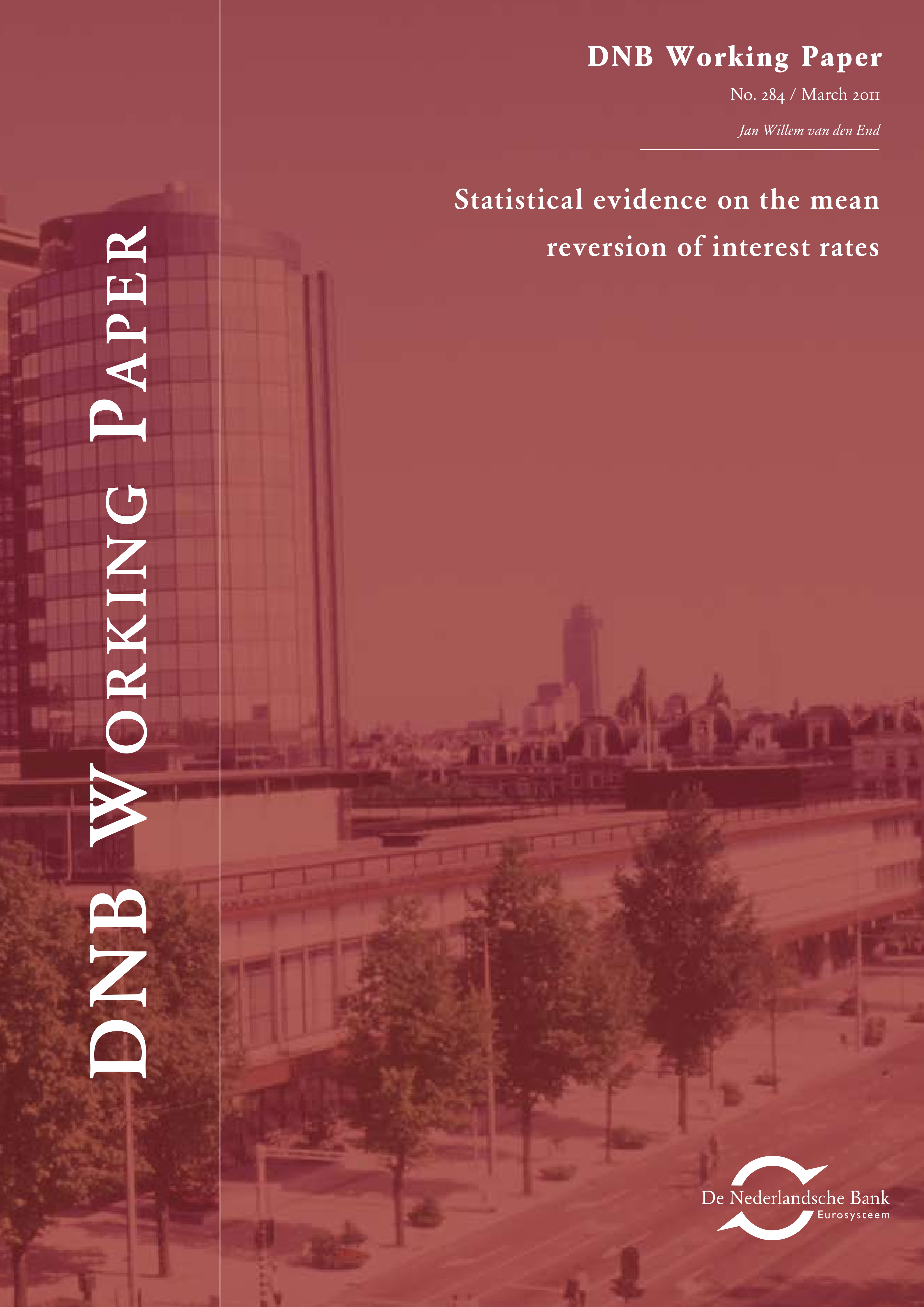

DNB Working Paper

No. 284 / March 2011

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DNB Working Paper

De Nederlandsche Bank
Eurosysteem

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* Views expressed are those of the author and do not necessarily reflect official positions of De Nederlandsche Bank.

Working Paper No. 284

March 2011

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Statistical evidence on the mean reversion of interest rates

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Abstract

Based on two hundred years of annual data of the Netherlands, Germany, US and Japan we analyse the mean reversion of long-term interest rates, by unit root tests over rolling windows and taking into account structural breaks and regime changes. While short-term rates and the yield curve tend to revert to their long-term average value, long-term rates can persistently deviate from it. At the outside, we only find weak statistical evidence for mean reversion of long-term rates. Outcomes of smooth transition autoregressive (STAR) models for long-term interest rates, indicate that the speed of mean reversion is regime dependent, being stronger when rates are far from their equilibrium value.

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Key words: interest rates, statistical methods, time-series models

JEL Codes: C22, C49, G12

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1. Introduction

Since the 1980's bond yields have trended downward and reached historic low levels in 2010. The 10-years nominal rate on German government bonds bottomed at 2% and US rates at 2.5% in October 2010, before edging slightly higher at the end of the year. Low interest rates are beneficial for households, businesses and governments, because they reduce their debt service. But there are also disadvantages. Low interest rates can create wrong incentives in the economy and foster financial imbalances. They make borrowing cheap and saving unattractive, and encourage market players to take greater financial risks, since low interest rates drive investors to higher yielding markets (Rajan, 2005). This search for yield is reinforced because low interest rates usually go hand in hand with rising asset prices (Borio and Zhu, 2008). Such market conditions were present before the recent credit crisis. The risk of another boom may seem far-off after three years of crisis, but it is precisely in a low interest rate environment that imbalances can develop, which will pose a threat to financial stability once interest rates rise again. Another disadvantage of a low interest rate environment is the pressure that it puts on institutional investors, like insurance companies and pension funds. They are confronted with low nominal returns on the fixed income portfolios and, even worse, with a higher mark-to-market value of their liabilities. Both factors squeeze their solvency position. The various challenges of low interest rates for the financial sector motivates our research question, which runs as three parts: i) how long will long-term interest rates remain low, ii) will they inevitably revert to their long-term equilibrium; in other words are long-term interest mean reverting or do they follow a random walk, and iii) how long would a mean reversion process take?

The research questions are addressed statistically, which means that we do not link interest rates explicitely to economic theory, but simply aim to model their empirical behaviour. The analysis is based on two hundred years of annual data of long-term interest rates of the Netherlands, Germany, US and Japan. We perform various unit root tests (taking into account structural breaks and regime changes) and conclude that mean reversion of long-term interest rates is not evident. Short-term rates and the yield curve have a stronger tendency to revert to their long-term average than long-term bond yields. The latter can persistently deviate from the long-run average. By using rolling windows and – more formally - by modelling long-term interest rates as a non-linear process, we find that the speed of mean reversion varies across time. In general, mean reversion is stronger when rates are either on a very high or on a very low level.

Our contribution to the literature is that we apply more recent and unconventional unit root tests to long-term bond yields, while existing studies commonly use conventional tests and concentrate on short-term interest rates. Moreover, we estimate non-linear smooth transition autoregressive (STAR) models for the interest rates of the four countries. STAR models have been widely used to model exchange rate behaviour but, as far as we know, only one other study applied this technique to interest rates (i.c. mortgage rates in Canada, by Liu, 2001).

The rest of the paper is organised as follows. Section 2 describes the findings in the literature on the mean reversion of interest rates. Section 3 presents the outcomes of various unit root tests. In section 4 we model the interest rate as a simple linear process and calculate its mean reverting half-life, while in section 5 we extent the analysis to non-linear models, to determine whether long-term rates behave differently in outer regimes. Section 6 concludes.

2. Mean reversion in the literature

2.1 Economic theory

According to economic theory it is plausible that interest rates are mean reverting, i.e that they revert to a long-term equilibrium level as time goes by. This level can be a based on fundamentals (relative mean reversion) or on an unspecified mean value (absolute mean reversion). Interest rates that are relative mean reverting reflect per capita economic growth, which is driven by technical progress in the long run. The real interest rate has a stable relation to that, according to growth theories (Barro and Sala-I-Martin, 2003). Empirical research indeed establishes a stable relationship between the real interest rate and economic growth (Lopez and Reyes, 2009). The nominal interest rate is also determined by inflation, which is volatile due to exogenous shocks in the economy, or due to monetary and fiscal policy measures. The Fisher equation says that inflation influences nominal rates, but not the real interest rate (Kwong, 2002). Therefore, it is more likely that real interest rates stay close to their equilibrium value than nominal interest rates.

