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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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Intergenerational Sharing of Unhedgeable Inflation Risk[a]

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Abstract

We explore how members of a collective pension scheme can share inflation risks in the absence of suitable financial market instruments. Using intergenerational risk sharing arrangements, risks can be allocated better across the various participants of a collective pension scheme than would be the case in a strictly individual- or cohort-based pension scheme, as these can only lay off risks via existing financial market instruments. Hence, intergenerational sharing of these risks enhances welfare. In view of the sizes of their funded pension sectors, this would be particularly beneficial for the Netherlands and the U.K.

Key words: pension funds, intergenerational risk sharing, unhedgeable inflation risk, incomplete markets, welfare loss.
JEL Codes: C61, E21, G11, G23.

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1 Introduction

One of the major debating points when it comes to the reform of collective pension arrangements is how to allocate investment risk to specific age-cohorts to better exploit the risk-bearing capacity of the individual age-cohorts. However, pension fund participants cannot be protected against all financial risks using the available market instruments. For example, inflation risk and long-term interest rate risk are largely unhedgeable financial risks, as the financial instruments necessary to (fully) hedge these risks are illiquid or non-existent in the market. Chen et al. (2020) indicate that for retirees the welfare loss from unhedgeable inflation risk can vary between 1%-8% of certainty-equivalent consumption depending on the participants' risk preferences. While inflation has undershot its target of below, but close to, 2% in the eurozone for a long time, recently inflation has jumped to levels that have become hard to imagine after the worldwide shift to independent central banks following inflation-targeting policies. The rise in inflation is likely a combination of ultraloose monetary policies, pent-up demand following corona and restricted supply due to corona lockdowns and the war in Ukraine. The future path of inflation is unclear. What is clear, though, is that inflation uncertainty has increased substantially and that the materialization of inflation risks can have large consequences for the returns on savings and disposable incomes, especially for the poorer parts of the population.

The current paper focuses on the question how participants of a collective pension scheme can share risks that cannot be hedged using existing market instruments, in particular inflation risks. In comparison with an individual-based pension scheme such risk-sharing arrangements within a collective scheme allow for a better allocation of risks, thereby raising participants' welfare, in particular by effectively mitigating financial market incompleteness.[1]

Many countries feature collective funded pension schemes.[2] These schemes typically aim to index benefits to wage inflation, price inflation or a combination of the two. However, such indexation tends to be conditional on the financial situation of the pension fund and, even in cases of unconditional indexation, protection against inflation risk cannot be perfect, if there is no external backstop guaranteeing the pensions. Hence, the participants in these schemes are generally exposed to inflation risk. The obvious remedy would be for the fund to hold financial instruments that hedge against inflation risk. However, in at least two dimensions financial markets tend to be incomplete with regard to inflation risks. First, there may be no swap contracts or index-linked bonds for the country-specific consumer price index (CPI).[3] This is the case, for example, for the Netherlands, where inflation is at best hedged using foreign indexed debt or European inflation swaps, which are only partially correlated with Dutch inflation (Chen et al., 2020).[4] Moreover, the liquidity and outstanding volumes of foreign index-linked bonds and inflation swaps are limited. Second, even if it were possible to perfectly hedge CPI inflation, workers and retirees would still be exposed to the inflation risks associated with their own specific consumption bundles, to the extent that these bundles differ from the bundle consumed by the average population member.

This raises the question to what extent an "internal" market for inflation risk hedging among pension fund participants can substitute for the relevant missing instruments in the financial market. Such an internal market should a priori have the potential to generate welfare gains, as

[1]Mandatory participation in such collective schemes may be essential to reap the benefits of intergenerational risk sharing, as individuals may decide to walk away from the arrangement when they have to pay. Beetsma et al. (2012) and Romp and Beetsma (2020) demonstrate that only in the presence of risk aversion it is possible to maintain a collective scheme based on voluntary participation.

[2]For example, Austria, Belgium, Czech Republic, Finland, France, Greece, Hungary, Iceland, Japan, Korea, Portugal, Slovenia, Spain, Switzerland, Turkey, the United States, Canada, Luxembourg and the Netherlands.

[3]Only France, the U.S. and the U.K. have liquid index-linked bonds markets. Other countries issuing index-linked bonds are Australia, Brazil, Canada, Germany, Greece, Iceland, Italy, Japan Spain and Sweden (Swinkels, 2012).

[4]Other examples of countries that aim at indexing pension benefits to inflation in the absence of suitable financial instruments to protect against inflation risk are Austria, Belgium, Czech Republic, Finland, Hungary, Korea, Luxembourg, Portugal, Slovenia, Switzerland and Turkey.

the indexation of working cohorts' pension entitlements to their nominal salary already provides substantial protection due to the correlation between nominal wage growth and inflation. This suggests that there is room for a mutually beneficial trade: the working generation receives a risk premium from retirees who buy the inflation risk protection from them and/or receive similar protection from future workers when they are retired themselves. In this paper we design such intergenerational risk sharing arrangements, which are mutually welfare improving by reducing consumption uncertainty. Part of their appeal is that they are easy to implement.

Most closely related to this paper is (Chen et al., 2020), which only identifies and quantifies the problem of having incomplete markets in terms of inflation risk, but does not design or analyse concrete risk-sharing arrangements. In this paper, we thus go beyond this earlier work by exploring how to design risk-sharing arrangements within collective funded pension arrangements that address the aforementioned market incompleteness. Related literature on funded pensions has studied the sharing of risks that are also traded on the market. Hence, this literature considers pension fund arrangements that can be replicated in financial markets. However, if pension contracts can be designed to share non-traded risks, pension funds have an edge as the only currently-existing institutions to provide protection against those risks.

The remainder of this paper is organized as follows. Section 2 developes our two overlapping generations framework. Section 3 presents the baseline results, while Section 4 presents the results assuming more realistic wage dynamics. Finally, Section 5 concludes the main text of the paper. The Appendix describes technical details on the solution method of the model.

2 Two Overlapping Generations Model

This section presents a simple two overlapping generations framework with unhedgeable inflation risk.

The economy The CPI is given by Π_t, which evolves as

$$\frac{d\Pi_t}{\Pi_t} = d\pi_{h,t} + \sigma_u dZ_{u,t}, \tag{1}$$

$$d\pi_{h,t} = (\bar{\pi} - \pi_{h,t})\, dt + \sigma_h dZ_{h,t}, \tag{2}$$

where $\sigma_h dZ_{h,t}$ denotes the hedgeable component of inflation risk and $\sigma_u dZ_{u,t}$ denotes the unhedgeable component. Both $Z_{h,t}$ and $Z_{u,t}$ are Brownian processes. For example, in the case of the Netherlands, eurozone inflation is hedgeable, while the Netherlands-specific deviation from eurozone inflation cannot be hedged in financial markets. We assume that $dZ_{h,t}$ and $dZ_{u,t}$ are independent of each other, so $dZ_{u,t}$ is unhedgeable indeed. The expected rate of hedgeable inflation ($\pi_{h,t}$) follows an Ornstein-Uhlenbeck process that is mean-reverting around $\bar{\pi}$.

Stock prices are also lognormally distributed with return given by

$$\frac{dS_t}{S_t} = \mu_S dt + \sigma_S dZ_{S,t}. \tag{3}$$

We do not consider interest rate risk and therefore assume that the bond market has a constant nominal return that is independent of the inflation trend.

$$\frac{dB_{N,t}}{B_{N,t}} = r dt. \tag{4}$$

We also consider a real bond

$$\frac{dB_{R,t}}{B_{R,t}} = (r - \bar{\pi})\, dt + d\pi_{h,t} \tag{5}$$

$$= (r - \pi_{h,t})\, dt + \sigma_h dZ_{h,t}. \tag{6}$$

Hence, the nominal return on the real bond moves with the hedgeable component of actual inflation.

Demography and wage income We refer to generation "t" as the generation that enters the pension fund as participants at time t. The generation retires at time $t + T^R$ and dies at time $t + T^D$. Hence, during the period t to $t + T^R$, it earns a wage income, while during the period $t + T^R$ to $t + T^D$, it is retired. The number of working years before retirement is half of the total time that each generation spends in the pension scheme, so $T^D = 2 * T^R$. We assume that the nominal wage rate $H_{t,\tau}$ of generation t at time τ is linked to the price index[5]

$$H_{t,\tau} = \begin{cases} \Pi_\tau, & \tau \in \left[t, t + T^R\right], \\ 0, & \tau \notin \left[t, t + T^R\right]. \end{cases} \tag{7}$$

We normalize the wage rate of generation 0 at time $t = 0$ to 1, i.e. $H_{0,0} = 1$. Moreover, all generations are of equal size, which we normalize to 1. Every T^R years a new generation enters the pension scheme, the working generation retires and the retired generation dies.

Preferences We assume that agents have a constant relative risk aversion (CRRA) level of $\gamma > 1$, i.e. period utility is given by $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$. Since the individual is concerned about her purchasing power, utility is obtained from nominal consumption divided by the price index.

Lifetime utility of an individual from generation t is given by

$$U_t = E_t \left[\int_t^{t+T^D} e^{-\rho(\tau - t)} u\left(\frac{C_{t,\tau}}{\Pi_{t,\tau}}\right) d\tau \right]. \tag{8}$$

Parameter ρ denotes the subjective discount rate and $C_{t,\tau}$ denotes consumption in nominal terms of generation t at time τ. Consumption in real terms is given by $\frac{C_{t,\tau}}{\Pi_{t,\tau}}$, where $\Pi_{t,\tau}$ can be the overall price index (i.e. $\Pi_{t,\tau} = \Pi_\tau \ \forall t$) or it can be a generation-specific price index, as defined in (20) at the end of this section.

The corresponding certainty equivalent consumption level is obtained by

$$U_t = \int_t^{t+T^D} e^{-\rho\tau} u\left(cec_t\right) d\tau \iff cec_t = \left(\frac{(1-\gamma)\rho U_t}{1 - e^{-\rho T^D}}\right)^{1/(1-\gamma)}. \tag{9}$$

This is the constant consumption level at each moment that would provide the individual the same expected utility as the expected utility that is derived from the set of future stochastic scenarios and resulting policy settings using (8).

Individual retirement plan ("autarky") "Autarky" is defined as the situation in which different generations do not engage in a risk sharing arrangement. When the generation enters the individual retirement plan at the start of its life, no wealth has been accumulated yet, i.e. $W_{t,t} = 0$. All savings take place via the pension plan.[6] Hence, the contribution (in nominal terms) at moment τ to the retirement plan of generation t is the nominal wage $(H_{t,\tau})$ that remains after consumption $(C_{t,\tau})$. The individual optimizes its contribution, $(H_{t,\tau} - C_{t,\tau})$, which

[5] This assumption is relaxed in Section 4 by assuming more realistic wage dynamics.

[6] Governments often make pension savings relatively attractive by providing certain tax advantages, such as the possibility to deduct pension contributions from income before taxes, while the pension benefits received later are taxed at a lower rate than the income tax rate at the time the contributions were made. We abstract from explicitly modelling such tax advantages.

is added to its accumulated nominal pension wealth, which evolves as

$$\frac{dW_{t,\tau}}{W_{t,\tau}} =\frac{H_{t,\tau}-C_{t,\tau}}{W_{t,\tau}}d\tau+\left(1-\theta_\tau^S-\theta_\tau^R\right)\frac{dB_{N,t}}{B_{N,t}}+\theta_\tau^S\frac{dS_t}{S_t}+\theta_\tau^R\frac{dB_{R,t}}{B_{R,t}}+\theta_\tau^S\sigma_S dZ_{S,\tau} \tag{10}$$

$$=\left(r+\theta_\tau^S\left(\mu_S-r\right)-\pi_{h,t}\theta_\tau^R+\frac{P_{t,\tau}}{W_{t,\tau}}\right)d\tau+\theta_\tau^S\sigma_S dZ_{S,t}+\theta_\tau^R\sigma_h dZ_{h,t},\forall\tau\in\left(t,t+T^D\right), \tag{11}$$

where θ_t^S is the fraction of wealth invested in stocks and θ_t^R the fraction invested in real bonds. The remainder is invested in nominal bonds. We restrict these allocations to $\theta_t^S,\theta_t^R\in[0,1]$ in line with the portfolios of real-life institutional investors.

From the moment of retirement $t+T^R$ of generation t, accumulated wealth is used to pay out pension benefits. Pension wealth continues to be invested in financial instruments during retirement.

