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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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Home biased expectations and macroeconomic imbalances in a monetary union^{*}

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Abstract

Under monetary union, economic dynamics may diverge across countries due to regional inflation differentials and a pro-cyclical real interest rate channel, yet stability is generally ensured through endogenous adjustment of the real exchange rate. The speed of adjustment depends, inter alia, on the way agents form expectations. We propose a model in which agents' expectations are largely based on domestic variables, and less so on foreign variables. We show that such home bias in expectations strengthens the real interest rate channel and causes country-specific shocks to generate larger and more prolonged macroeconomic imbalances.

Keywords: monetary union, macroeconomic imbalances, home biased expectations, E-stability.

JEL classifications: E03, F44, F45.

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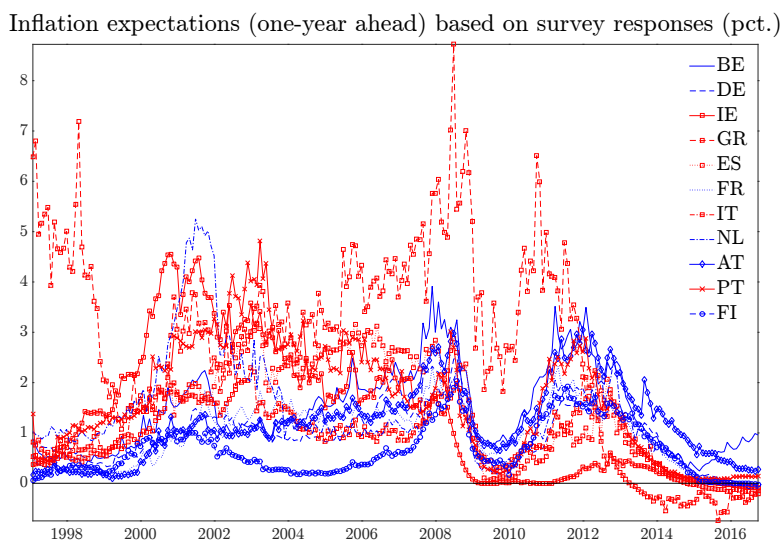
1 Introduction

Expectations play an important role in the design and evaluation of economic policy, which explains their prominence in many macroeconomic models. Inflation expectations, for instance, typically feed into actual inflation through price-setting and wage bargaining, and are therefore closely monitored by central banks who aim to maintain price stability (Orphanides and Williams, 2005; Bernanke, 2007; Mishkin, 2007). Recent studies suggest that inflation expectations in the euro area are well anchored around the European Central Bank's inflation target of close to, yet below, two percent (e.g. Beechey et al., 2011, Ehrmann et al., 2011, Galati et al., 2011, ECB, 2012, Autrup and Grothe, 2014). These results, however, are often based on aggregate expectations and do not take into account potentially important disparities across member countries.

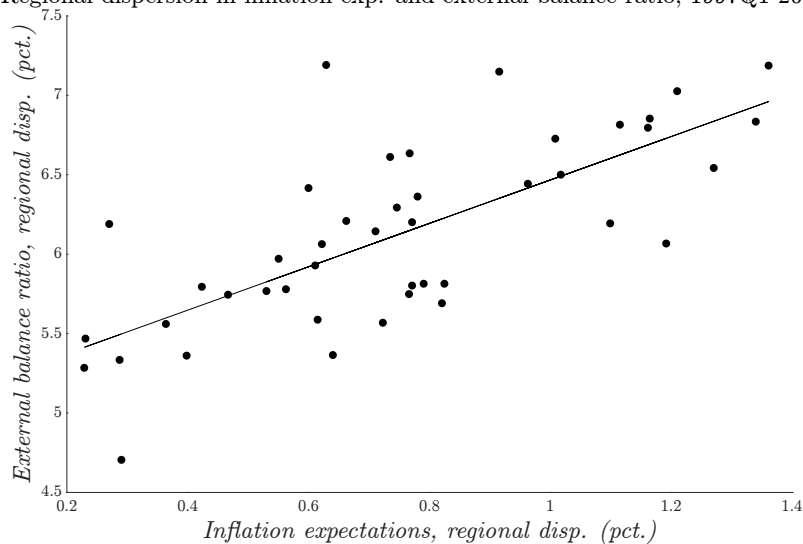
The top panel of Figure 1 shows the evolution of one-year ahead inflation expectations of households from different euro area countries, extracted from the European Commission's Consumer Survey. Importantly, the survey does not distinguish between domestic and non-domestic prices and thus refers to a general concept of the price level. As the figure makes clear, inflation expectations differ strongly across countries. Moreover, the dispersion is most pronounced during the first years of European monetary unification, a period characterized by large macroeconomic imbalances within the union. The bottom panel plots the regional dispersion in inflation expectations against the dispersion in the external balance (as a percentage of GDP), both measured by the unweighted cross-country standard deviation, for the period 1997Q1 to 2007Q4. The figure suggests that, during this period, larger cross-country differences in inflation expectations have been accompanied by more dispersed external balances.

In this paper, we study the implications of cross-country dispersion in expectations for macroeconomic imbalances in a monetary union. We propose a New Keynesian model for a two-country monetary union in which expectations are backward looking and updated periodically using a recursive least squares algorithm, as in Evans and Honkapohja (2001).

Figure 1: Expectations and cross-country dispersion in the euro area



Regional dispersion in inflation exp. and external balance ratio, 1997Q1-2007Q4



Note: The qualitative survey responses were extracted from the European Commission’s Consumer Survey and quantified using the probability approach as described by [Dias et al. \(2010\)](#). For more details, see [Appendix A](#). In the calculation of the standard deviation in inflation expectations and the external balance ratio, we excluded Greece due to some episodes of very high Greek inflation expectations. *Source:* European Commission’s Consumer Survey (inflation expectations), OECD Quarterly National Accounts dataset (external balance) and own calculations.

Moreover, we assume that agents, while acquiring information, focus more strongly on news about domestic variables than about foreign variables. It is because of this *home bias* that expectations may differ across member states of the monetary union. We show that a stronger degree of home bias in expectations leads to more amplified country-specific disturbances and larger and more prolonged macroeconomic imbalances.

To illustrate the relationship between macroeconomic imbalances and agents' expectations, we study the effects of a country-specific demand shock. If a country experiences a positive demand shock, inflation rises which raises inflation expectations and lowers the ex-ante real interest rate, which in turn fuels private spending and puts further pressure on inflation (expectations), which further lowers the real interest rate, etc. etc. We refer to this pro-cyclical channel as the *real interest rate channel*. Meanwhile, the rise in inflation leads to an appreciation of the real exchange rate, which reduces net exports and domestic output. Therefore, the real interest rate channel is countered by a counter-cyclical *real exchange rate channel*. The opposite occurs if a country were to experience a negative demand shock. Note that such country-specific shocks are amplified under monetary union, since the common central bank does not directly respond to national inflation, yet instead combats union-wide inflation.

The strength of the real interest rate channel relative to the real exchange rate channel depends, inter alia, on how expectations are formed and the extent by which agents internalize the effects of the real exchange rate. The backward-looking nature of expectations assumed in our model reinforces the feedback between inflation and inflation expectations, and thereby strengthens the real interest rate channel. Moreover, when expectations are home biased, agents do not fully observe the relative change in domestic versus foreign prices and thus underestimate the impact of the real exchange rate on the (domestic) economy. The home bias in expectations therefore weakens the real exchange rate channel and slows down the process of macroeconomic realignment. As a consequence, asymmetric disturbances generate larger and more prolonged macroeconomic imbalances if expectations are home biased than

if expectations are rational.

Our findings offer a possible explanation for the observed positive relationship between the dispersion in inflation expectations and external imbalances in the euro area, as shown in Figure 1. Moreover, they warrant greater attention from policymakers to the development of national expectations, even if aggregate expectations seem relatively stable. In fact, the degree of home bias in expectations across member countries helps determine whether inflation differentials are only temporary, and part of an endogenous adjustment process, or structural and potentially unstable. We provide empirical estimates for the degree of home bias in expectations for a group of euro area countries and find that home bias has indeed been strongest in times when external imbalances were particularly large.

Examination of the model's stability conditions under home biased expectations reveals important implications for monetary policy. We find that a higher degree of home bias in expectations reduces the likelihood that expectations converge to a fixed point. Because, in our model, the bias in expectations is permanent, the rational expectations equilibrium is never observed and so we define stability in expectations as convergence towards a so-called Restricted Perception Equilibrium or RPE (see also [Evans and Honkapohja, 2001](#), Chapters 3.6 and 13). Numerical simulations show that the RPE may not be reached if expectations are home biased, even in parameter regions where monetary policy yields a determinate rational expectations equilibrium. Intuitively, since the home bias in expectations increases the relative strength of the real interest rate channel, inflation expectations are more likely to become unanchored. Therefore, to guarantee an equilibrium that is both determinate and stable under learning, the central bank must adopt a more aggressive monetary stance with regards to inflation, beyond what is suggested by the Taylor-principle.

Our paper closely relates to the debate on the Walters critique ([Walters, 1994](#)) which centers around the power struggle between the real interest rate and real exchange rate channels, and the stability of monetary unions. The issue of internal adjustment under monetary union has been studied, among others, by [Angeloni and Ehrmann \(2007\)](#), [Deroose](#)

et al. (2008) and Allsopp and Vines (2010). In accordance with our results, Angeloni and Ehrmann (2007) show that inflation differentials in the euro area are amplified by the pro-cyclical real interest rate channel and find a dampening effect through changes in price competitiveness within the monetary union. Interestingly, the authors also mention the possibility of “a strong “home bias” in the mechanism driving inflationary expectations in the national economies” that could amplify inflation differentials (Angeloni and Ehrmann, 2007, p. 8). Deroose et al. (2008) and Allsopp and Vines (2010) focus on national (fiscal) policies and reforms that help increase market flexibility and thereby strengthen the real exchange rate channel.

Although there is ample empirical support for the presence of heterogeneity in expectations (see e.g. Carroll, 2003, Mankiw et al., 2004 and Pfajfar and Santoro, 2010), only a few studies relate such heterogeneity to the Walters critique. A notable exception is Carlin (2013), who uses a theoretical model to show that countries populated by non-rational wage-setters are more vulnerable to pro-cyclical real interest rate effects than countries in which wage-setting is more consistent with rational expectations. Likewise, Torižej (2010) uses a New Keynesian model to study asymmetric disturbances in a monetary union and finds macroeconomic volatility to be higher when agents use different (simple) forecasting rules than when expectations are rational. A key difference between these studies and the present paper, is that we allow agents’ learning methods to be *symmetric* across countries, since it is not a priori clear why these should be asymmetric. If expectations were to differ across countries, we believe a more natural and straightforward explanation would be that agents use different information sets, simply because agents are more likely to notice local rather than foreign shocks.

The rest of the paper is structured as follows. In the next section, we elaborate on the intuition underlying our assumption of home bias in expectations. We also present a case study in which we show that inflation expectations became more home biased in Spain during the Spanish housing boom that preceded the Great Recession. In Section 3, we

present the main building blocks of the model. In this section, we briefly review the learning methodology of [Evans and Honkapohja \(2001\)](#) and discuss how we introduce home bias in expectations. The results, based on impulse responses, are discussed in [Section 4](#) and the stability analysis is performed in [Section 5](#). Finally, [Section 6](#) concludes.

2 Home bias in expectations and the Spanish housing boom

Expectations regarding future economic conditions may differ across agents and countries for various reasons. Several studies have attempted to capture the heterogeneity in expectations and study its implications for policy. [Honkapohja and Mitra \(2006\)](#), for instance, introduce heterogeneity in expectations by allowing agents to use different forecasting methods that do not always yield the same predictions. [Branch and McGough \(2009\)](#) model heterogeneity in expectations by assuming a fraction of the population has rational expectations while the rest of the population forms expectations in an adaptive manner. Alternatively, in the sticky information model of [Mankiw and Reis \(2007\)](#), expectations are updated infrequently by a random, yet constant, fraction of the population, which generates dispersion in expectations throughout time. Expectations may differ, not only across agents, but also across countries due to differences in domestic policies and economic regimes that influence prior beliefs and the way agents update their expectations.

