inefficient.*

*Proof.* In the Appendix □

To illustrate this result it is useful to define two intuitive benchmark levels of the macro-prudential tax. First let us label as $\tau_p$ the level of tax at which the investment of a risk-shifting local bank would equal the efficient level of investment: $\frac{\tilde{\alpha}_H}{2(1+\tau_p)} = \frac{E(\alpha)}{2}$. This policy is optimal only if local banks continue to have risk-shifting incentives when $\tau = \tau_p$. To establish whether this is the case one needs to evaluate the second benchmark, $\tau_r$, the highest tax rate at which local banks have risk-shifting incentives: $\tilde{\alpha_L} = \frac{E(\alpha)}{2(1+\tau_r)}$.

**Lemma 7.** *The optimal macro-prudential tax set by the regulator depends on the relative size of inferred productivity parameters.*

- *If $2\tilde{\alpha}_L > E(\alpha)$: the private solution coincides with the efficient solution, so $\tau^* = 0$*
- *If $2\tilde{\alpha}_L < E(\alpha)$ and $2(\tilde{\alpha}_L\tilde{\alpha}_H) < E(\alpha)^2$, the regulator can achieve the efficient investment by setting $\tau^* = \tau_p = \frac{\tilde{\alpha}_H}{E(\alpha)} - 1$*
- *If $2\tilde{\alpha}_L < E(\alpha)$ and $2(\tilde{\alpha}_L\tilde{\alpha}_H) > E(\alpha)^2$, the efficient solution is unfeasible*
  - *if $2(\tilde{\alpha}_L\tilde{\alpha}_H)^2 > E(\alpha)^2[E(\alpha)^2 - 2\tilde{\alpha}_L^2]$, eliminating the risk-shifting is the best the regulator can do, so $\tau^* = \tau_r + \epsilon = \frac{E(\alpha)}{2\tilde{\alpha}_L} - 1 + \epsilon$*
  - *if $2(\tilde{\alpha}_L\tilde{\alpha}_H)^2 < E(\alpha)^2[E(\alpha)^2 - 2\tilde{\alpha}_L^2]$ minimizing the over-investment under risk-shifting is the best the regulator can do, so $\tau^* = \tau_r = \frac{E(\alpha)}{2\tilde{\alpha}_L} - 1$*

*Proof.* In the appendix □

The regulator is able to ensure efficient investment only if $\tau_p \leq \tau_r$. Otherwise, any policy rate above $\tau_r$ would be dominated, since it would lead to an inefficient fall in the local bank's investment. In this case the regulator compares two inefficient solutions: setting the requirement at $\tau_r$ so that to minimize the over-investment of risk-shifting local banks ($x_j = (\frac{\tilde{\alpha}_H}{2(1+\tau_r)})^2$) or just above that level to ensure that local banks do not risk-shift (in this case $x_j \approx (\frac{E(\alpha)}{2(1+\tau_r)})^2$). Since the objective of the regulator is to maximize the expected profits of the local banks, he imposes the requirement that results in the lowest deviation from the efficient solution.

Thus, in our simple setting the macro-prudential regulation may be unable to ensure that local banks invest efficiently. Interestingly, Lemma 7 shows that the policy may result in both over- and under-investment relative to the ex-ante efficient level. If ensuring the efficient investment level is impossible regulator may allow the local banks to risk-shift or tighten the macro-prudential policy so that to eliminate risk-shifting at a cost of inefficient under-investment. His preference over these two depends solely on the distance from the efficient investment level. This reflects the fact that we assumed no dead-weight loss from local bank's default, so that the only consideration of the social planner (and the regulator) is for the level of investment.

## 5. Extension: more signals vs more opacity

While in general prices offer some information on supply and demand, in our setup agents may observe only the asset price, as the cost of bank funding is unobservable as well as inelastic. [5] Less informed agents may have also non price information, and the natural variable is total amount of credit. This indicator may not be readily available nor easily measured (not least since banks report risk weighted rather than total assets). On the other hand, it has received considerably more attention since the crisis, and its realization may affect regulatory actions (with some delay) in the context of the countercyclical buffer in the Basel III framework.

This section studies how the inference process is improved when uninformed agents can observe total investment of large banks ($V = X + pY$). It first assesses as a benchmark the inference based solely on total investment, then considers the case when uninformed agents observe both quantity and price signals. For simplicity we assume the signal is precise. Next, we introduce additional uncertainty about banks' leverage, by assuming that large banks may be holding impaired assets on their balance sheets. We first study the investment choice of a bank holding a legacy asset. Then we analyze the effect on the inference: under price signal and both price and quantity signals.

