The economic impact of pricing CO_2 emissions: Input-Output analysis of sectoral and regional effects*

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1. Introduction

We aim to calculate the impact on the Dutch economy of energy policy scenarios, at the level of industries (1 or 2-digit). More in particular, we use a multiregional Input-Output (IO) model to calculate the sectoral price effects of a CO_2 tax. Furthermore, we are interested in the additional price effects of substitution. With substitution we mean using a different technology.

The standard IO price model can be used for scenario analyses, i.e. imposing a CO_2 tax. Calculations are typically based on fixed IO data for a recent year, however. Such an approach is very useful for short horizons in which the mixture of capital, labour and energy in production cannot be changed.

With longer horizons we need to take into account possible factor substitution effects of relative price changes, e.g. resulting from a CO_2 tax. CO_2 pricing aims at substitution from energy inputs towards more capital/labor intensive production. Additionally, CO_2 pricing may lead to a greening of the electricity mix.

In this report we describe extensions of the standard Leontief IO price model to take into account substitution effects. In a nutshell, the methodology consists of a variable IO model allowing for a more flexible production structure. We distinguish two types of substitution. First, we discuss substitution of primary inputs (capital-labor) for energy. Key input for this type of substitution is an assumption on substitution elasticities between production factors. The estimation of production functions helps us to identify substitution elasticities within sectors. Second, we consider the case of substitution between types of electricity production. The leading example is substituting electricity production generated by coal for electricity generated by gas.

The remainder of this report is as follows. In Section 2 we describe how conventional IO analysis can be used to evaluate the sectoral quantity and price effects of a CO_2 tax. In Section 3 we describe substitution between energy and capital/labor inputs. In Section 4 we discuss substitution between electricity input types. In Section 5 we describe how we constructed country level price aggregates to summarize IO results at the macro (country) level. Finally, in Section 6 we discuss estimation of quantity effects.

2. basic Input-Output model and pricing CO_2 emissions

2.1. quantity model

Consider an economy[1] of n sectors. Denote with x_i total output (production) for sector i. The following equation[2] describes how sector i distributes its product through intermediate sales to other sectors (z_{ij}) and final demand (f_i):

$$x_i = z_{i1} + z_{i2} + ... + z_{in} + f_i, \qquad i = 1, ..., n. \tag{2.1}$$

A fundamental assumption in conventional IO models is that z_{ij}, i.e. the interindustry flow from sector i to j, is entirely determined by the total output of sector j. In other words, the technical coefficients defined as:

$$a_{ij} = \frac{z_{ij}}{x_j}, \tag{2.2}$$

measure fixed relationships between a sector's output x_j and its inputs z_{ij}. In other words, IO analysis requires that a sector use inputs in fixed proportions. Consider, for example, the case of two inputs. Once the proportion z_{1j}/z_{2j} of inputs 1 and 2 is known, then additional amounts of input 1 or input 2 separately are useless for increasing output of sector j.

These fixed input-output ratios imply zero elasticity of substitution[3] between inputs in the production function. Ignoring the contribution of value added, the implicit form of the production function used in IO analysis is the Leontief production function:

$$x_j = \min\left\{\frac{z_{1j}}{a_{1j}}, ..., \frac{z_{nj}}{a_{nj}}\right\}. \tag{2.3}$$

This mathematical representation reflects the property of fixed proportions: increasing one input, while leaving the other inputs unchanged, will not increase output. Under this assumption of fixed technical coefficients (2.2), equation (2.1) can be expressed as:

$$x_i = a_{i1}x_1 + a_{i2}x_2 + ... + a_{in}x_n + f_i, \qquad i = 1, ..., n. \tag{2.4}$$

The matrix expression for (2.4) is:

$$x = Ax + f. \tag{2.5}$$

[1] For ease of exposition we abstract from multiple countries.

[2] This exposition uses notation from Miller and Blair (2009).

[3] We will relax the assumption of no substitution in later sections.

Solving for x leads to the familiar expression:

$$x = Lf, \tag{2.6}$$

where $L = (I - A)^{-1}$ is known as the Leontief inverse. A typical element l_{ij} of L measures how total output for sector i (x_i) depends on final demand for product j (f_j).