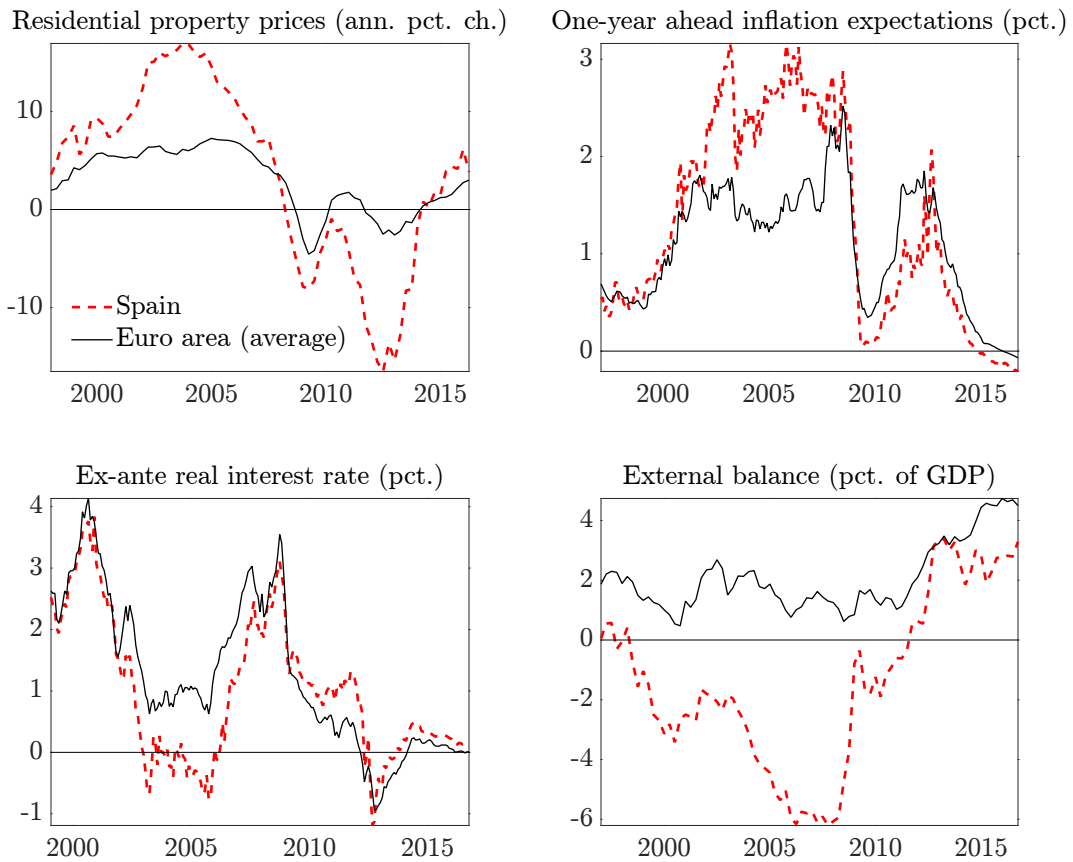
In this paper, we consider *home bias* as yet another potential source of heterogeneity in expectations. The idea is that agents, as they acquire information, focus more strongly on news about domestic variables than about foreign variables. There are several reasons for why agents might have such a home bias in the formation of expectations. First, the capacity to acquire and process information may be limited, and so agents might not use all information available when forming expectations (see [Sims, 2003](#)). Such ‘rational inattention’ can be particularly relevant with regards to foreign information, due to language barriers,

unfamiliarity with certain data sources, uncertainty about data quality, the importance of non-tradable goods sectors, etc. For these reasons, it might also take more time to process information of a foreign rather than a domestic nature, which increases the inertia in agents' expectations about foreign economic conditions. Second, expectations could be (come) home biased during episodes of large country-specific shocks during which agents are overly optimistic or pessimistic and exhibit myopic behavior. For instance, during a housing boom, agents may be inclined to focus more on domestic economic conditions rather than on foreign economic conditions, since the former is less likely to be affected by the latter in times of domestically driven buoyancy.

In this section, we offer some empirical evidence for the latter cause of home bias in expectations based on a case study of Spanish inflation expectations during the Spanish housing boom. The top-left panel of Figure 2 shows the remarkable rise in residential property prices in Spain between 2003 and 2006, which deviated strongly from average property prices in the euro area. During this time, Spanish inflation expectations increased sharply and were well above the euro area average (top-right panel), resulting in a below average real interest rate (lower-left panel). What is further apparent is that the divergent dynamics of Spanish housing prices, inflation expectations and the real interest rate vis-à-vis the rest of the euro area coincided with a deepening of Spain's trade deficit (lower-right panel).

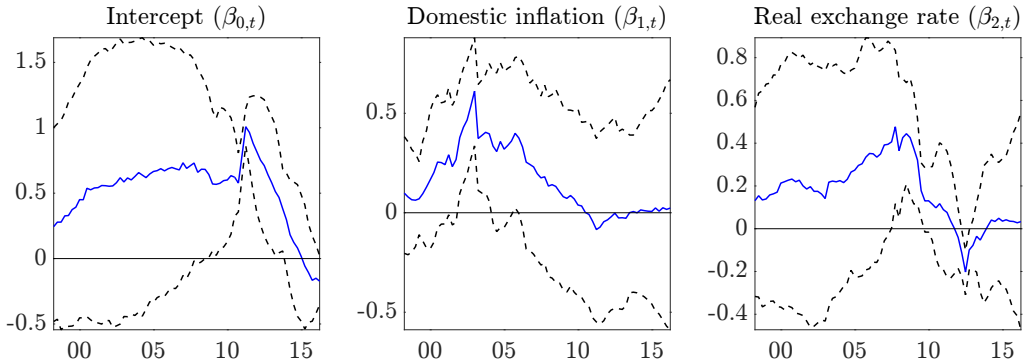
To see whether Spanish inflation expectations were more sensitive to domestic variables than to foreign variables during the housing boom, we regress expected inflation over the next four quarters, conditional on information available at quarter t , i.e. $\pi_{t+4|t}^e$, on a constant and one-period lags of the annual change in the GDP deflator, π_{t-1} , and the real effective exchange rate, Δq_{t-1} . The latter two regressors are proxies for the available information on domestic and foreign price changes, respectively. To track the contributions of both domestic and foreign variables to Spanish inflation expectations over time, we estimate the following

Figure 2: The Spanish housing boom



Note: The ex-ante real interest rate is calculated as the difference between the one-year nominal EURIBOR and one-year ahead inflation expectations. *Source:* ECB Statistical Data Warehouse (residential property price indices), European Commission's Consumer Survey (inflation expectations), Datastream (EURIBOR), OECD Quarterly National Accounts dataset (external balance).

Figure 3: Time-varying contributions to Spanish inflation expectations



Note: The figure shows estimates for the time-varying coefficients in Equation (1). Solid (dashed) lines reflect the 50th (16th and 84th) percentiles from the posterior distribution.

model with time-varying parameters:

$$\begin{aligned} \pi_{t+4|t}^e &= \beta_{0,t} + \beta_{1,t}\pi_{t-1} + \beta_{2,t}\Delta q_{t-1} + e_t & (1) \\ &= x_t\beta_t' + e_t, \quad e_t \sim \mathcal{N}(0, R), \end{aligned}$$

$$\beta_t = \beta_{t-1} + v_t, \quad v_t \sim \mathcal{N}(0, Q), \quad (2)$$

with $\text{cov}(e_t, v_t) = 0$ and $x_t = [1, \pi_{t-1}, \Delta q_{t-1}]'$, and where $\beta_t = [\beta_{0,t}, \beta_{1,t}, \beta_{2,t}]'$ are the time-varying parameters to be estimated. We use Bayesian estimation techniques and apply a Kalman filter with a diffuse prior to estimate the model (1)-(2).¹ Quarterly data for the GDP deflator and the real effective exchange rate, from 1998Q2 to 2016Q2, were obtained from Eurostat. The real effective exchange rate measures the trade-weighted change in relative unit labor costs prices vis-à-vis eighteen trading partners in the euro area.² For inflation expectations, we used quarterly averages of the monthly one-year ahead inflation expectations extracted from the European Commission's Consumer Survey.

Figure 3 shows the estimates for the time-varying coefficients in Equation (1). According

¹The prior distribution for the variance R is an Inverse Gamma distribution, i.e. $R \sim IG(T_0/2, D_0/2)$, where the degrees of freedom and scaling parameter are initialized at $T_0 = 1$ and $D_0 = 0.1$. The prior distribution for the variance-covariance matrix Q is an Inverse Wishart distribution, i.e. $Q \sim IW(Q_0, T_0)$, where Q_0 is initialized as the identity matrix. Remaining priors are $\beta_t = 0$, $R = 1$ and $Q = I_3$. A total of 12,000 draws were used for the Gibbs sampling algorithm, of which the first 1,000 draws were discarded. See Primiceri (2005) for further details on how to estimate time-varying parameter models.

²Using the real effective exchange rate based on relative consumer prices yields similar results.

to the figure, the contribution of domestic inflation to the dynamics of Spanish inflation expectations, as governed by $\beta_{1,t}$, increased during the Spanish housing boom (see middle panel). During the Great Recession, when euro area countries faced a common shock, the dynamics of Spanish inflation expectations were no longer driven by domestic inflation. Furthermore, there is little evidence that, during the housing boom, inflation expectations were influenced by changes in the real effective exchange rate (right panel). These results thus support our conjecture that changes in domestic variables are more important for the formation of (inflation) expectations than changes in foreign variables, at least in times when an economy faces large country-specific shocks.

3 The model

For our main analysis, we apply a New Keynesian model for a monetary union that consists of two countries, Home (H) and Foreign (F). In the first part of this section, we discuss how agents inside the model form expectations and provide details on how we introduce home bias in expectations. In the second part, we present the main building blocks of the rest of the model. In the third and last part, we discuss the model's implications for the relationship between home bias in expectations and the persistence of country-specific shocks.

3.1 How expectations are formed

Let x_t be a vector of endogenous variables, realized at t , and v_t a vector of exogenous variables. The model dynamics can then be summarized by the following two equations:

$$Ax_t = \sum_{i=\{H,F\}} B^i \tilde{E}_t^i x_{t+1} + Cx_{t-1} + Dv_t, \quad (3)$$

$$v_t = \rho v_{t-1} + \varepsilon_t, \quad (4)$$

where A , B^i , C and D are coefficient matrices of appropriate sizes, ϱ a diagonal matrix with all eigenvalues inside the unit circle, and ε_t a vector of i.i.d. normal shocks with mean zero and variance σ^2 . \tilde{E}_t^i is the expectations operator used by residents of country $i = \{H, F\}$. Throughout, we assume that agents living in the same country form identical expectations, whereas agents from different countries might form different expectations

The rational expectations solution to the model above has the following form:

$$x_t = \bar{\Lambda}_1 x_{t-1} + \bar{\Lambda}_2 v_t. \quad (5)$$

If expectations are rational, the coefficient matrices $\bar{\Lambda}_1$ and $\bar{\Lambda}_2$ are known to all agents. However, under the adaptive learning approach considered here, agents do not know the true coefficients matrices, yet must obtain estimates using the data available to them (see [Evans and Honkapohja, 2001](#)). In particular, agents proxy the rational expectations solution (5) using the following Perceived Law of Motion (PLM):

$$x_t = \Lambda_{0,t-1}^i + \Lambda_{1,t-1}^i x_{t-1} + \Lambda_{2,t-1}^i v_t. \quad (6)$$

Note that heterogeneity in beliefs about the economy's dynamics arises if agents obtain different estimates for $\Lambda_{j,t}^i$, with $j = \{0, 1, 2\}$.

Using past data, x_{t-1} , and information on new shocks, v_t , agents perform forecasts for x_{t+1} at t by applying their PLM (6).³ The forecasting equation is obtained by updating the PLM by one period:

$$\tilde{E}_t^i x_{t+1} = \left(I + \Lambda_{1,t-1}^i \right) \Lambda_{0,t-1}^i + \left(\Lambda_{1,t-1}^i \right)^2 x_{t-1} + \left(\Lambda_{1,t-1}^i + \varrho \right) \Lambda_{2,t-1}^i v_t, \quad (7)$$

where I is the identity matrix of conformable size.⁴ Using the forecasts generated by (7)

³The assumption that agents can observe shocks contemporaneously can be relaxed without doing much harm to our main results.

⁴Without loss of generality, we follow the literature and assume ϱ is known to all agents.

to replace the expectations terms $\tilde{E}_t^i x_{t+1}$ in (3) yields the so-called Actual Law of Motion (ALM):

$$x_t = \sum_{i=\{H,F\}} F^i \left(I + \Lambda_{1,t-1}^i \right) \Lambda_{0,t-1}^i + F^i \left(\Lambda_{1,t-1}^i \right)^2 x_{t-1} + \sum_{i=\{H,F\}} F^i \left(\Lambda_{1,t-1}^i + \varrho \right) \Lambda_{2,t-1}^i v_t + G x_{t-1} + H v_t, \quad (8)$$

where $F^i \equiv A^{-1}B^i$, $G \equiv A^{-1}C$ and $H \equiv A^{-1}D$. The ALM determines the realization of the endogenous variables conditional on agents' expectations and the state variables.