Nonetheless, it is plausible that nominal interest rates are mean reverting as well, since the zero rate forms a lower bound, while central banks with monetary policy measures prevent that inflation (and thereby the interest rate) overshoots at the upside. Moreover, an excessively high interest rate would dampen economic growth, causing a cyclical decline of the interest rate. Hence, nominal rates will revert to the mean if a sufficiently long horizon is considered. For these reasons, mean reversion is included in many finance models for the interest rate. One of the earliest models is the Vasicek model (Vasicek, 1977), which describes the evolution of an unspecified short-term interest rate in a stochastic differential equation,

$$\Delta r_t = \alpha \ (\beta - r_t) \ dt \ + \sigma \, dW_t \tag{1}$$

where α, β and σ are non-negative constants and r_t is the current level of interest rate. Parameter β is the long run interest rate and α the adjustment parameter, which determines the speed of change towards β. In case of mean reversion $\alpha > 0$. This implies that if the interest rate is above the long run mean ($r_t > \beta$), the drift is negative so that the rate will be pushed closer to the level β on average. Likewise, if the rate is lower than the long run mean, ($r_t < \beta$), the drift is positive so that the rate will be pushed to the level β. Factor σdW_t represents market risk, which randomly influences the interest rate according to a Brownian motion (W_t represents a Wiener process of the random market risk factor and σ measures the volatility of the randomness).[1] While the Vasicek model applies to the evolution of the short-term interest rate, we will use the mean reversion theory of the model to analyse the evolution of long-term interest rates in section 4.

In structural macro models of central banks the short-term interest rate is commonly based on a Taylor rule, in which the policy rate is a function of the deviation from the inflation target and output gap (Taylor, 1993). Implicitly this assumes that the interest rate is mean reverting (if the output gap can be stabilised and the inflation target is constant). In central bank models, the long-term interest rate is commonly derived from the short-term interest rate (see for instance Christoffel et al., 2008). The bond yield, and thereby the yieldcurve, follows from expectations about the short-term interest rate plus for instance a premium for uncertainty or liquidity risk.

2.2 Statistical evidence

Although economic theory assumes that nominal interest rates are (in the long run) mean reverting, empirical research is not conclusive. The mainstream literature concludes that the unit root hypothesis can not be rejected with regard to long-term bond yields, which implies that they would not be mean reverting (Rose, 1998; Stock and Watson, 1988; Campell and Shiller, 1991; Siklos and Wohar, 1997). These studies are mostly based on conventional unit root tests, such as the Dickey Fuller test (Dickey and Fuller, 1979). Interest rates usually exhibit a strong persistence and it can take a long time before they revert to a long-term equilibrium level. This shows up in the poor statistical evidence of mean reversion. More recent literature investigates the unit root hypothesis by fractional integrated techniques that apply differencing to time series by an order smaller than or greater than one (Baum, 2000 and Gil-Alana, 2004). These studies find that shocks to interest rates have a long memory, which explains their (close to) random walk behaviour. Mean reversion of real interest rates is neither a foregone conclusion in the literature (Rose, 1988). It is commonly found that real interest rates are highly persistent as well, with real interest rates to stay above or below a long-term average level for a long time (Neely and Rapach, 2008).

Studies on short-term interest rates sometimes do find mean reversion (for instance Wu and Zhang, 1996 and Wu and Chen, 2001). They investigate the behaviour of money market rates in various countries, using alternative unit root tests. The studies assume that there is too little information to find mean reversion with conventional tests. Therefore statistical procedures are applied to enhance the power of the unit root tests. In the literature, the evidence of mean reversion is most clear with regard to the yield curve (see for instance Shiller et al., 1983 and Seo, 2003). A steep yield

[1] The Brownian motion is a continuous stochastic time process, with *dt* the time interval over which the interest rate moves.

curve implies that the short-term rate will increase, causing a flattening of the yield curve, by which it reverts back to an average level.

3. Testing the random walk hypothesis

3.1 Data

To test whether interest rates are mean reverting, we use annual data of long-term interest rates in the US, Japan, Germany and the Netherlands. We constructed series of interest rates with the longest possible history. Since there is no single consistent dataset of historic interest rates available for the four countries considered, we had to combine and link data from various sources, such as data collections from other studies and data series from central banks and statistical offices (see Table 1). So we obtained annual data of long-term interest rate series for the Netherlands and the US back to 1800; for Germany back to 1821 and for Japan back to 1930. Interest rates in the latter two countries are missing for several years (in Germany: 1915-18, 1922-23, 1940-47 and 1954-55 and in Japan: 1964-65). As the common definition of long-term interest rates we use the representative yield on long-term government bonds. This is usually the yield on the most liquid long-term bond market segment in a country. The maturity of the representative bond sometimes changed in the countries considered, which implies that our dataset is not entirely consistent in terms of maturities.[2] Since the 1960's, the representative bond yield usually is the 10 years rate. We paid heed to possible statistical breaks in the data series which could relate to the linking of interest rates from various sources with different maturities, by checking the change of interest rates in years where two different sources had to be linked. This does not point to worrying statistical breaks (interest rate changes in the years concerned remain within one standard deviation and/or are in line with rate changes in other countries).

Measured from 1800, the median of the long-term government bond yield of the Netherlands was lower compared to the other three countries, while the Dutch rate was also the least volatile (see Table 1[3]). Since the early 1980s, the Japanese interest rate has been lowest. The real interest rate - taken as the difference between the nominal long-term interest rate and realised cpi-inflation[4] - was

[2] For Netherlands from 1986 onward the 10-years rate is used, in 1950-1985 the rate on five longest maturing government bonds and in 1814-1949 the rate on perpetual government bonds. For Germany, from 1990 onward the 10-years rate is used, in 1960-1989 the 9 to 10+ years rate, in the period before we used rates taken from Bordo and Jonung (1987) and in 1815-1880 from Homer and Sylla (2005). For Japan from 1966 onward the 10-years rate is used, in 1930-1966 we used rates taken from Bordo and Jonung (1987). For the US from 1953 onward the 10-years rate is used, in 1800-1952 the 10-years constant maturity yield.

[3] We concentrate on the median of the 10-years government bond yield in stead of the average, to eliminate the influence of outliers.

[4] This method derives the ex-post real rate. The ex-ante real rate is the ex-post real rate plus the difference between the realised and expected inflation.

close to real GDP growth rates (Table 1)[5]. The high volatility of real interest rates, compared to nominal interest rates, relates to strong swings in inflation, which had negative peaks in the 19th century and was extremely high in the 1920's (foremost in Germany). The influence of inflation on real interest rates comes to the fore in the standard deviation of real rates, which is much higher in the whole sample period, due to years of deflation and hyperinflation, than in the 1950-2010 period, when extreme inflation regimes were absent (Table 1). The fluctuations of inflation did not entirely show up in nominal bond yields. Nonetheless, we find a close long-term relationship between nominal bond yields and inflation according to cointegration tests, except for Japan (Table 2).[6]

Figure 1 shows that three distinct episodes can be distinguished. Since the 1980's, bond yields have trended downward to an historical low level in 2010. This is the period of the Great Moderation, in which the increasing confidence in central banks to maintain price stability was reflected in declining inflation. Another driving factor was the globalisation trend, among which the increasing supply of cheap products from Asia, which kept inflation low. The period before (between the 1950's until 1981) was characterised by upward trending interest rates. This related to the Keynesian experiment and the effects of the oil crises on inflation. In the period preceding the interest rate 'bubble' of the past fifty years, graphical inspection does not reveal a clear trend in interest rates (a slight downward trend is only visible in US rates).

3.2 Unit root tests

To assess whether long-term nominal interest rates are mean reverting, or whether they follows a random walk, we conduct various unit root tests on the government bond rates of the Netherlands, US, Germany and Japan. In general, the outcomes do not lead to a clear conclusion (Table 2). First we apply Ng and Perron (NP) tests (Ng and Perron, 2001). They distinguish from conventional Dickey Fuller test by using Generalised Least Squared (GLS) so that serial correlation does not affect the asymptotic distribution of the test statistic. Furthermore, the NP test is based upon de-trended data, so that 'unknown' trends or structural breaks do not influence the test outcomes. This is important because the interest rate series may contain structural breaks due to regime shifts in monetary policy or fundamental changes in the functioning of financial markets. Both features are included in the NP test statistics MZ_t (a modified version of the Phillips-Perron test statistic, Phillips and Perron, 1988) and MP_T (a modified version of the ERS Point Optimal statistic, which takes into account structural breaks, Elliot et al., 1996).