In the remainder capital letters denote nominal amounts, while lowercase letters denote real amounts. The real wealth dynamics are given by

$$dw_{t,\tau} =d\left(W_{t,\tau}/\Pi_\tau\right) \tag{12}$$

$$=W_{t,\tau}d\left(1/\Pi_\tau\right)+\left(1/\Pi_\tau\right)dW_{t,\tau}+d\left(W_{t,\tau},1/\Pi_\tau\right) \tag{13}$$

$$\iff \frac{dw_{t,\tau}}{w_{t,\tau}}=\left(r-\bar{\pi}+\theta_\tau^S\left(\mu_S-r\right)+\left(1-\theta_\tau^R\right)\left(\pi_{h,t}+\sigma_h^2\right)+\sigma_u^2\right)d\tau+$$

$$\ldots\theta_\tau^S\sigma_S dZ_{S,t}-\left(1-\theta_\tau^R\right)\sigma_h dZ_{h,t}-\sigma_u dZ_{u,t}+\frac{h_{t,\tau}-c_{t,\tau}}{w_{t,\tau}}d\tau,\forall\tau\in\left(t,t+T^D\right). \tag{14}$$

where $h_{t,\tau}=0,\forall\tau\in\left(t+T^R,t+T^D\right)$. Appendix A.1 describes how to solve the model under autarky regarding the consumption - savings decision and the decision to allocate the savings over the different assets..

Intergenerational risk sharing Suppose the different generations participating in the pension fund have a risk sharing arrangement for the unhedgeable component of inflation risk, where the working generation transfers an amount $f_{t,s}$ to the retirees. This can be an infinite horizon risk sharing arrangement, where each generation shares risks with another generation. It could also be a one-off arrangement, in which current workers and retirees agree on a risk sharing arrangement for as long as both are alive. In the one-off arrangement these generations can negotiate a risk premium that the working generation receives for insuring the retirees against unhedgeable inflation risk. Also in the infinite horizon setting a risk premium can be considered. However, current workers will retire at some moment and, hence, benefit from the insurance provided by the new generation of workers, so over a lifetime it could be fair not to have a risk premium paid by the retired generation.[7]

The real wealth dynamics of retirees from generation t become for all $\tau>t+T^R$:

$$\frac{dw_{t,\tau}}{w_{t,\tau}}=\left(r-\bar{\pi}+\theta_\tau^S\left(\mu_S-r\right)+\left(1-\theta_\tau^R\right)\left(\pi_{h,t}+\sigma_h^2\right)+\sigma_u^2\right)d\tau+$$

$$\ldots\theta_\tau^S\sigma_S dZ_{S,\tau}-\left(1-\theta_\tau^R\right)\sigma_h dZ_{h,\tau}-\sigma_u dZ_{u,\tau}-\frac{c_{t,\tau}}{w_{t,\tau}}d\tau+\frac{df_{t,\tau}}{w_{t,\tau}},\forall\tau\in\left(t+T^R,t+T^D\right). \tag{15}$$

Assume the following functional form for the transfer from the generation that enters the pension scheme at $t+T^R$ to generation t:

$$df_{t,\tau}=-w_{t,\tau}\theta_\tau^u\mu_{rp}d\tau+\theta_\tau^u w_{t,\tau}\sigma_u dZ_{u,\tau},\forall\tau\in\left(t+T^R,t+T^D\right), \tag{16}$$

[7]This is approximately correct with low interest rates, but when discounting is taken into account as well, this could introduce a pay-as-you-go transfer.

where $\theta^u \in [0,1]$ denotes the weight that is applied in the agreement. The contract consists of a fixed leg and a floating leg. The fixed leg is given by the parameter $\mu_{rp} \geq 0$, which can be considered a risk premium that the retired generation pays to the working generation. The floating leg is given by the unhedgeable risk $(\sigma_u dZ_{u,\tau})$. If $\theta^u = 0$, we have $df_{t,s} = 0$, so no risk transfer takes place. If $\theta^u = 1$, all unhedgeable inflation risk is transferred from the retired generation to the working generation, as this implies $df_{t,s} = w_{t,\tau} \left(-\mu_{rp} d\tau + \sigma_u dZ_{u,\tau}\right)$.

Hence, under the above risk sharing arrangement retirees' real wealth evolves as

$$dw_{t,\tau} = w_{t,\tau}\left(r - \bar{\pi} + \theta^S_\tau\left(\mu_S - r\right) + \left(1-\theta^R_\tau\right)\left(\pi_{h,t} + \sigma_h^2\right) + \sigma_u^2 - \theta^u\mu_{rp}\right)d\tau - c_{t,\tau}d\tau + \quad (17)$$

$$\ldots w_{t,\tau}\left[\theta^S_\tau \sigma_S dZ_{S,\tau} - \left(1-\theta^R_\tau\right)\sigma_h dZ_{h,\tau} + \left(\theta^u - 1\right)\sigma_u dZ_{u,\tau}\right], \quad (18)$$

while that of the working generation evolves as:

$$\begin{aligned} dw_{t,\tau} =& w_{t,\tau}\left[r - \bar{\pi} + \theta^S_\tau\left(\mu_S - r\right) + \left(1-\theta^R_\tau\right)\left(\pi_{h,t} + \sigma_h^2\right) + \sigma_u^2\right]d\tau + \\ &\ldots w_{t,\tau}\left[\theta^S_\tau \sigma_S dZ_{S,\tau} - \left(1-\theta^R_\tau\right)\sigma_h dZ_{h,\tau} - \sigma_u dZ_{u,\tau}\right] + \left(1 - c_{t,\tau}\right)d\tau + \\ &\ldots \left(w_{t-T^R,\tau}\theta^u\mu_{rp}d\tau - \theta^u w_{t-T^R,\tau}\sigma_u dZ_{u,\tau}\right). \end{aligned} \quad (19)$$

Appendix A.2 describes how to solve the model with intergenerational risk-sharing, such that participants of the pension fund optimize the consumption - savings decision and the decision to allocate the savings over the different assets.

Age-dependent inflation We now consider the case in which different generations face a different price index to evaluate their purchasing power. In particular, retirees typically have a different consumption basket than the working generation. For example, Stewart (2008) shows that the CPI for elderly and the CPI for workers have moved differently over time. Munnell and Chen (2015) argue that the reference index that is applied to adjust the Social Security in the U.S. should be linked to an index that gives more weight to costs related to medical care, as this is more relevant for the elderly. Chen et al. (2020) estimate that the standard deviation of the quarterly difference between the CPI of workers and elderly is 0.30%, based on data from the Bureau of Labor Statistics (2018). Figure 1 shows for the U.S. the CPIs of the elderly (CPI-E) and the workers (CPI-W).

Nominal spending of generation t at time τ is denoted by $C_{t,\tau} = c_{t,\tau}\Pi_{t,\tau}$, with

$$\Pi_{t,\tau} = \begin{cases} \Pi_{W,\tau}, & \tau \in \left[t, t+T^R\right], \\ \Pi_{R,\tau}, & \tau \in \left(t+T^R, t+T^D\right). \end{cases} \quad (20)$$

We follow the approach by Corsetti et al. (2008) for setting the price of different consumption baskets. In their setup, there are two countries and the price index of the consumption basket is a geometrically-weighted average of the price indices of the individual countries' production. Here, the CPI is a geometrically weighted average of the prices of the workers' and retirees' consumption baskets with weights $\left(T^D - T^R\right)/T^D$ and T^R/T^D, respectively, since all generations are equally sized:[8]

$$\Pi_t = \Pi_{W,t}^{\frac{T^R}{T^D}} \Pi_{R,t}^{\frac{T^D - T^R}{T^D}}. \quad (21)$$

The difference between worker and retiree inflation is given by

$$d\log\left(\frac{\Pi_{W,t}}{\Pi_{R,t}}\right) = \sigma_R dZ_{R,t}. \quad (22)$$

[8]The CPI without any further addition refers to the consumer price index of the entire economy.

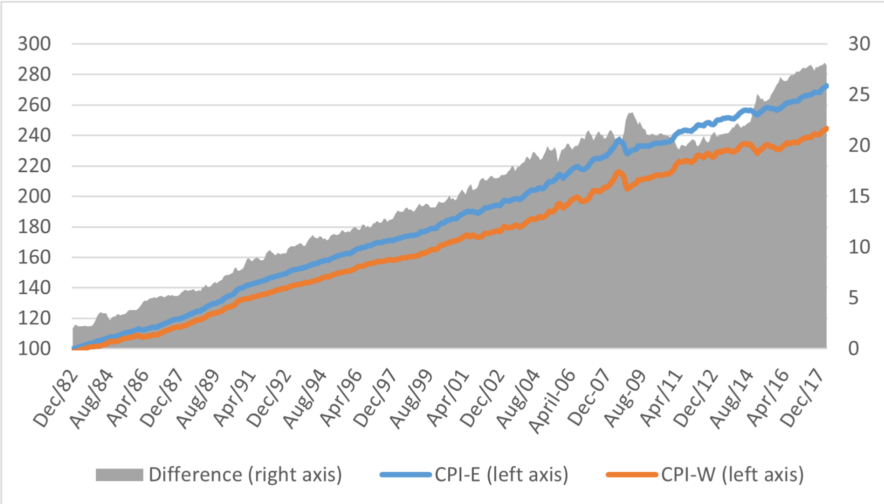

Figure 1: Consumer price index of elderly (CPI-E), of workers (CPI-W) and their difference, based on data over December 1982 - April 2018 from the Bureau of Labor Statistics (2018).

We estimate a standard deviation of this difference of 0.60% per annum, similar to the quarterly estimate by Chen et al. (2020). When worker inflation is lower than CPI inflation, retiree inflation exceeds CPI inflation, and vice versa. Hence, the idiosyncratic component of the age-specific inflation rates, which cannot be hedged with existing financial market instruments, can potentially be offset through a risk-sharing arrangement among the generations. To accommodate this possibility we consider the following functional form for the transfer from the generation that enters at $t+T^R$ to generation t, which generalises (16):

$$df_{t,\tau} = w_{t,\tau}\theta^u\left(-\mu_{rp}d\tau + \sigma_u dZ_{u,\tau} - \frac{1}{2}\sigma_R dZ_{R,\tau}\right), \forall\tau \in \left(t+T^R, t+T^D\right), \tag{23}$$

The wealth dynamics of the retirees become now

$$\begin{aligned} dw_{t,\tau} =& w_{t,\tau}\left(r - \bar{\pi} + \theta^S_\tau\left(\mu_S - r\right) + \left(1-\theta^R_\tau\right)\left(\pi_{h,t} + \sigma_h^2\right) + \sigma_u^2 - \theta^u\mu_{rp}\right)d\tau - c_{t,\tau}d\tau + \\ &\ldots w_{t,\tau}\left[\theta^S_\tau\sigma_S dZ_{S,\tau} - \left(1-\theta^R_\tau\right)\sigma_h dZ_{h,\tau} + \left(\theta^u - 1\right)\sigma_u dZ_{u,\tau} - \frac{1}{2}\theta^u\sigma_R dZ_{R,\tau}\right] \end{aligned} \tag{24}$$

while the wealth dynamics of the workers become

$$\begin{aligned} dw_{t,\tau} =& w_{t,\tau}\left[r - \bar{\pi} + \theta^S_\tau\left(\mu_S - r\right) + \left(1-\theta^R_\tau\right)\left(\pi_{h,t} + \sigma_h^2\right) + \sigma_u^2\right]d\tau + \\ &\ldots w_{t,\tau}\left[\theta^S_\tau\sigma_S dZ_{S,\tau} - \left(1-\theta^R_\tau\right)\sigma_h dZ_{h,\tau} - \sigma_u dZ_{u,\tau}\right] + \left(1 - c_{t,\tau}\right)d\tau + \\ &\ldots\left(w_{t-T^R,\tau}\theta^u\mu_{rp}d\tau - \theta^u w_{t-T^R,\tau}\sigma_u dZ_{u,\tau} + \frac{1}{2}\theta^u w_{t-1,\tau}\sigma_R dZ_{R,\tau}\right). \end{aligned} \tag{25}$$

3 Results

3.1 Social welfare evaluation

We evaluate the risk sharing arrangements in terms of social welfare (SW), which is the welfare of a social planner who takes into account the utility of both the current and future generations.