Having observed x_t , agents aim to improve their forecasts by updating $\Lambda_{j,t}^i$ using the following two updating equations:

$$\Lambda_t^i = \Lambda_{t-1}^i + \gamma_t M_t^{-1} z_t \left(x_t - \Lambda_{t-1}^i z_t \right), \quad (9)$$

$$M_t = M_{t-1} + \gamma_t \left(z_t z_t' - M_{t-1} \right), \quad (10)$$

where $\Lambda_t^i \equiv [\Lambda_{0,t}^i, \Lambda_{1,t}^i, \Lambda_{2,t}^i]$ and $z_t \equiv [1, x_{t-1}, v_t]'$. Equation (9) shows that the extent by which the coefficients are updated depends on the size of the forecast error, $x_t - \Lambda_{t-1}^i z_t$. The weight on the forecast error is determined by M_t , i.e. the moment matrix of z_t , and the gain parameter γ_t , which controls the speed with which agents learn. In what follows, we assume a constant gain, i.e. $\gamma_t = \gamma$ for all t , such that agents discard old data from their sample and assign more weight to new information (as is done in rolling window regressions).⁵

Home bias in expectations is introduced by allowing agents to weigh domestic and non-domestic variables differently in their information sets. Specifically, let $x_{d,t}$ denote a subset of x_t containing only domestic variables, and $x_{n,t}$ a subset containing non-domestic variables. The data vector can then be partitioned as $x_t = [x_{d,t}, x_{n,t}]'$. Similarly, partition the exogenous variables into domestic and non-domestic shocks, i.e. $v_t = [v_{d,t}, v_{n,t}]'$. Using this

⁵As discussed in Branch et al. (2014), adaptive learning models that use a constant gain perform better in terms of empirical fit than models using a time-varying gain.

transformation, we can rewrite the PLM:

$$x_t = \Lambda_{0,t-1}^i + \Lambda_{1,t-1}^i \begin{bmatrix} I & \mathbf{0} \\ \mathbf{0} & I\omega \end{bmatrix} \begin{bmatrix} x_{d,t-1} \\ x_{n,t-1} \end{bmatrix} + \Lambda_{2,t-1}^i \begin{bmatrix} I & \mathbf{0} \\ \mathbf{0} & I\omega \end{bmatrix} \begin{bmatrix} v_{d,t} \\ v_{n,t} \end{bmatrix}, \quad (11)$$

and the updating equations:

$$\Lambda_t^i = \Lambda_{t-1}^i + \gamma \left(M_t^i \right)^{-1} \tilde{z}_t^i \left(\tilde{x}_t^i - \Lambda_{t-1}^i \tilde{z}_t^i \right), \quad (12)$$

$$M_t^i = M_{t-1}^i + \gamma \left(\tilde{z}_t^i \left(\tilde{z}_t^i \right)' - M_{t-1}^i \right), \quad (13)$$

with $\mathbf{0}$ the zero matrix of conformable size, and where $\omega \in [0, 1]$. If $\omega < 1$, non-domestic variables are given a lower weight than domestic variables and we say that expectations are home biased. The tildes above the data vectors, \tilde{x}_t^i and \tilde{z}_t^i , denote the fact that, due to this home bias, the data used by the agents is different from the total available data. By limiting the amount of data used, the home bias parameter ω affects the moment matrix of the data, M_t^i , which in turn alters the way agents update their estimates of Λ_t^i by (12) and (13). Hence, agents' beliefs about economic dynamics are shaped by the degree of home bias in expectations.⁶ Note that, if $\omega = 1$, there is no home bias in expectations and, because all agents use the same information set, expectations are the same across countries.

In Section 2, we offered a number of reasons for why agents may be home biased when acquiring and processing new information. In the model, we circumvent the task of building the micro-foundations that explain such behavior, and instead control the degree of home bias in expectations by a single parameter, ω . We therefore do not need to alter the functional form of the PLM and updating equations, and can easily isolate the effects of home bias in expectations by considering different values for ω .

⁶Due to the self-referential nature of the system, in which realized variables and expectations affect each other, the regressors in the PLM are endogenous and so the Frisch-Waugh-Lovell theorem does not apply here. Hence, one cannot simply rewrite the updating equations in such a way that the effect of the home bias parameter on the estimated coefficients is nullified.

3.2 Building blocks of the model

The model economy, which resembles that of [Benigno \(2004\)](#), consists of two countries, Home and Foreign, who form a monetary union. Each country is populated by a continuum of households. Households that live in Home are indexed by $h \in [0, s)$, while those that live in Foreign are indexed by $f \in [s, 1]$. The parameter s measures the relative size of Home. The two countries interact on international goods and asset markets. Goods are produced by firms that are owned by the households of the country in which the firm is domiciled. Asset markets are complete, yet international risk sharing is not perfect due to cross-country heterogeneity in expectations. A common central bank sets monetary policy and targets union-wide aggregates. We discuss each of these three agents (the household, firm and central bank) separately. The market-clearing conditions are deferred to [Appendix B](#).

3.2.1 Household preferences

A representative household living in country i chooses consumption, c_t^i , and labor supply, n_t^i , while aiming to maximize expected life-time utility:

$$\tilde{E}_t^i \sum_{t=0}^{\infty} \beta^t z_{D,t}^i \left(\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \frac{(n_t^i)^{1+\varphi}}{1+\varphi} \right), \quad (14)$$

where $\beta \in (0, 1)$ denotes the household's discount factor, $\sigma > 0$ the coefficient of relative risk aversion and $\varphi > 0$ the inverse elasticity of labor supply. Labor is supplied only to domestic firms. $z_{D,t}^i$ is a demand shock that evolves according to a stationary AR(1) process. Importantly, demand shocks are allowed to be correlated across countries.

The household receives labor income, $W_t^i n_t^i$ where W_t^i denotes the nominal wage rate, and firm profits from domestic intermediate goods firms, \mathcal{P}_t^i . Further, households have access to a complete set of one-period Arrow-Debreu securities, $B_t^i(\xi_{t+1})$, each unit of which pays out one unit of currency when state ξ_{t+1} occurs.

Let $P_{i,t}^i$ denote the aggregate producer price index (PPI) of country i , P_t^i the consumer price index (CPI) and $Q_t(\xi_{t+1}|\cdot)$ the price of an Arrow-Debreu security. The flow budget constraint faced by the household for every event ξ_t at any time t is then given by

$$P_t^i c_t^i + \sum_{\xi^{t+1}} Q_t(\xi_{t+1}|\cdot) B_t^i(\xi_{t+1}) = W_t^i n_t^i + B_{t-1}^i + P_{i,t}^i \mathcal{P}_t^i. \quad (15)$$

We assume initial wealth to be identical across agents, i.e. $B_{-1}^i = \bar{B}$ for all i .

Maximizing the objective function (14), subject to the budget constraint (15) and the transversality condition $\lim_{T \rightarrow \infty} Q_{t,T} B_T^i = 0$, and taking prices, the wage rate, asset prices, and initial asset holdings as given, yields the conditions that pin down the optimal supply of labor:

$$(n_t^i)^\varphi = w_t^i (c_t^i)^{-\sigma}, \quad (16)$$

and the optimal intertemporal allocation of wealth between consumption and savings:

$$1 = \beta \tilde{E}_t^i \left[\frac{R_t}{\pi_{t+1}^i} \frac{z_{D,t+1}^i}{z_{D,t}^i} \left(\frac{c_{t+1}^i}{c_t^i} \right)^{-\sigma} \right], \quad (17)$$

where $w_t^i \equiv W_t^i/P_t^i$ denotes the real wage rate, $\pi_t^i \equiv P_t^i/P_{t-1}^i$ gross CPI inflation, and R_t is the risk-free nominal interest rate, set by the central bank, satisfying $1/R_t = \sum_{\xi_{t+1}} Q_t(\xi_{t+1})$.

Combining Equation (17) for both countries yields the ‘imperfect international risk-sharing condition’:⁷

$$\frac{z_{D,t}^H (c_t^H)^{-\sigma}}{z_{D,t}^F (c_t^F)^{-\sigma}} = q_t^{-1} \left\{ \frac{\tilde{E}_t^H \left[\frac{z_{D,t+1}^H (c_{t+1}^H)^{-\sigma}}{P_{t+1}^H} \right]}{\tilde{E}_t^F \left[\frac{z_{D,t+1}^F (c_{t+1}^F)^{-\sigma}}{P_{t+1}^F} \right]} \right\}, \quad (18)$$

where $q_t \equiv P_t^F/P_t^H$ denotes the real exchange rate. The term in curly brackets on the right-hand side of (18) reflects the relative expected marginal utility with respect to consumption across the two countries. If expectations are homogeneous, i.e. $\tilde{E}_t^H = \tilde{E}_t^F$, then due to the assumption of complete asset markets and symmetric initial conditions, this term

⁷See Appendix C for a brief derivation.

equals unity and perfect risk sharing is ensured through appropriate adjustments in the real exchange rate. However, under heterogeneous expectations, expected marginal utility, and hence the stochastic discount factor, may differ across countries. The additional term in curly brackets in (18) accounts for this difference. Note that, the stronger is the degree of home bias in expectations, the more dispersed are the stochastic discount factors and the more distorted is the risk sharing condition.

Consumption, c_t^i , consists of bundles of domestically produced final goods, $c_{i,t}^i$, and imported final goods, $c_{j,t}^i$, with $i, j = \{H, F\}$ and $i \neq j$. These bundles are aggregated according to the following technology:

$$c_t^i \equiv \left[(1 - \bar{\alpha}^i)^{\frac{1}{\eta}} (c_{i,t}^i)^{\frac{\eta-1}{\eta}} + (\bar{\alpha}^i)^{\frac{1}{\eta}} (c_{j,t}^i)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad i \neq j, \quad (19)$$

where $\eta \geq 1$ denotes the trade elasticity and $\bar{\alpha}^i$ determines the degree of country openness. Specifically, $\bar{\alpha}^H \equiv (1 - s)\alpha$ and $\bar{\alpha}^F \equiv s\alpha$, with $\alpha \in (0, 1)$. The final goods are, in turn, composites of differentiated intermediate goods, which are assembled according to standard Dixit-Stiglitz aggregators. The corresponding demand schedules and price indices are standard and presented in Appendix B.

3.2.2 Firm production and price setting

Differentiated intermediate goods $y_t^i(z)$ are produced by monopolistic wholesale firms, indexed by $z = \{h, f\}$, using the constant returns to scale production function $y_t^i(z) = n_t^i(z)$. Following Calvo (1983), wholesale firms can change their price only with fixed probability $1 - \theta \in (0, 1)$. Firms that can change their price do so to maximize expected discounted profits, subject to a demand schedule, the production function and a condition that pins down equilibrium demand for labor, i.e. $mc_t^i = (P_t^i/P_{i,t}^i) w_t^i$, where mc_t^i denotes real marginal

costs. The resulting optimal price, \bar{P}_t^i , is given by

$$\bar{P}_t^i = \frac{\epsilon}{\epsilon - 1} \frac{\tilde{E}_t^i \sum_{k=0}^{\infty} \theta^k Q_{t+k} (\xi_{t+k+1} | \cdot) \left(P_{i,t+k}^i \right)^{1+\epsilon} y_{t+k}^i m c_{t+k}^i}{\tilde{E}_t^i \sum_{k=0}^{\infty} \theta^k Q_{t+k} (\xi_{t+k+1} | \cdot) \left(P_{i,t+k}^i \right)^\epsilon y_{t+k}^i}, \quad (20)$$

where $\epsilon > 0$ measures the elasticity of substitution between intermediate goods. Firms that are unable to change their price keep prices fixed at last period's PPI.

3.2.3 The central bank and monetary policy

A common central bank sets the gross nominal interest rate according to the following rule:

$$\frac{R_t}{R} = \left(\frac{\pi_{MU,t}}{\pi_{MU}} \right)^{\phi_\pi} \left(\frac{y_{MU,t}}{y_{MU}} \right)^{\phi_y}, \quad (21)$$

where union-wide aggregates are weighted by country size, i.e. $\pi_{MU,t} \equiv s\pi_t^H + (1-s)\pi_t^F$ and $y_{MU,t} \equiv sy_t^H + (1-s)y_t^F$, and variables without a t subscript denote steady-state values.

3.3 Monetary union and the persistence of asymmetric shocks

The monetary policy rule (21) implies that asymmetric shocks may not always prompt a sufficiently strong response from the central bank if union-wide aggregates remain unchanged. This might be the case, for instance, if shocks are negatively correlated across countries or borne from relatively small regions within the union. It is because of this inability to tailor monetary policy to country-specific disturbances that temporary shocks can have long-lasting effects.

To better understand the peculiar nature of the propagation of shocks under monetary union, consider the linear version of the Home Euler equation (17), rewritten using the condition for CPI inflation, which is given by (38) in Appendix B:

$$\sigma \hat{c}_t^H = \sigma \tilde{E}_t^H \hat{c}_{t+1}^H - \left(\hat{R}_t - \tilde{E}_t^H \hat{\pi}_{H,t+1}^H \right) + \hat{\alpha}^H \left(\tilde{E}_t^H \hat{q}_{t+1} - \hat{q}_t \right) - \tilde{E}_t^H \Delta \hat{z}_{D,t+1}^H,$$

with $\hat{\alpha}^H \equiv \bar{\alpha}^H / (1 - \bar{\alpha}^H - \bar{\alpha}^F) > 0$, and where variables with a hat denote percentage deviations from steady state. A positive shock to Home demand raises expected real marginal costs which, by the price-setting condition of the firm (20), leads to higher expected producer price inflation, $\tilde{E}_t^H \hat{\pi}_{H,t+1}^H$. Higher expected inflation reduces the ex ante real interest rate, $\hat{R}_t - \tilde{E}_t^H \hat{\pi}_{H,t+1}^H$, which prompts an increase in household consumption, \hat{c}_t^H , and a further rise in aggregate demand. We refer to this pro-cyclical channel as the *real interest rate channel*. Under monetary union, this channel can be quite strong (and is generally stronger than under regimes characterized by flexible exchange rates), since the central bank does not target the nominal interest rate, \hat{R}_t , to stabilize inflation in a specific region of the union. Rather, the central bank is concerned only with inflation at the union-wide level, as indicated by (21). Therefore, even if short-lived, country-specific shocks can have strong persistent effects on the economy.

The rise in inflation, however, also leads to an expected appreciation of the real exchange rate—i.e. \hat{q}_t is expected to fall. Due to its negative effect on aggregate domestic income, a real exchange rate appreciation reduces consumption. This counter-cyclical channel is referred to as the *real exchange rate channel*, and its strength is increasing in the degree of country openness, governed by α . Particularly, the more integrated are international goods markets, the more sensitive are net exports, aggregate income and consumption to changes in the real exchange rate.