According to the MZ_t statistic, the unit root hypothesis (H0: interest rate is random walk) can not be rejected, except for Germany, which suggests that in three countries there is no mean reversion, i.e. the interest rates are non-stationary in terms of levels. The outcomes of the NP tests on the first

[5] With the exception of Japan, which is caused by the difference in available interest rate and GDP data. The relatively high standard deviation of GDP growth in the countries is to a large extent explained by World War II.
[6] Because the cointegration coefficient is not equal to 1 by definition, it can not be concluded that the real interest rate is stationary.

differences of the long-term rates indicate that these are stationary (see Table 2). This implies that the long-term interest rates are I(1), except for the Netherlands where rates seem to be I(2). The outcomes of the MP_T statistic do not change the main conclusion. Hence, also when taking into account structural breaks, random walk behaviour of bond yields can not be rejected, except for Germany (Table 2). Connolly et al. (2007) come to a similar conclusion for US bond yields, by showing that the evidence of long memory remains strong for yields after accounting for potential structural changes.

It is possible that the long-term interest rate is mean reverting around local trends; i.e. is trend-stationary in particular periods. This is tested by the Ng and Perron method applied to the three distinct periods described in section 3.1. The outcomes in Table 2 indicate that there is some stronger statistical evidence for rejecting random walk behaviour when testing for the three periods separately. Especially in the interim period (1951-1981), a random walk is rejected for German and US interest rates at a 1% confidence level and in the last period (since 1982) a random walk is rejected for Dutch and Japanese interest rates at a 5% and 1% confidence level. Trend stationarity in sub-sample periods is also found by Perron (1989) for US bond yields in 1930-1945 and 1946-1970. He concludes that the random walk hypothesis cannot be rejected for the sub-sample before 1929, but can be rejected for the sub-samples thereafter.

Next, we test whether random walk behaviour changes across time, by investigating whether long-term interest rates are trend-stationary in certain periods and not in others. This is applied by conducting Ng and Perron tests based on 30-years (overlapping) rolling windows. According to the MZ_t statistic, in most of the 30-years periods the bond yields behave like a random walk (i.e. are not mean reverting), again except for Germany (Figures 2.a-d). In case of the Dutch interest rate, the test indicates mean reversion in 40% of all periods and in just 10% of the periods in the case of US rates (measured at 5% confidence interval).

Finally, the previous tests could have too little power to reject the random walk hypothesis because of a lack of information in the data (Verbeek, 2008). For that reason we apply an alternative test which takes mean reversion as the null hypothesis (Kwiatkowski et al., 1992[7]). This so called KPSS test applied to the Netherlands and US - the two countries with the longest historic data series - indicates that mean reversion of long-term rates can not be rejected (Table 2). To draw this conclusion, a sufficiently long data series is required. Figures 3.a-b show that only after 120 to 160 annual observations the hypothesis that the interest rate is mean reverting is accepted at a 5% confidence interval. During shorter time spans the interest rate can overshoot or undershoot, for instance in periods with high or low inflation or imbalanced government finances. The outcomes resembles the

[7] The KPSS unit root test takes the null hypothesis of stationarity against the unit root as alternative. The idea behind the test is that a time series can be broken down in a deterministic trend, a random walk component and a stationary residual term. The null hypothesis (i.e. trend stationarity) says that the variance of the random walk component is equal to zero. It is a Lagrange Multiplier test, with a 5% critical value for the test parameter equal to 0.463 (and 0.347 at a 10% confidence interval). The test is asymptotic valid, which means that the critical boundaries are independent of the length of the time series.

findings by Gil-Alana (2004), who concludes from unit root tests on fractional integrated annual long-term US interest rates that a unit root seems plausible for the 1940-2000 period, but that mean reverting behaviour is evident when using a longer span of the data (i.e. 1798-2000 period).

4. Linear approximations of mean reversion

The Vasicek model presented in section 2.1 could also give an indication on the strength of mean reversion of interest rates and how long this process would take. We estimate an alternative version of this linear model,

$$\Delta r_t = c + \alpha \, (\beta - r_t) \, dt + \varepsilon_t \tag{2}$$

in which the adjustment parameter α measures the speed of mean reversion of interest rate r_t in one year time. As long-term equilibrium rate β we use the median of the long-term bond yield, since the average is influenced by outliers. The median rate β is a constant, measured over the whole sample period.[8]

The estimation results in Table 3 suggests that there is mean reversion in long-term interest rates (α is positive), but that this effect is not significant (α is not significantly different from zero, except for Germany). This reflects that mean reversion is a slow process. The estimated adjustment coefficient for the Netherlands implies that if the interest rate would deviate 100 basis points from its long-term level, each year 4.5 basis points of this deviation is adjusted on average. This means that in 15 years, half of the deviation is corrected (half-life). A comparable half-life is found if we estimate equation 2 for the panel of four countries together (Table 3).

The adjustment coefficient is also estimated on the basis of 30 years (overlapping) rolling windows[9], to assess whether the speed of adjustment changes across time. Studies that look at mean reversion of stock returns find that in periods with high financial tensions (due to wars, crises) mean reversion is stronger than in normal market conditions (Spierdijk et al., 2010). Using rolling windows also takes into account possible regime shifts in capital markets or monetary policy that could distort the outcomes.

Figures 4.a-d show that in most time periods long-term interest rates tend to revert to their long-term median levels (parameter α is positive), although this adjustment process is not statistically

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[8] The model has also been estimated with median β measured over 30 years rolling windows, but this does not lead to materially different outcomes compared to estimates with a constant parameter β (only for the US the adjustment coefficient is somewhat higher).

[9] Studies that look at mean reversion of stock returns also apply a window of about 30 years (see for instance Spierdijk et al., 2010). This window is chosen because a shorter period leaves less degrees of freedom for regression analysis and has more volatility, while a much longer period smoothes the outcomes too much.

significant in most periods. Occasionally the interest rate moves further away from this level ($\alpha < 0$), which implies mean aversion in stead of mean reversion. That happened at the beginning of the 1920's, when interest rates shot up. Graphical inspection suggests that the interest rate tends to revert to its mean if it is on a very high or on a very low level. Years in which bond yields deviate widely from their long-term median level are sometimes followed by mean reversion in subsequent years (see Figures 4.a-d; the adjustment coefficient α occasionally rises after the shaded time periods, such as after the 1980's).

The yield curve (long-term bond yield minus short-term money market rate[10]) exhibits more convincing mean reversion than the long-term interest rate (Figures 5.a-d). Estimations of the adjustment coefficient α suggest that if the curve would be 100 basis points steeper or flatter than its long-term level, each year 40 basis points are corrected on average. This relatively high speed of mean reversion is due to the mean reversion of short-term interest rates which, differently from long-term rates, is significant in most periods.[11] Although short and long terms interest rates in Germany and Netherlands are cointegrated (see Table 2), the rates have a different short-term dynamics. Hence, adjustments of the deviations between short and long-term interest rates (the yield spread) are predominantly driven by changes in the short-term rate.[12] A similar result is found by Campbell and Shiller (1991), who conclude from a model of the interest rate term structure that variations in the long-short spread were due primarily to sudden movements in short rates. The finding that the short-term rate tends to revert quicker to its equilibrium rate than the long-term rate could be explained by monetary policy measures, through which central banks combat inflation that under or overshoots the target rate.

[10] Short-term interest rates are available since 1814 for the Netherlands, since 1870 for the US, since 1880 for Germany and since 1946 for Japan. The short-term interest rates is the representative money market rate or, if that is not available, the official policy rate.

[11] The adjustment coefficient α in Equation 3 of the short-term rate in the Netherlands is 0.17 on average, which implies a half-life of 3.7 years.

[12] Estimations of a Vector Error Correction (VEC) model, including a long-term relationship between short-term and long-term rates, confirm that adjustments in the deviation between both rates are primarily driven by changes in the short-term rate. Short-term rates of the Netherlands and Germany are stationary, according to the Ng Perron unit root test; US and Japanese short-term rates are not.

5. Regime switching models

5.1 Model classes

The observation in the previous section, that long-term interest rates tend to revert quicker to the median from extreme high or low levels, suggests that rates adjust in a non-linear way. Such behaviour can be explored by regime switching models, in which the dynamics of a variable depend on being in an inner or outer regime. It assumes that mean reversion is more likely in the outer regime, while in the inner regime the interest rate can move randomly. A prominent class of regime switching models for financial market prices are Threshold Autoregressive Models (TAR) models (see Franses and Van Dijk, 2000). These models discriminate between large and small deviations, by which mean reversion depends on the distance of a variable from an equilibrium value. A regime is determined by a value of the transition variable relative to a threshold value. If the transition variable is a lagged value of the time series itself, the model is a Self-Exciting TAR (SETAR). Relaxing the rather strict assumption of a discrete transition from one regime to the other argues for applying a smooth transition autoregressive (STAR) model, which is a two-regime SETAR model with smooth transition between regimes, and a continuously varying and bounded adjustment speed. Exponential STAR (ESTAR) models are widely used in exchange rate studies (see for instance Schnatz, 2007). ESTAR models assume two (an upper and lower) thresholds and symmetric adjustments of positive and negative deviations from equilibrium. To investigate non-linearity in the mean reversion of interest rates, we estimate ESTAR models for the long-term interest rate of the Netherlands, Germany, US and Japan. We opt for this model since it is likely that interest rates change smoothly between high and low level regimes, as do exchange rates. As far as we know there is only one other study that applies STAR models to interest rates; Liu (2001) estimates such a model for Canadian mortgage rates. Other studies have applied Markov-switching models to interest rates to capture regime shifts and non-linear behaviour (Hamilton, 1989; Gray, 1996; Chua and Suardi, 2007). In this class of models the volatility is assumed to be regime dependent. This approach is commonly applied to short-term money market rates, which are usually more volatile than long-term interest rates.