Concretely, social welfare at time t is defined as

$$SW_t \equiv U_{t-T^R} + \sum_{s\in\mathcal{H}_t} e^{-\zeta(s-t)} U_s, \tag{26}$$

$$\mathcal{H}_t \equiv \left\{ t + n * T^R : n \in \mathbb{N} \cup \{0\} \right\}. \tag{27}$$

The discounting parameter $\zeta > 0$ determines how much weight is given to the future generations relative to the current generations. The future generations at time t, which are generations t to infinity, are weighted by $e^{-\zeta(s-t)}$. Hence, this weight converges to zero as s goes to infinity. The corresponding certainty equivalent social welfare is derived as follows:

$$SW_t = \int_0^{T^D-T^R} e^{-\rho\tau} u\left(cew_t\right) d\tau + \sum_{s\in\mathcal{H}} \int_0^{T^D} e^{-\zeta(s-t)} e^{-\rho\tau} u\left(cew_t\right) d\tau \tag{28}$$

$$\Longleftrightarrow\ u\left(cew_t\right) = \rho SW_t \left(\left[1 - e^{-\rho\left(T^D - T^R\right)}\right] + \left[1 - e^{-\rho T^D}\right] \frac{e^{-\zeta T^R}}{1 - e^{-\zeta T^R}} \right)^{-1} \tag{29}$$

$$\Longleftrightarrow\ cew_t = \left[(1-\gamma)\,\rho SW_t \left(\left[1 - e^{-\rho\left(T^D - T^R\right)}\right] + \left[1 - e^{-\rho T^D}\right] \frac{e^{-\zeta T^R}}{1 - e^{-\zeta T^R}} \right)^{-1} \right]^{1/(1-\gamma)} \tag{30}$$

3.2 Parameterisation

Table 1 reports the calibration of the parameters. We set the parameter for the expected hedgeable inflation component at $\bar{\pi} = 2\%$, in line with ECB's inflation target. The volatility of unhedgeable inflation is estimated by Chen et al. (2020) at 0.72% per annum, so we set the volatility of the unhedgeable inflation component at $\sigma_u = 0.72\%$. We set the nominal interest rate to $r = 2\%$, in line with the euro swap rates with tenor 10 to 20 years as of 22-7-2022. This way, we have a zero real interest rate on average ($r - \bar{\pi} = 0$). The parameters σ_h, σ_S and μ_S are chosen in line with the estimations by Brennan and Xia (2002). We normalize the real wage rate to $h_{t,\tau} = 1$ per annum. The total time an individual spends in the pension scheme is set to $T^D = 60$ years. The number of working years before retirement is half of the total time spent in the pension scheme, so $T^R = 30$ years. The constant relative risk aversion is set to $\gamma = 5$ in the benchmark calculations, but will be varied in Section 3.7.1. We assume that the subjective discount rate of the agents, ρ, and the subjective discount rate of the social planner, ζ, are both equal to the nominal interest rate, i.e. $\rho = \zeta = r$. Our baseline analysis assumes that working and retired generations feature the same consumption package and thus the same inflation rate, i.e. $\sigma_R = 0\%$. This allows us to explore the cost of age-dependent inflation with standard deviation ($\sigma_R = 0.60\%$ per annum) in Section 3.6 onward. We derive the results using 10,000 simulation paths of the economy.

3.3 Autarky

By applying the solution method described in Appendix A.1, we obtain the optimal savings and investment allocations under autarky. Figure 2 shows the corresponding real wealth and consumption evolution over 60 years based on the optimal allocations. Wealth accumulates through saving out of wage income during the first 30 years and the returns on those savings. The last 30 years of life wealth gradually decreases towards zero. The median consumption level is about 0.6 in the beginning and increases to 1.3 close to the end of life. As individuals becomes older the bandwith around their median consumption widens. In the final period of their life the 5th and 95th percentiles of 10,000 simulation paths of consumption are about 0.6, respectively 2.6. The rise in average consumption over the lifetime is driven by the fact that

Table 1: Parameters

Symbol	Parameter	Description
$\bar{\pi}$	0.02	expected hedgeable inflation
σ_h	0.027	volatility hedgeable inflation
σ_u	0.0072	volatility unhedgeable inflation
μ_{rp}	$[0, \gamma\sigma_u^2]$	risk premium IGR contract
μ_S	0.074	expected equity return
σ_S	0.158	volatility equity return
r	0.02	nominal interest rate
$h_{t,\tau}$	1	real wage income
T^R	30	number of years working before retirement
T^D	60	number of years in the pension scheme
γ	5	constant relative risk aversion
ρ	0.02	subjective discount rate
ζ	0.02	social planner's discount rate
σ_R	0.006	standard deviation of difference between price index of retirees and workers

consumption becomes more uncertain with age and the individual is risk averse, leading her to build up pre-cautionary savings that are released as she grows older. The certainty equivalent consumption level is 0.697. This implies that the optimal portfolio allocations and consumption result in a lifetime certainty equivalent consumption that is slightly larger than two thirds of annual real wage income.

At the start of the career the optimal fraction of wealth invested in equity is at its upper bound of 100%, while the optimal fraction in real bonds is at its lower bound of 0%. In line with standard lifecycle portfolio theory, the (relatively) safe human capital is largest at the start of the career and decreases with age. As a consequence, the financial wealth is invested less risky later in life. Hence, the optimal fraction invested in equity decreases with age, while the optimal fraction invested in real bonds increases with age.

3.4 One-off risk sharing arrangement

Now we consider a situation in which workers cover the unhedgeable inflation of the retirees. In itself this is welfare improving for the retirees and welfare deteriorating for workers. However, to compensate the latter group, they receive a risk premium from the retirees in return ($\mu_{rp} > 0$). This reduces the appetite of the retirees to shift unhedgeable risk to workers, but makes the latter more willing to do so.

We vary both parameters μ_{rp} and θ^u to investigate the corresponding welfare effects for both generations in Figure 3. The left-hand graph represents the working generation, while the right-hand graph represents the generation that is retired when the IGR arrangement is introduced. For $\theta^u = 0$ there is no IGR, so we are back at the autarky case and the welfare gain is 0% irrespective of the risk premium. For other allocations of the IGR contract ($\theta^u > 0$), we observe that there are welfare gains for both generations. A risk-sharing arrangement with a larger risk premium is welfare improving for the working generation and welfare deteriorating for the retired generation ceteris paribus.

Figure 4 presents the optimal allocation of the IGR contract (θ^u) given the risk premium (μ_{rp}). From the left-hand panel we observe that the optimal degree of unhedgeable inflation risk sharing increases with the risk premium for the working generation, while the right panel shows the opposite for the retired generation.

Workers prefer a higher risk premium, while retirees prefer a zero risk premium. The question

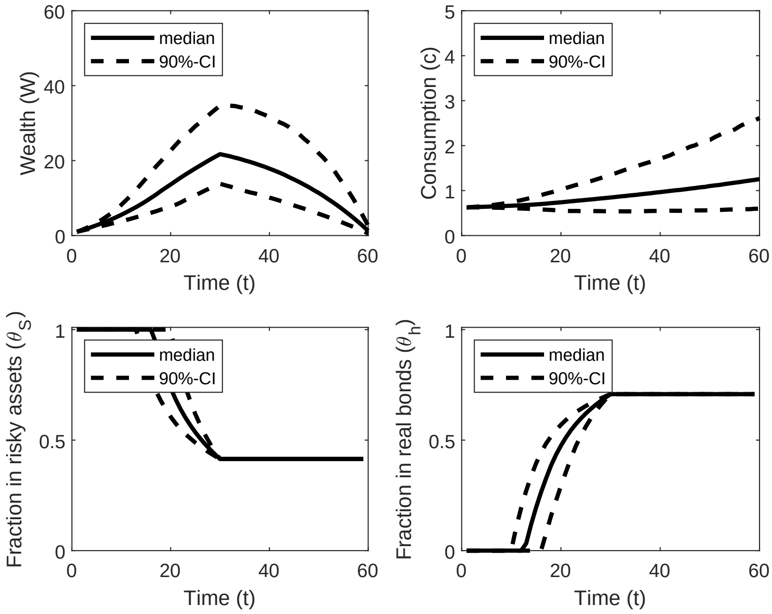

Figure 2: Wealth development and consumption. The solid line is the median. The dashed lines represent the 5th and 95th percentiles of 10,000 simulated paths.

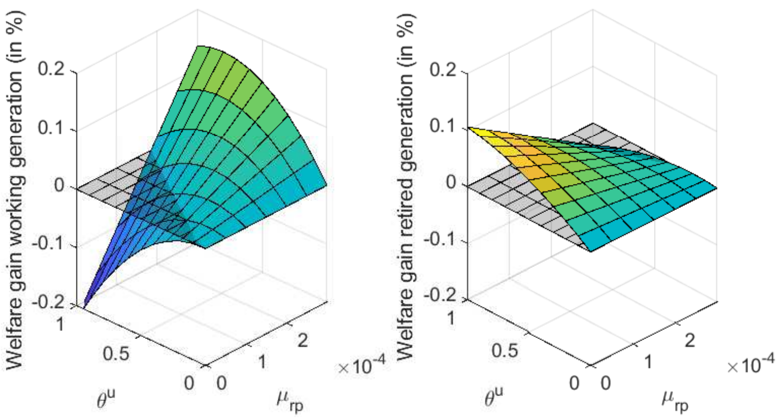

Figure 3: Welfare gain for different combinations of the one-off IGR contract (θ^u) and the risk premium (μ_{rp}).

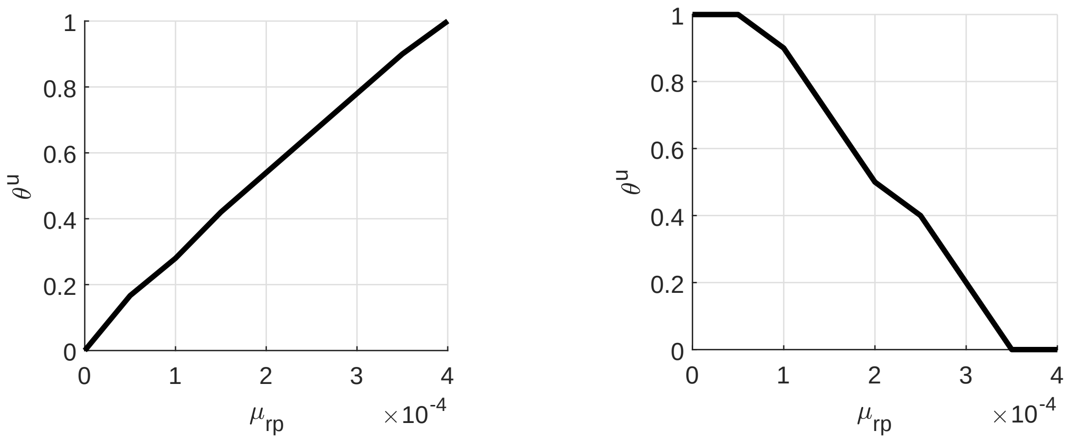

Figure 4: Optimal value of the one-off IGR contract (θ^u) given the risk premium (μ_{rp}). The left-hand panel shows the optimal allocation for workers, while the right-hand panel shows the optimal allocation for the retirees.

is whether there is a set of parameter combinations for which both groups simultaneously enjoy a welfare gain. In the left-hand panel of Figure 5 we show for each generation the area for which it experiences a positive welfare gain, while the right-hand panel depicts the area for which both generations enjoy positive welfare gains. The areas are obtained for stepsizes of 10 percentage-points for the allocation of the IGR contract θ^u and stepsizes of 0.005 percentage-points for the risk premium μ_{rp}. For $\theta^u \in [10\%, 100\%]$ and for $\mu_{rp} \in [0.005\%, 0.030\%]$ there exist IGR arrangements that make both workers and retirees better off. However, it is up to the social planner to make a tradeoff between the welfare gains of the different generations. The left-hand panel of Figure 6 shows the social planner's welfare gains. The right-hand panel shows the socially-optimal values of θ^u for given risk premia. They rise with the risk premium and range from $\theta^u = 30\%$ to $\theta^u = 80\%$.

3.5 Infinite horizon risk sharing

During working life a generation (partially) covers the unhedgeable inflation risk of the current retired. Subsequently, when the current workers have retired, the next generation of workers (partially) covers the unhedgeable inflation risk of the former. This goes on forever and is hence referred to as infinite horizon risk sharing. We would expect all current and future generations to benefit from such an arrangement.

Figure 7 shows the welfare effects of varying the parameters μ_{rp} and θ^u. The left-hand graph represents the working generation, while the right-hand graph represents the generation that is retired when the IGR arrangement is introduced. With $\theta^u = \mu_{rp} = 0$ there is no IGR, so the welfare gain is 0% irrespective of the risk premium. For other allocations of the IGR contract ($\theta^u > 0$), we again observe that there are welfare gains for both generations currently alive. Again a risk sharing arrangement with a larger risk premium is welfare deteriorating for the retired generation, while it is welfare improving for the current working generation and future generations, ceteris paribus.[9]

[9]The evaluation for future generations is (essentially) the same as for working generations. They both face an entire lifetime in the pension scheme with the IGR arrangement and the same economy. There is only one (relatively minor) difference and that is that the hedgeable component of inflation $(\pi_{h,t})$ starts at its mean at time $t = 0$, so $\pi_{h,0} = \bar{\pi}$, while for future generations there are different starting positions for this inflation component.