#### 5.0.1. Obervable total investment

The total investment by large banks is given by:

$$V(s,\alpha) = \begin{cases} (\frac{\alpha}{2})^2 + q(\alpha)R & \text{if } s = s^L \\ (1-\psi(\alpha,s))(\frac{\alpha}{2})^2 + \psi(\alpha,s)s^H & \text{if } s = s^H \end{cases} \tag{2}$$

When the available funding is low the total investment reflects the quality of the available opportunities. If the funding supply is abundant, some large banks are risk-shifting, using up all the available funding to scale up their investment, while others invest only in the productive technology.

When uninformed agents observe only the quantity signal (and no prices), they infer the productivity level from the total investment equation (2). It is straightforward to show that in this case their inference may be confused.

[5] Even if bank shares were traded, in a context without short sales their pricing may still not be informative, as their value rises both with lending quality and risk shifting.

This inference error is resolved once uninformed agents observe both total investment and the asset price. Inference based on each of the variables will give two estimates of productivity. If bank lending is driven by demand factors, the estimates of "safe" productivity level will match $\hat{\alpha}(V, s^L) = \hat{\alpha}(p, s^L)$. If the decisions by banks are driven by abundant funding supply, the estimates that assume risk-taking by large banks will overlap: $\hat{\alpha}(V, s^H) = \hat{\alpha}(p, s^L)$. Thus, if uninformed agents have access to precise information about the price of the risky asset and the total investment by global banks, they are able to correctly infer the aggregate productivity. Ultimately, observing enough variables enables to solve the system of unknown economic conditions.[6]

## 5.1. Additional balance sheet opacity

This section studies the effects of further bank balance-sheet opacity on the inference process. In particular we question the assumption that the initial leverage of banks is common knowledge. Throughout this section we use the assumptions on the structure of the shocks and the relation between the expected return of the asset and the productivity from the illustrative case discussed in previous sections.

Assume that global banks enter date $t = 0$ holding some amount $\lambda$ of unreported impaired assets, which generate certain losses at $t = 1$. For simplicity we assume that banks are homogeneous with respect to their holdings of this legacy asset. The size of the future losses is drawn from a binomial distribution at the beginning of $t = 0$:

$$\lambda = \begin{cases} 0 & \text{with prob. } \kappa \\ \bar{\lambda} & \text{with prob. } 1 - \kappa \end{cases}$$

Since bank balance sheets are opaque, uninformed agents do not observe its impact on bank leverage. This assumption reflects a realistic view of banks' discretion regarding recognizing losses on assets.

[6]In all fairness to empirical researchers, uninformed agents in our model may come to such precise estimates because the postulated view of economic rationality involves not just rational updating, but a precise knowledge of all probability distributions and precise value of all economic parameters.

**The problem of the global banks**

The future losses on the legacy asset modify the maximization problem of large banks.

$$\max_{x_i,y_i,s_i} q(\alpha)(\alpha\sqrt{x_i} + R(y) - s_i - \lambda) + (1 - q(\alpha))max[\alpha\sqrt{x_i} - \lambda - s_i, 0]$$

subject to:

$$x_i + py_i = s_i$$

$$s_i \leq s$$

The legacy asset affects banks' investment strategy by changing its risk taking incentives. Higher future losses on existing assets lower the value of banks' equity, making a risk shifting strategy relatively more attractive. Limited liability ensures that banks do not internalize the low equity value in the default state, thus the risk shifting threshold is shifted downwards.

**Lemma 8.** *When the bank holds legacy asset $\lambda$, it invests according to the risk shifting strategy (as defined in Lemma 1) whenever the available funding exceeds the individual risk shifting threshold defined as:*

$$\hat{s}_\lambda(\alpha, p, \lambda) = \begin{cases} (1 + q(\alpha)(\frac{\alpha}{2})^2(\frac{p}{q(\alpha)R}) - \lambda & \textit{if } p < q(\alpha)R \\ (\frac{\alpha}{2})^2 \frac{\frac{p}{R} - q(\alpha)(\frac{p}{R})^2}{q(\alpha)(1-\frac{p}{R})} - \frac{1-q(\alpha)}{q(\alpha)(\frac{R}{p}-1)}\lambda & \textit{if } p \geq q(\alpha)R \end{cases}$$

*Otherwise, it invests according to the solvent strategy (as defined in Lemma 1).*

*Proof.* In the Appendix □

Legacy assets do not affect investment choices within bank strategies, but change their relative profitability. When hidden losses are large, the risk-shifting strategy becomes dominant at lower levels of funding supply.

**Equilibrium investment and prices**

Since losses on impaired assets do not affect banks' strategies, the minimum unconstrained level of funding supply remains unchanged. The legacy asset affects the equilibrium outcome through the impact on the fair-price risk-shifting threshold. The fair-price risk-shifting threshold can now be defined as:

$$\hat{s}^*_\lambda(\alpha, \lambda) = \hat{s}(\alpha, q(\alpha)R, \lambda) = \hat{s}^*(\alpha) - \lambda \tag{3}$$

Future losses on the legacy asset shift the fair-price risk-shifting threshold downwards one-to-one. At the fair asset price, a unit increase in expected losses on the impaired assets affects the risk-taking incentives in the same manner as a unit increase in available funding. Both work by increasing banks leverage.