2.2. price model

A second set of n equations, which is closely related to (2.1), describes how sectoral output x_j is divided among value added v_j and intermediate inputs $z_{1j}, ..., z_{nj}$:

$$x_j = z_{1j} + z_{2j} + ... + z_{nj} + v_j, \qquad j = 1, ..., n. \tag{2.7}$$

Note the transpose of the subscripts, i.e. output is now defined as the sum of the column inputs and value added. Value added is produced by the primary inputs capital and labor. Dividing all elements in (2.7) by sectoral total output x_j, we have:

$$1 = a_{1j} + a_{2j} + ... + a_{nj} + \frac{v_j}{x_j}, \qquad j = 1, ..., n, \tag{2.8}$$

where we exploited the definition of the technical coefficients (2.2). The elements on the right-hand side reflect how much of each input is used to produce a single unit of output from industry j. The term $\frac{v_j}{x_j}$ is the value added content of output for sector j. The model (2.7) is expressed in monetary terms, hence it can be split into separate price and quantity[4] components:

$$x_j p_j = z_{1j} p_1 + z_{2j} p_2 + ... + z_{nj} p_n + v_j, \qquad j = 1, ..., n. \tag{2.9}$$

Dividing all elements in (2.9) by sectoral total output x_j, we have:

$$p_j = a_{1j} p_1 + a_{2j} p_2 + ... + a_{nj} p_n + \frac{v_j}{x_j}, \qquad j = 1, ..., n. \tag{2.10}$$

Output prices p_j are equal to the cost of production and (using matrix notation) the IO price model becomes:

$$p = A' p + v_c, \tag{2.11}$$

[4] There is a slight abuse of notation in the sense that we don't use separate symbols for quantities compared with the value transactions before.

where $v_c = \left(\frac{v_1}{x_1}, ..., \frac{v_n}{x_n}\right)'$ is the vector of value added content of output. Solving for p leads to:

$$p = L'v_c, \tag{2.12}$$

which describes how output prices depend on primary input prices. This structure can be used to evaluate how changes in value added lead to changes in sectoral unit costs and therefore output prices. The price model (2.12) is known as the cost-push IO model as opposed to the demand-pull quantity model in (2.6).

2.3. CO_2 price effects

We use the IO price model (2.12) to calculate direct and indirect price effects of a carbon tax.[5] In the IO price model the carbon tax can be modeled as a tax on intermediate inputs or value added. Although environmental corporate income taxes do exist, most of the environmental taxes apply to the purchase of an intermediate input. Typical examples are coal, oil and gas. Fullerton (1995) gives an overview of environmental taxes for the US as well as an unifying framework for analyzing their effects in the IO price model. Assuming that each intermediate input has its own tax rate we rewrite (2.10) as:

$$p_j = a_{1j}\left(1+\tau_1\right)p_1 + a_{2j}\left(1+\tau_2\right)p_2 + ... + a_{nj}\left(1+\tau_n\right)p_n + v_{cj}, \qquad j = 1, ..., n, \tag{2.13}$$

where τ_j, $j = 1, ..., n$, are tax rates. Defining

$$T = \begin{bmatrix} 1+\tau_1 & 0 & \cdots & 0 \\ 0 & 1+\tau_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 1+\tau_n \end{bmatrix}, \tag{2.14}$$

we can then express the IO price model including taxes as:

$$p = \left(I - A'T\right)^{-1} v_c. \tag{2.15}$$

Instead, if the tax applies to value added we rewrite (2.10) as:

$$p_j = a_{1j}p_1 + a_{2j}p_2 + ... + a_{nj}p_n + v_{cj}\left(1+\tau_j\right), \qquad j = 1, ..., n, \tag{2.16}$$

[5] Due to (2.8) all baseline prices are set equal to 1.

resulting in:

$$p = (I - A')^{-1} T v_c. \tag{2.17}$$

Summarizing, a tax on intermediate inputs can be seen as changing A, while an income tax changes v_c. In general the two types of taxes will lead to different cost increases in the IO price model. We will focus on the latter in the remaining of the analysis. To determine the sectoral tax rate τ_j we first calculate the total tax revenues, which are the product of the uniform CO_2 price tr (euro per kg) and the sectoral CO_2 emissions $co2_j$ (kg):

$$t_j = tr \times co2_j, \qquad j = 1, ..., n. \tag{2.18}$$

The sectoral tax rate is then defined as the total tax revenue per unit value added:

$$\tau_j = \frac{t_j}{v_j}, \qquad j = 1, ..., n, \tag{2.19}$$

which is the relative change in value added as a result of the CO_2 tax.