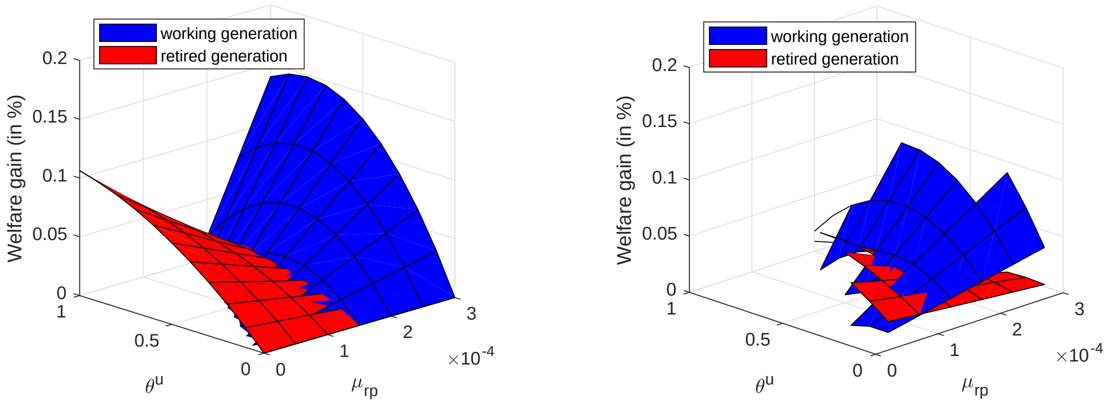

Figure 5: Welfare gain for different combinations of the one-off IGR contract (θ^u) and the risk premium (μ_{rp}). The left-hand panel depicts the areas with positive welfare gains of workers and of retirees. The right-hand panel depicts the area for which both groups experience positive welfare gains.

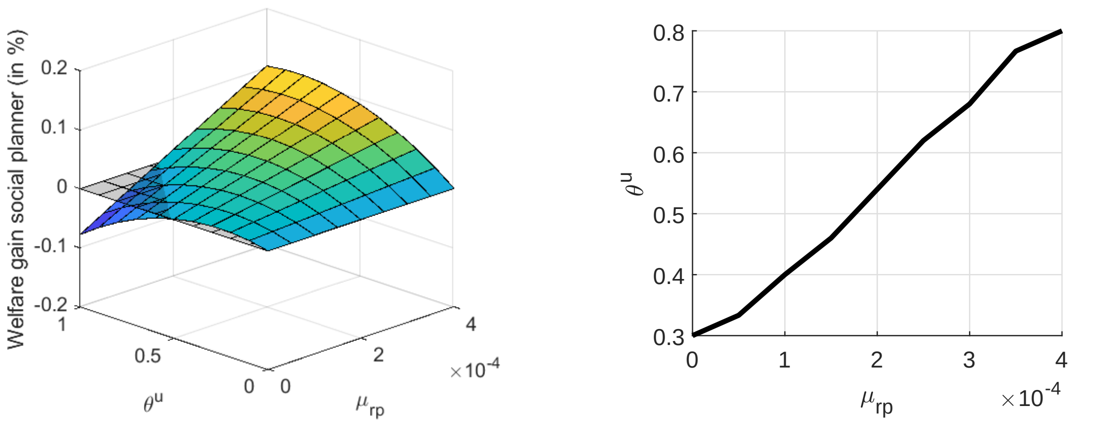

Figure 6: Social welfare evaluation of the one-off IGR contract. The left panel shows the social welfare gain for different combinations of the IGR contract (θ^u) and the risk premium (μ_{rp}). The right panel shows the socially-optimal value of the IGR contract (θ^u) given the risk premium (μ_{rp}).

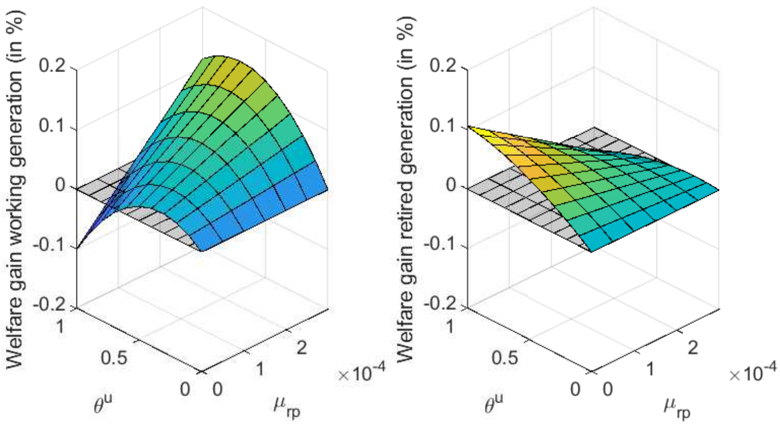

Figure 7: Welfare gains for different allocations of the infinite horizon IGR contract (θ^u) and different risk premia (μ_{rp}).

Figure 8 depicts the optimal allocations of the IGR contract (θ^u) given the risk premium (μ_{rp}) for current working and future generations (left-hand panel) and for the current retired generation (right-hand panel). For the former group the optimal value of θ^u increases with the risk premium, while for the latter group it falls with the risk premium.

The left-hand panel of Figure 9 depicts the areas with positive welfare gains for the currently-working and future generations and for the currently-retired generation, while the right-hand panel of Figure 9 depicts the area for which all generations on net benefit from the IGR arrangement. The results indicate that for $\theta^u \in [10\%, 100\%]$ and for $\mu_{rp} \in [0\%, 0.03\%]$ there exist IGR arrangements that make all generations better off. This range is wider than under the setting with one-off risk sharing, as the welfare gains under the infinite horizon risk sharing setting are larger as well. In effect, risks can be spread over a larger set of generations.

There is one combination depicted for which all generations enjoy a welfare gain larger than 0.05%. This is the case for $\theta^u = 60\%$ and $\mu_{rp} = 0.01\%$. However, it is up to a social planner to make a tradeoff between the welfare gains of the different generations in order to determine the social optimum. In the left-hand panel of Figure 10 the social planner's welfare gains from the IGR arrangement are shown. The right-hand panel shows the socially-optimal allocation of the IGR contract given the risk premium. The socially-optimal value of θ^u increases with the risk premium and varies between $\theta^u = 50\%$ and $\theta^u = 70\%$.

3.6 Age-dependent inflation

We extend the analysis by assuming age-dependent inflation by setting parameter σ_R to 0.60%. This way we have two sources of unhedgeable inflation risk, for which we investigate the welfare gains from the different risk-sharing arrangements.

3.6.1 Autarky

The certainty-equivalent consumption level with identical consumption baskets of workers and retirees was 0.697 in autarky (see Section 3.3). If we allow for age-dependent inflation, the certainty-equivalent consumption level is marginally lower. The welfare loss associated with age-dependent inflation is -0.03% and that from both sources of unhedgeable risk together is -0.16%.

The left-hand panel of Figure 11 shows the welfare levels over the remaining lifetime under different settings for unhedgeable inflation, by considering a complete market (solid red line), only

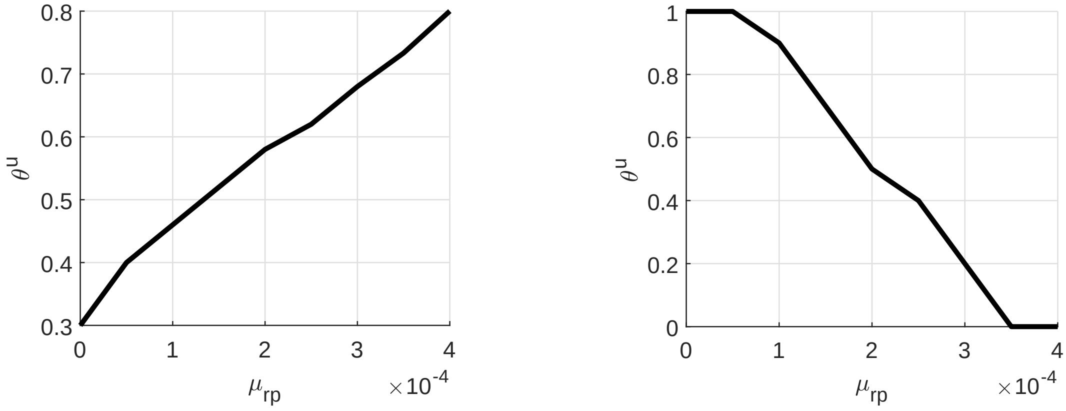

Figure 8: Optimal allocation of the infinite horizon IGR contract (θ^u) given the risk premium (μ_{rp}). The left-hand panel shows the optimal allocation for the current working generation and future generations, while the right-hand panel shows the optimal allocation for the current retired generation.

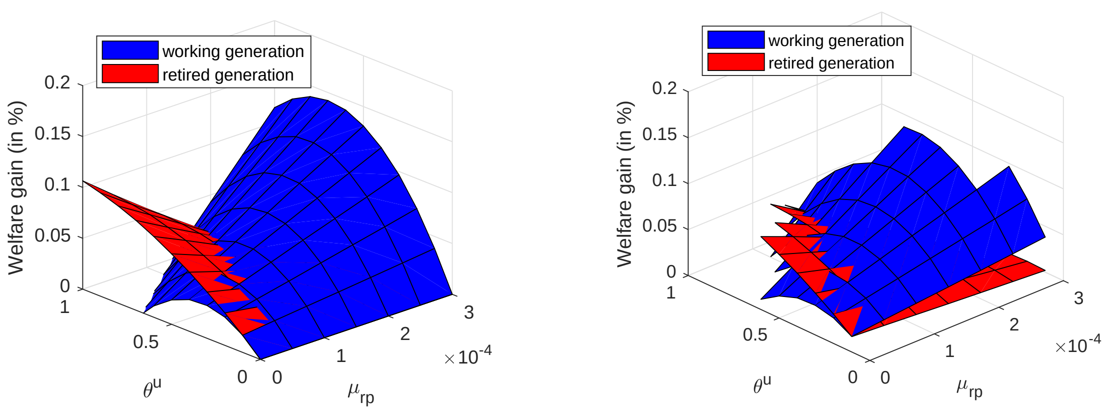

Figure 9: Welfare gain for different allocations of the infinite horizon IGR contract (θ^u) and different risk premia (μ_{rp}). The left panel presents the positive welfare gains of the current working generation and future generations and of the current retired generation. The right panel presents the welfare gains if these are strictly positive for all current and future generations.

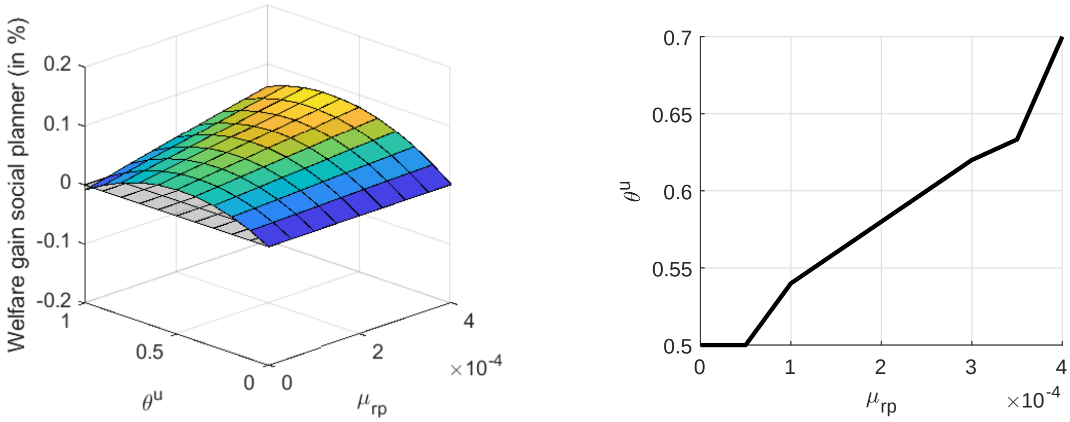

Figure 10: Social welfare evaluation of the infinite horizon IGR contract. The left-hand panel shows the social welfare gain relative to no IGR for different allocations of the IGR contract (θ^u) and different risk premia (μ_{rp}). The right-hand panel shows the socially optimal allocation of the IGR contract (θ^u) given the risk premium (μ_{rp}).

CPI inflation containing an unhedgeable component (dashed red line), only the age-dependent inflation rate containing an unhedgeable component (solid black line) and both sources of unhedgeable inflation risk being present (dashed black line). Welfare is lowest in the latter case. The right-hand panel of Figure 11 shows the welfare effects relative to the complete market setting. The welfare loss from the unhedgeable component of CPI inflation risk increases with age, because it is more difficult to smooth the price fluctuations when one grows older and the working generation is partially hedged through their salary income which is assumed to be a constant real wage. Moreover, the working generation is currently exposed to the price index for workers, but also exposed to the price index for retirees in the future, while the retired generation is affected by the price index for retirees only. Being exposed to both price indices dampens the risk exposure, as these indices move in opposite directions (see (21)). At the retirement age (i.e. $T^R = 30$), the welfare loss from both sources of unhedgeable inflation risk is 0.5% and the welfare loss ten years before passing away (i.e. $T^D - 10 = 50$) is 0.9%.