Note that the strength of the real interest rate channel relative to the real exchange rate channel depends, inter alia, on the degree of home bias in expectations. If the home bias is strong, expectations do not respond much to changes in the real exchange rate, since the real exchange rate is a non-domestic variable and agents ignore such variables. As a consequence, agents do not fully internalize real exchange rate fluctuations when making consumption decisions, which weakens the real exchange rate channel.

Moreover, recall from the imperfect risk sharing condition (18) that the home bias in expectations creates a wedge between Home and Foreign agents' stochastic discount factors,

which limits risk sharing opportunities. In the absence of home bias in expectations, the real exchange rate ensures, through its effect on expected future income, a tight relationship between Home and Foreign consumption levels and, hence, perfect risk sharing. Yet, when expectations are home biased, and real exchange rate adjustments not fully internalized, the real exchange rate is less able to perform its stabilizing role and consumption levels may become more dispersed across countries as economies face idiosyncratic shocks.

4 The effects of an asymmetric shock

4.1 Strategy and calibration

To illustrate the relationship between macroeconomic imbalances and home biased expectations, we study the effects of a country-specific, and hence asymmetric, demand shock. The reason for focusing on asymmetric, rather than symmetric, shocks is twofold. First, given that the common central bank only has one instrument to target union-wide aggregates, i.e. the nominal interest rate, monetary policy is unable to optimally respond to country-specific shocks, as any centralized monetary action would necessarily entail unwanted outcomes in some parts of the monetary union. Second, as mentioned in Section 2, expectations are more likely to be (come) home biased in the event of large country-specific shocks, to the extent they cause agents to view domestic economic conditions as being detached from the foreign world. If, on the other hand, member countries were confronted with the same shock, then monetary policy would be better able to absorb the shock, and prevent divergent economic dynamics, which would also help anchor expectations. Therefore, in what follows, we consider shocks that are perfectly negatively correlated across two equally-sized countries, i.e. $\text{corr}(z_{D,t}^H, z_{D,t}^F) = -1$ and $s = 0.5$, such that, following such shocks, union-wide aggregates and monetary policy are left unchanged.

The impulse responses are calculated following the approach suggested by [Eusepi and Preston \(2011\)](#). In particular, the model is simulated twice, whereby in each round the

Table 1: Benchmark calibration

Model parameters		
β	Household discount factor	0.99
$1/\varphi$	Frisch elasticity of labor supply	1
$1/\sigma$	Elasticity of intertemporal substitution	1
α	Trade openness	0.25
θ	Probability of non-price adjustment	0.75
η	Trade elasticity	1
ϕ_π	Monetary response to inflation	1.5
ϕ_y	Monetary response to output	0.25
s	Relative size of Home	0.5
ρ_D	AR(1) coefficient of demand shock	0.9
Learning parameters		
T	Simulation length	500
γ	Gain parameter	0.01
ω	Home bias parameter	1

economy is exposed to a series of small uncorrelated shocks, drawn randomly from a Normal distribution, for T periods, during which agents periodically update their beliefs according to the learning algorithm described in Section 3. In following with the literature, we assume that, at the start of each simulation round, agents' beliefs about the economy coincide with those under rational expectations. In one of the two simulations, the economy is hit by the shock of interest at period $T - K$, where K is the impulse response horizon. The impulse responses are then calculated as the difference between the two simulation rounds from period $T - K$ to T . This approach helps track the response of the economy *while* agents are still updating their beliefs.

To perform the simulations, we calibrate the model parameters at a quarterly frequency for t . For most parameters, we use values that are commonly found in the literature. However, we set the parameter governing country openness, α , somewhat lower than what is typically used in the literature to be consistent with the idea of expectations being home biased. We set $\alpha = 0.25$, which matches the average import-to-output ratio in Spain during its housing boom. For an overview of the benchmark calibration, see Table 1.

With regards the learning parameters, we set the simulation length T equal to 500 and

the gain parameter γ to 0.01. Our choice for γ lies in the middle of existing empirical findings, with somewhat higher estimates reported for professional forecasters (Branch and Evans, 2006) and lower estimates for households (Pfajfar and Santoro, 2010). The home bias parameter ω is initially set to unity, implying domestic and non-domestic variables are treated equally in agents' information sets and expectations are not home biased. Our objective is then to compare the impulse responses under the benchmark calibration with those generated using lower values for ω .

In order to get a sense of empirically plausible values for ω , we fit simulated inflation forecasts generated by the learning model (11)-(13) to data on household inflation expectations in the euro area. As in Section 2, we conjecture that four-quarter ahead inflation expectations in country i , $\tilde{E}_t^i \pi_{t+4}^i$, are formed based on past changes in domestic prices and past changes in relative prices vis-à-vis foreign trading partners:

$$\tilde{E}_t^i \pi_{t+4}^i = \Lambda_{0,t-1}^i + \Lambda_{1,t-1}^i \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \log(P_{i,t-1}^i/P_{i,t-5}^i) \times 100 \\ \log(q_{t-1}/q_{t-5}) \times 100 \end{bmatrix},$$

where $P_{i,t}^i$ is the producer price index and q_t the real effective exchange rate. Consistent with our theoretical setup, the degree of home bias in expectations is captured by the parameter ω . In each period t , the coefficient matrices $\Lambda_{0,t-1}^i$ and $\Lambda_{1,t-1}^i$ are updated using Equations (12) and (13) by evaluating the forecasts against actual data on CPI inflation. Quarterly data, from 1998Q1 to 2016Q2, for the PPI, real effective exchange rate and CPI are obtained from Eurostat. We also consider a shorter sample that excludes the crisis years. The simulations are performed many times, each time using different values for $\omega \in [0, 1]$ and $\gamma \in [0, 1]$, and compared against actual inflation expectations extracted from the European Commission's Consumer Survey, $\pi_{i,t+4|t}^e$. Particularly, we calculate the Mean Square Comparison Error ($MSCE^i$), i.e.

$$MSCE^i = \frac{1}{T} \sum_{t=1}^T \left(\pi_{i,t+4|t}^e - \tilde{E}_t^i \pi_{t+4}^i \right)^2,$$

Table 2: Empirical estimates for the home bias and gain parameters in the euro area

<i>Country</i>	1998Q1-2016Q2		1998Q1-2008Q2	
	<i>Home bias, ω</i>	<i>Gain, γ</i>	<i>Home bias, ω</i>	<i>Gain, γ</i>
Belgium	0.00	0.031	0.00	0.039
Germany	0.37	0.063	0.26	0.060
Ireland	0.00	0.032	0.00	0.041
Greece	0.03	0.043	0.00	0.044
Spain	0.60	0.008	0.44	0.008
France	0.69	0.017	0.00	0.020
Italy	0.40	0.007	0.00	0.010
Netherlands	0.71	0.021	0.10	0.027
Austria	0.44	0.019	0.64	0.023
Portugal	0.00	0.042	0.45	0.024
Finland	1.00	0.004	0.76	0.008

where t ranges from 1998Q1 to 2016Q2, and report the home bias and gain parameters that yield the lowest $MSCE^i$.^{8,9}

The results are reported in Table 2. The second column shows the estimates for the home bias parameter ω for the full sample period and suggests that there are large differences in the estimated degree of home bias in expectations across countries. In some countries, such as Finland, France and the Netherlands, expectations are best captured when using a relatively large value for ω , implying that consumers internalize, at least to some extent, foreign price developments when forming inflation expectations. However, other countries, like Belgium, Ireland and Portugal, exhibit a strong degree of home bias in expectations, as expectations are best explained if ω is set to zero. The estimates for ω for the shorter sample, shown in the fourth column, are generally smaller than those found using the full sample. The shorter sample, by not including the crisis years, is dominated by episodes of local, rather than common, shocks, which may explain the stronger degree of home bias in expectations. These results are in line with those from the case study in Section 2, where we observed an increase in the degree of home bias in Spanish inflation expectations during the Spanish

⁸For each simulation, we initialize the coefficient and moment matrices by setting them equal to zero and the identity matrix, respectively. These are used to obtain inflation forecasts for the first four quarters.

⁹Figure 8 in Appendix E plots the simulated inflation forecasts against the actual inflation expectations.

housing boom and a fall in home bias in the years thereafter. Given the dispersion in the estimates, we consider a wide range of values for ω in the analysis below.

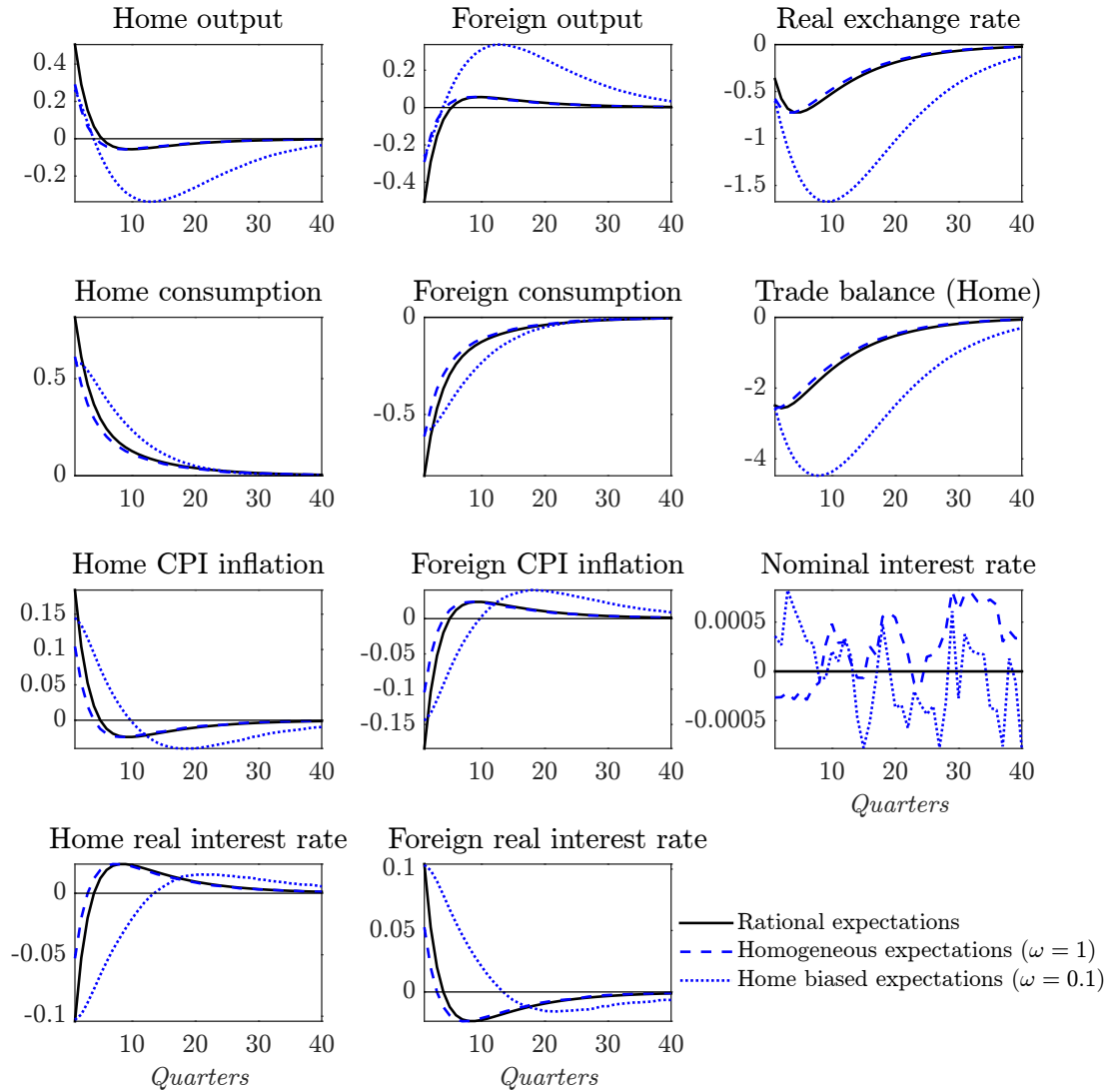
4.2 Impulse responses

The responses of the endogenous variables to a one standard deviation demand shock are displayed in Figure 4. The solid lines are the responses under rational expectations, the dashed lines the responses under adaptive and homogeneous expectations, and the dotted lines the responses under adaptive and home biased expectations. The shock is assumed to be positive in Home and negative in Foreign. Therefore, consumption rises in Home, which leads to an increase in Home output and CPI inflation, whereas Foreign consumption, output and inflation fall in response to the shock.

Since the shock has opposing effects across countries, union-wide inflation and output remain unchanged and so the short-term nominal interest rate set by the central bank does not respond (much) to the shock. Consequently, the real interest rate falls in Home, which further stimulates the initial demand shock and the consequent rise in Home consumption and inflation. As discussed in Section 3.3, this pro-cyclical real interest rate channel arises from the household's Euler equation that links expected consumption growth to the ex ante real interest rate. In Foreign, the opposite occurs and economic activity is suppressed by a fall in Foreign inflation (expectations) and an increase in the real interest rate. Thus, even under rational expectations, the asymmetric demand shock generates persistent macroeconomic imbalances within the monetary union, due to the real interest rate channel.