**Proposition 3.** *Assume that bank's existing assets generate losses equal to $\lambda$. Whenever the available funding is above the fair-price risk-shifting threshold defined in (3) mixed risk-shifting equilibrium emerges. The price of the speculative asset is above its fundamental value and is given by:*

$$p^* = \hat{p}^*(\alpha, s, \lambda)$$

*With the fraction of risk-shifting banks given by: $\psi(\alpha, s, \lambda) = \frac{p^*}{s-(\frac{\alpha p^*}{2R})^2}$. Otherwise, the economy is in a pure solvent equilibrium and the asset is fairly priced at $p^* = q(\alpha)R$.*

*Proof.* In the Appendix □

When banks hold the legacy asset, the solvent equilibrium with a fairly priced speculative asset becomes more rare. A lower risk-shifting threshold expands the range of productivity-supply realizations for which the economy ends in a risk-shifting equilibrium with under-priced speculative asset [7]. Consequently banks may have risk-shifting incentives even when the realized funding shock is low $s = s^L$.

**Corollary 3.** *A risk-shifting equilibrium may emerge when funding supply is low if the size of legacy asset is sufficiently large: $\overline{\lambda} > \hat{s}^*(\alpha) - s^L(\alpha)$*

Since, the losses on the legacy asset do not affect individual bank's strategy, asset prices are affected only if in absence of the losses the economy would be in a mixed risk-shifting equilibrium. In this case, the equilibrium price ensures that banks are indifferent between the risk shifting and the solvent investment. Higher losses on impaired assets lower the risk-shifting threshold for any given price. Thus, holding the supply and productivity fixed, a higher price is necessary to make banks indifferent between the two strategies. Consequently more banks risk shift in equilibrium compared to the benchmark with no losses

[7]The risk-shifting threshold with legacy asset holding increases in productivity under the conditions of Lemma 2 are satisfied.

**Inference problem by uninformed**

The inference problem of the smaller players takes a different form depending on the maximum potential losses from the impaired assets ($\overline{\lambda}$).

If the maximum losses are not too large ($\overline{\lambda} < \hat{s}^*(\alpha) - s^L$ for all $\alpha \in [\underline{\alpha}, \overline{\alpha}]$) banks always invest safely in a low funding state. Thus, when uninformed agents observe a price such that $p \in (p^*(\underline{\alpha}, s^L, \lambda), p^*(\overline{\alpha}, s^H, \bar{\lambda}))$, they know that it can result from:

- A solvent equilibrium with high productivity and low funding supply (and either high or low losses from the legacy asset), which occurs with probability $1 - \rho$
- A risk-shifting equilibrium with low productivity, high funding supply and low future losses from the legacy asset, which occurs with probability $\rho\kappa$
- A risk-shifting equilibrium with low productivity, high funding supply and high future losses from the legacy asset, which occurs with probability $\rho(1 - \kappa)$

Thus, the posterior beliefs about the productivity of the uninformed agents can be expressed as:

$$P(\alpha|p) = \begin{cases} \rho(1-\kappa) & \text{if } \hat{\alpha}(p, s^H, \overline{\lambda}) \\ \rho\kappa & \text{if } \hat{\alpha}(p, s^H, 0) \\ 1-\rho & \text{if } \hat{\alpha}(p, s^L, \overline{\lambda}) \end{cases}$$

The uncertainty about the initial leverage of large banks additionally confuses the inference from prices. However, if uninformed agents also observe the signal on the quantity of total investment by banks, their inference problem is resolved. They can now recognize whether large banks invested safely (as the price- and quantity-derived estimates should match). Moreover, if some of the large banks are risk-shifting, only the true productivity and size of future losses from the legacy asset are consistent with both the price and the quantity equations.

If the maximum value of legacy asset is sufficiently large ($\overline{\lambda} > \hat{s}^*(\alpha) - s^L(\alpha)$), banks may be risk-shifting even when the available funding is low. Let's assume that the above condition is satisfied for all productivity values in some subset of the range $\alpha \in [\alpha_L, \alpha_H]$ (where $[\alpha_L, \alpha_H] \subset [\underline{\alpha}, \overline{\alpha}]$). Then, we can define a range of prices $p \in (p^c_{min}, p^c_{max})$ for which the uninformed agents form four estimates of the productivity using the following as the bounds:

$$p_{min}^c = \max[p^*(\alpha_L, s^L, \overline{\lambda}), p^*(\underline{\alpha}, s^H, \overline{\lambda}],\ p_{max}^c = \min[p^*(\alpha_H, s^L, \overline{\lambda}), p^*(\overline{\alpha}, s^L, 0)]$$

