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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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Exclusive Portfolio Dealing and Market Inefficiency[☆]

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Abstract

We rationalize exclusive portfolio dealing in a novel three-period partial equilibrium framework populated by a representative, risk-neutral seller and a small number of *ex ante* identical broker-dealers. Endowed with independent, uncertain demand for a representative asset, the broker-dealers may compete in prices for exclusivity. If no exclusivity is granted, due to either the lack or seller rejection of offers, the seller enters a second-price auction with a zero-loss reserve price. While seller profits are constant under exclusivity (Bertrand Paradox), auction profits increase in the number of broker-dealers. Therefore, exclusivity arises in equilibrium only for a seller with at most two broker-dealers, reducing the trade frequency by one-third. The results are robust to endogenizing the number of broker-dealers and to allowing for the *ex post* asymmetry in asset demand. Exclusivity, however, does not arise when the auction features a seller-optimal reserve price. We motivate and conclude with an application to the security lending market.

Key Words: Exclusive Dealing; Intermediated Markets; Competition; Market Efficiency
JEL Classification: G14; G24; D43; D86
Declarations of Interest: none

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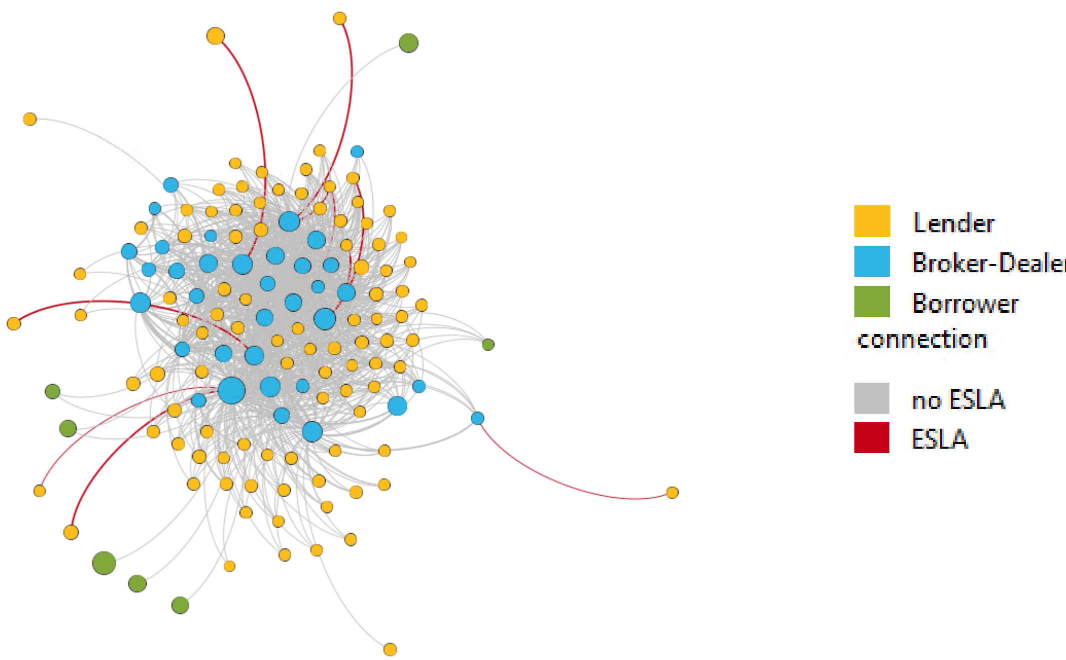
1. Introduction

The phenomenon of exclusive dealing in intermediated markets — such as classic and over-the-counter (OTC) stock markets, secondary bond markets, standardized futures and forward markets, real estate markets, or security lending markets — is anecdotally common, yet systematically overlooked in existing literature. As housing market participants, we are familiar with a single realtor exclusively representing the private home owner willing to sell or the real estate investor looking to rent out multiple units in their apartment complex.

Utilizing a novel transaction-level data set of the EU-based equity lending market,[1] we document similar trading patterns: We find an intermediated market (see Figure 1), where 20% of lenders grant a single broker-dealer exclusive access to their equity portfolio (see Figure 2). On the buyer side, some home-seekers *ex ante* contractually limit themselves to buy via a single realtor. Similarly, we find a small fraction of the ultimate equity borrowers formally seek exclusivity with a broker-dealer. Perhaps surprisingly, most equity borrowers without formalized exclusivity appear nevertheless to only engage with a single broker-dealer, indicating independent demand.

Figure 1: Network Plot of the EU-based Equity Lending Market

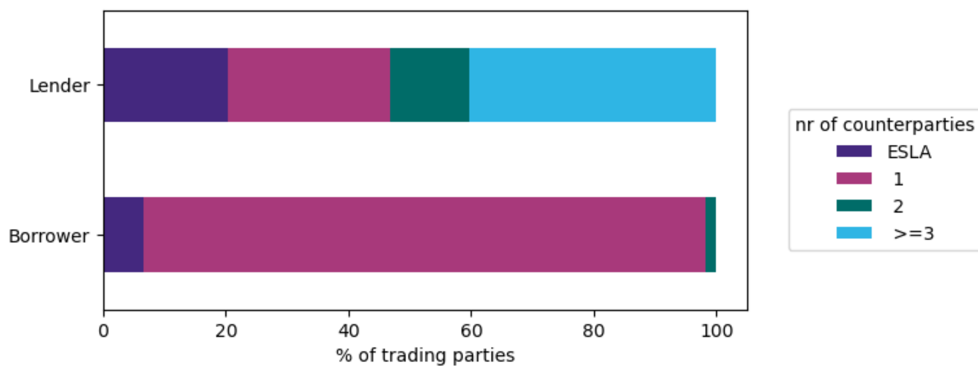


Note: *Figure 1 is based on 37 broker-dealers, 287 borrowers and 3821 lenders. Individual lenders and borrowers with the same combination of counterparties are joined and the dots reflect the logarithmic size of their joined trading volume.*

[1]The newly available and confidential Security Financing Transactions Regulation (SFTR) data allows us to observe individual equity lending contract terms, including counterparty LEIs, maturity, ISIN of the underlying, lending fees, and most importantly a flag for exclusivity.

Markets, where broker-dealers intermediate between sellers looking to sell (part of) their asset portfolio, and buyers looking to purchase selected assets, are typically studied in either a search and matching framework inspired by Duffie et al. (2005), or a network setting with (partially) endogenous links (see, e.g., Atkeson et al. (2015), Gofman (2014) and subsequent works). While complementary in nature, both approaches typically abstract from exclusive dealing to maintain tractability. Additionally, insights are typically limited to a specific pricing mechanism, such as e.g. Nash bargaining in the case of the search and matching models.

Figure 2: Relative Share of Trading Parties per Type



Note: *Figure 2 is based on 287 borrowers and 3821 lenders.*

The core theoretical contribution of this paper is the development of a novel three-period partial equilibrium framework that models trading between a representative seller and competing broker-dealers where all equilibrium outcomes solely depend on two parameters: The number of broker-dealer connections and the broker-dealers' private and independent demand uncertainty around a common reference value. As the revenue equivalence holds, this results in an intuitive environment that captures a wide range of pricing mechanisms including, but not exclusive to, first- and second-price auctions, and competition in prices. Further, the model is easily adaptable to application-specific characteristics by, for example, endogenizing connections, introducing relational frictions or adding complexity to the pricing mechanism. This framework may thus be of interest when studying the impact of different pricing frictions on market efficiencies in a variety of intermediated markets, such as the real-estate sector, over-the-counter markets for derivatives, stock and bond, or multi-product banking.

In this paper, specifically, we utilize the model to rationalize exclusive dealing in intermediated markets and to reveal how exclusivity influences individual and aggregate market outcomes. We show that exclusive dealing is granted by sellers with initial access to at most two broker-dealers. Further, we show that exclusive dealing is anti-competitive in nature and reduces trading volume by up to one third at the seller level. Through a series of extensions, we highlight both that the model remains solvable and that the results are generally robust if complexity is added. Finally, we apply the model insights to the EU-based equity lending market and show that exclusivity-induced inefficiencies aggregate to up to €42.30 bn annually in foregone trading.

These theoretical insights apply to any intermediated market with risk-neutral portfolio holders selling assets to broker-dealers with private, independent, and uncertain demand. Here, sellers face a trade-off between granting exclusivity to the entire asset portfolio up front or waiting until demand realizes for asset-by-asset competition to take place. The *ex ante* competition in prices for exclusivity is fierce (Bertrand paradox), yet realized demand is limited *ex post* to a single broker-dealer. Jointly, this ensures that seller payoffs under exclusivity are independent of the seller's number of broker-dealer connections. Alternatively, the seller may wait until broker-dealer demand has realized to host auctions on an asset level. Here, competition intensifies as more broker-dealers realize a positive demand and, thus, participate in a single auction. Hence, seller payoffs increase in the number of broker-dealer connections. Ultimately, only the least connected sellers find exclusivity to be more profitable. Crucial here is that exclusivity is negotiated prior to trading, and exclusive contracts are binding such that termination triggers compensation to the loss-incurring party.

Formally, we propose a three-period model that is populated by a representative, risk-neutral seller with exogenously given access to $N \geq 2$ competing broker-dealers seeking to maximize profits. At $t = 0$, the seller is endowed with a portfolio, whose size is normalized to one and that is characterized by a representative asset of known reference value.[2] Further, the broker-dealers are each endowed with an uncertain ask price (buying demand) that will be drawn independently from an identical uniform distribution with mean equal to the reference value.[3] Thus, broker-dealers are symmetrical *ex ante* in expected demand, but not *ex post* in realized demand. At $t = 1$, before the demand realizes, the broker-dealers choose whether to compete in offering an exclusive access agreement that specifies a bid price paid upon trading and a lump-sum transfer paid regardless of trade.[4] At $t = 2$, the demand realizes and each broker-dealer draws an independent ask price that can be either above or below the reference value. If exclusivity was granted, the exclusive dealer buys the representative asset when

[2]These two assumptions are without loss of generality: The portfolio size enters linearly in all equations. The transaction price is independent of the actual asset value but rather determined by the uncertainty of the ask price around the value.

[3]It follows directly from this that: [1] pricing is determined by demand uncertainty around the reference and value and [2] the representative seller may but not necessarily must experience excess purchase demand for the representative asset.

[4]This structure of the exclusive access agreement is inspired by the demand-boost theory of exclusive dealing by Calzolari et al. (2020).

able to realize a positive bid-ask spread given the agreed-upon bid price. The lump-sum transfer is paid independently of the actual purchase decision. If no exclusivity was granted, the asset is traded through a representative second-price auction where all N broker-dealers may participate.[5] Here, we assume that broker-dealers are unaware of each other's realized ask prices and, thus, possess private information when submitting their auction bid.[6]

To derive when exclusive dealing arises in equilibrium, we apply the notion of termination-proof subgame perfect Nash equilibrium (SPNE): At $t = 1$, both the seller and the broker-dealer must agree to exclusivity, anticipating auction and market exclusion, respectively, in the alternative. At $t = 2$, neither party can have an incentive to terminate the contract conditional on compensating the counterparty for the potential losses from the breach. The concept of termination proof equilibrium is novel in this paper as the more common concept of renegotiation-proof equilibrium, see e.g. Segal and Whinston (2000), is not applicable in this setting: Allowing for a lump-sum transfer in the exclusive dealing contract ensures that one agent's gain in results in an equivalent loss for the counterparty. As such, there do not exist mutually beneficial deviations.

Whether the SPNE with termination proof exclusivity or the representative auction arises thus depends on the representative seller's and broker-dealers' relative expected profits under exclusivity and auction. Under exclusivity, the competition in prices (bid price and lump-sum transfer) ensure that the Betrand Paradox applies: We have full-profit path through, i.e. all broker-dealers make zero profits and the representative seller expects to receive the trading surplus from the exclusive dealer. Under the auction, the expected seller (broker-dealer) profits increase (decrease) in the number of broker-dealer connections, as the higher the number of connections the higher the chance of excess demand in the second-price auction driving up prices. Termination costs arise when the seller expects higher profits under exclusivity than under the auction, yet the exclusive dealer terminates the contract.

Then comparing the sub-game outcomes, we find the auction SPNE to be unique for a seller with at least three broker-dealer connections, we find the auctions SPNE to be unique: For $N \geq 4$, the seller always rejects offers for exclusivity, as she can benefit from increased competition at the auction. For $N = 3$, the seller initially grants exclusive dealing rights, however, the exclusive dealer finds the benefits of terminating larger than the cost to compensate the seller for losses. Only for $N = 2$, there exists a SPNE with termination proof exclusivity. Here, the seller strictly prefers to grant exclusivity, and the chosen broker-dealer is indifferent between terminating the agreement or not. For completeness, we show that for

[5]The revenue equivalence principle holds and the assumption of second-price auctions is without loss of generality. Equal profits are obtained in a first-price auction or a Bertrand competition setting.

[6]This is in alignment with both Duffie et al. (2014) and Babus and Kondor (2018), studying the implications of private information for competition in OTC market settings.

$N = 1$, both the seller and the broker-dealer are indifferent between allocation with exclusive access or auction: The broker-dealer is a monopolist who extracts all profits in either case. In short, only sellers with one or two broker-dealers grant exclusivity.

From a normative perspective, we are foremost interested in whether a market is efficient: Is the asset traded when at least one broker-dealer observes a positive bid-ask spread? We confirm that the auction SPNE meets the first-best benchmark: Every broker-dealer with positive demand (ask price above value) bids in the auction. The same holds for the SPNE under $N = 1$ with termination proof exclusivity, as the broker-dealer is a monopolist regardless. However, for $N = 2$ the SPNE with termination proof exclusivity is inefficient relative to the auction SPNE and the first-best benchmark, as the total trading volume is reduced by one third: Only 50% of the assets in the portfolio are traded, while the uniform distribution of the ask prices with a mean equal to public value displays a trading probability of 75%.

We challenge the robustness of our results in three theoretical extensions: Endogenous number of broker-dealers, advantaged exclusive dealer, and seller-optimal reserve price. First, we relax the assumption of an exogenously given number of broker-dealer connections and, instead, allow connections to be a costly choice of the seller at $t = 0$. For a linear cost in new connections, we find a unique, profit-maximizing number of broker-dealer connections that decreases in the size of the marginal cost. Thus, for a sufficiently high cost, $N = 2$ is optimal and exclusivity may arise in equilibrium. Notably, we show that $N = 3$ is never optimal since the seller correctly anticipates that the exclusive dealer would terminate. In the second extension, we show that an exclusive dealer strictly prefers not to terminate the exclusive contract when surprised *ex post* negotiations with a favorable ask price distribution.[7] In the third extension, we provide the seller with a commitment device to her optimal auction reserve in the spirit of Levin and Smith (1996). Here, we confirm that, if possible, the seller commits to a reserve price strictly above what is required to break-even. The presence of such a reserve price boosts the seller's auction profits to such a degree that termination is no longer sufficiently costly for the exclusive dealer. Thus, any exclusive dealer would terminate the contract, and exclusivity does not arise in equilibrium.

Finally, we illustrate the practical importance of our theoretical findings by predicting the impact of exclusive dealing on aggregate inefficiency in the EU-based equity lending market. Efficiency is of a particular concern here, as sufficient asset lending supply is crucial for short sales to eliminate arbitrage opportunities, thereby ensuring that the law of one price holds in our financial markets. We predict the trading volume in a counterfactual market without any exclusivity by applying a bootstrapping strategy on the observed

[7] We explicitly refrain from *ex ante* asymmetry, as this introduces an informed principle.

lender-level trading in 2021. In simple terms, we randomly assign a share $x \in [0,1]$ of the observed lenders with exclusive security lending agreements (ESLAs) two broker-dealers in a hypothetical counterfactual auction. The remaining share $1 - x$ remains with a monopolistic broker-dealer in the counterfactual. Subsequently, we treat lenders with two assigned broker-dealers with a 50% increase in trading volume, while those with one broker-dealer remain untreated in trading volume. Repeating this for $100,000$ bootstraps, we obtain both a predicted increase in total trading volumes and the associated confidence intervals. Each 0.1 unit increase in x, predicts a statistically significant €4.23 bn increase in annual trading volume.

Related Literature Rationalizing the existence of exclusivity and characterizing associated outcomes in intermediated markets, our paper is foremost an addition to the existing literature on pricing in OTC markets. Here, the seminal paper by Duffie et al. (2005), studying pricing in a search-and-matching framework, has sparked a rich debate on relevant pricing frictions. Typically, the subsequent articles in this literature study trading in a dynamic setting (Gabrovski and Kospentaris, 2021). As we propose a three-period framework, a full review of the literature goes beyond the scope of this paper. Instead, we focus on reviewing the papers closest to us without implying anything as to the general relevance of the omitted works. Modeling a choice between exclusivity and auction relates to the frameworks proposed by Babus and Kondor (2018) and Dugast et al. (2022). Both papers study dealers' choices between bilateral trading and trading via centralized platforms in inter-dealer markets. Novel relative to their papers is both our shift in focus to transactions between the initial seller and the broker-dealer, and the generalization to asset portfolios rather than a single asset. We reinforce their findings that less connected agents prefer the bilateral option, in our case exclusivity, while more connected agents prefer the centralized auction.

Building on the established notion that broker-dealers have private information on their borrowing demand (Babus and Kondor, 2018; Duffie et al., 2014), we introduce private and independent demand uncertainty that is symmetric around a common reference value. However, we deviate from the focus on sophisticated traders that acquire knowledge over time and clear the market via double auctions. Instead, we opt for tractable second-price auctions to clear the market after demand is realized. Note that as the demand of broker-dealers is independent, the revenue equivalence holds and results are robust to other pricing mechanism. More crucially, we introduce *ex ante* market clearing via exclusivity before the uncertainty over the broker-dealers' private demand is realized. Here, we follow Babus and Kondor (2018) and assume that the *ex ante* competition for (future) trading is in prices. Deviating from their set-up, we shift the focus from quantity limitations on an individual asset level competition for exclusivity over the entire portfolio. To the best of our knowledge, we are the first to model and, thereby, to rationalize why portfolio holders engage in exclusive dealing in equilibrium.

By comparing the SPNEs with and without exclusivity, we can show that exclusivity leads to a 33% trading reduction on a seller-level — a sizable market inefficiency. Our paper thus adds a counter-perspective to the theoretical literature on non-exclusive contracting in financial markets. Here, papers by for example Attar et al. (2014), Biais et al. (2000), Bizer and DeMarzo (1992), Detragiache et al. (2000), Kahn and Mookherjee (1998), Pauly (1978), and van Boxtel et al. (2020) either imply or show directly that exclusive financing is not obtainable in equilibrium, yet is efficiency improving and desirable. Crucially, we deviate in set-up and assume that exclusive agreements are binding *ex post* in a sense that termination is costly. Other differences assumptions are that we consider private information of value on the buyer (broker-dealer) rather than the seller side (Biais et al., 2000), simultaneous trades rather than sequential transactions (Bizer and DeMarzo, 1992; Detragiache et al., 2000; van Boxtel et al., 2020), non-divisible stocks instead of divisible goods (Attar et al., 2011), and the absence of moral hazard (Kahn and Mookherjee, 1998; Pauly, 1978). Unifying our model with one or more of the above-mentioned alternative settings is a natural next step and, over time, will hopefully lead to a fruitful discussion in this line of research.

Modeling exclusive dealing as a contract that spans a portfolio of assets (goods), in limited capacity, relates to the literature on exclusive contracting between retailers and manufacturers (Bernheim and Whinston, 1998; Calzolari and Denicolò, 2013; Calzolari and Denicolò, 2015; Mathewson and Winter, 1987).[8] Most notably, we deviate from the assumption of substitute goods common to this literature. Instead, we consider exclusive contracting over a portfolio, where the representative asset poses a distinct good. Following Calzolari et al. (2020), we allow the broker-dealers to compete for exclusivity by setting both a transaction fee (bid price) and a lump-sum transfer. Considering a multi-product setting, we are able confirm their result that in equilibrium exclusive contracts have a zero bid premium above reference value to boost demand and full profit-pass-through via the lump-sum transfer. In both our and their paper, this demand-boosting leaves the buyer (broker-dealer) with exclusive rights worse off than in the alternative equilibrium. Here, we would like to highlight that in our setting price competition ensures demand boosting *within* an exclusive dealing agreement but reduces overall trading vis-à-vis the auction. Mostly focused on consumer welfare, this reduction in quantities is of secondary interest in the classic Industrial

[8] See Armstrong and Wright (2007) for exclusive contracting by two-sided platforms, where the platform does not interpose as an intermediary and therefore does not become the direct counterparty to both the supply and demand side.

Organization literature. However, it adds substantial value to the market micro-structure literature concerned with the efficiency of financial markets.

With earliest works by D'Avolio (2002) and Duffie et al. (2002), the security lending market has taken a center stage in the financial literature on market (in-)efficiency. The paper by Duffie et al. (2002) studies the lending price formation in a search-and-bargaining model, where pessimists are matched with both lenders and optimists over time, thereby being able to short sell. For tractability, they abstract from the role of broker-dealers intermediating between lenders and borrowers. Taking a complementary approach by introducing broker-dealers that profit from the bid-ask spread, we prove that our model nests a security lending market with intermediation.[9]

With access to transaction-level data covering the EU-based equity lending market, we can show that the individual-level inefficiency due to exclusive dealing effects 20% of all lenders. Aggregating this reduction over all lenders, we can predict the total market inefficiencies due to exclusivity. To the best of our knowledge we are the first to obtain a micro-founded empirical estimate of such inefficiencies. With this, we provide a novel micro-foundation for short sale constraints, whose market impact has been widely discussed (Asquith et al., 2005; Bai et al., 2006; D'Avolio, 2002; Gutierrez et al., 2018; Nagel, 2005; Nezafat et al., 2017).

2. Model Environment

The model is populated by a representative, risk-neutral seller (she), who is endowed with an asset portfolio. The portfolio is normalized to unit size and contains a single, representative asset with a positive and known value.[10] Furthermore, there exist N for-profit broker-dealers (he/they) that are each endowed with an independent and uncertain demand for the representative asset.[11] In the first period, and before the asset demand realizes, the broker-dealers may compete for exclusive access to the seller's portfolio. An exclusivity contract specifies a bid price paid upon purchase and a lump-sum transfer paid independently of trade. In a second period, each broker-dealer draws an independent ask price for the asset from an identical uniform distribution with mean equal to the asset's value. If exclusivity was granted, the exclusive dealer pays the lump-sum transfer, but only purchases the asset

[9] Specifically, the security lending market is captured by assuming the absence of broker-dealer default, for example due to correctly priced collateral, such that the lender's participation constraint reduces to a zero reference value (a parameter in our model).