3. substitution from energy to capital-labor

The conventional IO model assumes fixed technical coefficients. Output changes are solely due to changes in final demand (income effect) and output changes are independent of price changes. There are a number of alternative ways to relax the restrictive assumption of a zero elasticity of substitution. First, the Leontief production function (2.3) can be replaced with another production function, which explicitly allows for substitution. Examples are generalized Leontief, Cobb-Douglas and CES production functions. Klijs et al. (2015) apply a non-linear IO model for economic impact analysis in the region Zeeland. Second, conventional IO analysis can be combined with a Computational General Equilibrium (CGE) model. The IO analysis then provides volume effects, while the CGE model quantifies price effects. For example, in the EXIOMOD model developed by Bulavskaya et al. (2016) the production technology is modeled as a nested CES production function. In particular, energy can be substituted to the aggregate labor-capital input. Also there is substitution possible between energy types (electricity and petroleum products).

An advantage of non-linear IO or CGE models is that substitution is endogenously determined. A major disadvantage of non-linear IO models is that solving a large number of nonlinear equations is numerically challenging. In order to maintain the sectoral aggregation level and to avoid computational difficulties due to non-linearities, we maintain the linear IO framework. To relax the restrictive assumption of a zero elasticity of substitution, we will make changes to the technical coefficients. The resulting variable input-output model (Liew, 1984) is a conventional IO model, but it also contains a substitution effect:[6]

$$\Delta x = L \cdot \Delta A \cdot x + L \cdot \Delta f. \tag{3.1}$$

Output changes are dependent on price changes via a change in the technical coefficients (ΔA). For example, substitution away from energy would make the technical coefficient of this input lower. The price version of the variable input-output model can be expressed in a similar way as:

$$\Delta p = L' \cdot \Delta A' \cdot p + L' \cdot \Delta v_c. \tag{3.2}$$

The second term in (3.2) summarizes the price effects from the conventional Leontief price model (2.12), while the first term is again measuring the substitution effect.

In case of energy versus capital-labor substitution, we use the estimated substitution elasticities from sectoral production functions to determine the particular change in the entries of A. We illustrate the approach with a numerical example. We use a simplified IO model with only 3 sectors producing intermediate inputs: (1) energy; (2) agriculture; (3) manufacturing. This results in the following price model:

$$a_{11}p_1 + a_{21}p_2 + a_{31}p_3 + v_{c1} = p_1, \tag{3.3}$$

$$a_{12}p_1 + a_{22}p_2 + a_{32}p_3 + v_{c2} = p_2, \tag{3.4}$$

$$a_{13}p_1 + a_{23}p_2 + a_{33}p_3 + v_{c3} = p_3. \tag{3.5}$$

We assume that value added consists of a composite capital-labor input and tax:

$$v = p_z z + t, \tag{3.6}$$

$$v_c = p_z \frac{z}{x} + \frac{t}{x} = z_c p_z + t_c. \tag{3.7}$$

The CO_2 tax applied to value added then amounts to a change in the tax t, while the price and quantity of the composite capital-labor input stays unchanged. Suppose the CO_2 tax

[6] This expression follows from taking the total differential of the quantity IO model (2.5).

changes the relative energy price p_1/p_z with c_1. We will analyze how this relative price change of energy leads to substitution. Throughout the analysis we assume that the substitution elasticities of the agricultural and manufacturing inputs are zero, hence their relative price effects do not matter for substitution.