Meanwhile, the rise in Home inflation leads to an appreciation of the real exchange rate, which dampens the economic boom through a fall in the trade balance and a contraction in aggregate demand. Conversely, the real exchange rate depreciates from the perspective of Foreign, which helps recover economic activity. Hence, the destabilizing real interest rate channel is offset, endogenously, through a stabilizing real exchange rate channel that aids in restoring the trade imbalance.

Figure 4: Responses to an asymmetric demand shock



Note: Responses are expressed in percentage deviations from steady state. The demand shock is perfectly negatively correlated across the two countries, i.e. $\text{corr}(z_{D,t}^H, z_{D,t}^F) = -1$, with Home (Foreign) experiencing a positive (negative) shock.

The change in the real exchange rate induces firms in Home to lower their prices in order to remain competitive in international goods markets. This effect can be inferred from the linearized price-setting condition for intermediate goods firms in Home:

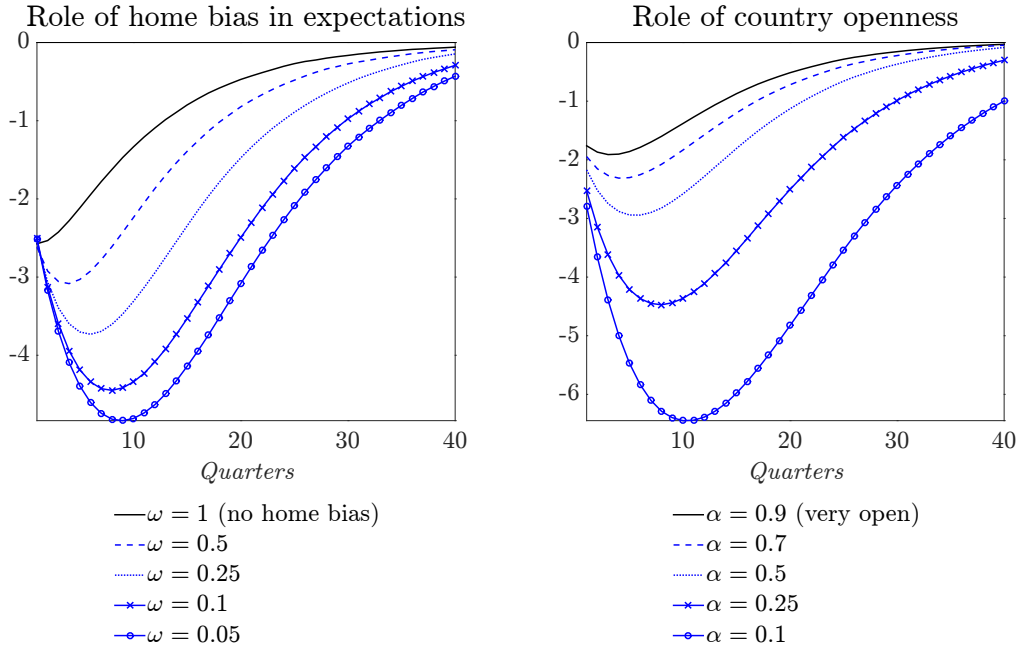
$$\hat{\pi}_{H,t}^H = \beta \tilde{E}_t^H \hat{\pi}_{H,t+1}^H + \frac{(1-\theta)(1-\beta\theta)}{\theta} (\varphi \hat{y}_t^H + \sigma \hat{c}_t^H + \hat{\alpha}^H \hat{q}_t).$$

An appreciation of the real exchange rate, i.e. a fall in \hat{q}_t , reduces real marginal costs through a decline in aggregate demand, which causes firms to adjust prices downwards and domestic inflation, $\hat{\pi}_{H,t}^H$, to fall. Foreign firms behave in the opposite direction. This nominal price adjustment counteracts the effects of the real interest rate channel such that, over time, the real interest rate rises in Home and falls in Foreign, until the regional dispersion in consumption levels disappears.

Note that the strength of this adjustment mechanism depends on the degree of country openness and price stickiness. The more open the economy is to foreign trade, i.e. the higher is α , the more sensitive aggregate demand and price-setting decisions are to changes in the real exchange rate. Moreover, the more flexible are prices, i.e. the lower is θ , the faster do prices adjust for a given change in the real exchange rate. In both cases, macroeconomic convergence results more rapidly following an asymmetric demand shock.

Under home biased expectations, the responses of the endogenous variables are more pronounced and persistent than under rational and homogeneous expectations. In generating the impulse responses under home biased expectations, we set ω equal to 0.1. Due to the home bias, agents do not fully internalize the relative change in domestic versus foreign prices and, as a consequence, underestimate the effects of the real exchange rate on the (domestic) economy, as explained in Section 3.3. Home bias in expectations thus weakens the real exchange rate channel. Moreover, as agents focus more on domestic economic conditions, inflation expectations in Home (Foreign) remain high (low), and the real interest rate low (high), for a much longer period than in the absence of home bias in expectations, which

Figure 5: Home's trade balance response to an asymmetric demand shock



strengthens the real interest rate channel. Due to the change in the relative strengths of the two channels, the demand shock has a more amplified effect under home biased expectations than under rational or homogeneous expectations, which is reflected by a sharper adjustment of the real exchange rate and larger macroeconomic imbalances.

The main message conveyed by these impulse responses is that real interest rate differentials across countries in a monetary union can lead to more prolonged macroeconomic imbalances if agents, while forming expectations, rely mostly on information inferred from domestic variables, while ignoring foreign and union-wide variables. The amplification of the effects of real interest rate differentials is due to a pro-cyclical real interest rate channel that undercuts a stabilizing counter-cyclical real exchange rate channel. To overcome the home bias in agents' expectations, a more extreme adjustment in nominal prices is required, given that, under monetary union, the nominal exchange rate is fixed and monetary policy tools are powerless (or at least inefficient) in the face of asymmetric shocks.

In the left panel of Figure 5, we analyze further the implications of home bias in ex-

pections for macroeconomic imbalances. We use Home’s trade balance as a measure of macroeconomic imbalances and focus on its response to an asymmetric demand shock under different values for the home bias parameter, ω . The results suggest that, for a greater degree of home bias in expectations, i.e. for lower values of ω , macroeconomic imbalances are larger and macroeconomic adjustment is slower. The intuition follows from our discussion above: the home bias causes agents to ignore relative international price changes and underestimate the impact of the real exchange rate on economic dynamics, which weakens the real exchange rate channel and strengthens the real interest rate channel. These results are in line with the empirically observed positive relationship between cross-country dispersion in inflation expectations and external balances in the euro area, as shown in the bottom panel of Figure 1: the stronger is the home bias in expectations, the more dispersed are expectations across countries and the greater are trade imbalances once countries face asymmetric shocks.

In the right panel of Figure 5, we consider the role of country openness. We again simulate an asymmetric demand shock and plot the response of Home’s trade balance, but now consider different degrees of country openness, governed by α , while keeping the home bias parameter fixed at $\omega = 0.1$. The results convey the following message: the more the economy is exposed to foreign trade, i.e. the higher is α , the stronger is the effect of the real exchange rate on economic conditions and therefore the quicker are agents to discover the counteracting forces of the real exchange rate. Hence, even if *expectations* are home biased, a sharp adjustment of the real exchange rate following an asymmetric demand shock can be avoided if *consumption* is not.

5 Stability and implications for monetary policy

In this section, which consists of two parts, we investigate the stability of monetary union under home biased expectations and the role for monetary policy. In the first part, we derive

the conditions for E-stability of the model.¹⁰ In the second part, we perform numerical simulations to illustrate how the requirements for monetary policy to guarantee E-stability change when considering different degrees of home bias in expectations. We use the insights from the stability analysis to explain how monetary policy can mitigate the pro-cyclical effects of the real interest rate channel and promote macroeconomic stability.

In what follows, we use the concept of decreasing (rather than constant) gain learning, as it allows us to consider the possibility of convergent beliefs.

5.1 The Restricted Perception Equilibrium

To derive the requirements for E-stability, we start by inserting agents' forecasting equation (7) into the state-space representation of the model, given by (8), to find the following ALM:

$$\begin{aligned}
 x_t &= \left[\sum_{j=H,F} F^j (I + \Lambda_1^j) \Lambda_0^j, \sum_{j=H,F} F^j (\Lambda_1^j)^2 \Omega_x^j + G, \sum_{j=H,F} F^j (\Lambda_1^j \Lambda_2^j + \Lambda_2^j \varrho) \Omega_v^j + H \right] \begin{bmatrix} 1 \\ x_{t-1} \\ v_t \end{bmatrix} \\
 &\equiv T(\Lambda^H, \Lambda^F) z_t,
 \end{aligned} \tag{22}$$

where $\Omega_k^i = \begin{bmatrix} I & \mathbf{0} \\ \mathbf{0} & I\omega \end{bmatrix}$, with $i = \{H, F\}$ and $k = \{x, v\}$, are matrices of conformable sizes that determine by how much the non-domestic subset of either x_t or v_t is weighted in agents' PLM, see (11). The assumption of a potential home bias in agents' expectations features a crucial departure from the standard adaptive learning literature, namely that agents in country i observe only an imperfect version of the ALM, which is equal to Equation (22) multiplied by Ω_x^i . Accordingly, agents face the following mapping from their PLM, as given

¹⁰We focus on the notion of *strong* E-stability, which assumes agents include both predetermined and non-predetermined variables in the data vector x_t when performing forecasts using their PLM (11). Given this notion, agents' forecasting models are likely to be over-parameterized, since only the state variables truly matter for the dynamics of the economy. Under weak E-stability, x_t only includes state variables.

by (11), to their *observed* ALM:

$$T^i(\Lambda^i) = \begin{bmatrix} \Omega_x^i \left[\sum_{j=H,F} F^j (I + \Lambda_1^j) \Lambda_0^j \right] \\ \Omega_x^i \left[\sum_{j=H,F} F^j (\Lambda_1^j)^2 \Omega_x^j + G \right] \\ \Omega_x^i \left[\sum_{j=H,F} F^j (\Lambda_1^j \Lambda_2^j + \Lambda_2^j \varrho) \Omega_v^j + H \right] \end{bmatrix}'. \quad (23)$$

Evidently, if non-domestic variables are weighted differently in agents' information sets than domestic variables, i.e. if $\omega < 1$, then the coefficients of the PLMs across countries are mapped into different observed ALMs, which implies that, in general, $\Lambda^H \neq \Lambda^F$. As a result, expectations will differ across countries in steady state which prevents the economy from converging towards the Rational Expectations Equilibrium (REE). Instead, the economy reaches a so-called *Restricted Perception Equilibrium* (RPE).¹¹ A necessary condition for the existence of such a RPE is that the mapping function, $T^i(\Lambda^i)$, has a fixed point, i.e. $\hat{\Lambda}^i = [\hat{\Lambda}_0^i, \hat{\Lambda}_1^i, \hat{\Lambda}_2^i]$, satisfying

$$\hat{\Lambda}_0^i = \Omega_x^i \left[\sum_{j=H,F} F^j (I + \hat{\Lambda}_1^j) \hat{\Lambda}_0^j \right], \quad (24)$$

$$\hat{\Lambda}_1^i = \Omega_x^i \left[\sum_{j=H,F} F^j (\hat{\Lambda}_1^j)^2 \Omega_x^j + G \right], \quad (25)$$

$$\hat{\Lambda}_2^i = \Omega_x^i \left[\sum_{j=H,F} F^j (\hat{\Lambda}_1^j \hat{\Lambda}_2^j + \hat{\Lambda}_2^j \varrho) \Omega_v^j + H \right]. \quad (26)$$

Because Equation (25) includes a quadratic matrix expression, multiple solutions for $\hat{\Lambda}_1^i$ may exist. On the other hand, $\hat{\Lambda}_2^i$ is uniquely pinned down by Equation (26) for any given $\hat{\Lambda}_1^i$. Further, note that the only solution for $\hat{\Lambda}_0^i$ is the zero vector.

The relevant ordinary differential equations that govern the E-stability properties of any

¹¹For a detailed discussion, see Evans and Honkapohja (2001, Chapters 3.6 and 13).