[10] This common value can be interpreted as the average perceived market value of future cash flows from all market participants.

[11] For now, N is treated as a parameter. N is endogenized as part of the extensions in Section 5.

when observing a positive bid-ask spread. In the absence of exclusivity, the seller hosts a second-price auction to sell the asset. The timing is summarized in Table 1 below.

Table 1: Model Timing

	Broker-Dealer	Seller
$t = 0$	• Endowed with independent and uncertain demand for each asset	• Endowed with asset portfolio
$t = 1$	• Anticipate its asset demand • Offer competitive exclusive access agreements: – Uniform bid-premium above value – Lump-sum transfer	• Anticipates profits with and without exclusive access • Decides whether to enter an agreement or reject all offer
$t = 2$	• Demand uncertainty realizes • With exclusive access agreement: – Holder buys assets with positive bid-ask-spread and pays lump-sum transfer – Other broker-dealers remain inactive • Without exclusive access agreement: – Broker-dealers bid asset-by-asset in a second price auction – Highest bidder gets to buy asset	• With exclusive access agreement: – Receives lump-sum transfers – Receives uniform bid price for all sold assets • Without exclusive access agreement: – Offers each asset via a second price auction – Sells asset to highest bidder

Seller At $t = 0$, the risk-neutral seller is endowed with an asset portfolio. Without loss of generality, we can normalize the portfolio size to one and let v denote the value of a representative asset. The seller is assumed to be profit-maximizing and sufficiently liquid to theoretically hold the assets for the long run. However, she is willing to sell the asset to a broker-dealer when offered a favorable bid price b:

$$b \geq v. \tag{1}$$

Broker-Dealers There exist $N \geq 2$ profit-maximizing broker-dealers, which we label with sub-scripts $n \in \{1, 2, \ldots, n, \ldots, N\}$.[12] The broker-dealers intermediate sales between the seller and (potential) buyers. We abstract from a detailed analysis on the buyer side and simply assume that each broker-dealer n draws an independent ask price a_n from a value-dependent uniform distribution at $t = 2$:

$$a_n \sim U(v - a, v + a). \tag{2}$$

[12] The case for $N = 1$ is discussed separately further below.

To realize such ask price, they must buy the asset from the seller at a bid price b_n (more below). For now, notice that broker-dealers are willing to buy and resell the asset when able to realize a positive bid-ask spread π_n:

$$\pi_n = a_n - b_n \geq 0. \tag{3}$$

Exclusive Dealing Anticipating their ask prices, broker-dealers may compete for exclusivity over the seller's portfolio by offering competitive exclusivity contracts at $t = 1$. Each exclusive contract specifies a uniform bid price premium p_n^E paid above value v.[13] The bid price $b_n^E(v)$ under exclusive access is, thus:

$$b_n^E(v) = v + p_n^E. \tag{4}$$

In addition, the exclusive contract specifies a lump-sum transfer T_n^E. Being *ex ante* identical, broker-dealers compete in prices jointly on both the bid-premium and the lump-sum transfer. Throughout the paper, we refer to the broker-dealer that has offered and was granted exclusive access as the (single) exclusive dealer.

At $t = 2$, the exclusive dealer pays both the bid price and the lump-sum transfer: While $b_n^E(v)$ is paid upon purchase of the asset, T_n^E is always paid. We assume that the exclusive contract is binding such that either nonpayment of T_n^E or one-sided termination entitles the counterparty to receive compensation equal to the lost profits. Capturing their legal complexity, exclusive contracts can neither be offered nor entered at $t = 2$. Assuming no termination, the exclusive dealer purchases the asset at $t = 2$, when:

$$\pi_n = a_n - b_n^E(v) > 0. \tag{5}$$

Second-Price Auction In the absence of exclusivity at $t = 2$, the seller hosts a standard second-price auction (Vickrey auction).[14] Simultaneously, each broker-dealer observes his ask price realizations a_n and decides whether and how much to bid (b_n).

With a small abuse of notation, we denote by $\max_{k \neq n} b_k$ the largest value of all other k submitted bids. For a representative auction, a broker-dealer n's profit (bid-ask spread) can be characterized by the following step function:

[13]Note that, for the uniform bid-premium assumption to be reasonable, ask price uncertainty a around the true value must independent of the actual value. If we were to relax the latter assumption, we would have to carefully evaluate the uniform bid-premium assumption.

[14]Given the iid property of ask price draws across broker-dealers, the revenue equivalence theorem holds and results are robust to assuming first-price auction or Bertrand competition with public information, instead.

$$\pi_n = \begin{cases} a_n - \max_{k \neq n} b_k & b_n > \max_{k \neq n} b_k \geq v \\ a_n - v & b_n > v \geq \max_{k \neq n} b_k \\ 0 & \max_{k \neq n} b_k \geq b_n \geq v \\ 0 & \text{otherwise} \end{cases}. \tag{6}$$

Case one and two in the profit function (6) reflect the broker-dealer n submitting the winning bid. Then, the bid-ask spread is the difference between the ask price and second highest bid above v (case one) or the difference between the ask price and v (case 2). Here, we follow standard convention and assume that the probability of having an equal highest and second highest bid is zero. The third case in (6) reflects that profits are zero if the broker-dealer n participated with a weakly positive bid, but did not have the highest bid. Finally, the fourth line in (6) reflects that the broker-dealer n receives zero profits when refraining from submitting a bid or bidding below value.

Equilibrium Notion We apply the notion of subgame perfect Nash equilibrium (SPNE). We start by assuming that exclusivity was not granted at $t = 1$. Arriving immediately at $t = 2$, we derive each broker-dealer's optimal auction bid and compute the respective expected profits of the seller and broker-dealers. Subsequently, we derive the exclusive agreement terms offered at $t = 1$. Here, we take into account that sellers can reject all offers when expected payoffs are lower than the auction profits, which serve as her reservation utility. Finally, we identify the SPNEs characterized by the broker-dealers' strategic choices of offering exclusive access agreements and check whether the SPNE is termination proof (more below).

3. Equilibria

3.1. Second-Price Auction

Optimal Auction Bids Let us assume that we have arrived at $t = 2$, no exclusivity was granted and ask-prices have realized. Then, we can derive the optimal auction bids. Here, broker-dealers decide on an asset level whether to participate in the (representative) auction and, conditional on participation, what to bid. For participation, recall a broker-dealer requires a positive bid-ask spread (see Equation (7)). Simultaneously, the seller requires a bid-price weakly above value v to sell the asset (see Equation (8)). Combining these two conditions in Equation (9), it is easy to see that broker-dealers participate in the representative auction if he draws an ask-price weakly larger than the asset value:

$$\pi_n = a_n - b_n \geq 0, \tag{7}$$

$$b_n \geq v, \tag{8}$$

$$a_n \geq b_n \geq v. \tag{9}$$

As is standard in second-price auctions, the broker-dealer bids the entire ask-price upon participation, i.e. whenever observing an ask-price realization above value ($a_n \geq v$). Bidding the entire ask-price is the highest possible bid that still ensures a weakly positive spread π_n, while simultaneously maximizing the chances of winning.

$$b_n^* = a_n \quad \text{if} \quad a_n \geq v. \tag{10}$$

For an ask-price realization below reservation price v, bidding truthfully will not lead to a broker-dealer winning the auction. Bidding more than the realized ask-price enhances the chance of winning, but leads to a loss upon success. Therefore, broker-dealers refrain from bidding whenever they observe an ask-price below value v:

$$b_n^* = \varnothing \quad \text{if} \quad a_n < v. \tag{11}$$

A broker-dealer's optimal bidding strategy is thus:

$$b_n^* = \begin{cases} a_n & a_n \geq v \\ \varnothing & \text{otherwise} \end{cases}. \tag{12}$$

Ask-Price Decomposition To ease the notation for the rest of the paper, note that the ask-price a_n can be decomposed into a value component and an uncertainty component $\tilde{a}_n$ around that value:

$$a_n = v + \tilde{a}_n \quad \text{where} \quad \tilde{a}_n \sim U(-a, a). \tag{13}$$

Then, the broker-dealer participates in the auction by bidding truthfully:

$$a_n \geq v, \tag{14}$$

$$\tilde{a}_n \geq 0. \tag{15}$$

Notationally, it is further useful to sort the broker-dealer draws in descending order:

$$\tilde{a}_1 \geq \tilde{a}_2 \geq \ldots \geq \tilde{a}_N. \tag{16}$$

Expected Seller Profits Given the optimal bidding strategy, we can move to $t = 1$ and derive the seller's expected profits. From a second-price auction with at least two bidders, the seller realizes the difference between the second highest bid and true value v as profit:

$$\pi_S = \max_{k \neq n} b_k - v = v + \tilde{a}_2 - v = \tilde{a}_2 \quad \text{if} \quad \tilde{a}_2 \geq 0. \tag{17}$$

In the case of none of one bidder, the seller leaves empty handed:

$$\pi_S = 0 \quad \text{if} \quad \tilde{a}_2 < 0. \tag{18}$$

For a portfolio size normalized to one, the seller's total expected profits are thus:

$$\mathbb{E}_1 \Pi_S = Pr(\tilde{a}_2 \geq 0) \mathbb{E}_1[\tilde{a}_2 \mid \tilde{a}_2 \geq 0]. \tag{19}$$

The uncertainty component $\tilde{a}_2$ represents the second order statistic given N draws from the same uniform distribution $U(-a, a)$. Using well-established order statistic results, we can derive the cdf and pdf of $\tilde{a}_2$. Denoted $F_2(\tilde{a}_2)$ and $F_2(\tilde{a}_2)$, respectively, they are:

$$F_2(\tilde{a}_2) = \left[\frac{\tilde{a}_2 + a}{2a}\right]^{N-1} \left[N + (1 - N)\frac{\tilde{a}_2 + a}{2a}\right], \tag{20}$$

$$f_2(\tilde{a}_2) = N(N-1)\frac{1}{2a}\left[\frac{\tilde{a}_2 + a}{2a}\right]^{N-2} \left[1 - \frac{\tilde{a}_2 + a}{2a}\right]. \tag{21}$$

Without going too much into analytical details here, we can derive the expected profits as:

$$\mathbb{E}_1 \Pi_S = Pr(\tilde{a}_2 \geq 0) \mathbb{E}_1[\tilde{a}_2 \mid \tilde{a}_2 \geq 0], \tag{22}$$

$$= (1 - F_2(0)) \int_0^a \tilde{a}_2 f_2 \frac{f_2(\tilde{a}_2)}{1 - F_2(0)} d\tilde{a}_2, \tag{23}$$

$$= \frac{a}{2^N}\left[\frac{(2^N(N-3) + N + 3)}{(N+1)}\right]. \tag{24}$$

Expected Broker-Dealer Profits A broker-dealer profits from the representative auction only when realizing the highest ask-price draw. Given the independent draws assumption a broker-dealer has the highest draw with a probability $1/N$. Conditional on having the highest draw, the expected profits depend not only on whether $\tilde{a}_1$ larger or smaller than zero, but also whether $\tilde{a}_2$ larger on smaller than zero. Note here that by definition $\tilde{a}_1 \geq \tilde{a}_2$.

$$\mathbb{E}_1 \Pi_n = \frac{1}{N}\Big[Pr(\tilde{a}_1 \geq \tilde{a}_2 \geq 0) \mathbb{E}_1[\tilde{a}_1 - \tilde{a}_2 \mid \tilde{a}_1 \geq \tilde{a}_2 \geq 0]$$
$$+ Pr(\tilde{a}_1 \geq 0 \geq \tilde{a}_2) \mathbb{E}_1\left[\tilde{a}_1 \mid \tilde{a}_1 \geq 0 \geq \tilde{a}_2\right]$$

$$+ Pr(\tilde{a}_1 < 0) \cdot 0 \Big]. \tag{25}$$

To derive the above expression (25) in closed form, we also need to define the distribution for the highest draw $\tilde{a}_1$ conditional on $\tilde{a}_1$ being larger than $\tilde{a}_2$. Quite intuitively, the largest draw from the uniform distribution must be uniformly distributed above the second largest draw, given that both draws are independent. We denote the conditional cdf and pdf with $F_{1|2}(\tilde{a}_1)$ and $f_{1|2}(\tilde{a}_1)$, respectively. Then:

$$F_{1|2}(\tilde{a}_1) = \frac{\tilde{a}_1 - \tilde{a}_2}{a - \tilde{a}_2}, \tag{26}$$

$$f_{1|2}(\tilde{a}_1) = \frac{1}{a - \tilde{a}_2}. \tag{27}$$

The first line in expression (25) represents the case where at least two brokers participate in the auction. In that case, the winning broker-dealer realizes the difference between the highest and second highest bid ($\tilde{a}_1 - \tilde{a}_2$). Again, without going too much into the analytical solutions, and given the just defined distributional properties, the two elements:

$$Pr(\tilde{a}_1 \geq \tilde{a}_2 \geq 0)\mathbb{E}[\tilde{a}_1 \mid \tilde{a}_1 \geq \tilde{a}_2 \geq 0] = (1 - F_2(0)) \int_0^a \frac{f_2(\tilde{a}_2)}{1 - F_2(0)} \int_{\tilde{a}_2}^a \tilde{a}_1 f_{1|2}(\tilde{a}_1) d\tilde{a}_1 d\tilde{a}_2, \tag{28}$$

$$= \frac{a}{2^{N+1}} \frac{(N-1)(2^{N+1} - N - 2)}{N+1}. \tag{29}$$

$$Pr(\tilde{a}_1 \geq \tilde{a}_2 \geq 0)\mathbb{E}[\tilde{a}_2 \mid \tilde{a}_1 \geq \tilde{a}_2 \geq 0] = Pr(\tilde{a}_2 \geq 0)\mathbb{E}_1[\tilde{a}_2 \mid \tilde{a}_2 \geq 0], \tag{30}$$

$$= \frac{a}{2^N} \left[\frac{(2^N(N-3) + N + 3)}{(N+1)} \right]. \tag{31}$$

The second line in (25) represents the case, where only the highest draw is above v (i.e. $\tilde{a}_1 \geq 0 \geq \tilde{a}_2$) and, thus, the winning broker-dealer can realize his whole ask-price premium. Utilizing again the above defined functional properties and a similar logic, we can obtain the following expression:

$$Pr(\tilde{a}_1 \geq 0 \geq \tilde{a}_2)\mathbb{E}_1 \left[\tilde{a}_1 \mid \tilde{a}_1 \geq 0 \geq \tilde{a}_2 \right] = \frac{N}{2^N} \frac{a}{2}. \tag{32}$$

Finally, the third line in (25) represents the case, where even the highest draw is insufficient for participation. In that case, the broker-dealer makes zero. This is equally true for all the (omitted) cases where he does not have the highest draw. Adding the three components derived above and dividing by N, yields the following total expected broker-dealer profits:

$$\mathbb{E}_1 \Pi_n = \frac{a}{2^N} \frac{2^{N+1} - N - 2}{N(N+1)}. \tag{33}$$

Note that identical analytical results for seller and broker-dealer profits can be obtained in an alternative mathematical approach relying on the binomial theorem: One can treat the number of draws $\tilde{a}_n$ above and below zero as success and failure, which follow a binomial distribution with N draws and a success probability of one-half. Given this, one can express the expected highest and second highest draw as a function of the number of successes. The derivations can be found in the Analytical Appendix C.

Sub-game Outcomes We summarize the sub-game outcomes under the second-price auction in Lemma 1 below. We would like to highlight that both the seller's and broker-dealer's expected profits do not depend on the underlying asset value v but are simply determined by the bounds a on the uncertainty $\tilde{a}_n$ around that value. Intuitively, profits in our model stem solely from the bid-ask spread around the value but not from the value itself. We acknowledge that this result crucially depends on the assumption that the upper and lower bounds on the ask-price uncertainty $\tilde{a}_n$ do not depend on v.

***Lemma* 1.** *The broker-dealers' optimal bidding strategy in the representative second-price auction at time $t = 2$ is:*

$$b_n^* = \begin{cases} a_n & a_n \geq v \\ \varnothing & otherwise \end{cases}. \tag{34}$$

Given the truthful bidding upon participation, the seller and broker-dealers expect the following total profits, respectively:

$$\mathbb{E}_1\Pi_S = \frac{a}{2^N}\frac{2^N(N-3)+N+3}{N+1}, \qquad \mathbb{E}_1\Pi_n = \frac{a}{2^N}\frac{2^{N+1}-N-2}{N(N+1)} \quad \forall n \in N. \tag{35}$$

3.2. Exclusive Dealing

The seller compares the expected auction profits with the expected profits given the available exclusive offers at $t = 1$. We denote all prices and profits associated with an exclusive agreement with an additional superscript ^{E}.

Profit Functions We start by deriving the general expressions for the expected seller and broker-dealer profits. For this, let us assume that the seller has agreed to grant exclusivity to a generic broker-dealer n against a promised lump-sum transfer T_n^E and bid-price $b_n^E(v)$ that specifies an asset-independent bid-premium p_n^E above value:

$$b_n^E(v) = v + p_n^E. \tag{36}$$

Recall the decomposition of the ask-price into a value component and an uncertainty $\tilde{a}_n$ around that value:

$$a_n = v + \tilde{a}_n \quad \text{where} \quad \tilde{a}_n \sim U(-a, a). \tag{37}$$

Then, for a given asset, the broker-dealer with exclusivity purchases an asset at $t = 2$, if:

$$\pi_n = a_n - b_n^E(v) \geq 0. \tag{38}$$

$$\tilde{a}_n \geq p_n^E. \tag{39}$$

Accounting for the likelihood of purchase, and given the unit-sized portfolio, the seller's and agreement-holder's aggregate expected profits under an exclusive access are:

$$\mathbb{E}_1 \Pi_S^E = Pr(\tilde{a}_n \geq p_n^E) p_n^E + T_n^E = \frac{a - p_n^E}{2a} p_n^E + T_n^E, \tag{40}$$

$$\mathbb{E}_1 \Pi_n^E = Pr(\tilde{a}_n \geq p_n^E) \mathbb{E}_1[\tilde{a}_n - p_n^E \mid \tilde{a}_n \geq p_n^E] = \frac{(a - p_n^E)_2}{4a} - T_n^E. \tag{41}$$

Studying expression (40), we observe that the seller's expected profits are monotonically increasing in T_n^E. The profits are, however, nonlinear in the bid-premium: A higher p_n^E increases the revenue from a single transaction, but reduces the probability of said transaction taking place. This non-linearity must be taken into account when deriving the equilibrium prices T_n^{E*} and p_n^{E*} under exclusivity.

Competitive Exclusivity We start by arguing that any equilibrium with an active exclusivity agreement must have two or more broker-dealers competing over it and that the exclusive dealer makes zero profits (Bertrand Paradox). Let us initially assume that a single broker-dealer offers a profitable agreement for a given bid-premium p_n^E and lump sum transfer T_n^E. Then any other broker-dealer has the incentive to offer the same bid-premium but a slightly higher lump-sum transfer to attract the seller instead. Otherwise, he would make zero profits when losing the agreement to a competitor. This applies for all bid-premia and transfer combinations, where the agreement holder makes a positive profit. Therefore, any two or more broker-dealers offering a competitive agreement must make zero profits in equilibrium.

For completeness, assume that a single broker-dealer offers an agreement where he makes zero profits. Without competition, he then has an incentive to lower lump-sum transfers to keep some of the gains of trade. However, as just argued, such deviation is not an equilibrium either as then again the competitor(s) have incentive to offer slightly favorable terms, thereby attracting the seller. Hence, any SPNE with exclusive dealing is necessarily characterized by at least two agreements, where the offered bid-premium and lump-sum transfer leave the broker-dealer with zero profits.

Equilibrium Prices Inserting the zero-profit condition into the exclusive dealer's expected profits (Equation (41)) yields the following equilibrium lump-sum transfer:

$$T_n^{E*} = \frac{(a - b_n^E)_2}{4a}. \tag{42}$$

We then insert the equilibrium lump sum transfer (42) into the expected seller profits (Equation (40)). By equating the associated first-order condition with respect to p_n^E to zero, we can show that $p_n^E = 0$ is the unique seller profit-maximizing bid price:

$$\mathbb{E}_1\Pi_S^E = \frac{a - p_n^E}{2a} p_n^E + \frac{(a - p_n^E)_2}{4a}, \tag{43}$$

$$\frac{\partial \mathbb{E}_1\Pi_S^E}{\partial p_n^E} = -\frac{2}{4a} p_n^E = 0, \tag{44}$$

$$p_n^{E*} = 0. \tag{45}$$

Expected Profits Inserting our optimal pricing into equations (40) and (41) results in the following expected total seller and broker-dealer profits, respectively:

$$\mathbb{E}_1\Pi_S^E = \frac{a}{4}, \tag{46}$$

$$\mathbb{E}_1\Pi_n^E = 0 \quad \forall n \in N. \tag{47}$$

Seller Participation In a final step, we must verify that the expected profits of the seller $\mathbb{E}_1\Pi_S^E$ under exclusivity (Equation (46)) are greater than or at least equal to the expected profits $\mathbb{E}_1\Pi_S$ from the representative auction (Equation (35)). Alternatively, the seller prefers to reject the agreement offers at $t = 1$ and simply lets the auction take place. Figure 3 displays the two respective profit functions.