The substitution elasticity of energy with repect to the composite capital-labor input measures the responsiveness of the ratio in which factors are used to the ratio of factor prices. For each sector this is defined as:

$$\sigma_j = \frac{\frac{\Delta(z_{cj}/a_{1j})}{z_{cj}/a_{1j}}}{\frac{\Delta(p_1/p_z)}{p_1/p_z}}, \qquad j = 1, 2, 3. \tag{3.8}$$

Because z_{cj} and a_{1j} have the same denominator x_j, the ratio z_{cj}/a_{1j} is the ratio of the capital-labor and energy inputs for sector j.

Define $c_1 = \frac{\Delta(p_1/p_z)}{p_1/p_z}$ as the relative price change.[7] Given (3.8) this will lead to a relative change of capital/labor-energy factor shares of:

$$\frac{\Delta\left(z_{cj}/a_{1j}\right)}{z_{cj}/a_{1j}} = \sigma_j c_1. \tag{3.9}$$

Defining

$$\frac{\Delta\left(z_{cj}/a_{1j}\right)}{z_{cj}/a_{1j}} = \frac{z_{cj}^1/a_{1j}^1}{z_{cj}^0/a_{1j}^0} - 1, \tag{3.10}$$

we have

$$\frac{z_{cj}^1}{z_{cj}^0} = \left(1 + \sigma_j c_1\right) \frac{a_{1j}^1}{a_{1j}^0}. \tag{3.11}$$

We consider two scenarios. First, suppose there is substitution between energy and the capital-labor input. Then we have

$$a_{1j}^1 + z_{cj}^1 = a_{1j}^0 + z_{cj}^0. \tag{3.12}$$

Equations (3.11) and (3.12) can be solved for z_{c1}^1 and a_{11}^1. After some algebra we have:

$$\frac{z_{cj}^1}{z_{cj}^0} = \frac{\left(a_{1j}^0 + z_{cj}^0\right)\left(1 + \sigma_j c_1\right)}{a_{1j}^0 + z_{cj}^0(1 + \sigma_j c_1)}, \tag{3.13}$$

$$\frac{a_{1j}^1}{a_{1j}^0} = \frac{a_{1j}^0 + z_{cj}^0}{a_{1j}^0 + z_{cj}^0(1 + \sigma_j c_1)}, \tag{3.14}$$

[7] Note that in the IO price model the price of value added is exogenous, hence c_1 is actually the relative change in p_1 or $c_1 = \frac{\Delta p_1}{p_1}$.

where the right hand side can be calculated from the data. Second, assume that $\Delta z_{cj} = 0$ then the relative change originates solely from a change in the technical coefficient a_{1j}. In other words, we consider the case of pure technological progress. We then have:

$$\frac{a_{1j}^1}{a_{1j}^0} = \frac{1}{1 + \sigma_j c_1}, \tag{3.15}$$

hence the absolute change in the technical coefficient is

$$\Delta a_{1j} = -\frac{\sigma_j c_1}{1 + \sigma_j c_1} a_{1j}, \tag{3.16}$$

which can again be calculated with the data. Given that $\sigma_j \geq 0$ and $a_{1j} \geq 0$, the sign of the absolute changes critically depends on the sign of c_1. If $c_1 \geq 0$, i.e. the energy price increases, production will become more energy efficient or $\Delta a_{1j} \leq 0$, and vice versa.

Consider the following numerical example:

$$Z = \begin{bmatrix} 10 & 10 & 30 \\ 10 & 20 & 20 \\ 10 & 30 & 30 \end{bmatrix}, \quad f = \begin{bmatrix} 30 \\ 50 \\ 50 \end{bmatrix}, \quad v = \begin{bmatrix} 50 \\ 40 \\ 40 \end{bmatrix}, \quad x = \begin{bmatrix} 80 \\ 100 \\ 120 \end{bmatrix},$$

hence the matrix of technical coefficients and its Leontief inverse become:

$$A = \begin{bmatrix} \frac{1}{8} & \frac{1}{10} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{8} & \frac{3}{10} & \frac{1}{4} \end{bmatrix},$$

$$\begin{aligned} L &= \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{8} & \frac{1}{10} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{8} & \frac{3}{10} & \frac{1}{4} \end{bmatrix} \right)^{-1} \\ &= \begin{bmatrix} 1.2632 & 0.34450 & 0.49761 \\ 0.26316 & 1.4354 & 0.40670 \\ 0.31579 & 0.63158 & 1.5789 \end{bmatrix}, \end{aligned}$$

$$v_c = \begin{bmatrix} \frac{5}{8} \\ \frac{2}{5} \\ \frac{1}{3} \end{bmatrix}.$$