RPE are derived in Appendix F and given by

$$\frac{d\Lambda^i}{d\tau} = \varpi^i \left[T^i (\Lambda_{t-1}^H, \Lambda_{t-1}^F) (\Omega_x^i)' - (\Omega^i)' \Lambda_{t-1}^i \right], \quad (27)$$

with

$$\Omega^i \equiv \begin{bmatrix} I & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Omega_x^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \Omega_v^i \end{bmatrix},$$

and where ϖ^i is defined as the asymptotic speed of adjustment in the learning algorithm, as described in [Honkapohja and Mitra \(2006\)](#). Since we assume agents in each country apply the same type of learning algorithm, $\varpi^H = \varpi^F = \varpi$. Further, denote any possible RPE that satisfies the conditions (24)-(26) by $(\hat{\Lambda}^H, \hat{\Lambda}^F)$. Then, by linearizing around the RPE and vectorizing the right-hand side of the differential equations (27), one obtains

$$\begin{bmatrix} \frac{d\text{vec}(\Lambda^H)}{d\tau} \\ \frac{d\text{vec}(\Lambda^F)}{d\tau} \end{bmatrix} = \varpi \Xi \begin{bmatrix} \text{vec}(\Lambda^H) \\ \text{vec}(\Lambda^F) \end{bmatrix}, \quad (28)$$

where τ expresses ‘notional time’ as in [Evans and Honkapohja \(2001\)](#) and Ξ is given by

$$\Xi = \begin{bmatrix} (\Omega_x^H \otimes I) D_{\Lambda^H} T - I \otimes (\Omega^H)' & (\Omega_x^H \otimes I) D_{\Lambda^F} T \\ (\Omega_x^F \otimes I) D_{\Lambda^H} T & (\Omega_x^F \otimes I) D_{\Lambda^F} T - I \otimes (\Omega^F)' \end{bmatrix}, \quad (29)$$

with

$$D_{\Lambda^i} T (\hat{\Lambda}^H, \hat{\Lambda}^F) = \begin{bmatrix} \Omega_x^i F^i (I + \hat{\Lambda}_1^i) & (\hat{\Lambda}_0^i)' \otimes \Omega_x^i F^i & \mathbf{0} \\ \mathbf{0} & (\Omega_x^i)' \otimes \Omega_x^i F^i \hat{\Lambda}_1^i + \\ & (\Omega_x^i)' (\hat{\Lambda}_1^i)' \otimes \Omega_x^i F^i & \mathbf{0} \\ \mathbf{0} & (\Omega_v^i)' (\hat{\Lambda}_2^i)' \otimes \Omega_x^i F^i & (\Omega_v^i)' \otimes \Omega_x^i F^i \hat{\Lambda}_1^i + \\ & & (\Omega_v^i)' \varrho' \otimes \Omega_x^i F^i \end{bmatrix}. \quad (30)$$

We are now ready to present the requirements for E-stability of the model under home biased expectations:

Proposition 1. *The RPE of the model with home bias in expectations is E-stable, if and only if the eigenvalues of Ξ have negative real parts.*

5.2 Monetary requirements for (E-)stability

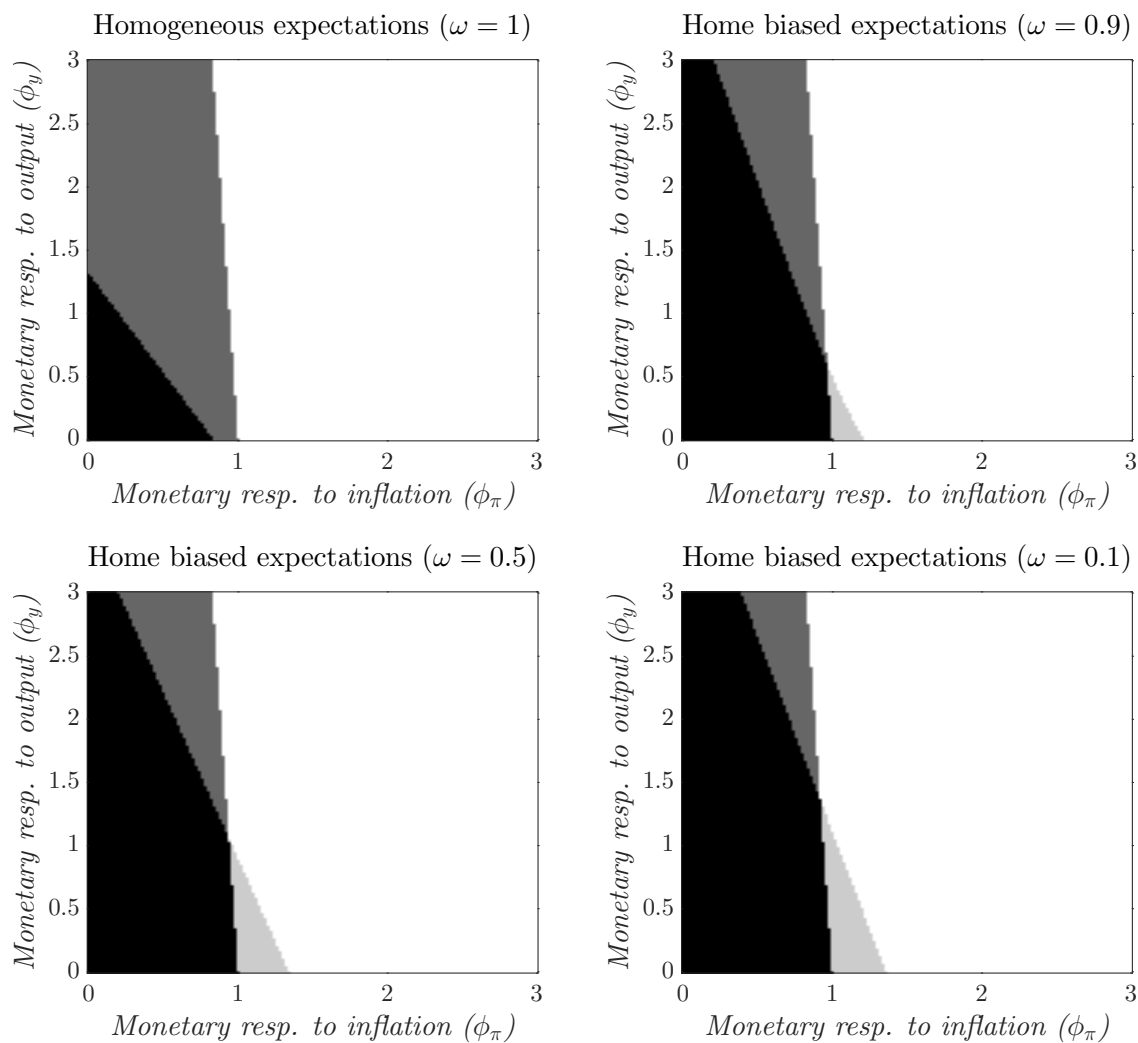
Figure 6 shows the conditions for E-stability from Proposition 1 as a function of the monetary policy parameters $\phi_\pi \in [0, 3]$, which governs the monetary response to inflation, and $\phi_y \in [0, 3]$, which governs the monetary response to output.¹² In the white area, monetary policy yields stable and unique rational expectations equilibria. These equilibria are also E-stable, i.e. learnable, RPEs. In the dark gray area, equilibrium is indeterminate, yet the RPE is E-stable. In the light gray area, the REE is determinate, whereas the RPE is *not* E-stable. Finally, the black region indicates the parameter space in which monetary policy is unable to deliver a determinate REE nor an E-stable RPE.

In the upper-left panel of Figure 6, expectations are adaptive, yet not home biased and therefore homogeneous across countries. In this case, we find that the conditions for determinacy are similar to, yet somewhat weaker than, those under rational expectations, which imply that the Taylor-principle is a sufficient condition for determinacy. To satisfy the Taylor-principle, the central bank must respond to increases (decreases) in inflation by raising (lowering) the real interest rate, which requires $\phi_\pi > 1$. If expectations are adaptive, determinacy may also be guaranteed if ϕ_π is set slightly below unity, provided the central bank aggressively responds to output fluctuations. Under these conditions, equilibrium is E-stable. However, if ϕ_π is set too far below unity, equilibrium is no longer determinate and can be E-unstable if the monetary response to output is sufficiently weak.

The remaining panels of Figure 6 show that the determinacy and E-stability require-

¹²We deviate from the benchmark calibration and set $\alpha = 0.6$ to ensure existence of a RPE over the whole grid of policy parameters when calculating the determinacy and E-stability conditions.

Figure 6: Determinacy and E-stability for different degrees of home bias in expectations



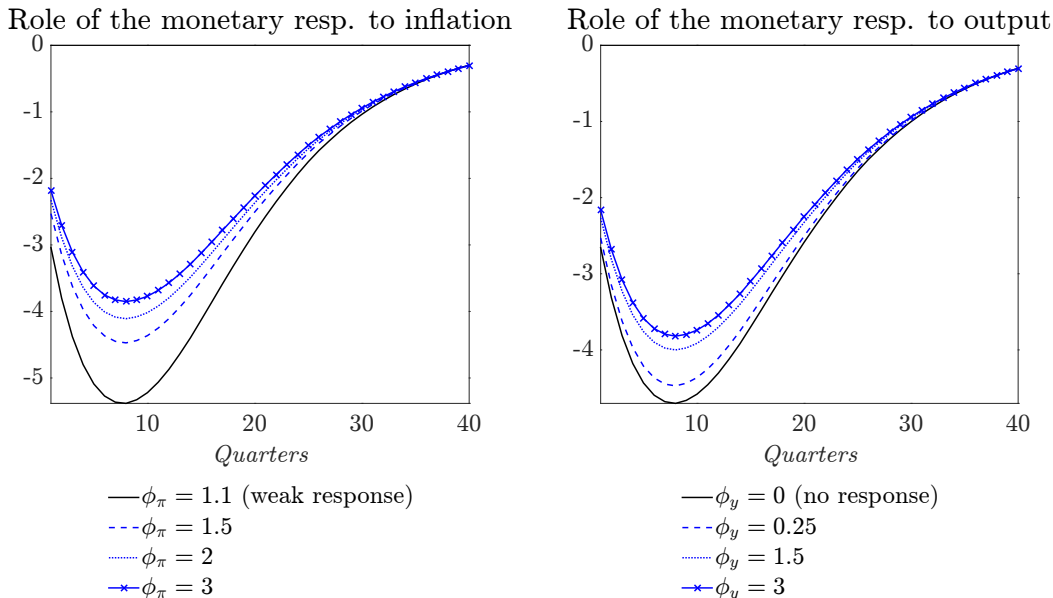
Note: The figure shows the conditions for E-stability and determinacy as a function of the monetary policy parameters ϕ_π and ϕ_y , and conditional on different degrees of home bias in expectations. White = determinate Rational Expectations Equilibrium (REE) and E-stable Restricted Perception Equilibrium (RPE); dark gray = indeterminate REE and E-stable RPE; light gray = determinate REE and E-unstable RPE; black = indeterminate REE and E-unstable RPE.

ments for monetary policy change in the presence of home bias in expectations. In fact, the monetary union is more likely to fall in regions characterized by E-unstable RPEs, the greater is the degree of home bias in expectations, which is reflected by the light-gray region that expands as ω falls. To guarantee an equilibrium that is both determinate and E-stable, the central bank must respond more aggressively to changes in inflation or to changes in output (or both) than is implied by the Taylor-principle. Thus, the monetary policy trade-off between inflation and output stabilization becomes more severe for greater degrees of asymmetry in information sets across countries.

These results are closely related to the Walters critique (Walters, 1994). Alan Walters, in his plea against UK membership of the European Monetary System, argued that monetary unions are inherently unstable due to the inability of member countries to, unilaterally, offset country-specific shocks through an adjustment in the nominal interest rate or nominal exchange rate. Accordingly, countries belonging to a monetary union would be more susceptible to self-strengthening inflationary or deflationary cycles than countries operating under flexible exchange rates. Those who challenge the Walters critique point to the stabilizing effects of the real exchange rate channel that, at least in the long run, dominate the pro-cyclical effects of the real interest rate channel. According to our results from Section 4, both channels are relevant for macroeconomic stability and the relative strengths of the two channels are affected by the degree of home bias in expectations.

The results presented in this section suggest that monetary unions may indeed be more likely to be unstable, thereby supporting the Walters critique, yet only insofar expectations are home biased. If expectations are not home biased, we find that the stability and determinacy requirements for monetary policy are not that different from those pertaining to economies outside monetary union (Clarida et al., 1997). Yet, if expectations are subject to home bias, stability of the union is threatened. In that case, to avoid unstable macroeconomic outcomes, the central bank must adopt a more aggressive stance towards inflation and output.

Figure 7: Macroeconomic imbalances and monetary policy



Note: The figure shows the response of Home’s trade balance to an asymmetric demand shock under home biased expectations, with $\omega = 0.1$.

To see whether a more aggressive monetary policy stance helps mitigate macroeconomic imbalances under home biased expectations, we again examine the effects of an asymmetric demand shock on Home’s trade balance, yet this time for different values of the monetary policy parameters ϕ_π and ϕ_y . The remaining parameters are kept at their benchmark values given by Table 1, yet the degree of home bias in expectations is set to $\omega = 0.1$. The results are shown in Figure 7.¹³

According to the figure, a more aggressive response of the central bank to changes in either inflation (i.e. higher values for ϕ_π , see left panel) or output (higher values for ϕ_y , right panel) reduces the trade balance response to the asymmetric demand shock. Recall that, following the shock, the nominal interest rate is left unchanged as union-wide aggregates remain constant. However, different monetary policy stances still can deliver different macroeconomic dynamics, even without changing the nominal interest rate, due to their

¹³In generating Figure 7, we consider a learning algorithm with decreasing gain. However, the results are nearly identical under a constant gain of $\gamma = 0.01$, which was the benchmark calibration of the gain parameter used in Section 4.

effect on agents' expectations *before* the shock hits the economy. For instance, if agents in Home expect, based on past experiences, a strong contractionary monetary response to an increase in aggregate demand, their inflation expectations remain close to the inflation target, even if a demand shock raises domestic inflation and expectations are home biased. Thus, a more aggressive monetary policy stance weakens the feedback between inflation expectations and the real interest rate, thereby weakening the real interest rate channel and allowing the real exchange rate to stabilize economic conditions more promptly.

Although the benefits of having anchored inflation expectations, in terms of promoting macroeconomic stability, are well understood, the results shown here provide an additional argument in favor of tightly anchored inflation expectations that applies particularly to monetary unions: not only does a credible inflation target prevent expectations from drifting too far from steady state, it also helps prevent expectations from *diverging* too far across regions within the union following asymmetric shocks.