Intuitively, the second-price auction yields higher pay-offs the more broker-dealers compete. Due to the Bertrand Paradox, exclusive access profits are, of course, independent of N. Hence, it is not surprising that only sellers with few broker-dealer connections prefer to grant exclusive access:

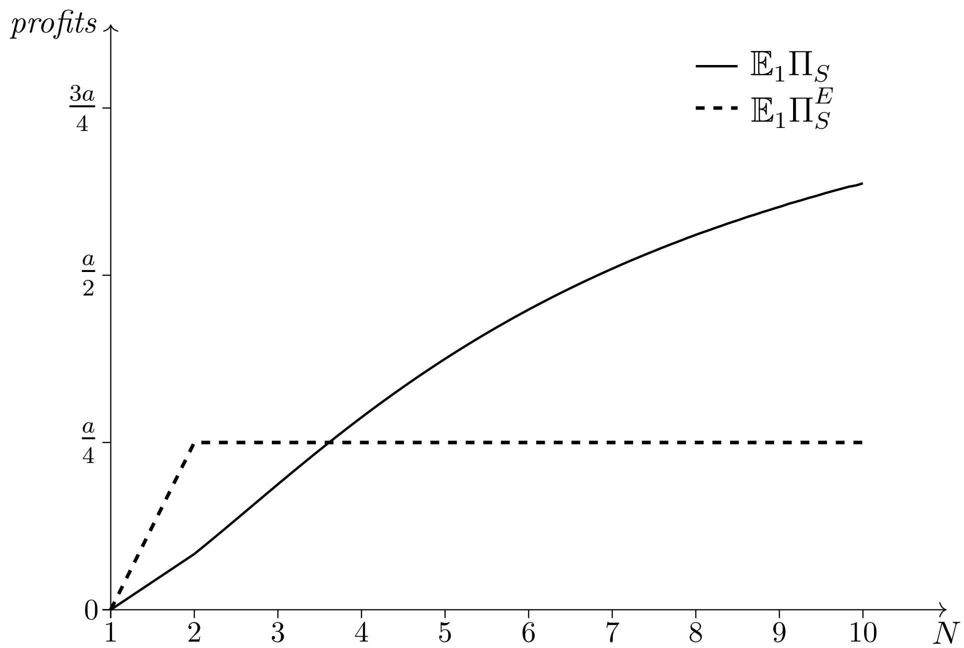
$$\mathbb{E}_1\Pi_S \leq \mathbb{E}_1\Pi_S^E, \tag{48}$$

$$\frac{a}{2^N} \frac{2^N(N-3) + N + 3}{N+1} \leq \frac{a}{4}, \tag{49}$$

$$N \leq 3. \tag{50}$$

Lemma 2. *For $N \leq 3$, competitive exclusive dealing may arise as a sub-game outcome, where:*

Figure 3: Expected Seller Profits



Note: *Figure 3 displays the expected seller profits on the y-axis as a function of the number of broker-dealer connections N on the x-axis. The dashed line represents expected seller profits under exclusivity and the solid line represents expected seller profits under the representative auction.*

$$b_n^{E*} = v, \qquad p_n^{E*} = 0, \qquad T_n^{E*} = \frac{a}{4}, \tag{51}$$

$$\mathbb{E}_1\Pi_S^E = \frac{a}{4}, \qquad \mathbb{E}_1\Pi_n^E = 0 \quad \forall n. \tag{52}$$

Exclusivity arises neither monopolistically nor when $N \geq 4$ as a sub-game outcome.

Off Path Monopoly Off the equilibrium path, we may observe a single (monopolistic) broker-dealer offering an exclusive access agreement. Immediately below, we will see that this is a relevant deviation, wherefore we briefly summarize these (off-path) expected profits here.

In this case, the monopolistic broker-dealer simply charges the profit maximizing combination of p_n^E and T_n^E constrained by the lender's (binding) participation constraint:

$$\mathbb{E}_1\Pi_n^E = \max_{p_n^E, T_n^E} \frac{(a - p_n^E)_2}{4a} - T_n^E, \tag{53}$$

$$s.t.$$

$$\mathbb{E}_1\Pi_S^E = \mathbb{E}_1\Pi_S. \tag{54}$$

Solving the broker-dealer's constraint maximization in (53) above, we find that setting $p_n^{E*} = 0$ is again optimal. Furthermore, optimal lump-sum transfers are set just such that the seller's participation constraint just binds given $p_n^E = 0$. This ensures that the lender agrees to exclusive access and all other broker-dealers are left empty handed. Remark 1 below summarizes the bid-prices, transfer, and resulting expected profits, whenever only a single broker-dealer n requests exclusive access.

Remark 1. *If offered a single exclusive access agreement, the lender always accepts, and the optimal fee and expected profits are:*

$$b_n^{E*} = v, \qquad p_n^{E*} = 0, \qquad T_n^{E*} = \mathbb{E}_1\Pi_S, \tag{55}$$

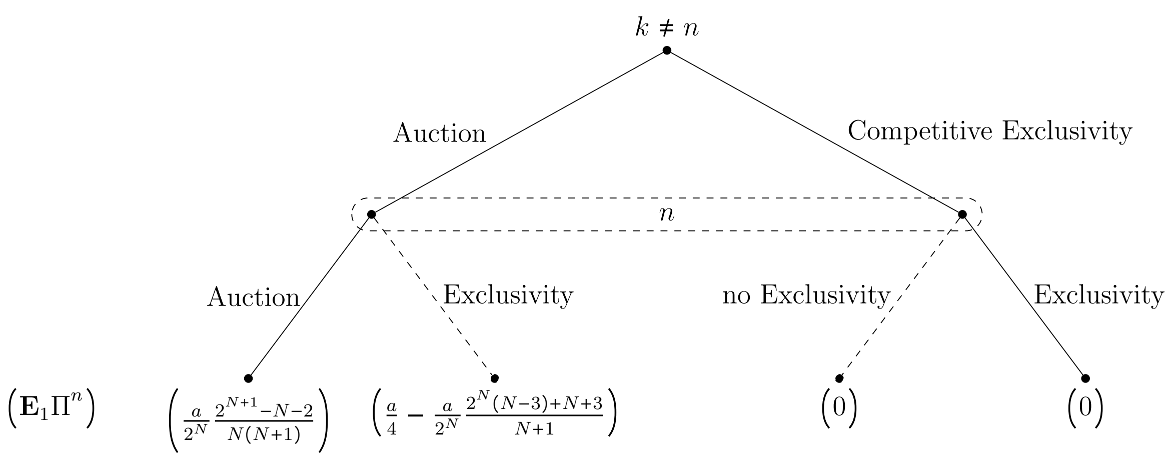
$$\mathbb{E}_1\Pi_S^E = \mathbb{E}_1\Pi_S, \qquad \mathbb{E}_1\Pi_n^E = \frac{a}{4} - T_n^{E*}, \qquad \mathbb{E}_1\Pi_k^E = 0 \quad \forall k \neq n \in N. \tag{56}$$

3.3. The Termination Proof SPNEs

We now turn to deriving the termination proof SPNE in this market. Here, we start by deriving when, depending on N, the auctions versus exclusive dealing are candidate SPNE. Consequently, we explain and apply the equilibrium refinement of termination proofness to any candidate exclusive access SPNE.

Candidate SPNE From the above studied sub-games, we have two types of candidate SPNEs: an auction SPNE and competitive exclusive dealing SPNE. Comparing the above-derived expected profits from auctions versus exclusivity, we now determine the broker-dealers' equilibrium exclusivity offer strategy. For this, we compare a representative broker-dealer n's expected profits when (correctly) anticipating none versus at least one other broker-dealer competing for exclusive dealing. Such expected profits are summarized in Figure 4 below.

Figure 4: A Broker-Dealer's profits given the Others' Choices



The two sub-game outcomes derived above are indicated by the solid branches in Figure 4. Let us start by considering the left branch of the tree. Here, we consider a broker-dealer

n that anticipates that no other broker-dealer $k \neq n$ has offered exclusivity. Conditional on correctly anticipating such, the broker-dealer has a choice between moving onto the auction or offering monopolistic exclusivity. Comparing the two left-most profits in Figure 4: $\mathbb{E}_1[\Pi_n \mid \text{Auctions}]$ and $\mathbb{E}_1[\Pi_n^E \mid \text{Auctions}]$, we reach the following two (in-)equalities:

$$\mathbb{E}_1\Pi_n \geq \mathbb{E}_1[\Pi_n^E \mid \text{Monopoly}] \qquad \forall\ N > 2, \tag{57}$$

$$\mathbb{E}_1\Pi_n = \mathbb{E}_1[\Pi_n^E \mid \text{Monopoly}] \qquad \text{if } N = 2. \tag{58}$$

Intuitively, the two inequalities state that a broker-dealer n, anticipating all other broker-dealers to prefer the auctions, also (weakly) prefers the auctions over offering a monopolistic exclusive access agreement. More formally, conditional on no other broker-dealer offering exclusivity, a single broker-dealer has no incentive to deviate by offering monopolistic exclusive dealing. Because all broker-dealers are symmetric, we can directly conclude from (57) and (58) that there thus always exists an auction candidate SPNE where broker-dealers refrain from competing for exclusive access. Furthermore, combining this with the insights from Lemma 2, we know that the auction SPNE is unique for $N \geq 4$.

***Lemma* 3.** *There always exists a candidate auction SPNE. It is the sole candidate SPNE for $N \geq 4$.*

The second type of candidate SPNEs is characterized by the right branch in Figure 4: At least two broker-dealers compete for exclusive dealing in equilibrium. Comparing the profits in the right branch of Figure 4, a broker-dealer is always indifferent between competing for exclusivity or not, conditional on anticipating at least one other competitive agreement. Again, relying on symmetry, this holds also for all other broker-dealers. Consequently, there exist at most four of such competitive agreement SPNEs: one for each broker-dealer pair and one for all three broker-dealers. Recall from from Lemma 2 that such exclusive access SPNEs only exist for $N \leq 3$, as otherwise the seller rejects any agreement in favor of the auctions.

***Lemma* 4.** *For $N \leq 3$, there exists multiple candidate SPNEs characterized by least two broker-dealers offering competitive exclusive access agreements.*

For completeness, also recall that monopolistic exclusive dealing has been ruled out in Lemma 2. To avoid repetition, this option is not explored further as a candidate SPNE in this section.

Terminations A common concern in the exclusive contracting literature is whether the candidate SPNE is renegotiation proof (Segal and Whinston, 2000): Sellers and broker-dealers renegotiating contract terms after signing but before ask-prices realize. In this model, mutually beneficial renegotiations are not possible as we allow for exclusive pricing via both a quantity dependent bid-premium and a lump-sum transfer. The latter ensures that any sure gain from exclusivity for the broker-dealer always results in a sure loss to the seller and vice versa. [15] We thought of a closely related and more relevant concept in this setting: SPNEs must be termination proof. The concept of termination proof SPNEs requires that neither counterparty has the incentive to single-handedly terminate the exclusive dealing contract in favor of triggering the auction. Crucially, we assume that contracts are binding by law in the sense that one-sided termination triggers counterparty compensation for losses from comparing profits under exclusivity and auctions.

Definition 1. *A termination proof SPNE with exclusive dealing requires that neither of the contracting parties has an incentive to single-handedly end exclusivity when required to compensate the counterparty for expected losses due to termination.*

As the seller always has the opportunity to decline exclusivity offers in favor of the auction at $t = 1$, the seller never has an incentive to terminate *ex post.* For broker-dealers, the Bertrand Paradox ensures that even the exclusive dealer makes zero profits. However, in the auction candidate SPNE, every broker-dealer is expected to make strictly positive profits (see Figure 5).

However, termination of exclusivity not only results in auction profits $\mathbb{E}_1\Pi_n$ but also triggers seller compensation for her losses. The losses are the difference between the seller's expected profits $\mathbb{E}_1\Pi_S^E$ under exclusive access and profits $\mathbb{E}_1\Pi_S$ from the auction.

An equilibrium is, thus, termination proof when the broker-dealer's cost of termination exceeds the benefits:

$$\mathbb{E}_1\Pi_S^E - \mathbb{E}_1\Pi_S \geq \mathbb{E}_1\Pi_n - \mathbb{E}_1\Pi_n^E. \tag{59}$$

Inequalities (60) and (61) consider profit costs versus gains from termination when $N = 3$ and when $N = 2$, respectively:

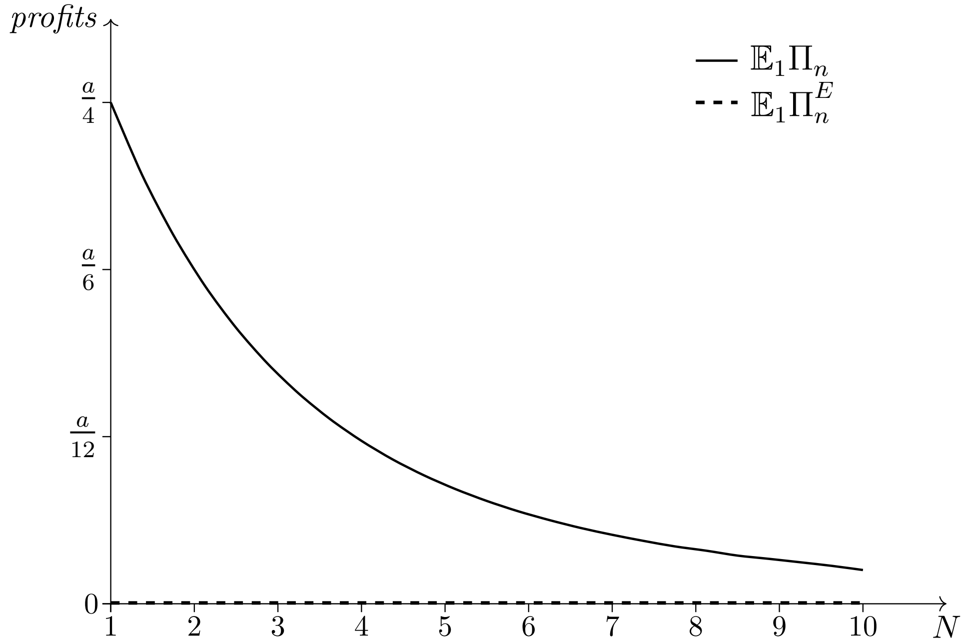
$$N = 3: \qquad \mathbb{E}_1\Pi_S^E - \mathbb{E}_1\Pi_S = \frac{a}{4} - \frac{3a}{16} \leq \frac{11a}{96} - 0 = \mathbb{E}_1\Pi_n - \mathbb{E}_1\Pi_n^E. \tag{60}$$

$$N = 2: \qquad \mathbb{E}_1\Pi_S^E - \mathbb{E}_1\Pi_S = \frac{a}{4} - \frac{a}{12} = \frac{a}{6} - 0 = \mathbb{E}_1\Pi_n - \mathbb{E}_1\Pi_n^E. \tag{61}$$

As inequality (60) highlights, the multiple agreement SPNE is not termination proof for $N = 3$. Here, the seller's expected profits from the auctions are just 25% smaller than those under exclusive access. As compensation is small, the broker-dealer prefers to terminate to terminate the exclusivity contract, thereby triggering an auction SPNE. Therefore, for $N \geq 3$ the auction SPNE is the unique termination proof SPNE.

[15] Note that in a more restrictive environment, where only quantity depending pricing is allowed, i.e. $T_n^E = 0$ by assumption, expected seller profits are an u-shaped function in p_n^E. Thus, the seller profit-maximizing bid-premium leaves strictly positive profits for the broker-dealer. Here, depending on parameters, renegotiation may be mutually beneficial ex post.

Figure 5: Expected Broker-Dealer Profits



Note: *Figure 5 displays a single broker-dealer's expected profits on the y-axis as a function of the representative seller's number of broker-dealer connections N on the x-axis. The dashed line represents expected broker-dealer profits under exclusivity and the solid line represents expected broker-dealer profits under the representative auction.*

However, for $N = 2$, an additional competitive exclusive dealing SPNE exists. As Equation (61) highlights, here the agreement-holder is just indifferent between entering the auctions and paying the punishment or not and hence does not choose to terminate. In Appendix 5.2, we show that the agreement-holders has a strict preference for not terminating in case he *ex post* observes higher ask-prices than anticipated *ex ante*.

Of course, the auction SPNE is termination proof by assumption, as no agreement can be offered or entered into at $t = 2$.

***Proposition* 1.** *There always exists a termination proof auction SPNE, which is unique for $N \geq 3$. For $N = 2$, there is also a termination proof SPNE with both broker-dealers offering competitive agreements for exclusive access.*

For completeness, we would briefly like to comment on the possibility of the exclusive dealer terminating after ask-prices realize. Of course, one may simply argue that such late termination does not allow for sufficient time for the seller to host an auction. In that case, the exclusive dealer gains nothing from terminating, but must still compensate the seller. Hence, he is left with a sure loss and would never terminate.

Alternatively, we could assume that the portfolio has enough different assets so that the law of large numbers holds. In this case, expected and realized profits are approximately equal. Thus, the above-derivative conditions apply.

Mixed Strategy in Exclusive Dealing As we will rely on this in one of the extensions, we would like to briefly mention that there exists a mixed strategy equilibrium in exclusive dealing for $N = 2$. Here, we allow the broker-dealers to offer the following exclusive dealing contract: The bid-premium is always zero. If the seller receives an additional exclusive dealing offer, she gets a lump-sum transfer equal T_n^E to $a/4$, i.e. the price competition case from *Lemma 2*. If the seller does not receive another exclusive dealing offer, the broker-dealer pays a lump sum $T_n^E = a/12$, i.e. the monopolistic exclusive dealing case in Remark 1. Given the resulting expected profits, it is easy to show that each broker-dealer offers exclusivity with a probability one-half. To see this, denote the probability of broker-dealers $n = 1$ and $n = 2$ to offer exclusivity with μ_1 and μ_2, respectively. Then, the broker-dealers' equilibrium probability with which to offer exclusive dealing is:

$$\mu_1\mu_2\mathbb{E}_1\Pi_1^E + \mu_1(1-\mu_2)\mathbb{E}_1[\Pi_1^E \mid \text{Monopoly}] = (1-\mu_1)\mu_2\cdot 0 + (1-\mu_1)(1-\mu_2)\mathbb{E}_1\Pi_1, \quad (62)$$

$$\mu_1\mu_2\cdot 0 + \mu_1(1-\mu_2)\frac{a}{6} = (1-\mu_1)\mu_2\cdot 0 + (1-\mu_1)(1-\mu_2)\frac{a}{6}, \quad (63)$$

$$\mu_1(1-\mu_2)\frac{a}{6} = (1-\mu_1)(1-\mu_2)\frac{a}{6}, \quad (64)$$

$$\mu_1^* = \frac{1}{2} = \mu_2^*. \quad (65)$$

The last equality holds due to symmetry between broker-dealers, therefore $\mu_1 = \mu_2$. Given the equilibrium probabilities, it is easy to compute expected profits under mixed strategies:

$$\mathbb{E}_1\Pi_S^M = \frac{1}{4}\mathbb{E}_1\Pi_S^E + \frac{3}{4}\mathbb{E}_1\Pi_S = \frac{a}{8}, \quad (66)$$

$$\mathbb{E}_1\Pi_n^M = \frac{1}{4}\mathbb{E}_1[\Pi_1^E \mid \text{Monopoly}] + \frac{1}{4}\mathbb{E}_1\Pi_n = \frac{a}{12}. \quad (67)$$

***Lemma* 5.** *For $N = 2$, there exists termination proof SPNE in mixed strategies, where each broker-dealer offers exclusivity with probability $\mu = 1/2$. The expected seller and broker-dealer profits are:*

$$\mathbb{E}_1\Pi_S^M = \frac{a}{8}, \qquad \mathbb{E}_1\Pi_n^M = \frac{1}{12}. \quad (68)$$

For completeness, note that there exists a mixed strategy equilibrium for $N = 2$, where each broker-dealer offers an agreement with probability one-half.[16]

Monopolistic Broker-Dealer For completeness, we also derive the SPNE with a single operating broker-dealer. For $N = 1$, the derivations are rather trivial as the broker-

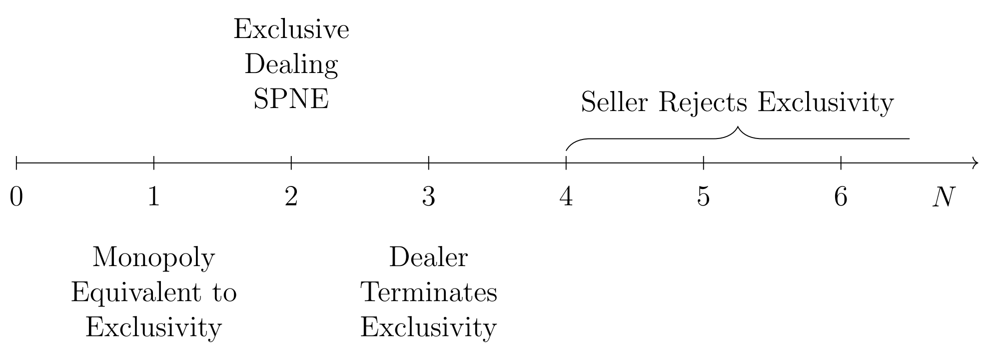
[16] In the mixed strategy SPNE, the uniform agreement bid-price is always zero. Transfers are, however, conditional on the sellers total agreement offers.

dealer is a monopolist and simply ensures that the seller just participates. Hence, the seller makes zero profits from lending and the broker-dealer realizes the entire ask-price for every asset where such is positive. An agreement may be offered and granted, but neither changes profits nor fees. Hence, an agreement comes at no benefit to the seller.

Remark 2. *For $N = 1$, the seller is indifferent between being offered an EAA or not, since the broker-dealer is a monopolist that always extracts all transaction surplus:*

$$b = b^E = v, \qquad T^E = \mathbb{E}_1 \Pi_S = 0, \qquad \mathbb{E}_1 \Pi_n = \frac{a}{4}. \tag{69}$$

Figure 6: Exclusivity in Equilibrium



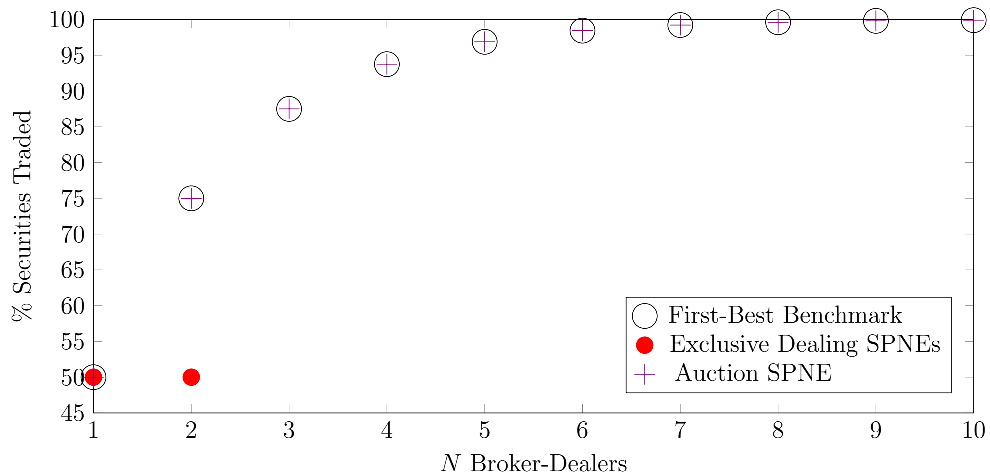
Note: *Figure 6 summarizes when exclusivity may (not) arise in SPNE as a function of broker-dealer connections N.*

4. Market Inefficiency

From a market efficiency perspective, each asset in the seller's portfolio where at least one of the N broker-dealers observes an above value ask-price should be traded. Therefore, the first-best benchmark implies that an asset is traded with probability $1 - 0.5^N$. At the portfolio level, this aggregates to a total of $(1 - 0.5^n) * 100$ percentage of assets traded. The hollow circles in Figure 7 below indicate such percentages as a function of N.