If the observed price is in these bounds, the uninformed agents know that it can result from:

- A solvent equilibrium with high productivity, low funding supply, and low future losses from the legacy asset, which occurs with probability $(1-\rho)\kappa$
- A risk-shifting equilibrium with moderate productivity, low funding supply and high future losses from the legacy asset, which occurs with probability $(1-\rho)(1-\kappa)$
- A risk-shifting equilibrium with low productivity, high funding supply and low future losses from the legacy asset, which occurs with probability $\rho\kappa$
- A risk-shifting equilibrium with low productivity, high funding supply and high future losses from the legacy asset, which occurs with probability $\rho(1-\kappa)$

Thus, the posterior beliefs about the productivity of the uninformed agents can be expressed as:

$$P(\alpha|p) = \begin{cases} \rho(1-\kappa) & \text{if } \hat{\alpha}(p, s^H, \overline{\lambda}) \\ \rho\kappa & \text{if } \hat{\alpha}(p, s^H, 0) \\ (1-\rho)(1-\kappa) & \text{if } \hat{\alpha}(p, s^L, \overline{\lambda}) \\ (1-\rho)\kappa & \text{if } \hat{\alpha}(p, s^L, 0) \end{cases}$$

The possibility of risk-shifting at the low funding supply leads smaller players to form an additional estimate of productivity. Also in this setting, observing an additional signal about the total investment allows to perfectly infer the underlying productivity if the economy is in a solvent equilibrium. That's because the price and quantity equations in the solvent equilibrium do not depend on the size of the funding shock or the legacy asset. However, because smaller agents are now unsure whether a risk shifting equilibrium is driven by high supply or large future losses on legacy asset, precise inference may be impossible. In particular, the inference will be confused when the price and total investment levels are such that:

$$p = p^*(\hat{\alpha}_U, s^L, \overline{z}) = p^*(\hat{\alpha}_D, s^H, 0)$$
$$V = V^*(\hat{\alpha}_U, s^L, \overline{z}) = p^*(\hat{\alpha}_D, s^H, 0)$$

In this case, the posterior beliefs of small players about the productivity are given by:

$$P(\alpha|p,V) = \begin{cases} \frac{(1-\rho)*\kappa}{(1-\rho)*\kappa+\rho*(1-\kappa)} & \text{if } \alpha = \hat{\alpha}_U \\ \frac{\rho*(1-\kappa)}{(1-\rho)*\kappa+\rho*(1-\kappa)} & \text{if } \alpha = \hat{\alpha}_D \end{cases}$$

We establish that such combinations may exist in our setting by solving the model numerically.

Thus, the additional source of opacity on bank's balance sheet's further complicates the inference, so that even if additional signals are available to the uninformed agents confusion may remain. This example highlight that in reality, the high degree of complexity and opacity of bank balance sheets is likely to distort the inference of market participants, econometricians and policymakers alike. Comparing the cases with three and four productivity estimates we can notice that a core to ensuring confusion with additional signals is for the element of the balance sheet that is assumed to be opaque be able to influence the risk-shifting incentives and the outcomes under risk-shifting.

## 6. Conclusions

We argue that opaqueness of intermediary balance sheets (in terms of asset values, funding volume and actual leverage) may add noise to asset prices, and explain why rational market participants underestimate risk in credit booms (and may overestimate it when bank funding is scarce). In our setting, a large expansion of available funding boosts banks' incentives to speculate. Other agents seeking to infer the underlying credit quality from asset prices may be unable to interpret it correctly. As a consequence whenever the price in a risk-shifting equilibrium increases in productivity, uninformed market participants over-invest precisely at the time when large banks are risk-shifting, potentially adding to financial instability.

We study the consequences of the imperfect inference when the uninformed agents are local banks. The limited liability combined with high uncertainty about the productivity values may encourage risk-shifting by local banks. In this case they invest excessively precisely when the the global economy is in the bad boom, amplifying the over-investment as well as losses. We show that in some circumstances a local regulator can restore efficiency by imposing a high macro-prudential requirement.

Finally, we show that making more realistic assumptions about the bank balance sheet opacity (for example assuming unobservability of bank capitalization) further complicates the inference.

Thus, observing additional signals on the economy may be insufficient to resolve the inference problem.

The analysis offers a broader classification of forms of risk shifting in terms of awareness (deliberate versus confused) and in terms of channels (excessive lending or speculation) some rational foundations to understand the emerging evidence on underestimation of risk in credit booms.

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# A. Appendix

## A.1. Lemma 1 and 2

We derive Lemma 1 and Lemma 2 using the general formulation with $\gamma \in (0, 1)$.