Suppose we introduce a CO_2 tax of 0.10 Euro per kg, hence we have a tax rate vector equal to:

$$tr = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}.$$

In combination with a hypothetical CO_2 emissions (in kg) vector:

$$co2 = \begin{bmatrix} 100 \\ 20 \\ 50 \end{bmatrix},$$

we have that the change in valued added (i.e. total paid tax) becomes:

$$\Delta v = \Delta t = \begin{bmatrix} 10 \\ 2 \\ 5 \end{bmatrix}.$$

Using the conventional price model (2.12) we have the following price change:

$$\begin{aligned} \Delta p &= \begin{bmatrix} 1.2632 & 0.34450 & 0.49761 \\ 0.26316 & 1.4354 & 0.40670 \\ 0.31579 & 0.63158 & 1.5789 \end{bmatrix}^T * \begin{bmatrix} \frac{1}{80} & 0 & 0 \\ 0 & \frac{1}{100} & 0 \\ 0 & 0 & \frac{1}{120} \end{bmatrix} * \begin{bmatrix} 10 \\ 2 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} 0.17632 \\ 0.098086 \\ 0.13612 \end{bmatrix}, \end{aligned}$$

which means a 17.6% price increase for the energy sector and smaller price changes (9.8 and 13.6%) for the other 2 sectors.

We next calculate the substitution effect. We assume a substitution elasticity $\sigma_1 = 0.5$ for the energy sector and $\sigma_2 = \sigma_3 = 0$ for the agricultural and manufacturing sectors. The price of the capital-labor input is exogenous and fixed. Given that all baseline prices were standardized at 1, the relative price change then becomes

$$c_1 = \frac{\Delta\left(p_1/p_z\right)}{p_1/p_z} = \frac{\Delta p_1}{p_1} = 0.17632.$$

We analyze the two scenarios described above. First, assume that there is substitution and we calculate the new shares of energy and capital-labor in the energy sector according to (3.13) and (3.14):

$$\begin{aligned} z_{c1}^{1} &= \frac{5}{8} * \frac{\left(\frac{1}{8}+\frac{5}{8}\right) * (1+0.5*0.17632)}{\frac{1}{8}+\frac{5}{8}*(1+0.5*0.17632)} \\ &= 0.63355, \end{aligned}$$

$$\begin{aligned} a_{11}^{1} &= \frac{1}{8} * \frac{\frac{1}{8}+\frac{5}{8}}{\frac{1}{8}+\frac{5}{8}*(1+0.5*0.17632)} \\ &= 0.11645. \end{aligned}$$

Using the variable input-output model (3.2), we calculate the price change due to substitution as follows:

$$\begin{aligned} \Delta p &= \begin{bmatrix} 1.2632 & 0.34450 & 0.49761 \\ 0.26316 & 1.4354 & 0.40670 \\ 0.31579 & 0.63158 & 1.5789 \end{bmatrix}^{T} * \begin{bmatrix} -0.00855 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} -0.0108 \\ -0.0029455 \\ -0.0042546 \end{bmatrix}. \end{aligned}$$

In this numerical example the substitution effect is much smaller than the original price effect. The use of more capital-labor instead of energy in the energy sector leads to an additional 1% decrease in the price of energy, while even smaller price decreases result for agriculture and manufacturing.

When we assume that there has been technological progress in the energy sector from decreasing the energy input according to (3.16), we have

$$\begin{aligned} \Delta a_{11} &= -\frac{0.5*0.17632}{1+0.5*0.17632} * \frac{1}{8} \\ &= -0.010127. \end{aligned}$$

Using the variable input-output model (3.2), the new technical coefficient for the energy input in the energy sector implies a price change of:

$$\Delta p = \begin{bmatrix} 1.2632 & 0.34450 & 0.49761 \\ 0.26316 & 1.4354 & 0.40670 \\ 0.31579 & 0.63158 & 1.5789 \end{bmatrix}^T * \begin{bmatrix} -0.010127 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0.012792 \\ -0.0034888 \\ -0.0050393 \end{bmatrix}.$$

Compared to substitution, pure technological progress leads to a somewhat stronger decrease in prices (e.g. the energy price drops 1.3%) mitigating the original CO_2 price effect.