The auction SPNE meets such a first-best benchmark: Every broker-dealer with ask-prices weakly above value v participates in the auction. Thus, all assets with at least one realized ask-price of above v are traded. This is indicated by the blue crosses in Figure 7 overlapping exactly with the first-best. For $N = 2$, there exists also an exclusive access SPNE, where both broker-dealers offer an agreement, and a single one is ultimately selected. Upon realization, the agreement-holder purchases each asset with probability one-half: the probability of an ask-price weakly above value. For the entire portfolio, this implies that 50% of all assets are traded, as indicated by the red dots in Figure 7. Here, it can easily be seen that the exclusive access SPNE does not meet the first-best benchmark.

Figure 7: Percentage of Traded assets in SPNE



Note: *Figure 7 displays the expected share of securities traded on the y-axis as a function of the representative seller's number of broker-dealer connections N on the x-axis. The hollow circles represent trading in efficient market (first-best benchmark), the crosses represent trading in the auction SPNE and the red and filled circles represent trading in the termination proof exclusive dealing SPNE.*

Corollary 1. *The auction SPNE always meets the first-best benchmark.*
The exclusive access SPNE experiences a one-third lower trading volume relative to the first-best for $N = 2$ but meets the first-best benchmark for $N = 1$.

5. Extensions

5.1. Endogenous N

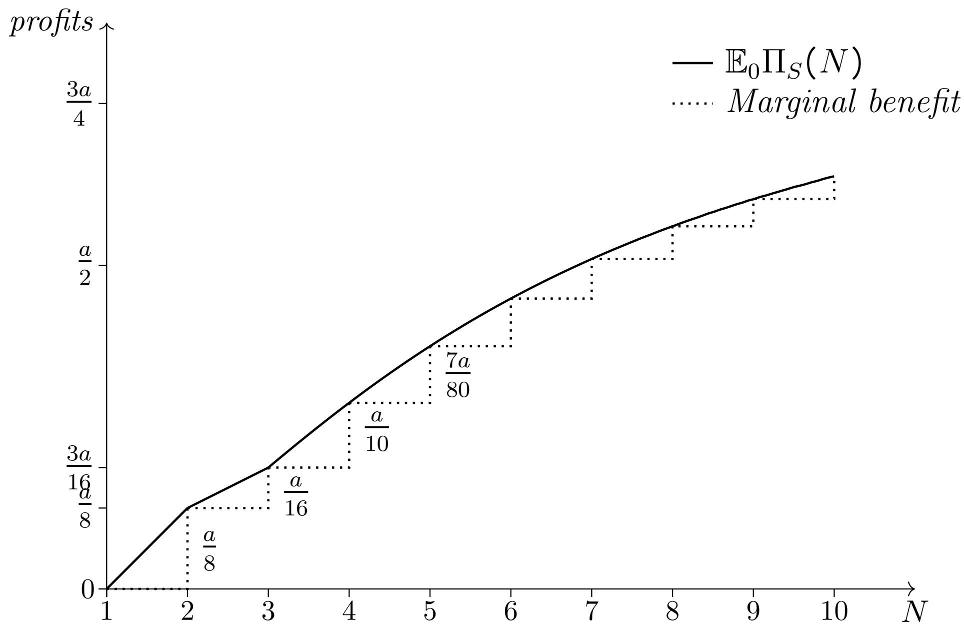
The focus of the baseline model lies on rationalizing when exclusivity arises rather than understanding what determines the seller's number of broker-dealer connections: A valid research question on its own. For tractability, we therefore assumed that the seller's number of broker-dealers is an exogenously given parameter N. In this extension, we would like to relax this assumption in a simple and straightforward fashion to push the limits of the model presented here. It is neither our intention to provide a mechanism for connection building nor suggest definitive determinants.

Imagine a slightly refined period $t = 0$, where the seller is endowed with the asset portfolio, contains the representative asset with value v, and a single broker-dealer connection $N = 1$. Before moving to $t = 1$, the seller must decide how many additional broker-dealer connections to establish. Each additional connection comes at a linear cost $c \geq 0$. Total costs $C(N)$ for N broker-dealers are, thus:

$$C(N) = c \cdot (N - 1) \quad \text{where} \quad c \geq 0. \tag{70}$$

The seller decides how many connections to establish by comparing the marginal costs and marginal benefits of each additional connection. The expected profits, underlying the marginal benefits, are shown in Figure 8. Here, Remark 1 determines the profits under $N = 1$: A monopolistic broker-dealer appropriates all games of trade and leaves the seller with zero profits. For $N = 2$, the termination proof SPNE with exclusive dealing or the auction SPNE may arise. Furthermore, a (unique) mixed strategy SPNE exists. As the (lack of) exclusive dealing is not guaranteed, we assume that the seller anticipates the SPNE in mixed strategies between exclusivity and auction. The associated profits are derived in *Lemma 5*. Finally, the auction SPNE is unique for $N \geq 3$ and the profits follow the function stated in *Lemma 1* deriving the sub-game outcome.

Figure 8: Expected Seller Profits at $t = 0$



Note: *Figure 8 displays the representative seller's expected expected profits on the y-axis as a function of the number of broker-dealer connections N on the x-axis. The solid line represents the expected profits at t = 0, assuming an equilibrium in mixed strategies between exclusive dealing and auctions for N = 2, while the dotted line represents the marginal benefit in profits from increasing N by one.*

Comparing the profits, its apparent that the largest benefit stems from adding a second broker-dealer, i.e. from $N = 1$ to $N = 2$. Intuitively, the seller moves from facing a monopoly to having broker-dealers compete. This puts an upper limit on cost c for connecting to an additional broker-dealer:

$$\mathbb{E}_0\Pi_S(N = 2) - \mathbb{E}_0\Pi_S(N = 1) \geq c, \tag{71}$$

$$\frac{a}{8} \geq c. \tag{72}$$

Thus, for $c > a/8$, the seller does not engage with more than one broker-dealer, while for $c < a/8$, the seller connects with at least one additional broker-dealer or two in total.

The number of total connections depends on the added benefits. Here, we can further see that the benefits from $N = 2$ to $N = 3$ are actually smaller than those from $N = 3$ to $N = 4$.

$$\mathbb{E}_0\Pi_S(N = 3) - \mathbb{E}_0\Pi_S(N = 2) = \frac{a}{16}, \tag{73}$$

$$\mathbb{E}_0\Pi_S(N = 4) - \mathbb{E}_0\Pi_S(N = 3) = \frac{a}{10}. \tag{74}$$

Intuitively, this implies that a seller never engages with $N = 3$ broker-dealers but directly jumps to $N = 4$, whenever:

$$\mathbb{E}_0\Pi_S(N = 4) - \mathbb{E}_0\Pi_S(N = 2) \geq 2c, \tag{75}$$

$$\frac{23a}{80} - \frac{a}{8} \geq 2c, \tag{76}$$

$$\frac{13a}{160} \geq c. \tag{77}$$

For all intermediate levels, $c \in [a/8, 13a/40)$, the seller chooses to connect with two broker-dealers.

Remark 3. *For $N \geq 4$, the expected seller profit functions are strictly concave and have the following properties:*

$$\frac{\partial \mathbb{E}_0\Pi_S(N \geq 4)}{\partial N} > 0, \qquad \frac{\partial^2 \mathbb{E}_0\Pi_S(N \geq 4)}{\partial N^2} < 0, \qquad \lim_{N\to\infty} \mathbb{E}_0\Pi_S(N \geq 4) = a. \tag{78}$$

Now for all $c < 13a/80$, we can utilize the functional property of the expected profits derived in Remark 2: Subtracting the weakly convex cost function from the concave profit function results in a strictly concave function with a global maximum N^*. Furthermore, this global maximum decreases in c, i.e. the slope of the linear function.

Lemma 6. *For a linear cost function for broker-dealer connections $C(N) = c(N - 1)$, the seller chooses:*

- *$N^* = 1$ if $c > a/8$.*
- *$N^* = 2$ if $c \in [a/8, 13a/160)$.*
- *And a unique $N^* \geq 4$ for every $c \in [13a/160, 0]$.*

As a word of caution, we would like to point out here that the parameter c is given by the normalization of the portfolio size to one. Thus, c represents the true cost of connection divided by the seller's portfolio size and is intrinsically dependent on that. It may, of course, depend on further seller characteristics, such as asset types, business model, or location.

5.2. *Advantaged Exclusive Dealer*

In the baseline model, we argue that the appropriate time of concern for termination of exclusivity is after the initial competition stage but before ask-prices realize. Thus, at the stage of termination, broker-dealers are still symmetric. In this extension, we check whether the competitive exclusive dealing SPNE is still termination proof when the exclusive dealer realizes he is in a favorable position after negotiations have taken place.

More explicitly, we ask whether the exclusive dealer terminates when he realizes he enjoys a positively biased ask-price draw and, thus, expects to purchase more on average. We assume that the exclusive dealer e enjoys a cumulative probability of obtaining a draw $\tilde{a}_e$ above 0 that is ϵ above one-half (the baseline). Then:

$$Pr(\tilde{a}_e \geq 0) = \frac{1}{2} + \epsilon, \qquad Pr(\tilde{a}_e < 0) = \frac{1}{2} - \epsilon. \tag{79}$$

To maintain tractability, we assume that, conditional on having a positive or negative draw, the probability density is still uniform. Given this, the refined density functions become:

$$f(\tilde{a}_e) = \begin{cases} \frac{0.5+\epsilon}{a} & \tilde{a}_e \geq 0, \\ \frac{0.5-\epsilon}{a} & \tilde{a}_e < 0 \end{cases}, \tag{80}$$

$$F(\tilde{a}_e) = \begin{cases} 0.5 - \epsilon + \frac{0.5+\epsilon}{a}\tilde{a}_e & \tilde{a}_e \geq 0 \\ \frac{(0.5-\epsilon)(\tilde{a}_e+a)}{a} & \tilde{a}_e < 0 \end{cases}. \tag{81}$$

For all other broker-dealers, we assume that their draws (on average) realize as anticipated, such that realized and expected profits are equal. Furthermore, we explicitly keep the assumption of *ex ante* identical broker-dealers, i.e., we do not have an informed principal at the stage of competition over exclusivity.

Assume that a broker-dealer arrives at $t = 2$ and observes the bias described above in his draws. Furthermore, we assume exclusivity was granted and a contract with the following bid-premium and lump-transfer was signed:

$$p_n^{E*} = 0, \qquad T_n^{E*} = \frac{Sa}{4}. \tag{82}$$

Then, given the revised ask-price distribution, expected profits become:

$$\mathbb{E}_2\Pi_e^E = Pr(\tilde{a}_e \geq 0)\mathbb{E}[\tilde{a}_e - 0 \mid \tilde{a}_e \geq 0] - \frac{a}{4} = \left(\frac{1}{2} + \epsilon\right)\frac{a}{2} - \frac{a}{4} = \frac{a\epsilon}{2}. \tag{83}$$

To derive the auction profits, recall their general functional form:

$$\begin{aligned}\mathbb{E}_1\Pi_e = Pr(\tilde{a}_e = \tilde{a}^1)\Big[& Pr(\tilde{a}^1 \geq \tilde{a}^2 \geq 0)\mathbb{E}[\tilde{a}^1 - \tilde{a}^2 \mid \tilde{a}^1 \geq \tilde{a}^2 \geq 0] \\ & + Pr(\tilde{a}^1 \geq 0 \geq \tilde{a}^2)\mathbb{E}_1\left[\tilde{a}^1 \mid \tilde{a}^1 \geq 0 \geq \tilde{a}^2\right] \\ & + Pr(\tilde{a}^1 < 0) \cdot 0\Big]. \end{aligned} \tag{84}$$

As $\tilde{a}_e$ is still uniformly distributed above zero, the expected profits and their likelihoods within the brackets of equation (84) do not change. However, what changes relative to the baseline model is the likelihood $Pr(\tilde{a}_e = \tilde{a}^1)$ of winning the auction. In the baseline, winning was equally likely for each broker-dealer and occurred with probability $1/N$. Now, we must account for the likelihood of $\tilde{a}_e$ being higher than the largest $\tilde{a}_{1'}$ of $N-1$ draws from our original distribution. The highest value of $N-1$ from a uniform distribution has the following cdf and pdf:

$$F_{1'}(\tilde{a}_{1'}) = \left[\frac{\tilde{a}_{1'} + a}{2a}\right]^{N-1}, \tag{85}$$

$$f_{1'}(\tilde{a}_{1'}) = \frac{N-1}{[2a]^{N-1}}(\tilde{a}_{1'} + a)^{N-2}. \tag{86}$$

Now, the biased broker-dealer wins with the following probability:

$$Pr(\tilde{a}_e = \tilde{a}^1) = Pr(\tilde{a}_e \geq \tilde{a}_{1'}), \tag{87}$$

$$= \frac{1 + \epsilon(2 - 2^{2-N})}{N}. \tag{88}$$

Above, we have shown that the exclusive dealing is only termination proof when $N = 2$. Inserting this into the above probability of winning, expected broker-dealer profits under the representative auction become:

$$\mathbb{E}_2\Pi_e = \frac{1+\epsilon}{2}\frac{a}{3}. \tag{89}$$

To check whether the competitive exclusive dealing SPNE remains termination proof, we must again compare the benefits with the cost from compensating the seller upon termination. Here, we assume that the seller is unaware of the broker-dealer's beneficial draw. Then, we can show that he does not terminate whenever $\epsilon > 0$:

$$\mathbb{E}_2\Pi_e - \mathbb{E}_2\Pi_e^E \leq \mathbb{E}_2\Pi_S^E - \mathbb{E}_2\Pi_S, \tag{90}$$

$$\frac{(1+\epsilon)a}{6} - \frac{a\epsilon}{2} \leq \frac{a}{6}, \tag{91}$$

$$0 \leq \epsilon. \tag{92}$$

From inequality (92), we can conclude that when enjoying a positively biased draw with $\epsilon \geq 0$ and $N = 2$, the broker-dealer strictly prefers not to terminate for $N = 2$.

We have also shown that exclusivity is initially offered but eventually terminated if $N = 3$. Repeating the above exercise, we get the following condition for non-termination:

$$\mathbb{E}_2\Pi_e - \mathbb{E}_2\Pi_e^E \leq \mathbb{E}_2\Pi_S^E - \mathbb{E}_2\Pi_S, \tag{93}$$

$$\frac{1+\frac{3}{2}\epsilon}{3}\frac{11a}{32} - \frac{a\epsilon}{2} \leq \frac{a}{4} - \frac{3a}{16}, \tag{94}$$

$$\frac{10}{63} \leq \epsilon. \tag{95}$$

Notice in the above inequality (95) that only the left-most and right-most terms, i.e., the auction terms, have changed when moving to $N = 3$. Due to the Bertrand Paradox, profits under exclusivity are independent of N. What we can see here is that if ϵ is sufficiently large, then exclusive dealing may even be a termination proof under $N = 3$. Now, of course, whether an increased likelihood by roughly one-six of obtaining an above-value draw is reasonable remains up for debate.

Lemma 7. *For $N = 2$ and an $\epsilon \geq 0$ increase in the likelihood to draw an ask-price above asset value, the exclusive dealing SPNE is termination proof.*
For $N = 3$ and an $\epsilon \geq 10/63$ increase in the likelihood to draw an ask-price above asset value, an additional termination proof exclusive dealing SPNE emerges.

5.3. Endogenous Reserve Price

In the baseline model, we have assumed that the seller agrees to a transaction whenever the bid-price exceeds v. In other words, her participation constraint cannot be violated. From a theoretical perspective, this is equivalent to either assuming an exogenously given auction reserve price r equal to the true value v or the lack of a commitment device to a higher reserve price.

We acknowledge that there exists a literature on optimal auction reserve price setting by sellers. Here, Levin and Smith (1996) establish that in private-value auctions, such as ours, the seller sets an optimal reserve price r^* strictly above her valuation that is independent of the number of bidders:

$$r^* = v + \frac{1 - F_v(r^*)}{f_v(r^*)}. \tag{96}$$

In our setting, private valuations are ask-price draws with uniform distributions symmetric around v with the following cdf and pdf:

$$F(a_n) = \frac{a_n - v + a}{2a}, \tag{97}$$

$$f(a_n) = \frac{1}{2a}. \tag{98}$$

Inserting the functional forms of the ask-price distribution yields the following closed-form expression for the optimal reserve price:

$$r^* = v + \frac{1 - \frac{r^* + a - v}{2a}}{\frac{1}{2a}}, \tag{99}$$

which solving for r^* reduces to:

$$r^* = v + \frac{a}{2}. \tag{100}$$

Because both v and ask-price distributions are common knowledge in this market, the reserve price does not reveal additional information to broker-dealers. Thus, truthful bidding for $a_n \geq r^*$ and non- participation otherwise is still optimal. The broker-dealer's optimal bidding function is:

$$b^{n*} = \begin{cases} a_n & a_n \geq r^* \\ \varnothing & a_n < r^* \end{cases}. \tag{101}$$

Notice that, similar to the ask-price, we can decompose the reserve price into a value component and an uncertainty component $\tilde{r}^*$:

$$\tilde{r}* = r^* - v = \frac{a}{2}. \tag{102}$$

Using this notation, expected seller profits from the representative auction are:

$$\mathbb{E}_1 \Pi_S = Pr(\tilde{a}_2 > \tilde{r}^*)\mathbb{E}[\tilde{a}_2 \mid \tilde{a}_2 > \tilde{r}^*] + Pr(\tilde{a}_1 > \tilde{r}^* > \tilde{a}_2)\tilde{r}^*, \tag{103}$$

$$= \frac{a}{2^{2N+1}} \frac{2^{2N+1}(N-3) + 3^{N-1}(N^2 + N + 18)}{2(N+1)}. \tag{104}$$

These auction profits exceed the exclusive dealing profits for all $N = 3$:

$$\mathbb{E}_1 Pi_S^E \geq \mathbb{E}_1 Pi_S, \tag{105}$$

$$\frac{a}{4} \geq \frac{a}{2^{2N+1}} \frac{1}{2(N+1)} \left[2^{2N+1}(N-3) + 3^{N-1}(N^2 + N + 18)\right], \tag{106}$$

$$2 \geq N. \tag{107}$$

Intuitively, the seller's auction profits must be larger in this extension than in the baseline: Setting a reserve price above v enables the seller to capture profits even in the case of only one broker-dealer bidding. As we show with the inequality, (105) this benefit is sufficiently large for the seller to reject exclusivity at $N \geq 3$ instead of $N \geq 4$ in the baseline.

We are thus left with checking whether for $N = 2$ the exclusive dealing remains negotiation proof by comparing broker-dealers benefits and costs from terminating. The broker-dealer's bid-ask-spread under an endogenous reserve price is:

$$\pi^{n*} = \begin{cases} \tilde{a}^1 - \tilde{a}^2 & \tilde{a}_n = \tilde{a}^1 \ \& \ \tilde{a}^2 \geq \tilde{r}^* \\ \tilde{a}^1 - \tilde{a}^2 & \tilde{a}_n = \tilde{a}^1 \ \& \ \tilde{a}^2 < \tilde{r}^* \\ 0 & otherwise \end{cases}. \tag{108}$$

Accounting for the likelihood of winning is equal to one-half given the two broker-dealers, a broker-dealer's expected profits are:

$$\mathbb{E}_1\Pi_n = \frac{1}{2}Pr(\tilde{a}^1 > \tilde{a}^2 > \tilde{r}^*)\mathbb{E}[\tilde{a}^1 - \tilde{a}^2 \mid \tilde{a}^1 > \tilde{a}^2 > \tilde{r}^*]$$

$$+ \frac{1}{2}Pr(\tilde{a}^1 > \tilde{r}^* > \tilde{a}^2)\mathbb{E}[\tilde{a}^1 - \tilde{r}^* \mid \tilde{a}^1 > \tilde{r}^* > \tilde{a}^2], \tag{109}$$

$$= \frac{5a}{96}. \tag{110}$$

Finally, we can show that exclusive dealing is not termination proof as the benefits out-weight the cost:

$$\mathbb{E}_1\Pi_n - \mathbb{E}_1\Pi_n^E \geq \mathbb{E}_1\Pi_S^E - \mathbb{E}_1\Pi_S, \tag{111}$$

$$\frac{5a}{96} - 0 \geq \frac{a}{4} - \frac{5a}{24}, \tag{112}$$

$$5 \geq 4. \tag{113}$$

***Lemma* 8.** *The auction SPNE is unique whenever a seller can credibly commit to an endogenous auction reserve price r^*, where:*

$$r^* = v + \frac{a}{2}. \tag{114}$$

There are, of course, multiple reasons why a seller setting such a reserve price is not possible. For one, trading platforms typically hosting such auctions are, of course, interested in their trading volume rather than seller profits. Given that above value reserve prices reduce trading frequency, it may not be in their incentive to allow for such. There is also an experimental literature, where, for example, Davis et al. (2011) show that even if possible, sellers typically do not set the rationally optimal reserve price.

6. Application: The Security Lending Market

In this final section, we would like to briefly illustrate how the above derived theoretical insights can be applied to a concrete market: The EU-based equity lending market. The section is organized as follows: First, we confirm theoretically that our model nests the equity lending market; then, we perform a counterfactual exercise assuming away all exclusive dealing. Additionally, the Empirical Appendix A contains a detailed description of the data set, and matches empirical insights with model inputs and outputs.

6.1. A Model for Security Lending

On first thought, equity lending transactions seem to differ slightly from the setup described above, as the underlying asset (stock) is ultimately returned from the borrower to the broker-dealer and then from the broker-dealer to the lender. Thus, bid and ask prices in this market, here denoted b' and a_n', respectively, reflect lending fees rather than purchase prices. Assuming away broker-dealer defaults for simplicity,[17] the participating constraint of a representative equity lender is:

$$b' \geq 0. \tag{115}$$

Ultimate borrowers borrow equity from broker-dealers' predominantly for short-sell motives. As such, broker-dealers' ask prices $a_n{}'$ represent the ultimate borrowers' perceived over/under valuation in the market:

$$a_s^{n\prime} \sim U(-a, a). \tag{116}$$

Given bid and ask prices, a broker-dealer intermediates if:

$$\pi_n' = a_n' - b_n' \geq 0. \tag{117}$$

Combining the lender and broker-dealer' participation constraints, trade happens if:

[17]This is equivalent to assuming that correctly priced collateral is posted for every sale, generating the same expected lender pay-offs Baklanova et al., 2019. Investigating potential collateral mispricing frictions in the security lending market is beyond the scope of this paper.

$$\pi_n' = a_n - b_n' \geq 0, \tag{118}$$

$$b_n' \geq 0, \tag{119}$$

$$a_n' \geq b_n' \geq 0. \tag{120}$$

Comparing equation (120) with its counterpart (9)) in the baseline model, one can easily see that the equity lending market is a special case of the baseline model where $v = 0$. Therefore, the baseline model nests the equity lending market. Note further that no equilibrium outcome characterizing the baseline model depends on v. Therefore, *Proposition 1*, and its preceding and consequent results hold without loss of generality.