**Optimal investment strategy** Let's first consider the case of a solvent bank (limited liability doesn't play a role). The bank will prefer investment into risky funding over forgoing the funding supply whenever $\frac{q(\alpha)R}{p} > 1$. Plugging the funding constraint into the utility function and taking FOC yields:

$$\alpha x^{\gamma-1} - \frac{q(\alpha)R}{p} = 0 \;\Rightarrow\; x = (\frac{\alpha\gamma p}{q(\alpha)R})^{\frac{1}{1-\gamma}},\; y = \frac{s-x}{p}$$

If $\frac{q(\alpha)R}{p} < 1$, bank prefers to forgo the available funding as the risky asset has a negative NPV. Thus the maximization problem is solved by

$$\alpha x^{\gamma-1} - 1 = 0 \;\Rightarrow\; x = (\alpha\gamma)^{\frac{1}{1-\gamma}},\; y = 0$$

If $\frac{q(\alpha)R}{p} = 1$, bank is indifferent between accepting the available funding and investing in the risky asset. His investment into productive technology is given by: $x = (\frac{\alpha\gamma p}{q(\alpha)R})^{\frac{1}{1-\gamma}} = (\alpha\gamma)^{\frac{1}{1-\gamma}}$. However, in order to remain solvent he cannot invest more than $y = \frac{\alpha x_s^{\gamma} - x_s}{p}$ into risky asset. A higher investment would result in a risk of default.

Now, we turn to the case when bank risks default. In this case limited liability plays a role and the bank maximizes the problem given below:

$$max_{x,y} q(\alpha)(\alpha x^{\gamma} + Ry - x - py)$$

$$\text{s.t:} x + py \leq s$$

The bank prefers to invest in the risky asset whenever the price is lower than the upside return $p < R$. Thus, we plug in the funding constraint and take a FOC with respect to $x$:

$$\alpha x^{\gamma-1} - \frac{R}{p} = 0 \;\Rightarrow\; x = (\frac{\alpha\gamma p}{R})^{\frac{1}{1-\gamma}},\; y = \frac{s-x}{p}$$

Now, the above investment results in a risk of default only if $\alpha x^{\gamma} - x < 0$. So we need $s > (\alpha\gamma)^{\frac{1}{1-\gamma}} \frac{1}{\gamma} (\frac{p}{R})^{\frac{\gamma}{1-\gamma}} = s_r(\alpha, p)$ for this strategy to exist.

**Individual risk-shifting threshold determination:**

If both strategies can be played ($s > s_r(\alpha, p)$), the bank prefers a strategy that gives him a higher expected profit. We consider two cases: when $p \geq (\alpha)R$ and when $p < q(\alpha)R$.

If $p \geq (\alpha)R$ the risk-shifting strategy profits and solvent strategy profits are given by:

$$\Pi_s = \alpha(\alpha\gamma)^{\frac{\gamma}{1-\gamma}} - (\alpha\gamma)^{\frac{1}{1-\gamma}}$$

$$\Pi_r = q(\alpha)[\alpha((\frac{\alpha\gamma p}{q(\alpha)R})^{\frac{\gamma}{1-\gamma}} - \frac{R}{p}(s - (\frac{\alpha\gamma p}{q(\alpha)R})^{\frac{1}{1-\gamma}}) - s]$$

Thus to have $\Pi_r > \Pi_s$ we need: $s > (\alpha\gamma)^{\frac{1}{1-\gamma}} \frac{1-\gamma}{\gamma} \frac{\frac{p}{R} - q(\alpha)(\frac{p}{R})^{\frac{1}{1-\gamma}}}{q(\alpha)(1-\frac{p}{R})} = \hat{s}(\alpha p | p \geq q(\alpha)R)$

We can show that $\hat{s}(\alpha, p | p \geq q(\alpha)R) > s_r(\alpha, p)$

Let $\frac{p}{R} = t$. If the risky strategy when it is possible for all $p > q(\alpha)R$:

$$t^{\frac{\gamma}{1-\gamma}} < (1-\gamma)\frac{t - q(\alpha)t^{\frac{1}{1-\gamma}}}{q(\alpha)(1-t)}$$
$$t > \gamma t + (1-\gamma t)q(\alpha)t^{\frac{\gamma}{1-\gamma}}$$

Now let's focus on the case when $p > q(\alpha)R$, we can represent $q(\alpha) = \xi(\alpha)t$ where $0 < \xi(\alpha) < 1$.

$$t > \gamma t + (1-\gamma t)\xi(\alpha)t^{\frac{1}{1-\gamma}}$$
$$t - \xi(\alpha)t^{\frac{1}{1-\gamma}} > \gamma(t - \xi(\alpha)t^{\frac{\gamma}{1-\gamma}})$$