4. substitution between energy types

To model substitution between energy types we construct hypothetical values for intermediary sales Z, value added v, final demand f and total production x. Denote this new technology by $\bar{Z}$, $\bar{f}$, $\bar{v}$, and $\bar{x}$. The new matrix of technical coefficients $\bar{A}$ and its Leontief inverse $\bar{L}$ are constructed from $\bar{Z}$ and $\bar{x}$ in the usual way. Furthermore, we update the vector of value added content in production $\bar{v}_c$ using $\bar{v}$ and $\bar{x}$. For the alternative technology the price model is:

$$\bar{p} = \bar{L}'\bar{v}_c. \tag{4.1}$$

We then evaluate for this alternative technology what the change in value added content $\Delta\bar{v}_c$ is and calculate:

$$\Delta\bar{p} = \bar{L}'\Delta\bar{v}_c. \tag{4.2}$$

as the change in prices including substitution. This is merely evaluating what the price increase of a CO_2 tax would be given that we are in the new hypothetical technology already. The substitution effect is then equal to $\Delta\bar{p} - \Delta p$.

The construction of the new technology is illustrated by means of a numerical example. Suppose we want to analyse the price effects of a CO_2 tax allowing for the possibility that coal driven electricity power plants are substituted by plants using gas. We use a simplified IO model with only 3 sectors producing intermediate inputs: (1) electricity generated by coal; (2) electricity generated by gas; (3) manufacturing. We furthermore assume that value added consists of a composite capital-labor input and tax as in (3.6)-(3.7), hence we have again the price model (3.3)-(3.5).

Consider the following numerical example:

$$Z = \begin{bmatrix} 10 & 10 & 30 \\ 10 & 20 & 20 \\ 10 & 30 & 30 \end{bmatrix}, \quad f = \begin{bmatrix} 30 \\ 50 \\ 50 \end{bmatrix}, \quad v = \begin{bmatrix} 50 \\ 40 \\ 40 \end{bmatrix}, \quad x = \begin{bmatrix} 80 \\ 100 \\ 120 \end{bmatrix}. \tag{4.3}$$

We consider the substitution effect of shutting down the whole 'coal' electricity sector. This implies that all electricity, which formerly has been produced by coal, is now produced by gas. Define the matrix S as follows:

$$S = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The matrix S sets all quantities (intermediary sales, value added and production) in the 'coal' sector equal to zero, while at the same time attributing the aggregate over both energy sectors to the 'gas' sector. In other words, it is assumed that production totals are unchanged and there only is a redistribution between electricity sectors. We then have:

$$\bar{Z} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 50 & 50 \\ 0 & 40 & 30 \end{bmatrix}, \quad \bar{f} = \begin{bmatrix} 0 \\ 80 \\ 50 \end{bmatrix}, \quad \bar{v} = \begin{bmatrix} 0 \\ 90 \\ 40 \end{bmatrix}, \quad \bar{x} = \begin{bmatrix} 0 \\ 180 \\ 120 \end{bmatrix},$$

hence the new matrix of technical coefficients is:

$$\bar{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.28 & 0.42 \\ 0 & 0.22 & 0.25 \end{bmatrix}.$$

Using the IO price model we can now calculate the price changes for the hypothetical technology with (4.2).

5. country level prices

In our multiregional IO model we have $r = 1, ..., c$ countries and $i = 1, ..., n$ sectors. Compared to the standard IO model introduced earlier the multiregional IO set up needs some additional notation.[8] In the standard IO model the basic equation for the distribution of the product of sector i is:

$$x_i = z_{i1} + z_{i2} + ... + z_{in} + f_i, \qquad i = 1, ..., n. \tag{5.1}$$

In a multicountry set up this is generalized to:

$$x_i^r = \sum_{s=1}^{c} \sum_{j=1}^{n} z_{ij}^{rs} + f_i^r, \qquad s = 1, ..., c; i = 1, ..., n, \tag{5.2}$$

where superscripts (r, s) denote the region and subscripts (i, j) sectors. The element z_{ij}^{rs} is intermediary sales from sector i in country r to sector j in country s, while f_i^r is final demand in sector i of country r. Total production in sector i of country r is denoted by x_i^r.