Corollary 2. *The equity lending market nests in the model environment described above and is captured by setting $v = 0$. Thus, all baseline results hold without loss of generality.*

Having shown that the security lending market is an application of our model, we can utilize the available data equity lending transactions to predict the aggregate market inefficiency stemming from exclusivity: How much do individual reductions in trading volumes impact aggregate trading volumes?

6.2. Model-Implied Counterfactual Trading

As part of the Security Financing Transactions Regulation (SFTR) database, available to euro zone central banks, we observe all equity lending transactions entered and matured in 2021 with at least one counterparty based in the European Union. For each transaction, the data set contains over 100 contract fields including: counterparty ids (LEIs), lending fee, collateral alsotype and amount, maturity, and, most importantly, an exclusivity flag. The exclusivity flag indicates whether a transaction between two counterparties was covered by an exclusive security lending agreement (ESLA).

After an initial round of data cleaning (see Appendix A), we keep transaction between lenders (corporate entities with $\geq 99\%$ lending transactions) and broker-dealers (corporate entities with at least 100 counterparties and both borrowing and lending).[18] Finally, we aggregate the transactions on a lender level.[19] Table 2 displays the *observed* lending by lenders with and without active ESLAs in 2021, including the average lender's number of ISINs, average value per ISIN and total loan value given ESLA status. Summing up all lenders' total loan values, we obtain a total market size of €$1,144,34$ bn annually.

As a natural follow up question of your inefficiency result in *Corollary 1*— for lenders

[18] We drop transactions by private clients, between broker-dealers and ultimate borrowers and between any market participant and small traders.

[19] For transactions performed by a subsidiary, we utilize the LEI of the ultimate parent. Intra-group trades were dropped.

Table 2: Observed Lending by ESLA Status

	Without ESLA		With ESLA	
	Mean	SD	Mean	SD
Avg. Number of ISINs	83.99	255.90	29.75	103.51
Avg. Volume per ISIN (€ mn)	21.08	790.51	2.15	7.19
Total Trading Volume (€ mn)	347.58	1635.57	110.47	512.32
Number of Lenders	3046		775	

with two (one) broker-dealer connection ESLAs reduce individual lending by 33% (0%) — is to which extend the observed ESLAs reduce aggregate trading in our market. To quantify the aggregate inefficiency, we perform a counterfactual analysis that assumes away all observed ESLAs in the data and subsequently predicts the aggregate increase in total lending volume.

To obtain the counterfactual aggregate lending, we want to predict how many ISINs each lender with an ESLA would have lent out in the absence of such. Based on the results of *Corollary 1*, we assume that one-and-a-half times the original ISINs are traded in the counterfactual with two broker-dealer connections while trading volumes are unchanged in the counterfactual case of a single broker-dealer connection. The core challenge is that neither the model nor the data allow us to infer which of the lenders with an ESLA would have two broker-dealer connections in the counterfactual.

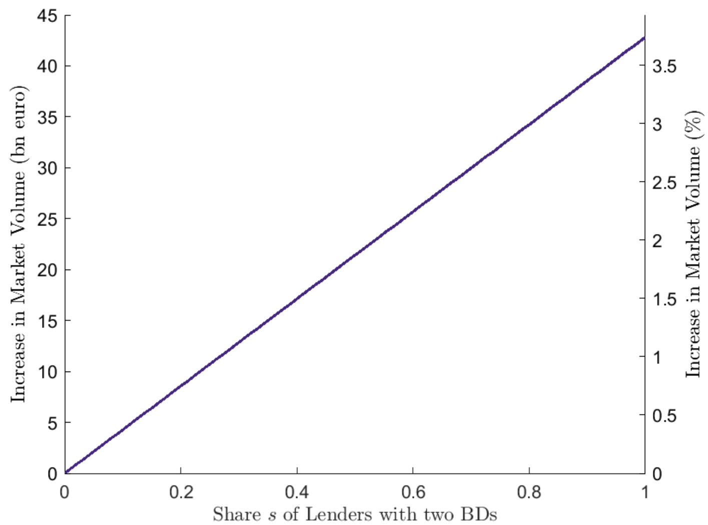
To account for this, we simply obtain predictions for different shares $x \in [0, 1]$ of lenders that we randomly assign two broker-dealers in the counterfactual and the associated trading volume increase. The remaining share $1 - x$ of lenders with ESLAs is then assumed to continue having one broker-dealer connection even in the absence of exclusivity. Formally, assume that a lender is randomly indexed with $l \in \{1, \ldots, l, \ldots, L\}$ and let us denote her total number of traded ISINs with I_l. In addition, let the superscript ^{C} denote the counterfactual. Then:

$$\forall l \leq x \cdot L: \quad I_l^C = I_l \cdot 1.5, \tag{121}$$

$$\forall l > x \cdot L: \quad I_l^C = I_l. \tag{122}$$

Subsequently, we multiply each lender's counterfactual size I^C by her (true) average loan value per ISIN to obtain the counterfactual total portfolio value. Summing over all lenders in the market and subtracting the original total market volume, we obtain the aggregate inefficiency. We repeat this process over 100,000 Monte Carlo simulations to obtain both average inefficiency and bootstrapped standard errors. A detailed description of the algorithm applied can be found in Appendix A.4.

Figure 9: Predicted Aggregate Inefficiency of ESLAs



Note: *Figure 9 displays the predicted increase in market volume in bn euro (left axis) and percentage (right axis) as an average of 100,000 bootstraps in which each a share $s \in [0,1]$ of the 775 lenders with ESLA is randomly treated with two broker-dealer connections and trading via auctions in a counterfactual.*

Figure 9 displays the predicted average reduction in trading volumes due to ESLAs across the $100,000$ Monte Carlo draws. We omit the (bootstrapped) confidence intervals, since they are so narrow that they would be invisible. Needless to say that all predictions are significant. Standard errors can be found in Appendix A.4.

We find that whenever at least some lenders with ESLAs would instead trade with two broker-dealers, the overall trading volume increases significantly. Not surprisingly, this increase in total volume is bigger the more lenders are assumed to have two broker-dealer connections in the counterfactual. What is surprising is the magnitude. Even when only one in ten lenders are assigned two broker-dealers in the counterfactual, the total market volume increases by €4.28 bn. If all lenders with active ESLAs had access to two competing broker-dealers, market volume would even increase by €42.30 bn annually or 3.74%.

Conjecture 1. *In the counterfactual, ruling out ESLAs would likely result in additional lending in the billions of euro.*

7. Conclusion

In this paper, we show that exclusivity over a portfolio is granted to intermediating broker-dealers by sellers with at most two broker-dealer connections. However, this reduces trading volume by up to a third at the seller level. In financial markets, such as the security lending market, these can aggregate to billions of euro in foregone trading volume annually.

To derive these insights, we develop a novel three-period partial equilibrium framework, where a representative seller has access to a small number of identical broker-dealers. The seller is endowed with a representative asset, while the broker-dealers are each endowed with uncertain asset demand. The seller's value of the asset is public knowledge and the broker-dealers are *ex ante* symmetric. Before the demand realizes, broker-dealers may compete in prices for exclusivity. If exclusivity is granted, the exclusive dealer buys the asset when a (weakly) positive bid-ask-spread realizes. When exclusivity was either not granted or terminated early, broker-dealers bid in second-price auctions for those securities, where they realize an ask-price above seller value.

Due to the nature of price competition, the seller's profits under exclusivity are identical if there are at least two broker dealers: The Bertrand Paradox applies and we have full profit pass through. The profits in the second-price auction naturally increase in the number of broker-dealers: More potential participants imply a higher likelihood of at least two bids and a higher expected value of the second-highest bid. The seller thus faces a trade-off between full profit pass through from the exclusive dealer or partial profit pass through in the second-price auction that increases in bidders. Ultimately, this leads to exclusivity being granted by sellers with at most two broker-dealer connections. Here, the equilibria with exclusive dealing and with the auction coexist. For all sellers with more than two broker-dealers, the auction equilibrium is unique.

The auction equilibrium meets the first-best of an efficient market: A security is always traded when at least one of the broker-dealers can realize a positive bid-ask spread. The exclusive dealing equilibrium, however, does not meet the first best: The security is traded only if the exclusive dealer realizes a positive bid-ask spread. Comparing the exclusive dealing and auction equilibrium for sellers with two broker-dealers, we find a 33% lower trading likelihood in the former. Applying this to all sellers in the EU-based security lending market, where we observe exclusivity on the transaction level, we find that exclusivity poses the risk of reducing trading by several billion euros: €4.28 bn if 10% of lenders would face two competing broker-dealers instead and €42.30 bn if 100% of lenders had access to two broker-dealer.

The results are robust to both endogeneizing the number of broker-dealers and introducing an exclusive dealer that surprisingly enjoys a higher ask-prices than anticipated. However, exclusivity is ruled out as an equilibrium, when providing the seller with a commitment devise to her optimal auction reserve price: This price is strictly above the one implied by the participation constraint, thereby, resulting in higher profit-pass through from broker-dealers to the seller. Ultimately, this is good news, as it does not require prohibiting exclusive dealing to improve exclusivity. Simply, allowing sellers to set minimum reserve prices, when auctioning their portfolio on trading platforms, is equally potent.

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Appendix A. Empirical Appendix

A.1. Data Description

We are able to apply the model to the EU-based security lending market, since recently the Securities Financing Transaction Regulation (SFTR) has come into force. As part of this regulation, EU-based counterparties are required to report all transactions regarding repurchase agreements, margin lending, security buy-and-sell back, and securities or commodities lending transaction on a daily basis. The data reported contains ca. 100 contract fields and, in addition, 800 enrichment fields for every single transaction since mid-2020. Thus, it provides a rich source of information for both policy makers and researchers alike. For this paper, the data set is particularly useful, since market participants are required to flag whether the transaction is covered by an exclusive security lending agreement (ESLA).

For this exercise, we focus entirely on the subset of non-cleared equity lending transactions. Simplified, the raw data contains both a daily stock and flow report from execution until maturity submitted for each of their individual transactions. First, we combine all daily reports in a single observation utilizing the unique transaction identifier and collect contract variables of particular interest: loan volume, quantity, prices and fees, collateral, realized maturity, both lending and borrowing side counterparty Legal Entity Identifiers (LEIs) and, most importantly the ESLA-flag. Next, we limit our sample to transactions both entered and matured in 2021 to have consistent reporting standards throughout the sample. Subsequently, we drop all intra-group transactions, where the equity holder and the security receiver have the same parent LEI code. On the remaining transactions, we perform a series of additional quality checks and cleaning steps. A detailed description of the data set, the cleaning procedures and the output generation process can be found in a complementary market analysis (Kessler et al., 2023).[20]

In the market analysis, we identify five different types of market participants to showcase who lends to whom and under which conditions: borrower, broker-dealer, lender, private client, and trader. Types are assigned based on their aggregate trading patterns across all observations in our sample. All types, except private clients, are corporate entities that are identified by their LEI code. For borrowers and lenders, we observe that 99% of all their

[20]Given the explicit focus on ESLAs we perform an additional cleaning step, where we correct for misreporting of the ESLA-flag variable. For lenders with more than one counterparty, we replace ESLA="TRUE" with "FALSE", when less than 10% of the transactions are (falsely) reported to be covered by an ESLA. If the reverse holds, we remove the lender from the sample. For lenders with a single counterparty, we replace ESLA="TRUE" with "False" when less than 40% are covered by an ESLA. Vice versa, we replace ESLA="FALSE" with "TRUE" when more than 60% of transactions are covered by an ESLA. For the remaining misreporting, where lenders have between 40%-60% covered by an ESLA and only one counterparty, we simply drop the lender.

transactions are borrowing and lending, respectively. Broker-dealers and traders engage in both lending and borrowing. Broker-dealers have large trading volumes and more than a 100 counterparties, while traders are typically smaller agents that also own their own portfolio to lend. Private clients are typically natural persons not subject to reporting requirements.

To match the model, we will restrict our sample to lenders, broker-dealers and borrowers. We can consistently remove traders as 91% of the lenders interact only with broker-dealers. Of the remaining lenders, the majority interacts predominantly with broker-dealers. Therefore, we can discard traders without loss of generality. We also disregard private clients, as we cannot determine their total number of counterparties: We only observe the client ID as assigned by the reporting counterparty; therefore, we cannot identify them consistently across our sample. In (Kessler et al., 2023) we show that these private clients have negligible trading volume compared to lenders and can safely discard them.

An important characteristic of the contract, which determines, among others, the fee structure, is the type of collateral underlying. In the equity lending market, we observe three types of collateralization: none, basket, and cash. In the case of cash collateral, a net-rebate rate is reported instead of a lending fee. In (Kessler et al., 2023) we find that only 6.2% of the total transaction volume is collateralized with cash. Given the low market share, we abstract from further analysis of contracts with cash collateral and a rebate rate rather than a lending fee. Instead, we focus on the remaining transactions secured with either a collateral basket (85.9%) or without collateral (7.8%). Ultimately, we are left with 3,624,859 observations. Table 3 shows the characteristics of the market participants per counterparty type.

Table 3: Trading Party Characteristics

Type	Nr. Parties	Avg. Total Loan Value (€ mn)	Avg. Transaction Value (€ mn)	Avg. Nr. Transactions	Avg. Nr. Borrowers	Avg. Nr. Lenders
Borrower	287	3071.15	1.89	5219.29		1.02
Broker-Dealer	37	90254.84	1.29	117584.90	18.28	381.24
Lender	3821	299.49	13.60	366.69	3.59	

Before moving on, please note that we continuously ensure data confidentiality by making sure that each reported statistic contains at least three lender and broker-dealers each, and no two market participants make up more than 85%. Tables, where only aggregated statistics are shown, are derived on data points between the $1s^{st}$ and 99^{th} percentiles to ensure that outliers are not driving the results. All figures are produced using data points between the 10^{th} and 90^{th} percentiles.

A.2. Matching Model Inputs

In the model, we have risk-neutral owners of a security portfolio and a small number of distinct intermediaries with private, independent demand. As demand is uncertain, the portfolio owner must decide upfront whether to sell/lend the securities exclusively or competitively. As we show in the paragraphs below, these assumptions match the security lending market quite well.

Risk-Neutral Lender In in the raw data, we observe two types of lenders: private clients, identified by a broker-dealer issued id, and corporate clients, identified by a LEI. Due to a very small market share and negligible trading volumes (Kessler et al., 2023), we drop private clients from our final sample. Thus, we are left with 3821 registered corporate entities that are typically well-diversified lenders with rather large and frequent trades (see Table 3). Therefore, we see the assumption of risk neutrality to be well justified.

Representative Lender In the model, we further assume that the lender is representative — a short hand for broker-dealers engaging in many lenders that are unlikely to strategically interact with each other and only make up a small portion of a broker-dealer's portfolio. From Table 3, we can see that the average broker-dealer has over 381 lender-counterparties. As such, strategic interaction between lenders is unlikely, especially in the opaque over-the-counter security lending market. Thus, the assumption of representativeness is justified from an empirical perspective.

Small N Additionally, Table 3 shows that lenders without ESLAs trade only with a selected handfull of broker-dealers, and rarely more than three. On the demand side, we predominantly observe single-homing: 92% of all borrowers trade with only a single broker-dealer. Thus, the assumption of private borrowing demand of broker-dealers seems justified.

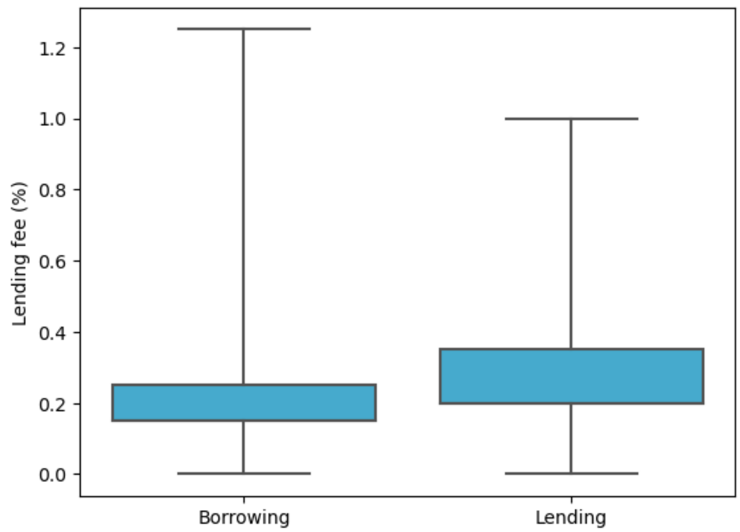
Independent Demand Further, we assume in the model that broker-dealers have independent demand. As we can see from Table 3, the average ultimate borrower has indeed only one broker-dealer connection.

Profitable Intermediation We model broker-dealers as market makers that profit from intermediating between lenders and borrowers. Figure 10 shows that broker-dealers receive higher lending fees for lending, than they pay for borrowing. The median lending fee paid and received by broker-dealers is 0.15 and 0.20 respectively.[21] Thus, we can indeed assume that they engage in profitable intermediation. As a caveat, we would like to mention that not all stocks that are borrowed by broker-dealers from ultimate lenders are lend out to ultimate borrowers and not all lend out stocks are initially borrowed. This could

[21] Note here that the whiskers of the two boxplots in Figure 10 indicate the 10^{th} and 90^{th} percentiles to ensure confidentiality.

indicate that they use some of the borrowed stocks for short-selling while simultaneously lending out of their own inventory. For this paper, we focus on their role as market makers, which should on average be profitable.

Figure 10: Broker-Dealer Lending Fee Distribution



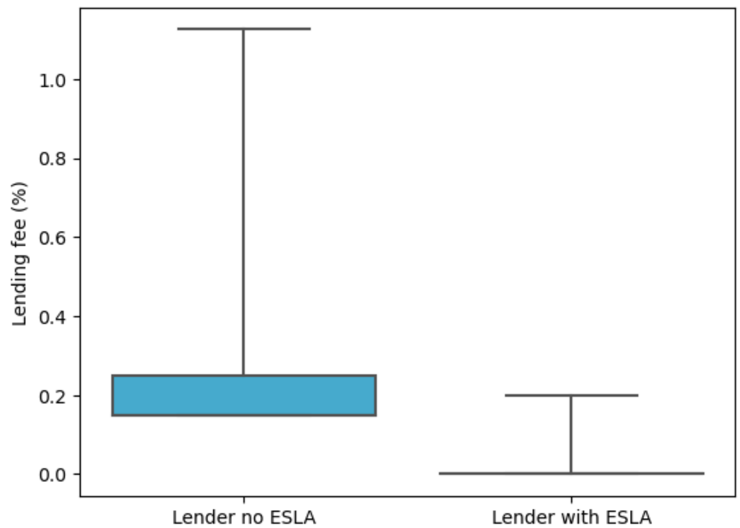
Note: *Figure 10 is based on 37 broker dealers with 2,126,924 borrowing and 2,223,718 lending transactions.*

Timing The timing difference between ESLA and auction offers cannot be inferred directly from the data. Here, we must rely on anecdotal evidence from practitioners highlighting the typically lengthy bilateral negotiations required to set up Master agreements, such as the Global Master Securities Lending agreement (GMSLA) provided by the International Capital Market Association (ICMA) that cover the exclusive access (ICMA, n.d.). Negotiating such complex legal documents must be compared to the relative ease with which large-scale lending is conducted over trading platforms where portfolio holders can announce individual stock level auctions. Examples of such trading platforms are the BlackRock Security Lending platform, Sharegain and FIS Securities Lending Platform. All three exclusively serve institutional clients. In a complimentary paper on corporate bonds, Hendershott et al. (2021) show that sellers predominantly sell to their existing broker-dealer connections, even when selling via an auction platform.

A.3. Matching Model Outcomes

Naturally, our analysis leads to the question: To what extent are reported fees fitting the model outcomes? All lenders in our final sample receive a lending fee for each transaction from the winning broker-dealer. Lending fees are typically reported as an annualized percentage of loan value rather than euro or dollar amounts. Figure 11 displays the box plots for lending fees with and without ESLAs, where the whiskers represent the 10th and 90th percentile.

Figure 11: Lending Fee Distribution by ESLA



Note: *Figure 11 is based on 1,246,039 transactions of 3046 lenders without ESLAs and 155,102 transactions of 775 lenders with ESLAs.*

To match the observed fees with the model prices expressed in units, we transform the observed the lending fee (f^*) expressed as a percentage paid on loan value (V) given q units of stick into a bid price b^* per single unit of asset:

$$b^* = \frac{f}{100}\frac{V}{q}. \tag{123}$$

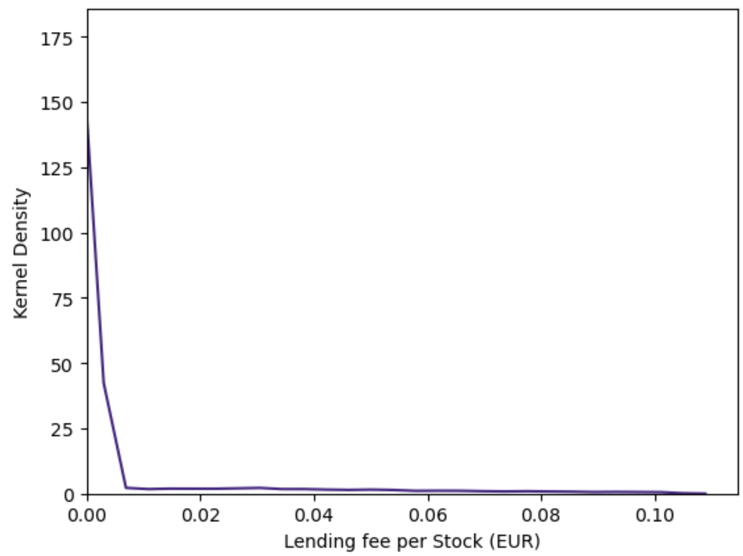
To verify the model prices fit the data, we only focus on comparing equilibrium outcomes for $N \leq 2$ with contract terms from lenders with at most two counterparties, as we have shown that ESLAs arise only for sellers with two or less counterparties. Furthermore, in the case of lenders with three or more broker-dealers theoretical price predictions are cumbersome and without any substantial additional insights. Given this selection, we derive three model implications that we match with realized and observed lending using visual output.

Exclusive Security Lending Agreements We start by considering the subset of transactions by exclusivity-granting lenders. From Lemma 3, we know that lenders with ESLAs should pay zero bid-prices. Of course, this simplifies reality, as we assume in the model that lenders face zero marginal transaction costs. Softening our expectations slightly, the first model implication (M1) states that per-unit lending fees should be close to zero when covered by ESLAs.