As $t < 1$, $t < t^{\frac{1}{1-\gamma}}$ and $\xi(\alpha) < 1$

$$t - \xi(\alpha)t^{\frac{1}{1-\gamma}} > 0 \Rightarrow$$
$$t - \xi(\alpha)t^{\frac{1}{1-\gamma}} > \gamma(t - \xi(\alpha)t^{\frac{1}{1-\gamma}}) > 0$$

And further

$$\gamma(t - \xi(\alpha)t^{\frac{1}{1-\gamma}}) > \gamma t^{\frac{\gamma}{1-\gamma}}(t-1) < 0$$

When $p < q(\alpha)R$, profits of risk-shifting and solvent strategy are given by:

$$\Pi_r = q(\alpha)[\alpha(\frac{\alpha\gamma p}{R})^{\frac{\gamma}{1-\gamma}} + \frac{R}{p}(s - (\frac{\alpha\gamma p}{R})^{\frac{1}{1-\gamma}}) - s]$$
$$\Pi_s = \alpha(\frac{\alpha\gamma p}{q(\alpha)R})^{\frac{\gamma}{1-\gamma}} + \frac{q(\alpha)R}{p}(s - (\frac{\alpha\gamma p}{q(\alpha)R})^{\frac{1}{1-\gamma}}) - s$$

So bank prefers the risky strategy if $s > (\alpha\gamma)^{\frac{1}{1-\gamma}}(\frac{p}{R})^{\frac{\gamma}{1-\gamma}} \frac{1-\gamma}{\gamma} \frac{q^{\frac{-\gamma}{1-\gamma}}(\alpha) - q(\alpha)}{1-q(\alpha)} = \hat{s}(\alpha, p | p < q(\alpha)R)$. We show that $\hat{s}(\alpha, p | p < q(\alpha)R) > s_r(\alpha, p)$.

$$(\alpha\gamma)^{\frac{1}{1-\gamma}}(\frac{p}{R})^{\frac{\gamma}{1-\gamma}} \frac{1-\gamma}{\gamma} \frac{q^{\frac{-\gamma}{1-\gamma}}(\alpha) - q(\alpha)}{1-q(\alpha)} > (\alpha\gamma)^{\frac{1}{1-\gamma}}(\frac{p}{R})^{\frac{\gamma}{1-\gamma}} \frac{1}{\gamma}$$
$$(1-\gamma)(q^{\frac{-\gamma}{1-\gamma}} - q) > 1 - q$$

Let's denote $z(q) = (1-\gamma)(q^{\frac{-\gamma}{1-\gamma}} - q)$, $w(q) = 1 - q$. First we observe that: $z(1) = w(1)$. Second:

$$\frac{\partial z(q)}{\partial q} = (1-\gamma)(-\frac{\gamma}{1-\gamma}q^{\frac{-1}{1-\gamma}} - 1) = -(\gamma q^{\frac{-1}{1-\gamma}} + 1 - \gamma)$$
$$\frac{\partial w(q)}{\partial q} = -1$$

In the range of $q \in (0,1)$ $\frac{\partial z(q)}{\partial q} < \frac{\partial w(q)}{\partial q}$. The two functions are both strictly decreasing in this range, and cross at $q = 1$. That means that for any $q \in (0,1)$ $z(q) > w(q)$.

Substituting $\gamma = \frac{1}{2}$ yields the results in the text.

**Comparative statics on the individual risk-shifting threshold**

The partial derviative of the individual risk-shifting threshold with respect to prices is positive if

$$1 - \gamma > q(\frac{p}{R})^{\frac{\gamma}{1-\gamma}}(1 - \frac{p}{R}\gamma)$$

Let us define the right-side of the inequality: $q(\frac{p}{R})^{\frac{\gamma}{1-\gamma}}(1 - \frac{p}{R}\gamma = z(p)$ and show that it is increasing in $p$:

$$\frac{\partial z(p)}{\partial p} = q(\frac{p}{R})^{\frac{\gamma}{1-\gamma}}\frac{\gamma}{(1-\gamma)p}(1 - \frac{p}{R}\gamma) - \frac{\gamma}{R}q(\frac{p}{R})^{\frac{\gamma}{1-\gamma}} = \frac{q}{(1-\gamma)p}\gamma(\frac{p}{R})^{\frac{\gamma}{1-\gamma}}(1 - \frac{p}{R}) > 0$$

Thus, the risk shifting threshold is increasing in price if $1 - \gamma > z(p)$ for highest feasible $p$. Since when price exceeds the upside return of the risky asset the risk-shifting threshold is infinitely large, we focus on $p = R$ as the maximum price. If $1 - \gamma > z(R)$ then $1 - \gamma > z(p)\forall p$:

$$1 - \gamma > z(R) = q(1-\gamma) \Rightarrow \frac{\partial \hat{s}}{\partial p} > 0 \forall p$$

For $p < q(\alpha)R$ the relation is straightforward. The relationship between the productivity and the risk-shifting threshold follows immediately from the first order derivatives.