6 Conclusion

Although the importance of expectations for policy design and evaluation is broadly acknowledged in the literature, attention is often focused on rational or homogeneous expectations at the aggregate level, thereby ignoring potentially important differences in expectations across agents. This approach does not seem appropriate when studying asymmetric shocks under monetary union, where forces that provoke divergences in economic dynamics and expectations across member countries are likely to be strong.

In this paper, we studied the role of home bias in expectations for macroeconomic imbalances in a monetary union. We provided empirical evidence that suggests expectations can become more home biased towards domestic variables in times of large country-specific shocks. We then applied a New Keynesian model for a two-country monetary union that features such home bias in expectations and examined the effects of asymmetric shocks. We

found that home bias in expectations aggravates pro-cyclical real interest rate effects, causing macroeconomic imbalances be more pronounced. In particular, the expectations bias shifts agents' attention from international price competitiveness effects to domestic real interest rate effects. As a result, overheating economic conditions in countries with above-average inflation tend to persist, since the relatively low real interest rate keeps economic activity elevated despite the appreciation of the real exchange rate. In contrast, countries with below-average inflation and high real interest rates experience more protracted recessions. In both cases, the real exchange rate must adjust more strongly than under rational or homogeneous expectations in order to normalize economic conditions. We further showed that, to prevent expectations from drifting too far from the currency area's inflation target, monetary policy must target inflation and output more aggressively, especially if the degree of home bias in expectations is large.

It is well established that cross-country inflation differentials under monetary union may reflect, at least in part, the endogenous response of the real exchange rate to asymmetric shocks. In fact, such a self-correcting mechanism forms an integral part of the smooth functioning of monetary unions ([Angeloni and Ehrmann, 2007](#)). Hence, it is not always clear whether a central bank should care about dispersion in inflation rates across member countries. However, the results presented in this paper point towards a potential risk embedded in divergent inflationary dynamics under monetary union that is related to the way agents form expectations. To distinguish between regional economic disparities that are either temporary, and part of a necessary adjustment process, or structural, and potentially unstable, the central bank should monitor expectations at both the aggregate and national level. Internalizing interregional dispersion into the monetary policy strategy, then, is warranted insofar expectations are sensitive to past domestic economic conditions and ignorant of international spillover effects. Alternatively, fiscal policy could be employed to offset the bias in expectations at the national level, for instance by changing import tariffs or export subsidies to replicate a real exchange rate adjustment. Whether such policies are able to

fully undo the effects of home biased expectations is a question we leave for future research.

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A Quantifying inflation expectations from qualitative survey data

In the paper, we used data on inflation expectations that were extracted from the European Commission’s Consumer Survey. In this survey, households are asked to answer questions about their views on economic and financial conditions. Questions 5 and 6 of the Consumer Survey are about recent and future price developments. Question 5 asks how consumers have perceived price developments over the past twelve months. The survey offers six possible answers: “They have (1) risen a lot, (2) risen moderately, (3) risen slightly, (4) stayed about the same, (5) fallen, or (6) don’t know”. Question 6 asks how consumers expect prices to develop in the next twelve months as compared to the past twelve months. Again, six possible answers are offered: “They will (1) increase more rapidly, (2) increase at the same rate, (3) increase at a slower rate, (4) stay about the same, (5) fall, or (6) don’t know”.

Following [Batchelor and Orr \(1988\)](#), [Berk \(1999\)](#) and [Dias et al. \(2010\)](#), we use the probability approach to quantify these qualitative survey responses, which consists of three steps. In the first step, we calculate *actual inflation*, $\pi_{i,t}$, for each country i as the annual growth rate of the Harmonized Index of Consumer Prices, taken from Eurostat. In the second step, we transform the series of actual inflation into *perceived inflation*, $\pi_{i,t}^P$, using the shares $S_{m,t}$ of the responses to Question 5 belonging to answer $m = 1, 2, \dots$. We use the Hodrick-Prescott filter to obtain the trend component of $\pi_{i,t}$, i.e. $\pi_{i,t}^{HP}$, where we set the smoothing parameter equal to 14,400. Then, we calculate perceived inflation as

$$\pi_{i,t}^P = \frac{-Z_{3,t} - Z_{4,t}}{Z_{1,t} + Z_{2,t} - Z_{3,t} - Z_{4,t}} \pi_{i,t}^{HP}, \quad (31)$$

where the thresholds $Z_{m,t}$ are defined as

$$Z_{1,t} = \mathcal{F}_t^{-1}(1 - S_{1,t}), \quad (32)$$

$$Z_{2,t} = \mathcal{F}_t^{-1}(1 - S_{1,t} - S_{2,t}), \quad (33)$$

$$Z_{3,t} = \mathcal{F}_t^{-1}(1 - S_{1,t} - S_{2,t} - S_{3,t}), \quad (34)$$

$$Z_{4,t} = \mathcal{F}_t^{-1}(S_{5,t}), \quad (35)$$

and where \mathcal{F} denotes the cumulative Normal standard distribution function. Finally, in the third step, we calculate *expected inflation* over the next twelve months, conditional on information available at month t , i.e. $\pi_{i,t+12|t}^e$, as

$$\pi_{i,t+12|t}^e = \frac{-Z_{3,t} - Z_{4,t}}{Z_{1,t} + Z_{2,t} - Z_{3,t} - Z_{4,t}} \pi_{i,t}^P, \quad (36)$$

where the thresholds $Z_{m,t}$ are again defined by Equations (32)-(35), yet are now calculated using the statistical distribution of the responses to Question 6.

B Optimal demand, price indices and market clearing

This section presents the demand schedules for Home and Foreign final and intermediate goods, the consumption and producer price indices, and the market clearing conditions.

Total consumption c_t^i consists of Home and Foreign final goods, denoted by $c_{i,t}^i$ and $c_{j,t}^i$, respectively, with $i, j = \{H, f\}$ and $i \neq j$. Maximizing c_t^i , given by (19), subject to appropriate expenditure constraints and taking prices as given, yields the following demand schedules:

$$c_{i,t}^i = (1 - \bar{\alpha}^i) \left(\frac{P_{i,t}^i}{P_t^i} \right)^{-\eta} c_t^i, \quad c_{j,t}^i = \bar{\alpha}^i \left(\frac{P_{j,t}^i}{P_t^i} \right)^{-\eta} c_t^i, \quad i \neq j, \quad (37)$$

and consumer price indices:

$$P_t^i = \left[(1 - \bar{\alpha}^i) (P_{i,t}^i)^{1-\eta} + \bar{\alpha}^i (P_{j,t}^i)^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad i \neq j. \quad (38)$$

In turn, final goods consist of bundles of differentiated intermediate goods, $c_{H,t}^i(h)$ and $c_{F,t}^i(f)$, which are assembled using the following aggregators:

$$c_{H,t}^i \equiv \left[\left(\frac{1}{s} \right)^{\frac{1}{\epsilon}} \int_0^s c_{H,t}^i(h)^{\frac{\epsilon-1}{\epsilon}} dh \right]^{\frac{\epsilon}{\epsilon-1}}, \quad c_{F,t}^i \equiv \left[\left(\frac{1}{1-s} \right)^{\frac{1}{\epsilon}} \int_s^1 c_{F,t}^i(f)^{\frac{\epsilon-1}{\epsilon}} df \right]^{\frac{\epsilon}{\epsilon-1}}. \quad (39)$$

Maximizing these bundles yields the following optimal demand conditions:

$$c_{H,t}^i(h) = \frac{1}{s} \left(\frac{P_t^i(h)}{P_{H,t}^i} \right)^{-\epsilon} c_{H,t}^i, \quad c_{F,t}^i(f) = \frac{1}{1-s} \left(\frac{P_t^i(f)}{P_{F,t}^i} \right)^{-\epsilon} c_{F,t}^i, \quad (40)$$

and producer price indices:

$$P_{H,t}^i = \left(\frac{1}{s} \int_0^s P_t^i(h)^{1-\epsilon} dh \right)^{\frac{1}{1-\epsilon}}, \quad P_{F,t}^i = \left(\frac{1}{1-s} \int_s^1 P_t^i(f)^{1-\epsilon} df \right)^{\frac{1}{1-\epsilon}}. \quad (41)$$

Assuming the law of one price holds for all goods, we have $P_{H,t}^H = P_{H,t}^F = P_{H,t}$ and $P_{F,t}^H = P_{F,t}^F = P_{F,t}$. Using the demand schedules in (40) and appropriate aggregators for national output, we can derive the goods market clearing conditions for Home:

$$y_t^H = \left(\frac{P_{H,t}}{P_t^H} \right)^{-\eta} \left[(1 - \bar{\alpha}^H) c_t^H + \frac{1-s}{s} \bar{\alpha}^F q_t^\eta c_t^F \right], \quad (42)$$

and Foreign:

$$y_t^F = \left(\frac{P_{F,t}}{P_t^F} \right)^{-\eta} \left[(1 - \bar{\alpha}^F) c_t^F + \frac{s}{1-s} \bar{\alpha}^H q_t^{-\eta} c_t^H \right]. \quad (43)$$

Finally, labor market clearing satisfies

$$n_t^i = y_t^i \mathcal{D}_t^i, \quad (44)$$

where \mathcal{D}_t^i is a measure of price dispersion.

C International risk sharing

In this section, we provide a brief derivation of the imperfect international risk sharing condition (18), which closely follows Wang (2008).

Subject to the budget constraint (15), the household aims to maximize utility (14), which we denote here as $U_t^i(c_t^i, n_t^i, z_{D,t}^i)$. Let $\mu^i(\xi_{t+1}|\cdot)$ be the probability that event ξ_{t+1} occurs at $t+1$ as perceived by agents from country $i = \{H, F\}$. The household then solves the following Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \sum_{\xi_{t+1}} \beta^t \mu^i(\xi_{t+1}|\cdot) \left\{ U_t^i(c_t^i, n_t^i, z_{D,t}^i) + \lambda_t \left[W_t^i n_t^i + B_{t-1}^i + P_{i,t}^i \psi_t^i - \sum_{\xi_{t+1}} Q_t(\xi_{t+1}|\cdot) B_t^i(\xi_{t+1}) - P_t^i c_t^i \right] \right\}. \quad (45)$$

Combine the first-order conditions with respect to c_t^i and $B_t^i(\xi_{t+1})$ to obtain

$$\frac{U_{c,t}^i Q_t(\xi_{t+1}|\cdot)}{P_t^i} = \beta \mu^i(\xi_{t+1}|\cdot) \frac{U_{c,t+1}^i(\xi_{t+1})}{P_{t+1}^i(\xi_{t+1})},$$

with $U_{c,t}^i$ the marginal utility of consumption. Since this condition holds for all i , we can write

$$\frac{P_t^F U_{c,t}^H}{P_t^H U_{c,t}^F} = \frac{\mu^H(\xi_{t+1}|\cdot) U_{c,t+1}^H(\xi_{t+1}) (P_{t+1}^H(\xi_{t+1}))^{-1}}{\mu^F(\xi_{t+1}|\cdot) U_{c,t+1}^F(\xi_{t+1}) (P_{t+1}^F(\xi_{t+1}))^{-1}}, \quad (46)$$

which implies

$$\begin{aligned} \frac{P_t^F U_{c,t}^H}{P_t^H U_{c,t}^F} &= \frac{\mu^H(\xi_{1,t+1}|\cdot) U_{c,t+1}^H(\xi_{1,t+1}) (P_{t+1}^H(\xi_{1,t+1}))^{-1}}{\mu^F(\xi_{1,t+1}|\cdot) U_{c,t+1}^F(\xi_{1,t+1}) (P_{t+1}^F(\xi_{1,t+1}))^{-1}} \\ &= \frac{\mu^H(\xi_{2,t+1}|\cdot) U_{c,t+1}^H(\xi_{2,t+1}) (P_{t+1}^H(\xi_{2,t+1}))^{-1}}{\mu^F(\xi_{2,t+1}|\cdot) U_{c,t+1}^F(\xi_{2,t+1}) (P_{t+1}^F(\xi_{2,t+1}))^{-1}} \\ &= \dots \\ &= \frac{\mu^H(\xi_{m,t+1}|\cdot) U_{c,t+1}^H(\xi_{m,t+1}) (P_{t+1}^H(\xi_{m,t+1}))^{-1}}{\mu^F(\xi_{m,t+1}|\cdot) U_{c,t+1}^F(\xi_{m,t+1}) (P_{t+1}^F(\xi_{m,t+1}))^{-1}}. \end{aligned}$$

In other words, relative expected marginal utilities across countries must be equal for all m possible events occurring at time $t + 1$. Denote $P_t^F U_{c,t}^H / (P_t^H U_{c,t}^F) = X$, such that (46) implies

$$\begin{aligned} \mu^H(\xi_{t+1}|\cdot) U_{c,t+1}^H(\xi_{t+1}) (P_{t+1}^H(\xi_{1,t+1}))^{-1} &= X \cdot \mu^F(\xi_{1,t+1}|\cdot) U_{c,t+1}^F(\xi_{1,t+1}) (P_{t+1}^F(\xi_{1,t+1}))^{-1}, \\ &\dots = \dots \\ \mu^H(\xi_{m,t+1}|\cdot) U_{c,t+1}^H(\xi_{m,t+1}) (P_{t+1}^H(\xi_{m,t+1}))^{-1} &= X \cdot \mu^F(\xi_{m,t+1}|\cdot) U_{c,t+1}^F(\xi_{m,t+1}) (P_{t+1}^F(\xi_{m,t+1}))^{-1}. \end{aligned}$$