M1 Per-unit lending fees of transactions under ESLAs should be close or equal to zero.

Testing *M1* statistically using conventional methods is difficult, as the lending fees do not follow a normal distribution: They have both a clear cutoff and a mass point at zero. To verify *I1*, we instead rely on visual output by estimating the Kernel density over all per-unit lending fees paid by lenders with ESLA. As Figure 12 below shows, the transaction fees under exclusive agreements are indeed very close to zero. In fact, 84% of all transactions have zero lending fee and all are less than 11 cents.

Figure 12: Kernel Density of the Per-Unit Fees under Exclusivity



Note: *Figure 12 is based on 155,102 transactions of 775 lenders with ESLAs.*

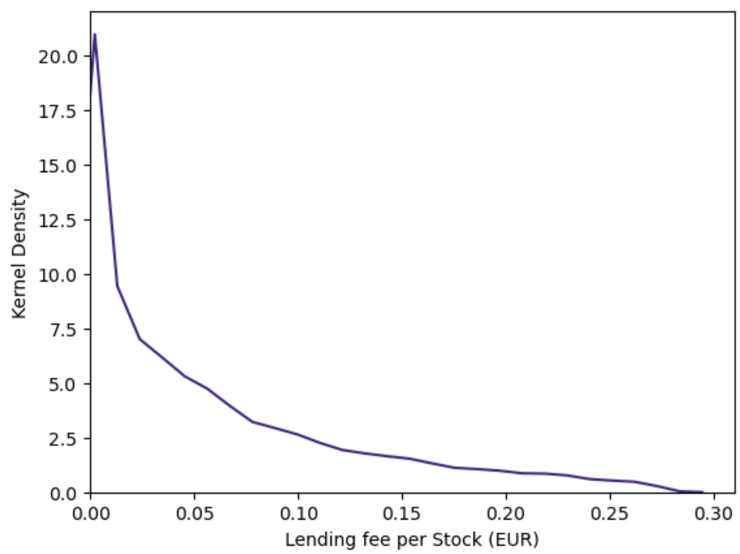
Auctions In the absence of agreements, a lender's realized fee distribution depends on the number of broker-dealer connections. As the number of lender with exactly N broker-dealers decreases sharply for $N \geq 3$, we abstract from those cases. Instead, we focus on the cases $N = 1$ and $N = 2$ separately.

For $N = 1$, the baseline model predicts that transactions have a zero lending fee, since the single broker-dealer acts as a monopolist. Testing such hypothesis in the data is challenging though, as we are only observing the realized number of counterparties and not the (hypothetically) available number. As such, we adjust our expectations and the second model implication (M2) states that lenders with a single active broker-dealer connection should receive predominantly zero lending fees.

M2 Outside ESLAs, transactions of lenders with a single broker-dealer have more often than not zero bid-prices.

To verify *M2*, we estimate the Kernel density over all per-unit lending fees paid by lenders with a single broker-dealer connection. Displayed in Figure 13, we observe that fees are likely to be zero or close to zero. However, we also see that a non-negligible number of lenders indeed makes a profit. This could be due to nonzero lending costs or relational factors outside the security lending market. In the latter case, their restriction to one broker-dealer might be by choice. A lender requesting offers from multiple broker-dealers, but ultimately selecting one, allows her to nevertheless enjoy the benefits of (some) competition without paying e.g. on-boarding costs. Unfortunately, the data does not show how many counterparties they considered ex ante.

Figure 13: Kernel Density of Per-Unit Fees without exclusivity and $N = 1$



Note: *Figure 13 is based on 112,130 transactions of 1010 lenders without ESLAs and* $N = 1$.

For lenders with two broker-dealers ($N = 2$), obtaining the distribution of realized bid-prices b^* we can rely on *Lemma 1*: The broker-dealers bid truthfully in the second price auction. Thus, the realized bid price is equal to the second highest draw a_2 whenever $a_2 > 0$ and is equal to zero when a_2 is smaller or equal to zero. Relying on the derived distributional properties of the second highest draw (see Section 3.1), this leads to the following theoretical bid price pdf:

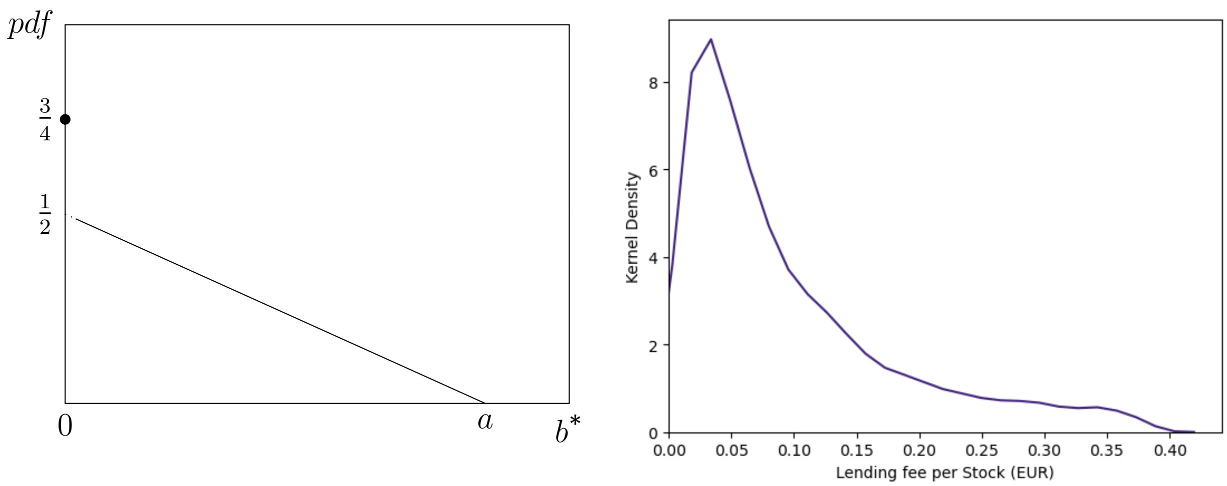
$$pdf(b^*) = \begin{cases} \frac{a-b^*}{2a^2} & b^* > 0 \\ \frac{3}{4} & b^* = 0 \end{cases}. \tag{124}$$

The density function, shown in Figure 14a below, takes the value $3/4$ at exactly zero. This is cumulative density of broker-dealer 1 drawing an ask price below 0 but not broker-dealer 2, broker-dealer 2 drawing an ask price below 0 but not broker-dealer 1, and both broker-dealer's drawing an ask price below 0. In all three cases, the second highest draw is below or equal to zero.

For a marginal increase in bid prices, we observe an immediate jump downwards to 1/2. This jump reflects that a zero bid-price is more frequent relative to all other positive bid-prices due to the absence of second bids whenever one of the two broker-dealers draws a negative ask-price. For every other value above zero, the density is a linearly decreasing function with a slope equal to $-1/2a^{-2}$. The model implication *M3* below summarizes such behavior.

M3 *For transactions of lenders trading with two broker-dealers, zero is the most frequent realized bid-price, followed by an immediate downward jump and consequent decrease.*

Figure 14: Bid-Price Distributions without exclusivity and $N = 2$



(a) PDF of Bid-Prices

(b) Kernel Density ($N = 2$)

Note: *Figure 14b is based on 29,011 transactions of 494 lenders without ESLAs and* $N = 2$.

Notice here that *M3* intentionally does not describe the decrease in density for increasing fees as linear, although the theoretical model predicts this. Again, this is motivated by the fact that we only observe the realized but not the initially available number of broker-dealers. In reality, some lenders could access offers from three (or more) broker-dealers, but chose to engage with two for simplicity. This increases the density of fees the closer they are to zero. Figure 14b displays the estimated Kernel densities using the transactions of sellers who trade with two broker-dealers.

As a small caveat, we did not estimate a discrete Kernel with a jump at zero, but rather relied on the standard assumption of continuity common to available statistical packages. Nevertheless, we can clearly see that values just above zero indeed have the highest density. Furthermore, the density declines for higher values. However, such a decline is convex rather than linear. This is additional evidence that some sellers may choose to let several broker-dealers compete but ultimately only engage with two.

A.4. Aggregate Market Inefficiency

In this section, we describe our algorithm to compute the predicted increase in trading due to the counterfactual assumption of eliminating ESLAs. In a first step, we calculate the portfolio size of each lender in our data. Here, we decided to measure portfolio size by the number of distinct ISINs a lender trades the average annual volume per ISIN. This aggregation of trades on an ISIN level is motivated by the fact that loans are often split into multiple identical transactions with the same borrower. The portfolios of the lenders without ESLAs are temporarily put aside. For those with ESLAs, we perform the following bootstrapping algorithm:

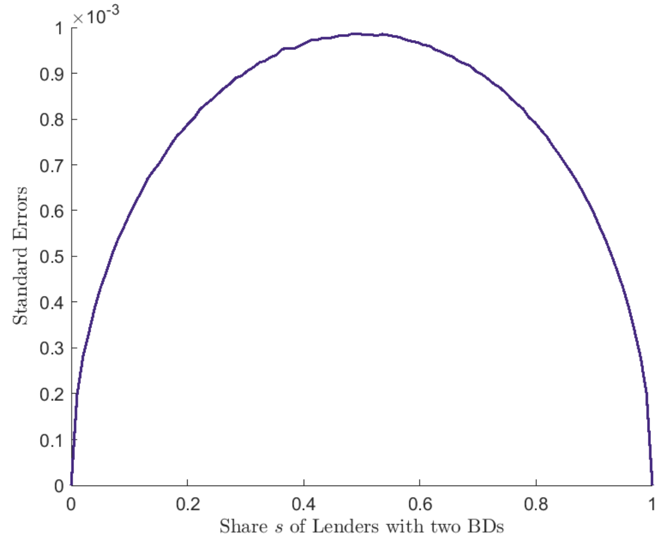
1. We define a grid of 100 values denoted x falling in equal steps between zero and one: $x \in [0, 1]$. Each x represents the share of lenders having two versus one connected broker-dealers in the counterfactual.
2. For each share x defined in the grid, we repeat the following 100,000 times:
   (a) Draw a random number from a standard uniform distribution for each lender.
   (b) Sort the lenders by their draw.
   (c) Assign the first x share of lenders two counterfactual broker-dealers and the remaining $1 - x$ one.
   (d) For those where we assign two broker-dealers, we follow Corollary 1 and predict a 50% increase in the traded portfolio size.
   (e) We compute the counterfactual aggregate portfolio by summing the counterfactual portfolios of the ESLA holders and the actual portfolios of the non-ESLA holders.
   (f) We obtain the percentage increase in the counterfactual market volume (MV^C relative to the true market volume (MV):

$$\%MV = \frac{MV^C - MV}{MV} \times 100. \tag{125}$$

3. From the 100,000 bootstraps, we calculate the average predicted increase in total portfolio and the bootstrapped standard errors. With both, we can obtain the 5^{th} and 95^{th} confidence intervals.

Figure 9 in the main text shows the predicted increases in traded portfolios in euro and as percentage of market size, but omits confidence intervals due to their extremely small size. Figure 15 below plots the standard errors of the percentage increase. Note that, as

Figure 15: Bootstrapped Standard Errors around Aggregate Predictions



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expected, the standard errors increase for medium shares x around 0.5. Here, there simply is higher variation between the bootstraps when assigning one versus two counterparties at random. Nevertheless, the standard errors (se) always remain below 0.001.

We can use the above standard errors to test for significance of the predictions. Here, we test:

H0: The predicted percentage increase in market value is equal to zero.

Ha: The predicted percentage increase in market value is not equal to zero.

To test such hypothesis, we perform a t-test with the following t-statistic for each $x \in (0,1)$:[22]

$$t_x = \frac{\%MV - 0}{se}. \tag{126}$$

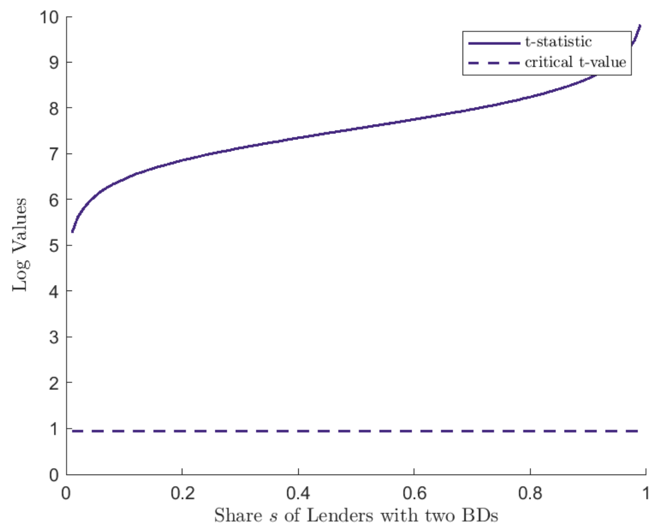
The critical $t - value$ is taken from a Student-t distribution with $100,000 - 1$ degrees of freedom and assuming a 1% significance level:

$$t - crit = 2.58. \tag{127}$$

The lowest t-value from the bootstrapping is equal to 194.84 at $x = 0.0101$. All other t-values are substantially larger. To display the t-statistics and critical t-value in the same Figure, the natural logarithm was taken. As one can see from Figure 16, all predictions are significant at the 1% level.

[22] Please note that testing significance for $x = 0$ and $x = 1$ is trivial, as there is no uncertainty. All lenders are assigned $N = 1$ and $N = 2$, respectively.

Figure 16: Bootstrapped Standard Errors around Aggregate Predictions



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Appendix B. Proofs

This section derives the results presented in Sections 2 and 4 in consecutive order. For ease of reading, note that superscript $_n$ denotes broker-dealer related variables, superscript $_S$ denotes seller related variables, subscript ^{E} and denotes everything related to exclusive dealing. Further, we decompose the ask-price into a value and uncertainty component:

$$a_n = v + \tilde{a}_n \quad \text{where} \quad \tilde{a}_n \sim U(-a, a). \tag{128}$$

Finally, we sort the broker-dealer draws of $\tilde{a}_n$ in descending order such that:

$$\tilde{a}_1 > \tilde{a}_2 > \ldots > \tilde{a}_n. \tag{129}$$

Here, and in all further sections, we apply the assumption that the likelihood of two equal draws is zero.

B.1. Auction Sub-Game Outcome

Proof for *Lemma 1*:

1. We start by assuming that we have arrived at $t = 2$ without any exclusive dealing.

2. Next, recall that a risk-neutral seller only provides the security when receiving a weakly above value bid-price: $b \geq v$.

This lower limit on the price results in broker-dealers participating only when $a_n \geq 0$, as they would otherwise realize the loss:

$$\pi_n = a_n - b_n \geq 0, \tag{130}$$

$$a_n \geq b_n \geq v. \tag{131}$$

.

Therefore, the optimal bidding strategy whenever $a_n < v$ is abstinence and:

$$b_n = \varnothing \quad \forall a_n < v. \tag{132}$$

3. If $a_n \geq v$, truthful bidding is the optimal strategy.

3.1. First, assume that all other broker-dealers $k \neq n$ draw a below value ask-price $a_k < v$ and thus $b_k = \varnothing$. Then, n is indifferent between truthful bidding, overbidding and underbidding (above v). In either case, he would always pay zero and make a profit $\pi_n = a_n - v \geq 0$. Underbidding below v is not optimal, as it would leave a sure profit on the table.

3.2. Second, assume that at least one other broker-dealer k participates in the auction bidding a generic $b_k < a^n$. Here, the broker-dealer find truthful bidding optimal. Conditional on winning, n is indifferent between truthful bidding or bidding slightly less: He would win, pay b_k regardless and make a positive return. However, underbidding below b_k is not optimal, as he would lose and forego a profitable transaction. Bidding truthfully maximizes the chance of winning without risking any losses or extra cost.

3.3 Finally, assume that n participates in the auction bidding a generic $b_k > a^n$. Then the broker-dealer n is indifferent between truthful bidding and underbidding: He would lose regardless of making zero profits. With sufficient overbidding, the broker-dealer would win the auction. However, because $b_k > a^n$, he would make a loss. This is less optimal than the zero profits from truthful bidding.

3.4. With the same arguments applied for all other (symmetric) broker-dealers, we can conclude that the optimal bidding strategy is:

$$b_n = \begin{cases} a_n & a_n \geq v \\ \varnothing & otherwise \end{cases}. \tag{133}$$

3.5. Given truthful bidding, the bid ask-spread can be rewritten as follows:

$$\pi_n = \begin{cases} v + \tilde{a}_1 - \tilde{a}_2 & \tilde{a}_n = \tilde{a}_1 > \tilde{a}_2 \geq v \\ v + \tilde{a}_1 - v & \tilde{a}_n = \tilde{a}_1 > v \geq \tilde{a}_2 \\ 0 & otherwise \end{cases}. \tag{134}$$

4. As we can see from Equation (134), the bid-ask spread ultimately depends on the

distribution of the second highest draw of the uncertainty component (second order statistics). Let us denote the cdf and pdf of $\tilde{a}_2$ with $F_2(\tilde{a}_2)$ and $f_2(\tilde{a}_2)$ respectively. Applying well-established order static properties, these distributions are:

$$F_2(\tilde{a}_2) = \left[\frac{\tilde{a}_2 + a}{2a}\right]^{N-1} \left[N + (1-N)\frac{\tilde{a}_2 + a}{2a}\right], \tag{135}$$

$$f_2(\tilde{a}_2) = N(N-1)\frac{1}{2a}\left[\frac{\tilde{a}_2 + a}{2a}\right]^{N-2}\left[1 - \frac{\tilde{a}_2 + a}{2a}\right]. \tag{136}$$

Furthermore, the spread depends on the conditional distribution of the highest draw $\tilde{a}_1$ given the second highest draw $\tilde{a}_2$. Because our ask-price draws are iid from a uniform distribution, by assumption $\tilde{a}_1$ must be uniformly distributed above $\tilde{a}_2$. Denote the cdf and pdf of conditional distribution of $\tilde{a}_1$ given $\tilde{a}_2$ with $F_{1|2}(\tilde{a}_1)$ and $f_{1|2}(\tilde{a}_1)$ respectively. Here:

$$F_{1|2}(\tilde{a}_1) = \frac{\tilde{a}_1 - \tilde{a}_2}{a - \tilde{a}_2}, \tag{137}$$

$$f_{1|2}(\tilde{a}_1) = \frac{1}{a - \tilde{a}_2}. \tag{138}$$

5. With the optimal bids and order statistic distributions in mind, we now turn to calculating the broker-dealers' expected profits.

5.1. The equation below states the expected broker-dealer profits from a representative broker-dealer n:

$$\begin{aligned}\mathbb{E}_1\Pi_n = \frac{1}{N}\Big[& Pr(\tilde{a}_1 > \tilde{a}_2 \geq 0)\mathbb{E}[\tilde{a}_1 - \tilde{a}_2 \mid \tilde{a}_1 > \tilde{a}_2 \geq 0] \\ & + Pr(\tilde{a}_1 \geq 0 \geq \tilde{a}_2)\mathbb{E}_1\left[\tilde{a}_1 \mid \tilde{a}_1 \geq 0 \geq \tilde{a}_2\right] \\ & + Pr(\tilde{a}_1 < 0)\cdot 0\Big].\end{aligned} \tag{139}$$

First, note that $1/N$ represents the probability that a broker-dealer n has the highest draw (i.e., $\tilde{a}_n = \tilde{a}_1$).