3.6.2 Risk sharing

In Sections 3.4 and 3.5 we evaluated risk-sharing when both workers and retirees featured the same consumption bundle. The welfare gains from our risk-sharing arrangement are larger in the presence of age-dependent inflation, as there is an additional source of unhedgeable risk to be shared – see Figure 12, which shows the welfare gains from IGR under both the one-off and infinite-horizon arrangement. Varying the risk premium (μ_{rp}) has a relatively small welfare effect for both the working and retired generations, while the allocation of the IGR contract (θ^u) has a relatively substantial effect. When the allocation to the IGR contract is zero ($\theta^u = 0$) there is no welfare gain from risk sharing. For a strictly positive allocation to the IGR contract ($\theta^u > 0$) we see that the workers' welfare gain from the risk sharing arrangement becomes larger at first, but falls in θ^u for values of θ^u close to 1, which implies that too much risk is shifted from retirees to workers compared to what is optimal for workers.

The largest additional welfare gain from age-dependent inflation when compared to the case without age-dependent inflation is obtained at around $\theta^u = 30\%$ for the working generation and around $\theta^u = 80\%$ for the retired generation. However, the additional welfare gains from IGR are relatively small, which can be explained by the fact that the initial loss from age-dependent inflation in autarky is also small.

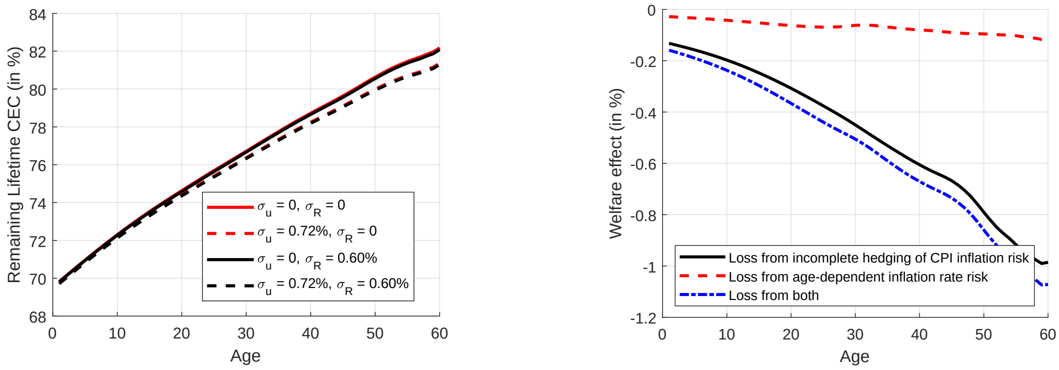

Figure 11: Remaining lifetime welfare. The left-hand panel depicts the welfare levels in terms of certainty equivalent consumption (CEC) for different unhedgeable inflation settings. The right-hand panel depicts the welfare losses associated with the different sources of unhedgeable risk, where the complete market setting is considered as the base case.

The results from the one-off risk sharing arrangement (left panel) and infinite horizon risk sharing arrangement (middle panel) are quite similar, as shown in Figure 12. The additional welfare gains from IGR are slightly larger for the working generation under infinite horizon risk sharing, as they now also benefit from protection against age-dependent inflation when they have retired themselves.

In order to measure the adequacy of the risk-sharing arrangements, we can relate the welfare gains of the risk-sharing arrangement to the welfare loss of the unhedgeable risks: i.e. $\left(\frac{gain}{-loss} - 1\right) * 100\%$. The loss from both sources of risk combined is -0.16% for the working generation and -0.52% for the retired generation. Figure 13 shows that an adequate policy for intergenerational risk sharing can more than cover the welfare loss from unhedgeable risk for the working generation and future generations (left-hand panel: one-off risk sharing; middle panel: infinite horizon risk sharing), as the subsidy (μ_{rp}) from the retired generation to the working generation allows the working generation to invest more wealth over a longer period. For the retired generation (right panel) only up to 20% of the welfare loss can be covered, but only for a risk premium that is close to zero.

3.6.3 Varying risk sharing by source of unhedgeable inflation risk

So far, parameter θ^u captured the common degree of sharing of both sources of unhedgeable inflation risk. Now we make the arrangement more sophisticated by allowing for different degrees of sharing of the two types of unhedgeable inflation risk. This yields the following transfer from generation t (workers) to generation $t - T^R$ (retirees):

$$df_{t,\tau} = w_{t,\tau}\left(-\left(\theta^{u,1} + \theta^{u,2}\right)\mu_{rp}d\tau + \theta^{u,1}\sigma_u dZ_{u,\tau} - \frac{1}{2}\theta^{u,2}\sigma_R dZ_{R,\tau}\right), \forall\tau \in \left(t + T^R, t + T^D\right), \tag{31}$$

where parameter $\theta^{u,1}$ captures the degree to which the unhedgeable component of CPI inflation is shared, while parameter $\theta^{u,2}$ denotes the degree of intergenerational sharing of age-specific inflation risk.

We start with the situation without a risk premium, i.e. $\mu_{rp} = 0$, which is shown in the top line of Figure 14. The welfare gain of the working generation is increasing in the parameters $\theta^{u,1}$ and $\theta^{u,2}$ for small values of these parameters, but decreasing for larger values. The welfare gains are more sensitive to the degree of sharing of CPI inflation $\left(\theta^{u,1}\right)$ than to the degree of sharing

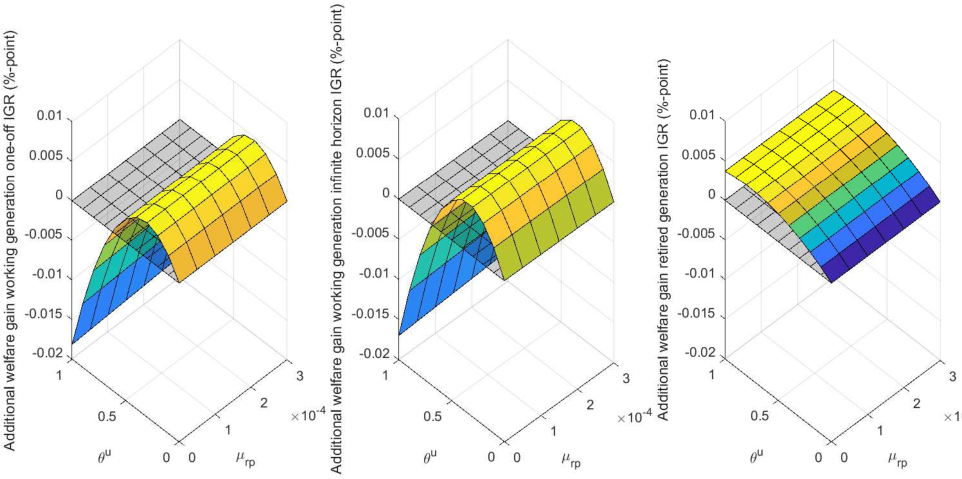

Figure 12: The panels show the additional welfare gain for different allocations of the IGR contract (θ^u) and different risk premia (μ_{rp}) comparing a setting with age-dependent inflation and a setting without age-dependent inflation.

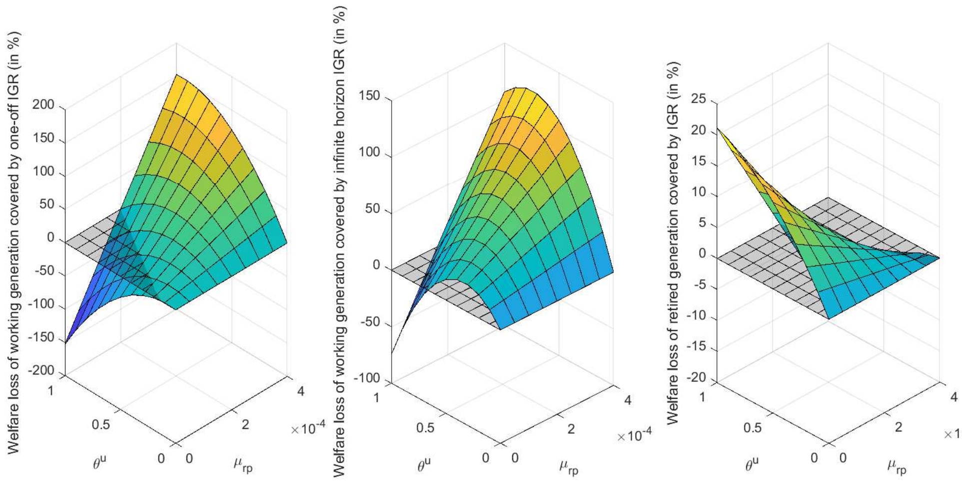

Figure 13: The extent of welfare loss that is covered by the welfare gain from intergenerational risk sharing for different allocations of the IGR contract (θ^u) and different risk premia (μ_{rp}).

Table 2: Welfare effect from unhedgeable inflation risk.

Parameter setting	σ_u(in %)	σ_R(in %)	Lifetime welfare effect (in %)			
			$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	0.72	0	-0.08	-0.13	-0.18	-0.27
Age-dependent inflation risk	0	0.60	-0.01	-0.03	-0.04	-0.07
Both sources of unhedgeable inflation risk	0.72	0.60	-0.09	-0.16	-0.22	-0.33
	σ_u(in %)	σ_R(in %)	Welfare effect during retirement (in %)			
			$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	0.72	0	-0.37	-0.47	-0.53	-0.64
Age-dependent inflation risk	0	0.60	-0.01	-0.06	-0.10	-0.18
Both sources of unhedgeable inflation risk	0.72	0.60	-0.38	-0.52	-0.62	-0.78

Note: the welfare effect is relative to the setting without unhedgeable inflation risk, i.e., $\sigma_u = \sigma_R = 0$*. The welfare effects during retirement are evaluated at the moment of retirement.*

of age-specific inflation risk $\left(\theta^{u,2}\right)$. This is in line with the fact that the loss from unhedgeable CPI inflation risk is larger than the loss associated with age-specific inflation risk.

Introducing a risk premium raises the benefit of the risk-sharing arrangement for workers (see the second line with $\mu_{rp} = 0.01\%$). This effect rises with the size of the risk premium (see the bottom line with $\mu_{rp} = 0.02\%$). This positive welfare effect for the workers is particularly strong with one-off risk sharing, as with infinite horizon risk sharing workers will have to pay the risk premium when they have retired themselves. The opposite holds for the current retirees: a larger risk premium lowers their welfare benefit from risk sharing.

3.7 Sensitivity analysis

3.7.1 Varying risk aversion

This subsection investigates the welfare losses from unhedgeable inflation risk without and with risk sharing when we vary the degree of relative risk aversion. Table 2 shows the results. The lifetime loss in the absence of risk sharing ranges from about -0.09% for relative risk aversion level $\gamma = 3$ to -0.33% for $\gamma = 10$. For individuals at the start of their retirement the welfare effect ranges from -0.38% for $\gamma = 3$ to -0.78% for $\gamma = 10$. The corresponding welfare losses are larger for retirees, but remain below 1%.

3.7.2 Higher levels of inflation and volatility

After many years of being low and stable, inflation has now risen to levels rarely seen over the past decades. While inflation is projected to gradually fall towards its target, the uncertainty around the expected inflation path is much higher than before. Hence, we investigate the risk-sharing arrangements in a setting with twice the baseline inflation volatility, so $\sigma_h = 5.4\%$, $\sigma_u = 1.44\%$ and $\sigma_R = 1.2\%$. In order to maintain the ratio of expected inflation over volatility, we also double the expected inflation rate to $\bar{\pi} = 4\%$. For consistency, we do the same for the nominal interest rate and the subjective discount rates, i.e. $\bar{\pi} = \rho = \zeta = r$. By comparing Figure 15 with Figure 11, we observe that the welfare losses from unhedgeable risk have doubled as well.