5.2 Estimation procedure

Following the procedure proposed by Teräsvirta (1994) and Franses and Van Dijk (2000) for estimating STAR models, we first specify linear AR models of order *p*, for the first differences of long-term interest rates,

$$\Delta r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 \Delta r_{t-1} + \ldots + \beta_p \Delta r_{t-p} + \varepsilon_t \qquad (3)$$

The lag length p is determined by the common outcome of different information criteria (Akaike, Schwarz and Hannan-Quinn information criterion). Taking into account the heteroscedasticity in the residuals, the models are estimated with heteroscedasticity and autocorrelation consistent covariances (HAC). The estimation results in Table 4 show that the autoregressive terms are statistically significant at different lag levels p for the four countries considered. The linear models have low explanatory power and the parameters of the 'level variable' r_t are close to zero and least significant, in line with the outcomes in section 3 that long-term interest rates are a (near) random walk.

The second step involves testing for the presence of non-linearities in the adjustment of long-term interest rates. This is achieved by performing the following auxiliary regression (which is the third order Taylor expansion of $F(.)$ in equation 6, see Teräsvirta, 1994),

$$\Delta r_t = \beta_0 + \beta_1' x_t + \beta_2' x_t q_{t-d} + \beta_3' x_t q^2_{t-d} + \beta_4' x_t q^3_{t-d} + \eta_t \tag{4}$$

with x_t the regressors from equation (3), i.e. $(\Delta r_t, \ldots, \Delta r_{t-p})'$ and $\beta_j = (\beta_{1,j}, \ldots, \beta_{p,j})'$, j = 1,...,4. Rejecting that the non-linear terms β_2 to β_4 are (jointly) zero would support the presence of remaining non-linearities in the AR models. The regression includes the transition variable q_t which is a lagged value of the time series itself (r_t), as in the SETAR model. Parameter d implies that the rise in mean reversion occurs with a delay. As the delay d is unknown it is commonly tested for plausible values. The null hypothesis of linearity is rejected at different values of d in case of the Netherlands, Germany and the US; only for Japan a linear model specification seems appropriate (Table 5).

5.3 Estimation results

Based on the results in the previous section, we estimate STAR models for long-term nominal rates of the Netherlands, US, Germany and Japan.

$$\Delta r_t = \beta_0 + \beta' x_t + (\theta' x_t)\, F(q_{t-d}, \gamma, c) + \varepsilon_t \tag{5}$$

with β' and θ' the vectors of coefficients, x_t the vector of explanatory variables, i.e. the lag of r_t and lags of Δr_t, $F(.)$ the bounded transition function that ranges between 0 and 1, parameter γ indicating the smoothness of the transition from one regime to the other, q_{t-d} the transition variable (the level of the long-term interest rate lagged d periods) and c the threshold value. When $F(.)$ = 0 the change of the interest rate is only explained by the linear part of the model, i.e. $\beta' x_t$. When the adjustment of the interest rate is very fast then $F(.)$ converges to 1. In the ESTAR model $F(.)$ is an U-shaped exponential function,

$$F(.) = (1 - exp\{-\gamma [q_{t-d} - c]\}^2) \quad , \gamma > 0 \tag{6}$$

When estimating the ESTAR model, the delay parameter *d* is determined by the lowest p-values in Table 5 (*d*=2 for the US and *d*=1 for the other countries), as suggested by Schnatz (2007). The model is estimated with both a fixed threshold value (*c*) and a time-varying threshold, as in Liu (2001). In the time-varying model the threshold *c* is not estimated but replaced by the *t* year backward moving average of the transition variable (i.e. long-term interest rate). The length of the moving average window is based on a grid search of *t* that renders the best model fit and it turns out that *t* = 8 years is most appropriate for the four countries countries considered. The advantage of the time-varying threshold is that it takes into account shifts in the equilibrium interest rate, which for instance could relate to changes in the monetary policy regime.

The ESTAR models are estimated by non-linear least-squares, imposing the standard restrictions that $\theta_{2..5} = - \beta_{2..5}$ and $\theta_1 = - (1 + \beta_1)$, which implies that the process of Δr_t in the outer regime, when *F(.)* = 1, is white noise (see Michael et al, 1997). The results in Table 6 present the restricted estimates. Comparing the R-squared statistics of Table 6 with those in Table 4 indicates that the non-linear models have higher explanatory power than the linear AR models. It confirms the value added of taking into account the (different) behaviour of interest rates in the outer regimes. As indicated by the t-values of the estimators (including γ) and the R squared, the fixed threshold model has a lower fit than the time-varying model (except for Japan, but we concluded that the linear specification is more appropriate than the ESTAR model for this country). Therefore the time-varying specification is our preferred model. Table 6 shows that the smoothness parameter γ is significant in most cases in the time-varying model (except for Japan), but the estimated coefficient has low values, indicative of the slow adjustment of long-term interest rates. According to the time-varying model, the adjustment speed is relatively highest for Dutch rates and lowest for Japanese rates (Figure 6). The Figure depicts the outcomes of the transition function *F(.)* and shows that a deviation of the Dutch interest rate by 3 percentage points from its moving average raises the transition function to 0.6. The historic pattern of the transition function confirms that in episodes when interest rates moved far away from their moving averages, the adjustment was stronger (Figures 7.a-d). The adjustment speed peaked in the 1980's (when interest rates were on historic high levels) and in the 1930's in Germany. The outcomes are in line with Liu (2001), who find that mortgage interest rate changes are relatively more significant in response to large than to small deviations of the equilibrium value.

5.4 Half-lives

To express the adjustment speed resulting from non-linear models in half-life values, Generalised Impulse Response Functions (GIRF) are commonly used, see Franses and Van Dijk (2000). The GIRF is defined as the average difference between two realizations of the stochastic process ($\Delta r_{t=h}$) which start with identical histories up to time t-1 (ω_{t-1}, initial conditions). The first realization is hit by an arbitrary shock $\varepsilon_t = \delta$ at time t, while the other one is not,

$$\mathrm{GIRF}_{\Delta r}(h, \delta, \omega_{t-1}) = E[\, \Delta r_{t+h} \mid \varepsilon_t = \delta, \omega_{t-1} \,] - E[\, \Delta r_{t+h} \mid \omega_{t-1} \,] \qquad (7)$$

Given that δ and ω_{t-1} are single realizations of random variables, the GIRF itself is a random variable. In equation (7) the expectation of Δr_t is conditional only with respect to the shock and the history; all shocks that might occur in intermediate periods are, in effect, averaged out. To differentiate between high and low interest rate (outer) regimes, we consider subsets of shocks and histories, defined as *S* and *H*, respectively, so that the conditional GIRF is given by $\mathrm{GIRF}_{\Delta r}$ (h, S, H). This property is useful to associate histories with a high interest rate with positive shocks and low interest regimes with negative shocks (as suggested by Holt and Graig, 2006). Values of the normalized initial shock are set equal to $\delta / \sigma_\varepsilon = \pm 3, 2, 1$, where σ_ε is the estimated standard deviation of the residuals from the STAR models. The different values of δ allow for comparing the persistence of large and small shocks. Table 8 reports the half-lives of shocks according to the time-varying ESTAR model, that is, the time needed for the cumulative $\mathrm{GIRF}_{\Delta r} < ½\ \delta$[13]. The Table shows that in most cases the half-life is shorter for large shocks than for small shocks, which is most obvious for Dutch interest rates (see also Figures 8.a-h as illustrations of the GIRF). Hence, the GIRF confirms that mean reversion of interest rates is regime dependent; i.e. it is more likely that rates revert to their average value if they are far from their equilibrium value. This result concurs with outcomes of Markov-switching models for short-term interest rate in other studies, which show that the rate reverts to its long-run mean when the levels and volatility are high, but behaves like a random walk process during periods when the interest rate level and volatility are low (Chua and Suardi, 2007).