## A.2. Derivation of Proposition 1 and Lemma 3

The equilibrium level of speculative price equalizes its demand and supply: $D_y = S_y$, so it depends on banks' investment choice. Three types of equilibria may emerge:

- A pure solvent equilibrium in which all banks play the solvent strategy and the price is $p = \min[q(\alpha)R, s - (\frac{\alpha\gamma p}{q(\alpha)R})^{\frac{1}{1-\gamma}}]$
- A pure risk-shifting equilibrium in which all banks take risk and the price is $p = s - (\frac{\alpha\gamma p}{R})^{\frac{1}{1-\gamma}}$
- A mixed equilibrium in which fraction $\hat{\psi}(\alpha, s)$ of banks risk shifts while the others choose the solvent strategy. The speculative price ensures all banks are indifferent: $p^*$ is such that $s = \hat{s}(\alpha, p^*)$.

We can use the inverse of the individual risk-shifting threshold: $\hat{p}(\alpha, s)$ to define the equilibrium investment. For any $\alpha$ and $s$ the equilibrium

- If $p_s^* > \hat{p}(\alpha, s)$ all banks prefer solvent investment.
- If $p_r^* > \hat{p}(\alpha, s) > p_s^*$ there is a mixed risk-shifting in equilibrium.
- If $\hat{p}(\alpha, s) > p_r^* > p_s^*$ there is a pure risk-shifting equilibrium.

In the general case with $\gamma \in (0, 1)$, we are unable to find the closed form solutions for this problem and we are only able to implicitly define the equilibrium. As long as for a given productivity $\alpha$ the functions are such that $\hat{p}(s) = p_s^*$ only for one value $s$, there exists a unique risk-shifting threshold corresponding to that value $s$. If the functional forms are such that the two functions cross more than once, there may be several thresholds resulting in several ranges of funding supply values for which the solvent or risk-shifting equilibrium occurs.

For the case with $\gamma = \frac{1}{2}$, $\hat{p}(s) = p_s^*$ cross only once. We can show that $\hat{s}(p = q(\alpha)R) > s_{min}$ The funding necessary for the fair pricing of the asset is lower than the risk-shifting threshold evaluated at the fair asset price (the highest price feasible under solvent equilibrium) if and only if $(\frac{\alpha}{2})^2 > R$. If this condition is satisfied, there is no scope for risk-shifting when $s < \hat{s}(p = q(\alpha)R)$ so that we know that for some funding supply $s$ $\hat{p}(s) = p_s^* = q(\alpha)R$. Otherwise we can find a threshold price level $p = (1 + q - q(\frac{2}{\alpha})^2 R)qR$ that ensures that $\hat{s} = p + (\frac{\alpha p}{2qR})^2$.

We can show that if $(\frac{\alpha}{2})^2 > R$ then $p_r^*(s) > \hat{p}(s)$, so that it is impossible for all banks to have risk-shifting incentives. Thus, only a mixed risk-shifting equilibrium can emerge in which the fraction of banks risk-shifting is such that the sum of individual demands brings about the price at which banks are indifferent between the two strategies.

### A.3. Derivation of Proposition 2 and Lemma 4

The result on the confused inference follows straightforwardly from the example discussed in the text. In this illustratory case the confusion region of prices is given by: $\underline{p} = \hat{p}(\underline{\alpha}, s^H)$ and $\overline{p} = q(al\hat{p}ha)R$. Since asset prices are assumed to increase in productivity: $\overline{p} > \underline{p}$, so that the confusion region is non-empty.

The confusion may also emerge for other parameter ranges. If the equilibrium price is decreasing in productivity in the risk-shifting equilibrium we will have that $\underline{p} = q(al\hat{p}ha)R$ and $\overline{p} = \min[\hat{p}(\underline{\alpha}, s^H), p_s^*(\overline{\alpha})]$.

The relative size of the inferred productivity values is lower in the high funding state whenever the equilibrium price is an increasing function of productivity. A higher funding shifts parts of the curve of the price as a function of productivity upwards. This upward shift means that for the same value of productivity the equilibrium price is higher. Since equilibrium price increases in productivity, we can only restore the same price with a lower level of productivity.