The welfare effects from risk-sharing in the setting with higher inflation volatility are shown in Figure 16 for different allocations of the IGR contract and different risk premia. For the

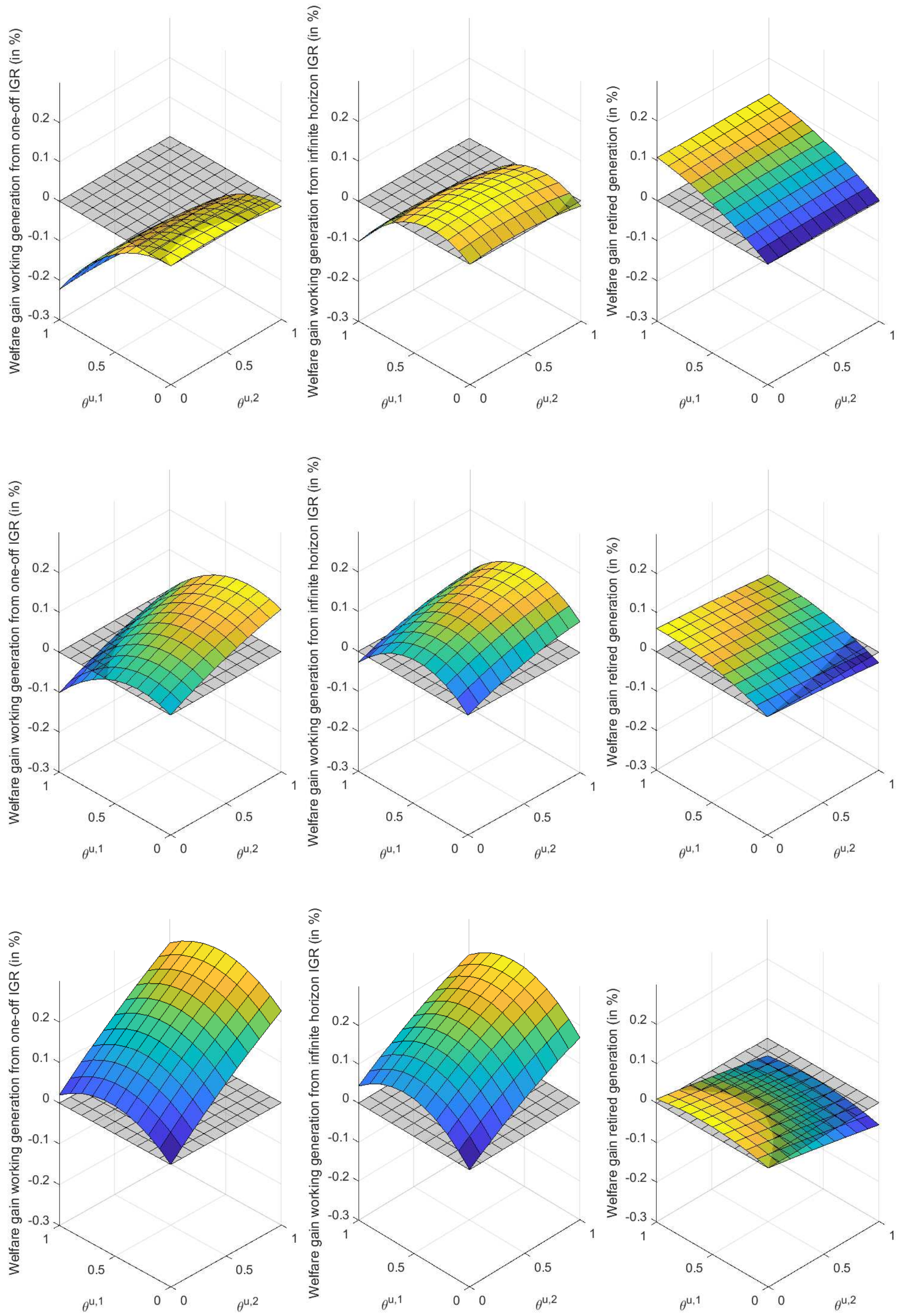

Figure 14: Welfare gain from intergenerational risk sharing for different allocations of IGR contract $\theta^{u,1}$ and $\theta^{u,2}$. Top graphs: $\mu_{rp} = 0\%$; middle graphs: $\mu_{rp} = 0.01\%$; bottom graphs: $\mu_{rp} = 0.02\%$.

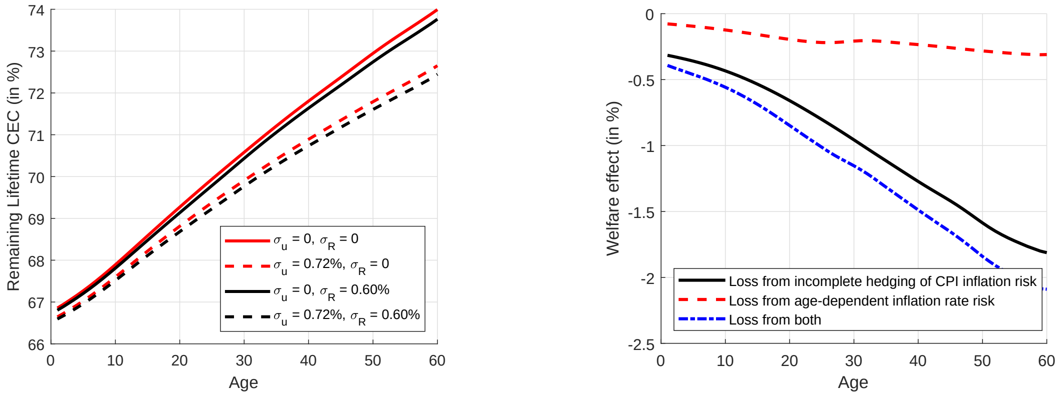

Figure 15: Remaining lifetime welfare in the setting with a doubling in the average level of inflation and its volatility. The left-hand panel depicts the welfare levels in terms of certainty equivalent consumption (CEC) for different settings for unhedgeable inflation. The right-hand panel depicts the welfare losses (calculated as a percentage fall in CEC) associated with the different settings for unhedgeable inflation, where the complete market setting is considered as the base case.

retirees the welfare gains have more than doubled. Consider for example the retirees' welfare gain of 0.33% with $\theta^u = 100\%$ and $\mu_{rp} = 0$, which is about three times the welfare gain under the initial parameter setting. For the working generation the optimal welfare gain is about the same, but it is obtained for a lower value for the degree of risk-sharing θ^u, since there is a trade-off between absorbing less risk of the retired when young (lower value of θ^u) versus shedding more risk after having transitioned into retirement (higher value of θ^u). Higher inflation volatility results in a higher probability of wealth and consumption getting close to zero, which with CRRA utility results in a high marginal utility of consumption. To limit the chances of this happening the working generation's optimal allocation θ^u of the risk-sharing contract falls when inflation volatility increases, while the total welfare gains from risk-sharing are preserved.

3.7.3 Three overlapping generations

In reality, working life is generally longer than the time spent in retirement. Hence, we now investigate a pension scheme with three overlapping generations, where generations work for $T^R = 40$ years and the retirement period is $T^D - T^R = 20$ years. In this setting every 20 years a new generation enters the pension scheme. Hence, there are 2 working generations and 1 retired generation at each moment in time. The intergenerational contract is between the retired generation and the youngest working generation only. The oldest working generation does not participate in the intergenerational risk sharing arrangement, because there is little benefit from absorbing the unhedgeable risk of the retired generation so close before one's own retirement. Also, since the remaining human capital is smaller, the risk absorption capacity is smaller.

Welfare under autarky is shown in Figure 17. Lifetime certainty equivalent consumption has increased from 0.697 to 0.866 compared to the benchmark setting, because wage income will be received over an extra ten years - see the left-hand panel of Figure 17. The welfare loss at retirement age remains about 0.5%, but the lifetime welfare loss from unhedgeable inflation risk shrinks from 0.16% to 0.09%, as the partial hedge from salary income is extended by ten more years - see the right-hand panel of Figure 17.

Figure 18 depicts the welfare gains from risk-sharing in this modified setting. The left-hand graph and the middle graph show the welfare gains of the working generation under, respectively, the one-off risk-sharing arrangement and the infinite horizon risk-sharing arrangement. Since

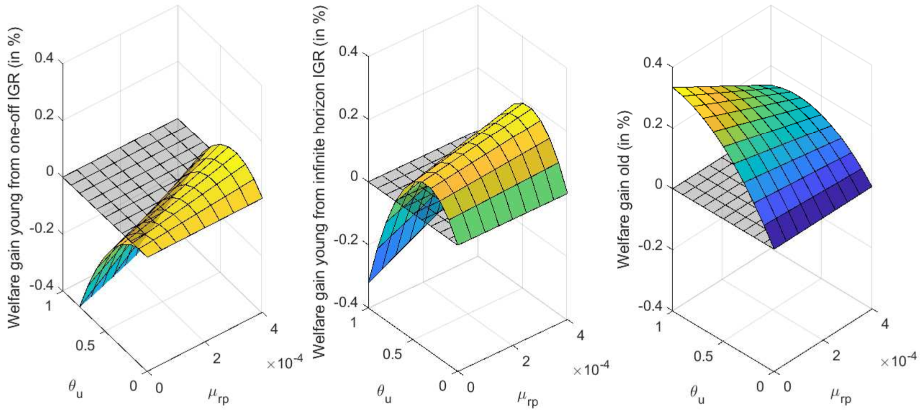

Figure 16: Welfare gain for different allocations of the IGR contract (θ^u) and different risk premia (μ_{rp}) in the setting with a higher average level of inflation and its volatility.

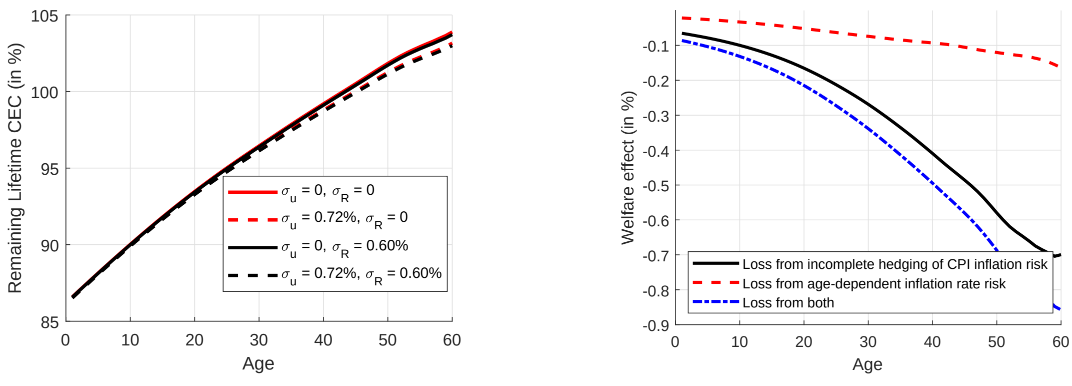

Figure 17: Remaining lifetime welfare in the setting with $T^R = 40$ years of working. The left-hand panel depicts the welfare levels in terms of certainty equivalent consumption (CEC) for different settings for unhedgeable inflation. The right-hand panel depicts the welfare losses (calculated as a percentage fall in CEC) associated with the different settings for unhedgeable inflation, where the complete market setting is considered as the base case.

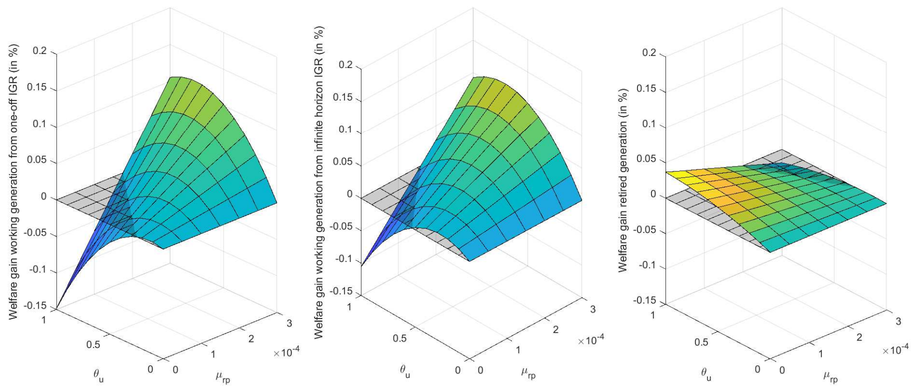

Figure 18: Welfare gain for different allocations of the IGR contract (θ^u) and different risk premia (μ_{rp}) in the 3 overlapping generation setting.

the welfare loss from unhedgeble risk is slightly lower, the welfare gains from risk-sharing are also slightly lower than for the setting with only two overlapping generation, but the general patterns are similar.

3.8 Age-dependent degree of risk-sharing

In order to investigate whether the risk-sharing arrangement can be improved further, we make parameter θ^u age-dependent. Therefore, we consider the following form:

$$\theta^u_{t,\tau} = \theta^u_{start}\frac{T^D - \tau + t}{T^D - T^R} + \theta^u_{end}\frac{\tau - t - T^R}{T^D - T^R}, \forall \tau \in \left(t + T^R, t + T^D\right). \tag{32}$$

Hence, at the retirement age $\tau - t = T^R$ the degree of risk-sharing is $\theta^u_{t,\tau} = \theta^u_{start}$ and at the age of death $\tau - t = T^D$ the degree of risk-sharing is $\theta^u_{t,\tau} = \theta^u_{end}$. During retirement the allocation linearly goes from θ^u_{start} to θ^u_{end}.