Prices can be summarized in various ways. We have an industry and country specific $nc \times 1$ price vector p. A typical element p_i^r in this column vector is the price in sector i of country r. At the country level we define three different prices: (1) GDP deflator; (2) export price; (3) consumer price.

The GDP deflator is defined as follows. We calculate the change in sectoral value added as a result of the carbon tax. As a result we have value added measured before and after tax, which we denote by the $nc \times 1$ vectors v_1 and v_0 respectively. Quantities are fixed in the IO model, hence changes in value added are solely due to price changes. Therefore, we calculate the relative change in the GDP deflator for country r as the relative change in total valued added aggregated over all sectors in country r:

$$GDP^r = \frac{\sum_{i=1}^{n} v_{i1}^r}{\sum_{i=1}^{n} v_{i0}^r}. \tag{5.3}$$

The exports for industry i in country r consists of two components, i.e. exports for final demand and intermediate exports. Define:

$$K_f = I_c \otimes \iota_n, \tag{5.4}$$

$$E_1 = (\iota_n \otimes \iota_c \iota_c' - K_f) \circ F, \tag{5.5}$$

where F is the $nc \times c$ matrix of final demand with typical element f_i^{rs}, i.e. sales of sector i in country r to final demand of sector i in country s. Note that total final demand of sector i in

[8] We use notation from Miller and Blair (2009).

country r is defined as the row sum of F, i.e. $f_i^r = \sum_{s=1}^{c} f_i^{rs}$. Furthermore, define:

$$K_\iota = I_c \otimes \iota_n \iota_n', \tag{5.6}$$

$$E_2 = (\iota_{nc}\iota_{nc}' - K_\iota) \circ Z, \tag{5.7}$$

where the typical element of Z is z_{ij}^{rs}. Then exports for final demand and intermediate exports are calculated as $e1 = E_1\iota_c$ and $e2 = E_2\iota_{nc}$ respectively, with typical elements:

$$\begin{aligned} e1_i^r &= \sum_{s=1}^{c} f_i^{rs} - f_i^{rr} \\ &= \sum_{s \neq r} f_i^{rs}, \end{aligned} \tag{5.8}$$

$$\begin{aligned} e2_i^r &= \sum_{j=1}^{n} \sum_{s=1}^{c} z_{ij}^{rs} - \sum_{j=1}^{n} z_{ij}^{rr} \\ &= \sum_{j=1}^{n} \sum_{r \neq s} z_{ij}^{rs}. \end{aligned} \tag{5.9}$$

The quantity $e1_i^r$ is total exports of sector i in country r due to final demand in the rest of the world. The quantity $e2_i^r$ is total exports of sector i in country r due to intermediate sales to all sectors in the rest of the world. Total exports of sector i in country r therefore is simply the aggregate of intermediate exports and final demand exports:

$$e_i^r = \left(e1_i^r + e2_i^r\right). \tag{5.10}$$

Quantities are fixed in the IO model, hence changes in exports are due to price changes. The relative price change for exports of country r is then calculated as:

$$EXP^r = \frac{\sum_{i=1}^{n} e_{i1}^r}{\sum_{i=1}^{n} e_{i0}^r}, \tag{5.11}$$

where e_{i1}^r and e_{i0}^r are the value of exports before and after tax.

To measure the price competitiveness of sector i in country j we calculate proceed as follows. Define the change in the relative export price for country r as

$$REP^r = EXP^r - \sum_{s \neq r} \omega^{rs} EXP^s, \tag{5.12}$$

with ω^{rs} the share of exports of country r to country s. Bilateral exports and export shares from country r to country s are calculated as:

$$e^{rs} = \sum_{i=1}^{n} f_i^{rs} + \sum_{i=1}^{n} \sum_{j=1}^{n} z_{ij}^{rs}, \tag{5.13}$$

$$\omega^{rs} = \frac{e^{rs}}{\sum_{s=1}^{c} e^{rs}}, \tag{5.14}$$

where the denominator is total exports for country r. Note that $\sum_{s=1}^{c} e^{rs} = \sum_{i=1}^{n} e_i^r$ and $\sum_{s=1}^{c} \omega^{rs} = 1$ by definition. An increase of REP^r means a decrease in the competitiveness of country r.