### A.4. Derivation of Lemma 5 and Lemma 6

There are two feasible investment choices by the local bank:

- It can invest like a firm (efficient): $x_k = (\frac{E(\alpha)}{2})^2$, which is a feasible solution as long as it ensures no risk of default by the bank: $\alpha_H \frac{E(\alpha)}{2}) - \frac{E(\alpha)}{2})^2 > 0$ and $\alpha_L \frac{E(\alpha)}{2}) - \frac{E(\alpha)}{2})^2 > 0$
- It can risk-shift, by gambling on the high realization of the productivity $x_k = (\frac{\alpha_H}{2})^2$, this solution is feasible only if at this level of investment banks have risk-shifting incentitves (that is they indeed risk default in the low productivity state): $\alpha_L \frac{\alpha_H}{2}) - \frac{\alpha_H}{2})^2 < 0$

Since the profits under the efficient investment choice are strictly higher than under risk-shifting, risk-shifting is only possible if the efficient investment is unfeasible: $\alpha_L \frac{E(\alpha)}{2}) - \frac{E(\alpha)}{2})^2 < 0 \Rightarrow 2\alpha_L < E(\alpha)$. If this condition is satisfied the feasibility condition for risk-shifting is always satisfied.

The optimal choice of the regulator is interesting only if the local banks have risk-shifting incentives .

$$\max_{\tau} \sum_{\tilde{\alpha} \in A} P(\tilde{\alpha}) \max[\tilde{\alpha}\sqrt{x} - (1+\tau)x + \tau x, 0]$$

Where: $x = \begin{cases} (\frac{E(\alpha)}{2(1+\tau)})^2 \text{ if } 2\tilde{\alpha}_L(1+\tau) > E(\alpha) \\ (\frac{\tilde{\alpha}_H}{2(1+\tau)})^2 \text{ if } 2\tilde{\alpha}_L(1+\tau) \leq E(\alpha) \end{cases}$.

Let's use the above to find the limit $\tau_r$ at which the bank switches from risk-shifting to a solvent investment: $\tau_r = \frac{E(\alpha)}{2\alpha_L} - 1$.

Assume that banks have risk-shifting incentives for all $\tau$, the FOC of the regulator problem yields:

$$\frac{\partial SW}{\partial \tau} = \frac{\partial x}{\partial \tau}[\frac{1}{2}E(\alpha)x^{-\frac{1}{2}} - 1] = 0$$

Thus, the optimal choice is such that: $E(\alpha) = 2x^{\frac{1}{2}} = \frac{\alpha^H}{2(1+\tau)} \Rightarrow \tau_p = \frac{\alpha^H}{E(\alpha)} - 1$. This solution is consistent only if at $\tau_p$ banks still have risk-shifting incentives:

$$2\tilde{\alpha}_L < (1 + \tau_p)E(\alpha)$$

Or simply: $\tau_p < \tau_r$.

If $\tau_r < \tau_p$ we are at a corner solution. If we increase the requirement marginally to $\tau_r + \epsilon$ bank's no longer risk-shift so their investment is given by: $x_j = (\frac{E(\alpha)}{2(1+\tau_r+\epsilon)})^2$. The investment is too low relative to the efficient level $x^* = \frac{E(\alpha)}{2}$. With this investment schedule any further increase in $\tau$ would reduce the investment making it even more inefficient. If we set $\tau_r$, banks risk-shift and invest according to: $x_j^* = (\frac{\tilde{\alpha}_H}{2(1+\tau_r)})^2$. Since under risk shifting the investment schedule leads to over investment, in this case we would like to choose a maximum $\tau = \tau_r$ to limit extend of inefficient overinvestment. The thresholds in terms of $\alpha_L$, $\alpha_H$ and $E(\alpha)$ follow from comparing the Social Planner utility under the two solutions.

### A.5. Derivation of Lemma 8 and Proposition 3

The presence of the legacy asset does not affect the FOCs of the problem. Thus, the solvent and the risk-shifting allocations are characterized as before. To derive the individual risk-shifting threshold let us compare the profits under the two allocations: If $p < qR$:

$$\Pi_s = \alpha * \frac{\alpha p}{2qR} + \frac{qR}{p}(s - (\frac{\alpha p}{2qR}^2) - s - \lambda$$
$$\Pi_r = q[\alpha * \frac{\alpha p}{2R} + \frac{R}{p}(s - (\frac{\alpha p}{2R})^2 - s - \lambda]$$

If $p = qR$:

$$\Pi_s = \alpha * \frac{\alpha p}{2qR} - s - \lambda$$
$$\Pi_r = q[\alpha * \frac{\alpha p}{2R} + \frac{R}{p}(s - (\frac{\alpha p}{2R})^2 - s - \lambda]$$

The threshold follows from setting the profits equal to each other.

In the presence of losses on legacy asset, the solvent investment price lower bound is shifted upwards: risk-shifting is preferred at higher prices levels than without the losses.

As before the highest feasible price under the all solvent investment is given by $p = q(\alpha)R$. Evaluating the risk-shifting threshold at the fair price, thus gives us the equilibrium risk-shifting threshold.

Since now the lowest price under which the risk shifting is possible is higher, more banks risk-shift in equilibrium. If the losses are not too large we still have that the resulting equilibrium is mixed as long as $p_r^* > \hat{p}$ for all $\alpha$.