6. Conclusion

The statistical analysis in the paper, based on two hundred years of interest rate data of the Netherlands, Germany, US and Japan, indicates that mean reversion of long-term interest rates is not evident from standard statistical tests. Short-term rates and yield curves have a stronger tendency to revert to their long-term average than long-term bond yields. The latter can persistently deviate from their average values. Mean reversion of Dutch and US long-term rates can only be accepted if sufficiently long data series are used. Hence, a main conclusion from various unit root tests, conducted over rolling windows and taking into account structural breaks and regime changes is that at the outside, only weak statistical evidence for mean reversion of long-term rates is present.

[13] The impulse responses for the level of the interest rate are constructed by cumulating the impulse responses for the first differences, that is $\mathrm{GIRF}_r = \Sigma^{h}_{j=o}\mathrm{GIRF}_{\Delta r.}$

The paper also analysed whether mean reversion is more likely in outer regimes, while allowing for random walk behaviour in inner regimes. Such non-linear behaviour is modelled for long-term interest rates by smooth transition autoregressive (STAR) models, with a time-varying moving average interest rate as threshold value. The estimation results indicate that the speed of mean reversion varies across time. In general, mean reversion is stronger when rates are far from their long-run average value. Simulations of the half-lives of the adjustment process indicate that the adjustment speed of Dutch and German interest rates is higher than of US and Japanese rates.

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ANNEX

Table 1. Descriptive statistics

Percentages based on annual data 1800-2010 (DE from 1821, Japan from 1930)

	Median				Stdev			
1800-2010	NL	DE	US	JP	NL	DE	US	JP
Long-term interest rate	4,3	4,4	5,0	5,5	1,6	1,7	2,0	2,2
Real interest rate[1]	3,0	3,1	3,3	1,7	6,3	6,2E+9	5,7	44,2
Real GDP-growth[2]	2,8	2,9	3,5	2,9	7,5	6,8	5,4	6,8
1950-2010								
Long-term interest rate	6,2	6,4	5,7	6,3	2,1	1,7	2,8	2,4
Real interest rate[1]	2,7	3,7	2,3	2,1	3,1	2,0	2,4	3,6
Real GDP-growth[2]	3,1	3,0	3,4	3,0	2,4	3,3	2,4	4,2

	Maximum				Minimum			
1800-2010	NL	DE	US	JP	NL	DE	US	JP
Long-term interest rate	11,6	10,4	13,9	8,9	2,6	2,5	1,7	1,0
Real interest rate[1]	27,2	110,0	21,7	24,3	-18,4	-85,5E+9	-20,0	-313,2
1950-2010								
Long-term interest rate	11,6	10,4	13,9	8,9	2,6	2,5	2,2	1,0
Real interest rate[1]	7,1	12,0	8,1	7,3	-11,7	-1,7	-5,4	-15,0

[1] Real rate is difference between nominal GDP-growth and realised cpi inflation;
the extreme values for DE relate to the hyperinflation in the 1920's and 1930's.

[2] Real GDP-growth, data available from 1870 for NL, from 1852 for DE, from 1871 for JP and US.

Sources: NL: Eurostat, DNB, CBS. DE: Eurostat, Bordo & Jonung, Homer & Sylla.
US: Federal Reserve, Global Financial Data. JP: BoJ, Bordo & Jonung.

Figure 1. Long-term interest rates

Percent, representative government bond rate

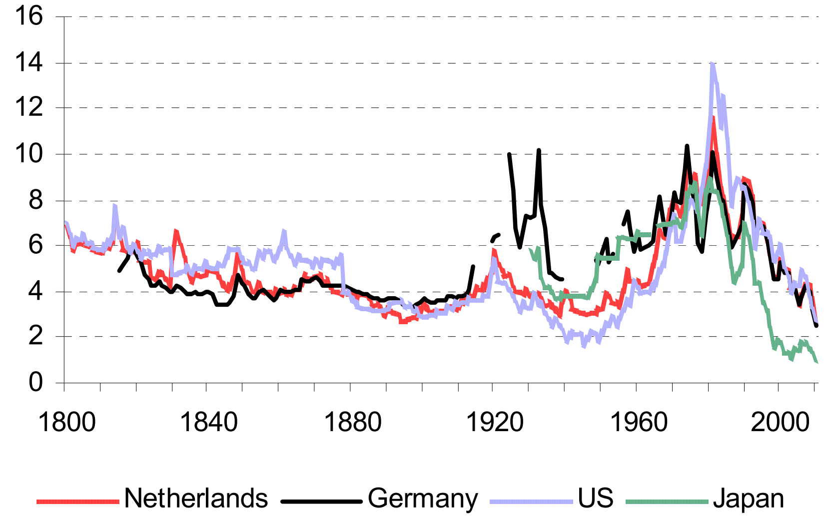


Table 2. Statistical tests

	NL	DE	US	JP
Cointegration test (long-term rate, inflation)[1]	37.0***	35.0***	26.0***	11.7
Cointegration test (long-term rate, ST rate)[1]	23.7***	19.7**	8.8	6.9
NP MZ_t unit root test (long-term rate)[2]	-1.25	-2.59***	-1.26	-0.38
NP MP_t unit root test (long-term rate)[2]	6.15	2.01**	-6.26	13.42
NP MZ_t unit root test (Δ long-term rate)[2]	-0.47	-6.17***	-3.71***	-4.23***
NP MZ_t unit root test (long-term rate)				
period 1800-1950[3]	-2.40	-1.78	-3.04**	-1.41
period 1951-1981[3]	1.11	-3.73***	-4.17***	-2.53
period 1982-2010[3]	-3.06**	-1.92	-2.13	-2.63*
KPSS unit root test (long-term rate)[4]	0.31	0.78***	0.23	0.35*

[1] Trace statistic, Johanson test for cointegration (constant, no trend),
H0: no cointegration. *** H0 rejected at 1% confidence level, ** 5%, * 10%.
[2] Ng Perron test-statistic (constant, no trend), H0: interest rate has unit root.
[3] Ng Perron test-statistic (test with constant and trend), H0: interest rate has unit root.
[4] LM-statistic Kwiatkowski-Phillips-Schmidt-Shin test, H0: interest rate is stationary,
*** H0 rejected at 1% confidence level, ** 5%, * 10%.
The number of lags is determined by information criteria.

Figure 2.a. Unit root test, long-term rate Netherlands
NP MZt test statistic, unit root test (incl trend), 30 yrs rolling window

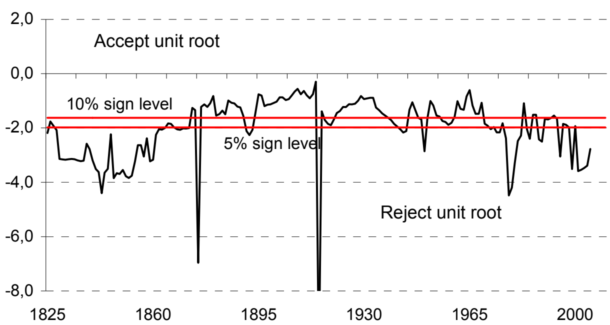


Figure 2.b. Unit root test, long-term rate US
NP MZt test statistic, unit root test (incl trend), 30 yrs rolling window

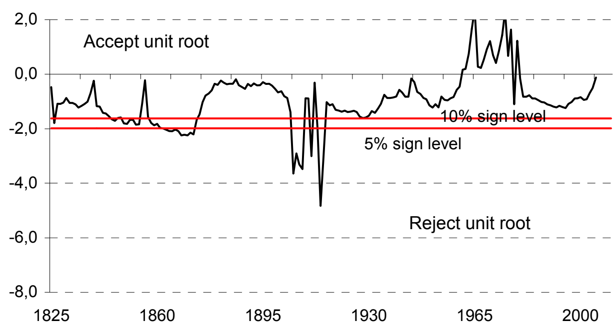


Figure 2.c. Unit root test, long-term rate Germany
NP MZt test statistic, unit root test (incl trend), 30 yrs rolling window

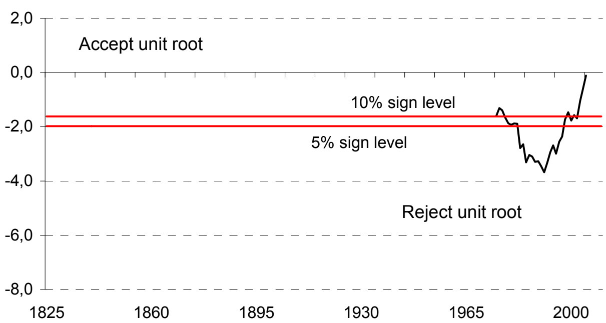


Figure 2.d. Unit root test, long-term rate Japan
NP MZt test statistic, unit root test (incl trend), 30 yrs rolling window