The second term is the expected difference between the first and second highest draws of the uncertainty component, conditional on both draws being above value. The third term is the expected value of the highest draw of the uncertainty component conditional on being above v but second highest draw being below v. The third term is the

5.2. A closed-form expression for the first term in Equation (139) can be derived in the following steps.

First, notice that the first term can be separated into two terms:

$$Pr(\tilde{a}_1 > \tilde{a}_2 \geq 0)\mathbb{E}[\tilde{a}_1 - \tilde{a}_2 \mid \tilde{a}_1 > \tilde{a}_2 \geq 0] =$$

$$Pr(\tilde{a}_1 > \tilde{a}_2 \geq 0)\mathbb{E}[\tilde{a}_1 \mid \tilde{a}_1 > \tilde{a}_2 \geq 0] - Pr(\tilde{a}_1 > \tilde{a}_2 \geq 0)\mathbb{E}[\tilde{a}_2 \mid \tilde{a}_1 > \tilde{a}_2 \geq 0]. \tag{140}$$

Deriving the first term in (140):

$$Pr(\tilde{a}_1 > \tilde{a}_2 > 0)\mathbb{E}[\tilde{a}_1 \mid \tilde{a}_1 > \tilde{a}_2 > 0], \tag{141}$$

$$= Pr(\tilde{a}_1 > \tilde{a}_2 \geq 0) \int_0^a f_2(\tilde{a}_2 \mid \tilde{a}_2 \geq 0) \int_{\tilde{a}_2}^a \tilde{a}_1 f_{1|2}(\tilde{a}_1) d\tilde{a}_1 d\tilde{a}_2, \tag{142}$$

$$= Pr(\tilde{a}_1 > \tilde{a}_2 > 0) \int_0^a f_2(\tilde{a}_2 \mid \tilde{a}_2 \geq 0) \frac{1}{a - \tilde{a}_2} \int_{\tilde{a}_2}^a \tilde{a}_1 d\tilde{a}_1 d\tilde{a}_2, \tag{143}$$

$$= Pr(\tilde{a}_1 > \tilde{a}_2 > 0) \int_0^a f_2(\tilde{a}_2 \mid \tilde{a}_2 \geq 0) \frac{1}{a - \tilde{a}_2} \left[\frac{a_2 - \tilde{a}^{2_2}}{2} \right] d\tilde{a}_2, \tag{144}$$

$$= Pr(\tilde{a}_1 > \tilde{a}_2 > 0) \int_0^a f_2(\tilde{a}_2 \mid \tilde{a}_2 \geq 0) \frac{a + \tilde{a}_2}{2} d\tilde{a}_2, \tag{145}$$

$$= Pr(\tilde{a}_1 > \tilde{a}_2 > 0) \int_0^a \frac{f_2(\tilde{a}_2)}{1 - F_2(0)} \frac{a + \tilde{a}_2}{2} d\tilde{a}_2, \tag{146}$$

$$= Pr(\tilde{a}_1 > \tilde{a}_2 > 0) \frac{1}{1 - F_2(0)} \frac{N(N-1)}{[2a]_n} \frac{1}{2} \int_0^a (\tilde{a}_2 + a)^{N-2} (a - \tilde{a}_2)(a + \tilde{a}_2) d\tilde{a}_2, \tag{147}$$

$$= Pr(\tilde{a}_1 > \tilde{a}_2 > 0) \frac{1}{1 - F_2(0)} \frac{N(N-1)}{[2a]_n} \frac{1}{2} \frac{a^{N+1}(2^{N+1} - N - 2)}{N(N+1)}. \tag{148}$$

Now, let us compute the probability:

$$Pr(\tilde{a}_1 > \tilde{a}_2 > 0) = \int_0^a f_2(\tilde{a}_2) \int_{\tilde{a}_2}^a f_{1|2}(\tilde{a}_1) d\tilde{a}_1 d\tilde{a}_2, \tag{149}$$

$$= \int_0^a f_2(\tilde{a}_2) 1 d\tilde{a}_2 = (1 - F_2(0)). \tag{150}$$

Finally, we can derive the first term in (140)

$$Pr(\tilde{a}_1 > \tilde{a}_2 > 0)\mathbb{E}[\tilde{a}_1 \mid \tilde{a}_1 > \tilde{a}_2 > 0] = \frac{a}{2^{N+1}} \frac{(N-1)(2^{N+1} - N - 2)}{N+1}. \tag{151}$$

Let us turn to the second term in Equation (140) and notice that by definition $\tilde{a}_1 > \tilde{a}_2$. Then:

$$Pr(\tilde{a}_1 > \tilde{a}_2 \geq 0)\mathbb{E}[\tilde{a}_2 \mid \tilde{a}_1 > \tilde{a}_2 \geq 0], \tag{152}$$

$$= Pr(\tilde{a}_2 \geq 0)\mathbb{E}[\tilde{a}_2 \mid \tilde{a}_2 \geq 0], \tag{153}$$

$$= (1 - F_2(0)) \int_0^a b_2 \frac{f_2(b_2)}{1 - F_2(0)} db_2, \tag{154}$$

$$= \frac{a}{2^N} \left[\frac{(2^N(N-3) + N + 3)}{(N+1)} \right]. \tag{155}$$

5.3. A closed-form expression for the second term in Equation (139) can be derived in the following steps.

$$Pr(\tilde{a}_1 \geq 0 \geq \tilde{a}_2)\mathbf{E}[\tilde{a}_1 \mid \tilde{a}_1 \geq 0 > \tilde{a}_2], \tag{156}$$

$$= Pr(\tilde{a}_1 \geq 0 \geq \tilde{a}_2) \int_{-a}^{a} f_2(\tilde{a}_2 \mid \tilde{a}_2 < 0) \int_{0}^{a} \tilde{a}_1 f_{1|2}(\tilde{a}_1 \mid \tilde{a}_1 \geq 0) d\tilde{a}_1 d\tilde{a}_2, \tag{157}$$

$$= Pr(\tilde{a}_1 \geq 0 \geq \tilde{a}_2) \int_{-a}^{0} f_2(\tilde{a}_2 \mid \tilde{a}_2 < 0) \frac{1}{1 - F_{1|2}(0)} \int_{0}^{a} \frac{\tilde{a}_1}{a - \tilde{a}_2} d\tilde{a}_1 d\tilde{a}_2, \tag{158}$$

$$= Pr(\tilde{a}_1 \geq 0 \geq \tilde{a}_2) \int_{-a}^{0} f_2(\tilde{a}_2 \mid \tilde{a}_2 < 0) \frac{1}{1 - F_{1|2}(0)} \frac{1}{a - \tilde{a}_2} \frac{a_2}{2} d\tilde{a}_2, \tag{159}$$

$$= Pr(\tilde{a}_1 \geq 0 \geq \tilde{a}_2) \int_{-a}^{0} f_2(\tilde{a}_2 \mid \tilde{a}_2 < 0) \frac{a - \tilde{a}_2}{a} \frac{1}{a - \tilde{a}_2} \frac{a_2}{2} d\tilde{a}_2, \tag{160}$$

$$= Pr(\tilde{a}_1 \geq 0 \geq \tilde{a}_2) \frac{a}{2} \int_{-a}^{0} \frac{f_2(\tilde{a}_2)}{F_2(0)} d\tilde{a}_2, \tag{161}$$

$$= \int_{a}^{0} f_2(\tilde{a}_2) \int_{0}^{a} f_{1|2}(\tilde{a}_1) d\tilde{a}_1 d\tilde{a}_2 \frac{a}{2}, \tag{162}$$

$$= \int_{-a}^{0} f_2(\tilde{a}_1)(1 - F_{1|2}(0)) d\tilde{a}_2 \frac{a}{2}, \tag{163}$$

$$= \int_{-a}^{0} f_2(\tilde{a}_1) \frac{a}{a - \tilde{a}_2} d\tilde{a}_2 \frac{a}{2}, \tag{164}$$

$$= \frac{a_2}{2} \frac{N(N-1)}{[2a]_n} \int_{-a}^{0} (\tilde{a}_2 + a)^{N-2} d\tilde{a}_2, \tag{165}$$

$$= \frac{a_2}{2} \frac{N(N-1)}{[2a]_n} \frac{a^{N-1}}{N-1}, \tag{166}$$

$$= \frac{N}{2^N} \frac{a}{2}. \tag{167}$$

5.4. Bringing everything together by adding/subtracting and dividing by N, a broker-dealer's expected profits from the auction are:

$$\mathbb{E}_1 \Pi_n = \frac{1}{N} Pr(\tilde{a}_1 > \tilde{a}_2 \geq 0)[\tilde{a}_1 - \tilde{a}_2 \mid \tilde{a}_1 > \tilde{a}_2 \geq 0] + \frac{1}{N} Pr(\tilde{a}_1 \geq 0 > \tilde{a}_2)\mathbb{E}[\tilde{a}_1 \mid \tilde{a}_1 > 0 > \tilde{a}_2], \tag{168}$$

$$= \frac{1}{N} \left[\frac{a}{2^{N+1}} \frac{(N-1)(2^{N+1} - N - 2)}{N+1} - \frac{a}{2^N} \left[\frac{(2^N(N-3) + N + 3)}{(N+1)} \right] + \frac{aN}{2^{N+1}} \right], \tag{169}$$

$$= \frac{a}{2^N N(N+1)} \left[\frac{(N-1)(2^{N+1} - N - 2)}{2} - 2^N(N-3) - N - 3 + \frac{N(N+1)}{2} \right], \tag{170}$$

$$= \frac{a}{2^N N(N+1)} \frac{1}{2} \left[(N-1)(2^{N+1} - N - 2) - 2^{N+1}(N-3) - 2N - 6 + N(N+1) \right], \tag{171}$$

$$=\frac{a}{2^N N(N+1)}\frac{1}{2}\Big[2^{N+1}(N-1-N+3)+(N-1)(-N-2)-2N-6+N(N+1)\Big], \tag{172}$$

$$=\frac{a}{2^N N(N+1)}\frac{1}{2}\Big[2^{N+2}-N_2-2N+2+N-2N-6+N_2+N\Big], \tag{173}$$

$$=\frac{a}{2^N N(N+1)}\frac{1}{2}\Big[2^{N+2}-2N-4\Big], \tag{174}$$

$$=\frac{a}{2^N N(N+1)}(2^{N+1}-N-2). \tag{175}$$

6. Finally, we derive the seller's expected profits:

$$\mathbb{E}_1\Pi_S = Pr(\tilde{a}_2 \geq 0)\mathbb{E}_1[\tilde{a}_2 \mid \tilde{a}_2 \geq 0] + Pr(\tilde{a}_2 < 0)\cdot 0, \tag{176}$$

$$=\frac{a}{2^N}\left[\frac{(2^N(N-3)+N+3)}{(N+1)}\right]. \tag{177}$$

B.2. Exclusive Dealing

Next, we move on to the proof for *Lemmas 2* and *3*. Before doing so, we briefly discuss the general properties of the seller's profits under exclusivity. As highlighted above, an ask-price draw can be decomposed into a fixed and an uncertain component:

$$a_n = v + \tilde{a}_n \quad \text{where} \quad \tilde{a}_n \sim U(-a,a). \tag{178}$$

Further, the bid-price under exclusivity is composed of a value component and a fixed bid-premium:

$$b_n^E(v) = v + p_n^E. \tag{179}$$

Assume that the seller has granted exclusivity with a given uniform bid-premium p_n^E. Then, the lending happens with probability:

$$Pr(\tilde{a}_n \geq p_n^E) = 1 - Pr(\tilde{a}_n < p_n^E) = 1 - \frac{p_n^E + a}{2a} = \frac{a - p_n^E}{2a}. \tag{180}$$

Given this, the seller expects a total payoff:

$$\mathbb{E}_1\Pi_S = (Pr(\tilde{a}_n > p_n^E)p_n^E + T_n^E = S\frac{a - p_n^E}{2a}p_n^E + T_n^E. \tag{181}$$

We can similarly derive the expected profits of the exclusive dealer and show that they are strictly decreasing in both p_n^E and T_n^E:

$$\mathbb{E}_1\Pi_n^E = Pr(\tilde{a}_n > b_n^E)\mathbb{E}_1\left[\tilde{a}_n - p_n^E \mid a_n > p_n^E\right] - T_n^E, \tag{182}$$

$$= \frac{a - p_n^E}{2a}\frac{a - p_n^E}{2} - T_n^E, \tag{183}$$

$$\frac{\partial \mathbb{E}_1\Pi_n^E}{\partial p_n^E} = S\frac{p_n^E - a}{2a} < 0, \tag{184}$$

$$\frac{\partial \mathbb{E}_1\Pi_n^E}{\partial T_n^E} = -1. \tag{185}$$

Proof for *Lemma 2*:

1. We start by establishing that the Bertrand Paradox holds: For $N \geq 2$ at least two broker-dealers offer competitive bid-premia and transfer combination that maximizes seller profit while leaving all broker-dealers with zero profits. Any other combination of prices cannot be sustained.

1.1. Assume that one broker-dealer sets a bid-premium and transfer combination that allows him to make strictly positive profits. Then, any of the other broker-dealers can offer the same bid-price and marginally higher lump-sum transfer. This would allow him to attract the seller instead while still making a positive, albeit slightly lower, profit. This rules out a monopolistic broker-dealer winning the exclusive dealing and making a profit.

1.2. Next, assume that two broker-dealers compete and the winning broker-dealer offers an agreement where bid-premium and lump-sum transfer is such that he would make a profit. Then the losing broker-dealer has an incentive to slightly underbid by offering an epsilon higher lump-sum transfer and, thereby, winning over the seller. Hence, this can also not be an equilibrium.

1.3. This logic of underbidding continues until two or more broker-dealers offer bid-premium and transfers that leave all of them with zero profits. Only here does there not exist an incentive to deviate. Both broker-dealers that have offered exclusivity and those that have not are indifferent to changing action, as they make zero profits regardless. Raising prices will not make them win the exclusive dealing, not offering exclusivity leaves them with losing to someone else, and offering/winning the exclusive dealing leaves them with zero profits. This is commonly referred to as the Bertrand Paradox.

1.5. For completeness note that a single broker-dealer offering exclusivity with terms that lead to zero profits is not an equilibrium strategy either. Correctly anticipating that no other broker-dealer enters, he has an incentive to reduce the lump-sum transfer and retain some profits. In return, per point 1.1, however, this is also ruled out as an equilibrium.

1.6. Concluding, any exclusive dealing equilibrium must have two or more broker-dealer competing and offering such bid-premium and lump-sum transfers, that they are left with zero profits. In other words, there is full profit pass through.

2. Next, we show that there exists a unique p_n^E and T_n^E combination that maximizes seller profits.

2.1. First, note that the lump sum transfer T_n^E must necessarily equate the chosen broker-dealer's expected profits to zero given the offered bid-premium p_n^E:

$$\mathbb{E}_1\Pi_n^E = \frac{a-b_n^E}{2a}\frac{a-b_n^E}{2} - T_n^E = 0, \tag{186}$$

$$T_n^E = \frac{a-b_n^E}{2a}\frac{a-b_n^E}{2}. \tag{187}$$

2.2. Inserting this into the seller's profit function, we can see that the seller profits are highest at $p_n^E = 0$:

$$\mathbb{E}_1\Pi_S^E = \max_{p_n^E} S\frac{a-p_n^E}{2a}p_n^E + \frac{a-p_n^E}{2a}\frac{a-p_n^E}{2}, \tag{188}$$

$$\frac{\partial\mathbb{E}_1\Pi_S^E}{\partial p_n^E} = -2\frac{S}{4a}p_n^E = 0, \tag{189}$$

$$p_n^E = 0. \tag{190}$$

2.2. Inserting the solution back into the broker-dealers zero profit condition yields the the following transfers:

$$T_n^E = \frac{a-p_n^E}{2a}\frac{a-p_n^E}{2} = \frac{a}{4}. \tag{191}$$

3. Having derived optimal bid-price and lump-sum transfer, we can calculate the expected broker-dealer seller profits:

3.1. Trivially, all broker-dealers make zero profits. This is regardless whether they offered and subsequently granted the exclusive dealing or not.

$$\mathbb{E}_1\Pi_n^E = 0 \quad \forall n. \tag{192}$$

3.2. The seller then expects the following profit:

$$\mathbb{E}_1\Pi_S^E = T_n^E = \frac{a-p_n^E}{2a}\frac{a-p_n^E}{2} = \frac{a}{4}. \tag{193}$$

4. Finally, notice that the auction pay-offs serve as a participation constraint for the seller. She may always reject the exclusivity offer and try her luck at the auctions. Hence, exclusive dealings are only ever accepted when expected payoffs exceed those from the S auctions:

$$\mathbb{E}_1\Pi_S^E \geq \mathbb{E}_1\Pi_S, \tag{194}$$
$$\frac{a}{4} \geq \frac{a}{2^N}\frac{2^N(N-3)+N+3}{N+1}, \tag{195}$$
$$3 \geq N. \tag{196}$$

Hence, sellers accept competitive exclusive dealing only if connected with less than four broker-dealers.

Proof for Remark 1:

1. For completeness, we also derive optimal fee terms in case of a single exclusive dealing offer. Notice that those will be ruled out as potential SPNE.

$$p_n^E = 0, \qquad T_n^E = \mathbb{E}_1\Pi_S^E = \mathbb{E}_1\Pi_S, \qquad \mathbb{E}_1\Pi_n^E = \frac{a}{4} - T_n^E, \qquad \mathbb{E}_1\Pi_k^E = 0 \quad \forall k \neq n \in N. \tag{197}$$

2. We start by assuming only one broker-dealer offering exclusivity and correctly anticipating the other broker-dealer not to. Note that this is happens only off the equilibrium path.

3. Being a monopolist, the broker-dealer then sets the lowest combination of bid-price and transfer possible that still motivate the seller to grant the exclusive dealing. Hence, he equates the sellers participation constraint:

$$S\frac{a-b_n^E}{2a}p_n^E + T_n^E = \frac{a}{2^N}\frac{2^N(N-3)+N+3}{N+1}, \tag{198}$$
$$T_n^E = \frac{a}{2^N}\frac{2^N(N-3)+N+3}{N+1} - S\frac{a-p_n^E}{2a}p_n^E. \tag{199}$$

4. Inserting (199) into the sellers profit maximization problem yields:

$$\mathbb{E}_1\Pi_n^E = \max_{p_n^E \geq 0} \frac{a-p_n^E}{2a}\frac{a-p_n^E}{2} - \frac{a}{2^N}\frac{2^N(N-3)+N+3}{N+1} + S\frac{a-p_n^E}{2a}b_n^E. \tag{200}$$

And the FOC wrt. p_n^E is equal to:

$$\frac{\partial \mathbb{E}_1\Pi_n^E}{\partial p_n^E} = -\frac{S}{a4}2p_n^E = 0, \tag{201}$$
$$p_n^E = 0. \tag{202}$$

5. An optimal bid-price p_n^E implies a lump-sum transfer:

$$T_n^E = \frac{a}{2^N}\frac{2^N(N-3)+N+3}{N+1}. \tag{203}$$

6. The respective total payoffs are thus:

$$\mathbb{E}_1\Pi_S^E = \frac{a}{12}, \tag{204}$$

$$\mathbb{E}_1\Pi_n^E = \frac{a}{4} - \frac{a}{2^N}\frac{2^N(N-3)+N+3}{N+1}. \tag{205}$$

7. Because the transfers take the participation constraint into account, the sellers always accept.

8. For the broker-dealers, the profits from exclusive dealing decrease in N. Here, offering a single exclusive dealing is only optimal as long it is better than the auction payoffs.

$$\mathbb{E}_1\Pi_n^E = \frac{a}{4} - \frac{a}{2^N}\frac{2^N(N-3)+N+3}{N+1} \geq \frac{a}{2^N}\frac{2^{N+1}-N-2}{N(N+1)}, \tag{206}$$

$$N \leq 2. \tag{207}$$

B.3. Candidate SPNEs

Proof for *Lemma 3*:

1. Let us start by assuming we are at $t = 1$ and no other broker-dealer offers exclusivity. Then, a representative broker-dealer n does not have an incentive to deviate by offering monopolistic exclusivity instead:

$$\mathbb{E}_1\Pi_n \geq \mathbb{E}_1[\Pi_n^E \mid \text{Monopoly}], \tag{208}$$

$$\frac{a}{2^N N(N+1)}(2^{N+1}-N-2) \geq \frac{a}{4} - \frac{a}{2^N}\frac{2^N(N-3)+N+3}{N+1}. \tag{209}$$

Lets start by showing that the above equation holds with strict equality for $N = 2$:

$$\frac{1}{2_2 2(2+1)}(2^3-2-2) = \frac{1}{4} - \frac{1}{2_2}\frac{2_2(2-3)+2+3}{2+1}, \tag{210}$$

$$\frac{4}{24} = \frac{1}{4} - \frac{1}{12}, \tag{211}$$

$$\frac{2}{12} = \frac{3}{12} - \frac{1}{12}, \tag{212}$$

$$0 = 0. \tag{213}$$

Next, let us show that it holds with strict equality for $N > 0$. For this purpose, we rearrange the inequality to:

$$\mathbb{E}_1\Pi_n \geq \mathbb{E}_1[\Pi_n^E \mid \text{Monopoly}], \tag{214}$$

$$\frac{a}{2^N N(N+1)}(2^{N+1}-N-2) \geq \frac{a}{4} - \frac{a}{2^N}\frac{2^N(N-3)+N+3}{N+1}, \tag{215}$$

$$4N(N+2) + 2^N(N(3N-13)+8) - 8 \geq 0. \tag{216}$$

We know that for $N = 2$, the above equation holds with equality. Then, one can simply treat it as a function of N, where it is sufficient to show that it is strictly increasing for $N > 2$.

$$f(N) = 4N(N + 2) + 2^N(N(3N - 13) + 8) - 8, \tag{217}$$

$$f'(N) = 8(N + 1) + 2^N(-13 + 6N) + 2^N(8 + N(-13 + 3N))log(2) > 0 \quad \forall N \approx> 1.6537. \tag{218}$$

The exact analytical expression is quite complex and does not add value in this study. The interested reader is encouraged to plug in the equation in appropriate software such as Wolfram Mathematica.

2. Now, let us move to $t = 2$. By assumption, we have ruled out the possibility that the exclusivity can be entered at $t = 2$. Hence, arriving at $t = 2$ without exclusive dealing automatically triggers the auctions.

1. From *Lemma 1*, we know that both the seller and all broker-dealers make a strictly positive profit from the auctions. Therefore, no one has an incentive to not participate, conditional on all other agents participating.

2. From *Lemma 2*, we know that the seller rejects any competitive exclusive dealings for $N \geq 4$.

3. Combining all three, there always exists a candidate auction SPNE and that it is the only candidate for $N \geq 4$.

Proof for *Lemma 4*:

1. From *Lemma 2*, we know that the seller rejects any competitive exclusive dealing for $N \geq 4$, but accepts those for $N \leq 3$.

2. Here, conditional on some or all other broker-dealers making a competitive exclusive dealing, a single exclusive dealing is indifferent between making also a competitive exclusive dealing offer or not: he would be left with zero profits in either case.

3. As all broker-dealers are symmetric, this holds for every broker-dealer. Hence, an candidate SPNE with competitive exclusive dealings exist.

4. Note, here for $N = 2$ is requires both broker-dealers making a competitive exclusive dealing. For $N = 3$, it only requires two out of three making one.

B.4. Termination Proof SPNEs

Finally, we turn to test whether our candidate SPNEs are termination proof. This requires that the exclusive dealer has no incentive to terminate the exclusive dealing at $t = 2$, compensating the seller for potential losses.

Proof for *Proposition 1*:

1. By assumption, the auction SPNE is termination proof. Once all agents have arrived at $t = 2$, no further exclusive dealing can be offered and/or accepted. Hence, in the absence of exclusive dealings being offered at $t = 1$ the auctions remain unchallenged at $t = 2$.

2. Assume instead that exclusivity was granted at $t = 1$. The exclusive dealer can terminate the contract at $t = 2$, triggering the auction SPNE. However, he must compensate the seller for the profit losses due to breach of contact. The exclusive dealer thus terminates if:

$$\mathbb{E}_1\Pi_n - \left(\mathbb{E}_1\Pi_S^E - \mathbb{E}_1\Pi_S\right) \leq \mathbb{E}_1\Pi_n^E. \tag{219}$$

3. Let us start by assuming $N = 3$ and insert this into (219). It can easily be shown that the inequality is violated:

$$\frac{Sa11}{96} - \left(\frac{a}{4} - \frac{Sa3}{16}\right) > 0 = \mathbb{E}_1\Pi_n^E. \tag{220}$$

This implies that for $N = 3$, the exclusive dealer terminates the exclusive dealing and triggers the auction SPNE instead. Hence, for $N = 3$ exclusive dealings are not termination proof. Hence, the auction SPNE is the sole SPNE.

4. Inserting $N = 2$ into (219) instead yields in an equality:

$$\frac{a}{6} - \left(\frac{a}{4} - \frac{a}{23}\right) = 0 = \mathbb{E}_1\Pi_n^E. \tag{221}$$

Hence, for $N = 2$, the equilibrium is termination proof.

5. Just to provide some intuition, the punishment for termination decreases faster than the benefits from termination. And hence, for $N = 3$, the exclusive dealer gains slightly less from termination than for $N = 2$, but pays substantially less punishment.

Finally, we conclude with studying the case of a single connected broker-dealer and $N = 1$.

Proof for Remark 2: For $N = 1$, the seller is indifferent between being offered exclusivity or not, as the broker-dealer is a monopolist that always extracts all transaction surplus:

$$b = b^E = v, \qquad T^E = 0 \mathbb{E}_1\Pi_S = 0, \qquad \mathbb{E}_1\Pi_n = \frac{a}{4}. \tag{222}$$

Proof:

1. Starting with the auctions at $t = 2$. Given that there is no competitive bid, the broker-dealer always pays a bid-prices equal to v such that the seller's participation constraint is

just binding. He realizes the whole ask-price premium as profits for every purchased security. This leads to:

$$\Pi_n = Pr(a_n > 0)\mathbb{E}_1[a_n \mid a^0] = \frac{a}{4}, \tag{223}$$

$$\Pi_S = 0. \tag{224}$$

2. Because the auction profits serve as the sellers outside value, the broker-dealer can offer exclusivity with zero bid-premium and lump-sum transfer. The profits of both agents remain the same.