We start with the case of a zero risk premium, i.e. $\mu_{rp} = 0$, which is shown in the top line of Figure 19, followed by the introduction of a gradually increasing risk premium that benefits the working generation (second to fourth line in the figure). The results do not indicate that θ^u_{start} and θ^u_{end} should differ much from each other to obtain higher welfare gains. We can argue that when approaching retirement it is optimal for the working cohort to have relatively less risk-sharing with the retirees. Since the allocation θ^u represents the fraction of unhedgeable risk of the retired generation that is shared with the working generation, this already holds with a constant degree of risk-sharing, as the wealth of retirees that is subject to unhedgeable risk decreases and reaches zero at the age of death, while the wealth of the working generation increases. Table 3 shows that in most cases a virtually constant degree of risk-sharing would be optimal.

For a given risk premium (μ_{rp}) we can determine the optimal certainty equivalent social welfare measured by (30) for each intergenerational risk-sharing arrangement. Table 4 shows the corresponding welfare gains compared to autarky. The welfare gains with an age-dependent degree of risk-sharing do not indicate much improvement compared to benchmark case with constant degree of risk-sharing. However, varying risk sharing by source of risk (Section 3.6.3) can further improve the welfare gains, because with a positive risk premium, social welfare gains become twice as large with infinite horizon risk-sharing and three times as large with the one-off risk sharing arrangement compared to the benchmark case without varying the degree of risk sharing by source of risk. In the social optimal IGR arrangements all of the the age-depent

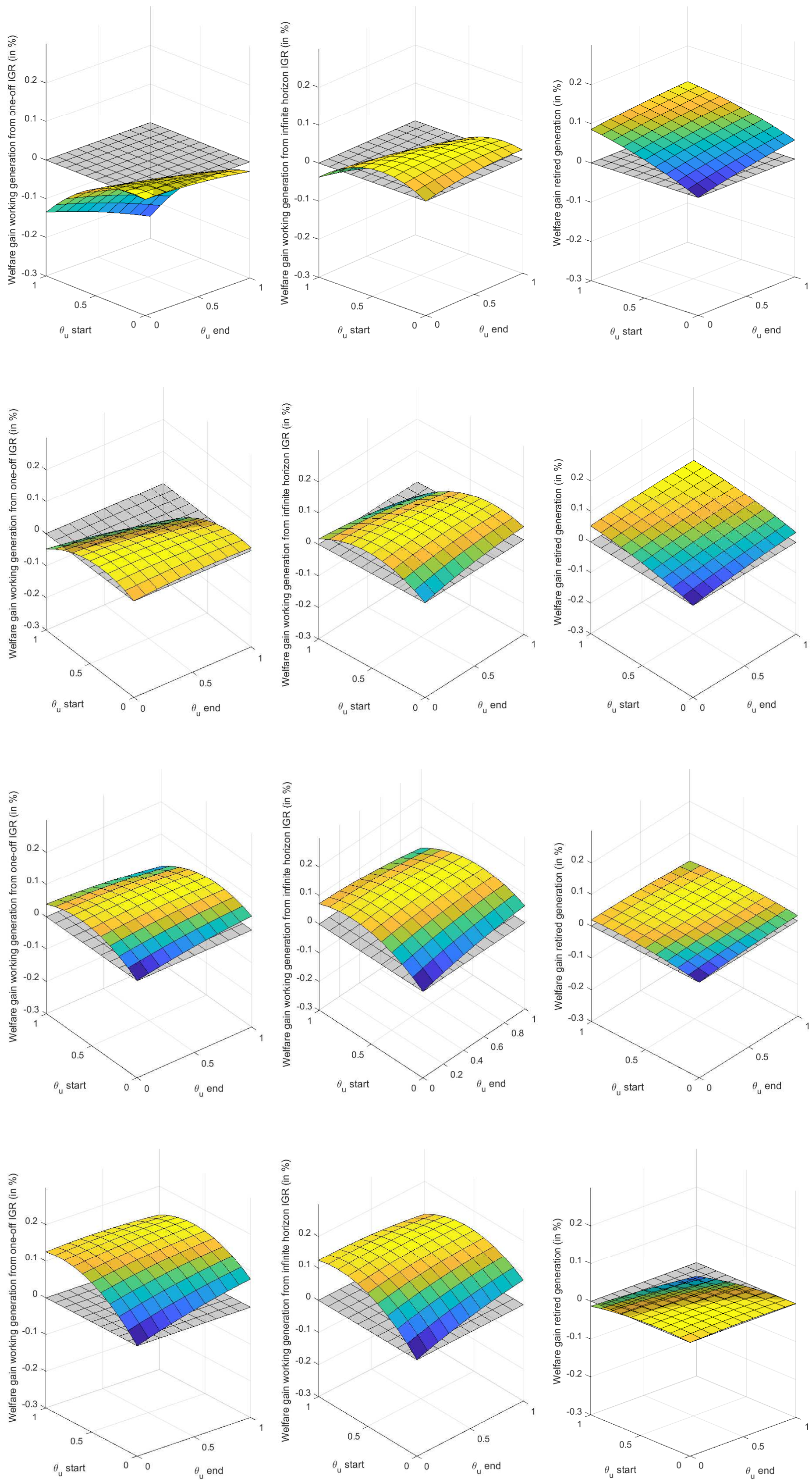

Figure 19: Welfare gain from intergenerational risk sharing for different degrees of risk-sharing over one's lifetime. Top line: $\mu_{rp} = 0\%$; second line: $\mu_{rp} = 0.01\%$; third line: $\mu_{rp} = 0.02\%$; bottom line: $\mu_{rp} = 0.03\%$.

Table 3: Optimal start and end allocations of the IGR contract. Values are rounded to the nearest 10%.

risk premium	working generation one-off IGR θ^u_{start}	θ^u_{end}	working generation infinite horizon IGR θ^u_{start}	θ^u_{end}	retired generation θ^u_{start}	θ^u_{end}
$\mu_{rp} = 0\%$	10%	0%	30%	40%	100%	100%
$\mu_{rp} = 0.01\%$	30%	20%	40%	50%	90%	90%
$\mu_{rp} = 0.02\%$	50%	60%	50%	60%	60%	60%
$\mu_{rp} = 0.03\%$	60%	100%	60%	70%	20%	20%

Table 4: Social welfare gain at the optimal degree of risk sharing in percent increase in certainty-equivalent consumption for the different risk-sharing arrangements compared to autarky

One-off risk-sharing arrangement	$\mu_{rp} = 0\%$	$\mu_{rp} = 0.01\%$	$\mu_{rp} = 0.02\%$	$\mu_{rp} = 0.03\%$
Benchmark (Section 3.6.2)	0.014	0.029	0.05	0.075
Varying risk sharing by source (Section 3.6.3)	0.014	0.072	0.145	0.224
Age-dependent degree of risk-sharing (Section 3.8)	0.014	0.029	0.05	0.076
Infinite horizon risk-sharing	$\mu_{rp} = 0\%$	$\mu_{rp} = 0.01\%$	$\mu_{rp} = 0.02\%$	$\mu_{rp} = 0.03\%$
Benchmark (Section 3.6.2)	0.048	0.057	0.067	0.077
Varying risk sharing by source (Section 3.6.3)	0.049	0.084	0.129	0.175
Age-dependent degree of risk-sharing (Section 3.8)	0.049	0.058	0.067	0.077

inflation risk is shared, while about half of the unhedgeable inflation risk of the overall price index is shared.

4 Stochastic real wage with productivity shocks

In this subsection we drop the assumption of a constant real wage, i.e. $h_{t,\tau} = 1$, allowing for a more realistic setting in which the nominal wage is not perfectly linked to the price index. Following Cocco et al. (2005) we assume that the real wage evolves as:

$$h_{t,\tau} = \begin{cases} \exp\left(g_{\tau-t} + \kappa_{t,\tau} + \nu_\tau\right), & \tau \in \left[t, t+T^R\right], \\ 0, & \tau \notin \left[t, t+T^R\right]. \end{cases} \tag{33}$$

Again t refers to the generation and τ to the period in time. Further, $g_{\tau-t}$ refers to the deterministic age component of the real wage profile, which typically increases with age due to career making. The processes $\kappa_{t,\tau}$ and ν_τ represent an idiosyncratic shock to generation t and a shock for all generations in period τ, respectively. We assume that the deterministic part is given by a polynomial function of age $\tau - t$:

$$g_{\tau-t} = \theta_0 + \theta_1 (\tau - t) + \theta_2 \frac{(\tau-t)^2}{10} + \theta_3 \frac{(\tau-t)^3}{100}. \tag{34}$$

We obtain the following estimates using data for the Netherlands on average income per age cohort of the working population (CBS, 2019)

$$\hat{\theta}_0 = 3.3634, \quad \hat{\theta}_1 = 0.0674, \quad \hat{\theta}_2 = -0.0271, \quad \hat{\theta}_3 = 0.0025. \tag{35}$$

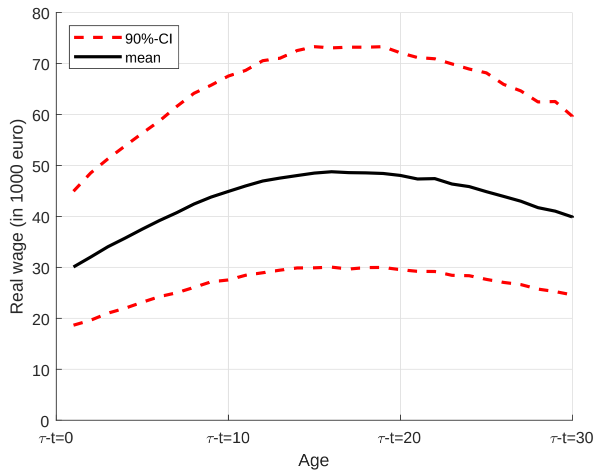

Figure 20: Stochastic real wage income

The idiosyncratic risk is modelled as

$$\kappa_{t,\tau} \sim N\left(0, \sigma_\kappa^2\right), \tag{36}$$

where we take the same value as in Cocco et al. (2005), which is $\sigma_\kappa^2 = 0.0738$. We can consider this idiosyncratic risk as a (labour) productivity shock. Figure 20 shows the estimated stochastic real wage income as a function of age for the case without unhedgeable risk and persistent risk in the real wage, i.e., $\sigma_u = 0$ and $\nu_t = 0$.

For the persistent shock of real labor income, we assume that ν_τ represents the part of unexpected inflation that is not captured by nominal wage arrangements:

$$\nu_\tau = -\left(1-\eta\right)\left(\sigma_h dZ_{h,\tau} + \sigma_u dZ_{u,\tau}\right) + \nu_{\tau-1}, \text{ with } \eta \in [0,1] \text{ and } \nu_0 = 0. \tag{37}$$

For $\eta = 1$ there is no persistent risk but only idiosyncratic real wage risk, i.e., $h_{t,\tau} = \exp\left(g_{\tau-t} + \kappa_{t,\tau}\right)$. For $\eta < 1$, there is nominal rigidity in wages, similar to the staggered nominal-wage contract first formulated by Taylor (1979), as part of the unexpected inflation (i.e. $\sigma_h dZ_{h,\tau} + \sigma_u dZ_{u,\tau}$) is not captured in the nominal wage contracts. Collective labour contracts are typically revised every two to four years and, therefore, unexpected inflation is only gradually made up for to protect the real wage. Nominal rigidity increases the market incompleteness of unhedgeable inflation risk, as the nominal wages become less of an implicit hedge.

4.1 Autarky

Figure 21 assumes persistent risk ($\eta = 0$). The welfare effects are similar to those presented in Figure 11; the welfare losses from both sources of unhedgeable risk combined are -0.15% over the entire lifetime and -0.52% at the retirement age (i.e. $T^R = 30$). For other degrees of persistence, for example $\eta = 0.5$ or $\eta = 1$, the corresponding welfare losses are almost the same.