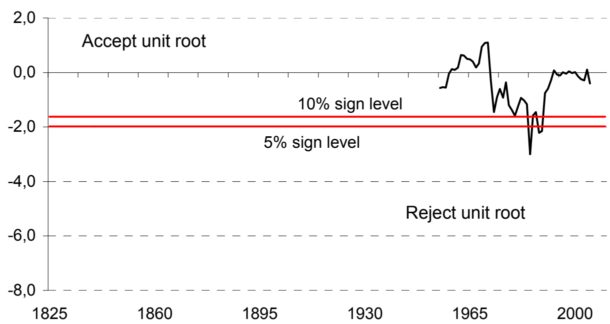


Figure 3.a. KPSS test, long-term rate Netherlands
LM statistic at increasing number of observations

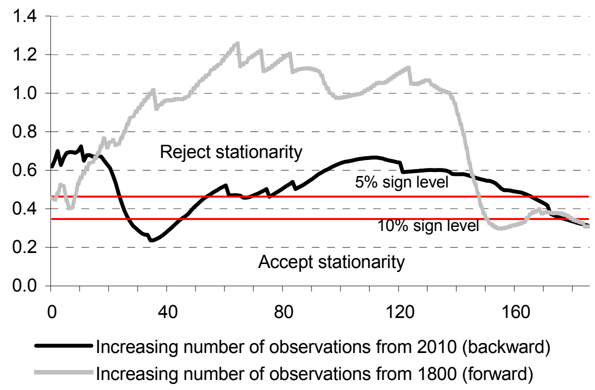


Figure 3.b. KPSS test, long-term rate US
LM statistic at increasing number of observations

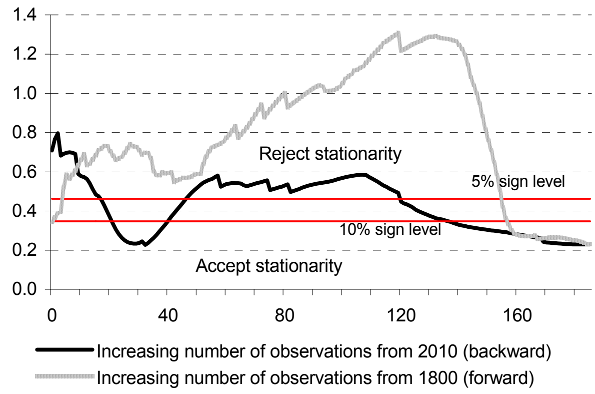


Table 3. Estmation results mean reversion model

Outcomes Equation 2 for adjustment parameter, based on annual data 1800-2010 (DE from 1821, Japan from 1930)

	Coefficient[1] α	Confidence interval upper-, lower boundary 95%	Half-life[3] in years
Netherlands	0,045	(-0.017, 0.107)	15,1
Germany	0,091	(0.016, 0.166)	7,3
US	0,038	(-0.044, 0.121)	17,9
Japan	0,015	(-0.038, 0.067)	45,9
Panel, 4 countries[2]	0,048	(-0.010, 0.107)	14,1

[1] Outcomes OLS regression with HAC conistent covariances (Newey-West).

[2] Outcomes panel estimations with White robust covariances.

[3] Half-life is calculated as log (0.5) / log (1-α), as in Spierdijk et al., 2010.

Figure 4.a. Adjustment speed long-term rate NL
Coefficient α and confidence interval, 10-years rate

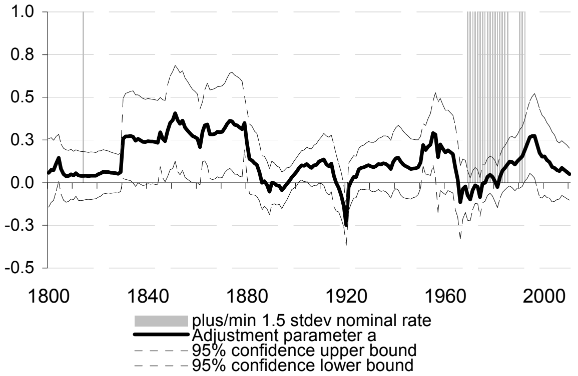


Figure 4.b. Adjustment speed long-term rate DE
Coefficient α and confidence interval, 10-years rate

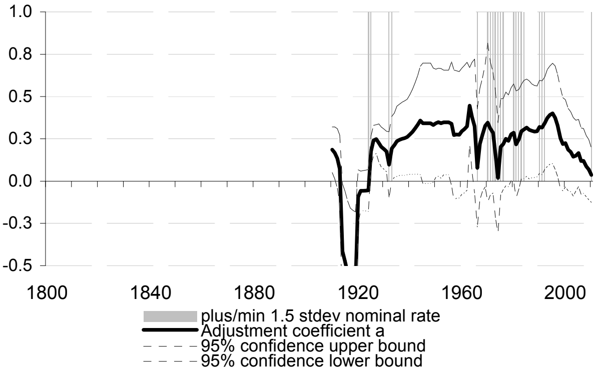


Figure 4.c. Adjustment speed long-term rate US
Coefficient α and confidence interval, 10-years rate

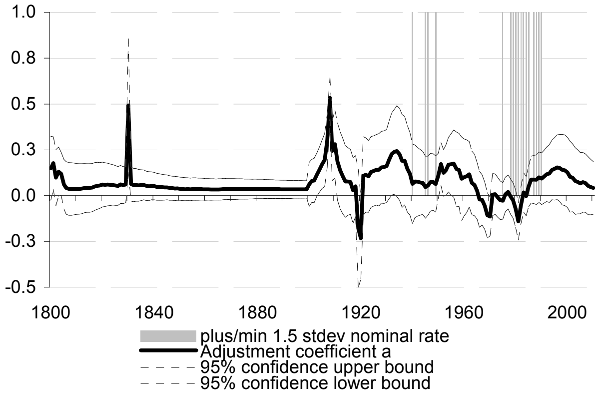


Figure 4.d. Adjustment speed long-term rate JP
Coefficient α and confidence interval, 10-years rate

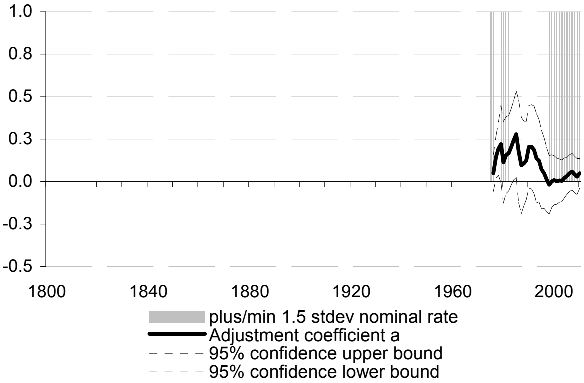


Figure 5.a. Adjustment speed yieldcurve NL
Coefficient α and confidence interval, 10-yrs rate - 3 mt rate

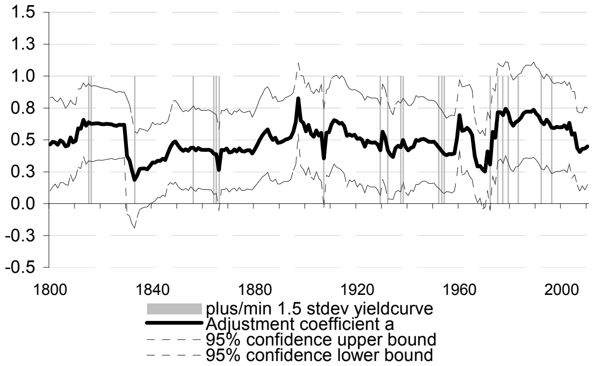


Figure 5.b. Adjustment speed yieldcurve DE
Coefficient α and confidence interval, 10-yrs rate - 3 mt rate

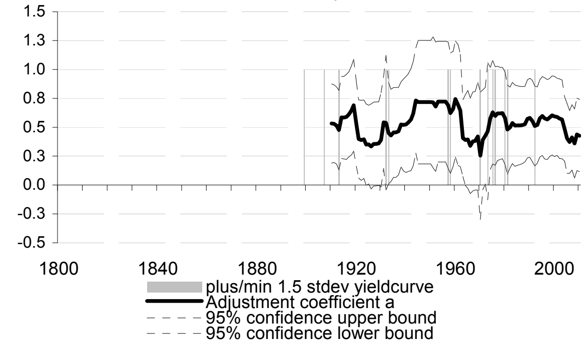


Figure 5.c. Adjustment speed yieldcurve US
Coefficient α and confidence interval, 10-yrs rate - 3 mt rate