The consumer price of country r is defined as a weighted average its production prices:

$$pc^r = \sum_{i=1}^{n} w_i^r p_i^r, \tag{5.15}$$

where the weights w_i^r represent the share of consumption for each good i with respect to total final demand in country r:

$$w_i^r = \frac{f_i^r}{\sum_{i=1}^{n} f_i^r}. \tag{5.16}$$

The relative price change for the consumer price of country r is then calculated as:

$$PC^r = \frac{pc_1^r}{pc_0^r}. \tag{5.17}$$

Similar calculations lead to export and consumer prices for aggregated sectors.

6. quantity effects

To acquire a broader view on the possible effects of the CO_2 tax, we also estimate the impact of cost increases on final demand, exports and production for the Netherlands. Because in the conventional IO model price changes are independent from quantity changes, we estimate the latter using external information on price elasticities. More specifically, to calculate quantity changes in (domestic) final demand we use sectoral price elasticities of demand, which indicate the percentage change in sectoral final demand as a result of a percentage change in costs. Regarding exports (intermediate and final demand) we exploit sectoral export price elasticities, which indicate the percentage change in sectoral exports as a result of a percentage change in the relative export price. Table 1 reports the calibrated price elasticities, which are based on an extensive review of the recent empirical literature. The percentage quantity change (final demand or exports) is then estimated as the product of the relevant sectoral price elasticity and the percentage change in costs (final demand or relative export price) as calculated by the IO price model. Note that this method delivers an upper bound on the expected quantity effects as it assumes a full pass-through of costs to output prices as well as price inelastic supply curves.

To calculate the effects on production, we use the predicted domestic demand and export quantity effects in combination with the IO quantity model (2.6), which models the relation between final demand f and production x. Because we use a multiregional IO model, part of the intermediate exports, i.e. intermediate sales to all sectors in the rest of the world $e2_i^r$, are not in f, but in the intermediate sales. From the multiregional IO table we therefore construct a national IO table for the Netherlands, which includes all exports ($e1_i^r$ and $e2_i^r$) in final demand in the usual way. We then apply the quantity model (2.6) to calculate the change in production x as a result of the change in final demand f, which now consists of domestic final demand and total exports (intermediate and final demand).

Table 1: calibrated sectoral price elasticities

sector	demand	exports
Agriculture, forestry and fishing	-0.90	-1.55
Mining and quarrying	-0.30	-2.00
Food, drink and tobacco	-0.20	-1.12
Textile, clothing and leather-industry	-0.50	-1.64
Wood, paper and graphical industry	-0.50	-1.64
Oil industry	-0.50	-1.64
Chemical industry	-1.20	-2.04
Pharmaceutical industry	-1.20	-2.04
Rubber, plastic and other non-metallic mineral products	-0.30	-1.64
Basic metals and metal products	-0.80	-1.98
Computer, electronic and optical products	-0.50	-1.64
Electrical equipment	-0.50	-1.64
Machine-industry	-0.50	-1.64
Automobile, shipping and aircraft-industry	-0.50	-1.64
Other industry and repair	-0.50	-1.64
Energy companies	-0.10	-2.00
Water and sewerage-treatment	-0.10	-2.00
Construction	-1.30	-2.00
Trade and repair	-0.30	-1.65
Transportation	-2.00	-1.65
Horeca	-2.00	-1.65
Information and communication	-2.00	-1.65
Financial services	-0.10	-1.65
Real estate services	-1.00	-1.65
Professional, scientific and technical activities	-1.00	-1.65
Administrative and support services	-1.00	-1.65
Public administration and defence	-0.40	-1.65
Education	-1.00	-1.65
Healthcare	-0.20	-1.65
Culture, sports and recreation	-1.30	-1.65
Other services	-0.30	-1.65

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