Aggregating over all events, we then have

$$\begin{aligned} &\frac{E_t^H \left[\frac{U_{c,t+1}^H}{P_{t+1}^H} \right]}{E_t^F \left[\frac{U_{c,t+1}^F}{P_{t+1}^F} \right]} \\ &= \frac{\sum_{\xi_{t+1}} \mu^H(\xi_{t+1}|\cdot) U_{c,t+1}^H(\xi_{t+1}) (P_{t+1}^H(\xi_{t+1}))^{-1}}{\sum_{\xi_{t+1}} \mu^F(\xi_{t+1}|\cdot) U_{c,t+1}^F(\xi_{t+1}) (P_{t+1}^F(\xi_{t+1}))^{-1}} \\ &= \frac{\mu^H(\xi_{1,t+1}|\cdot) U_{c,t+1}^H(\xi_{1,t+1}) (P_{t+1}^H(\xi_{1,t+1}))^{-1} + \dots + \mu^H(\xi_{m,t+1}|\cdot) U_{c,t+1}^H(\xi_{m,t+1}) (P_{t+1}^H(\xi_{m,t+1}))^{-1}}{\mu^F(\xi_{1,t+1}|\cdot) U_{c,t+1}^F(\xi_{1,t+1}) (P_{t+1}^F(\xi_{1,t+1}))^{-1} + \dots + \mu^F(\xi_{m,t+1}|\cdot) U_{c,t+1}^F(\xi_{m,t+1}) (P_{t+1}^F(\xi_{m,t+1}))^{-1}} \\ &= \frac{X \cdot \left[\mu^F(\xi_{1,t+1}|\cdot) U_{c,t+1}^F(\xi_{1,t+1}) (P_{t+1}^F(\xi_{1,t+1}))^{-1} + \dots + \mu^F(\xi_{m,t+1}|\cdot) U_{c,t+1}^F(\xi_{m,t+1}) (P_{t+1}^F(\xi_{m,t+1}))^{-1} \right]}{\mu^F(\xi_{1,t+1}|\cdot) U_{c,t+1}^F(\xi_{1,t+1}) (P_{t+1}^F(\xi_{1,t+1}))^{-1} + \dots + \mu^F(\xi_{m,t+1}|\cdot) U_{c,t+1}^F(\xi_{m,t+1}) (P_{t+1}^F(\xi_{m,t+1}))^{-1}} \\ &= X \\ &= \frac{P_t^F U_{c,t}^H}{P_t^H U_{c,t}^F}, \end{aligned}$$

which implies

$$\frac{U_{c,t}^H}{U_{c,t}^F} = \frac{E_t^H \left[U_{c,t+1}^H \frac{P_t^H}{P_{t+1}^H} \right]}{E_t^F \left[U_{c,t+1}^F \frac{P_t^F}{P_{t+1}^F} \right]}.$$

Finally, using Equation (14), we obtain Equation (18).

D Linearization

The model is linearized around a deterministic, zero-inflation steady state. Let variables with a hat denote the percentage deviation of the corresponding variable from its steady-state value, i.e. $\hat{x}_t = (x_t - x)/x$ for any generic variable x_t . A scaled-down version of the linear model is then given by

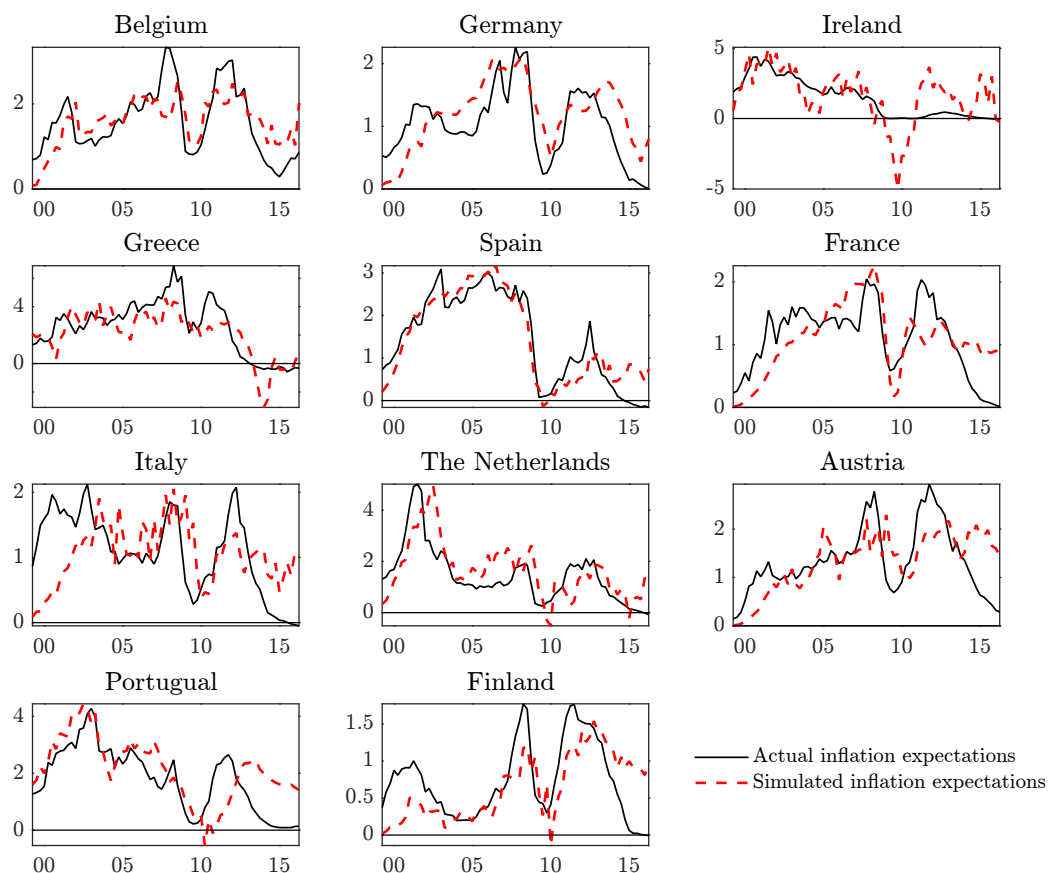
$$\begin{aligned}
\hat{y}_t^H &= (1 - \bar{\alpha}^H) \frac{c^H}{y^H} \hat{c}_t^H + \frac{1-s}{s} \bar{\alpha}^F \frac{c^F}{y^H} \hat{c}_t^F + \Theta_H \hat{q}_t, \\
\sigma \hat{c}_t^H &= \sigma \tilde{E}_t^H \hat{c}_{t+1}^H - \left[\phi_\pi (s \hat{\pi}_t^H + (1-s) \hat{\pi}_t^F) + \phi_y (s \hat{y}_t^H + (1-s) \hat{y}_t^F) - \tilde{E}_t^H \hat{\pi}_{t+1}^H \right] - \tilde{E}_t^H \Delta \hat{z}_{D,t+1}^H, \\
\hat{\pi}_t^H &= \hat{\pi}_{H,t}^H + \hat{\alpha}^H (\hat{q}_t - \hat{q}_{t-1}), \\
\hat{\pi}_{H,t}^H &= \beta \tilde{E}_t^H \hat{\pi}_{H,t+1}^H + \kappa (\varphi \hat{y}_t^H + \sigma \hat{c}_t^H + \hat{\alpha}^H \hat{q}_t), \\
\hat{y}_t^F &= (1 - \bar{\alpha}^F) \frac{c^F}{y^F} \hat{c}_t^F + \frac{s}{1-s} \bar{\alpha}^H \frac{c^H}{y^F} \hat{c}_t^H - \Theta_F \hat{q}_t, \\
\hat{c}_t^F &= \hat{c}_t^F + \tilde{E}_t^F \hat{c}_{t+1}^F - \tilde{E}_t^H \hat{c}_{t+1}^H + \frac{1}{\sigma} \left(\tilde{E}_t^F \hat{\pi}_{t+1}^F - \tilde{E}_t^H \hat{\pi}_{t+1}^H + \tilde{E}_t^H \Delta \hat{z}_{D,t+1}^H - \tilde{E}_t^F \Delta \hat{z}_{D,t+1}^F \right), \\
\hat{\pi}_t^F &= \hat{\pi}_{F,t}^F - \hat{\alpha}^F (\hat{q}_t - \hat{q}_{t-1}), \\
\hat{\pi}_{F,t}^F &= \beta \tilde{E}_t^F \hat{\pi}_{F,t+1}^F + \kappa (\varphi \hat{y}_t^F + \sigma \hat{c}_t^F - \hat{\alpha}^F \hat{q}_t), \\
\hat{q}_t &= \hat{q}_{t-1} + \hat{\pi}_t^F - \hat{\pi}_t^H, \\
\hat{z}_{D,t}^i &= \rho_D \hat{z}_{D,t-1}^i + \varepsilon_{D,t}^i,
\end{aligned}$$

where $\kappa \equiv (1 - \theta)(1 - \beta\theta)/\theta$ and $\hat{\alpha}^i = \bar{\alpha}^i / (1 - \bar{\alpha}^F - \bar{\alpha}^H)$, with $i = \{H, F\}$, and where

$$\begin{aligned}
\Theta_H &\equiv \eta \frac{\hat{\alpha}^H}{\bar{\alpha}^H} \left[\bar{\alpha}^H (1 - \bar{\alpha}^H) \frac{c^H}{y^H} + (1 - \bar{\alpha}^F) \frac{1-s}{s} \bar{\alpha}^F \frac{c^F}{y^H} \right], \\
\Theta_F &\equiv \eta \frac{\hat{\alpha}^F}{\bar{\alpha}^F} \left[\bar{\alpha}^F (1 - \bar{\alpha}^F) \frac{c^F}{y^F} + (1 - \bar{\alpha}^H) \frac{s}{1-s} \bar{\alpha}^H \frac{c^H}{y^F} \right].
\end{aligned}$$

E Additional graphs

Figure 8: Actual and simulated inflation expectations in the euro area (pct.)



Note: The figure shows the simulated inflation expectations (dashed) with the lowest Mean Square Comparison Error with regards to actual inflation expectations (solid). The latter were extracted from the European Commission's Consumer Survey.

F Derivation of Equation (27)

To derive the ordinary differential equation (27), first consider (12) and substitute in for \tilde{x}_t^i by multiplying (22) by Ω_x^i to find

$$\begin{aligned}
\Lambda_t^i &= \Lambda_{t-1}^i + \gamma_t (M_t^i)^{-1} \tilde{z}_t^i \left[\Omega_x^i T(\Lambda_{t-1}^H, \Lambda_{t-1}^F) z_t - \Lambda_{t-1}^i \tilde{z}_t^i \right]' \\
&= \Lambda_{t-1}^i + \gamma_t (M_t^i)^{-1} \Omega^i z_t \left[\Omega_x^i T(\Lambda_{t-1}^H, \Lambda_{t-1}^F) z_t - \Lambda_{t-1}^i \Omega^i z_t \right]' \\
&= \Lambda_{t-1}^i + \gamma_t (M_t^i)^{-1} \Omega^i z_t \left\{ \left[\Omega_x^i T(\Lambda_{t-1}^H, \Lambda_{t-1}^F) - \Lambda_{t-1}^i \Omega^i \right] z_t \right\}' \\
&= \Lambda_{t-1}^i + \gamma_t (M_t^i)^{-1} \Omega^i z_t z_t' \left[\Omega_x^i T(\Lambda_{t-1}^H, \Lambda_{t-1}^F) - \Lambda_{t-1}^i \Omega^i \right]'.
\end{aligned}$$

Note that

$$E \left[(M_t^i)^{-1} \Omega^i z_t z_t' \left(\begin{array}{c} \left\{ \Omega_x^i \left[F^H (I + \Lambda_1^H) \Lambda_0^H + F^F (I + \Lambda_1^F) \Lambda_0^F \right] - \Lambda_0^i \right\}' \\ \left\{ \Omega_x^i \left[F^H (\Lambda_1^H)^2 \Omega_x^H + F^F (\Lambda_1^F)^2 \Omega_x^F + G \right] - \Omega_x^i \Lambda_1^i \right\}' \\ \left\{ \Omega_x^i \left[F^H (\Lambda_1^H \Lambda_2^H + \Lambda_2^H \varrho) \Omega_v^H + F^F (\Lambda_1^F \Lambda_2^F + \Lambda_2^F \varrho) \Omega_v^F + H \right] - \Omega_v^i \Lambda_2^i \right\}' \end{array} \right) \right].$$

Let $E[zz'] \equiv \lim_t E[z_t z_t']$. Then, we obtain the following ODEs:

$$\frac{d\Lambda_i}{d\tau} = \varpi^i (M_t^i)^{-1} \Omega^i E[zz'] \left[\Omega_x^i T(\Lambda_{t-1}^H, \Lambda_{t-1}^F) - \Lambda_{t-1}^i \Omega^i \right]', \quad (47)$$

$$\frac{dM^i}{d\tau} = \varpi^i \left[\Omega^i E[zz'] - M^i \right]. \quad (48)$$

From Equation (48), it is evident that $M^i \rightarrow \Omega^i E[zz']$, which implies that Equation (47) reduces to Equation (27).

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