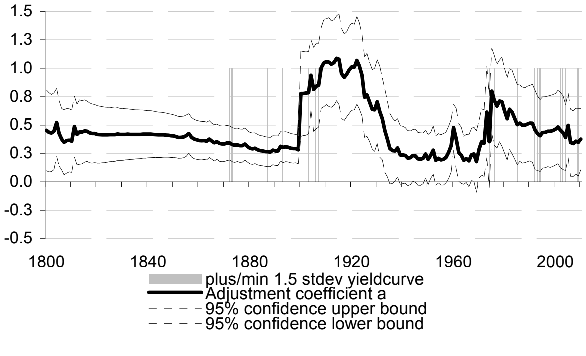


Figure 5.d. Adjustment speed yieldcurve JP
Coefficient α and confidence interval, 10-yrs rate - 3 mt rate

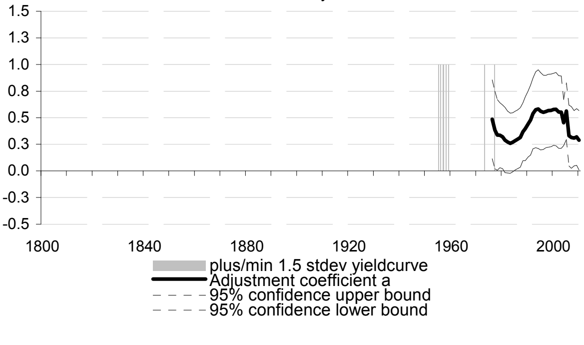


Table 4. Estimation results linear models

Coefficients (t-values in parentheses)[1]

	NL	DE	US	JP
	Δr_t	Δr_t	Δr_t	Δr_t
β_0, *constant*	0.22*	0.15**	0.21*	0.13
	(-1.91)	(-2.52)	(-1.66)	(0.67)
β_1, r_{t-1}	-0.05*	-0.04**	-0.05*	-0.04
	(-1.82)	(-1.82)	(-1.64)	(-0.97)
β_2, Δr_{t-1}	0.23***	-	-	0.26**
	(3.10)			(1.99)
β_3, Δr_{t-2}	-0.16***	-	-	-
	(-4.64)			
β_4, Δr_{t-3}	-	-0.25***	0.14*	-
		(-4.85)	(1.64)	
β_5, Δr_{t-4}	-	-	-	-
β_6, Δr_{t-5}	-	-0.18***	-	-
		(-2.73)		
Adj. R^2	0.07	0.10	0.03	0.04
Heteroskedasticity test	78.53	40.24	90.19	1.42
F test statistic[2]	(0.00)	(0.00)	(0.00)	(0.24)
Serial correlation LM test	2.35	0.99	1.52	0.43
F test statistic[3]	(0.10)	(0.37)	(0.22)	(0.65)

[1] Outcomes OLS with Heteroskedasticity and Autocorrelation Consistent Covariance (HAC) or Newey-West estimator.
***, **, * is significant at 1%, 5%, 10% confidence level
[2] Breusch-Pagan-Godfrey test. H0: homoscedastic errors. P-values in parantheses.
[3] Breusch-Godfrey test. H0: no serial correlation. P-values in parantheses.

Table 5. Results of Terasvirta linearity tests

Chi-squared test statistic (p-values in parentheses)[1]

Transition variable	NL	DE	US	JP
r_{t-1}	2.05	30.68	4.95	1.32
	(0.04)	(0.00)	(0.00)	(0.26)
r_{t-2}	16.22	2.12	11.26	3.31
	(0.06)	(0.03)	(0.00)	(0.77)

[1] H0: AR model is linear.

Table 6. Estimation results ESTAR model
Coefficients (t-values in parentheses)[1]

	NL		DE		US		JP	
	Δr_t	Δr_t	Δr_t	Δr_t	Δr_t	Δr_t	Δr_t	Δr_t
d	*1*	*1*	*1*	*1*	*2*	*2*	*1*	*1*
Threshold	*Fixed*	*Varying*	*Fixed*	*Varying*	*Fixed*	*Varying*	*Fixed*	*Varying*
β_0, *constant*	5.25	-0.13	82.29	-0.23	-0.09	0.06	0.82	0.05
	(0.33)	(-0.90)	(0.13)	(-1.13)	(-0.12)	(0.26)	(0.08)	(0.38)
β_1, r_{t-1}	-0.58	0.03	-6.86	0.05	0.03	-0.02	-0.13	-0.02
	(-0.38)	(0.90)	(-0.14)	(0.97)	(0.22)	(-0.30)	(-0.08)	(-0.68)
β_2, Δr_{t-1}	0.31	0.41***	-	-	-	-	0.29**	0.27*
	(1.24)	(4.03)					(2.18)	(1.79)
β_3, Δr_{t-2}	-0.11	-0.06	-	-	-	-	-	-
	(-0.92)	(-0.93)						
β_4, Δr_{t-3}	-	-	-1.87	-0.25***	0.21	0.22	-	-
			(-0.14)	(-2.21)	(1.31)	(1.25)		
β_5, Δr_{t-4}	-	-	-	-	-	-	-	-
β_6, Δr_{t-5}	-	-	-1.61	-0.16***	-	-	-	-
			(-0.15)	(-1.61)				
γ	0.01	0.11***	0.01	0.07***	0.01*	0.04*	0.01	0.02
	(1.40)	(3.22)	(0.30)	(2.89)	(1.63)	(1.96)	(0.51)	(0.35)
c	10.53*	-	18.54	-	7.64***	-	6.8	-
	(1.66)	-	(0.46)	-	(5.69)	-	(0.51)	-
Adj. R squared	0.19	0.19	0.22	0.22	0.07	0.07	0.04	0.01
Heteroskedasticity test	25.01	13.19	10.50	11.00	19.22	15.83	1.27	2.47
F test statistic[2]	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.29)	(0.07)
Serial correlation LM test	1.89	1.52	4.4	8.17	1.96	1.58	0.15	0.26
F test statistic[3]	(0.15)	(0.22)	(0.01)	(0.00)	(0.14)	(0.21)	(0.86)	(0.78)

[1] Outcomes NLS with Heteroskedasticity and Autocorrelation Consistent Covariance (HAC) or Newey-West estimator.
Coefficients of vector θ' are determined by applying the standard restrictions that $\theta_{2..5} = -\beta_{2..5}$ and $\theta_1 = -(1+\beta_1)$, as in Schnatz (2006).
***, **, * is significant at 1%, 5%, 10% confidence level.
[2] Breusch-Pagan-Godfrey test. H0: homoscedastic errors. P-values in parantheses.
[3] Breusch-Godfrey test. H0: no serial correlation. P-values in parantheses.

Figure 6. Speed of adjustment ESTAR
Outcomes transition function on y-axis; deviation from threshold value in percentage points on x-axis

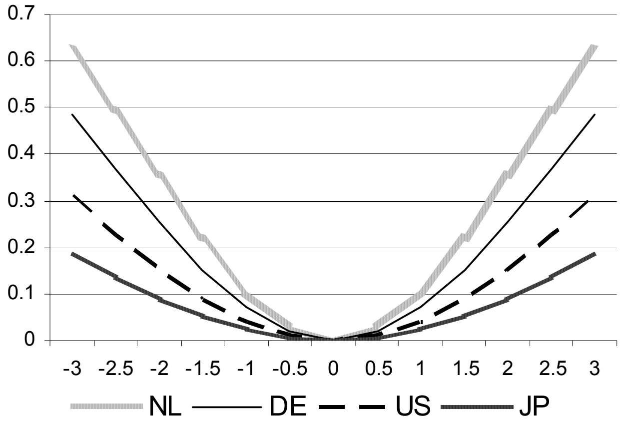


Figure 7.a. Historic adjustment speed NL (ESTAR)
Outcomes transition function on y-axis

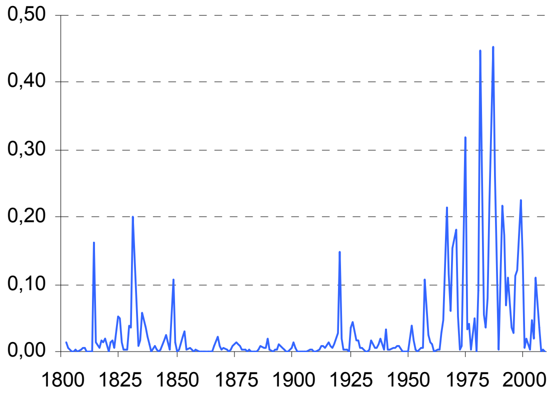


Figure 7.b. Historic adjustment speed US (ESTAR)
Outcomes transition function on y-axis

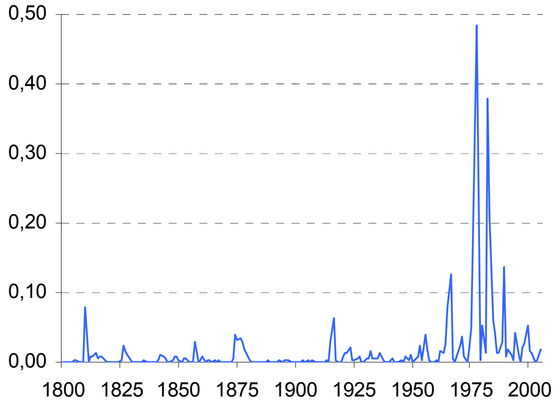


Figure 7.c. Historic adjustment speed DE (ESTAR)
Outcomes transition function on y-axis