B.5. Proof for the Extension: Endogenous Reservation Price

1. We assume that the reserve price is public knowledge, but does not impact the information set available to broker-dealers: v remains public knowledge. Therefore, broker-dealers bit truthfully in the following fashion:

$$b^{n*} = \begin{cases} a_n & a_n \geq r \\ & a_n < r \end{cases}. \tag{225}$$

Again, we can sort the bids in descending order, such that the bid-ask spread is, thus:

$$\pi^{n*} = \begin{cases} a^1 - a^2 & a_n = a^1 \,\&\, a^2 \geq r^* \\ a^1 - r^* & a_n = a^1 \,\&\, a^2 < r^* \\ 0 & otherwise \end{cases}. \tag{226}$$

2. Given truthful bidding, we can express the seller's expected profits as:

$$\mathbb{E}_1\Pi_S = Pr(a^1 < r)v + Pr(a^1 > r > a^2)r + Pr(a^1 > a^2 > r)\mathbb{E}_1[a^2 \mid a^1 > a^2 > r]. \tag{227}$$

In a next step, we can insert the (general) the cdf and pdf of the second and first order statistics, respectively:

$$F_2(a^2) = [F(a^2)]^{N-1}[N + (1-N)F(a^2)], \tag{228}$$

$$f_2(a^2) = N(N-1)f(a^2)\left[F(a^2)\right]^{N-2}\left[1 - F(a^2)\right], \tag{229}$$

,

$$F_1(a^1) = [F(a^1)]_n, \tag{230}$$

$$f_1(a^1) = Nf(a^1)[F(a^1)]^{N-1}. \tag{231}$$

Inserting this into the above seller profit function, yields:

$$
\begin{aligned}
\mathbb{E}_1\Pi_S =&[F(r)]_n v \\
&+ N\Big[F(r)\Big]^{N-1}\Big(1 - F(r)\Big)r \\
&+ N(N-1)\int_r^a a_n f(a_n)\Big[F(a_n)\Big]^{N-2}\Big[1 - F(a_n)\Big]da_n, \qquad (232)\\
=&[F(r)]_n v \\
&+ Nr\Big[F(r)\Big]^{N-1} - Nr\Big[F(r)\Big]^{N} \\
&+ N(N-1)\int_r^a a_n f(a_n)\Big[F(a_n)\Big]^{N-2}\Big[1 - F(a_n)\Big]da_n. \qquad (233)
\end{aligned}
$$

Then, the optimal reserve price, r^*, can be derived taking the first-order condition:

$$
\begin{aligned}
0 =&N[F(r)]^{N-1}f(r)v \\
&+ N\Big[F(r)\Big]^{N-1} + N(N-1)\Big[F(r)\Big]^{N-2}f(r)r \\
&- N\Big[F(r)\Big]^{N} - N^2\Big[F(r)\Big]^{N-1}f(r)r \\
&- N(N-1)f(r)\Big[F(r)\Big]^{N-2}\Big[1 - F(r)\Big]r. \qquad (234)
\end{aligned}
$$

Dividing both sides by $N\Big[F(r)\Big]^{N-2}$ yields:

$$
\begin{aligned}
0 =&F(r)f(r)v \\
&+ F(r) + (N-1)f(r)r \\
&- \Big[F(r)\Big]^2 - NF(r)f(r)r \\
&- (N-1)f(r)\Big[1 - F(r)\Big]r \qquad (235)\\
=&F(r)f(r)v + F(r) - \Big[F(r)\Big]^2 - F(r)f(r)r. \qquad (236)
\end{aligned}
$$

Rearranging for r^* yields:

$$
r^* = v + \frac{1 - F(r^*)}{f(r^*)}. \qquad (237)
$$

The above result is well-established and first derived by Levin and Smith, 1996. Relying on the functional properties of $a_n \sum U(v-a, v+a)$, we obtain a closed form solution:

$$r^* = v + \frac{1 - \frac{r^*-v+a}{2a}}{\frac{1}{2a}}, \tag{238}$$
$$r^* = v + \frac{a}{2}. \tag{239}$$

3. Conveniently, we can again decompose the optimal reserve price into a value component and an uncertainty component $\tilde{r}$:

$$\tilde{r}^* = r^* - v = \frac{a}{2}. \tag{240}$$

With this, we can rewrite seller profits as:

$$\mathbb{E}_1 \Pi_S = Pr(\tilde{a}_2 > \tilde{r}^*)\mathbb{E}[\tilde{a}_2 \mid \tilde{a}_2 > \tilde{r}^*] + Pr(\tilde{a}_1 > \tilde{r}^* > \tilde{a}_2)\tilde{r}^*. \tag{241}$$

Where the first term becomes:

$$Pr(\tilde{a}_2 > \tilde{r}^*)\mathbb{E}[\tilde{a}_2 \mid \tilde{a}_2 > \tilde{r}^*] \tag{242}$$
$$= \left(1 - F_2\left(\frac{a}{2}\right)\right) \int_{\frac{a}{2}}^{a} \tilde{a}_2 f_2 \frac{f_2(x)}{1 - F_2(\frac{a}{2})} d\tilde{a}_2, \tag{243}$$
$$= \int_{\frac{a}{2}}^{a} \tilde{a}_2 f_2(\tilde{a}_2) d\tilde{a}_2, \tag{244}$$
$$= \int_{\frac{a}{2}}^{a} \tilde{a}_2 N(N-1) \frac{1}{2a} \left[\frac{\tilde{a}_2 + a}{2a}\right]^{N-2} \left[1 - \frac{x+a}{2a}\right] dx, \tag{245}$$
$$= \frac{N(N-1)}{[2a]_n} \int_{\tilde{r}^*}^{a} x\,(x+a)^{N-2}\,(a-x) dx, \tag{246}$$
$$= \frac{N(N-1)}{[2a]_n} \frac{a^{N+1}}{2^{N+1} N(N+1)(N-1)} \left(2^{2N+1}(N-3) - 3^{N-1}(N^2+N-18)\right), \tag{247}$$
$$= \frac{a}{2^{2N+2}} \frac{2^{2N+1}(N-3) - 3^{N-1}(N^2+N-18)}{N+1}. \tag{248}$$

The second term becomes:

$$Pr(\tilde{a}_1 > \tilde{r}^* > \tilde{a}_2)\mathbb{E}_1[\tilde{r}^*] = \int_{-a}^{\frac{a}{2}} f_2(\tilde{a}_2) \int_{\frac{a}{2}}^{a} f_{1|2}(\tilde{a}_1) d\tilde{a}_1 d\tilde{a}_2 \frac{a}{2}, \tag{249}$$
$$= \int_{-a}^{\frac{a}{2}} f_2(\tilde{a}_1)(1 - F_{1|2}(\frac{a}{2})) d\tilde{a}_2 \frac{a}{2}, \tag{250}$$
$$= \int_{-a}^{\frac{a}{2}} f_2(\tilde{a}_1) \left(1 - \frac{\frac{a}{2} - \tilde{a}_2}{a - \tilde{a}_2}\right) d\tilde{a}_2 \frac{a}{2}, \tag{251}$$

$$= \int_{-a}^{\frac{a}{2}} f_2(\tilde{a}_1) \frac{a - \frac{a}{2}}{a - \tilde{a}_2} d\tilde{a}_2 \frac{a}{2}, \tag{252}$$

$$= \frac{a}{2} \frac{a - \frac{a}{2}}{[2a]_n} N(N-1) \int_{-a}^{\frac{a}{2}} (\tilde{a}_2 + a)^{N-2} d\tilde{a}_2, \tag{253}$$

$$= \frac{a}{2} \frac{a - \frac{a}{2}}{[2a]_n} N(N-1) \frac{3^{N-1}}{2^{N-1}} \frac{a^{N-1}}{N-1}, \tag{254}$$

$$= \frac{a}{2} \frac{a}{2} \frac{1}{[2a]_n} \frac{3^{N-1}}{2^{N-1}} \frac{N(N-1)}{N-1} a^{N-1}, \tag{255}$$

$$= \frac{a}{2^{2N+1}} 3^{N-1} N. \tag{256}$$

Total seller profits under the auctions are:

$$\mathbb{E}_1 \Pi_S = \frac{a}{2^{2N+2}} \frac{2^{2N+1}(N-3) - 3^{N-1}(N^2 + N - 18)}{N+1} + \frac{a}{2^{2N+1}} 3^{N-1} N, \tag{257}$$

$$= \frac{a}{2^{2N+1}} \frac{1}{2(N+1)} \left[2^{2N+1}(N-3) + 3^{N-1}(N^2 + N + 18) \right]. \tag{258}$$

4. Then, the seller prefers exclusive dealing if:

$$\mathbb{E}_1 \Pi_S \leq \mathbb{E}_1 \Pi_S^E, \tag{259}$$

$$\frac{a}{2^{2N+1}} \frac{1}{2(N+1)} \left[2^{2N+1}(N-3) + 3^{N-1}(N^2 + N + 18) \right] \leq \frac{a}{4}, \tag{260}$$

$$N < 3. \tag{261}$$

Thus, we have excluded exclusive dealing as a candidate SPNE for $N \geq 3$.
For $N = 2$, expected seller profits are:

$$\mathbb{E}_1 \Pi_S = \frac{5a}{24}. \tag{262}$$

5. Next, we must check whether for $N = 2$, the broker-dealer would terminate.
For $N = 2$, expected Broker-Dealer Profits are:

$$\mathbb{E}_1 \Pi_n = \frac{1}{2} Pr(\tilde{a}_1 > \tilde{a}_2 > \tilde{r}^*) \mathbb{E}[\tilde{a}_1 - \tilde{a}_2 \mid \tilde{a}_1 > \tilde{a}_2 > \tilde{r}^*] + \frac{1}{2} Pr(\tilde{a}_1 > \tilde{r}^* > \tilde{a}_2) \mathbb{E}[\tilde{a}_1 - \tilde{r}^* \mid \tilde{a}_1 > \tilde{r}^* > \tilde{a}_2]. \tag{263}$$

Where:

$$F_2(\tilde{a}_2) = \frac{(a + \tilde{a}_2)(3a - \tilde{a}_2)}{4a^2}, \tag{264}$$

$$f_2(\tilde{a}_2) = \frac{a - \tilde{a}_2}{2a^2}. \tag{265}$$

Recall further that:

$$f_{1|2}(x) = \frac{f(x)}{1 - F(\tilde{a}_2)} = \frac{1}{a - \tilde{a}_2}, \tag{266}$$

$$F_{1|2}(\tilde{a}_1) = \frac{x - \tilde{a}_2}{a - \tilde{a}_2}. \tag{267}$$

Then:

$$Pr(\tilde{a}_1 > \tilde{a}_2 > \tilde{r}^*)\mathbb{E}[\tilde{a}_1 \mid \tilde{a}_1 > \tilde{a}_2 > \tilde{r}^*], \tag{268}$$

$$= Pr(\tilde{a}_1 > \tilde{a}_2 \geq \tilde{r}^*) \int_{\frac{a}{2}}^{a} f_2(\tilde{a}_2 \mid \tilde{a}_2 > 0.5a) \int_{\tilde{a}_2}^{a} \tilde{a}_1 f_{1|2} d\tilde{a}_1 d\tilde{a}_2, \tag{269}$$

$$= Pr\left(\tilde{a}_1 > \tilde{a}_2 > \frac{a}{2}\right) \int_{\tilde{r}^*}^{a} f_2(\tilde{a}_2 \mid \tilde{a}_2 > \tilde{r}^*) \frac{1}{a - \tilde{a}_2} \left[\frac{a^2 - \tilde{a}_2^2}{2}\right] d\tilde{a}_2, \tag{270}$$

$$= Pr(\tilde{a}_1 > \tilde{a}_2 > \tilde{r}^*) \frac{1}{1 - F_2(\tilde{r}^*)} \int_{\frac{a}{2}}^{a} \frac{a - \tilde{a}_2}{[2a]^2}(a + \tilde{a}_2) d\tilde{a}_2, \tag{271}$$

$$= \frac{1}{4a^2}\frac{5a^3}{24}, \tag{272}$$

$$= \frac{5a}{96}. \tag{273}$$

Further:

$$= Pr(\tilde{a}_2 > \frac{a}{2})\mathbb{E}_1[\tilde{a}_2 \mid \tilde{a}_2 > \frac{a}{2}], \tag{274}$$

$$= \int_{\frac{a}{2}}^{a} \tilde{a}_2 \frac{a - \tilde{a}_2}{2a^2} d\tilde{a}_2, \tag{275}$$

$$= \frac{a}{24} \tag{276}$$

Finally,

$$Pr(\tilde{a}_1 \geq \frac{a}{2} \geq \tilde{a}_2)\mathbf{E}[\tilde{a}_1 \mid \tilde{a}_1 > \frac{a}{2} > \tilde{a}_2], \tag{277}$$

$$= Pr(\tilde{a}_1 \geq \frac{a}{2} \geq \tilde{a}_2) \int_{-a}^{\frac{a}{2}} f_2(\tilde{a}_2 \mid \tilde{a}_2 < \frac{a}{2}) \int_{\frac{a}{2}}^{a} \tilde{a}_1 f_{1|2}(\tilde{a}_1 \mid \tilde{a}_1 > \frac{a}{2}) d\tilde{a}_1 d\tilde{a}_2, \tag{278}$$

$$= Pr(\tilde{a}_1 \geq \frac{a}{2} \geq \tilde{a}_2) \int_{-a}^{\frac{a}{2}} f_2(\tilde{a}_2 \mid \tilde{a}_2 < \frac{a}{2}) \frac{1}{1 - F_{1|2}(\frac{a}{2})} \int_{\frac{a}{2}}^{a} \frac{\tilde{a}_1}{a - \tilde{a}_2} d\tilde{a}_1 d\tilde{a}_2, \tag{279}$$

$$= Pr(\tilde{a}_1 \geq \frac{a}{2} \geq \tilde{a}_2) \int_{-\frac{a}{2}a}^{\frac{a}{2}} f_2(\tilde{a}_2 \mid \tilde{a}_2 < \frac{a}{2}) \frac{1}{1 - F_{1|2}(\frac{a}{2})} \frac{1}{a - \tilde{a}_2} \frac{3a^2}{8} d\tilde{a}_2, \tag{280}$$

$$= Pr(\tilde{a}_1 \geq \frac{a}{2} \geq \tilde{a}_2) \int_{-a}^{\frac{a}{2}} f_2(\tilde{a}_2 \mid \tilde{a}_2 < \frac{a}{2}) \frac{2(a - \tilde{a}_2)}{a} \frac{1}{a - \tilde{a}_2} \frac{3a^2}{8} d\tilde{a}_2, \tag{281}$$

$$= Pr(\tilde{a}_1 \geq \frac{a}{2} \geq \tilde{a}_2) \frac{3a}{4} \int_{-a}^{\frac{a}{2}} \frac{f_2(\tilde{a}_2)}{F_2(\frac{a}{2})} d\tilde{a}_2, \tag{282}$$

$$= \int_{a}^{\frac{a}{2}} f_2(\tilde{a}_2) \int_{\frac{a}{2}}^{a} f_{1|2}(\tilde{a}_2) d\tilde{a}_1 d\tilde{a}_2 \frac{3a}{4}, \tag{283}$$

$$= \int_{-a}^{\frac{a}{2}} f_2(\tilde{a}_2)(1 - F_{1|2}(\frac{a}{2})) d\tilde{a}_2 \frac{3a}{4}, \tag{284}$$

$$= \int_{-a}^{\frac{a}{2}} f_2(\tilde{a}_1) \frac{a}{2(a - \tilde{a}_2)} d\tilde{a}_2 \frac{3a}{4}, \tag{285}$$

$$= \frac{3a^2}{8} \int_{-a}^{\frac{a}{2}} \frac{a - \tilde{a}_2}{2a^2} \frac{1}{a - \tilde{a}_2} d\tilde{a}_2, \tag{286}$$

$$= \frac{3}{16} \int_{-a}^{\frac{a}{2}} 1 d\tilde{a}_2, \tag{287}$$

$$= \frac{3}{16} \frac{3a}{2}. \tag{288}$$

$$Pr(\tilde{a}_1 \geq \frac{a}{2} \geq \tilde{a}_2) \frac{a}{2}, \tag{289}$$

$$= \frac{a}{2} \int_{a}^{\frac{a}{2}} f_2(\tilde{a}_2) \int_{\frac{a}{2}}^{a} f_{1|2}(\tilde{a}_2) d\tilde{a}_1 d\tilde{a}_2, \tag{290}$$

$$= \frac{a}{2} \int_{-a}^{\frac{a}{2}} f_2(\tilde{a}_1) \frac{a}{2(a - \tilde{a}_2)} d\tilde{a}_2, \tag{291}$$

$$= \frac{a^2}{4} \int_{-a}^{\frac{a}{2}} \frac{a - \tilde{a}_2}{2a^2} \frac{1}{a - \tilde{a}_2} d\tilde{a}_2, \tag{292}$$

$$= \frac{1}{8} \frac{3a}{2}. \tag{293}$$

Adding all elements, total broker-dealer profits are:

$$\mathbb{E}_1 \Pi_n = \frac{1}{2} \left[\frac{5a}{96} - \frac{a}{24} + \frac{9a}{32} - \frac{3a}{16} \right], \tag{294}$$

$$= \frac{5a}{96}. \tag{295}$$

6. Finally, we can show that exclusive dealing is not termination proof as:

$$\mathbb{E}_1 \Pi_n - \mathbb{E}_1 \Pi_n^E \geq \mathbb{E}_1 \Pi_S^E - \mathbb{E}_1 \Pi_S, \tag{296}$$

$$\frac{5}{96} \geq \frac{1}{4} - \frac{5}{24}, \tag{297}$$

$$\frac{5}{96} \geq \frac{24 - 20}{96}, \tag{298}$$

$$5 \geq 4. \tag{299}$$

Appendix C. Analytical Appendix

This section indicates how the expected profits under auctions can be derived using the binomial theorem instead of order statistics.

1. First note that the ask-prices are symmetric around the true value v. Thus, the probability with which a broker-dealer draws an ask-price above v and, thus, participates with probability $Pr(a^n > v) = 0.5$.

2. Conditional on participating, a broker-dealer's expected profits from the representative auction depend on the number of other participating broker-dealers. Each broker-dealer independently draws an above value bid-price and, thus, participates with probability one half. The total number k of other submitted submitted bid-prices, thus, follows a Bernoulli distribution with $N-1$ trials and $p = q = 0.5$. Thus, the probability of k is characterized by the pmf of a binomial distribution:

$$Pr(k) = \begin{pmatrix} N-1 \\ k \end{pmatrix} 0.5^{N-1} 0.5^{N-1}. \tag{300}$$

3. Next, recall that a broker-dealer makes a profit only if he has the highest bid conditional on k other bids, or $k+1$ bids in total. We know from 3. above that only broker-dealers with ask-prices above v particpate. Further, ask prices above v are uniformly distributed. Thus, having the highest of $k+1$ draws happens with probability:

$$Pr(a^n > \max_{k \neq n} b^k \geq v) = \frac{1}{k+1}. \tag{301}$$

4. Conditional on participating and winning the auction, the broker-dealer expects a bid-ask-spread equal to the difference between his winning bid and the second highest bid out of $k+1$ bids. Relying on the properties of the uniform distribution, the highest bid takes on the following value in expectations:

$$\mathbb{E}_1[a^n \mid a^n > \max_{k \neq n} b^k \geq v] = v + \frac{k+1}{k+2} a. \tag{302}$$

Next, the second highest of $k+1$ bids is equally easily characterized for a normal distribution as:

$$\mathbb{E}_1[\max_{k \neq n} b^k \mid \max_{k \neq n} b^k \geq v] = v + \frac{k}{k+2} a. \tag{303}$$

This leads to the following expected bid-ask-spread:

$$\mathbb{E}_1[a^n - \max_{k \neq n} b^k \mid a^n > \max_{k \neq n} b^k \geq v] = v + \frac{k+1}{k+2} a - \left(v + \frac{k}{k+2} a \right) = \frac{a}{k+2}. \tag{304}$$

5. Combining the elements from 1.-4., the broker-dealers expected profits from a single auction are:

$$\mathbb{E}_1\pi^n = Pr(a^n \geq v) \sum_K^{N-1} Pr(k) Pr(a^n > \max_{k \neq n} b^k \mid a^n > \max_{k \neq n} b^k \geq v \geq v)$$
$$\cdot \mathbb{E}_1[a^n - \max_{k \neq n} b^k \mid a^n > \max_{k \neq n} b^k \geq v], \tag{305}$$
$$= 0.5 \sum_k^{N-1} \binom{N-1}{k} 0.5^{N-1} 0.5^{N-1} \frac{1}{k+1} \frac{a}{k+1}. \tag{306}$$

Here, we can rely on mathematical software, such as Mathematica, to apply the binomial theorem to arrive at a closed form expression:

$$\mathbb{E}_1\pi^n = \frac{a}{2^N} \frac{2^{N+1} - N - 2}{N(N+1)}. \tag{307}$$

5. Applying a similar logic, we can derive the seller's expected profits. The seller expects positive profits equal to the second highest bid minus v only whenever two or more broker-dealers submit to the auction. Whenever, only one broker-dealer submits, the v is paid, leaving the seller with zero profits. The same holds true, when no broker-dealer participates in an auction. Denote with n the number of participating broker-dealers. For all auctions, the expected profits then become:

$$\mathbb{E}_1\Pi^S = \sum_{n=2}^{N} Pr(n) \mathbb{E}_1 \left[\max_{k \neq n} b^k - v \mid n \right], \tag{308}$$
$$= \sum_{n=2}^{N} \binom{N}{n} 0.5^N a \frac{n-1}{n+1} = S0.5^N a \left[\sum_{n=2}^{N} \binom{N,}{n} \frac{n}{n+1} - \sum_{n=2}^{N} \binom{N}{n} \frac{1}{n+1} \right], \tag{309}$$
$$= S0.5^N a \left[\frac{N^2 + 3N - 2^{N+2} + 4}{2(N+1)} - \frac{(N-1)\left(N - 2\left(2^N - 1\right)\right)}{2(N+1)} \right], \tag{310}$$
$$= \frac{a}{2^N} \frac{2^N(N-3) + N + 3}{N+1}. \tag{311}$$

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