4.2 Risk sharing

Figure 22 depicts the welfare gains for different risk sharing arrangements. The left-hand graph and the middle graph show the welfare gains of the working generation under, respectively, the one-off risk sharing arrangement and the infinite horizon risk sharing arrangement. In general the patterns are similar, but there are two main differences. First, introducing a risk premium

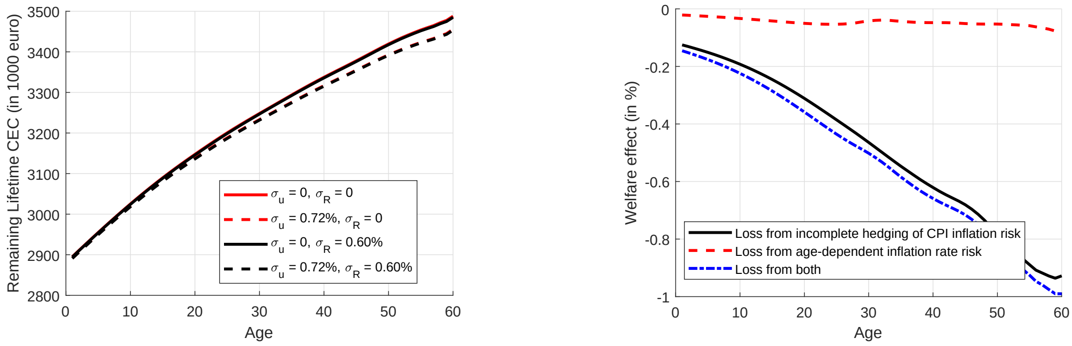

Figure 21: Remaining lifetime welfare with stochastic real wage and persistent risk ($\eta = 0$). The left-hand panel depicts the welfare levels in terms of certainty equivalent consumption (CEC) for different unhedgeable inflation settings. The right-hand panel depicts the welfare losses associated with the different sources of unhedgeable risk, where the complete market setting is considered as the base case.

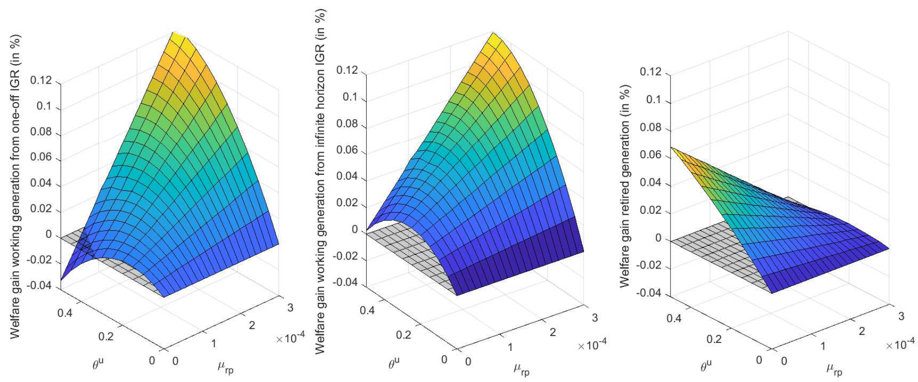

Figure 22: Welfare gain for different allocations of the IGR contract (θ^u) and different risk premia (μ_{rp}) in the stochastic real wage setting and persistent risk ($\eta = 0$).

has a stronger positive welfare effect for the working generation under the one-off risk sharing arrangement, as they only receive the risk premium, while under the infinite horizon arrangement they also have to pay the risk premium when they have retired themselves. Second, the welfare gains are larger under the infinite horizon risk-sharing arrangement, because in that setting the working generation also benefits form the protection against unhedgeble risk when they have retired. The right-hand graph shows the welfare gain from risk sharing of the current retired generation. Again, the risk premium (μ_{rp}) is welfare deteriorating for the retired generation.

The risk sharing arrangement is set up such that workers cover part of the unhedgeable inflation of the retirees. However, the former should also have the capacity to do so. As shown in Figure 20, the real wage is lowest early in working life. This might limit the degree to which the risks of the retirees can be hedged. Therefore, Figure 22 only presents the results for the allocation of the IGR contract $\theta^u \in [0.0, 0.5]$, because for values $\theta^u > 0.5$ the wealth of the youngest workers can become negative resulting in extremely low welfare. Hence, the optimal value of θ^u is lower than what we have obtained in Section 3. For workers the optimal one-off risk sharing allocation is in the range $\theta^u \in [0.15, 0.5]$ and the optimal infinite horizon risk sharing allocation is in the range $\theta^u \in [0.25, 0.5]$, depending on the risk premium.

The labour income of workers can absorb the unhedgeable inflation risk of the retirees to a lesser extent when there are persistent labour income shocks and when wage income is relatively low at the start of the working career. Hence, the welfare gains from the IGR contract are smaller in the presence of this more realistic representation of the real wage process than in the stylised case of a constant real wage considered in Section 3.6.2.[10]

5 Conclusion

We investigated several intergenerational risk sharing arrangements in a collective pension scheme aimed at reducing welfare losses from unhedgeable inflation risks. There were two sources of unhedgeable inflation risk: CPI inflation containing an unhedgeable component and age-dependent consumption bundles. Workers are better capable of absorbing both these risks, as their wage income is a natural hedge of inflation. Workers can benefit from the risk sharing arrangement by receiving a risk premium from the retirees and through an infinite horizon risk sharing contract by benefiting from shedding unhedgeable inflation risk when they have become retirees themselves. The welfare benefits from risk sharing are larger for higher levels of risk aversion. For a realistic real wage process the welfare benefits from the risk sharing arrangement are lower, because the amount of risk that can be shifted is limited by the earnings of individuals in their early working life. Still these benefits remain non-negligible.

With larger inflation volatility the welfare gains from intergenerational risk-sharing are larger. The downside of intergenerational risk-sharing is the risk of discontinuity, because when the transfer from one generation to the another becomes too large, members of the pension fund may become unwilling to participate. The intergenerational contract could then be teminated, for example under political pressure. Our model takes this implicitly into account, as the assumed utility function measures extremely low utility for consumption close to zero, which prevents contracts being optimal even with a small probability of such events. In practice, policy makers could prevent this by limiting the intergenerational transfers. For example, in the new Dutch pension contract the solidarity buffer that can be organized for intergenerational risk-sharing may not exceed 15% of the total assets of the pension fund. However, limiting the extent of risk-sharing also limits the welfare gains from risk-sharing. In our baseline analysis with transferring 100% of the unhedgeable risk from retirees to workers, the intergenerational transfers are in 90% of the simulations in the range of $[-1.2\%, +1.2\%]$ of the total assets of retirees having just retired. Since the wealth of the retirees decreases during retirement and the wealth of the working generation accumulates, this range shrinks to $[-0.07\%, +0.07\%]$ of the total assets of the pension fund during the last year of the IGR arrangement. Hence, we argue that with a buffer up to 15%, there is plenty of leeway to achieve intergenerational sharing of unhedgeable inflation risk.

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[10]We would expect the restriction on the above range for θ^u to become less relevant in a setting with a continuous inflow of new cohorts and outflow of the oldest cohorts.

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A Appendix

A.1 Solving method autarky

The Bellman equation is given by

$$V\left(w_{s,t},t\right)=\max_{\theta_t,c_t}u\left(c_{s,t}\right)dt+e^{-\rho dt}E_t\left[V\left(w_{s,t+dt},t+dt\right)\right]. \tag{38}$$

The corresponding first-order conditions with respect to the portfolio allocations are

$$0=\frac{\partial}{\partial\theta}e^{-\rho dt}E_t\left[V\left(w_{s,t+dt},t+dt\right)\right] \tag{39}$$

$$\iff 0=\frac{\partial}{\partial\theta}E_t\left[V\left\{\left(w_{s,t}-c_{s,t}\right)\left[1+\left(r-\bar{\pi}+\theta^S_\tau\left(\mu_S-r\right)+\left(1-\theta^R_\tau\right)\left(\pi_{h,t}+\sigma_h^2\right)+\sigma_u^2\right)d\tau+\right.\right.\right.$$
$$\left.\left.\left.\ldots\ \theta^S_\tau\sigma_S dZ_{S,\tau}-\left(1-\theta^R_\tau\right)\sigma_h dZ_{h,\tau}-\sigma_u dZ_{u,\tau}\right]+h_{t,\tau}dt,t+dt\right\}\right] \tag{40}$$

$$=\frac{\partial}{\partial\theta}E_t\left[V\left(\left(w_{s,t}-c_{s,t}\right)\left\{1+\left(r-\bar{\pi}\right)d\tau+\theta^S_\tau dX_S+\right.\right.\right. \tag{41}$$

$$\left.\left.\left.\ldots\ \left(1-\theta^R_\tau\right)\left(\pi_{h,t}+dX_h\right)+dX_u\right\}+h_{t,\tau}dt,t+dt\right)\right] \tag{42}$$

$$\iff 0=\begin{bmatrix} E_t\left[\left(w_{s,t}-c_{s,t}\right)dX_S V_w\left(w_{s,t+dt},t+dt\right)\right] \\ E_t\left[-\left(w_{s,t}-c_{s,t}\right)\left(\pi_{h,t}+dX_h\right)V_w\left(w_{s,t+dt},t+dt\right)\right]\end{bmatrix} \tag{43}$$

$$\text{with } dX_S=\left(\mu_S-r\right)d\tau+\sigma_S dZ_{S,\tau} \tag{44}$$

$$dX_i=\sigma_i^2 d\tau-\sigma_i dZ_{i,\tau},\quad i\in[h,u] \tag{45}$$

The first-order condition with respect to the optimal consumption is

$$c_{s,t}^{-\gamma}\frac{\Pi_t}{\Pi_{s,t}}=e^{-\rho dt}E_t\left[\left\{1+\left(r-\bar{\pi}\right)d\tau+\theta^S_\tau dX_S+\left(1-\theta^R_\tau\right)\left(\pi_{h,t}+dX_h\right)+dX_u\right\}V_w\left(w_{s,t+dt},t+dt\right)\right]. \tag{46}$$

The envelope theorem yields

$$V_w\left(w_{s,t+dt},t+dt\right)=\frac{\Pi_{t+dt}}{\Pi_{s,t+dt}}c_{s,t+dt}^{-\gamma} \tag{47}$$

implying the following first-order conditions

$$\begin{aligned}
0 &= \left[w_{s,t}-c_{s,t}\right]E_t\left[dX_S c_{s+1}^{-\gamma}\right] && (48)\\
0 &= -\left[w_{s,t}-c_{s,t}\right]E_t\left[dX_h c_{s+1}^{-\gamma}\right] && (49)\\
\frac{\Pi_t}{\Pi_{s,t+dt}}c_{s,t}^{-\gamma} &= e^{-\rho dt}E_t\left[\left\{1+\left(r-\bar{\pi}\right)d\tau+\theta_\tau^S dX_S+\left(1-\theta_\tau^R\right)\left(\pi_{h,t}+dX_h\right)+dX_u\right\}c_{s+1}^{-\gamma}\right]. && (50)
\end{aligned}$$

These we can solve numerically with backward recursion, using endogenuous gridpoints (Carroll, 2006).

A.2 Solving method IGR

The Bellman equation is again given by

$$V\left(w_{s,t},t\right)=\max_{\theta_t,c_t}u\left(c_{s,t}\right)dt+e^{-\rho dt}E_t\left[V\left(w_{s,t+dt},t+dt\right)\right]. \tag{51}$$

The first-order conditions with respect to the portfolio allocations are

$$\begin{aligned}
& 0=\frac{\partial}{\partial\theta}e^{-\rho dt}E_t\left[V\left(w_{s,t+dt},t+dt\right)\right] && (52)\\
\iff\ & 0=\frac{\partial}{\partial\theta}E_t\Big[V\Big\{\left(w_{s,t}-c_{s,t}\right)\Big[1+\Big(r-\bar{\pi}+\theta_\tau^S\left(\mu_S-r\right)+\left(1-\theta_\tau^R\right)\left(\pi_{h,t}+dX_h\right)+\sigma_u^2\Big)d\tau+\\
& \ldots\ \theta_\tau^S\sigma_S dZ_{S,\tau}-\left(1-\theta_\tau^R\right)\sigma_h dZ_{h,\tau}-\left(1-\theta^u\right)\sigma_u dZ_{u,\tau}-\theta^u\mu_{rp}d\tau-df_{t-T^R,\tau}\Big]+h_{t,\tau}dt,t+dt\Big\}\Big] && (53)\\
& =\frac{\partial}{\partial\theta}E_t\Big[V\Big\{\left(w_{s,t}-c_{s,t}\right)\Big[1+\left(r-\bar{\pi}\right)d\tau+\theta_\tau^S dX_S+\left(1-\theta_\tau^R\right)\left(\pi_{h,t}+dX_h\right)+\\
& \ldots\ dX_u+\theta^u\left(\sigma_u dZ_{u,\tau}-\mu_{rp}d\tau\right)\Big]-df_{t-T^R,\tau}+h_{t,\tau}dt,t+dt\Big\}\Big] && (54)\\
\iff\ & 0=\begin{bmatrix} E_t\left[\left(w_{s,t}-c_{s,t}\right)dX_S V_w\left(w_{s,t+dt},t+dt\right)\right]\\ E_t\left[-\left(w_{s,t}-c_{s,t}\right)\left(\pi_{h,t}+dX_h\right)V_w\left(w_{s,t+dt},t+dt\right)\right]\end{bmatrix}. && (55)
\end{aligned}$$

The remainder of the solving method is equivalent to the autarky case as shown in (46) to (50).

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