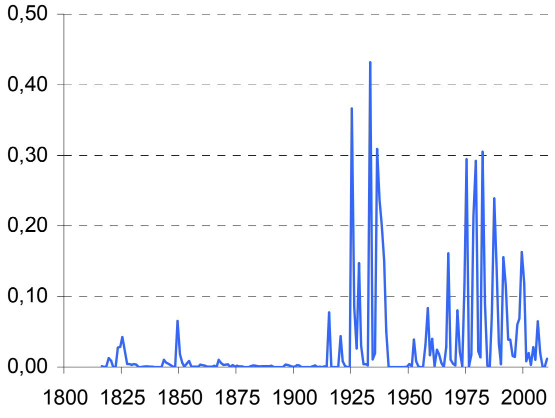


Figure 7.d. Historic adjustment speed JP (ESTAR)
Outcomes transition function on y-axis

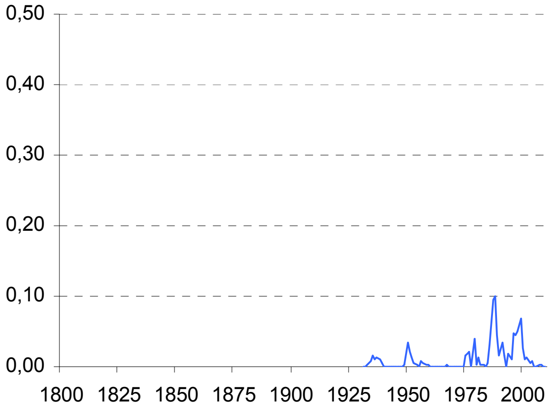


Table 7. Half-life, Generalised Impulse Responses, ESTAR
Half-life in years

	NL	DE	US	JP
shock	*Regime r_t > mav (r_t)*			
+ 1 s.e.	-	5	9	10
+ 2 s.e.	7	5	-	9
+ 3 s.e.	5	6	8	8
shock	*Regime r_t* < mav (r_t)			
- 1 s.e.	-	3	12	5
- 2 s.e.	15	3	13	4
- 3 s.e.	5	2	8	4

Figure 8.a. Impulse response function and half-life NL
Half-life of positive shock in higher outer regime (ESTAR)

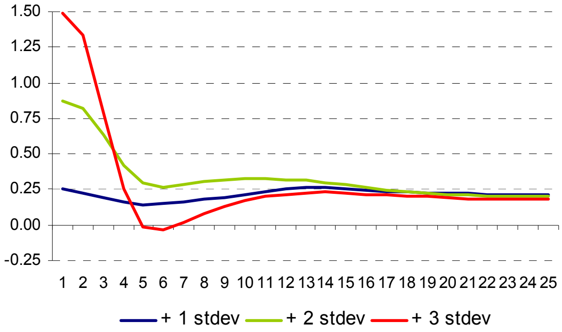


Figure 8.b. Impulse response function and half-life US
Half-life of positive shock in higher outer regime

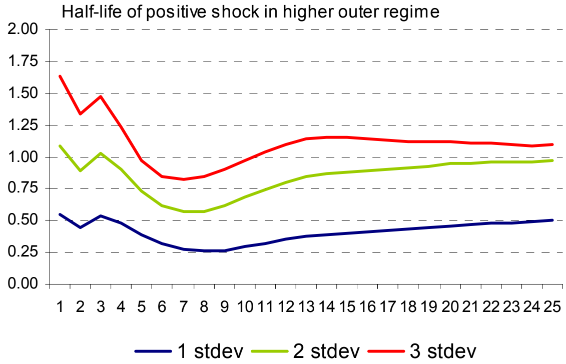


Figure 8.c. Impulse response function and half-life DE
Half-life of positive shock in higher outer regime (ESTAR)

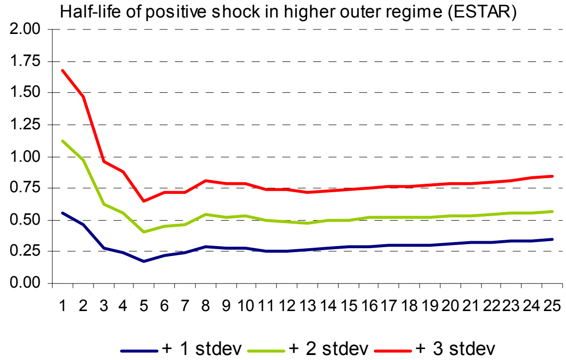


Figure 8.d. Impulse response function and half-life JP
Half-life of positive shock in higher outer regime (ESTAR)

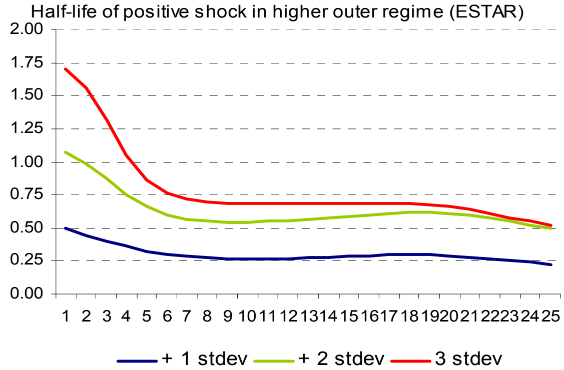


Figure 8.e. Impulse response function and half-life NL
Half-life of negative shock in lower outer regime (ESTAR)

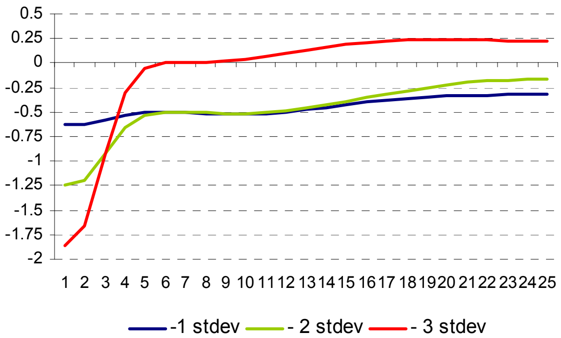


Figure 8.f. Impulse response function and half-life US
Half-life of negative shock in lower outer regime (ESTAR)

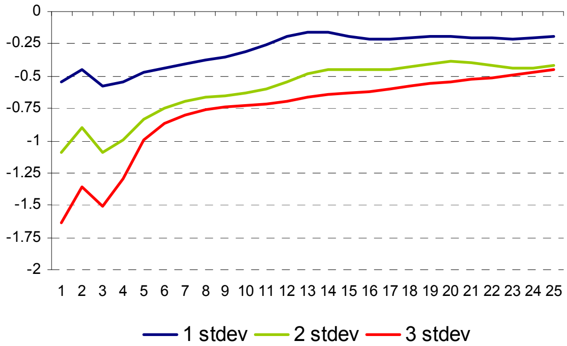


Figure 8.g. Impulse response function and half-life DE
Half-life of negative shock in lower outer regime (ESTAR)

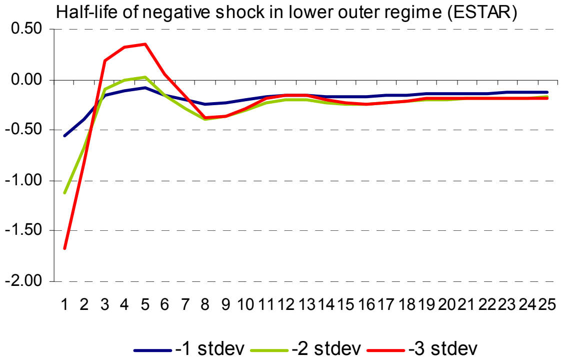
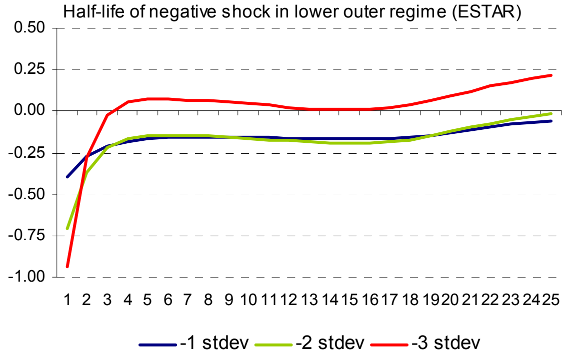


Figure 8.h. Impulse response function and half-life JP
Half-life of negative shock in lower outer regime (ESTAR)



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